Chapter 17 Electrostatic Potential and Capacitance

ELECTROSTATIC POTENTIAL

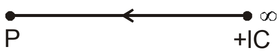
***Electric potential** at a point in an electric field is defined as the amount of work done in bringing a unit positive test charge from infinity to that point along any arbitrary path.* (Infinity is taken as point of zero potential). It is denoted by V ;

$$V = \frac{W}{q_0} = \frac{\text{work done in bringing unit positive test charge from infinity to some point}}{\text{unit positive test charge}}$$

Its **SI unit** is JC^{-1} or volt. It is a scalar quantity.

Also, **electric potential at any point** in an electric field is defined as the negative line integral of the electric field vector $\vec{E}$ from a point infinitely away from all charges to that point

i.e. $V = -\int_{\infty}^{r} \vec{E}.\vec{dr}$



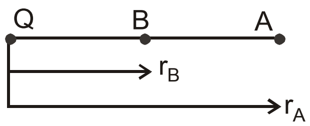
Potential due to a Point Charge

The **electric potential due to a point charge** q at separation r is given by

$$V = \frac{1}{4\pi\varepsilon}.\frac{q}{r}$$

(Please note that we have to write q with its sign in this formula)

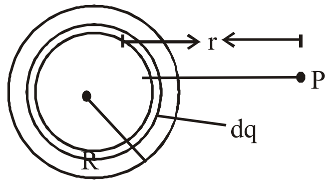
4F potential difference between two points is the work done in bringing unit positive charge from one point to another.



$$V_{AB} = V_B - V_A = -\int_{A}^{B} E.dr = \frac{-Q}{4\pi\varepsilon_o}.\left(\frac{l}{r_B} - \frac{l}{r_A}\right) \ J/C$$

Electric Potential due to Continuous Charge Distribution

The potential due to a continuous charge distribution is the sum of potentials of all the infinitesimal charge elements in which the distribution may be divided.



i.e. $V = \int dV = \int \frac{dq}{4\pi\varepsilon_o r}$ where $dV = \frac{dq}{4\pi\varepsilon_o r}$

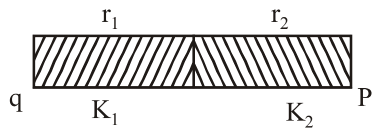
Potential due to a System of Charges

The electric potential due to a system of charges $q_1, q_2, ...q_n$ is $V = V_1 + V_2 + ... + V_n$

$$= \frac{1}{4\pi\varepsilon}.\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + ... + \frac{q_n}{r_n}\right) = \frac{1}{4\pi\varepsilon}\sum \frac{q_i}{r_i}$$

where r_i is the point from charge q_i and ε is the permitivity of medium in which the charges are situated.

Potential at any point P due to a point chage q at a distance $(r_1 + r_2)$ where r_1 is the thickness of medium of dielectric constant x_1 and r_2 is the thickness of the medium of dielectric constant k_2



$$V = k\frac{q}{r_1\sqrt{K_1} + r_2\sqrt{K_2}} \text{ where } k = \frac{1}{4\pi \in_0}$$

Relation between electric field and potential

The relation between electric field (E) and potential (V) is

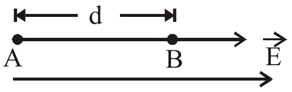
$$E = -\frac{dV}{dr}$$

For 3-D we can write

$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y} \text{ and } E_z = -\frac{\partial V}{\partial z}$$

So **electric field is equal to negative potential gradient.**

In this relation negative sign indicates that in the direction of electric field, potential decreases. Consider two points A and B situated in a uniform electric field at a distance d then,

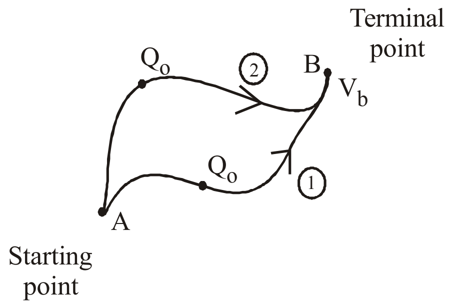


The potential difference between A and B is

$$V_{AB} = Ed$$

Conservative nature of electric field

The electric field is conservative in nature. In figure the work, W_{AB} has the same value whatever path is taken in moving the test charge.



so, $V_{AB} = V_B - V_A = \dfrac{W_{AB}}{Q}$

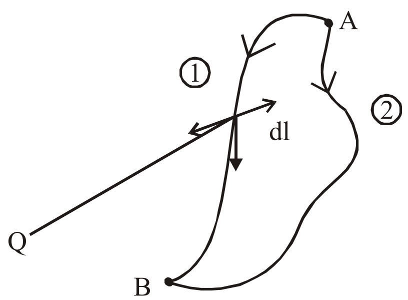
has the same value for any path between A and B and V_B and V_A are unique for the points A and B.

Note : We cannot find the absolute value of potential therefore conventionally, we take infinity as the point of zero potential. If need arises, we can assume any point to be the point of zero potential and find the potential of other points on this basis.

POTENTIAL ENERGY OF A SYSTEM OF CHARGES

Potential energy can be defined only for those forces, which are conservative, such as gravitational and electrostatic forces. *The potential energy of a charge between two points is defined as the amount of work done in bringing the charge from one point to another.*

i.e. $V_B - V_A = \dfrac{W_{ext}}{q} = -\int_A^B \vec{E}.\vec{dr}$

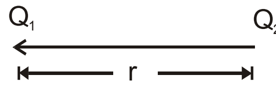


Calculation of external work done against the field and a point charge Q in moving a test charge q from A to B. For a conservative field the work done by any path is same. The sectional force is – qE.

If A is at infinity then at infinity since potential is zero we assume infinity as reference point, $V_A = 0$

$$\Rightarrow V_B = -q\int_{\infty}^{B} E.dl$$

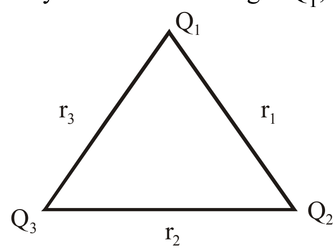
Potential energy of a system of two charges Q_1 and Q_2 is,



$$U = \frac{1}{4\pi \in_0}\frac{Q_1 Q_2}{r^2}$$

[*Please note that in this formula we have to write charges with sign*]

Potential energy of a system of three charges Q_1, Q_2 and Q_3



$$U = \frac{1}{4\pi \in_0}\left[\frac{Q_1 Q_2}{r_1} + \frac{Q_2 Q_3}{r_2} + \frac{Q_3 Q_1}{r_3}\right]$$

Keep in Memory

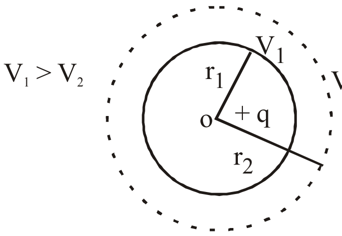
1. For an assembly of n charges [Total number of intersection ${}^nC_2 = \dfrac{n(n-1)}{2}$] the potential energy is

$$U = \frac{1}{2}\left[k\sum_{\substack{i,j\\ i\neq j}}^{n}\frac{q_i q_j}{r_{ij}}\right]$$

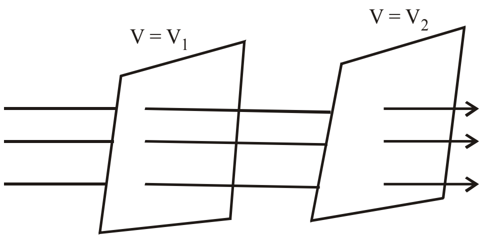
2. For a system of two charges.
   If U_{system} = –ve, then there is net force of attraction between the charges of the system.
   If U_{system} = +ve, then there is net force of repulsion between the charges of the system
   U_{system} = max   for unstable equilibrium
   U_{system} = min   for stable equilibrium
   Also $F = -\dfrac{dU}{dx} = 0$
3. The energy required to take away the charges of a dipole at infinite distance $U = k\dfrac{q^2}{2l}$
4. The work done when a charge q is moved across a potential difference of V volt is given by W = qV
5. When one electronic charge (1.6×10^{-19} coulomb i.e., charge of electron) is moved across one volt the work done is called **one electron volt (eV).** Thus
   $1eV = (1\ \text{volt}) \times (1.6\times10^{-19}\ \text{coulomb}) = 1.6\times10^{-19}$ joule.

EQUIPOTENTIAL SURFACE

It is that surface where the potential at any point of the surface has the same value. The electric lines of force and the equipotential surface are mutually perpendicular to each other. No work is done in moving a charge from one point to other on an equipotential surface. Work is done in moving a charge from one equipotential surface to another.



Spherical equipotential surface for point charge

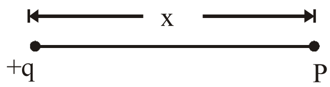


Plane equipotential surface for uniform field

- Equipotential surface do not cut each other.
- The density of the equipotential lines gives an idea of the strength of electric field at that point. Higher the density, larger is the field strength.

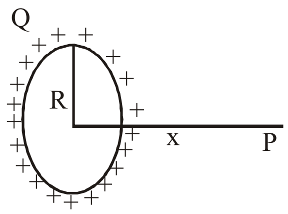
Potential Due to Various Charge Distribution

(i) **Electric potential due to isolated point charge**



$$V = k\frac{(+q)}{x}$$

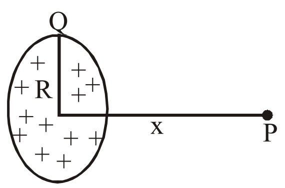
(ii) A **circular ring** of radius R with uniformly distributed charge Q



$$V = \frac{kQ}{\sqrt{R^2 + x^2}}$$

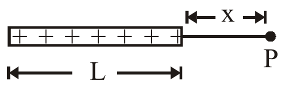
- Potential V does not depend on the way of charge distriubution on the ring (uniform / non-uniform).

(iii) **A circular disc** of radius R with uniformly distributed charge with surface charge density σ



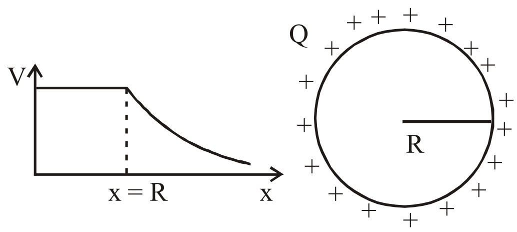
$$V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2 + x^2} - x\right]$$

(iv) A **finite length of charge** with linear charge density λ



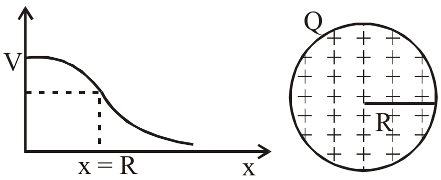
$$V = k\lambda \log_e\left(\frac{x + L}{x}\right)$$

(v) Due to **a spherical shell** of uniformly distributed charge with surface charge density σ



$$V_{in} = k\frac{Q}{R},\ V_{surface} = k\frac{Q}{R},\ V_{out} = k\frac{Q}{x}$$

(vi) Due to a **solid sphere** of uniformly distributed charge with volume charge density ρ.

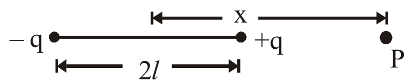


$$V_{centre} = k\frac{3Q}{2R},\ V_{in} = \frac{kQ(3R^2 - r^2)}{2R^3}$$

$$V_{surface} = k\frac{Q}{R},\ V_{out} = k\frac{Q}{x}$$

Potential due to Electric dipole

(a) Along axial line :



$$V_{axial} = k\frac{p}{x^2 - l^2}$$

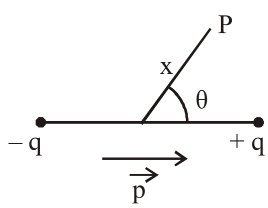
$$\text{when } x >> l\ V_{axial} = \frac{kp}{x^2}$$

(b) Along equatorial line :

$$V_{eq} = zero$$

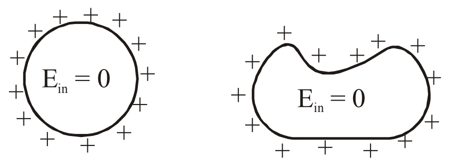
(c) At any point from the dipole :

$$V = k\frac{p\cos\theta}{(x^2 - l^2\cos^2\theta)}$$



Keep in Memory

1. Electric field inside a charged conductor is zero



Spherical conductor   Irregular shaped conductor

But in both the cases the potential at all the points of the surface will remain the same. But charges will have same distribution on spherical conductor and in case of irregularly shaped conductor the charge distribution will be non-uniform. At sharp points, charge density has greatest value.

2. Electronic lines of force are always perpendicular to the equipotential surfaces.
3. The work done in moving a charge from a point to the other on an equipotential surface is zero as the potential difference between the two points is zero.
4. The electric potential at a point due to a point charge decreases (or increases) by K-times if the distance between the charge and the point increases (or decreases) by K-times.
5. A ring with a charge distribution behaves as a point charge for the points very far from its centre.
6. The electric potential is constant inside a hollow charged sphere and it is also equal to its value on the surface but it varies inversely with the distance outside the sphere.
7. The electric potential at points inside a solid sphere has a non-zero value and decreases as we go from the centre outwards. It behaves as a point charge for the points outside the sphere.
8. The electric potential at a point due to a dipole varies directly with the dipole moment.

