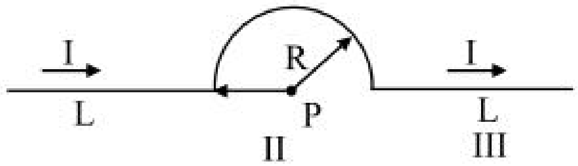
10 Magnetic Fields due to Electric Current

Multiple Choice Questions (1 Mark Each)

1. According to right hand rule, the direction of magnetic induction if the current is directed in anticlockwise direction is
   (A) perpendicular and inwards.
   (B) perpendicular and outwards.
   (C) same as current.
   (D) opposite to that of current.

2. A conductor has three segments; two straights of length L and a semi-circular with radius R. It carries a current I. What is the magnetic field B at point P?



(A) $\frac{\mu_0 I}{4\pi R}$ (B) $\frac{\mu_0}{4\pi}\frac{I}{R^2}$

(C) $\frac{\mu_0 I}{4R}$ (D) $\frac{\mu_0 I}{4\pi}$

Hint: Applying Biot-Savart law to the 3 sections of the wire.

For the section (i) and (iii) the angle between the current-length elements $I\,d\vec{l}$ and $\vec{R}$ is 180° and 0°, respectively.

$$\therefore \quad dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin(180)^\circ}{R^2} = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin(0)^\circ}{R^2} = 0$$

$$\Rightarrow B_I = B_{III} = 0$$

For section (ii), $d\vec{l}$ is always perpendicular to $\vec{R}$.

$$\therefore \quad (dB)_{II} = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin(90)^\circ}{R^2} = \frac{\mu_0}{4\pi}\frac{I\,dl}{R^2}$$

Integrating, $(B)_{II} = \frac{\mu_0}{4\pi}\frac{I}{R^2}\int_0^{\pi R} dl = \frac{\mu_0}{4\pi}\frac{I}{R^2}\,\pi R$

$\therefore \quad (B)_{II} = \frac{\mu_0}{4}\frac{I}{R}$

Total $B = B_I + B_{II} + B_{III} = 0 + \frac{\mu_0 I}{4R} + 0 = \frac{\mu_0 I}{4R}$

Direction of B at O is coming out of the plane of the paper.

*[**Note:** Answer calculated above is in accordance with textual methods of calculation.]*

3. A strong magnetic field is applied on a stationary electron. Then the electron
   (A) moves in the direction of the field.
   (B) remains stationary.
   (C) moves perpendicular to the direction of the field.
   (D) moves opposite to the direction of the field.

Hint: Magnetic force acts on a moving charge. Since, electron is stationary, no magnetic force will act upon it.

4. The force between two parallel current carrying conductors is F. If the current in each conductor is doubled, then the force between them becomes
   (A) 4F (B) 2F
   (C) F (D) F/4

Hint: $F = \frac{\mu_0}{4\pi}\,\frac{2I_1I_2}{r}$

After doubling current,

$F' = \frac{\mu_0}{4\pi}\,\frac{2\times 2I_1\times 2I_2}{r} = 4F$

5. Which of the following is not a unit of magnetic induction?
   (A) gauss (B) tesla
   (C) oersted (D) Wb/m^2

6. The magnetic dipole moment of current loop is independent of
   (A) number of turns.
   (B) area of loop.
   (C) current in the loop.
   (D) magnetic field in which it is lying.

7. Circular loop of radius 0.0157 m carries a current 2 A. The magnetic field at the centre of the loop is

(A) 1.57×10^{-3} Wb/m^2

(B) 8.0×10^{-5} Wb/m^2

(C) 2.0×10^{-3} Wb/m^2

(D) 3.14×10^{-1} Wb/m^2

Hint: $B_c = \frac{\mu_0 I}{2R} = \frac{4\pi \times 10^{-7} \times 2}{2 \times 1.57 \times 10^{-2}} = \frac{4 \times 3.14 \times 10^{-7} \times 2}{2 \times 1.57 \times 10^{-2}} = 8.0 \times 10^{-5}$ Wb/m^2

Very Short Answer (VSA) (1 Mark Each)

Q.1. What is Lorentz force?

Ans: When a charged particle moves through a region in which both electric and magnetic fields are present, then the net force experienced by that charged particle is sum of electrostatic force and magnetic force and is called as Lorentz force.

Q.2. What is solenoid?

Ans: A solenoid is a long, insulated copper wire closely wound on a hollow cylindrical glass or plastic tube in the form of a helix.

Q.3. What is toroid?

Ans: A toroid is a solenoid of finite length bent into a hollow circular tube.

