Chapter 9

Q58. Prove that:

$$\frac{(\sin A - 2\sin^3 A)}{(2\cos^3 A - \cos A)} = \tan A$$

Solution

LHS

$$\frac{(\sin A - 2\sin^3 A)}{(2\cos^3 A - \cos A)}$$

$$\frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$

$$\frac{\sin A(\sin^2 A + \cos^2 A - 2\sin^2 A)}{\cos A(2\cos^2 A - \sin^2 A - \cos^2 A)}$$

$$\frac{\sin A(\cos^2 A - \sin^2 A)}{\cos A(\cos^2 A - \sin^2 A)}$$

$$\frac{\sin A}{\cos A} = \tan A = RHS$$

Q59. Prove that: sec A (1 – sin A) (sec A + tan A) = 1

Solution

We have to prove sec A (1 – sin A) (sec A + tan A) = 1
We know that $\sec^2 A - \tan^2 A = 1$
So,
= (sec A – sec A sin A) (sec A + tan A)
$= \left(\sec A - \frac{1 \sin A}{\cos A}\right)(\sec A + \tan A)$
$= \left(\sec A - \frac{\sin A}{\cos A}\right)(\sec A + \tan A)$
= (sec A –tan A) (sec A + tan A)
$= \sec^2 A - \tan^2 A$
= 1 = RHS

Q60. If sin ɵ + cos ɵ = √3 then prove that tan ɵ + cot ɵ = 1

Solution

sin ɵ + cos ɵ = √3
$= (\sin\theta + \cos\theta)^2 = \sqrt{3}$
$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$
=> 2 sin ɵ cos ɵ = 2
=> sin ɵ cos ɵ = 1
$\Rightarrow \sin\theta\cos\theta = \sin^2\theta + \cos^2\theta$
$\Rightarrow 1 = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta + \cos\theta}$
=> tan ɵ + cot ɵ = 1

Q61. Show that: $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

Solution

$\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$
We know that
L.H.S = tan ɵ + tan ɵ
taking out tan ɵ as common
$1 + \tan^2\theta = \sec^2\theta$
i.e. $\tan^2\theta = \sec^2\theta - 1$
it can be written as
$= (\sec^2\theta - 1)\sec^2\theta$
Hence Proved

Q62. If secθ+tanθ=m, show that (m² - 1)/(m² + 1)=sinθ.

Solution

We have
$(m^2-1)=(\sec\theta+\tan\theta)^2-1$
$=\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta-1$
$=(\sec^2\theta-1)+\tan^2\theta+2\sec\theta\tan\theta$

$=2\tan^2\theta+2\sec\theta\tan\theta$ [$\because \sec^2\theta-1=\tan^2\theta$]
$=2\tan\theta(\tan\theta+\sec\theta)$. ...(i)

$(m^2+1)=(\sec\theta+\tan\theta)^2+1$

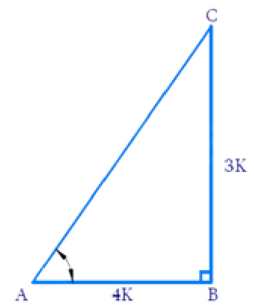
$=\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta + 1$
$=(1+\tan^2\theta)+ \sec^2\theta + 2\sec\theta\tan\theta$
$=2\sec^2\theta+2\sec\theta\tan\theta$ [$\because 1+\tan^2\theta=\sec^2\theta$]
$=2\sec\theta(\sec\theta+\tan\theta)$. ...(ii)

From (i) and (ii), we get
$(m^2-1)/(m^2+1) = \tan\theta/\sec\theta = (\sin\theta/\cos\theta \times \cos\theta) = \sin\theta$.

Q63. If 3 cot A = 4, check whether (1 - tan² A) / (1 + tan² A) = cos² A - sin² A or not

Solution

3 cot A = 4
Thus, cot A = 4/3
Let ΔABC be a right-angled triangle where angle B is a right angle.
cot A = side adjacent to ∠A / side opposite to ∠A = AB/BC = 4/3
Let AB = 4k and BC = 3k, where k is a positive integer.
By applying the Pythagoras theorem in ΔABC, we get,



$AC^2 = AB^2 + BC^2$
= (4k)2 + (3k)2
$= 16k^2 + 9k^2$
$= 25k^2$
$AC = \sqrt{25k^2}$
= 5k

Therefore,

tan A = side opposite to ∠A / side adjacent to ∠A = BC/AB

= 3k/4k = 3/4

sin A = side opposite to ∠A / hypotenuse = BC/AC

= 3k/5k = 3/5

cos A = side adjacent to ∠A / hypotenuse = AB/AC

= 4k/5k = 4/5

L.H.S = $(1 - \tan^2 A) / (1 + \tan^2 A)$

= $[1 - (3/4)^2] / [1 + (3/4)^2]$

= (1 - 9/16) / (1 + 9/16)

= (16 - 9) / (16 + 9)

= 7/25

R.H.S = $\cos^2 A - \sin^2 A$

$(4/5)^2 - (3/5)^2$

= 16/25 - 9/25

= (16 - 9) / 25

= 7/25

Therefore, $(1 - \tan^2 A) / (1 + \tan^2 A) = \cos^2 A - \sin^2 A$