7

Gravitation

Multiple Choice Questions (MCQs)

Q. 1 The earth is an approximate sphere. If the interior contained matter which is not of the same density everywhere, then on the surface of the earth, the acceleration due to gravity

(a) will be directed towards the centre but not the same everywhere

(b) will have the same value everywhere but not directed towards the centre

(c) will be same everywhere in magnitude directed towards the centre

(d) cannot be zero at any point

Ans. *(d)* If we assume the earth as a sphere of uniform density, then it can be treated as point mass placed at its centre. In this case acceleration due to gravity $g = 0$, at the centre.

It is not so, if the earth is considered as a sphere of non-uniform density, in that case value of g will be different at different points and cannot be zero at any point.

Q. 2 As observed from the earth, the sun appears to move in an approximate circular orbit. For the motion of another planet like mercury as observed from the earth, this would

(a) be similarly true

(b) not be true because the force between the earth and mercury is not inverse square law

(c) not be true because the major gravitational force on mercury is due to the sun

(d) not be true because mercury is influenced by forces other than gravitational forces

Ans. *(c)* As observed from the earth, the sun appears to move in an approximate circular orbit. The gravitational force of attraction between the earth and the sun always follows inverse square law.

Due to relative motion between the earth and mercury, the orbit of mercury, as observed from the earth will not be approximately circular, since the major gravitational force on mercury is due to the sun.

Q. 3 Different points in the earth are at slightly different distances from the sun and hence experience different forces due to gravitation. For a rigid body, we know that if various forces act at various points in it, the resultant motion is as if a net force acts on the CM (centre of mass) causing translation and a net torque at the CM causing rotation around an axis through the CM. For the earth-sun system (approximating the earth as a uniform density sphere).

(a) the torque is zero

(b) the torque causes the earth to spin

(c) the rigid body result is not applicable since the earth is not even approximately a rigid body

(d) the torque causes the earth to move around the sun

Ans. ***(a)*** As the earth is revolving around the sun in a circular motion due to gravitational attraction. The force of attraction will be of radial nature *i.e.*, angle between position vector r and force F is zero. So, torque $= |\tau| = |\mathbf{r} \times \mathbf{F}| = rF\sin 0° = 0$

Q. 4 Satellites orbitting the earth have finite life and sometimes debris of satellites fall to the earth. This is because

(a) the solar cells and batteries in satellites run out

(b) the laws of gravitation predict a trajectory spiralling inwards

(c) of viscous forces causing the speed of satellite and hence height to gradually decrease

(d) of collisions with other satellites

Ans. ***(c)*** As the total energy of the earth satellite bounded system is negative $\left(\frac{-GM}{2a}\right)$.

where, a is radius of the satellite and M is mass of the earth.

Due to the viscous force acting on satellite, energy decreases continuously and radius of the orbit or height decreases gradually.

Q. 5 Both the earth and the moon are subject to the gravitational force of the sun. As observed from the sun, the orbit of the moon

(a) will be elliptical

(b) will not be strictly elliptical because the total gravitational force on it is not central

(c) is not elliptical but will necessarily be a closed curve

(d) deviates considerably from being elliptical due to influence of planets other than the earth

Ans. ***(b)*** As observed from the sun, two types of forces are acting on the moon one is due to gravitational attraction between the sun and the moon and the other is due to gravitational attraction between the earth and the moon. Hence, total force on the moon is not central.

Q. 6 In our solar system, the inter-planetary region has chunks of matter (much smaller in size compared to planets) called asteroids. They

(a) will not move around the sun, since they have very small masses compared to the sun

(b) will move in an irregular way because of their small masses and will drift away into outer space

(c) will move around the sun in closed orbits but not obey Kepler's laws

(d) will move in orbits like planets and obey Kepler's laws

Ans. *(d)* Asteroids are also being acted upon by central gravitational forces, hence they are moving in circular orbits like planets and obey Kepler's laws.

Q. 7 Choose the wrong option.

(a) Inertial mass is a measure of difficulty of accelerating a body by an external force whereas the gravitational mass is relevant in determining the gravitational force on it by an external mass

(b) That the gravitational mass and inertial mass are equal is an experimental result

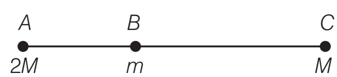
(c) That the acceleration due to gravity on the earth is the same for all bodies is due to the equality of gravitational mass and inertial mass

(d) Gravitational mass of a particle like proton can depend on the presence of neighbouring heavy objects but the inertial mass cannot

Ans. *(d)* Gravitational mass of proton is equivalent to its inertial mass and is independent of presence neighbouring heavy objects.

Q. 8 Particles of masses $2M$, m and M are respectively at points A, B and C with $AB = \frac{1}{2}(BC)$. m is much-much smaller than M and at time $t = 0$, they are all at rest as given in figure.

At subsequent times before any collision takes place.



(a) m will remain at rest

(b) m will move towards M

(c) m will move towards $2M$

(d) m will have oscillatory motion

Thinking Process

The particle B will move towards the greater force, between forces by A and B.

Ans. *(c)* Force on B due to $A = F_{BA} = \frac{G(2Mm)}{(AB)^2}$ towards BA

Force on B due to $C = F_{BC} = \frac{GMm}{(BC)^2}$ towards BC

As, $(BC) = 2AB$

$$\Rightarrow \quad F_{BC} = \frac{GMm}{(2AB)^2} = \frac{GMm}{4(AB)^2} < F_{BA}$$

Hence, m will move towards BA (*i.e.*, $2M$)

Multiple Choice Questions (More Than One Options)

Q. 9 Which of the following options are correct?

(a) Acceleration due to gravity decreases with increasing altitude

(b) Acceleration due to gravity increases with increasing depth (assume the earth to be a sphere of uniform density)

(c) Acceleration due to gravity increases with increasing latitude

(d) Acceleration due to gravity is independent of the mass of the earth

Thinking Process

Acceleration due to gravity is maximum on the surface of the earth, it decreases in both cases while going upward or at a depth.