Q.4. Calculate the value of magnetic field at a distance of 2 cm from a very long straight wire carrying a current 5 A.

Ans: $B = \frac{\mu_0 I}{2\pi R} \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 2 \times 10^{-2}} = 5 \times 10^{-5}$ T

Q.5. What happens to the magnetic field at the centre of a circular current carrying coil if we double the radius of the coil keeping the current unchanged?

Ans: Magnetic field at the centre of the circular coil, $B = \frac{\mu_0 I}{2r}$. Hence, if we double the radius, magnetic field at the centre of coil will become half its original value.

Q.6. A solenoid of length 50 cm of inner radius of 1 cm and is made up of 500 turns of copper wire for a current of 5 A in it. What will be magnitude of magnetic field inside the solenoid?

Ans: Magnitude of magnetic field inside the solenoid,

$$B = \mu_0 \frac{N}{l} i = 4\pi \times 10^{-7} \times \frac{500}{0.5} \times 5 = 6.284 \times 10^{-3} \text{ T}$$

Q.7. State the orientation of magnetic dipole with respect to magnetic field, which possess maximum magnetic potential energy.

Ans: When magnetic dipole moment and magnetic field are antiparallel to each other, magnetic potential energy of a magnetic dipole is maximum.

Short Answer I (SA1) (2 Marks Each)

Q.1. A toroid of 4000 turns has outer radius of 26 cm and inner radius of 25 cm. If the current in the wire is 10 A. Calculate the magnetic field of the toroid.

Solution:

Given: $R_1 = 0.25$ m; $R_2 = 0.26$ m;

$N = 4000$; $i = 10$ A

To find: Magnetic field of the toroid (B)

Formula: $B = \mu_0 \times \frac{N}{l} \times i$

where, l = mean length of toroid = $2\pi \frac{(R_1 + R_2)}{2}$

Calculation: From formula,

$$l = \pi (R_1 + R_2)$$
$$= \pi (0.25 + 0.26)$$
$$= \pi \times 0.51 \text{ m}$$

$$\therefore \quad B = \frac{(4\pi \times 10^{-7}) \times 4000 \times 10}{\pi \times 0.51} = \frac{16}{0.51} \times 10^{-3}$$

$$\therefore \quad B_{in} = \mathbf{3.137 \times 10^{-2} \text{ T}}$$

Ans: The magnetic field of the toroid is **3.137×10^{-2} T**.

Q.2. Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why?

Ans:

i. The magnetic field around the toroid consists of concentric circular lines of force around it. Thus, magnetic field in the interior of toroid is tangential to each loop.

ii. Whereas, the magnetic field produced by a solenoid is similar to the magnetic field of a bar magnet. One end of the solenoid coil acts as south pole and the other end acts as north pole with the field lines inside the solenoid remaining parallel. Thus, the magnetic field B is parallel to the axis of the solenoid.

As a result, magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid.

Q.3. A solenoid of length π m and 5 cm in diameter has winding of 1000 turns and carries a current of 5 A. Calculate the magnetic field at its centre along the radius.

Solution:

Given: $l = \pi$ m, diameter = 5 cm,

N = 1000 turns, i = 5 A

We know that, $\mu_0 = 4\pi \times 10^{-7}$ Tm/A

To find: Magnetic field (B)

Formulae: i. $n = \frac{N}{l}$

ii. $B = \mu_0 ni$

Calculation: From formula (i),

$$n = \frac{1000}{\pi}\ \frac{\text{turns}}{\text{m}}$$

From formula (ii),

$$B = 4\pi \times 10^{-7} \times \frac{1000}{\pi} \times 5$$

$$= 20 \times 10^{-7+3}$$

$$= \mathbf{2 \times 10^{-3}\ T}$$

Ans: The magnetic field is $\mathbf{2 \times 10^{-3}}$ **T**.

Q.4. Currents of equal magnitude pass through two long parallel wires having separation of 1.35 cm. If the force per unit length on each wire is 4.76 × 10^{-2} N/m, what is I?

Solution:

Given: $I_1 = I_2 = I$, $\frac{F}{L} = 4.76 \times 10^{-2}$ N

$d = 1.35$ cm $= 1.35 \times 10^{-2}$ m

To find: Electric current

Formula: $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$

Calculation: From formula,

$$4.76 \times 10^{-2} = \frac{4\pi \times 10^{-7} \times I \times I}{2 \times \pi \times 1.35 \times 10^{-2}}$$

$$\therefore \quad I^2 = \frac{4.76 \times 10^{-2} \times 1.35 \times 10^{-2}}{2 \times 10^{-7}} = 1.35 \times 2.38 \times 10^{-2-2+7}$$

$$I = \sqrt{1.35 \times 2.38 \times 10^3}$$

$$= \sqrt{13.5 \times 2.38} \times 10$$

$$= \left\{\text{anti}\log\left(\frac{1}{2}(\log 13.5 + \log 2.38)\right)\right\} \times 10$$

$$= \left\{\text{anti}\log\left(\frac{1}{2}(1.1303 + 0.3766)\right)\right\} \times 10$$

$= \{\text{antilog}\ (0.7535)\} \times 10$

$= 5.669 \times 10$

$=$ **56.69 A**

Ans: The electric current is **56.69 A**.

