CBSE Class 10 Maths Chapter 10- Circle

Objective Questions

Introduction to Circles

1. Two concentric circles of radii a and b (a > b) are given. Find the length of the chord of the larger circle which touches the smaller circle.

(A) $\sqrt{a^2 + b^2}$

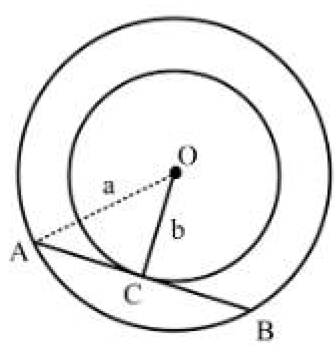
(B) $\sqrt{a^2 - b^2}$

(C) $2\sqrt{a^2 - b^2}$

(D) $2\sqrt{a^2 + b^2}$

Answer: (C) $2\sqrt{a^2 - b^2}$

Solution: Let O be the common center of the two circles and AB be the chord of the larger circle which touches the smaller circle at C.
Join OA and OC.
Then OC ⊥ AB
Let OA = a and OC = b.



Since OC ⊥ AB, OC bisects AB
[∵ perpendicular from the centre to a chord bisects the chord].

In right Δ ACO, we have

$OA^2=OC^2+AC^2$ [by Pythagoras' theorem]

$$\Rightarrow \quad AC = \sqrt{OA^2 - OC^2} = \sqrt{a^2 - b^2}$$
$$\therefore \quad AB = 2AC = 2\sqrt{a^2 - b^2} \quad [\because \text{C is the midpoint of AB}]$$

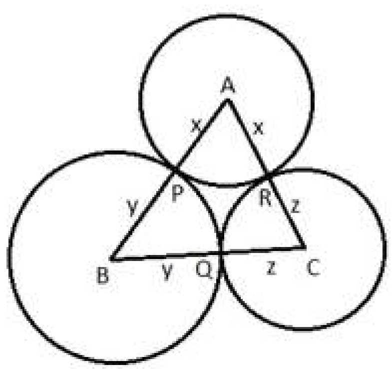
i.e., Length of the chord $AB = 2\sqrt{a^2 - b^2}$

2. Three circles touch each other externally. The distance between their centres is 5 cm, 6 cm and 7 cm. Find the radii of the circles.

(A) 2 cm, 3 cm, 4 cm
(B) 1 cm, 2 cm, 4 cm
(C) 1 cm, 2.5 cm, 3.5 cm
(D) 3 cm, 4 cm, 1 cm

Answer: (A) 2 cm, 3 cm, 4 cm

Solution: Consider the below figure wherein three circles touch each other externally.



Since the distances between the centres of these circles are 5 cm, 6 cm and 7 cm respectively, we have the following set of equations with respect to the above diagram:
$x+y = 5$ .....(1)

$y+z = 6$ ...... (2) ($\Rightarrow y=6-z$)... (2.1)

$x+z = 7$ .....(3)

Adding (1), (2) and (3), we have $2(x+y+z) =5+6+7=18$

$\Rightarrow x+y+z=9$.... (4)

Using (1) in (4), we have $5+z=9 \Longrightarrow z=4$

Now using, (3) $\Longrightarrow x=7-z=7-4=3$

And (2.1) $\Longrightarrow y=6-z=6-4=2$

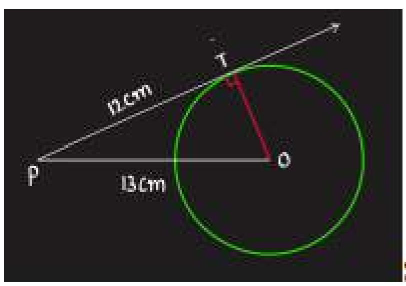
Therefore, the radii of the circles are 3 cm, 2 cm and 4 cm.

