Class 10th

Mathematics

Real Number

MOST IMPORTANT QUESTIONS

Multiple Choice Question (1 Mark)

1. $2\sqrt{3}$ is

(A) an integer
(B) a rational number
(C) an irrational number
(D) a whole number

Ans. **(C)**

Sol. Let us assume that $2\sqrt{3}$ is a rational number.

Now $2\sqrt{3}$ where r is rational number

or $\sqrt{3} = \frac{r}{2}$

Now, we know that $\sqrt{3}$ is an irrational number,

So, $\frac{r}{2}$ has to be irrational to make the equation true. This is a contradiction to our assumption. Thus, our assumption is wrong and $2\sqrt{3}$ is an irrational number.

Thus (C) is correct option

2. 225 can be expressed as

(A) 5×3^2 (B) $5^2 \times 3$
(C) $5^2 \times 3^2$ (D) $5^3 \times 3$

Ans. **(C)**

Sol. By prime factorization of 225, we have

$225 = 3 \times 3 \times 5 \times 5$

$= 3^2 \times 5^2$ or $5^2 \times 3^2$

Thus (C) is correct option

3. The sum of exponents of prime factors in the prime factorisation of 196 is

(A) 3 (B) 4
(C) 5 (D) 2

Ans. **(B)**

Sol. Prime factors of 196

$196 = 4 \times 49$

$= 2^2 \times 7^2$

The sum of exponents of prime factor is $2 + 2 = 4$.

Thus (B) is correct option.

4. The HCF and the LCM of 12, 21, 15 respectively are

(A) 3, 140 (B) 12, 420
(C) 3, 420 (D) 420, 3

Ans. **(C)**

Sol. We have $12 = 2 \times 2 \times 3$

$21 = 3 \times 7$

$15 = 3 \times 5$

HCF (12, 21, 15) = 3

LCM (12, 21, 15) = $2 \times 2 \times 3 \times 5 \times 7 = 420$

Thus (C) is correct option

5. The LCM of smallest two-digit composite number and smallest composite number is

(A) 12 (B) 4
(C) 20 (D) 44

Ans. **(C)**

Sol. Smallest two-digit composite number is 10 and smallest composite number is 4.

LCM (10,4) = 20

Thus (C) is correct option

6. HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, then the other number is

(A) 36 (B) 35
(C) 9 (D) 81

Ans. **(D)**

Sol. Let y be the second number.

Since, product of two numbers is equal to product of LCM and HCM

$54 \times y = \text{LCM} \times \text{HCF}$

$54 \times y = 162 \times 27$

$$y = \frac{162 \times 27}{54} = 81$$

Thus (D) is correct option

7. If two positive integers a and b are written as $a = x^3 y^2$ and $b = xy^3$, where x, y are prime numbers, then HCF (a, b) is

(A) xy (B) xy^2

(C) x^3y^3 (D) x^2y^2

Ans. (B)

Sol. We have $a = x^3y^2 = x \times x \times x \times y \times y$

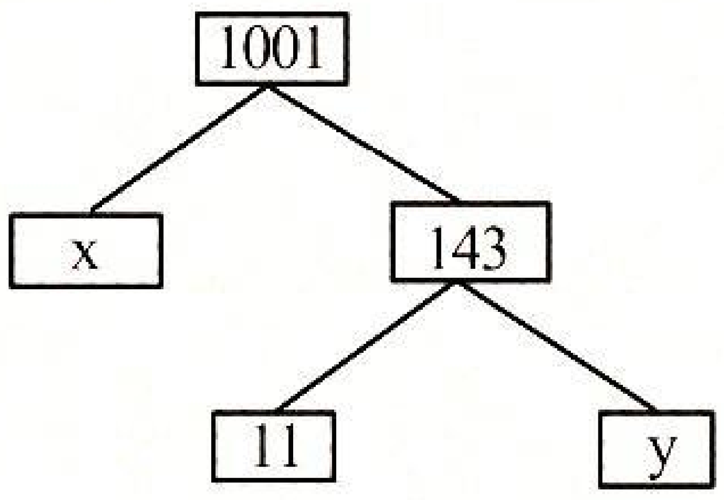
$b = xy^3 = x \times y \times y \times y$