COMMON DEFAULT

✘ ***Incorrect :*** Where electric field is zero, electric potential is also zero.

✔ ***Correct :*** It is not always correct, for example in a charged conducting shell, electric field inside the shell E = 0 but potential is not zero.

✘ ***Incorrect :*** Where electric potential is zero, electric field is also zero.

✔ ***Correct :*** It is not always correct. In the case of equitorial plane of an electric dipole the electric potential is zero but the electric field is non-zero.

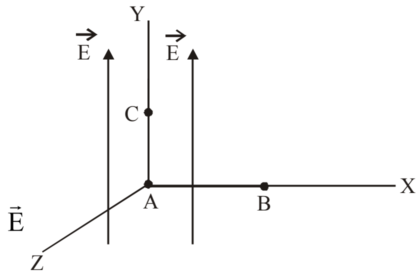
Example 1

A uniform electric field pointing in positive y-direction exists in a region. Let A be the origin, B be the point on the x-axis at x = 2 cm and C be the point on the y-axis at y = 2 cm. Then the potential at the points A, B and C satisfy:

(a) $V_A < V_B$ (b) $V_A > V_B$
(c) $V_A < V_C$ (d) $V_A > V_C$

Solution : (d)

As electric field represents the direction of motion of positive charge, which is from higher potential to lower potential, therefore, from fig, we find $V_A = V_B$ and $V_A > V_C$



Example 2

A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at x = + 1 cm and C be the point on the y-axis at y = + 1 cm. Then the potential at the points A, B and C satisfy

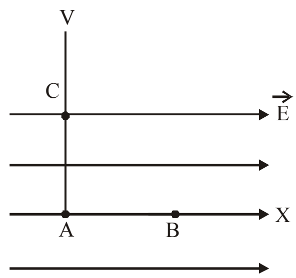
(a) $V_A < V_B$ (b) $V_A > V_B$
(c) $V_A < V_C$ (d) $V_A > V_C$

Solution : (b)

As $\vec{E}$ is directed along +ve direction of X-axis, therefore,

$V_A > V_B$

C is vertically above A. Therefore, $V_A = V_C$.



Example 3

Calculate the maximum voltage upto which a sphere of radius 2 cm can be charged in air under normal conditions, assuming that maximum electric intensity in air can be 3×10^6 *volt/m. Also, find the charge required to be given to the sphere.*

Solution :

We know that

$$\text{Electric intensity (E)} = \frac{\text{Electric Potential (V)}}{\text{Distance (r)}}$$

$\Rightarrow V = Er$ ……(1)

Given, $E = 3 \times 10^6$ volt/m, $r = 2\text{ cm} = 2 \times 10^{-2}$ m.

Substituting the above values in eq. (1), we get

$V = (3 \times 10^6 \text{ volt/m}) \times 2 \times 10^{-2}\text{ m} = 6 \times 10^4$ volt.

$\Rightarrow V = 60\text{ kV}$

Also, we know that the electric intensity on the surface of a charged sphere is given by

$$E = \frac{1}{4\pi \epsilon_0} \cdot \frac{Q}{r^2}; \quad Q = \text{charge on the sphere}$$

$$\Rightarrow 3 \times 10^6 = (9 \times 10^9) \times \frac{Q}{(2 \times 10^{-2})^2}$$

$$\Rightarrow Q = 1.33 \times 10^{-7} \text{ coulomb.}$$

ELECTROSTATICS OF CONDUCTORS

Conductor is a substance that can be used to carry or conduct electric charges. Metals like silver. Copper, aluminium etc. are good conductors of electricity.

Regarding electrostatics of conductors following points are worth noting.

(i) Inside a conductor, electric field is zero.

(ii) The interior of a conductor can have no excess charge in static situation.

(iii) Electric field at the surface of a charged conductor is $\vec{E} = \frac{\sigma}{\epsilon_0} \hat{n}$

where, s = surface charge density

$\hat{n}$ = unit vector normal to the surface in the outward direction.

(iv) Electric field just outside a charged conductor is perpendicular to the surface of the conductor at every point.

(v) Electrostatic potential is constant throughout the volume of the conductor and has the same value as on its surface.

(vi) Surface density of charge is different at different points.

CAPACITORS AND CAPACITANCE

***A capacitor** or **condenser** is a device that stores electrical energy.* It generally consists of two conductors carrying equal but opposite charges.

*The ability of a capacitor to hold a charge is measured by a quantity called the **capacitance.*** Let us consider two uncharged identical conductors X and Y and create a P.D. (Potential Difference) V between them by connecting with battery B as shown in figure.

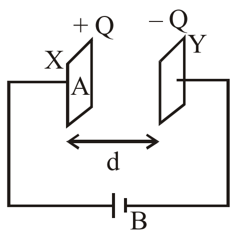


Fig- A capacitor consists of electrically insulated conductors carrying equal positive and negative charge

After connection with the battery, the two conductors X and Y have equal but opposite charges. Such a combination of charged conductors is a device called a capacitor. The P.D. between X and Y is found to be proportional to the charge Q on capacitor. *The capacitance C, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of P.D. between them.*

$$V \propto Q \Rightarrow \frac{Q}{V} = \text{constant} = (\text{Capacitance}).$$

Capacitance is always a positive quantity.

The **S.I. unit** of capacitance is coulomb per volt or farad (F). Further more, the value of ***capacitance depends** on size, shape, relative positions of plate, and the medium between the plates. The value of C does not depend on the charge of the plate or p.d. between the plates.*

ENERGY STORED IN A CAPACITOR

If Q is charge, V is p.d, C is the capacitance of the capacitor then the energy stored is $U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$

Sharing of Charges

When the two charged conductors of capacitances C_1 and C_2 at potentials V_1 and V_2 respectively, are connected by a conducting wire, the charge flows from higher to lower potential, until the potentials of the two conductors are equal.

The common potential after sharing of charges,

$$V = \frac{\text{Net charge}}{\text{Net capacitance}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

The charges after sharing on two conductors will be

$$Q_1' = C_1V \text{ and } Q_2' = C_2V \text{ i.e., } \frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{Q_1'}{Q_2'}$$

There is a loss of energy during sharing, converted to heat given by

$$\Delta U = U_{\text{initial}} - U_{\text{final}} = \left[\frac{1}{2}C_1{V_1}^2 + \frac{1}{2}C_2{V_2}^2\right] - \frac{1}{2}(C_1 + C_2)V^2$$

$$\text{or, } \Delta u = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}$$

PARALLEL PLATE CAPACITOR

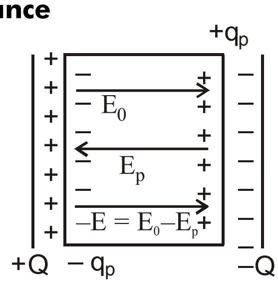
It consists of two parallel metallic plates of any shape, each of area A and at a distance d apart.

The capacitance of the capacitor is given by $C = \frac{\varepsilon_0 A}{d}$

Effect of Dielectric on Capacitance

When a dielectric slab is placed between the plates of a parallel plate capacitor, the charge induced on its plates due to polarisation of dielectric is $q_p = Q\left(1 - \frac{1}{K}\right)$

where K = dielectric constant.

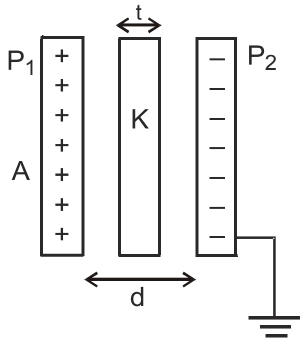


When an electric field is applied across a dielectric, induced charges appear on the surface of dielectric which is shown in the above figure. These induced charges produce their own field which acts in the opposite direction of the applied field. Hence, total field is reduced, i.e., $E_0 - E_p = E$, where E_0 is the applied field, E_p is the induced field and E is the resultant field.

E is given by $\frac{E_0}{K}$, where K is the dielectric constant.

If medium between the plates is having a dielectric of dielectric constant K then the capacitance is given $C = \left(\frac{K\varepsilon_o A}{d}\right)$

If the space between the plates is partly filled with dielectric then the capacitance of the capacitor will be given by,



$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)},$$

where t is the thickness of the dielectric with dielectric constant K.

Keep in Memory

1. The unit farad is quite a big unit for practical purposes. Even the capacitance of a huge body like earth is 711 μF.
2. A capacitor is a device which stores charges and produces electricity whenever required.
3. If the two plates of a capacitor is connected with a conducting wire, sparking takes place which shows that electrical energy is converted into heat and light energy.
4. A capacitor allows A.C. but doesn't allow D.C. to pass through it.
5. The capacitance of a capacitor increases with insertion of a dielectric between its plates and decreases with increase in the separation between the plates.
6. The capacitance of a capacitor increases K times if a medium of dielectric constant K is inserted between its plates.
7. The energy of a capacitor for a particular separation between the plates is the amount of work done in separating the two plates to that separation if they are made to touch to each other.
8. The loss of energy when the two charged conductors are connected by a wire doesn't depend on the length of the wire.

Example 4

A parallel plate capacitor is maintained at a certain potential difference. When a 3 mm slab is introduced between the plates, in order to maintain the same potential difference, the distance between the plates is increased by 2.4 mm. Find the dielectric constant of the slab.

Solution :

The capacity of a parallel plate capacitor in air is given by

$$C = \frac{\varepsilon_0 A}{d} \qquad ...(1)$$

By introducing a slab of thickness t, the new capacitance C′ becomes

$$C' = \frac{\varepsilon_0 A}{d' - t(1 - 1/K)} \qquad ...(2)$$

The charge (Q = CV) remains the same in both the cases. Hence

$$\frac{\varepsilon_o A}{d} = \frac{\varepsilon_o A}{d' - t(1 - 1/K)} \text{ or } d = d' - t\left(1 - \frac{1}{K}\right)$$

$d' = d + 2.4 \times 10^{-3}$ m, t = 3 mm = 3×10^{-3} m.

Substituting these values, we have

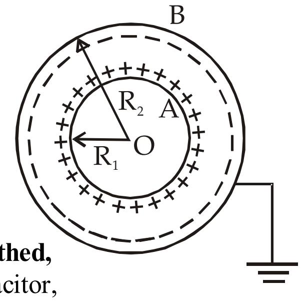
$$d = d + (2.4 \times 10^{-3}) - 3 \times 10^{-3}\left(1 - \frac{1}{K}\right)$$

or $\quad (2.4 \times 10^{-3}) = 3 \times 10^{-3}\left(1 - \frac{1}{K}\right)$

Solving it, we get K = 5.

SPHERICAL CAPACITOR

It consists of two concentric spherical conductors of radii R_1 and R_2. The space between two conductors is filled by a dielectric of dielectric constant K.



(a) **When outer conductor is earthed,**
Capacitance of spherical capacitor,

$$C = \frac{4\pi \epsilon_0 R_1 R_2}{R_2 - R_1} \quad \text{(without dielectric)}$$

$$C = \frac{(4\pi\varepsilon_o K)R_1 R_2}{(R_2 - R_1)} \quad \text{(with dielectric)}$$

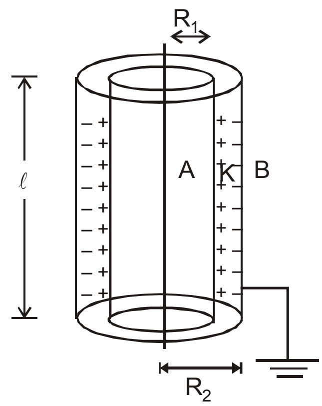
(b) **When inner sphere is earthed,**

$$C = \frac{4\pi\varepsilon_o K \, R_1 R_2}{R_2 - R_1} + 4\pi\varepsilon_o R_2$$

This is because the combination behaves as two capacitors in parallel, one is a capacitor formed by two concentric spherical shells and the other is an isolated spherical shell of radius R_2.

CYLINDRICAL CAPACITOR

It consists of two-coaxial cylindrical conductors of radii R_1 and R_2, the outer surface of outer conductor being earthed. The space between the two is filled with a dielectric of dielectric constant K.



The capacitance of cylindrical condenser of length ℓ

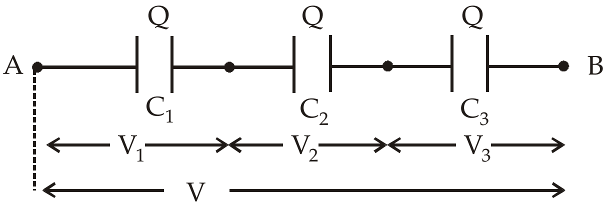
$$C = \frac{2\pi \epsilon_0 \ell}{\log_e\left(\frac{R_2}{R_1}\right)} \qquad \text{(without dielectric)}$$

$$C = \frac{2\pi \epsilon_o K\ell}{\log_e\left(\frac{R_2}{R_1}\right)} \qquad \text{(with dielectric)}$$

COMBINATION OF CAPACITORS

Series Combination

(i) In this combination, the positive plate of one capacitor is connected to the negative plate of the other.