3. A point P is 13 cm from the centre of the circle. The length of the tangent drawn from P to the circle is 12cm. Find the radius of the circle.

(A) 5cm
(B) 7cm
(C) 10cm
(D) 12cm

Answer: (A) 5cm

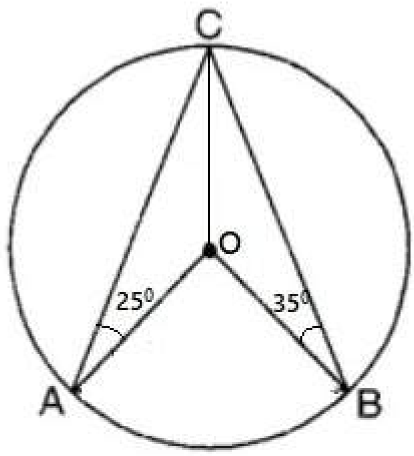
Solution:



Since,
tangent to a circle is perpendicular to the radius through the point of contact
So, $\angle OTP=90°$
So, in triangle OTP
$(OP)^2=(OT)^2+(PT)^2$
$13^2=(OT)^2+12^2$
$(OT)^2=13^2-12^2$
$OT^2=25$

OT= $\sqrt{25}$
OT=5
So, radius of the circle is 5 cm

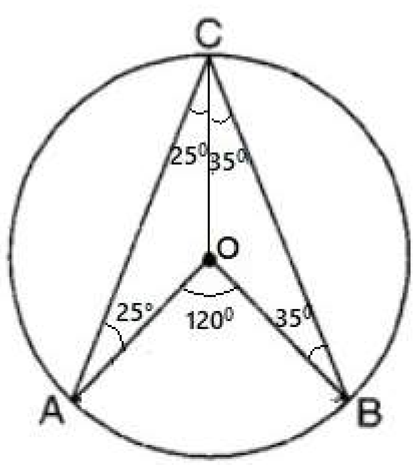
4. In the adjoining figure 'O' is the center of circle, ∠CAO = 25° and ∠CBO = 35°. What is the value of ∠AOB?



(A) 120°
(B) 110°
(C) 55°
(D) Data insufficient

Answer: (A) 120°

Solution:



In ΔAOC,

$OA=OC$ --------(radii of the same circle)

$\therefore \Delta AOC$ is an isosceles triangle

$\rightarrow \angle OAC=\angle OCA=25°$----- (base angles of an isosceles triangle)

In ΔBOC,

$OB=OC$ --------(radii of the same circle)

$\therefore \Delta BOC$ is an isosceles triangle

$\rightarrow \angle OBC=\angle OCB=35°$ -----(base angles of an isosceles triangle)

$\angle ACB=25°+35°=60°$

$\angle AOB=2\times\angle ACB$ ----(angle at the center is twice the angle at the circumference)

$= 2\times 60°$

$=120°$

5. A: What is a line called, if it meets the circle at only one point?

B: Collection of all points equidistant from a fixed point is _______.

1: Chord

2: Tangent

3: Circle

4: Curve

5: Secant

Which is correct matching?

(A) A-2; B-4
(B) A-5; B-4
(C) A-4; B-1
(D) A-2; B-3

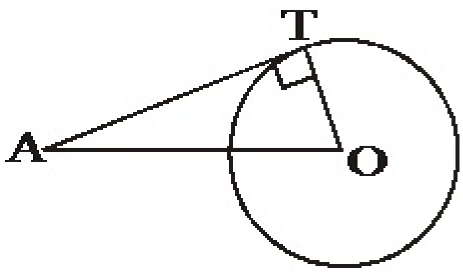
Answer: (D) A-2; B-3

Solution: Tangent is a line which touches the circle at only 1 point.

Collection of all points equidistant from a fixed point is called a circle.

Tangent to the Circle

6. A point A is 26 cm away from the centre of a circle and the length of tangent drawn from A to the circle is 24 cm. Find the radius of the circle.