$HCF\ (a, b) = HCF\ (x^3y^2, xy^2)$

$= x \times y \times y = xy^3$

Thus (B) is correct option

8. The values of x and y in the given figure are



(A) $x = 7, y = 13$

(B) $x = 13, y = 7$

(C) $x = 9, y = 12$

(D) $x = 12, y = 9$

Ans. (A)

Sol. $1001 = x \times 143 \Rightarrow x = 7$

$143 = y \times 11 \Rightarrow y = 13$

Hence, $x = 7, y = 13$

Thus (A) is correct option

9. If p_1 and p_2 are two odd prime numbers such that $p_1 > p_2$, then $p_1^2 - p_2^2$ is

(A) an even number

(B) an odd number

(C) an odd prime number

(D) a prime number

Ans. (A)

Sol. $p_1^2 - p_2^2$ is an even number

Let us takes $P_1 = 5$

and $P_2 = 3$

Then $p_1^2 - p_2^2 = 25 - 9 = 16$

16 is an even number.

Than (A) is correct option

10. The least number which is a perfect square and is divisible by each of 16,20 and 24 is

(A) 240 (B) 1600

(C) 2400 (D) 3600

Ans. (D)

Sol. The LCM of 16, 20 and 24 is 240. The least multiple of 240 that is a perfect square is 3600

Thus (D) is correct option.

11. **Assertion:** The HCF of two numbers is 5 and their product is 150, then their LCM is 30

Reason: For any two positive integers a and b, HCF (a, b) + LCM (a, b) = a × b

(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(B) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(C) Assertion (A) is true but reason (R) is false.

(D) Assertion (A) is false but reason (R) is true.

Ans. (C)

Sol. We have,

$LCM\ (a, b) \times HCF\ (a, b) = a \times b$

$LCM \times 5 = 150$

$$LCM = \frac{150}{5} = 30$$

Thus (C) is correct option.

Very Short Answer Type Question [2 Marks]

12. What is the HCF of smallest prime number and the smallest composite number?

Sol. Smallest prime number is 2 and smallest composite number is 4. HCF of 2 and 4 is 2.

13. a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then calculate the least prime factor of (a + b).

Sol. Here a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. The least prime factor of (a + b) would be 2.

14. Check whether 4^n can end with the digit 0 for any natural number n.

Sol. If the number 4^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2. That is, the prime factorization of 4^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $4^n = 2^{2n}$ is 2. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 4^n. So, there is no natural number n for which 4^n ends with the digit zero. Hence 4^n cannot end with the digit zero.

15. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two-given number

Sol. We have

$404 = 2 \times 2 \times 101$

$= 2^2 \times 101$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

HCF $(404, 96) = 2^2 = 4$

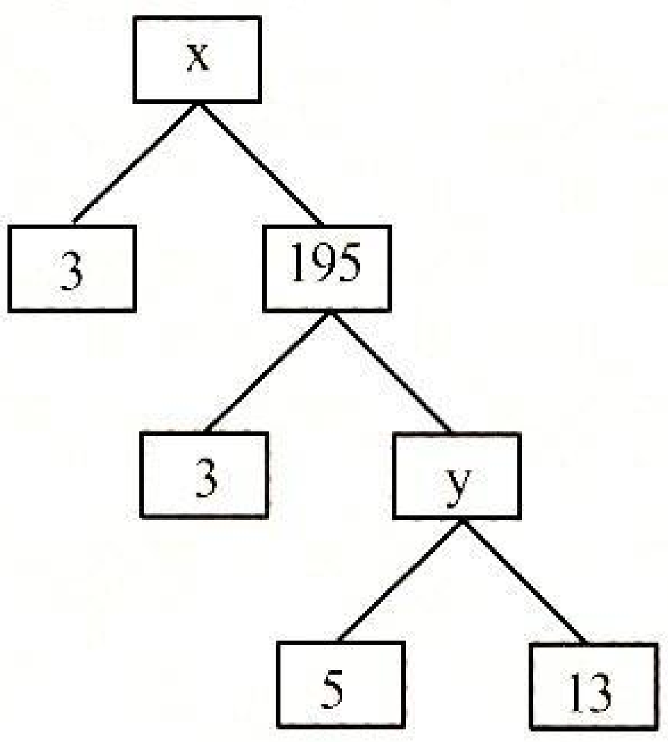
LCM $(404, 96) = 101 \times 2^5 \times 3 = 9696$

HCF × LCM $= 4 \times 9696 = 38784$

Also, $404 \times 96 = 38784$

Hence, HCF × LCM = Product of 404 and 96

16. Complete the following factor tree and find the composite number x.



Sol. We $y = 5 \times 13 = 65$

$x = 3 \times 195 = 585$

17. Explain why $(7 \times 13 \times 11) + 11$ and $(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 3$ are composite numbers.