(ii) The charges of individual capacitor are equal.

(iii) The potential difference is shared by the capacitors in the inverse ratio of their capacities

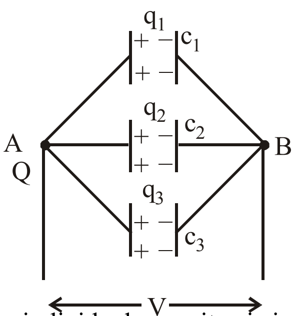
i.e. $Q = C_1V_1 = C_2V_2 = C_3V_3$

Hence $V = V_1 + V_2 + V_3$

(iv) The equivalent capacitance (C) between A and B is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots.. + \frac{1}{C_n}$$

Parallel Combination



(i) In this arrangement, +ve plates of all the condensers are connected to one point and negative plates of all the condensers are connected to the other point.

(ii) The Potential difference across the individual capacitor is same.

(iii) The total charge shared by the individual capacitor is in direct ratio of their capacities

i.e. $V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_3}{C_3}$

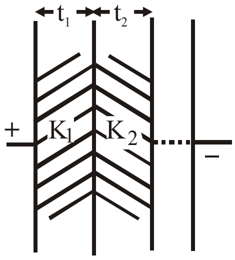
Hence, $Q = q_1 + q_2 + q_3$

(iv) The equivalent capacitance between A and B is $C_{eq} = C_1 + C_2 + C_3 + \ldots\ldots.. + C_n$

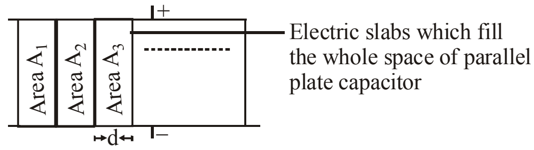
Keep in Memory

1. The capacitance of a parallel plate capacitor having a number of slabs of thickness t_1, t_2, t_3 .... and dielectric constant K_1, K_2, K_3 .... respectively between the plates is

$$C = \frac{\varepsilon_0 A}{\left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3} + \ldots..\right)}$$

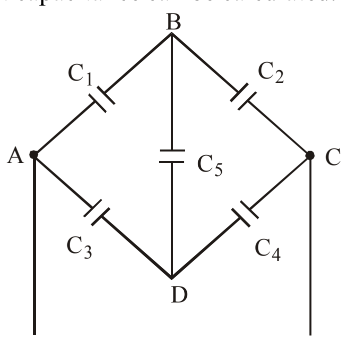


2. When a number of dielectric slabs of same thickness (d) and different areas of cross-section A_1, A_2, A_3 ... having dielectric constants K_1, K_2, K_3, ... respectively are placed between the plates of a parallel plate capacitor then the capacitance is given by



$$C = \frac{\varepsilon_0(K_1A_1 + K_2A_2 + K_3A_3 + \ldots.)}{d}$$

3. When five capacitors are connected in wheatstone bridge arrangement as shown, such that $\frac{C_1}{C_2} = \frac{C_3}{C_4}$, the bridge is balanced and C_5 becomes ineffective. No charge is stored on C_5. Therefore C_1, C_2 and C_3, C_4 are in series. The two series combinations are in parallel between A and C. Hence equivalent capacitance can be calculated.



RELATION BETWEEN THREE ELECTRIC VECTORS $\vec{D}$, $\vec{P}$ AND $\vec{E}$

If an electric field E is applied across a parallel plate capacitor filled with a dielectric of dielectric constant K (or permittivity ε), then

Polarisation P = induced charge per unit area (opposite to free charge) $= \frac{q'}{A}$

Electric displacement $D = \varepsilon E = \varepsilon_o E + P$

i.e. Polarisation $P = (\varepsilon - \varepsilon_o)E = (K\varepsilon_o - \varepsilon_o)E$

Electric susceptibility, $\chi_e = P/E$

Relation between dielectric constant K and electric susceptibility χ_e is

$$K = 1 + \frac{\chi_e}{\varepsilon_0}$$

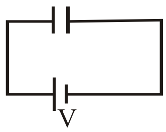
Effect of filling dielectric With battery connected

When there is no dielectric

Capacitance $C_0 = \dfrac{A\varepsilon_0}{d}$

Potential difference between the plates V

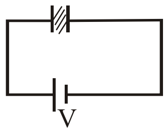
Charge on a plate Q = CV



Energy $E_0 = \dfrac{1}{2}C_0V^2$

Electric field $E_0 = \dfrac{V}{d}$

When dielectric is inserted



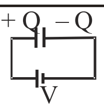
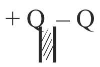
$C = \dfrac{KA\varepsilon_0}{d} = KC_0$

$Q = K\,C_0\,V = KQ_0$

$U = \dfrac{1}{2}KC_0V^2 = KE_0$

$E = \dfrac{V}{d} = E_0$

Effect of filling a dielectric in a capacitor after disconnection of battery

		
Capacitance	C_0	$C = K\,C_0$
Charge	Q_0	$Q = Q_0$
P.D	$V_0 = \dfrac{Q_0}{C_0}$	$V = \dfrac{Q_0}{C} = \dfrac{V_0}{K}$
Potential energy	$U_0 = \dfrac{1}{2}C_0V_0^2$	$U = \dfrac{1}{2}KC_0\left(\dfrac{V_0}{K}\right)^2 = \dfrac{U_0}{K}$

CHARGING AND DISCHARGING A CAPACITOR

Charging a Capacitor

When an uncharged capacitor is connected across a source of constant potential difference such as a cell, it takes a finite time to get fully charged, although this time interval may be small. This time-interval depends on the capacity of the capacitor and the resistance in the circuit.

During the period of charging :

1. The charge on the capacitor increases from ‘zero’ to the final steady charge.
2. The potential difference developed across the capacitor opposes the constant potential difference of the source.
3. The charge on the capacitor ‘grows’ only as long as the potential difference of source is greater than the potential difference across the capacitor. This transport of the charge from the source to the capacitor constitutes a transient current in the circuit.
4. As the charge on the capacitor increases, more energy is stored in the capacitor.
5. When the capacitor is fully charged, potential difference across the capacitor is equal to the potential difference of the source and the transient current tends to zero.

   If V_0 = constant potential difference of the source

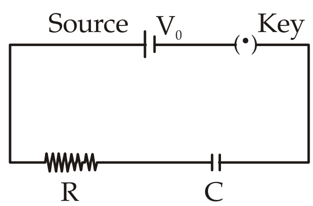
   R = pure resistance in the circuit

   C = capacity of the capacitor

   Q_0 = final charge on the capacitor, when fully charged

   q = charge on the capacitor at time ‘t’ from the starting of the charging

   V = potential difference across the capacitor at time ‘t’



Then $\dfrac{Q_0}{V_0} = \dfrac{q}{V} = C$

and i = current in the circuit at time ‘t’ = $\dfrac{dq}{dt}$

At time ‘t’ by Kirchhoff’s law

$V_0 - R\dfrac{dq}{dt} - \dfrac{q}{C} = 0$ i.e. $\dfrac{dq}{CV_0 - q} = \dfrac{1}{CR}dt$

Integrating and putting in the initial condition q = 0 at t = 0, we get

$$q = CV_0\left[1 - e^{-\frac{1}{CR}t}\right]$$

Special cases :

(i) At t = 0, q = 0.

(ii) When t increases, q increases.

(iii) As $t \to \infty$, $q \to Q_0 = CV_0$ $\therefore q = Q_0\left[1 - e^{-\frac{1}{CR}t}\right]$

(iv) At t = CR [‘CR’ has dimensions of time]

$$q = Q_0\left[1 - \frac{1}{e}\right] = 0.631Q_0$$

This value of t = CR is called the ‘**time constant**’ of the (CR) circuit.

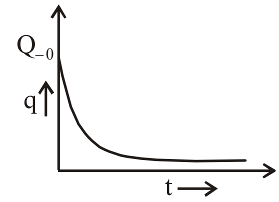
Discharging of a Capacitor

If after charging the capacitor, the source of constant potential difference is disconnected and the charged capacitor is shorted through a resistance ‘R’, then by Kirchhoff’s law, at time ‘t’ from the instant of shorting,

$$\frac{q}{C} + R\frac{dq}{dt} = 0$$

Putting,
(i) the initial condition, $q = Q_0$ at $t = 0$ and
(ii) the final condition, $q = 0$ at $t \to \infty$,
the solution to the above equation is

$$q = Q_0 e^{-\frac{1}{CR}t}$$



Keep in Memory

1. If n small drops each having a charge q, capacity 'C' and potential V coalesc to form a big drop, then
   (i) the charge on the big drop = nq
   (ii) capacity of big drop = $n^{1/3}\, C$
   (iii) potential of big drop = $n^{2/3}\, V$
   (iv) potential energy of big drop = $n^{5/3}\, U$
   (v) surface density of charge on the big drop = $n^{1/3}$ × surface density of charge on one small drop.
2. **Charged soap bubble :** Four types of pressure act on a charged soap bubble.
   (i) Pressure due to air outside the bubble P_O, acting inwards.
   (ii) Pressure due to surface tension of soap solution P_T, acting inwards.
   (iii) Pressure due to air inside the bubble, P_i, acting outwards.
   (iv) Electric pressure due to charging, $P_e = \dfrac{\sigma^2}{2\varepsilon_o}$, acting outwards.

   In equilibrium, $P_i + P_e = P_O + P_T$
   or, $P_i - P_O = P_T - P_e$
   or, $P_{excess} = P_T - P_e$

   $$\therefore \quad P_{excess} = \frac{4T}{r} - \frac{\sigma^2}{2\varepsilon_o}$$

   Where T = surface tension of soap solution,
   σ = surface charge density of bubble.
   If $P_i = P_O$ then $P_i - P_O = P_T - P_e = 0$ or $P_T = P_e$

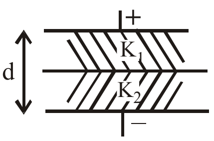
   $$\frac{4T}{r} = \frac{\sigma^2}{2\varepsilon_o} = \frac{1}{2\varepsilon_o}\left(\frac{q}{4\pi r^2}\right)^2$$

   Hence for maintaining the equilibrium of charged soap bubble, $\sigma = \sqrt{\dfrac{8\varepsilon_o T}{r}}$

   $$q = 8\pi r\sqrt{2\varepsilon_o rT}$$
3. Force of attraction between the plates of a parallel plate capacitor $= \dfrac{q^2}{2\varepsilon_o AK}$
   where, A = area of the plates of capacitor and
   K = dielectric constant of the medium filled between the plates.
   In terms of electric field, the force of attraction
   $$F = \frac{1}{2}\varepsilon_0 KE^2$$
4. **Uses of capacitor :**
   - In LC oscillators
   - As filter circuits
   - Tuner circuit in radio etc.
5. The total energy stored in an array of capacitors (in series or in parallel) is the sum of the individual energies stored in each capacitor.

COMBINATION OF CAPACITOR : EQUIVALENT CAPACITANCE

(i)



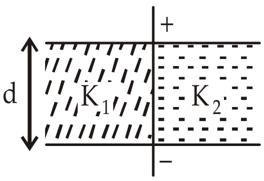
$\equiv$

$C_1 = K_1 \dfrac{A\varepsilon_0}{d/2}$, $C_2 = K_2 \dfrac{A\varepsilon_0}{d/2}$

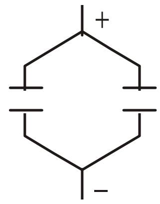
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\left[d = \frac{d}{2} + \frac{d}{2} \text{ and area of each plate} = A\right]$$

(ii)

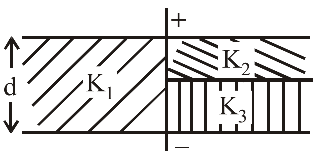


$$\equiv C_1 = \frac{K_1\left(\frac{A}{2}\right)\varepsilon_0}{d}$$

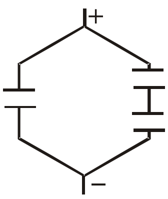


$$C_2 = \frac{K_2\left(\frac{A}{2}\right)\varepsilon_0}{d} \quad ; \quad C_{eq} = C_1 + C_2$$

(iii)