Q.5. Explain "Magnetic force never does any work on moving charges".

Ans:

i. Magnetic force is given by $\vec{F}_m = q\left(\vec{v} \times \vec{B}\right)$.

ii. This makes direction of magnetic force $\left(\vec{F}_m\right)$ perpendicular to direction of velocity of charged particles $\left(\vec{v}\right)$.

iii. Thus, magnetic force is in turn perpendicular to displacement of charged particles.

iv. According to properties of dot product, $\vec{F}_m \cdot \vec{v} = 0$, for any magnetic field $\vec{B}$.

Hence, magnetic force never does any work on moving charges.

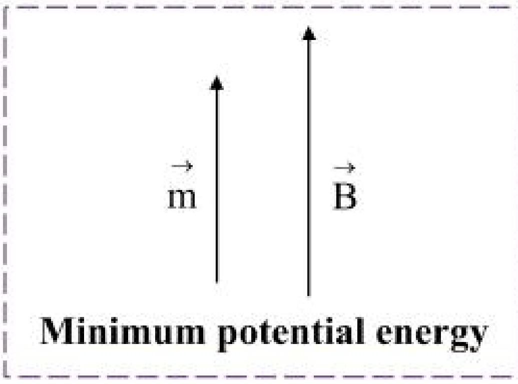
Q.6. State the conditions when magnetic potential energy of a magnetic dipole (current carrying coil) kept in uniform magnetic field be minimum and maximum.

Ans:

i. **Condition for minimum magnetic potential energy:**

In a magnetic field when $\vec{m}$ and $\vec{B}$ are parallel to each other, magnetic potential energy of a magnetic dipole is minimum.

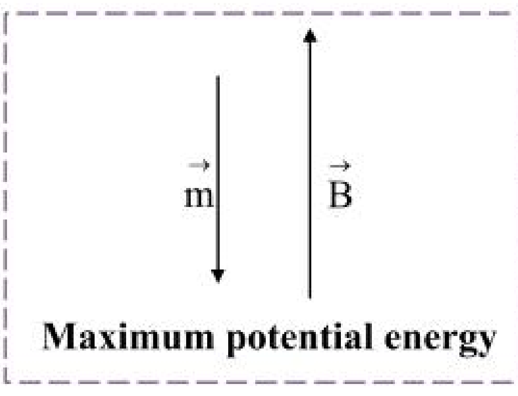
Mathematically, when $\theta = 0$, $U = -\, m\, B \cos 0° = -\, m\, B$



ii. Condition for maximum magnetic potential energy:

When $\vec{m}$ and $\vec{B}$ are antiparallel to each other, magnetic potential energy of a magnetic dipole is maximum.

Mathematically, when $\theta = 180°$, $U = -\, m\, B \cos 180° = m\, B$

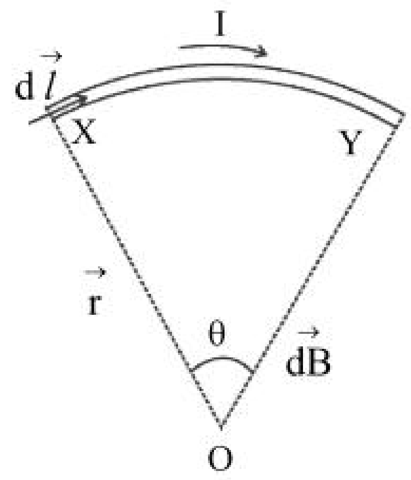


Q.7. Derive the expression for magnetic field produced by a current in a circular arc of wire.

Ans:

i. Consider circular arc of a wire (XY), carrying a current I.

ii. The circular arc XY subtends an angle θ at the centre O of the circle with radius r of which the arc is a part, as shown in figure below.



Current carrying wire of a shape of circular arc.

iii. Consider length element $d\vec{l}$ lying always perpendicular to $\vec{r}$.

