Trigonometry

Angle

The angle is defined as the amount of revolution that the revolving line makes with its initial position. From θ is positive if it is traced by revolving line in anticlockwise direction and is negative if it is covered in clockwise direction.

$1° = 60'$ (minute), $1'$ (min) $= 60''$ (sec)

1 right angle $= 90°$ (degrees) also 1 right angle $= \frac{\pi}{2}$ rad (radian)

One radian is the angle subtended at the centre of a circle, whose length is equal to the radius of the circle.

$$1 \text{ rad} = \frac{180°}{\pi} \approx 57°\ 17'\ 45'' \approx 57.3°$$

$$\pi = \left(\frac{22}{7}\right)$$

Trigonometric ratios (or T ratios)

Let two fixed lines XOX' and YOY' intersecting at right angles to each other at point O. Then

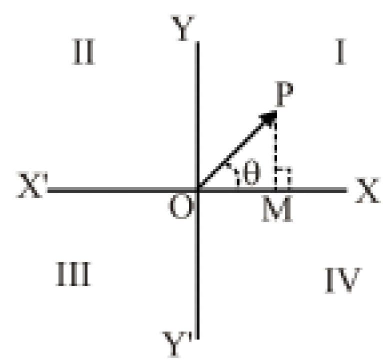
(i) Point O is called origin.

(ii) XOX' known as X-axis and YOY' are Y-axis.

(iii) Portions XOY, YOX', Y'OX' and XOY' are called I, II, III and IV quadrant respectively.

Consider that the revolving line OP has traced out angle θ.

(in I quadrant) in anticlockwise direction. From P, perpendicular PM on OX. Then, side OP (in front of right angle) is called hypotenuse, side MP (in front of angle θ) is called opposite side or perpendicular and side OM (making angle θ with hypotenuse) is called adjacent side or base.



The three sides of a right angled triangle are connected to each other through six different ratios, called trigonometric ratios or simply T-ratios.

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{MP}{OP} \qquad \cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{OM}{OP}$$

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{MP}{OM} \qquad \cot\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{OM}{MP}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{OP}{OM} \qquad \operatorname{cosec}\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{OP}{MP}$$

It can be easily proved that

$$\text{cosec}\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta + \sec^2\theta \qquad 1 + \cot^2\theta = \text{cosec}^2\theta$$

Illustration :

Given $\sin\theta = \frac{3}{5}$. *Find all the other T-ratios, if θ lies in the first quadrant.*

Sol. *In* ΔOMP, $\sin\theta = \frac{3}{5}$ *So* $MP = 3$ *and* $OP = 5$

$$\because OM = \sqrt{(5)^2 - (3)^2} = \sqrt{25-9} = \sqrt{16} = 4$$

Now, $\cos\theta = \frac{OM}{OP} = \frac{4}{5}$ $\qquad \tan\theta = \frac{MP}{OM} = \frac{3}{4}$ $\qquad \cot\theta = \frac{OM}{MP} = \frac{4}{3}$

$\sec\theta = \frac{OP}{OM} = \frac{5}{4}$ $\qquad \text{cosec}\,\theta = \frac{OP}{MP} = \frac{5}{3}$

The T-ratios of a few standard angles ranging from 0° to 180°

Angle (θ)	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	–1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	–1	$-\frac{1}{\sqrt{3}}$	0

$$\sin(90°-\theta) = \cos\theta \quad \cos(90°-\theta) = \sin\theta \quad \tan(90°-\theta) = \cot\theta$$

$$\sin(90°+\theta) = \cos\theta \quad \cos(90°+\theta) = -\sin\theta \quad \tan(90°+\theta) = -\cot\theta$$

$$\sin(-\theta) = -\sin\theta \quad \cos(-\theta) = \cos\theta \quad \tan(-\theta) = -\tan\theta$$

$$\sin(180°-\theta) = \sin\theta \quad \cos(180°-\theta) = \cos\theta \quad \tan(180°-\theta) = \tan\theta$$

$$\sin(180°+\theta) = -\sin\theta \quad \cos(180°+\theta) = -\cos\theta \quad \tan(180°+\theta) = \tan\theta$$

$$\sin(270°-\theta) = -\cos\theta \quad \cos(270°-\theta) = -\sin\theta \quad \tan(270°-\theta) = \cot\theta$$

$$\sin(270°+\theta) = -\cos\theta \quad \cos(270°+\theta) = \sin\theta \quad \tan(270°+\theta) = -\cot\theta$$

$$\sin(360°-\theta) = -\sin\theta \quad \cos(360°-\theta) = \cos\theta \quad \tan(360°-\theta) = -\tan\theta$$

Illustration :

Find the value of

(i) $\cos(-60°)$ (ii) $\tan 210°$ (iii) $\sin 300°$ (iv) $\cos 120°$

Sol. (i) $\cos(-60°) = \cos 60° = \frac{1}{2}$ (ii) $\tan 210° = (\tan 180° + 30°) = \tan 30° = \frac{1}{\sqrt{3}}$

(iii) $\sin 300° = \sin(270° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

(iv) $\cos 120° = \sin(180° - 60°) = -\cos 60° = -\frac{1}{2}$

A few important trigonometric formulae

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$	$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$
$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$	$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
$\sin 2A = 2 \sin A \cos A$	$\cos 2A = \cos^2 A - \sin^2 A$
$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$	$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

Small angle approximation :

θ is very small and it must be in radian when you are taking approximation.

$\sin\theta \simeq \theta$, $\tan\theta \simeq \theta$

$\sin\theta \simeq \tan\theta$.

$\cos\theta \simeq 1$

Illustration :

Evaluate $\sin 2°$

Sol. $2° = 2 \times \frac{\pi}{180}\text{ rad} = \frac{\pi}{90}\text{ rad}$

Now

$$\sin 2° = \sin\left(\frac{\pi}{90}\text{rad}\right) \simeq \left(\frac{\pi}{90}\right)$$

Illustration :

Evaluate $\sin 2° (1 - \cos 2°)$

Sol. $\sin 2° (1 - 1 + 2 \sin^2 1°)$

$$2 \sin 2° \sin^2 1° \simeq 2\left(2 \times \frac{\pi}{180°}\right)\left(\frac{\pi}{180°}\right)^2$$

$$= 4\left(\frac{\pi}{180°}\right)^3$$

Practice Exercise

Q.1 Find the value of

(i) $\cos(-30°)$ (ii) $\sin 120°$ (iii) $\sin 135°$ (iv) $\cos 120°$ (v) $\sin 270°$ (vi) $\cos 270°$

Ans. (i) $\frac{1}{\sqrt{3}}$ (ii) $\frac{\sqrt{3}}{2}$ (iii) $\frac{1}{\sqrt{2}}$ (iv) $-\frac{\sqrt{3}}{2}$ (v) -1 (vi) 0