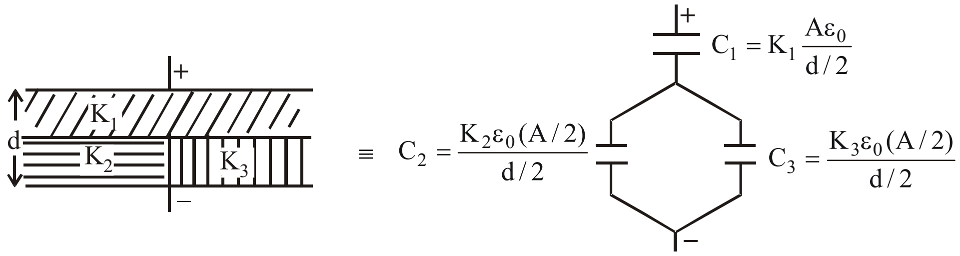
$$\equiv C_1 = \frac{K_1(A/2)\varepsilon_0}{d}$$



$$C_2 = \frac{K_2(A/2)\varepsilon_0}{d/2} \qquad C_3 = \frac{K_3(A/2)\varepsilon_0}{d/2}$$

$$C_{eq} = C_1 + \frac{C_2C_3}{C_2C_3}$$

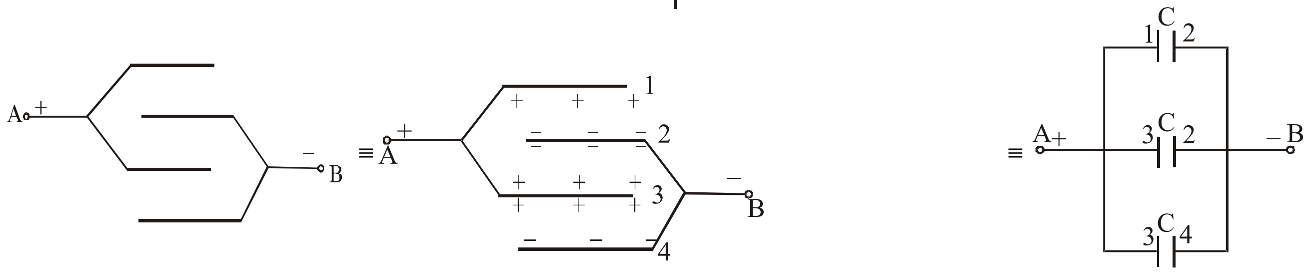
(iv)



$$C_1 = K_1\frac{A\varepsilon_0}{d/2} \qquad C_2 = \frac{K_2\varepsilon_0(A/2)}{d/2} \qquad C_3 = \frac{K_3\varepsilon_0(A/2)}{d/2}$$

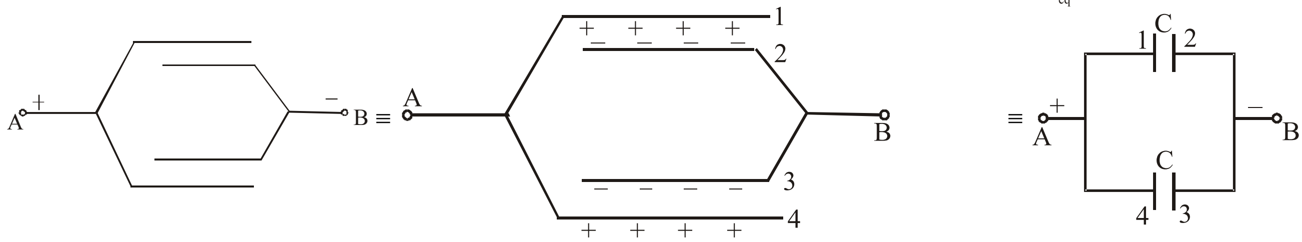
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2 + C_3}$$

(v)



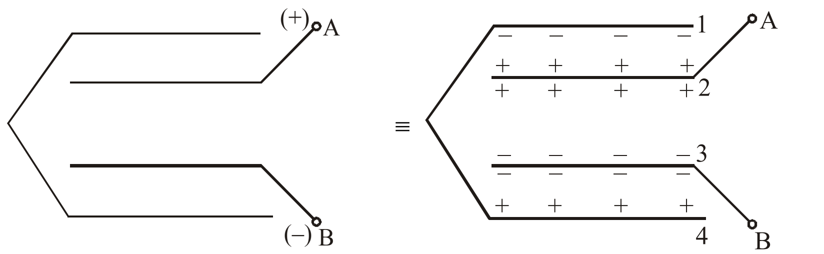
$C_{eq} = C + C + C = 3C$

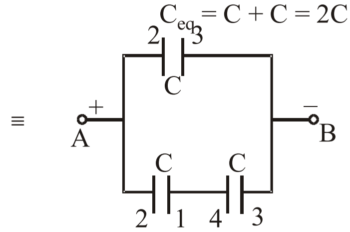
(vi)



$C_{eq} = C + C = 2C$

(vii)



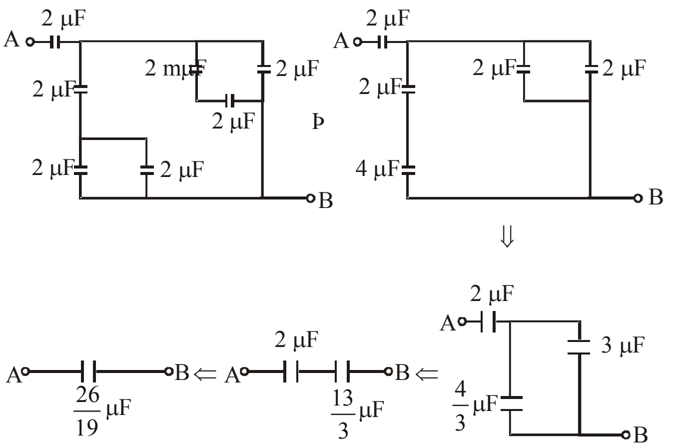


$$\frac{1}{C_{eq}} = C + \left[\frac{1}{\frac{1}{C} + \frac{1}{C}}\right] \text{ or, } C_{eq} = \frac{3C}{2}$$

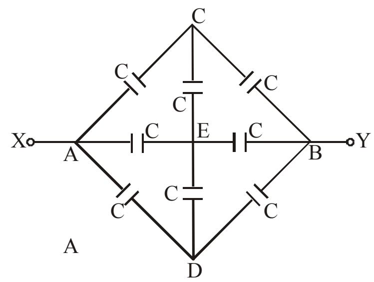
SOME METHODS OF FINDING EQUIVALENT CAPACITANCE

Method 1 : Successive Reduction

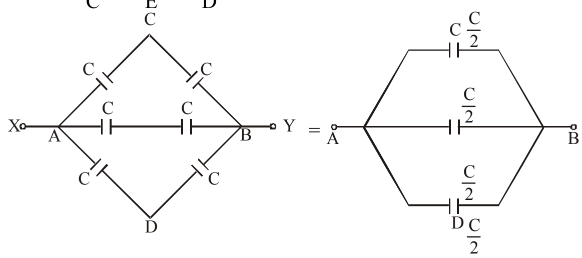
This method is applicable only when the capacitor can be clearly identified as in series or in parallel.

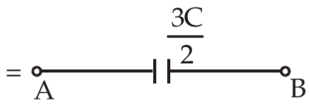


Method 2 : Using Symmetry

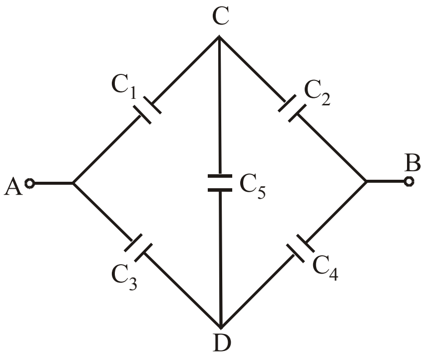


The above circuit is symmetrical about XAEBY axis. This is because the upper part of the circuit is mirror image of lower part. Therefore $V_C = V_E = V_D$. The circuit can be redrawn as





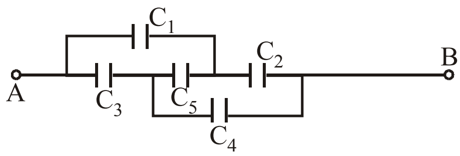
Method 3 : Wheatstone bridge



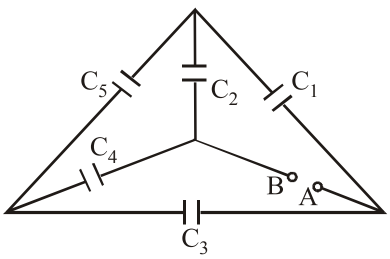
If $\frac{C_1}{C_3} = \frac{C_2}{C_4}$ then the wheatstone bridge is balanced. In this case there will be no charge accumulation in C_5 when battery is attached across A and B. Therefore the equivalent circuit is the capacitance C_1 and C_2 are in series. Similarly C_3 and C_4 is in series. Therefore the equivalent capacitance occurs between A and B is

$$\frac{1}{C_{eq}} = \frac{1}{C_1 + C_2} + \frac{1}{C_3 + C_4}$$

The other forms of wheatstone bridge are :



or



Method 4 : If none of the above method works, then we can use the method of Kirchhoff's laws - junction law and loop law.

SHARP POINT ACTION (CORONA DISCHARGE)

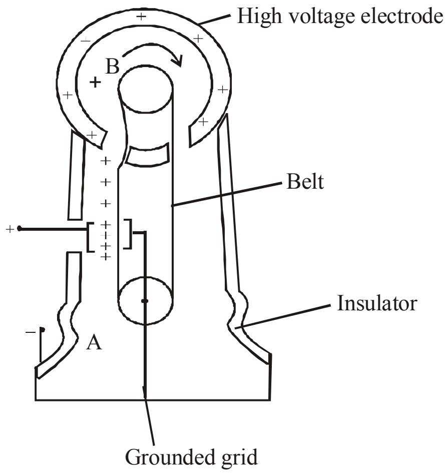
When the electric field (σ/ε_0) on a point on the surface of a conductor exceeds the electric strength of air, then the air becomes conducting and the surface of conductor loses charge. This action occurs usually at the sharp points of a conductor as here σ is high, thus creating high electric field. This phenomenon is also called **corona discharge**.

VAN DE GRAAF GENERATOR

R.J. Van de Graff in 1931 designed an electrostatic generator capable of generating very high potential of the order of 5×10^6 V, which was then made use of an accelerating charged particles so as to carry out nuclear reactions.

Principle : *When a charged conductor is placed in contact with the inside of a hollow conductor, all of the charge of first conductor is transferred to the hollow conductor. i.e., the charge on hollow conductor or its potential can be increased by any limit by repeating that processes.*

The basic fact of Van de Graaf generator is described in fig. (Charge is delivered continuously to a high voltage electrode on a moving belt of insulating material).



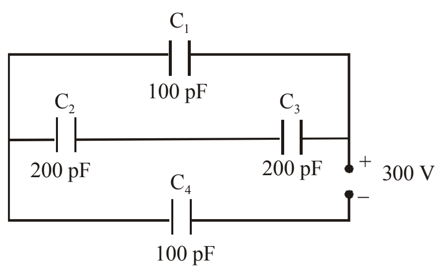
Schematic diagram of a Van de Graaf generator. Charge is transferred to hollow conductor at the top by means of a rotating belt. The charge is deposited on the belt at point A and is transferred to hollow conductor at point B.

The high voltage electrode is a hollow conductor mounted on an insulating medium. The belt is charged at A by means of **corona discharge** between comb-like metallic needles and a grounded grid. The needles are maintained at a positive potential of typically 10^4 eV. The positive charge on the moving belt is transferred to the high voltage electrode by second comb of needles at B.

Since the electric field inside the hollow conductor is negligible, the positive charge on the belt easily transfers to the high- voltage electrode, regardless of its potential. We can increase the potential of the high voltage electrode until electrical discharge occur through the air . The "breakdown" voltage of air is about 3×10^6 V/m.

Example 5

Obtain equivalent capacitance of the following network as shown in fig. For a 300 volt supply, determine the charge and voltage across each capacitor.



Solution :

As it is clear from fig, C_2 and C_3 are in series.

$$\therefore \quad \frac{1}{C_s} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{200} + \frac{1}{200}$$

$$= \frac{2}{200} = \frac{1}{100} \Rightarrow C_s = 100\,\text{pF}$$

Now, C_s and C_1 are in parallel.

$C_p = C_s + C_1 = 100 + 100 = 200$ pF.

Again, C_p and C_4 are in series. Their combined capacitance C is

$$\frac{1}{C} = \frac{1}{C_p} + \frac{1}{C_4} = \frac{1}{200} + \frac{1}{100} = \frac{3}{200}$$

$$C = \frac{200}{3} = 66.7 \text{ pF} = 66.7 \times 10^{-12} \text{ F}$$

As C_p and C_4 are in series.

$\therefore V_p + V_4 = 300$ volt.

Charge on C_4,

$$q_4 = CV = \frac{200}{3} \times 10^{-12} \times 300 = 2 \times 10^{-8} \text{C.}$$

Potential difference across C_4:

$$V_4 = \frac{q_4}{C_4} = \frac{2 \times 10^{-8}}{100 \times 10^{-12}} = 200 \text{ volt.}$$

$$V_p = 300 - V_4 = 300 - 200 = 100 \text{ volt}.$$

Potential difference across $C_1 = V_1 = V_p = 100$ volt.

Charge on

$C_1, q_1 = C_1 V_1 = 100 \times 10^{-12} \times 100 = 10^{-8}$ C

Potential difference across C_2 and C_3 in series = 100 volt

$V_2 = V_3 = 50$ volt

Charge on

$C_2 = q_2 = C_2 V_2 = 200 \times 10^{-12} \times 50 = 10^{-8}$ C

Charge on

$C_3 = q_3 = C_3 V_3 = 200 \times 10^{-12} \times 50 = 10^{-8}$ C

Example 6

Two isolated metallic solid spheres of radii R and 2 R are charged such that both of these have same charge density σ. The spheres are located far away from each other, and connected by a thin conducting wire. Find the new charge density on the bigger sphere.