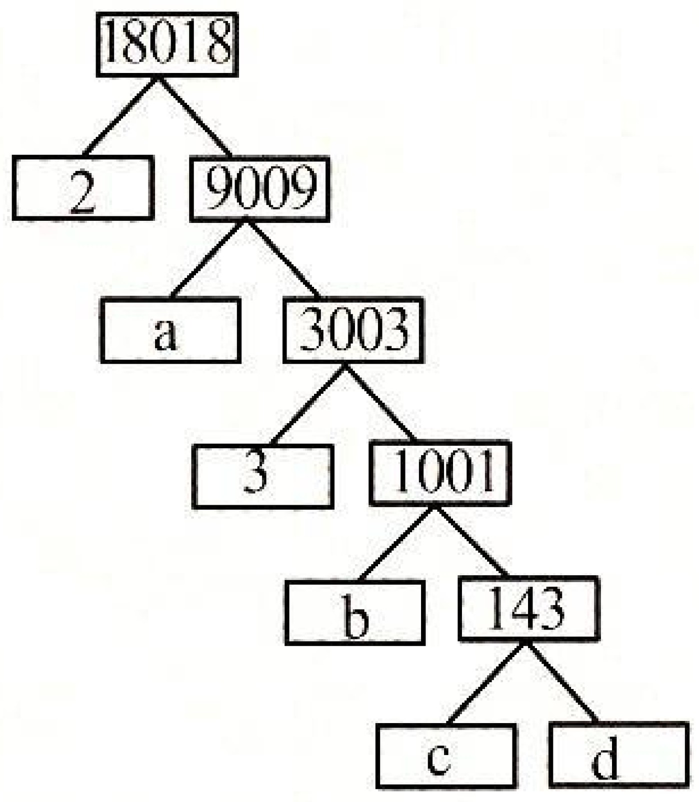
Sol. $(7 \times 5 \times 11) + 11 = 11 \times (7 \times 13 + 1)$

$(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 3 = 3(7 \times 6 \times 5 \times 4 \times 2 \times 1 + 1)$

$= 3 \times (1681) = 3 \times 41 \times 41$

Since given number have more than two prime factors, both number are composite.

18. Find the missing numbers a, b, c and d in the given factor tree:



Sol. We have $a = \frac{9009}{3003} = 3$

$b = \frac{1001}{143} = 7$

Since, $143 = 11 \times 13$,

Thus c = 11 and d = 13 or c = 13 and d = 11

19. Explain whether $3 \times 12 \times 101 + 4$ is a prime number or a composite number.

Sol $3 \times 12 \times 101 + 4 = 4(3 \times 3 \times 101 + 1)$

$= 4(909 + 1)$

$= 4(910)$

$= 2 \times 2 \times (10 \times 7 \times 13)$

$= 2 \times 2 \times 2 \times 5 \times 7 \times 13$

= a composite number

20. The length, breadth and height of a room are 8 m 50cm, 6m 25cm and 4m 75cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly.

Sol. Here we have to determine the HCF of all length which can measure all dimension.

Length, $l = 8\text{m } 50\text{cm} = 850\text{cm}$

$= 50 \times 17 = 2 \times 5^2 \times 17$

Breadth $b = 6\text{m } 25\text{cm} = 625\text{cm}$

$= 25 \times 25 = 5^2 \times 5^2$

Height, $h = 4\text{m } 75\text{cm} = 475\text{cm}$

$= 25 \times 19 = 5^2 \times 19$

HCF(l, b, h) = HCF (850, 625, 475)

$= \text{HCF}(2 \times 5^2 \times 17, 5^2 \times 5^2 \times 19)$

$= 5^2 = 25\text{cm}$

Thus 25 cm rod can measure the dimensions of the room exactly. This is longest rod that can measure exactly.