Ans. ***(a, c, d)***

Acceleration due to gravity at altitude h, $g_h = \dfrac{g}{(1+h/R)^2} \approx g\left(1-\dfrac{2h}{R}\right)$

At depth d, $g_d = g\left(1-\dfrac{d}{R}\right)$

In both cases with increase in h and d, g decreases.

At latitude ϕ, $g_\phi = g - \omega^2 R\cos^2\phi$

As ϕ increases g_ϕ increases.

Also, we can conclude from the formulae, that it is independent of mass.

Q. 10 If the law of gravitation, instead of being inverse square law, becomes an inverse cube law

(a) planets will not have elliptic orbits

(b) circular orbits of planets is not possible

(c) projectile motion of a stone thrown by hand on the surface of the earth will be approximately parabolic

(d) there will be no gravitational force inside a spherical shell of uniform density

Ans. ***(a, c)***

If the law of gravitation becomes an inverse cube law, then we can write, for a planet of mass m revolving around the sun of mass M,

$$F = \frac{GMm}{a^3} = \frac{mv^2}{a} \qquad \text{(where } a \text{ is radius of orbiting planet)}$$

$$\Rightarrow \quad v = \text{orbital speed} = \frac{\sqrt{GM}}{a} \Rightarrow v \propto \frac{1}{a}$$

Time period of revolution of a planet $T = \dfrac{2\pi a}{v} = \dfrac{2\pi a}{\dfrac{\sqrt{GM}}{a}} = \dfrac{2\pi a^2}{\sqrt{GM}}$

$$\Rightarrow \quad T^2 \propto a^4$$

Hence, orbit will not be elliptical. [for elliptical orbit $T^2 \propto a^3$]

As force $F = \left(\dfrac{GM}{a^3}\right)m = g'm$

where, $g' = \dfrac{GM}{a^3}$

As g', acceleration due to gravity is constant, hence path followed by a projectile will be approximately parabolic. (as $T \propto a^2$)

Q. 11 If the mass of the sun were ten times smaller and gravitational constant G were ten times larger in magnitude. Then,

(a) walking on ground would become more difficult

(b) the acceleration due to gravity on the earth will not change

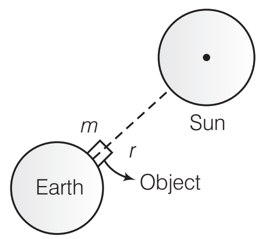
(c) raindrops will fall much faster

(d) airplanes will have to travel much faster

Ans. *(a, c, d)*

Given, $G' = 10G$

Consider the adjacent diagram.



Force on the object due to the earth $= \dfrac{G'M_e m}{R^2} = \dfrac{10GM_e m}{R^2}$ $[\because G' = 10G \text{ given}]$

$= 10\left(\dfrac{GM_e m}{R^2}\right)$

$= (10g)\,m = 10\,mg$ $\left[\because g = \dfrac{GM_e}{R^2}\right]$ ...(i)

Force on the object due to the sun $F = \dfrac{GM'_s m}{r^2}$

$= \dfrac{G(M_s)m}{10r^2}$ $\left[\because M'_s = \dfrac{M_s}{10} \text{ (given)}\right]$

As $r >> R$ (radius of the earth) $\Rightarrow$ F will be very small.

So, the effect of the sun will be neglected.

Now, as $g' = 10\,g$

Hence, weight of person $= mg' = 10\,mg$ [from Eq. (i)]

i.e., gravity pull on the person will increase. Due to it, walking on ground would become more difficult.

Critical velocity, v_c is proportional to g *i.e.*,

$v_c \propto g$

As, $g' > g$

$\Rightarrow$ $v_c' > v_c$,

Hence, rain drops will fall much faster.

To overcome the increased gravitational force of the earth, the aeroplanes will have to travel much faster.

Q. 12 If the sun and the planets carried huge amounts of opposite charges,

(a) all three of Kepler's laws would still be valid

(b) only the third law will be valid

(c) the second law will not change

(d) the first law will still be valid

Thinking Process

Electrostatic force of attraction acts between two opposite charges.

Ans. ***(a, c, d)***

Due to huge amounts of opposite charges on the sun and the earth electrostatic force of attraction will be large. Gravitational force is also attractive in nature have both forces will be added.

Both the forces obey inverse square law and are central forces. As both the forces are of same nature, hence all the three **Kepler's laws** will be valid.

Q. 13 There have been suggestions that the value of the gravitational constant G becomes smaller when considered over very large time period (in billions of years) in the future. If that happens, for our earth,

(a) nothing will change

(b) we will become hotter after billions of years

(c) we will be going around but not strictly in closed orbits

(d) after sufficiently long time we will leave the solar system

Ans. ***(c, d)***

We know that gravitational force between the earth and the sun.

$F_G = \frac{GMm}{r^2}$, where M is mass of the sun and m is mass of the earth.

When G decreases with time, the gravitational force F_G will become weaker with time. As F_G is changing with time. Due to it, the earth will be going around the sun not strictly in closed orbit and radius also increases, since the attraction force is getting weaker.

Hence, after long time the earth will leave the solar system.