Using Biot-Savart law, magnetic field produced at O is:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\vec{r}}{r^3}$$

$$dB = \frac{\mu_0}{4\pi} I \frac{dl\, r\sin 90^\circ}{r^3}$$

$$= \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2} \qquad \text{....(1)}$$

iii. Equation (1) gives the magnitude of the field. The direction of the field is given by the right-hand rule. Thus, the direction of each of the dB is into the plane of the paper. The total field at O is

$$B = \int dB = \frac{\mu_0}{4\pi} I \int_A^\theta \frac{dl}{r^2}$$

$$= \frac{\mu_0}{4\pi} I \int_A^\theta \frac{r\,d\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{I}{r}\theta \qquad \text{....(2)}$$

where, the angle θ is in radians.

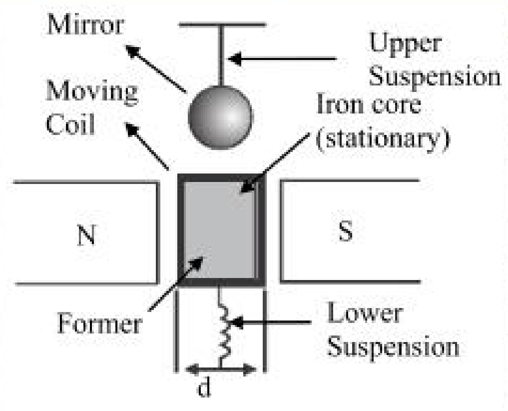
Short Answer II (SA2) (3 Marks Each)

Q.1. Explain construction and working of moving coil galvanometer.

Ans: Construction:

i. M.C.G. consists of a coil of several turns mounted (suspended or pivoted) in such a way that it can freely rotate about a fixed axis, in a radial uniform magnetic field.

ii. A soft iron cylindrical core makes the field radial and strong.



Moving coil galvanometer

Working:

i. The coil rotates due to a torque acting on it as the current flows through it. Torque acting on current carrying coil is $\tau = NIAB \sin\theta$.

Here $\theta = 90°$ as the field is radial.

$\therefore$ $\tau = NIAB$

where A is the area of the coil, B the strength of the magnetic field, N the number of turns of the coil and I the current in the coil.

ii. This torque is counter balanced by a torque due to a spring fitted at the bottom so that a fixed steady current I in the coil produces a steady angular deflection ϕ.

iii. Larger the current is, larger is the deflection and larger is the torque due to the spring. If the

deflection is ϕ, the restoring torque due to the spring is equal to $K\phi$ where K is the torsional constant of the spring.

Thus, $K\phi = NIAB$,

and the deflection $\phi = \left(\frac{NAB}{K}\right)I$

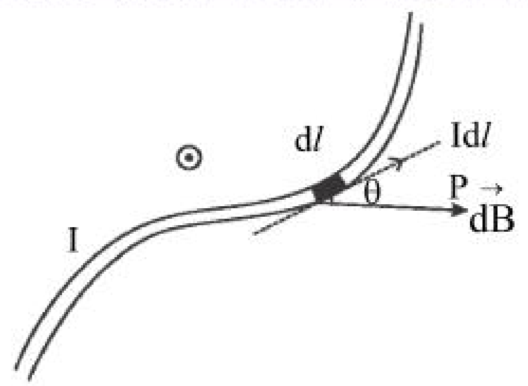
This means, the deflection ϕ is proportional to the current I i.e., $\phi \propto I$.

Q.2. Explain Biot Savart's law.

Ans:

i. Consider an arbitrarily shaped wire carrying a current I.

ii. Let dl be a length element along the wire. The current in this element is in the direction of the length vector $d\vec{l}$ which produces differential magnetic field $d\vec{B}$ directed into the plane of paper as shown in figure below:



Current carrying wire of arbitrary shape

iii. Consider point P at distance r from element dl. Net magnetic field at the point P can be obtained by integrating i.e., summing up of magnetic fields $d\vec{B}$ from these length elements.

iv. Experimentally, the magnetic fields $d\vec{B}$ produced by current I in the length element $d\vec{l}$ is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} \qquad ....(1)$$

where, θ is the angle between the directions of $d\vec{l}$ and $\vec{r}$,

μ_0 (permeability of free space) = $4\pi \times 10^{-7}$ T m/A ≈ 1.26×10^{-6} T m/A

v. The direction of $d\vec{B}$ is dictated by the cross product $d\vec{l} \times \vec{r}$.

Vectorially, $$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \vec{r}}{r^3} \qquad ....(2)$$

Equations (1) and (2) are known as the Biot-Savart law.

Q.3. A rectangular coil of 10 turns, each of area 0.05 m², is suspended freely in a uniform magnetic field of induction 0.01 T. A current of 30 μA is passed through it.

i. What is the magnetic moment of the coil?

ii. What is the maximum torque experienced by the coil?

