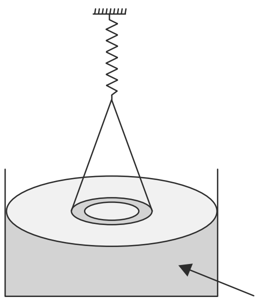
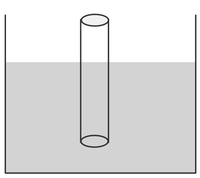
09 SURFACE TENSION

LEVEL 1

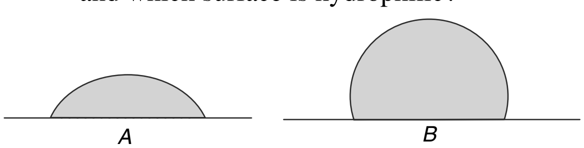
Q.1. A circular ring has inner and outer radii equal to 10 *mm* and 30 *mm* respectively Mass of the ring is $m = 0.7\ g$. It gently pulled out vertically from a water surface by a sensitive spring. When the spring is stretched 3.4 *cm* from its equilibrium position the ring is on verge of being pulled out from the water surface. If spring constant is $k = 0.7\ Nm^{-1}$ find the surface tension of water.



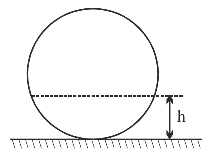
Q.2. A long thin walled capillary tube of mass *M* and radius *r* is partially immersed in a liquid of surface tension *T*. The angle of contact for the liquid and the tube wall is 30°. How much force is needed to hold the tube vertically? Neglect buoyancy force on the tube.



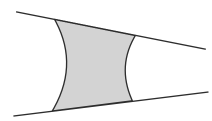
Q.3. (i) Water drops on two surfaces *A* and *B* has been shown in figure. Which surface is hydrophobic and which surface is hydrophilic?



(ii) A liquid is filled in a spherical container of radius *R* till a height *h*. In this position the liquid surface at the edges is also horizontal. What is the contact angle between the liquid and the container wall?



Q.4. A conical pipe shown in figure has a small water drop. In which direction does the drop will tend to move?



Q.5. A narrow tube of length *l* and radius *r* is sealed at one end. Its open end is brought in contact with the surface of water while the tube is held vertical. The water rises to a height *h* in the tube. The contact angle of water with the tube wall is θ, density of water is ρ and the atmospheric pressure is P_o. Find the surface tension of the liquid. Assume that the temperature of air inside the tube remains constant and the volume of the meniscus is negligible.

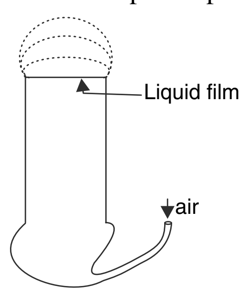
Q.6. The internal radius of one arm of a glass capillary *U* tube is r_1 and for the second arm it is $r_2 (> r_1)$. The tube is filled with some mercury having surface tension *T* and contact angle with glass equal to $90° + \theta$.

(a) It is proposed to connect one arm of the *U* tube to a vacuum pump- so that the mercury level in both arms can be equalized. To which arm the pump shall be connected?

(b) When mercury level in both arms is same, how much below the atmospheric pressure is the pressure of air in the arm connected to the pump?

Q.7. In a horizontal capillary tube, the rate of capillary flow depends on the surface tension force as well as the viscous force. Lueas and washburn showed that the length (x) of liquid penetration in a horizontal capillary depends on a factor (k) apart from time (t). The factor is given by $k = \left[\frac{rT\cos\theta}{2\eta}\right]^{\frac{1}{2}}$; where r, T, θ and η are radius of the capillary tube, surface tension, contact angle and coefficient of viscosity respectively. If the length of liquid in the capillary grows from zero to x_0 in time t_0, how much time will be needed for the length to increases from x_0 to $4x_0$.

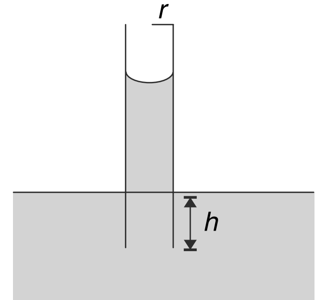
Q.8. A glass tube of radius R is covered with a liquid film at its one end. Air is blown slowly into the tube to gradually increase the pressure inside. What is the maximum pressure that the air inside the tube can have? Assume that the liquid film does not leave the surface (whatever its size) and it does not get punctured. Surface tension of the liquid is T and atmospheric pressure is P_o.



Q.9. Why bubbles can be formed using soap water but we do not have bubbles formed out of pure water?

Q.10. A tapering glass capillary tube A of length 0.1 m has diameters 10^{-3} m and 5×10^{4} m at the ends. When it is just immersed in a liquid at 0°C with larger radius in contact with liquid surface, the liquid rises 8×10^{-2} m in the tube. In another experiment, in a cylindrical glass capillary tube B, when immersed in the same liquid at 0°C, the liquid rises to 6×10^{-2} m height. The rise of liquid in tube B is only 5.5×10^{-2} m when the liquid is at 50°C. Find the rate at which the surface tension changes with temperature considering the change to be linear. The density of liquid is $(1/14) \times 10^{4}$ kg/m^3 and the angle of contact is zero. Effect of temperature on the density of liquid and glass is negligible.

Q.11. (i) One end of a uniform glass capillary tube of radius $r = 0.025$ cm is immersed vertically in water to a depth $h = 1cm$. Contact angle is 0°, surface tension of water is 7.5×10^{-2} N/m, density of water is $\rho = 10^3$ kg/m^3 and atmospheric pressure is $P_o = 10^5$ N/m^2

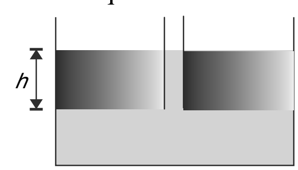


Find the excess pressure to be applied on the water in the capillary tube so that -

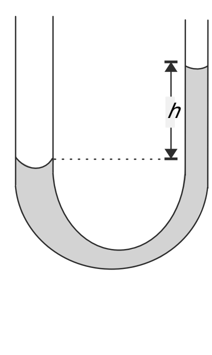
(a) The water level in the tube becomes same as that in the vessel.

(b) Is it possible to blow out an air bubble out of the tube by increasing the pressure?

(ii) A container contains two immiscible liquids of density ρ_1 and ρ_2 ($\rho_2 > \rho_1$). A capillary of radius r is inserted in the liquid so that its bottom reaches up to denser liquid and lighter liquid does not enter into the capillary. Denser liquid rises in capillary and attain height equal to h which is also equal to column length of lighter liquid. Assuming zero contact angle find surface tension of the heavier liquid.