Q. 14 Supposing Newton's law of gravitation for gravitation forces $\mathbf{F}_1$ and $\mathbf{F}_2$ between two masses m_1 and m_2 at positions $\mathbf{r}_1$ and $\mathbf{r}_2$ read

$$\mathbf{F}_1 = -\mathbf{F}_2 = -\frac{\mathbf{r}_{12}}{r_{12}^3} GM^2{}_0 \left(\frac{m_1 m_2}{M_0^2}\right)^n$$

where M_0 is a constant of dimension of mass, $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$ and n is a number. In such a case,

(a) the acceleration due to gravity on the earth will be different for different objects

(b) none of the three laws of Kepler will be valid

(c) only the third law will become invalid

(d) for n negative, an object lighter than water will sink in water

Ans. ***(a,c, d)***

Given,
$$\mathbf{F}_1 = -\mathbf{F}_2 = \frac{-\mathbf{r}_{12}}{r_{12}^3} GM_0^2 \left(\frac{m_1 m_2}{M_0^2}\right)^n$$

$$\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$$

Acceleration due to gravity, $g = \frac{|F|}{\text{mass}}$

$$= \frac{GM_0^2 (m_1 m_2)^n}{r_{12}^2 (M_0)^{2n}} \times \frac{1}{(\text{mass})}$$

Since, g depends upon position vector, hence it will be different for different objects. As g is not constant, hence constant of proportionality will not be constant in Kepler's third law. Hence, Kepler's third law will not be valid.

As the force is of central nature. $\left[\because \text{force} \propto \frac{1}{r^2}\right]$

Hence, first two Kepler's laws will be valid.

For negative n,

$$g = \frac{GM_0^2 (m_1 m_2)^{-n}}{r_{12}^2 (M_0)^{-2n}} \times \frac{1}{(\text{mass})}$$

$$= \frac{GM_0^{2(1+n)}}{r_{12}^2} \frac{(m_1 m_2)^{-n}}{(\text{mass})}$$

$$g = \frac{GM_0^2}{r_{12}^2} \left(\frac{M_0^2}{m_1 m_2} \right)^n \times \frac{1}{\text{mass}}$$

As $M_0 > m_1$ or m_2

$g > 0$, hence in this case situation will reverse *i.e.*, object lighter than water will sink in water.

Q. 15 Which of the following are true?

(a) A polar satellite goes around the earth's pole in north-south direction

(b) A geostationary satellite goes around the earth in east-west direction

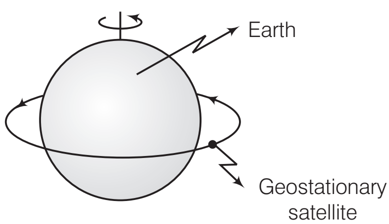
(c) A geostationary satellite goes around the earth in west-east direction

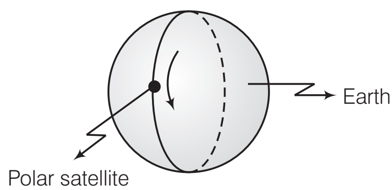
(d) A polar satellite goes around the earth in east-west direction

Ans. *(a, c)*

A geostationary satellite is having same sense of rotation as that of earth *i.e.*, west-east direction.

A polar satellite goes around the earth's pole in north-south direction.





Q. 16 The centre of mass of an extended body on the surface of the earth and its centre of gravity

(a) are always at the same point for any size of the body

(b) are always at the same point only for spherical bodies

(c) can never be at the same point

(d) is close to each other for objects, say of sizes less than 100 m

(e) both can change if the object is taken deep inside the earth

Ans. *(d)*

For small objects, say of sizes less than 100 m centre of mass is very close with the centre of gravity of the body. But when the size of object increases, its weight changes and its CM and CG become far from each other.

Very Short Answer Type Questions

Q. 17 Molecules in air in the atmosphere are attracted by gravitational force of the earth. Explain why all of them do not fall into the earth just like an apple falling from a tree.

Ans. Air molecules in the atmosphere are attracted vertically downward by gravitational force of the earth just like an apple falling from a tree. Air molecules move randomly due to their thermal velocity and hence the resultant motion of air molecules is not exactly in the vertical downward direction.

But in case of apple, only vertical motion dominates because of being heavier than air molecules. But due to gravity, the density of atmosphere increases near to the earth's surface.

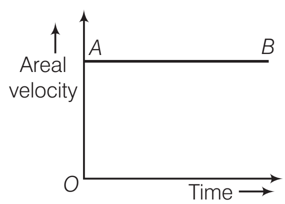
Q. 18 Give one example each of central force and non-central force.

Ans. **Example of central force** Gravitational force, electrostatic force etc.

Example of non-central force Nuclear force, magnetic force acting between two current carrying loops etc.

Q. 19 Draw areal velocity *versus* time graph for mars.

Ans. Areal velocity of a planet revolving around the sun is constant with time. Therefore, graph between areal velocity and time is a straight line (AB) parallel to time axis.(Kepler's second law).



Q. 20 What is the direction of areal velocity of the earth around the sun?

Ans. Areal velocity of the earth around the sun is given by

$$\frac{d\mathbf{A}}{dt} = \frac{\mathbf{L}}{2m}$$

where, $\mathbf{L}$ is the angular momentum and m is the mass of the earth.

But angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times m\mathbf{v}$

$\therefore$ Areal velocity $\left(\frac{dA}{dt}\right) = \frac{1}{2m}(\mathbf{r} \times m\mathbf{v}) = \frac{1}{2}(\mathbf{r} \times \mathbf{v})$

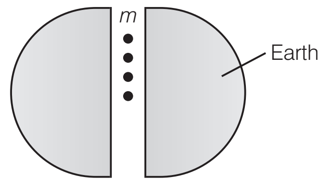
Therefore, the direction of areal velocity $\left(\frac{d\mathbf{A}}{dt}\right)$ is in the direction of $(\mathbf{r} \times \mathbf{v})$, *i.e.*, perpendicular to the plane containing $\mathbf{r}$ and $\mathbf{v}$ and directed as given by right hand rule.