21. Show that $5\sqrt{6}$ is an irrational number. Given that $\sqrt{6}$ is an irrational number.

Sol. Let $5\sqrt{6}$ be rational number, which can be expressed as $\frac{a}{b}$, where $b \neq 0$; a and b are co-primes.

Now $5\sqrt{6} = \frac{a}{b}$

$\sqrt{6} = \frac{a}{5b}$

Or, $\sqrt{6}$ = rational

But $\sqrt{6}$ is an irrational number, thus our assumption is wrong Hence, $5\sqrt{6}$ is an irrational number.

22. An army contingent of 612 members is to march behind an army band of 48 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Sol. Let the number of columns be x which is the largest number, which should divide both 612 and 48. It means x should be HCF of 612 and 48.

We can write 612 and 48 as follows

$612 = 2 \times 2 \times 3 \times 3 \times 17$

$48 = 2 \times 2 \times 2 \times 2 \times 3$

$\text{HCF}(612, 48) = 2 \times 2 \times 3 = 12$

Thus, HCF of 612 and 48 is 12 i.e. 12 columns are required.

23. Given that $\sqrt{5}$ is irrational, prove that $2\sqrt{5} - 3$ is an irrational number.

Sol. Assume that $2\sqrt{5} - 3$ is a rational number. Therefore, we can write it in the form $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

Now $2\sqrt{5} - 3 = \frac{p}{q}$

Rewriting the above expression as,

$2\sqrt{5} = \frac{p}{q} + 3$

$\Rightarrow \sqrt{5} = \frac{p + 3q}{2q}$

Here $\frac{p+3q}{2q}$ is rational because p and q are co-prime integers thus $\sqrt{5}$ should be a rational number. But $\sqrt{5}$ is irrational. This contradicts the given fact that $\sqrt{5}$ is irrational. Hence $2\sqrt{5} - 3$ is an irrational number.

24. 144 cartons of Coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If each stack is of the same height and if it equal contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

Sol. The required answer will be HCF of 144 and 90.

$144 = 2^4 \times 3^2$

$90 = 2 \times 3^2 \times 5$

$\text{HCF}(144, 90) = 2 \times 3^2 = 18$

Thus each stack would have 18 cartons.

25. Three bells toll at intervals of 9,12,15 minutes respectively. If they start tolling together, after what time will they next toll together?

Sol. The required answer is the LCM of 9, 12, and 15 minutes.

Finding prime factor of given number we have,

$9 = 3 \times 3 = 3^2$

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

$15 = 3 \times 5$

LCM (9, 12, 15) $= 2^2 \times 3^2 \times 5$

= 180 minutes

The bells will toll next together after 180 minutes.

26. The HCF of 65 and 117 is expressible in the form 65m – 117. Find the value of m. Also find the LCM of 65 and 117 using prime factorization method.

Sol. Finding prime factor of given number we have

$117 = 13 \times 3 \times 3$

$65 = 13 \times 5$

HCF (117, 65) = 13

LCM (117, 65) $= 13 \times 5 \times 3 \times 3 = 585$

HCF = 65m – 117

13 = 65m – 117

65m = 117 + 13 = 130

$$m = \frac{130}{65} = 2$$

27. Prove that $\sqrt{5}$ is an irrational number and hence show that $2-\sqrt{5}$ is also an irrational number.

Sol. Assume that $\sqrt{5}$ be a rational number then we have

$\sqrt{5} = \frac{a}{b}$, (a, b are co-primes and $b \neq 0$)

$a = b\sqrt{5}$

Squaring both the sides, we have

$a^2 = 5b^2$

Thus 5 is a factor of a^2 and in result 5 is also a factor of a.

Let a = 5c where c is some integer, then we have

$a^2 = 25c^2$

Substituting $a^2 = 5b^2$ we have

$5b^2 = 25c^2$

$b^2 = 5c^2$

Thus 5 is a factor of b^2 and in result 5 is also a factor of b.

Thus 5 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{5}$ is rational number is wrong. Hence $\sqrt{5}$ is irrational.

Let us assume that $2-\sqrt{5}$ be rational equal to a, then we have

$2-\sqrt{5} = a$

$2-a = \sqrt{5}$

Since we have assume 2 – a is rational, but $\sqrt{5}$ is not rational. Rational number cannot be equal to an irrational number. Thus $2-\sqrt{5}$ is irrational.