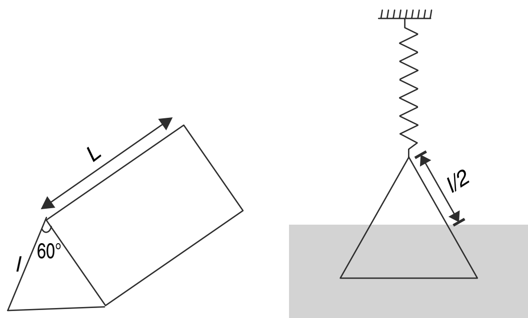


Q.12. The radii of two columns in a U tube are r_1 and r_2 ($r_1 > r_2$). A liquid of density ρ is filled in it. The contact angle of the liquid with the tube wall is θ. If the surface tension of the liquid is T then plot the graph of the level difference (h) of the liquid in the two arms versus contact angle θ. Plot the graph for angle θ changing from 0° to 90°. Assume the curved surface of meniscus to be part of a sphere.

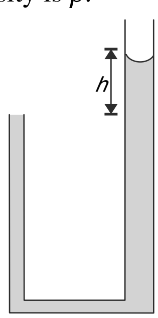


LEVEL 2

Q.13. A glass prism has its principal section in form of an equilateral triangle of side length l. The length of the prism is L (see fig.). The prism, with its base horizontal, is supported by a vertical spring of force constant k. Half the slant surface of the prism is submerged in water. Surface tension of water is T and contact angle between water and glass is 0°. Density of glass is d and that of water is ρ (< d). Calculate the extension in the spring in this position of equilibrium.



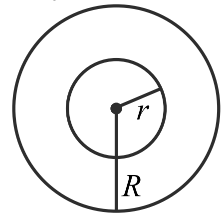
Q.14. Two capillaries of small cross section are connected as shown in the figure. The right tube has cross sectional radius R and left one has a radius of r (< R). The tube of radius R is very long where as the tube of radius r is of short length. Water is slowly poured in the right tube. Contact angle for the tube wall and water is $\theta = 0°$. Let h be the height difference between water surface in the right and left tube. Surface tension of water is T and its density is ρ.



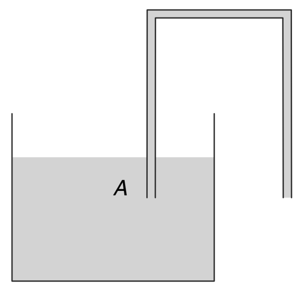
(a) Find the value of h if the water surface in the left tube is found to be flat.

(b) Find the maximum value of h for which water will not flow out of the left tube .

Q.15. A soap bubble of radius r is formed inside another soap bubble of radius R (> r). The atmospheric pressure is P_0 and surface tension of the soap solution is T. Calculate change in radius of the smaller bubble if the outer bubble bursts. Assume that the excess pressure inside a bubble is small compared to P_0.



Q.16. In the siphon shown in the figure the ends A and B of the tube are at same horizontal level. Water fills the entire tube but it does not flow out of the end B. With the help of a diagram show how the water surface at end B changes if the end B were slightly lower than the position shown.

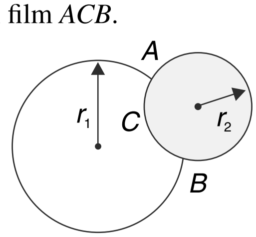


Q.17. A glass capillary tube sealed at the upper end has internal radius r. The tube is held vertical with its lower end touching the surface of water. Calculate the length (L) of such a tube for water in it to rise to a height h (< L). Atmospheric pressure is P_o and surface tension of water is T. Assume that water perfectly wets glass (Density of water = ρ)

Q.18. In the last question let the length of the tube be L and its outer radius be R. Water rises in it to a height h. Calculate the vertical force needed to hold the tube in this position. Mass of empty tube is M.

Q.19. A glass capillary tube is held vertical and put into contact with the surface of water in a tank. It was observed that the liquid rises to the top of the tube before settling to an equilibrium height h_0 in the tube. Assume that water perfectly wets glass and viscosity is small. Is the length of the capillary tube larger then $2h_0$?

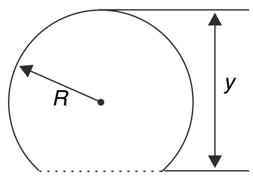
Q.20. Two soap bubbles of radii r_1 and r_2 are attached as shown. Find the radius of curvature of the common film ACB.



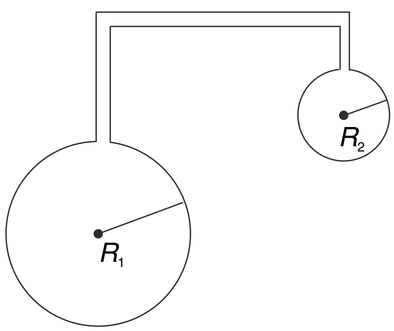
Q.21. (a) In the last question find the angle between the tangents drawn to the bubble surfaces at point *A*.

(b) In the above question assume that $r_1 = r_2 = r$. What is the shape of the common interface *ACB*? Find length *AB* in this case.

(c) With $r_1 = r_2 = r$ the common wall bursts and the two bubbles form a single bubble find the radius of this new bubble. It is given that volume of a truncated sphere of radius *R* and height *y* is $\frac{\pi}{3}y^2(3R - y)$ [see figure]



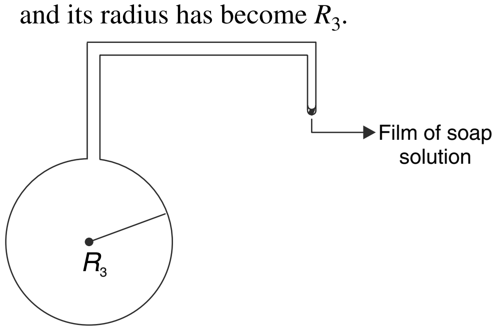
Q.22. Two soap bubbles of radius R_1 and R_2 ($< R_1$) are joined by a straw. Air flows from one bubble to another and a single bubble of radius R_3 remains.



(a) From which bubble does the air flow out ?

(b) Assuming no temperature change and atmospheric pressure to be P_o, find the surface tension of the soap solution.

Q.23. In the last problem, one of the bubbles supplies its entire air to the other bubble and a film of soap solution is formed at the end of the straw which keeps it closed. What is the radius of curvature of this film if the bigger bubble has grown in size and its radius has become R_3.



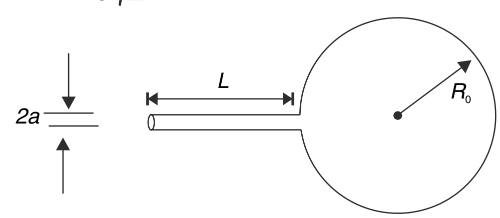
LEVEL 3

Q.24. Consider a rain drop falling at terminal speed. For what radius (R) of the drop can we disregard the influence of gravity on its shape? Surface tension and density of water are T and ρ respectively.

Q.25. A soap bubble has radius R and thickness of its wall is a. Calculate the apparent weight (= true weight - Buoyancy) of the bubble if surface tension of soap solution and its density are T and d respectively. The atmospheric pressure is P_0 and density of atmospheric air is ρ_0. By assuming $a = 10^{-6}\ m$, $R = 10\ cm$, $P_0 = 10^5\ Nm^{-2}$, $\rho_0 = 1.2\ kg\ m^{-3}$, $d = 10^3\ kg\ m^{-3}$, $T = 0.04\ Nm^{-1}$; show that the weight of the bubble is mainly because of water in the skin. What is weight of the bubble?