(A) $2\sqrt{313}$
(B) 12
(C) 7
(D) 10

Answer: (D) 10

Solution: Let O be the centre of the circle and let A be a point outside the circle such that OA = 26 cm.

Let AT be the tangent to the circle.

Then, AT = 24 cm. Join OT.

Since the radius through the point of contact is perpendicular to the tangent, we have ∠OTA = 90°. In right △ OTA, we have

$OT^2 = OA^2 - AT^2$

$= [(26)^2 - (24)^2] = (26 + 24)(26 - 24) = 100.$

=> OT = $\sqrt{100}$ = 10cm

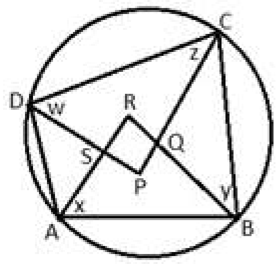
Hence, the radius of the circle is 10 cm.

7. The quadrilateral formed by joining the angle bisectors of a cyclic quadrilateral is a

(A) cyclic quadrilateral
(B) parallelogram
(C) square
(D) Rectangle

Answer: (A) cyclic quadrilateral

Solution:



ABCD is a cyclic quadrilateral $\therefore \angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

$1/2\angle A + 1/2\angle C = 90°$ and $1/2\angle B + 1/2\angle D = 90°$

$x + z = 90°$ and $y + w = 90°$

In $\triangle ARB$ and $\triangle CPD$, $x+y+\angle ARB = 180°$ and $z+w+\angle CPD = 180°$

$\angle ARB = 180° - (x+y)$ and $\angle CPD = 180° - (z+w)$

$\angle ARB + \angle CPD = 360° - (x+y+z+w) = 360° - (90+90)$

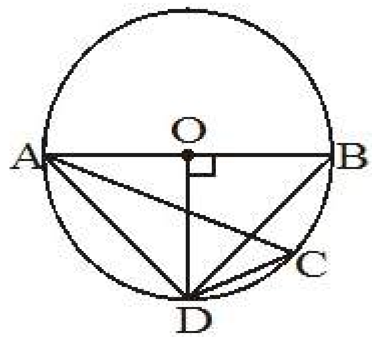
$= 360° - 180°$ $\angle ARB + \angle CPD = 180°$

$\angle SRQ + \angle QPS = 180°$

The sum of a pair of opposite angles of a quadrilateral PQRS is 180∘.

Hence PQRS is cyclic quadrilateral

8. In the given figure, AB is the diameter of the circle. Find the value of ∠ ACD

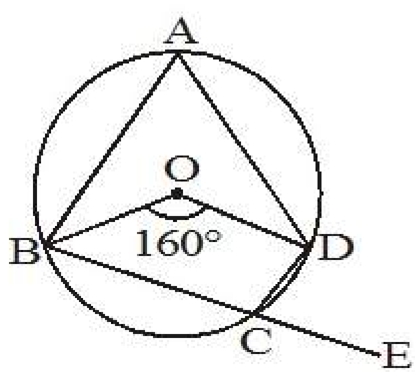


(A)25°
(B)45°
(C)60°
(D) 30°

Answer: (B) 45°

Solution: OB = OD (radius)
∠ ODB = ∠ OBD
∠ ODB + ∠OBD + ∠BOD = 180°
2∠ODB + 90° = 180°
∠ODB = 45°
∠OBD = ∠ACD (Angle subtended by the common chord AD)
Therefore ∠ACD = 45°

9. Find the value of ∠ DCE:



(A) 80°

(B) 75°
(C) 90°
(D) 100°

Answer: (A) 80°

Solution: ∠ BAD =1/2 BOD

∠ BAD =1/2(160°)