Q. 21 How is the gravitational force between two point masses affected when they are dipped in water keeping the separation between them the same?

Ans. Gravitational force acting between two point masses m_1 and m_2, $F = \frac{Gm_1m_2}{r^2}$, is independent of the nature of medium between them. Therefore, gravitational force acting between two point masses will remain unaffected when they are dipped in water.

Q. 22 Is it possible for a body to have inertia but no weight?

Ans. *Yes*, a body can have inertia (*i.e.*, mass) but no weight. Everybody always have inertia (*i.e.*, mass) but its weight (mg) can be zero, when it is taken at the centre of the earth or during free fall under gravity.



e.g., In the tunnel through the centre of the earth, the object moves only due to inertia at the centre while its weight becomes zero.

Q. 23 We can shield a charge from electric fields by putting it inside a hollow conductor. Can we shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?

Ans. A body cannot be shielded from the gravitational influence of nearby matter, because gravitational force between two point mass bodies is independent of the intervening medium between them.

It is due to the above reason, we cannot shield a body from the gravitational influence of nearby matter by putting it either inside a hollow sphere or by some other means.

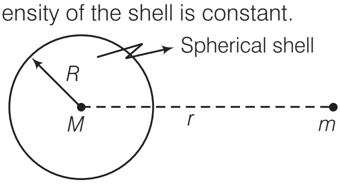
Q. 24 An astronaut inside a small spaceship orbitting around the earth cannot detect gravity. If the space station orbitting around the earth has a large size, can he hope to detect gravity?

Ans. Inside a small spaceship orbitting around the earth, the value of acceleration due to gravity g, can be considered as constant and hence astronaut feels weightlessness.

If the space station orbitting around the earth has a large size, such that variation in g matters in that case astronaut inside the spaceship will experience gravitational force and hence can detect gravity. *e.g.*, On the moon, due to larger size gravity can be detected.

Q. 25 The gravitational force between a hollow spherical shell (of radius R and uniform density) and a point mass is F. Show the nature of F *versus* r graph where r is the distance of the point from the centre of the hollow spherical shell of uniform density.

Ans. Consider the diagram, density of the shell is constant. Let it is ρ.



Mass of the shell = (density) × (volume)

$$= (\rho) \times \frac{4}{3}\pi R^3 = M$$

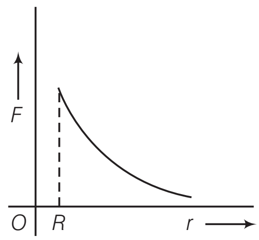
As the density of the shell is uniform, it can be treated as a point mass placed at its centre.

Therefore, F = gravitational force between M and $m = \frac{GMm}{r^2}$

$$F = 0 \text{ for } r < R \qquad (\textit{i.e.}, \text{ force inside the shell is zero})$$

$$= \frac{GM}{r^2} \text{ for } r \geq R$$

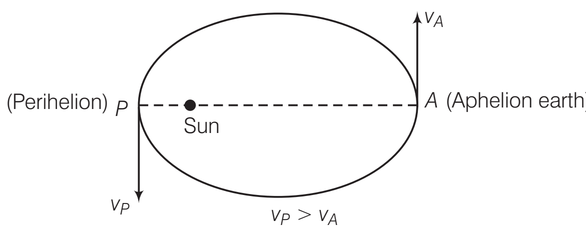
The variation of F *versus* r is shown in the diagram.



Note *When r tends to infinity, force tends to zero, also force is maximum on the surface of the hollow spherical shell.*

Q. 26 Out of aphelion and perihelion, where is the speed of the earth more and why?

Ans. Aphelion is the location of the earth where it is at the greatest distance from the sun and perihelion is the location of the earth where it is at the nearest distance from the sun.



The areal velocity $\left(\frac{1}{2}\mathbf{r} \times \mathbf{v}\right)$ of the earth around the sun is constant (Kepler's IInd law).

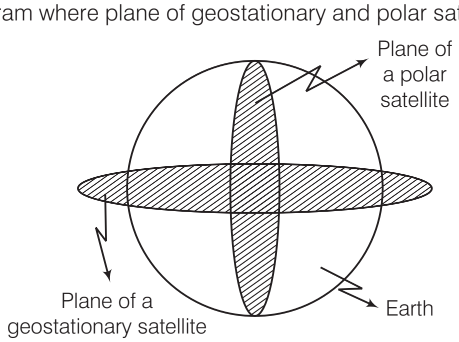
Therefore, the speed of the earth is more at the perihelion than at the aphelion.

Q. 27 What is the angle between the equatorial plane and the orbital plane of

(a) polar satellite?

(b) geostationary satellite?

Ans. Consider the diagram where plane of geostationary and polar satellite are shown.



Clearly

(a) Angle between the equatorial plane and orbital plane of a polar satellite is 90°.

(b) Angle between equatorial plane and orbital plane of a geostationary satellite is 0°.

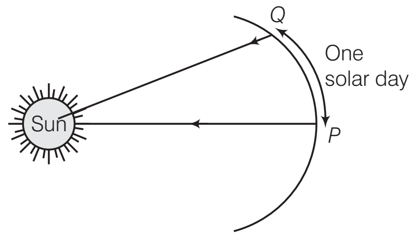
Short Answer Type Questions

Q. 28 Mean solar day is the time interval between two successive noon when sun passes through zenith point (meridian).

Sidereal day is the time interval between two successive transit of a distant star through the zenith point (meridian).