Q.26. A soap bubble is blown at the end of a capillary tube of radius a and length L. When the other end is left open, the bubble begins to deflate. Write the radius of the bubble as a function of time if the initial radius of the bubble was R_0. Surface tension of soap solution is T. It is known that volume flow rate through a tube of radius a and length L is given by Poiseuille's equation-

$$Q = \frac{\pi a^4 \Delta P}{8\eta L}$$



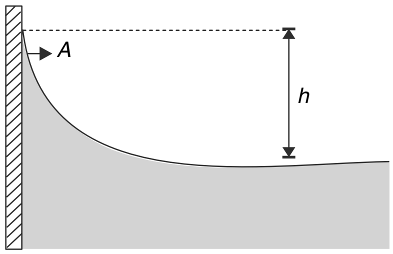
Where ΔP is pressure difference at the two ends of the tube and η is coefficient of viscosity. Assume that the bubble remains spherical.

Q.27. Two blocks are floating in water. When they are brought sufficiently close they are attracted to each other due to surface tension effects. When the experiment is repeated after replacing water with mercury, once again the two blocks are attracted. Explain the phenomena. It is given that water wets the material of the block where as mercury does not.

Q.28. A long thin string has a coat of water on it. The radius of the water cylinder is r. After some time it was found that the string had a series of equally spaced identical water drops on it. Find the minimum distance between two successive drops.

Q.29. A liquid having surface tension T and density ρ is in contact with a vertical solid wall. The liquid surface gets curved as shown in the figure. At the bottom the liquid surface is flat.

The atmospheric pressure is P_o.

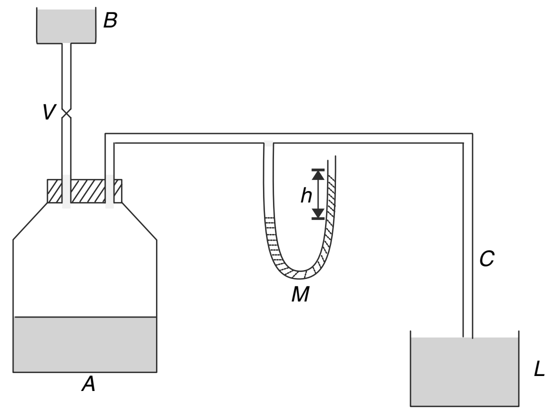


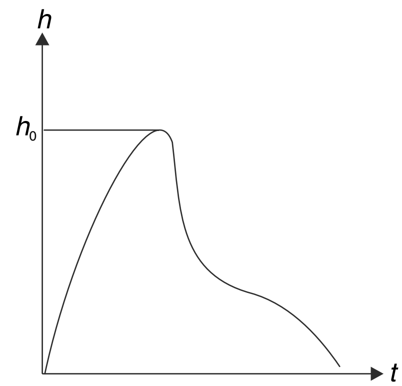
(i) Find the pressure in the liquid at the top of the meniscus (i.e. at A)

(ii) Calculate the difference in height (h) between the bottom and top of the meniscus.

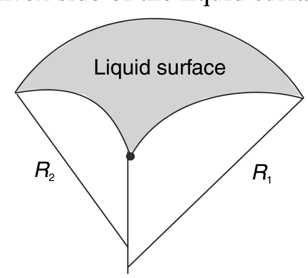
Q.30. Is it possible that water evaporates from a spherical drop of water just by means of surface energy supplying the necessary latent heat of vaporisation? The drop does not use its internal thermal energy and does not receive any heat from outside. It is known that water drops of size less than 10^{-6} m do not exist. Latent heat of vaporisation of water is $L = 2.3 \times 10^6$ Jkg^{-1} and surface tension is $T = 0.07$ Nm^{-1}.

Q.31. In the arrangement shown in the figure, A is a jar half filled with water and half filled with air. It is fitted with a leak proof cork. A tube connects it to a water vessel B. Another narrow tube fitted to A connects it to a narrow tube C via a water monometer M. The tip of the tube C is just touching the surface of a liquid L. Valve V is opened at time $t = 0$ and water from vessel B pours down slowly and uniformly into the jar A. An air bubble develops at the tip of tube C. The cross sectional radius of tube C is r and density of water is ρ. The difference in height of water (h) in the two arms of the manometer varies with time 't' as shown in the graph. Find the surface tension of the liquid L.

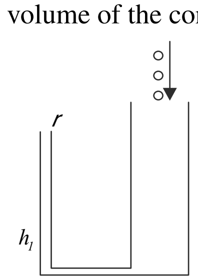




Q.32. A curved liquid surface has radius of curvature R_1 and R_2 in two perpendicular directions as shown in figure. Surface tension of the liquid is T. Find the difference in pressure on the concave side and the convex side of the liquid surface.



Q.33. A capillary tube of radius r and height h_1 is connected to a broad tube of large height as shown in the figure. Water is poured into the broad tube – drop by drop. Drops fall at regular intervals. Plot the variation of height of water in both tubes with time. Initially the tube and capillary are empty. Neglect the volume of the connecting pipe.



ANSWERS

1. $0.076\ Nm^{-1}$

2. $2\sqrt{3}\pi rT + Mg$

3. (i) $A \rightarrow$ hydrophilic , $B \rightarrow$ hydrophobic

(ii) $\cos^{-1}\left(\frac{R-h}{R}\right)$

4. Towards the tapered end.

5. $T = \frac{r}{2\cos\theta}\left[\frac{P_\circ h}{\ell - h} + \rho g h\right]$

6. (a) To capillary of smaller radius

(b) $\frac{2T\sin\theta(r_2 - r_1)}{r_1 r_2}$

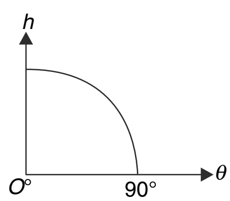
7. $15t_0$

8. $P_\circ + \frac{4T}{R}$

10. $-1.4\times10^{-4}\ \frac{N}{m^\circ C}$

11. (i) (a) 600 P_a (b) Yes. (ii) $T = \frac{r}{2}(\rho_2 - \rho_1)gh$

12.