∠ BAD = 80°

ABCD is a cyclic quadrilateral

∠ BAD + ∠ BCD = 180°

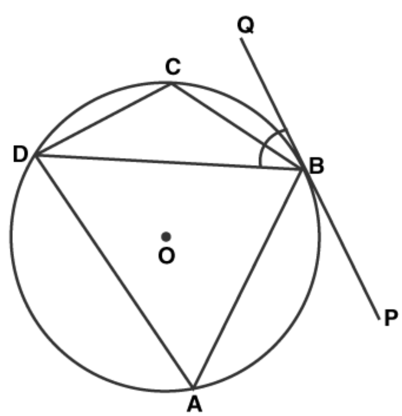
∠ BCD = 100°

∠ DCE = 180°- ∠ BCD

∠ DCE = 180°– 100°

∠ DCE = 80°

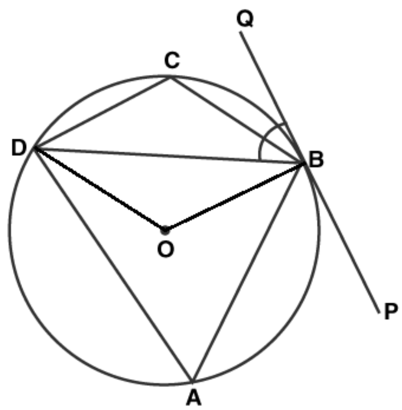
10. ABCD is a cyclic quadrilateral PQ is a tangent at B. If ∠DBQ = 65°, then ∠BCD is



(A) 35°
(B) 85°
(C) 90°
(D) 115°

Answer: (D) 115°

Solution:



Join OB and OD

We know that OB is perpendicular to PQ

$\angle OBD = \angle OBQ - \angle DBQ$

$\angle OBD = 90° – 65°$

$\angle OBD = 25°$

OB = OD (radius)

$\angle OBD = \angle ODB = 25°$

In $\triangle ODB$

$\angle OBD + \angle ODB + \angle BOD = 180°$

$25° + 25° + \angle BOD = 180°$

$\angle BOD = 130°$

$\angle BAD = 1/2\ \angle BOD$

(Angle subtended by a chord on the centre is double the angle subtended on the circle)

$\angle BAD = 1/2\ (130°)$

$\angle BAD = 65°$

ABCD is a cyclic quadrilateral

$\angle BCD + \angle BAD = 180°$

$\angle BCD + 65° = 180°$

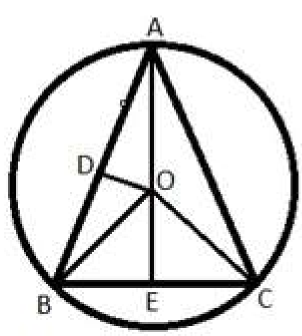
$\angle BCD = 115°$

11. In a circle of radius 5 cm, AB and AC are the two chords such that AB = AC = 6 cm. Find the length of the chord BC

(A) None of these
(B) 9.6cm
(C) 10.8cm
(D) 4.8cm

Answer: (B) 9.6cm

Solution:



Consider the triangles OAB and OAC are congruent as

AB=AC

OA is common

OB = OC = 5cm.

So $\angle OAB = \angle OAC$

Draw OD perpendicular to AB

Hence $AD = AB/2 = 6/2 = 3$ cm as the perpendicular to the chord from the center bisects the chord.

In $\triangle ADO$

$OD^2 = AO^2 - AD^2$

$OD^2 = 5^2 - 3^2$

$OD = 4$ cm

So Area of OAB = $1/2\ AB \times OD = 1/2\ 6 \times 4 = 12$ sq. cm. ….. (i)

Now AO extended should meet the chord at E and it is middle of the BC as ABC is an isosceles with AB= AC

Triangles AEB and AEC are congruent as

AB =AC

AE common,

$\angle OAB = \angle OAC$.

Therefore triangles being congruent, $\angle AEB = \angle AEC = 90°$

Therefore BE is the altitude of the triangle OAB with AO as base.