Solution:

Given: $N = 10$, $A = 0.05\ m^2$, $I = 30\ \mu A = 30 \times 10^{-6}\ A$

$B = 0.01\ T$

To find: i. Magnetic moment (m) of coil

ii. Maximum torque experienced (τ_{max})

Formulae: i. $m = NIA$

ii. $\tau_{max} = (NIA)B = mB$

Calculation: From formula (i),

$m = 10 \times 30 \times 10^{-6} \times 0.05$

$= 15 \times 10^{-6}\ Am^2$

$= \mathbf{15\ \mu Am^2}$

From formula (ii),

$\tau_{max} = 15 \times 10^{-6} \times 0.01$

$= 15 \times 10^{-8}$

$= \mathbf{1.5 \times 10^{-7}\ Nm}$

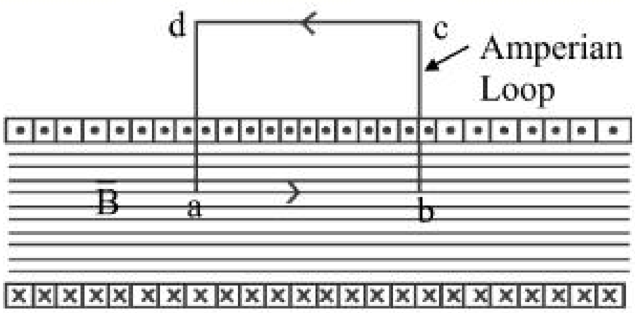
Ans: i. Magnetic moment of coil is **15 μAm²**.

ii. Maximum torque experienced by coil is $\mathbf{1.5 \times 10^{-7}\ Nm}$.

Q.4. Using Ampere's law, derive an expression for the magnetic induction inside an ideal solenoid carrying a steady current.

Ans:

i. Consider an ideal solenoid as shown in figure below.



Ampere's law applied to a part of a long ideal solenoid

ii. The dots (·) show that the current is coming out of the plane of the paper and the crosses (×) show that the current is going into the plane of the paper, both in the coil of square cross section wire.

iii. For the application of the Ampere's law, an Amperian loop is drawn as shown figure and box.

iv. Using Ampere's law,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

Over the rectangular loop abcd, the above integral takes the form

$$\int_a^b \vec{B} \cdot \vec{dl} + \int_b^c \vec{B} \cdot \vec{dl} + \int_c^d \vec{B} \cdot \vec{dl} + \int_d^a \vec{B} \cdot \vec{dl} = \mu_0 I$$

where, I is the net current encircled by the loop.

∴ $B L + 0 + 0 + 0 = \mu_0 I$ ....(1)

Here, the second and fourth integrals are zero because $\vec{B}$ and $\vec{dl}$ are perpendicular to each other. The third integral is zero because outside the solenoid, $B = 0$.

v. If the number of turns is n per unit length of the solenoid and the current flowing through the wire is i, then the net current coming out of the plane of the paper is

$I = nLi$

∴ Using equation (1)

$BL = \mu_0 nLi$

∴ $B = \mu_0 ni$ ....(2)

Equation (2) is the required expression.

Q.5. Derive an expression for the net torque on a rectangular current carrying loop placed in a uniform magnetic field with its rotational axis perpendicular to the field.

Ans:

i. Consider rectangular loop abcd placed in a uniform magnetic field $\vec{B}$ such that the sides ab and cd are perpendicular to the magnetic field $\vec{B}$ but the sides bc and da are not, as shown in figure (a) below:

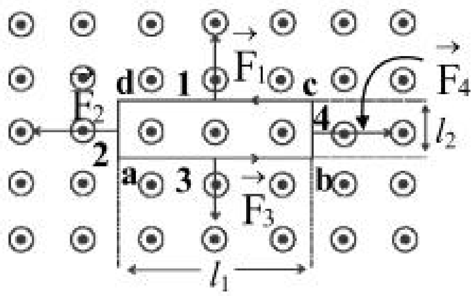


Fig (a): Loop abcd placed in a uniform

ii. The force $\vec{F_4}$ on side 4 (bc) will be

$$\vec{F_4} = Il_2 B \sin (90° - \theta)$$

iii. The force $\vec{F_2}$ on side 2 (da) will be equal and opposite to $\vec{F_4}$ and both act along the same line. Thus, $\vec{F_2}$ and $\vec{F_4}$ will cancel out each other.

iv. The magnitudes of forces $\vec{F_1}$ and $\vec{F_3}$ on sides 1 (cd) and 3 (ab) will be $Il_1 B \sin 90°$ i.e., $Il_1 B$. These two forces do not act along the same line and hence they produce a net torque.

v. This torque results into rotation of the loop so that the loop is perpendicular to the direction of $\vec{B}$, the magnetic field.