Solution :

Charge on smaller sphere, $q_1 = 4\pi R^2 \sigma$

Charge on bigger sphere, $q_2 = 4\pi(2R)^2 \sigma = 1\pi R^2 \sigma$

$\therefore$ Total charge, $q = q_1 + q_2 = 20\pi R^2 \sigma$

Combined capacity of two spheres,

$C = C_1 + C_2 = 4\pi\varepsilon_o R + 4\pi\varepsilon_o (2R) = 12\pi\varepsilon_o R$

After contact, charge is exchanged and a common potential V is reached.

$$V = \frac{\text{total charge}}{\text{total capacity}} = \frac{q}{C} = \frac{20\pi R^2 \sigma}{12\pi\varepsilon_o R} = \frac{5R\sigma}{3\varepsilon_o}$$

Now, charge on bigger sphere,

$$q_2' = C_2 V = 4\pi\varepsilon_o (2R) \times \frac{5R\sigma}{3\varepsilon_o} = \frac{40\pi R^2 \sigma}{3}$$

$\therefore$ Surface density of charge

$$\sigma_2' = \frac{q_2'}{\text{surface area}} = \frac{40\pi R^2 \sigma}{3(4\pi)(2R)^2} = \frac{5}{6}\sigma.$$

Example 7

Two insulated metal spheres of radii 10 cm and 15 cm charged to a potential of 150 V and 100 V respectively, are connected by means of a metallic wire. What is the charge on the first sphere?

Solution :

Here, $r_1 = 10$ cm, $r_2 = 15$cm

$V_1 = 150$ V, $V_2 = 100$ V

Common potential

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

$$= \frac{4\pi \epsilon_0 (r_1 V_1 + r_2 V_2)}{4\pi \epsilon_0 (r_1 + r_2)}$$

$$= 120 \text{ volt}$$

$$q_1 = C_1 V$$

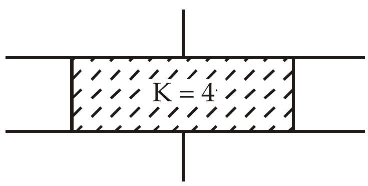
$$= 4\pi \epsilon_0 r_1 V$$

$$= \frac{10^{-1}}{9 \times 10^9} \times 12\text{C}$$

$$= \frac{12}{9 \times 10^9} \times 3 \times 10^9 \text{ esu} = 4 \text{esu}$$

Example 8

Consider a parallel plate capacitor of capacity 10 μF with air filled in the gap between the plates. Now one half of the space between the plates is filled with a dielectric of dielectric constant K = 4 as shown in fig.



The capacity of the capacitor changes to

(a) 25 μF *(b) 20 μF*

(c) 40 μF *(d) 5 μF*

Solution : (a)

The arrangement is equivalent to three capacitors in parallel

$$C_1 = \frac{\varepsilon_0 A/4}{d} = \frac{10}{4} = 2.5\,\mu F\,;$$

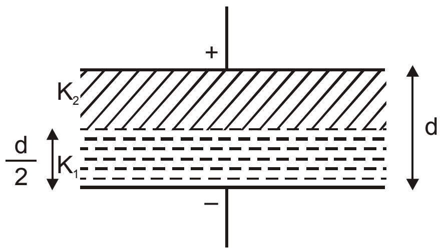
$$C_2 = \frac{K\varepsilon_0 A/2}{d} = 4 \times \frac{10}{2} = 20\,\mu F\,;$$

$$C_3 = \frac{\varepsilon_0 A/4}{d} = \frac{10}{2} = 2.5\,\mu F$$

$$\therefore\ C_{eq} = C_1 + C_2 + C_3 = 2.5 + 20 + 2.5 = 25\mu F$$

Example 9

A parallel plate capacitor is filled with dielectric as shown in fig. Its capacitance has ratio with that and without of dielectric as



(a) $(K_1 + K_2)$ (b) $\left(\dfrac{K_1+K_2}{K_1-K_2}\right)$

(c) $\left(\dfrac{2K_1K_2}{K_1+K_2}\right)$ (d) $\left(\dfrac{K_1+K_2}{K_1K_2}\right)$

Solution : (c)

Without dielectric, $C_0 = \dfrac{\varepsilon_0 A}{d}$

With dielectric as shown,

$$C_1 = \frac{K_1\varepsilon_0 A}{d/2} = 2K_1C_0; \qquad C_2 = \frac{K_2\varepsilon_0 A}{d/2} = 2K_2C_0;$$

As C_1, C_2 are in series,

$$\therefore\ \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_2 + C_1}{C_1 C_2}\,;$$

$$\therefore\ C_s = \frac{C_1C_2}{C_1+C_2} = \frac{2K_1C_0\,2K_0C_0}{2C_0(K_1+K_2)} = \frac{2K_1K_2C_0}{K_1+K_2}$$

$$\frac{C_s}{C_0} = \frac{2K_1K_2}{K_1+K_2}$$

Example 10

A capacitor of capacity 1 μF is connected in closed series circuit with a resistance of $10^7\ \Omega$, an open key and a cell of 2 V with negligible internal resistance.

(i) When the key is switched on at a time t = 0, find :

(a) the time constant for the circuit.

(b) the charge on the capacitor at steady state.

(c) time taken to deposit charge equalling half that at steady state.

(ii) If after fully charging the capacitor, the cell is shorted by zero resistance at time t = 0, find the charge on the capacitor at t = 50 s.

Solution :

(i) (a) Time constant = RC = $(10^7)(10^{-6}) = 10$ s.

(b) Q_0 = charge on capacitor at steady state $= V_0C = 2 \times 10^{-6} = 2\,\mu C$

(c) $q = \dfrac{1}{2}Q_0 = Q_0[1 - e^{-\frac{1}{CR}t}]$

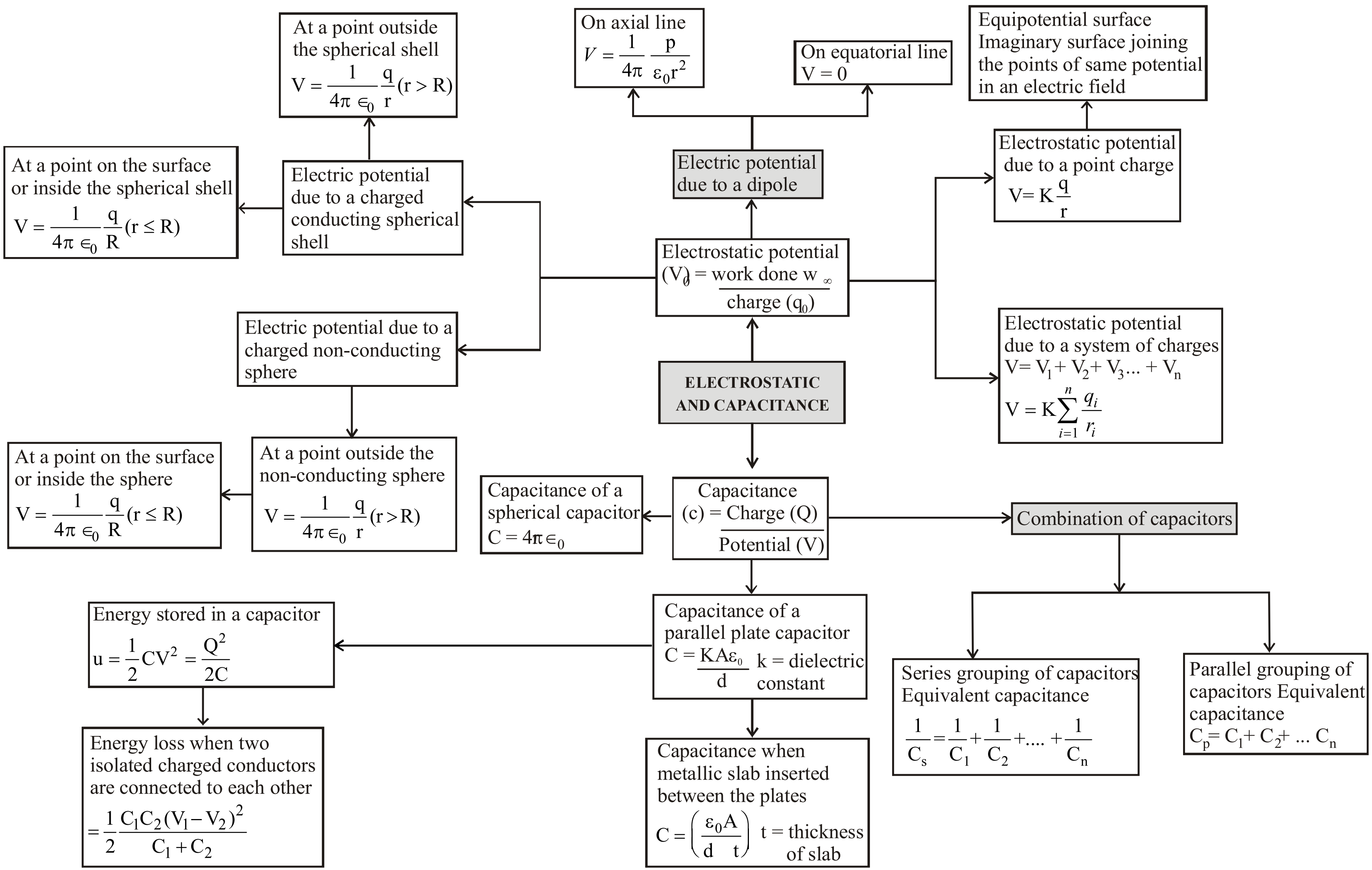
$$\Rightarrow e^{\frac{t}{10}} = 2 \ \text{ or, }\ t = 10 \times 2.306 \times \log_{10} 2 = 6.94\ \text{s.}$$

(ii) $q = Q_0 e^{-\frac{1}{CR}t}$

$$= (2\times10^{-6})(e^{-\frac{50}{10}}) = (2\times10^{-6})\left(\frac{1}{e^5}\right)$$

$$= 1.348 \times 10^{-8}\ C \qquad (\because e = 2.718)$$

CONCEPT MAP


ELECTROSTATIC AND CAPACITANCE
Electrostatic potential $(V_0) = \frac{\text{work done } w_\infty}{\text{charge } (q_0)}$
Electric potential due to a dipole
On axial line $V = \frac{1}{4\pi}\frac{p}{\varepsilon_0 r^2}$
On equatorial line $V = 0$
Electrostatic potential due to a point charge $V = K\frac{q}{r}$
Equipotential surface Imaginary surface joining the points of same potential in an electric field
Electrostatic potential due to a system of charges $V = V_1 + V_2 + V_3 \ldots + V_n$
$V = K\sum_{i=1}^{n}\frac{q_i}{r_i}$
Electric potential due to a charged conducting spherical shell
At a point outside the spherical shell $V = \frac{1}{4\pi \epsilon_0}\frac{q}{r}(r > R)$
At a point on the surface or inside the spherical shell $V = \frac{1}{4\pi \epsilon_0}\frac{q}{R}(r \le R)$
Electric potential due to a charged non-conducting sphere
At a point outside the non-conducting sphere $V = \frac{1}{4\pi \epsilon_0}\frac{q}{r}(r > R)$
At a point on the surface or inside the sphere $V = \frac{1}{4\pi \epsilon_0}\frac{q}{R}(r \le R)$
Capacitance $(c) = \frac{\text{Charge (Q)}}{\text{Potential (V)}}$
Capacitance of a spherical capacitor $C = 4\pi \epsilon_0$
Combination of capacitors
Series grouping of capacitors Equivalent capacitance $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$
Parallel grouping of capacitors Equivalent capacitance $C_p = C_1 + C_2 + \ldots C_n$
Capacitance of a parallel plate capacitor $C = \frac{KA\varepsilon_0}{d}$ k = dielectric constant
Capacitance when metallic slab inserted between the plates $C = \left(\frac{\varepsilon_0 A}{d \quad t}\right)$ t = thickness of slab
Energy stored in a capacitor $u = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$
Energy loss when two isolated charged conductors are connected to each other $= \frac{1}{2}\frac{C_1 C_2 (V_1 - V_2)^2}{C_1 + C_2}$

EXERCISE - 1
Conceptual Questions

1. A parallel plate capacitor is charged to a certain voltage. Now, if the dielectric material (with dielectric constant k) is removed then the

(a) capacitance increases by a factor of k
(b) electric field reduces by a factor k
(c) voltage across the capacitor decreases by a factor k
(d) None of these

2. Two identical conducting balls having positive charges q_1 and q_2 are separated by a distance r.If they are made to touch each other and then separated to the same distance, the force between them will be

(a) less than before (b) same as before
(c) more than before (d) zero

3. A sphere of radius R has uniform volume charge density. The electric potential at a points (r <R) is

(a) due to the charge inside a sphere of radius r only
(b) due to the entire charge of the sphere
(c) due to the charge in the spherical sheel of inner and outer radii r and R, only
(d) independent of r

4. Eight drops of mercury of equal radius and possessing equal charge combine to form a big drop. The capacitance of bigger drop as compared to each small drop is