Also this implies BE =EC or BC =2BE

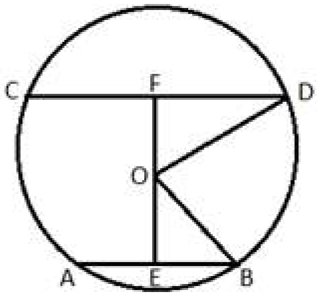
Therefore the area of the $\triangle$ OAB

$= ½ \times AO \times BE = ½ \times 5 \times BE = 12$ sq. cm as arrived in eq (i).

$BE = 12 \times 2/5 = 4.8$cm

Therefore $BC = 2BE = 2 \times 4.8$ cm $= 9.6$ cm.

12.

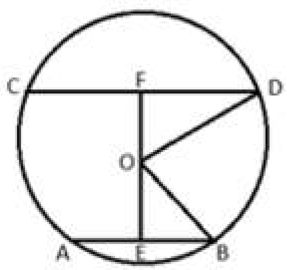


In a circle of radius 17 cm, two parallel chords are drawn on opposite sides of a diameter. The distance between the chords is 23 cm. If the length of one chord is 16 cm, then the length of the other is

(A) None of these
(B) 15cm
(C) 30cm
(D) 23cm

Answer: (C) 30cm

Solution:



Given that

OB = OD =17

AB = 16 ⇒ AE = BE = 8 cm as perpendicular from centre to the chord bisects the chords

EF = 23 cm

Consider △OEB

$OE^2 = OB^2 - EB^2$

$OE^2 = 17^2 - 8^2$

OE = 15 CM

OF = EF - OE

OF = 23 - 15

OF = 8 cm

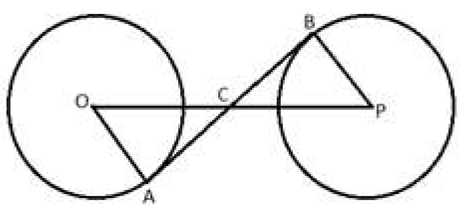
$FD^2 = OD2 - OF^2$

$FD^2 = 17^2 - 8^2$

FD = 15

Therefore CD = 2FD = 30 cm

13. The distance between the centres of equal circles each of radius 3 cm is 10 cm. The length of a transverse tangent AB is



(A) 10cm
(B) 8cm
(C) 6cm
(D) 4cm

Answer: (B) 8cm

Solution: $\angle OAC = \angle CBP = 90°$

$\angle OCA = \angle PCB$ (Vertically opposite angle)

Triangle OAC is similar to PBC

OA/PB = OC/PC

3/3 = OC/PC

OC = PC

But PO = 10 cm

Therefore OC = PC = 5cm

$AC^2 = OC^2 - OA^2$

$AC^2 = 5^2 - 3^2$

AC = 4 cm

Similarly BC = 4 cm

Therefore AB = 8 cm

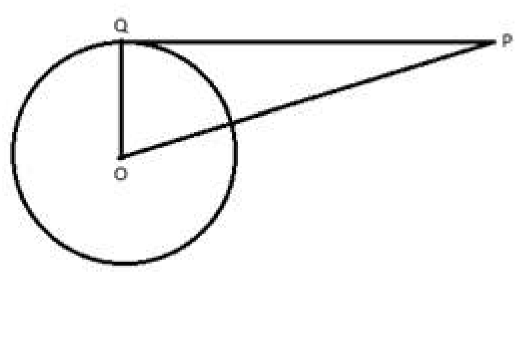
Theorems

14. A point P is 10 cm from the center of a circle. The length of the tangent drawn from P to the circle is 8 cm. The radius of the circle is equal to

(A) 4cm
(B) 5cm
(C) None of these
(D) 6cm

Answer: (D) 6cm

Solution:



Given that OP = 10 cm, PQ = 8 cm

As, tangent to a circle is perpendicular to the line joining the centre of the circle to the tangent at the point of contact to the circle.