13. $x = \frac{1}{K}\left[\frac{\sqrt{3}}{4}l^2 L\,d.g - \frac{3\sqrt{3}}{16}l^2 L\rho g + \sqrt{3}TL + Tl\right]$

14. (a) $h = \frac{2T}{R\rho g}$ (b) $\frac{2T}{\rho g}\left(\frac{r+R}{rR}\right)$

15. $\Delta r = \frac{4Tr}{3P_0 R}$

16. The radius of curvature decreases

17. $L = \frac{P_\circ hr}{2T - \rho grh} + h$

18. $Mg + \pi P_\circ\left[R^2 - \frac{Lr^2}{L-h}\right] + 2\pi(R+r)T$

19. No, $l < 2h_0$

20. $\frac{r_1 r_2}{r_1 - r_2}$

21. (a) 120° (b) $\sqrt{3}r$ (c) $\frac{3r}{2(2)^{1/3}}$

22. (a) From smaller bubble (b) $T = \frac{P_\circ\left(R_3^3 - R_1^3 - R_2^3\right)}{4\left(R_1^2 + R_2^2 - R_3^2\right)}$

23. R_3

24. $R << \sqrt{\frac{T}{\rho g}}$

25. $\frac{16\pi}{3}\frac{R^2\rho_\circ}{P_\circ}g + 4\pi R^2 a.d.g$

26. $R = R_0\left[1 - \frac{a^4 Tt}{2\eta L R_0^4}\right]^{\frac{1}{4}}$

28. $\frac{9}{2}r$

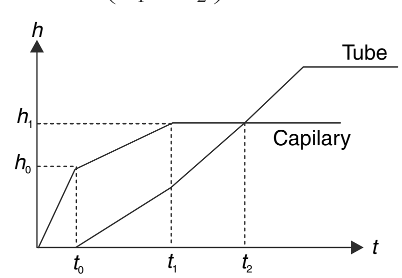
29. (i) $P_o - \rho gh$ (ii) $\sqrt{\frac{2T}{\rho g}}$

30. No

31. $\frac{\rho g h_0 r}{2}$

32. $\Delta P = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

33.



SOLUTIONS

1. When the ring is about to leave the water surface, surface tension force on it is

$F_{ST} = 2\pi RT + 2\pi rT = 2\pi(R + r)T$

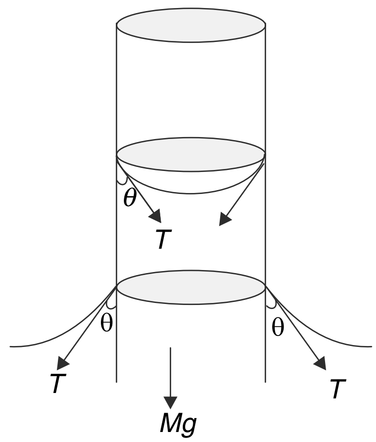
Spring force $F_S = kx$

$\therefore\ kx = 2\pi(R + r)T + mg$

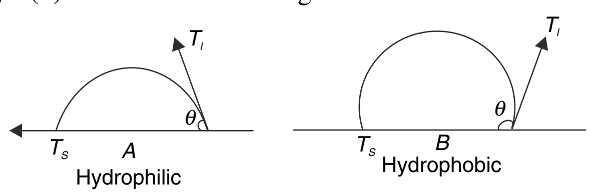
$$\therefore T = \frac{kx - mg}{2\pi(R+r)} = \frac{0.7\times3.4\times10^{-2} - 7\times10^{-4}\times9.8}{2\times3.14\times(30+10)\times10^{-3}} = 0.076\ Nm^{-1}$$

2. Force = Mg + force due to surface tension on the inner wall + force due to surface tension on the outer wall

$= Mg + 2\pi rT\cos\theta + 2\pi rT\cos\theta = Mg + 2\sqrt{3}\pi rT$



3. (i) The correct contact angle (θ) has been shown in figure

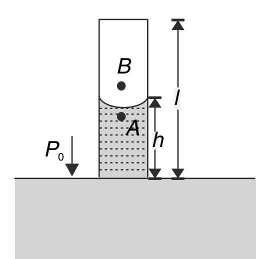


If θ is acute the surface is hydrophilic (i.e. water wets the surface) and if θ is obtuse the surface is hydrophobic.

(ii) Draw a tangent on the container wall at point of contact. Angle between this tangent and the liquid surface is the contact angle.

4. Pressure difference on two sides of a curved surface is inversely proportional to the radius of curvature.

5. $P_A = P_\circ - \rho gh$



Pressure at B is higher than at A by $\frac{2T}{r}\cos\theta$

$$\therefore P_B = P_A + \frac{2T\cos\theta}{r} = P_\circ - \rho gh + \frac{2T\cos\theta}{r}$$

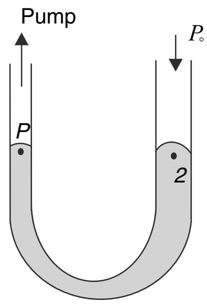
$P_1V_1 = P_2V_2$ for air inside the tube

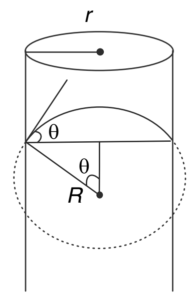
$$P_\circ A\ell = \left[P_\circ - \rho gh + \frac{2T\cos\theta}{r}\right]A(\ell - h)$$

$$\Rightarrow \frac{2T\cos\theta}{r} = \frac{P_\circ\ell}{\ell - h} - P_\circ + \rho gh = \frac{P_\circ h}{\ell - h} + \rho gh$$

$$\Delta P = T\left(\frac{1}{R_1}+\frac{1}{R2}\right) \Rightarrow T = \frac{r}{2\cos\theta}\left[\frac{P_\circ h}{\ell - h}+\rho g h\right]$$

6. Relation between radius of curvature (R) of the Hg surface and tube radius (r) is $\frac{r}{R} = \sin\theta$





$P_1 = P_2$

$$P + \frac{2T\sin\theta}{r_1} = P_\circ + \frac{2T\sin\theta}{r_2}; \quad \therefore\ P_\circ - P = 2T\sin\theta\left(\frac{1}{r_1}-\frac{1}{r_2}\right) = 2T\sin\theta\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

7. One can easily show that dimension of the factor k are: $[k] = \left[M^\circ L^1 T^{-\frac{1}{2}}\right]$

If $x = kt^a$, then $a = \frac{1}{2}$

$\therefore\ x \propto t^{\frac{1}{2}}$

Time needed for x to go from zero to x_0 is t_0

Time for x to grow from zero to $4x_0$ will be $16t_0$

$\therefore$ required answer is $15t_0$

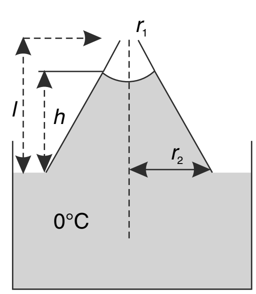
9. When water sprays from a tap in a small container, one can see bubbles, but they burst very soon. This is because surface tension of water is high and it tends to draw water molecules into the bulk, to the point where thickness of bubble wall is too thin to remain intact. On the other hand, the surface tension of soap water is much lower. The molecules of the bubble are less stressed and can last longer.