(a) 16 times (b) 8 times
(c) 4 times (d) 2 times

5. The capacitance of a metallic sphere is $1\mu F$, then it's radius is nearly

(a) 1.11 m (b) 10 m
(c) 9 km (d) 1.11 cm

6. Three charges 2 q, – q and – q are located at the vertices of an equilateral triangle. At the centre of the triangle

(a) the field is zero but potential is non-zero
(b) the field is non-zero, but potential is zero
(c) both field and potential are zero
(d) both field and potential are non-zero

7. Two conducting spheres of radii r_1 and r_2 are equally charged. The ratio of their potentials is

(a) r_1/r_2 (b) r_2/r_1 (c) r_1^2/r_2^2 (d) r_2^2/r_1^2

8. The electric potential due to a small electric dipole at a large distance r from the centre of the dipole is proportional to

(a) r (b) 1/r (c) $1/r^2$ (d) $1/r^3$

9. An electron of mass m and charge e is accelerated from rest through a potential difference V in vacuum. Its final speed will be

(a) $\sqrt{\frac{2eV}{m}}$ (b) $\sqrt{\frac{eV}{m}}$ (c) eV/2m (d) eV/m

10. A positive point charge q is carried from a point B to a point A in the electric field of a point charge + Q at O. If the permitivity of free space is ε_0, the work done in the process is given by

(a) $\frac{qQ}{4\pi\varepsilon_o}\left(\frac{1}{a}+\frac{1}{b}\right)$ (b) $\frac{qQ}{4\pi\varepsilon_o}\left(\frac{1}{a}-\frac{1}{b}\right)$

(c) $\frac{qQ}{4\pi\varepsilon_o}\left(\frac{1}{a^2}-\frac{1}{b^2}\right)$ (d) $\frac{qQ}{4\pi\varepsilon_o}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)$

11. Two concentric, thin metallic spheres of radii R_1 and R_2 $(R_1 > R_2)$ bear charges Q_1 and Q_2 respectively. Then the potential at distance r between R_1 and R_2 will be $\left(k=\frac{1}{4\pi\varepsilon_0}\right)$

(a) $k\left(\frac{Q_1+Q_2}{r}\right)$ (b) $k\left(\frac{Q_1}{r}+\frac{Q_2}{R_2}\right)$

(c) $k\left(\frac{Q_2}{r}+\frac{Q_1}{R_1}\right)$ (d) $k\left(\frac{Q_1}{R_1}+\frac{Q_2}{R_2}\right)$

12. Force between two plates of a capacitor is

(a) $\frac{Q}{\varepsilon_0 A}$ (b) $\frac{Q^2}{2\varepsilon_0 A}$

(c) $\frac{Q^2}{\varepsilon_0 A}$ (d) None of these

13. An alpha particle is accelerated through a potential difference of 10^6 volt. Its kinetic energy will be

(a) 1 MeV (b) 2 MeV (c) 4 MeV (d) 8 MeV

14. Two capacitors of capacitances C_1 and C_2 are connected in parallel across a battery. If Q_1 and Q_2 respectively be the charges on the capacitors, then $\frac{Q_1}{Q_2}$ will be equal to

(a) $\frac{C_2}{C_1}$ (b) $\frac{C_1}{C_2}$ (c) $\frac{C_1^2}{C_2^2}$ (d) $\frac{C_2^2}{C_1^2}$

15. A system of two parallel plates, each of area A, are separated by distances d_1 and d_2. The space between them is filled with dielectrics of permittivities ε_1 and ε_2. The permittivity of free space is ε_0. The equivalent capacitance of the system is

(a) $\frac{\varepsilon_1\varepsilon_2 A}{\varepsilon_2 d_1+\varepsilon_1 d_2}$ (b) $\frac{\varepsilon_1\varepsilon_2\varepsilon_0 A}{\varepsilon_1 d_1+\varepsilon_2 d_2}$

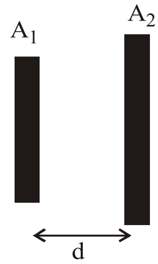
(c) $\frac{\varepsilon_0 A}{\varepsilon_1 d_1+\varepsilon_2 d_2}$ (d) $\frac{\varepsilon_0 A}{\varepsilon_1 d_2+\varepsilon_2 d_1}$

16. A large insulated sphere of radius r charged with Q units of electricity is placed in contact with a small insulated uncharged sphere of radius r′ and is then separated. The charge on smaller sphere will now be

(a) $Q(r'+r)$ (b) $Q(r+r')$

(c) $\dfrac{Q}{r'+r}$ (d) $\dfrac{Qr'}{r'+r}$

17. The capacitance of the capacitor of plate areas A_1 and A_2 ($A_1 < A_2$) at a distance d, as shown in figure is



(a) $\dfrac{\epsilon_0 (A_1 + A_2)}{2d}$

(b) $\dfrac{\epsilon_0 A_2}{d}$

(c) $\dfrac{\epsilon_0 \sqrt{A_1 A_2}}{d}$

(d) $\dfrac{\epsilon_0 A_1}{d}$

18. When air is replaced by a dielectric medium of force constant K, the maximum force of attraction between two charges, separated by a distance

(a) decreases K-times (b) increases K-times

(c) remains unchanged (d) becomes $\dfrac{1}{K^2}$ times

19. A conductor carries a certain charge. When it is connected to another uncharged conductor of finite capacity, then the energy of the combined system is

(a) more than that of the first conductor
(b) less than that of the first conductor
(c) equal to that of the first conductor
(d) uncertain

20. The magnitude of the electric field E in the annular region of a charged cylindrical capacitor

(a) is same throughout
(b) is higher near the outer cylinder than near the inner cylinder
(c) varies as $\dfrac{1}{r}$, where r is the distance from the axis
(d) varies as $\dfrac{1}{r^2}$, where r is the distance from the axis

21. A parallel plate condenser with oil between the plates (dielectric constant of oil K = 2) has a capacitance C. If the oil is removed, then capacitance of the capacitor becomes

(a) $\sqrt{2}\,C$ (b) $2C$ (c) $\dfrac{C}{\sqrt{2}}$ (d) $\dfrac{C}{2}$

22. An air capacitor C connected to a battery of e.m.f. V acquires a charge q and energy E. The capacitor is disconnected from the battery and a dielectric slab is placed between the plates. Which of the following statements is correct ?

(a) V and q decrease but C and E increase
(b) V remains unchange, but q, E and C increase
(c) q remains unchanged, C increases, V and E decrease
(d) q and C increase but V and E decrease.

23. Two parallel metal plates having charges + Q and – Q face each other at a certain distance between them. If the plates are now dipped in kerosene oil tank, the electric field between the plates will

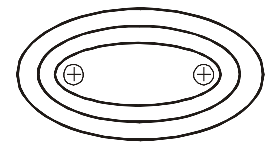
(a) remain same (b) become zero
(c) increases (d) decrease

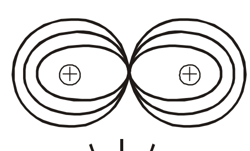
24. A parallel plate condenser has a uniform electric field E(V/m) in the space between the plates. If the distance between the plates is d(m) and area of each plate is A(m^2) the energy (joules) stored in the condenser is

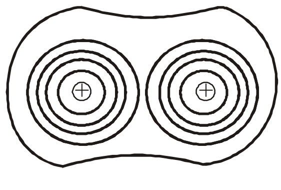
(a) E^2Ad/ϵ_0 (b) $\dfrac{1}{2}\epsilon_0 E^2$

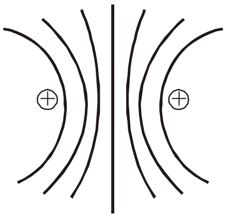
(c) $\epsilon_0 EAd$ (d) $\dfrac{1}{2}\epsilon_0 E^2Ad$

25. Which of the following figure shows the correct equipotential surfaces of a system of two positive charges?

(a)


(b)


(c)


(d)


EXERCISE - 2
Applied Questions

1. The positive terminal of 12 V battery is connected to the ground. Then the negative terminal will be at

(a) –6 V (b) +12 V (c) zero (d) –12 V

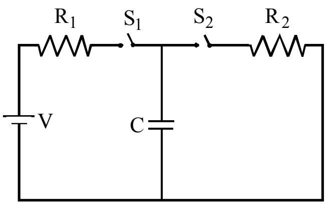
2. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. The potential at a distance of 2 cm from the centre of the sphere is

(a) zero (b) 10 V (c) 4 V (d) 10/3 V

3. Find the dipole moment of a system where the potential 2.0×10^{-5} V at a point P, 0.1m from the dipole is 3.0×10^4. (Use $\theta = 30°$).

(a) 2.57×10^{-17} Cm (b) 1.285×10^{-15} Cm
(c) 1.285×10^{-17} Cm (d) 2.57×10^{-15} Cm

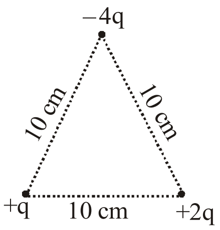
4. A battery of e.m.f. V volt, resistors R_1 and R_2, a condenser C and switches S_1 and S_2 are connected in a circuit shown. The condenser will get fully charged to V volt when



(a) S_1 and S_2 are both closed
(b) S_1 and S_2 are both open
(c) S_1 is open and S_2 is closed
(d) S_1 is closed and S_2 is open

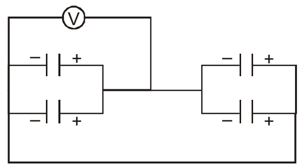
5. The electric potential at the surface of an atomic nucleus (Z = 50) of radius of 9×10^{-15} m is
(a) 80 V (b) 8×10^6 V (c) 9 V (d) 9×10^5 V

6. Three point charges +q, + 2q and – 4q where q = 0.1 μC, are placed at the vertices of an equilateral triangle of side 10 cm as shown in figure. The potential energy of the system is



(a) 3×10^{-3} J (b) -3×10^{-3} J
(c) 9×10^{-3} J (d) -9×10^{-3} J

7. The four capacitors, each of 25 μ F are connected as shown in fig. The dc voltmeter reads 200 V. The charge on each plate of capacitor is



(a) $\pm 2 \times 10^{-3}$ C (b) $\pm 5 \times 10^{-3}$ C
(c) $\pm 2 \times 10^{-2}$ C (d) $\pm 5 \times 10^{-2}$ C

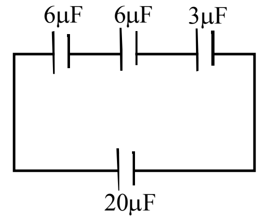
8. An air capacitor of capacity C = 10 μF is connected to a constant voltage battery of 12 volt. Now the space between the plates is filled with a liquid of dielectric constant 5. The (additional) charge that flows now from battery to the capacitor is
(a) 120 μC (b) 600 μC (c) 480 μC (d) 24 μC

9. A capacitor is charged to store an energy U. The charging battery is disconnected. An identical capacitor is now connected to the first capacitor in parallel. The energy in each of the capacitors is
(a) 3 U/2 (b) U (c) U/4 (d) U/2

10. Two capacitors when connected in series have a capacitance of 3 μF, and when connected in parallel have a capacitance of 16 μF. Their individual capacities are
(a) 1 μF, 2 μF (b) 6 μF, 2 μF
(c) 12 μF, 4 μF (d) 3 μF, 16 μF

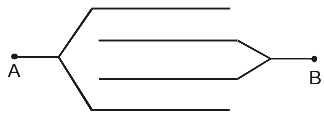
11. The capacity of a parallel plate condenser is 10 μF, when the distance between its plates is 8 cm. If the distance between the plates is reduced to 4 cm, then the capacity of this parallel plate condenser will be
(a) 5 μF (b) 10 μF (c) 20 μF (d) 40 μF

12. The capacitor, whose capacitance is 6, 6 and 3μF respectively are connected in series with 20 volt line. Find the charge on 3μF.