Angle OQP = 90^2

Applying Pythagoras theorem to triangle OPQ

$OQ^2 + QP^2 = OP^2$

$OQ^2 + 8^2 = 10^2$

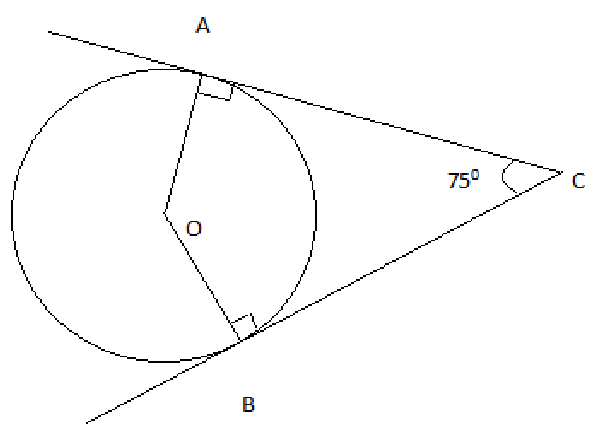
$OQ^2 = 100-64$

$=36$

OQ = 6 cm.

Ans: Radius of the circle is 6 cm.

15. In fig, O is the centre of the circle, CA is tangent at A and CB is tangent at B drawn to the circle. If ∠ACB = 75°, then ∠



AOB=

(A) 75°
(B) 85°
(C) 95°
(D) 105°

Answer: (D) 105°

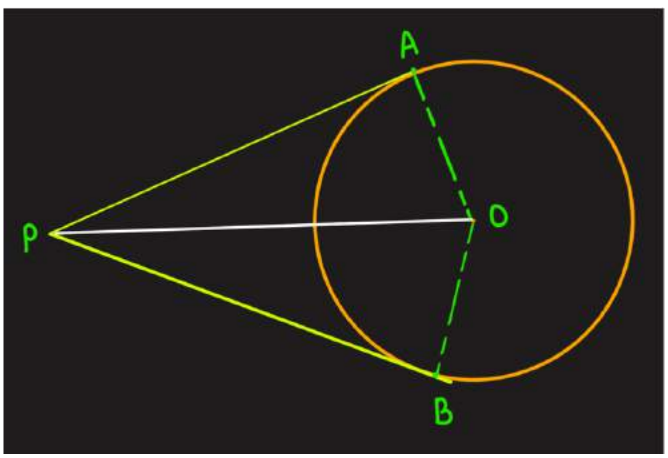
Solution: ∠OAC = ∠OBC = 90°

∠OAC + ∠OBC + ∠ACB + ∠AOB = 360° ..... (sum of angles of a quadrilateral)

90° + 90° + 75° + ∠AOB = 360°

∠AOB = 105°

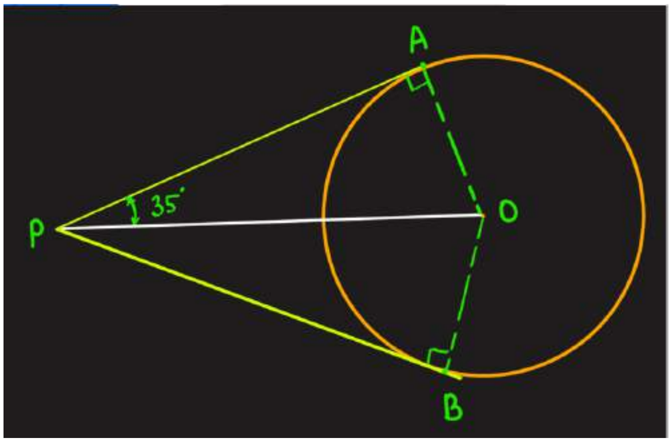
16. PA and PB are the two tangents drawn to the circle. O is the centre of the circle. A and B are the points of contact of the tangents PA and PB with the circle. If ∠OPA = 35°, then ∠POB =