10. The situation is shown in figure.

Let r_1 and r_2 be radii of upper and lower ends of the conical capillary tube. The radius r at the meniscus is given by

$$r = r_1 + (r_2 - r_1)\left(\frac{l-h}{l}\right)$$

$$= (2.5\times10^{-4}) + (2.5\times10^{-4})\left(\frac{0.1-0.08}{0.1}\right) = 3.0\times10^{-4}\ m$$



The surface tension at 0°C is given by

$$T_0 = \frac{rh\rho g}{2} = \frac{(3.0\times10^{-4})(8\times10^{-2})(1/4\times10^{4})(9.8)}{2} = 0.084$$

For tube B; $\frac{T_0}{T_{50}} = \frac{h_0}{h_{50}} = \frac{6\times10^{-2}}{5.5\times10^{-2}} = \frac{12}{11}$

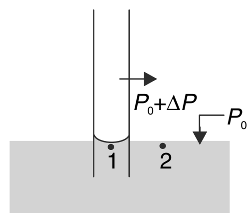
Or, $T_{50} = \frac{11}{12}\times T_0 = \frac{11}{12}\times 0.084 = 0.077 \quad N/m$

Considering that change in surface tension as linear, the change in surface tension with temperature is given by

$$\alpha = \frac{T_{50}-T_0}{50} = \frac{0.077-0.084}{50} = -1.4\times10^{-4}\ \frac{N}{m^\circ C}$$

11. (i) (a) $P_1 = P_2$

$$P_\circ + \Delta P - \frac{2T}{R} = P_\circ$$



[R = radius of curvature of the curved surface. Pressure on convex side is lesser by $\frac{2T}{R}$ than pressure on concave side]

For $\theta = 0°$; $R = r$

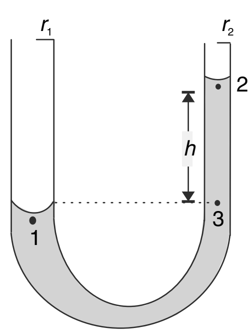
$$\Delta P = \frac{2T}{r} = \frac{2\times7.5\times10^{-2}}{2.5\times10^{-4}} = 600\ Pa$$

(ii) $P_0 + \rho_1 gh - \rho_2 gh + \frac{2T}{r} = P_0$

$$T = \frac{r}{2}(\rho_2 - \rho_1)gh$$

12. $P_1 = P_3$

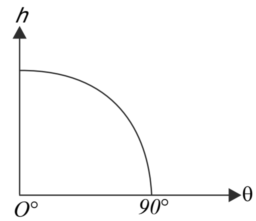
$$P_\circ - \frac{2T}{R_1} = P_2 + \rho gh \quad \Rightarrow \quad P_\circ - \frac{2T}{R_1} = P_\circ - \frac{2T}{R_2} + \rho gh$$



R_1 & R_2 are radii of curvature of the meniscus and it is known that $R = \frac{r}{\cos\theta}$

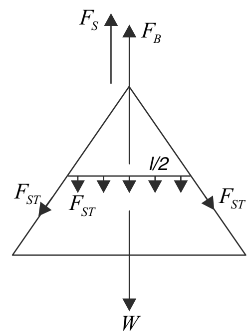
$$\rho gh = 2T\left[\frac{1}{R_2} - \frac{1}{R_1}\right] \quad \Longrightarrow \quad h = \frac{2T\cos\theta}{\rho g}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$h = \frac{2T}{\rho g}\left(\frac{r_1 - r_2}{r_1 r_2}\right)\cos\theta \qquad \therefore\ h \propto \cos\theta$$



13. Volume of prism , $V = \frac{\sqrt{3}}{4} l^2 L$

Volume of submerged part, $V' = \frac{\sqrt{3}}{4} l^2 L - \frac{\sqrt{3}}{4}\left(\frac{l}{2}\right)^2 L \ = \frac{3\sqrt{3}}{16} l^2 L$



For equilibrium -

Spring force (F_S) + Buoyancy (F_B) = weight (W) + surface tension force (F_{ST})

$$kx + \frac{3\sqrt{3}}{16} l^2 L \rho g = \frac{\sqrt{3}}{4} l^2 L\, dg + 2.TL\cos 30° + 2T\frac{l}{2}$$

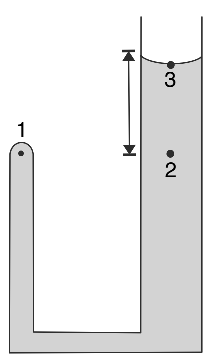
$$x = \frac{1}{K}\left[\frac{\sqrt{3}}{4} l^2 L\, d.g - \frac{3\sqrt{3}}{16} l^2\ L\rho g + \sqrt{3}TL + Tl\right]$$

14. When water on two sides has same level (with left tube completely filled) the radius of curvature of the water surface on two sides will be same - equal to *R*. As more water is added, the surface in the left tube gets flatter, then becomes completely flat and then becomes convex up.

Figure shows change in curvature of the left surface as height of water in the right tube increases.

In extreme case left surface is hemispherical (radius = r) with convex side up. After this, water starts flowing out of the tube.

(a) The meniscus in the right tube is hemispherical (because $\theta = 0°$) . Radius of curvature of the surface is = R



When surface on left is flat, pressure at point 1 (just below the surface) is atmospheric pressure (P_o)

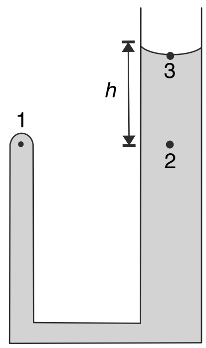
$P_1 = P_2$

$$P_\circ = P_\circ - \frac{2T}{R} + \rho g h \quad \Rightarrow h = \frac{2T}{R\rho g}$$

(b) In extreme case, when water does not come out of left tube, the surface gets convex with radius of curvature r

$P_1 = P_2$

$$P_\circ + \frac{2T}{r} = P_\circ - \frac{2T}{R} + \rho g h \qquad \Rightarrow \frac{2T}{\rho g}\left[\frac{r+R}{rR}\right] = h$$



15. P_0 = atmospheric pressure

$P_0 + \frac{4T}{R} =$ pressure inside the larger bubble.

$P_0 + \frac{4T}{R} + \frac{4T}{r} =$ pressure inside the smaller bubble.

After the larger bubble bursts, the new pressure inside the smaller bubble is $= P_0 + \frac{4T}{r}$

Using $\quad P_2V_2 = P_1V_1$

$$\left(P_0 + \frac{4T}{r}\right)V_2 = \left(P_0 + \frac{4T}{R} + \frac{4T}{r}\right)\frac{4}{3}\pi r^3$$

$$\frac{4}{3}\pi r_1^3 = \left(1 + \frac{\frac{4T}{R}}{P_0 + \frac{4T}{r}}\right)\frac{4}{3}\pi r^3$$

Where r_1 is new radius of the bubble.

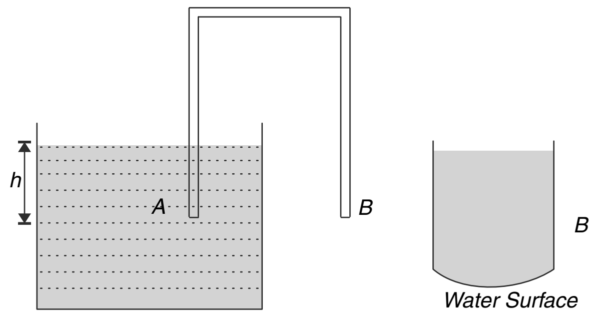
$$r_1 \simeq r\left(1 + \frac{4T}{RP_0}\right)^{1/3} \qquad \left[\because\ P_0 + \frac{4T}{r} \simeq P_0\right]$$

$$\simeq r\left[1 + \frac{4T}{3RP_0}\right] \qquad \left[\because\ \frac{4T}{RP_0} << 1\right]$$

$$\therefore \qquad r_1 - r = \frac{4Tr}{3P_0 r} \qquad \Rightarrow \Delta r = \frac{4Tr}{3P_0 R}$$

16. Pressure at end B inside the tube is

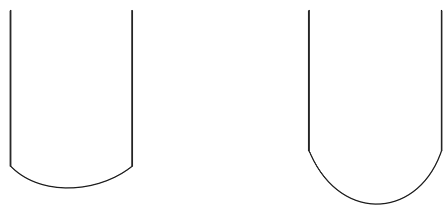
$P_B = P_o + \rho g h \qquad [P_o$ = atmospheric pressure]



$\therefore P_B - P_o = \rho gh$

$\therefore \frac{2T}{r} = \rho gh$ [r = radius of curvature of the surface]

With increase in h, the curvature of water surface increases (i.e. r decreases) so as to prevent water from flowing out.