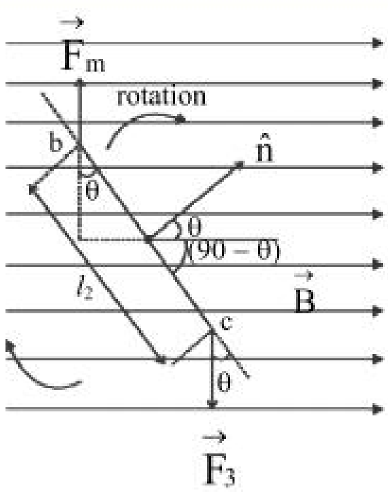


Fig (b): Side view of the loop abcd at an angle θ

vi. Now the moment arm is $\frac{1}{2}(l_2 \sin\theta)$ about the central axis of the loop.

Hence, torque τ due to forces $\vec{F}_1$ and $\vec{F}_3$ will be

$$\tau = \left(Il_1 B \frac{1}{2} l_2 \sin\theta\right) + \left(Il_1 B \frac{1}{2} l_2 \sin\theta\right)$$

$= Il_1 l_2 B \sin\theta$

vii. If the current carrying loop is made up of multiple turns N, in the form of a flat coil, the total torque will be

$\tau' = N\tau = N I l_1 l_2 B \sin\theta$

$\tau' = (NIA)B \sin\theta$; where, A is the are enclosed by the coil $= l_1 l_2$

This is the required expression.

Q.6. A circular loop of radius 9.7 cm carries a current 2.3 A. Obtain the magnitude of the magnetic field (i) at the centre of the loop (ii) at a distance of 9.7 cm from the centre of the loop but on the axis.

Solution:

Given: $R = 9.7 \text{ cm} = 9.7 \times 10^{-2}$ m,

$I = 2.3$ A,

$z = 9.7 \text{ cm} = 9.7 \times 10^{-2}$ m

To find: Magnetic field

i. at the centre of the loop

ii. on the axis at a distance

Formulae: i. $B_c = \frac{\mu_0 I}{2R}$ ii. $B_a = \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}}$

Calculation:

From formula (i),

$$B_c = \frac{4\pi \times 10^{-7} \times 2.3}{2 \times 9.7 \times 10^{-2}}$$

$$= \frac{2 \times 3.142 \times 2.3}{9.7} \times 10^{-7+2}$$

$= \{\text{antilog}\,[\log 2 + \log 3.142 + \log 2.3 - \log 9.7]\} \times 10^{-5}$

$= \{\text{antilog}\,(0.3010 + 0.4972 + 0.3617 - 0.9868)\} \times 10^{-5}$

$= \{\text{antilog}\,(0.1731)\} \times 10^{-5}$

$= 1.489 \times 10^{-5}$

$\approx 1.49 \times 10^{-5}$ T

= 14.9 μT

From formula (ii),

$$B_a = \frac{4\pi \times 10^{-7} \times 2.3 \times \left(9.7 \times 10^{-2}\right)^2}{2\left[\left(9.7 \times 10^{-2}\right)^2 + \left(9.7 \times 10^{-2}\right)^2\right]^{3/2}}$$

$$= \frac{4\pi \times 10^{-7} \times 2.3 \times \left(9.7 \times 10^{-2}\right)^2}{2 \times 2^{3/2} \times \left(9.7 \times 10^{-2}\right)^3}$$

$$= \frac{\pi \times 10^{-7} \times 2.3}{2^{1/2} \times \left(9.7 \times 10^{-2}\right)}$$

$= 5.268 \times 10^{-6}$ T

= **5.268 μT**

Ans: i. Magnetic field at the centre of **14.9 μT.**

ii. Magnetic field on the axis at a distance of 9.7 cm from the centre is **5.268 μT.**

Q.7. The magnetic field at the centre of a circular loop of radius 12.3 cm is 6.4×10^{-6} T. What will be the magnetic moment of the loop?