(A) 55°
(B) 65°
(C) 85°
(D) 75°

Answer: (A) 55°

Solution: ∠OAP =∠OBP = 90°



$\angle AOP = 180° - 35° - 90°$

$\angle AOP = 55°$

OA = OB

AP = PB

OP is common base

Therefore $\triangle OAP \cong \triangle OBP$

$\angle AOP = \angle BOP$

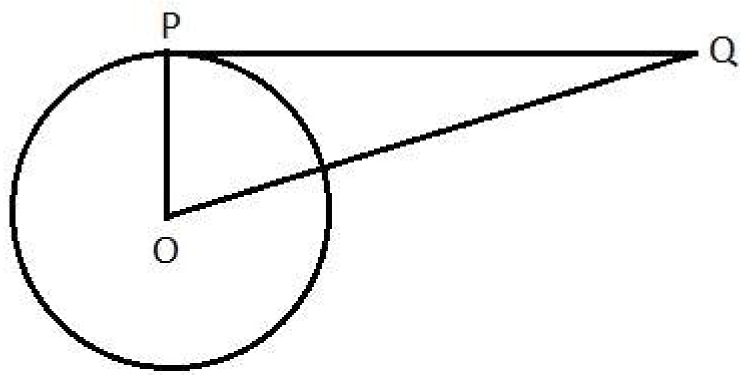
Ans: $\angle BOP = 55°$

17. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the center O at a point Q such that OQ =13 cm. Length PQ is:

(A) $\sqrt{119}$
(B) 8.5cm
(C) 13cm
(D) 12cm

Answer: (D) 12cm

Solution:



Given that OP = 5 cm, OQ = 13 cm

To find PQ

Applying Pythagoras theorem to triangle OPQ

$OP^2 + QP^2 = OQ^2$

$5^2 + QP^2 = 13^2$

$QP^2 = 169 – 25 = 144$

QP = $\sqrt{144}$ cm

QP =12 cm

18. The length of the tangent from a point A to a circle, of radius 3 cm, is 4 cm. The distance of A from the centre of the circle is

(A) $\sqrt{7}$

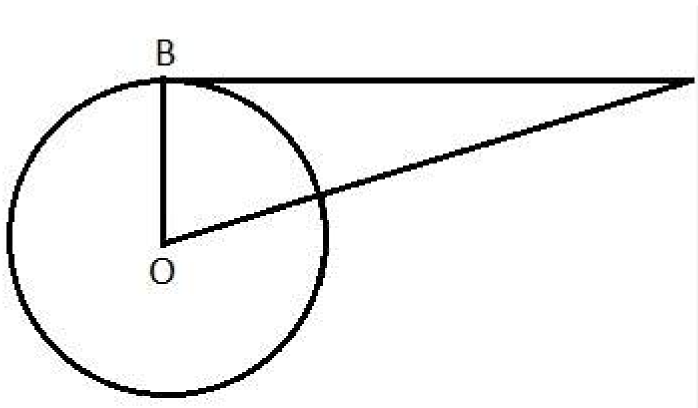
(B) 7cm

(C) 5cm

(D) 25cm

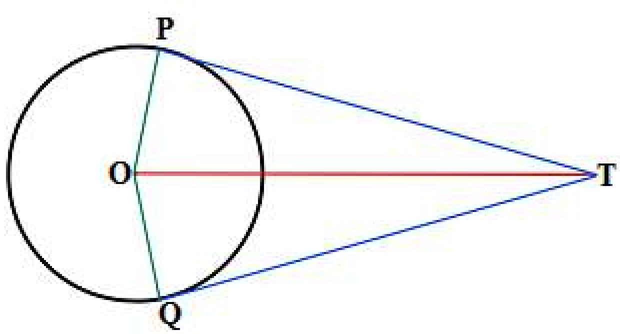
Answer: (C) 5cm

Solution:



Given that AB = 4 cm, OB =3 cm
To find OA
Applying Pythagoras theorem to triangle OAB
$OB^2 + AB^2 = OA^2$
$3^2 + 4^2 = OA$
$OA^2 = 25$
OA = 5 cm
Therefore the distance of A from the centre of the circle is 5 cm.