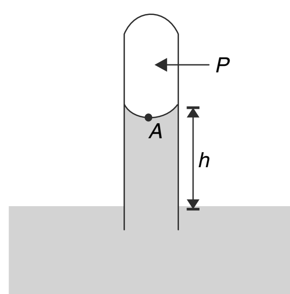


The figure shows the change in surface as B is lowered.

17. When the tube is brought into contact with water, it is filled with air at atmospheric pressure. When water rises to a height h, the air pressure (P) is given by

$PA(L - h) = P_oAL$

$\therefore P = \frac{P_o L}{L - h}$



Radius of curvature of meniscus $R = r$, since contact angle is zero.

Pressure at A is $P_A = P_o - \rho gh$

$\therefore P = P_A + \frac{2T}{r}$

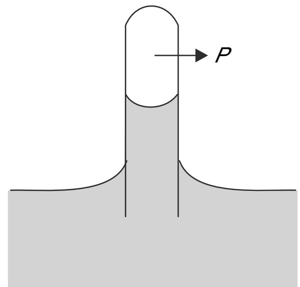
$\therefore \frac{P_o L}{L - h} = P_o - \rho gh + \frac{2T}{r} \quad \Rightarrow P_o\left(\frac{L}{L-h} - 1\right) = \frac{2T}{r} - \rho gh$

$\Rightarrow \frac{P_o h}{L - h} = \frac{2T}{r} - \rho gh \quad \Rightarrow L - h = \frac{P_o h}{\frac{2T}{r} - \rho gh}$

$\Rightarrow L = h + \frac{P_o r h}{2T - \rho ghr}$

18. Surface tension force on outer wall of the tube is $2\pi RT\ (\downarrow)$

Surface tension force (at meniscus) on the inner tube wall is $2\pi rT\ (\downarrow)$



Force due to air pressure inside the tube $\pi r^2 P(\uparrow)$

Force due to atmospheric pressure = $\pi R^2 P_o(\downarrow)$

For equilibrium of the tube, let the upward force needed be F

$$F + P\pi r^2 = Mg + \pi R^2 P_o + 2\pi RT + 2\pi rT$$

$$F = Mg + \pi R^2 P_\circ - \frac{P_\circ L\pi r^2}{L-h} + 2\pi(R+r)T$$

$$F = Mg + \pi P_\circ\left[R^2 - \frac{Lr^2}{L-h}\right] + 2\pi(R+r)T$$

19. It is known that $h_0 = \dfrac{2T}{\rho g r}$ [Here $\cos\theta = 1$]

If the tube has large length and viscosity is not there, the upward pulling surface tension force can cause the water to rise to a maximum height h. Where

$$F\,h = \pi r^2 h\,\rho . g\,\frac{h}{2}$$

Where left side is work done by surface tension force and right side is the rise in potential energy of the water in the glass tube

$$(2\pi rT\cos\theta)h = \frac{1}{2}\pi r^2 \rho g h^2$$

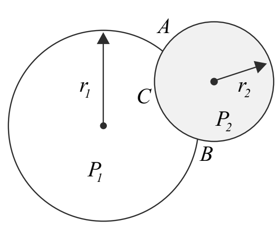
$\cos\theta = 1 \quad \therefore \dfrac{4T}{\rho g r} = h$

$\therefore h = 2h_0$

In presence of viscosity $h < 2h_0$. If water touches the brim of the tube, it means length of tube must be less than $2h_0$

20. Let the radius of curvature of surface ACB be R.

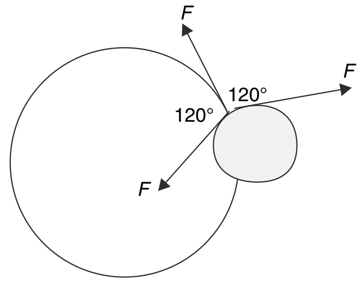
$$P_2 - P_1 = \frac{4T}{R}$$



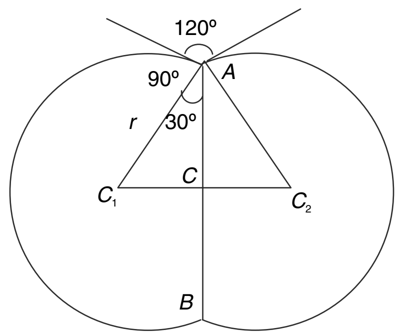
[$\dfrac{4T}{R}$ because there are two surfaces in the wall ACB]

$$\therefore \frac{4T}{r_2} - \frac{4T}{r_1} = \frac{4T}{R} \qquad \Rightarrow R = \frac{r_1 r_2}{r_1 - r_2}$$

21. (a) Consider a particle on the common meeting line of the three surface. It experience three forces of same magnitude (surface tension is same for all three films) and is in equilibrium. This is possible only if the three forces are at 120° to each other



(b) When $r_1 = r_2 = r$ the pressure on two sides of the common wall is same. The common wall remains flat in shape of a disc. AB is diameter of the disc.



$AC = r\cos 30° = \frac{\sqrt{3}r}{2}$ $\quad \Rightarrow AB = 2AC = \sqrt{3}r$

(c) In above figure $CC_1 = r\sin 30° = \frac{r}{2}$

$\therefore\ y = r + \frac{r}{2} = \frac{3r}{2}$

$\therefore$ volume of each bubble $= \frac{\pi}{3}\left(\frac{3r}{2}\right)^2\left(3r - \frac{3r}{2}\right) = \frac{9\pi r^3}{8}$

If radius of new bubble = R, then $\frac{4}{3}\pi r^3 = 2 \times \frac{9\pi r^3}{8}$

$R = \frac{3r}{2(2)^{1/3}}$

22. (a) As excess pressure for a soap bubble is $(4T / r)$ and external pressure p_0,

$p_i = p_0 + (4T / r)$

Pressure inside the smaller bubble is higher, Hence, air flows from smaller bubble to the larger one.

(b) The larger bubble grows in size till the entire air of smaller bubble is transferred into it.