By drawing appropriate diagram showing the earth's spin and orbital motion, show that mean solar day is 4 min longer than the sidereal day. In other words, distant stars would rise 4 min early every successive day.

Ans. Consider the diagram below, the earth moves from the point P to Q in one solar day.



Every day the earth advances in the orbit by approximately 1°. Then, it will have to rotate by 361° (which we define as 1 day) to have the sun at zenith point again.

$\because$ 361° corresponds to 24 h.

$\therefore$ 1° corresponds to $\frac{24}{361} \times 1 = 0.066\,\text{h} = 3.99\,\text{min} \approx 4\,\text{min}$

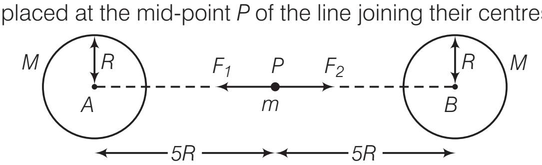
Hence, distant stars would rise 4 min early every successive day.

Q. 29 Two identical heavy spheres are separated by a distance 10 times their radius. Will an object placed at the mid-point of the line joining their centres be in stable equilibrium or unstable equilibrium? Give reason for your answer.

Thinking Process

To determine the nature of equilibrium, we have to displace the object through a small distance, from the middle point and then force will be calculated in displaced position.

Ans. Let the mass and radius of each identical heavy sphere be M and R respectively. An object of mass m be placed at the mid-point P of the line joining their centres.



Force acting on the object placed at the mid-point,

$$F_1 = F_2 = \frac{GMm}{(5R)^2}$$

The direction of forces are opposite, therefore net force acting on the object is zero.

To check the stability of the equilibrium, we displace the object through a small distance x towards sphere A.

Now, force acting towards sphere A, $F_1' = \frac{GMm}{(5R - x)^2}$

Force acting towards sphere B, $F_2' = \frac{GMm}{(5R + x)^2}$

As $F_1' > F_2'$, therefore a resultant force $(F_1' - F_2')$ acts on the object towards sphere A, therefore object start to move towards sphere A and hence equilibrium is unstable.

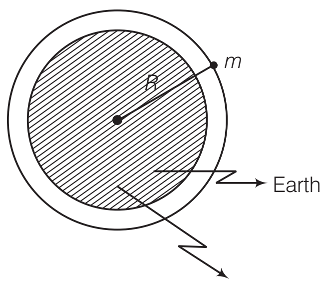
Q. 30 Show the nature of the following graph for a satellite orbitting the earth.

(a) KE *versus* orbital radius R

(b) PE *versus* orbital radius R

(c) TE *versus* orbital radius R

Ans. Consider the diagram, where a satellite of mass m, moving around the earth in a circular orbit of radius R.

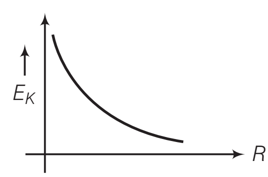


Orbital speed of the satellite orbitting the earth is given by $v_o = \sqrt{\frac{GM}{R}}$

where, M and R are the mass and radius of the earth.

(a) $\therefore$ KE of a satellite of mass m, $E_K = \frac{1}{2} m v_o^2 = \frac{1}{2} m \times \frac{GM}{R}$

$$\therefore \quad E_K \propto \frac{1}{R}$$

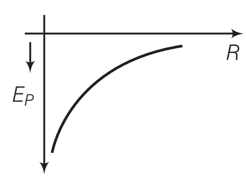


It means the KE decreases exponentially with radius.

The graph for KE *versus* orbital radius R is shown in figure.

(b) Potential energy of a satellite $E_P = -\frac{GMm}{R}$

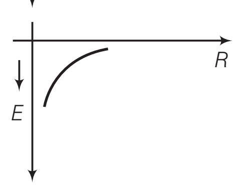
$$E_P \propto -\frac{1}{R}$$



The graph for PE *versus* orbital radius R is shown in figure.

(c) Total energy of the satellite $E = E_K + E_P = \frac{GMm}{2R} - \frac{GMm}{R}$

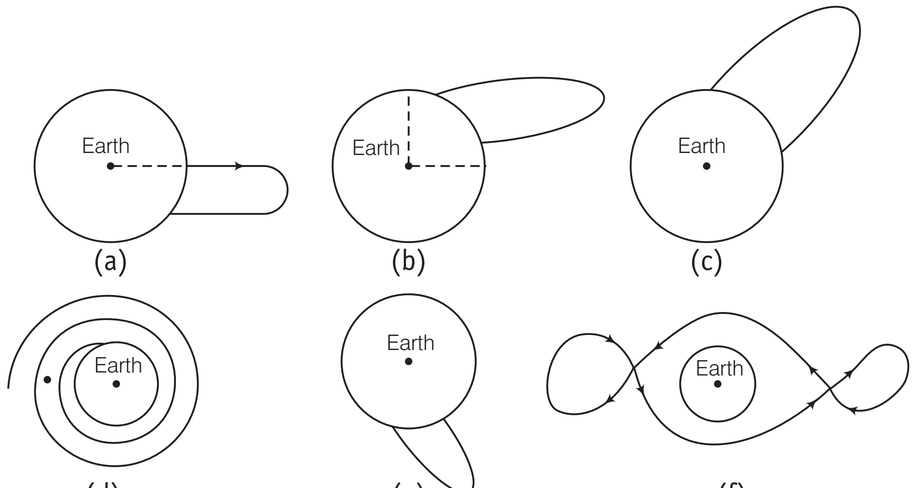
$$= -\frac{GMm}{2R}$$



The graph for total energy *versus* orbital radius R is shown in the figure.