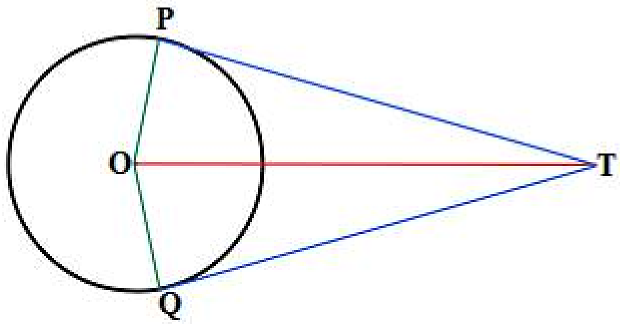
19. If TP and TQ are two tangents to a circle with center O such that ∠POQ = 110°, then, ∠PTQ is equal to:



(A) 90°
(B) 80°
(C) 70°
(D) 60°

Answer: (C)70°

Solution:



We know that $\angle OQT = \angle OPT = 90°$.

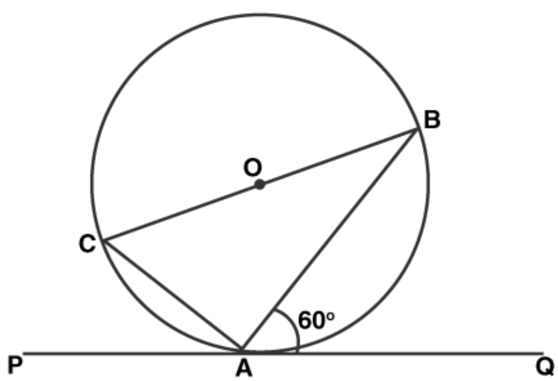
Also $\angle OQT + \angle OPT + \angle POQ + \angle PTQ = 360°$.

$\angle PTQ = 360° - 90° - 90° - 110°$

$= 70°$

$\therefore \angle PTQ = 70°$

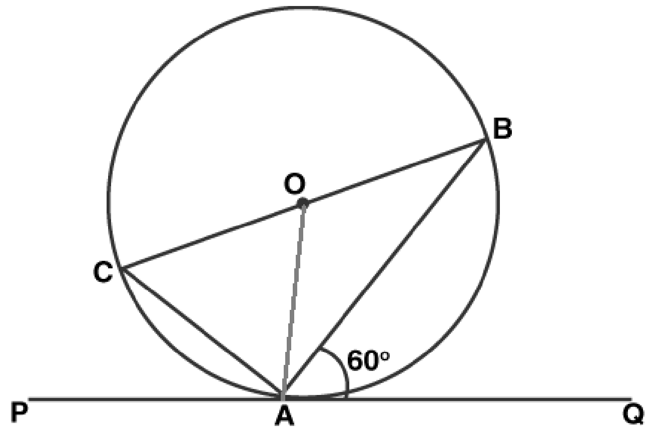
20. In the given figure, PAQ is the tangent. BC is the diameter of the circle. $\angle BAQ = 60°$, find $\angle ABC$:



(A) 25°
(B) 30°
(C) 45°
(D) 60°

Answer: (B) 30°

Solution:



Join OA

As the tangent at any point of a circle is perpendicular to the radius through the point of contact

$\angle OAQ = 90°$

$\angle OAB = \angle OAQ - \angle BAQ$

$\angle OAB = 90° – 60°$

$\angle OAB = 30°$

OA = OB (radius)

$\angle OAB = \angle OBA$

Therefore $\angle OBA = 30°$

$\angle ABC = 30°$