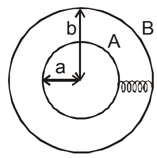
(a) 30 μc
(b) 60 μF
(c) 15 μF
(d) 90 μF

13. Four metallic plates each with a surface area of one side A, are placed at a distance d from each other. The two outer plates are connected to one point A and the two other inner plates to another point B as shown in the figure. Then the capacitance of the system is



(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{2\varepsilon_0 A}{d}$ (c) $\frac{3\varepsilon_0 A}{d}$ (d) $\frac{4\varepsilon_0 A}{d}$

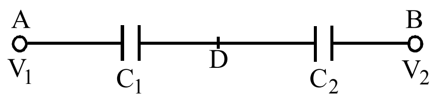
14. Two spherical conductors A and B of radii a and b (b>a) are placed concentrically in air. The two are connected by a copper wire as shown in figure. Then the equivalent capacitance of the system is



(a) $4\pi\varepsilon_0 \frac{ab}{b-a}$
(b) $4\pi\varepsilon_0 (a+b)$
(c) $4\pi\varepsilon_0 b$
(d) $4\pi\varepsilon_0 a$

15. A ball of mass 1 g carrying a charge 10^{-8} C moves from a point A at potential 600 V to a point B at zero potential. The change in its K.E. is
(a) -6×10^{-6} erg (b) -6×10^{-6} J
(c) 6×10^{-6} J (d) 6×10^{-6} erg

16. Two capacitors C_1 and C_2 in a circuit are joined as shown in figure. The potentials of points A and B are V_1 and V_2 respectively; then the potential of point D will be



(a) $\frac{(V_1+V_2)}{2}$ (b) $\frac{C_2V_1+C_1V_2}{C_1+C_2}$
(c) $\frac{C_1V_1+C_2V_2}{C_1+C_2}$ (d) $\frac{C_2V_1+C_1V_2}{C_1+C_2}$

17. A parallel plate capacitor with air between the plates is charged to a potential difference of 500V and then insulated. A plastic plate is inserted between the plates filling the whole gap. The potential difference between the plates now becomes 75V. The dielectric constant of plastic is
(a) 10/3 (b) 5 (c) 20/3 (d) 10

18. The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by a 400 volt supply. How much electrostatic energy is stored by the capacitor?
(a) 2.55×10^{-6} J (b) 1.55×10^{-6} J
(c) 8.15×10^{-6} J (d) 5.5×10^{-6} J

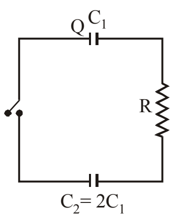
19. From a supply of identical capacitors rated 8 mF, 250V, the minimum number of capacitors required to form a composite 16 mF, 1000V is
(a) 2 (b) 4 (c) 16 (d) 32

20. Calculate the area of the plates of a one farad parallel plate capacitor if separation between plates is 1 mm and plates are in vacuum
(a) $18 \times 10^8\ m^2$ (b) $0.3 \times 10^8\ m^2$
(c) $1.3 \times 10^8\ m^2$ (d) $1.13 \times 10^8\ m^2$

21. A one microfarad capacitor of a TV is subjected to 4000 V potential difference. The energy stored in capacitor is
(a) 8 J (b) 16 J
(c) 4×10^{-3} J (d) 2×10^{-3} J

22. Two capacitors, $C_1 = 2\mu F$ and $C_2 = 8\ \mu F$ are connected in series across a 300 V source. Then
(a) the charge on each capacitor is 4.8×10^{-4} C
(b) the potential difference across C_1 is 60 V
(c) the potential difference across C_2 is 240 V
(d) the energy stroed in the system is 5.2×10^{-2} J

23. Two capacitors C_1 and $C_2 = 2C_1$ are connected in a circuit with a switch between them as shown in the figure. Initially the switch is open and C_1 holds charge Q. The switch is closed. At steady state, the charge on each capacitor will be



(a) Q, 2Q (b) $\frac{Q}{3}, \frac{2Q}{3}$
(c) $\frac{3Q}{2}, 3Q$ (d) $\frac{2Q}{3}, \frac{4Q}{3}$

24. Two capacitors of capacitance C are connected in series. If one of them is filled with dielectric substance k, what is the effective capacitance ?
(a) $\frac{kC}{(1+k)}$ (b) $C(k+1)$
(c) $\frac{2kC}{1+k}$ (d) None of these

25. The potential at a point x (measured in μ m) due to some charges situated on the x-axis is given by $V(x) = 20/(x^2 - 4)$ volt
The electric field E at x = 4 μ m is given by
(a) (10/9) volt/μ m and in the +ve x direction
(b) (5/3) volt/μ m and in the –ve x direction
(c) (5/3) volt/μ m and in the +ve x direction
(d) (10/9) volt/μ m and in the –ve x direction

26. Capacitance (in F) of a spherical conductor with radius 1 m is
(a) 1.1×10^{-10} (b) 10^6
(c) 9×10^{-9} (d) 10^{-3}

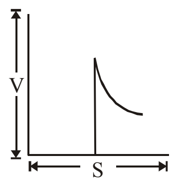
27. Two metal pieces having a potential difference of 800 V are 0.02 m apart horizontally. A particle of mass 1.96×10^{-15} kg is suspended in equilibrium between the plates. If e is the elementary charge, then charge on the particle is
(a) 8 (b) 6 (c) 0.1 (d) 3

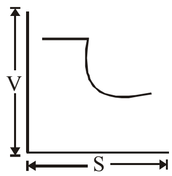
28. Identical charges – q each are placed at 8 corners of a cube of each side b. Electrostatic potential energy of a charge + q which is placed at the centre of cube will be
(a) $\frac{-4\sqrt{2}\,q^2}{\pi\,\varepsilon_0 b}$ (b) $\frac{-8\sqrt{2}\,q^2}{\pi\,\varepsilon_0 b}$
(c) $\frac{-4q^2}{\sqrt{3}\,\pi\,\varepsilon_0 b}$ (d) $\frac{-8\sqrt{2}\,q^2}{\pi\varepsilon_0 b}$

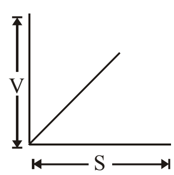
29. A charge +q is fixed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$, ... upto ∞ on X-axis and charge –q is fixed on each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$, ... upto ∞ . Here x_0 is a positive constant. Take the potential at a point due to a charge Q at a distance r from it to be $\frac{Q}{4\pi\varepsilon_0 r}$. Then the potential at the origin due to above system of charges will be
(a) zero (b) $\frac{q}{8\pi\varepsilon_0 x_0 \log_e 2}$
(c) infinity (d) $\frac{q \log_e 2}{4\pi\varepsilon_0 x_0}$

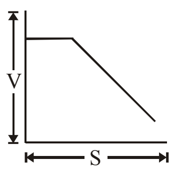
30. Two equally charged spheres of radii a and b are connected together. What will be the ratio of electric field intensity on their surfaces?
(a) $\frac{a}{b}$ (b) $\frac{a^2}{b^2}$ (c) $\frac{b}{a}$ (d) $\frac{b^2}{a^2}$

31. In a hollow spherical shell, potential (V) changes with respect to distance (s) from centre as

(a)


(b)


(c)


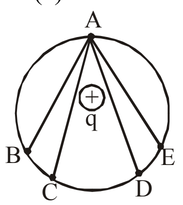
(d)


32. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance 2C is similary charged to a potential difference 2V. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is
(a) zero (b) $\frac{3}{2}CV^2$ (c) $\frac{25}{6}CV^2$ (d) $\frac{9}{2}CV^2$

33. A solid conducting sphere having a charge Q is surrounding by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surfaces is

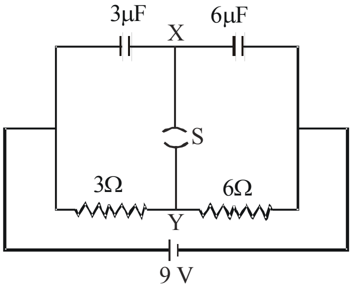
(a) V (b) 2V (c) 4V (d) -2 V

34. In the electric field of an point charge q, a certain charge is carried from point A to B, C, D and E. Then the work done is



(a) least along the path AB
(b) least along the path AD
(c) zero along any one of the path AB, AC, AD andAE
(d) least along AE

35. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is



(a) 0 (b) 54 μC (c) 27μC (d) 81 μC

36. If a slab of insulating material 4×10^{-5} m thick is introduced between the plates of a parallel plate capacitor, the distance between the plates has to be increased by 3.5×10^{-5} m to restore the capacity to original value. Then the dielectric constant of the material of slab is

(a) 8 (b) 6 (c) 12 (d) 10

37. Three capacitors each of capacity 4μF are to be connected in such a way that the effective capacitance is 6 μF. This can be done by

(a) connecting two in parallel and one in series
(b) connecting all of them in series
(c) connecting them in parallel
(d) connecting two in series and one in parallel

38. If we increase 'd' of a parallel plate condenser to '2d' and fill wax to the whole empty space between its two plate, then capacitance increase from 1pF to 2pF. What is the dielectric constant of wax?

(a) 2 (b) 4 (c) 4 (d) 8

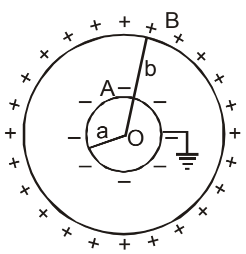
39. Two spherical conductors A and B of radii a and b (b > a) are placed concentrically in air. B is given charge +Q and A is earthed. The equivalent capacitance of the system is

(a) $4\pi\varepsilon_0 \dfrac{ab}{b-a}$

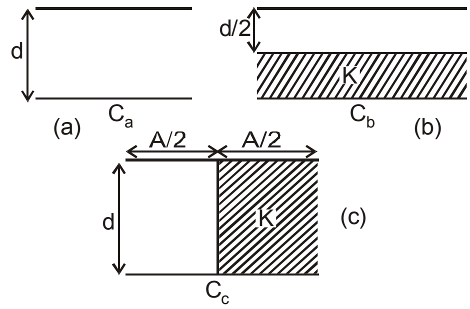
(b) $4\pi\varepsilon_0 (a+b)$

(c) $4\pi\varepsilon_0 b$

(d) $4\pi\varepsilon_0 \left(\dfrac{b^2}{b-a}\right)$

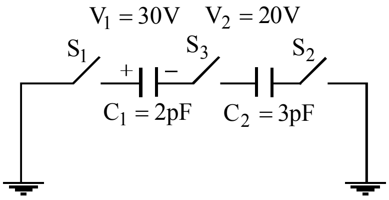


40. The capacitance of a parallel plate capacitor is C_a (Fig. a). A dielectric of dielectric constant K is inserted as shown in fig (b) and (c). If C_b and C_c denote the capacitances in fig (b) and (c), then



(a) both $C_b, C_c > C_a$ (b) $C_c > C_a$ while $C_b > C_a$
(c) both $C_b, C_c < C_a$ (d) $C_a = C_b = C_c$

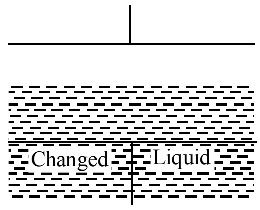
41. In the circuit shown, which of the following statements is true if V_1 (potential across C_1) is 30 V and V_2 (potential across C_2) is 20 V?



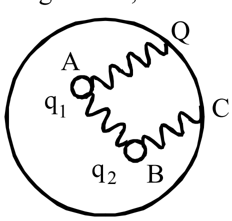
(a) With S_1 closed, $V_1 = 15$ V, $V_2 = 25$ V
(b) With S_3 closed, $V_1 = V_2 = 25$ V
(c) With S_1 and S_2 closed, $V_1 = V_2 = 0$
(d) With S_1 and S_3 closed, $V_1 = 30$ V, $V_2 = 20$ V

42. A parallel plate capacitor is located horizontally such that one of the plates is submerged in a liquid while the other is above the liquid surface. When plates are charged the level of liquid

(a) rises
(b) falls
(c) remains unchanged
(d) may rise or fall depending on the amount of charge



43. Two small conductors A and B are given charges q_1 and q_2 respectively. Now they are placed inside a hollow metallic conductor C carrying a charge Q. If all the three conductors A, B and C are connected by a conducting wire as shown, the charges on A, B and C will be respectively



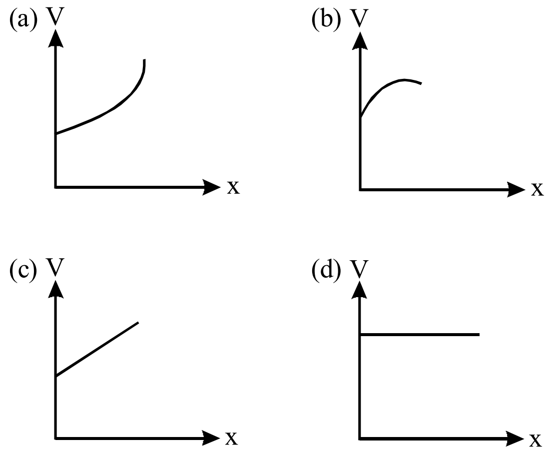
(a) $\dfrac{q_1+q_2}{2}, \dfrac{q_1+q_2}{2}, Q$

(b) $\dfrac{Q+q_1+q_3}{3}, \dfrac{Q+q_1+q_2}{3}, \dfrac{Q+q_1+q_2}{3}$

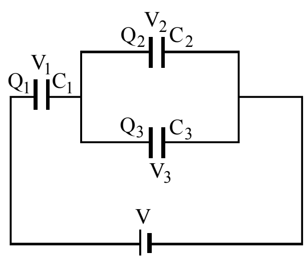
(c) $\dfrac{q_1+q_2+Q}{2}, \dfrac{q_1+q_2+Q}{2}, 0$

(d) $0, 0, Q+q_1+q_2$

44. Between the plates of a parallel plate capacitor dielectric plate is introduced just to fill the space between the plates. The capacitor is charged and later disconnected from the battery. The dielectric plate is slowly drawn out of the capacitor parallel to plates. The plot of the potential difference V across the plates and the length of the dielectric plate drawn out is

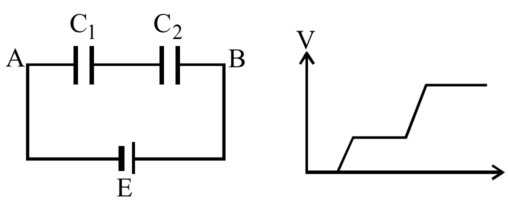


45. Three capacitors C_1, C_2 and C_3 are connected to a battery as shown. With symbols having their usual meanings, the correct conditions are