Note *We should keep in mind that Potential Energy (PE) and Kinetic Energy (KE) of the satellite-earth system is always negative.*

Q. 31 Shown are several curves [fig. (a), (b), (c), (d), (e), (f)]. Explain with reason, which ones amongst them can be possible trajectories traced by a projectile (neglect air friction).

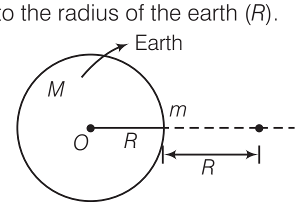


Ans. The trajectory of a particle under gravitational force of the earth will be a conic section (for motion outside the earth) with the centre of the earth as a focus. Only (c) meets this requirement.

***Note** The trajectory of the particle depends upon the velocity of projection. Depending upon the magnitude and direction of velocity it may be parabolic or elliptical.*

Q. 32 An object of mass m is raised from the surface of the earth to a height equal to the radius of the earth, that is, taken from a distance R to $2R$ from the centre of the earth. What is the gain in its potential energy?

Ans. Consider the diagram where an object of mass m is raised from the surface of the earth to a distance (height) equal to the radius of the earth (R).



Potential energy of the object at the surface of the earth $= -\dfrac{GMm}{R}$

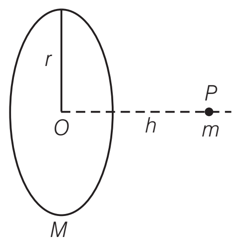
PE of the object at a height equal to the radius of the earth $= -\dfrac{GMm}{2R}$

$\therefore$ Gain in PE of the object $= \dfrac{-GMm}{2R} - \left(-\dfrac{GMm}{R}\right)$

$$= \frac{-GMm + 2GMm}{2R} = +\frac{GMm}{2R}$$

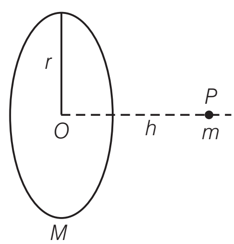
$$= \frac{gR^2 \times m}{2R} = \frac{1}{2}mgR \qquad (\because GM = gR^2)$$

Q. 33 A mass m is placed at P a distance h along the normal through the centre O of a thin circular ring of mass M and radius r (figure).



If the mass is moved further away such that OP becomes $2h$, by what factor the force of gravitation will decrease, if $h = r$?

Ans. Consider the diagram, in which a system consisting of a ring and a point mass is shown.



Gravitational force acting on an object of mass m, placed at point P at a distance h along the normal through the centre of a circular ring of mass M and radius r is given by

$$F = \frac{GMmh}{(r^2 + h^2)^{3/2}} \quad \text{(along } PO\text{) ...(i)}$$

When mass is displaced upto distance $2h$, then

$$F' = \frac{GMm \times 2h}{[r^2 + (2h)^2]^{3/2}} \quad [\because h = 2r]$$

$$= \frac{2GMmh}{(r^2 + 4h^2)^{3/2}} \quad \text{...(ii)}$$

When $h = r$, then from Eq.(i)

$$F = \frac{GMm \times r}{(r^2 + r^2)^{3/2}} \Rightarrow F = \frac{GMm}{2\sqrt{2}r^2}$$

and
$$F' = \frac{2GMmr}{(r^2 + 4r^2)^{3/2}} = \frac{2GMm}{5\sqrt{5}r^2} \quad \text{[From Eq. (ii) substituting } h = r\text{]}$$

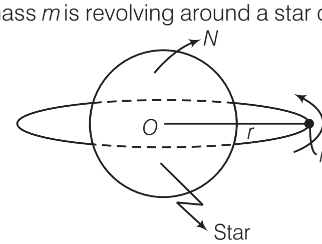
$\therefore$
$$\frac{F'}{F} = \frac{4\sqrt{2}}{5\sqrt{5}}$$

$\Rightarrow$
$$F' = \frac{4\sqrt{2}}{5\sqrt{5}} F$$

Long Answer Type Questions

Q. 34 A star like the sun has several bodies moving around it at different distances. Consider that all of them are moving in circular orbits. Let r be the distance of the body from the centre of the star and let its linear velocity be v, angular velocity ω, kinetic energy K, gravitational potential energy U, total energy E and angular momentum l. As the radius r of the orbit increases, determine which of the above quantities increase and which ones decrease.

Ans. The situation is shown in the diagram, where a body of mass m is revolving around a star of mass M.



Linear velocity of the body $v = \sqrt{\frac{GM}{r}}$

$\Rightarrow$ $v \propto \frac{1}{\sqrt{r}}$

Therefore, when r increases, v decreases.

Angular velocity of the body $\omega = \frac{2\pi}{T}$

According to Kepler's law of period,

$$T^2 \propto r^3 \quad \Rightarrow \quad T = kr^{3/2}$$

where k is a constant

$\therefore$ $\omega = \frac{2\pi}{kr^{3/2}} \Rightarrow \omega \propto \frac{1}{r^{3/2}}$ $\left(\because \omega = \frac{2\pi}{T}\right)$

Therefore, when r increases, ω decreases.

Kinetic energy of the body $K = \frac{1}{2}mv^2 = \frac{1}{2}m \times \frac{GM}{r} = \frac{GMm}{2r}$

$\therefore$ $K \propto \frac{1}{r}$

Therefore, when r increases, KE decreases.

Gravitational potential energy of the body,

$$U = -\frac{GMm}{r} \Rightarrow U \propto -\frac{1}{r}$$

Therefore, when r increases, PE becomes less negative *i.e.,* increases.

Total energy of the body $E = \text{KE} + \text{PE} = \frac{GMm}{2r} + \left(-\frac{GMm}{r}\right) = -\frac{GMm}{2r}$

Therefore, when r increases, total energy becomes less negative, *i.e.*, increases.