Solution:

Given: $B = 6.4 \times 10^{-6}$ T,

$R = 12.3$ cm

$= 12.3 \times 10^{-2}$ m

To find: Magnetic moment (m)

Formula: $$B = \frac{\mu_0 I R^2}{2\left(z^2 + R^2\right)^{3/2}}$$

Calculation: From formula,

$$B = \frac{\mu_0 I \pi R^2}{2\pi\left(z^2 + R^2\right)^{3/2}}$$

$$= \frac{\mu_0 m}{2\pi\left(z^2 + R^2\right)^{3/2}} \quad \ldots[\because m = I(\pi R^2)]$$

$$B = \frac{\mu_0 m}{2\pi R^3} \quad \ldots(\because z = 0)$$

$$\therefore \quad m = \frac{B \times 2\pi R^3}{\mu_0}$$

$$= \frac{6.4 \times 10^{-6} \times 2 \times \pi \times \left(12.3 \times 10^{-2}\right)^3}{4\pi \times 10^{-7}}$$

$$= 3.2 \times 10^{-6+7-6} \times (12.3)^3$$

$$= \{\text{antilog}(\log 3.2 + 3 \log 12.3)\} \times 10^{-5}$$

$$= \{\text{antilog}(0.5051 + 3.2697)\} \times 10^{-5}$$

$$= \{\text{antilog}(3.7748)\} \times 10^{-5}$$

$$= 5.954 \times 10^3 \times 10^{-5}$$

$$= \mathbf{5.954 \times 10^{-2}\ Am^2}$$

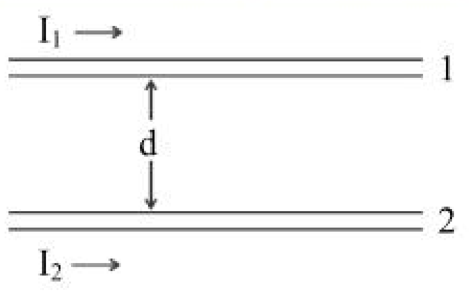
Ans: Magnetic moment of the loop is **5.954×10^{-2} Am^2**.

Long Answer (LA) (4 Marks Each)

Q.1. Show that currents in two long, straight, parallel wires exert forces on each other. Derive the expression for the force per unit length on each conductor.

Ans: Case I: Both wires carry current in same direction.

i. Consider two long parallel wires separated by distance d and carrying current I_1 and I_2 respectively same direction as same as shown in figure below:



Two long parallel wires, distance d apart

ii. The magnetic field at the second wire due to the current I_1 in the first one, according to Biot – Savart's law is

$$B = \frac{\mu_0 I_1}{2\pi d} \quad \text{....(1)}$$

iii. By the right-hand rule, the direction of this field is into the plane of the paper.

iv. Force on the wire 2, because of the current I_2 and the magnetic field B due to current in wire 1, applying Lorentz force law is,

$$F = I_2 \left(\frac{\mu_0 I_1}{2\pi d}\right)\int dl \qquad ....(2)$$

The direction of this force is towards wire 1, i.e., it will be attractive force.

v. Force of attraction per unit length of the wire will be

$$\frac{F}{L} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} \qquad ....(3)$$

Case II: Two wires carry current in opposite direction.

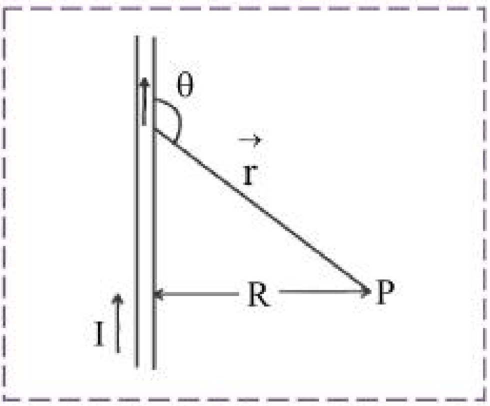
Force is of repulsive nature between antiparallel currents and magnitude of force of repulsion per unit length is, $\left|\frac{F}{L}\right| = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$

Q.2. Using Biot Savart's law, obtain the expression for the magnetic induction near a straight infinitely long current carrying wire.

Ans:

i. Consider a straight wire of length l carrying current I.

ii. Let a point P situated at a perpendicular distance R from the wire as shown below.



iii. Consider infinitesimal length $\vec{dl}$ of wire carrying current I, then current element $= I\vec{dl}$.

iv. Current element is situated at distance r from point P making an angle θ, as shown in figure above.

v. Using Biot Savart law, magnetic field, produced $\vec{dB}$ at P due to current element $I\vec{dl}$ is,

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\sin\theta}{r^2} \qquad ....(1)$$

vi. According to properties of cross-product, $\vec{dl} \times \vec{r}$ indicates direction of $\vec{dB}$, in this case, is into the plane of paper.