$p_1 = \left[p_0 + \frac{4T}{R_1}\right], \quad p_2 = \left[p_0 + \frac{4T}{R_2}\right]$ and $p_3 = \left[p_0 + \frac{4T}{R_3}\right]$ ...(i)

and $V_1 = \frac{4}{3}\pi R_1^3, \quad V_2 = \frac{4}{3}\pi R_2^3$ and $V_3 = \frac{4}{3}\pi R_3^3$ ...(ii)

Now, as mass is conserved, $\quad n_1 + n_2 = n_3$

i.e., $\frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2} = \frac{P_3V_3}{RT_3}$ $\quad \left[\text{as } PV = nRT, \text{ i.e., } n = \frac{PV}{RT}\right]$

At temperature is constant, i.e., $T_1 = T_2 = T_3$, the above expression reduces to

$$P_1V_1 + P_2V_2 = P_3V_3$$

Which, in the light of Eqn. (i) and (ii) becomes

$$\left[P_0 + \frac{4T}{R_1}\right]\left[\frac{4}{3}\pi R_1^3\right] + \left[P_0 + \frac{4T}{R_2}\right]\left[\frac{4}{3}\pi R_2^3\right] = \left[P_0 + \frac{4T}{R_3}\right]\left[\frac{4}{3}\pi R_3^3\right]$$

i.e., $4T(R_1^2 + R_2^2 - R_3^2) = P_0(R_3^3 - R_1^3 - R_2^3)$

i.e., $T = \dfrac{P_0(R_3^3 - R_1^3 - R_2^3)}{4(R_1^2 + R_2^2 - R_3^2)}$.

23. Hint: the pressure must be same everywhere inside the straw and the bubble.

24. Since the drag force balances the gravity, the necessary condition must be that the variation in hydrostatic pressure inside the drop should be negligible compared to the excess pressure due to surface tension

$$\rho g 2R << \frac{2T}{R} \qquad \Rightarrow R << \sqrt{\frac{T}{\rho g}}$$

25. Excess pressure inside the soap bubble $\Delta P = \dfrac{4T}{R}$

From ideal gas equation, pressure of a gas (at a given temperature) is directly proportional to its density

$P \propto \rho$

$\therefore \dfrac{\Delta P}{P_\circ} = \dfrac{\Delta\rho}{\rho_\circ}$ where P_o and ρ_o are pressure and density of atmospheric air.

$\Rightarrow \Delta\rho = \dfrac{\rho_\circ}{P_\circ}\Delta P$

Apparent weight = True weight – Buoyancy

= weight of air inside Bubble – Buoyancy + weight of water skin

$$= \frac{4}{3}\pi R^3\left(\rho_\circ + \Delta\rho\right)g - \frac{4}{3}\pi R^3\rho_\circ g + 4\pi R^2 a\, d.g$$

$$= \frac{4}{3}\pi R^3\Delta\rho g + 4\pi R^2 a\, d.g$$

$$= \frac{4}{3}\pi R^3\,\frac{\rho_\circ}{P_\circ}\,\frac{4T}{R}\,g + 4\pi R^2.a.d.g$$

$$= \frac{4}{3}\times 3.14\times(0.1)^2\times\frac{1.2}{10^5}\times 4\times .04\times 10 + 4\times 3.14\times(0.1)^2\times 10^{-6}\times 10^3\times 10$$

$= 0.8 \times 10^{-6} + 1.3 \times 10^{-3} \simeq 1.3 \times 10^{-3}\, N$

26. $\Delta P = \dfrac{4T}{R} \quad \Rightarrow \quad Q = \dfrac{\pi a^4 \Delta P}{8\eta L}$

$\Rightarrow \dfrac{dV}{dt} = \dfrac{\pi a^4 T}{2\eta LR}$; But $V = \dfrac{4}{3}\pi R^3$

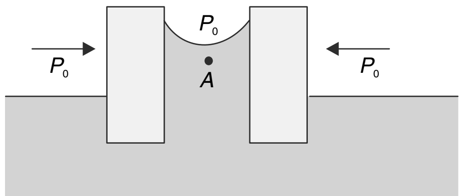
$$\frac{dV}{dt} = \frac{dV}{dR}\frac{dR}{dt} = 4\pi R^2\frac{dR}{dt}$$

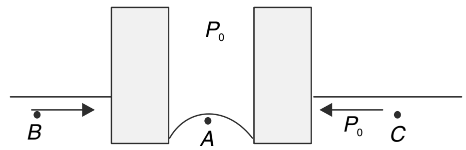
$$\therefore\ -4\pi R^2\frac{dR}{dt} = \frac{\pi a^4 T}{2\eta LR}$$

$$-4\int_{R_0}^{R} R^3 dR = \frac{a^4 T}{2\eta L}\int_0^t dt \qquad \Rightarrow R^4 - R_0^4 = \frac{a^4 T}{2\eta L}t$$

$$\therefore\ R = \left(R_0^4 - \frac{a^4 T}{2\eta\, L}\ t\right)^{\frac{1}{4}} = R_0\left(1 - \frac{a^4 Tt}{2\eta\, LR_0^4}\right)^{\frac{1}{4}}$$

27. Water rises between the blocks due to capillary effect. Pressure below the curved water surface (like at a point A shown in figure) is below the atmospheric pressure.





But outer walls of the blocks experience atmospheric pressure. This causes the blocks to be pushed towards each other. In case of Hg, the situation is as shown in the figure. At points like B and C the pressure is higher than atmospheric pressure which pushed the blocks closer.

28. Consider a length L of the string. Total surface energy of water on it is $E = 2\pi rLT$

If spherical drops are of radius R and separation between two successive drops is l then conservation of volume gives

$$\frac{4}{3}\pi R^3 \frac{L}{l} = \pi r^2 L \qquad \Rightarrow \frac{4}{3}R^3 = r^2 l \qquad \text{.................(i)}$$

The total surface energy of liquid drops over a length L is

$$E' = 4\pi R^2.T.\frac{L}{l} = 4\pi\left(\frac{3r^2 l}{4}\right)^{2/3}.T\frac{L}{l} \qquad \text{[using (i)]}$$

$$\therefore\ E' = 4\pi\left(\frac{3r^2}{4}\right)^{2/3}\frac{TL}{l^{1/3}}$$

The final surface energy must be less than the original surface energy

$$\therefore\ E' < E$$

$$4\pi\left(\frac{3r^2}{4}\right)^{2/3}\frac{TL}{l^{1/3}} < 2\pi rLT$$

$$2\left(\frac{3r^2}{4}\right)^{2/3} < r\,l^{1/3} \qquad \Rightarrow \qquad 2\left(\frac{3}{4}\right)^{2/3} r^{1/3} < l^{1/3}$$

$$8\times\frac{9}{16}\times r < l \quad \Rightarrow \quad \frac{9}{2}r < l$$

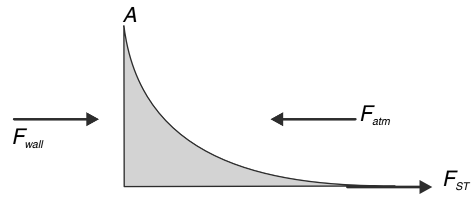
29. (i) Pressure at the lowest level of meniscus is $= P_o$ [$\because$ liquid surface is flat]

$\therefore$ Pressure in liquid at A is $P_A = P_o - \rho gh$

(ii) We will consider the horizontal equilibrium of the liquid in the meniscus. We will consider a depth L perpendicular to the plane of the figure.