Angular momentum of the body $L = mvr = mr\sqrt{\frac{GM}{r}} = m\sqrt{GMr}$

$\therefore$ $L \propto \sqrt{r}$

Therefore, when r increases, angular momentum L increases.

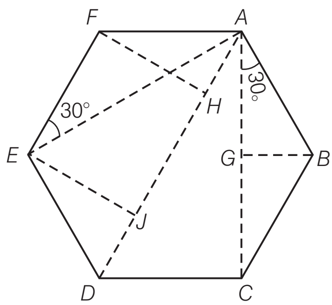
Note *In this case, we have not considered the sun-object system as isolated and the force on the system is not zero. So, angular momentum is not conserved.*

Q. 35 Six point masses of mass m each are at the vertices of a regular hexagon of side l. Calculate the force on any of the masses.

Thinking Process

To calculate resultant force, we will apply principle of superposition i.e., net force will be equal to sum of individual forces by each point mass (m).

Ans. Consider the diagram below, in which six point masses are placed at six vertices A, B, C, D, E and F.



$$AC = AG + GC = 2AG$$
$$= 2l\cos 30° = 2l\sqrt{3}/2$$
$$= \sqrt{3}l = AE$$
$$AD = AH + HJ + JD$$
$$= l\sin 30° + l + l\sin 30° = 2l$$

Force on mass m at A due to mass m at B is, $f_1 = \dfrac{Gmm}{l^2}$ along AB.

Force on mass m at A due to mass m at C is, $f_2 = \dfrac{Gm \times m}{(\sqrt{3}l)^2} = \dfrac{Gm^2}{3l^2}$ along AC.

$[\because AC = \sqrt{3}l]$

Force on mass m at A due to mass m at D is, $f_3 = \dfrac{Gm \times m}{(2l)^2} = \dfrac{Gm^2}{4l^2}$ along AD. $[\because AD = 2l]$

Force on mass m at A due to mass m at E is, $f_4 = \dfrac{Gm \times m}{(\sqrt{3}l)^2} = \dfrac{Gm^2}{3l^2}$ along AE.

Force on mass m at A due to mass m at F is, $f_5 = \dfrac{Gm \times m}{l^2} = \dfrac{Gm^2}{l^2}$ along AF.

Resultant force due to f_1 and f_5 is, $F_1 = \sqrt{f_1^2 + f_5^2 + 2f_1 f_5 \cos 120°} = \dfrac{Gm^2}{l^2}$ along AD.

$[\because$ Angle between f_1 and $f_5 = 120°]$

Resultant force due to f_2 and f_4 is, $F_2 = \sqrt{f_2^2 + f_4^2 + 2f_2 f_4 \cos 60°}$

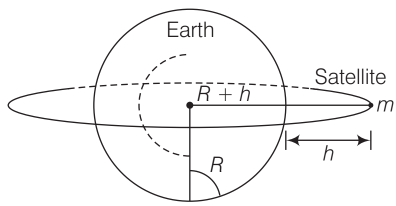
$$= \frac{\sqrt{3}Gm^2}{3l^2} = \frac{Gm^2}{\sqrt{3}l^2} \text{ along } AD.$$

So, net force along $AD = F_1 + F_2 + F_3 = \dfrac{Gm^2}{l^2} + \dfrac{Gm^2}{\sqrt{3}l^2} + \dfrac{Gm^2}{4l^2} = \dfrac{Gm^2}{l^2}\left(1 + \dfrac{1}{\sqrt{3}} + \dfrac{1}{4}\right).$

Q. 36 A satellite is to be placed in equatorial geostationary orbit around the earth for communication.

(a) Calculate height of such a satellite.

(b) Find out the minimum number of satellites that are needed to cover entire earth, so that atleast one satellite is visible from any point on the equator.



$[M = 6 \times 10^{24}$ kg, $R = 6400$ km, $T = 24$ h, $G = 6.67 \times 10^{-11}$ SI unit]

Ans. Consider the adjacent diagram

Given, mass of the earth $M = 6 \times 10^{24}$ kg

Radius of the earth, $R = 6400 \text{ km} = 6.4 \times 10^6$ m

Time period $T = 24$ h

$= 24 \times 60 \times 60 = 86400$ s

$G = 6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2$

(a) Time period $T = 2\pi\sqrt{\dfrac{(R+h)^3}{GM}}$ $\left[\because v_o = \sqrt{\dfrac{GM}{R+h}} \text{ and } T = \dfrac{2\pi(R+h)}{v_o}\right]$

$\Rightarrow$ $T^2 = 4\pi^2 \dfrac{(R+h)^3}{GM} \Rightarrow (R+h)^3 = \dfrac{T^2 GM}{4\pi^2}$

$\Rightarrow$ $R + h = \left(\dfrac{T^2 GM}{4\pi^2}\right)^{1/3} \Rightarrow h = \left(\dfrac{T^2 GM}{4\pi^2}\right)^{1/3} - R$

$\Rightarrow$ $h = \left[\dfrac{(24 \times 60 \times 60)^2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{4 \times (3.14)^2}\right]^{1/3} - 6.4 \times 10^6$

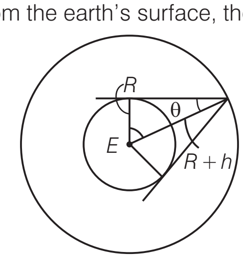
$= 4.23 \times 10^7 - 6.4 \times 10^6$

$= (42.3 - 6.4) \times 10^6$

$= 35.9 \times 10^6$ m

$= 3.59 \times 10^7$ m

(b) If satellite is at height h from the earth's surface, then according to the diagram.