vii. Summing up all current elements from upper half of infinitely long wire,

$$B_{upper} = \int_0^\infty dB = \frac{\mu_0}{4\pi}\int_0^\infty \frac{I\,\vec{dl}\,\sin\theta}{r^2} \quad \text{....(2)}$$

viii. Taking into account symmetry of wire, current elements in lower half of infinitely long wire will also contribute same as upper half.

i.e., $B_{lower} = B_{upper}$ ....(3)

ix. Adding contributions from lower and upper part, total magnetic field point P is

$$B = 2\int_0^\infty dB \quad \text{....[using equation (2)]}$$

$$= \frac{2\mu_0}{4\pi}\int_0^\infty \frac{I\,dl\sin\theta}{r^2} \quad \text{....[using equation (1)]}$$

But $r = \sqrt{l^2 + R^2}$ and

$$\sin\theta = \sin(\pi - \theta)$$

$$= \frac{R}{r}$$

$$= \frac{R}{\sqrt{l^2 + R^2}}$$

$\therefore$ $$B = \frac{\mu_0 I}{2\pi}\int_0^\infty \frac{R\,dl}{\left(l^2 + R^2\right)\sqrt{l^2 + R^2}}$$

$$= \frac{\mu_0 I}{2\pi}R\int_0^\infty \frac{dl}{\left(l^2 + R^2\right)^{\frac{3}{2}}}$$

Solving the integration,

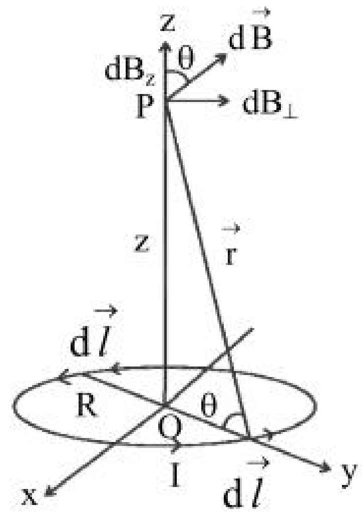
$$B = \frac{\mu_0 I}{2\pi}R \times \frac{1}{R^2} = \frac{\mu_0 I}{2\pi R} \quad \text{....(4)}$$

This is the equation for magnetic field at a point situated at a perpendicular distance R from infinitely long wire carrying current I.

Q.3. Derive an expression for axial magnetic field produced by current in a circular loop.

Ans:

i. Consider loop of radius R carrying current I placed in x–y plane with its centre at origin O as shown in figure below.



Magnetic field on the axis of a circular current loop of radius R

ii. Let point P can be on z–axis at distance $\vec{r}$ from line element $d\vec{l}$ of the loop.

iii. Using Biot-Savart law, the magnitude of the magnetic field dB is given by

$$dB = \frac{\mu_0}{4\pi} I \frac{\left|d\vec{l} \times \vec{r}\right|}{r^3}$$

iv. Any element $d\vec{l}$ will always be perpendicular to the vector $\vec{r}$ from the element to the point P. The element $d\vec{l}$ is in the x-y plane, while the vector $\vec{r}$ is in the y-z plane. Hence

$$d\vec{l} \times \vec{r} = dl\, r$$

$$\therefore \quad dB = \frac{\mu_0}{4\pi} I \frac{dl}{r^2}$$

but $r^2 = R^2 + z^2$

$$\therefore \quad dB = \frac{\mu_0}{4\pi} I \frac{dl}{\left(z^2 + R^2\right)}$$

v. Now, direction of $d\vec{B}$ is perpendicular to the plane formed by $d\vec{l}$ and $\vec{r}$. Its z component is dB_z and the component perpendicular to the z-axis is $dB_\perp$. The components $dB_\perp$ when summed over, yield zero as they cancel out due to symmetry. Hence, only z component remains

vi. The net contribution along the z axis is obtained by integrating $dB_z = dB \cos\theta$ over the entire loop.

From figure,

$$\cos\theta = \frac{R}{r} = \frac{R}{\sqrt{z^2 + R^2}}$$

$$\therefore \quad B_z = \int dB_z = \frac{\mu_0}{4\pi} I \int \frac{dl}{\left(z^2 + R^2\right)} \cos\theta$$

$$= \frac{\mu_0}{4\pi} I \int \frac{R\,dl}{\left(z^2 + R^2\right)^{\frac{3}{2}}}$$

$$= \frac{\mu_0}{4\pi} \times \frac{IR}{\left(z^2 + R^2\right)^{\frac{3}{2}}} \times 2\pi R$$

$$B_z = \frac{\mu_0}{2} \times \frac{IR^2}{\left(z^2 + R^2\right)^{\frac{3}{2}}}$$

This is the magnitude of the magnetic field due to current I in the loop of radius R, on a point at P on the z axis of the loop.