Force due to wall $F_{wall} = P_{av}\,hL$

$$= \left(\frac{P_\circ + P_\circ - \rho gh}{2}\right)hL = \left(P_\circ - \frac{\rho gh}{2}\right)hL$$



Force due to atmosphere $F_{atm} = P_o hL$

Force due to surface tension $F_{ST} = T.L$

$\therefore F_{wall} + F_{ST} = F_{atm}$

$$\left(P_\circ - \frac{\rho gh}{2}\right)hL + T.L = P_\circ hL$$

$$\Rightarrow P_\circ h - \frac{\rho gh^2}{2} + T = P_\circ h \qquad \therefore\ h = \sqrt{\frac{2T}{\rho g}}$$

30. Consider a water drop of radius R

Surface area $S = 4\pi R^2$

If radius decreases by ΔR, the surface area changes by $\Delta S = -8\pi R\Delta R$

$\therefore$ Loss in surface energy $\Delta E = 8\pi R\Delta R.T$

The volume of the drop $V = \frac{4}{3}\pi R^3$

$\Delta V = 4\pi R^2 \Delta R$

$\therefore$ Energy required for evaporation of a water layer ΔR thick is

$\Delta E' = \rho\Delta VL = 4\pi R^2\Delta R.\ \rho.L$

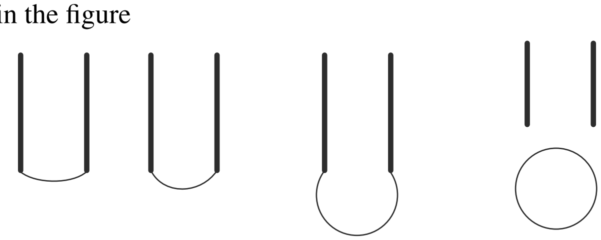
Loss in surface energy can cause water to evaporate if $\Delta E > \Delta E'$

$$8\pi R\ \Delta R\mathrm{T} > 4\pi R^2\ \Delta R\rho L \quad \Rightarrow R < \frac{2T}{\rho L} = \frac{2\times 0.07}{10^3 \times 2.3 \times 10^6}$$

$6 \times 10^{-11}\ m$

Drops of this size do not exist. In fact the above number is close to size of one molecule of water.

31 When water level rises in A, the air pressure rises. This blows a bubble at the tip of C. The monometer reading gives the pressure difference of the air inside the container and the atmospheric pressure (P_o). The various stages of bubble has been shown in the figure

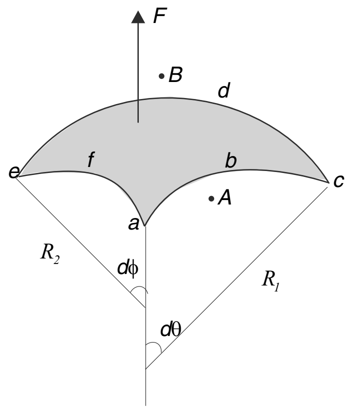


The inside pressure is maximum when radius of the bubble is equal to the radius of the capillary (i.e. when the bubble is hemispherical)

$P_{max} - P_o = \rho gh_o$

$$\Rightarrow \frac{2T}{r} = \rho gh_0 \quad \Rightarrow T = \frac{\rho gh_0 .r}{2}$$

32. Consider a small patch on the liquid surface. Angle subtended by arc abc at the centre of curvature is $d\theta$ and the angle subtended by arc $a\,f\,e$ at its centre of curvature is $d\phi$.



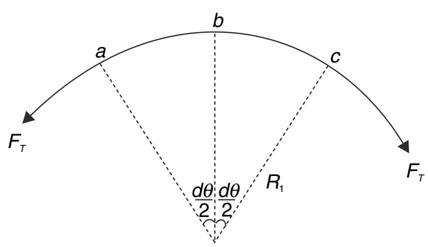
Area of the patch

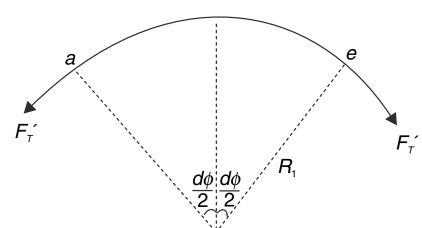
$dS = (R_1 d\theta)(R_2 d\phi)$ ....(i)

Pressure on concave side (at point A) is P_A and pressure on convex side (at point B) is P_B. Net outward force on the patch due to pressure difference is $F = (P_A - P_B)\, dS$ .... (ii)

This force is balanced by surface tension force. Look at the two figures shown. F_T and F_T' are surface tension force on arcs of length $R_2 d\phi$ and $R_1 d\theta$ respectively.

∴ Net inward force on the patch of liquid surface is





$$= 2F_T \sin\frac{d\theta}{2} + 2F'_T \sin\frac{d\phi}{2} = F_T d\theta + F'_T d\phi \quad [\because \sin d\theta \simeq d\theta]$$

$$= T.R_2 d\phi.d\theta + TR_1 d\theta.d\phi$$

$$= T(R_1 + R_2)d\phi.d\theta = T\left(\frac{R_1 + R_2}{R_1 R_2}\right)dS \quad \text{[using (i)]}$$

This force balances the force given by equation (ii)

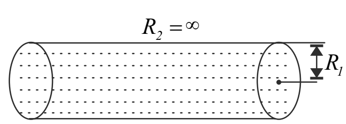
$$(P_A - P_B)\,dS = T\left(\frac{R_1 + R_2}{R_1 R_2}\right)dS \quad \Rightarrow \quad P_A - P_B = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Note: If the surface is spherical $R_1 = R_2 = R$ (say)

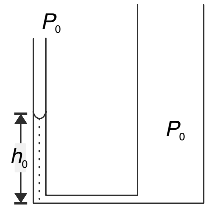
$$\therefore P_A - P_B = \frac{2T}{R}$$

If the liquid surface is cylindrical

$$R_1 = R;\ R_2 = \infty \quad \therefore P_A - P_B = \frac{T}{R}$$



33. First the water level rises in the capillary till the time the pressure at the bottom of the capillary becomes equal to atmospheric pressure.



$$P_O - \frac{2T\cos\theta}{r} + \rho g h_0 = P_O \quad \Rightarrow \quad h_0 = \frac{2T\cos\theta}{r\rho g}$$

After this the height difference between levels of water in the capillary and the tube remains constant at h_0. It means height in both of them increases by same amount when some water is poured. At time t_1 water touches the brim of the capillary. When more water is poured the radius of curvature of the water surface in the meniscus of the capillary changes (surface gets flatter) and the water level in the wide tube continues to rise. Note that the level of water in the broad tube will now rise at a slightly faster pace as quantity of water is not increasing in the capillary. The time t_2 when water level becomes same in both tubes the meniscus surface become flat (as in the broad tube). Thereafter, as the height of water increases in broad tube, the meniscus gets convex. After time t_2 water begins to flow out of the capillary and water level in both tubes becomes constant.