$$\cos\theta = \frac{R}{R+h} = \frac{1}{\left(1+\frac{h}{R}\right)} = \frac{1}{\left(1+\frac{3.59\times10^7}{6.4\times10^6}\right)}$$

$$= \frac{1}{1+5.61} = \frac{1}{6.61} = 0.1513 = \cos 81°18'$$

$$\theta = 81°18'$$

$\therefore$ $$2\theta = 2\times(81°18') = 162°36'$$

If n is the number of satellites needed to cover entire the earth, then

$$n = \frac{360°}{2\theta} = \frac{360°}{162°36'} = 2.31$$

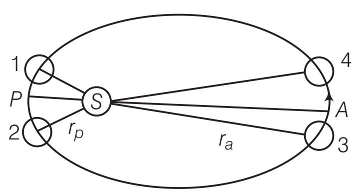
$\therefore$ Minimum 3 satellites are required to cover entire the earth.

Q. 37 Earth's orbit is an ellipse with eccentricity 0.0167. Thus, the earth's distance from the sun and speed as it moves around the sun varies from day-to-day. This means that the length of the solar day is not constant through the year. Assume that the earth's spin axis is normal to its orbital plane and find out the length of the shortest and the longest day. A day should be taken from noon to noon. Does this explain variation of length of the day during the year?

Thinking Process

As the earth orbits the sun, the angular momentum is conserved and areal velocity is constant.

Ans. Consider the diagram. Let m be the mass of the earth, v_p, v_a be the velocity of the earth at perigee and apogee respectively. Similarly, ω_p and ω_a are corresponding angular velocities.



Angular momentum and areal velocity are constant as the earth orbits the sun.

At perigee, $r_p^2\,\omega_p = r_a^2\,\omega_a$ at apogee ...(i)

If a is the semi-major axis of the earth's orbit, then $r_p = a(1-e)$ and $r_a = a(1+e)$ ...(ii)

$\therefore$ $$\frac{\omega_p}{\omega_a} = \left(\frac{1+e}{1-e}\right)^2, e = 0.0167$$ [from Eqs. (i) and (ii)]

$\therefore$ $$\frac{\omega_p}{\omega_a} = 1.0691$$

Let ω be angular speed which is geometric mean of ω_p and ω_a and corresponds to mean solar day,

$\therefore$ $$\left(\frac{\omega_p}{\omega}\right)\left(\frac{\omega}{\omega_a}\right) = 1.0691$$

$\therefore$ $$\frac{\omega_p}{\omega} = \frac{\omega}{\omega_a} = 1.034$$

If ω corresponds to 1° per day (mean angular speed), then $\omega_p = 1.034°$ per day and $\omega_a = 0.967°$ per day. Since, $361° = 24$, mean solar day, we get $361.034°$ which corresponds to 24 h, 8.14" (8.1" longer) and 360.967° corresponds to 23 h 59 min 52" (7.9" smaller).

This does not explain the actual variation of the length of the day during the year.

Q. 38 A satellite is in an elliptic orbit around the earth with aphelion of $6R$ and perihelion of $2R$ where $R = 6400$ km is the radius of the earth. Find eccentricity of the orbit. Find the velocity of the satellite at apogee and perigee. What should be done if this satellite has to be transferred to a circular orbit of radius $6R$?

[$G = 6.67 \times 10^{-11}$ SI unit and $M = 6 \times 10^{24}$ kg]

Ans. Given, r_p = radius of perihelion $= 2R$

r_a = radius of apnelion $= 6R$

Hence, we can write

$$r_a = a(1+e) = 6R \quad \text{...(i)}$$
$$r_p = a(1-e) = 2R \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$\text{eccentricity, } e = \frac{1}{2}$$

By conservation of angular momentum, angular momentum at perigee = angular momentum at apogee

$$\therefore \quad mv_p r_p = mv_a r_a$$
$$\therefore \quad \frac{v_a}{v_p} = \frac{1}{3}$$

where m is mass of the satellite.

Applying conservation of energy, energy at perigee = energy at apogee

$$\frac{1}{2}mv_p^2 - \frac{GMm}{r_p} = \frac{1}{2}mv_a^2 - \frac{GMm}{r_a}$$

where M is the mass of the earth.

$$\therefore \quad v_p^2\left(1 - \frac{1}{9}\right) = -2GM\left(\frac{1}{r_a} - \frac{1}{r_p}\right) = 2GM\left(\frac{1}{r_p} - \frac{1}{r_a}\right) \quad \left(\text{By putting } v_a = \frac{v_p}{3}\right)$$

$$v_p = \frac{\left[2GM\left(\frac{1}{r_p} - \frac{1}{r_a}\right)\right]^{1/2}}{\left[1 - (V_a / V_p)^2\right]^{1/2}} = \left[\frac{\frac{2GM}{R}\left(\frac{1}{2} - \frac{1}{6}\right)}{\left(1 - \frac{1}{9}\right)}\right]^{1/2}$$

$$= \left(\frac{2/3}{8/9}\frac{GM}{R}\right)^{1/2} = \sqrt{\frac{3}{4}\frac{GM}{R}} = 6.85 \text{ km/s}$$

$$v_p = 6.85 \text{ km/s}, \ v_a = 2.28 \text{ km/s}$$

For circular orbit of radius r,

$$v_c = \text{orbital velocity} = \sqrt{\frac{GM}{r}}$$

For $$r = 6R, \ v_c = \sqrt{\frac{GM}{6R}} = 3.23 \text{ km/s.}$$

Hence, to transfer to a circular orbit at apogee, we have to boost the velocity by $\Delta = (3.23 - 2.28) = 0.95$ km/s. This can be done by suitably firing rockets from the satellite.