(a) $Q_1 = Q_2 = Q_3$ and $V_1 = V_2 = V$
(b) $V_1 = V_2 = V_3 = V$
(c) $Q_1 = Q_2 + Q_3$ and $V = V_1 = V_2$
(d) $Q_2 = Q_3$ and $V_2 = V_3$

46. Figure (i) shows two capacitors connected in series and connected by a battery. The graph (ii) shows the variation of potential as one moves from left to right on the branch AB containing the capacitors. Then



(i) (ii)

(a) $C_1 = C_2$
(b) $C_1 < C_2$
(c) $C_1 > C_2$
(d) C_1 and C_2 cannot be compared

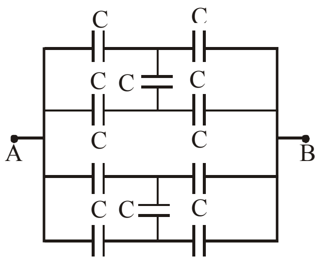
47. Two vertical metallic plates carrying equal and opposite charges are kept parallel to each other like a parallel plate capacitor. A small spherical metallic ball is suspended by a long insulated thread such that it hangs freely in the centre of the two metallic plates. The ball, which is uncharged, is taken slowly towards the positively charged plate and is made to touch that plate. Then the ball will
(a) stick to the positively charged plate
(b) come back to its original position and will remain there
(c) oscillate between the two plates touching each plate in turn
(d) oscillate between the two plates without touch them

48. Two parallel plate capacitors of capacitances C and 2C are connected in parallel and charged to a potential difference V. The battery is then disconnected and the region between the plates of the capacitor C is completely filled with a material fo dielectric constant K. The potential difference across the capacitors now becomes

(a) $\frac{3V}{K+2}$ (b) KV (c) $\frac{V}{K}$ (d) $\frac{3}{KV}$

49. A parallel plate capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with this capacitor are given by Q_0, V_0, E_0, and U_0 respectively. A dielectric slab is now introduced to fill the space between the plates with the battery still in connection. The corresponding quantities now given by Q, V, E and U are related to the previous ones as
(a) $Q > Q_0$ (b) $V > V_0$ (c) $E > E_0$ (d) $U < U_0$

50. The effective capacitance of combination of combination of equal capacitors between points A and B shown in figure is



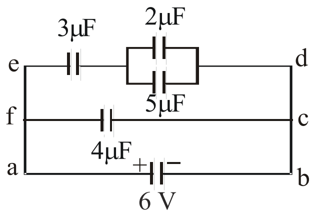
(a) C (b) 2C (c) 3C (d) $\frac{C}{2}$

51. A parallel plate capacitor of plate area A and plate separation d is charged to potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of capacitor so as to fill the space between the plates. If Q, E and W denote respectively, the magnitude of charge on each plate electric field between the plates (after the slab is inserted), and work done on the system, in question, in the process of inserting the slab, then which is wrong ?

(a) $Q = \frac{\varepsilon_0 AV}{d}$ (b) $Q = \frac{\varepsilon_0 KAV}{d}$

(c) $E = \frac{V}{Kd}$ (d) $W = \frac{\varepsilon_0 AV^2}{2d}\left(1 - \frac{1}{K}\right)$

52. In the circuit given below, the charge in μC, on the capacitor having 5 μF is



(a) 4.5 (b) 9 (c) 7 (d) 15

53. If a charge – 150 nC is given to a concentric spherical shell and a charge +50 nC is placed at its centre then the charge on inner and outer surface of the shell is

(a) –50 nC, –100 nC (b) +50 nC, –200 nC
(c) –50 nC, –200 nC (d) 50 nC, 100 nC

54. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be

(a) 1/2 (b) 1 (c) 2 (d) 1/4

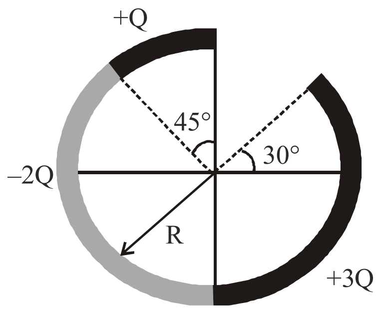
55. Four point charges q, q, q and – 3q are placed at the vertices of a regular tetrahedron of side L. The work done by electric force in taking all the charges to the centre of the tetrahedron is (where $k = \frac{1}{4\pi\varepsilon_0}$)

(a) $\frac{6kq^2}{L}$ (b) $\frac{-6kq^2}{L}$ (c) $\frac{12kq^2}{L}$ (d) zero

56. Two identical particles each of mass m and having charges –q and +q are revolving in a circle of radius r under the influence of electric attraction. Kinetic energy of each particle is $\left(k = \frac{1}{4\pi\varepsilon_0}\right)$

(a) $kq^2/4r$ (b) $kq^2/2r$ (c) $kq^2/8r$ (d) kq^2/r

57. Figure shows three circular arcs, each of radius R and total charge as indicated. The net electric potential at the centre of curvature is



(a) $\frac{Q}{2\pi\varepsilon_0 R}$

(b) $\frac{Q}{4\pi\varepsilon_0 R}$

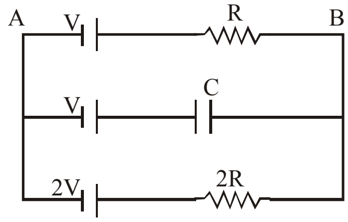
(c) $\frac{2Q}{\pi\varepsilon_0 R}$

(d) $\frac{Q}{\pi\varepsilon_0 R}$

58. If the potential of a capacitor having capacity 6 μF is increased from 10 V to 20 V, then increase in its energy will be

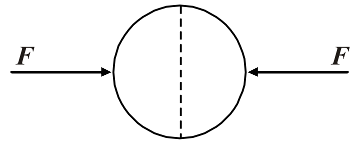
(a) 4×10^{-4} J (b) 4×10^{-4} J
(c) 9×10^{-4} J (d) 12×10^{-6} J

59. In the given circuit with steady current, the potential drop across the capacitor must be



(a) $\frac{2V}{3}$ (b) $\frac{V}{3}$ (c) $\frac{V}{2}$ (d) V

60. A uniformly charged thin spherical shell of radius *R* carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force *F* (see figure). *F* is proportional to



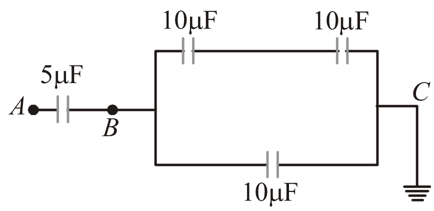
(a) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (b) $\frac{1}{\varepsilon_0}\sigma^2 R$

(c) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (d) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

61. A dielectric slab of thickness *d* is inserted in a parallel plate capacitor whose negative plate is at $x = 0$ and positive plate is at $x = 3d$. The slab is equidistant from the plates. the capacitor is given some charge. As one goes from 0 to $3d$

(a) the magnitude of the electric field remains the same
(b) the direction of the electric field remains the same
(c) the electric potential decreases continuously
(d) the electric potential increases at first, then decreases and again increases

62. In the given circuit if point *C* is connected to the earth and a potential of +2000V is given to the point A, the potential at B is



(a) 1500V (b) 1000V (c) 500V (d) 400V

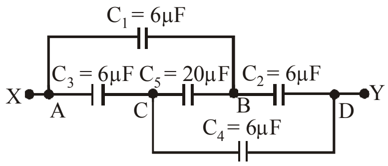
63. A 4 μF capacitor, a resistance of 2.5 MΩ is in series with 12V battery. Find the time after which the potential difference across the capacitor is 3 times the potential diference across the resistor. [Given In (b) = 0.693]

(a) 13.86s (b) 6.93 *s* (c) 7*s* (d) 14 *s*

64. If a capacitor 900 μF is charged to 100 V and its total energy is transferred to a capacitor of capacitance 100 μF then its potential is

(a) 200 V (b) 30 V (c) 300 V (d) 400 V

65. What is the effective capacitance between points X and Y?



(a) 24 μF
(b) 18 μF
(c) 12 μF
(d) 6 μF

66. In a parallel plate capacitor, the distance between the plates is d and potential difference across plates is V. Energy stored per unit volume between the plates of capacitor is

(a) $\frac{Q^2}{2V^2}$ (b) $\frac{1}{2}\varepsilon_0\frac{V^2}{d^2}$

(c) $\frac{1}{2}\frac{V^2}{\varepsilon_0 d^2}$ (d) $\frac{1}{2}\varepsilon_0\frac{V^2}{d}$

67. A capacitor C_1 is charged to a potential difference V. The charging battery is then removed and the capacitor is connected to an uncharged capacitor C_2. The potential difference across the combination is

(a) $\frac{VC_1}{(C_1+C_2)}$ (b) $V\left(1+\frac{C_2}{C_1}\right)$

(c) $V\left(1+\frac{C_1}{C_2}\right)$ (d) $\frac{VC_2}{(C_1+C_2)}$

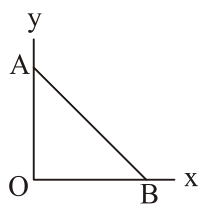
68. As per this diagram a point charge +q is placed at the origin O. Work done in taking another point charge – Q from the point A [coordinates (0, a)] to another point B [coordinates (a, 0)] along the straight path AB is

(a) zero

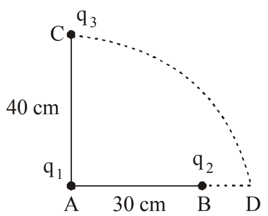
(b) $\left(\frac{-qQ}{4\pi\varepsilon_0}\frac{1}{a^2}\right)\sqrt{2}a$

(c) $\left(\frac{qQ}{4\pi\varepsilon_0}\frac{1}{a^2}\right)\cdot\frac{a}{\sqrt{2}}$

(d) $\left(\frac{qQ}{4\pi\varepsilon_0}\frac{1}{a^2}\right)\cdot\sqrt{2}a$

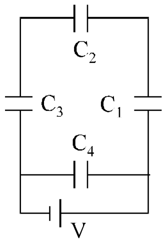


69. Two charges q_1 and q_2 are placed 30 cm apart, as shown in the figure. A third charge q_3 is moved along the arc of a circle of radius 40 cm from C to D. The change in the potential energy of the system is $\frac{q_3}{4\pi\epsilon_0}k$, , where k is



(a) $8q_1$ (b) $6q_1$ (c) $8q_2$ (d) $6q_2$

70. A network of four capacitors of capacity equal to $C_1 = C$, $C_2 = 2C$, $C_3 = 3C$ and $C_4 = 4C$ are conducted to a battery as shown in the figure. The ratio of the charges on C_2 and C_4 is



(a) 4/7 (b) 3/22 (c) 7/4 (d) 22/3

71. A series combination of n_1 capacitors, each of value C_1, is charged by a source of potential difference 4 *V*. When another parallel combination of n_2 capacitors, each of value C_2, is charged by a source of potential difference *V*, it has the same (total) energy stored in it, as the first combination has. The value of C_2 , in terms of C_1, is then

(a) $\frac{2C_1}{n_1 n_2}$ (b) $16\frac{n_2}{n_1}C_1$ (c) $2\frac{n_2}{n_1}C_1$ (d) $\frac{16C_1}{n_1 n_2}$

72. A condenser of capacity C is charged to a potential difference of V_1. The plates of the condenser are then connected to an ideal inductor of inductance L. The current through the inductor when the potential difference across the condenser reduces to V_2 is

(a) $\left(\frac{C(V_1^2-V_2^2)}{L}\right)^{1/2}$ (b) $\left(\frac{C(V_1-V_2)^2}{L}\right)^{1/2}$

(c) $\frac{C(V_1^2-V_2^2)}{L}$ (d) $\frac{C(V_1-V_2)}{L}$

DIRECTIONS (for Qs. 73 to 75) : Each question contains STATEMENT-1 and STATEMENT-2. Choose the correct answer (ONLY ONE option is correct) from the following-

(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

73. **Statement 1 :** Each of the plates of a parallel-plate capacitor is given equal positive charge Q. The charges on the facing surfaces will be same.

Statement 2 : A negative charge (–Q) will be induced on each of the facing surfaces.

74. **Statement 1 :** Electric potential and electric potential energy are different quantities.

Statement 2 : For a system of positive test charge and point charge electric potential energy = electric potential.

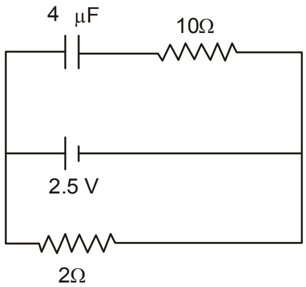
75. **Statement I :** Two equipotential surfaces cannot cut each other.

Statement II : Two equipotential surfaces are parallel to each other.

EXERCISE - 3
Exemplar & Past Years NEET/AIPMT Questions

Exemplar Questions

1. A capacitor of 4 μF is connected as shown in the circuit. The internal resistance of the battery is 0.5Ω. The amount of charge on the capacitor plates will be



(a) 0 μC (b) 4 μC (c) 16 μC (d) 8 μC

2. A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge

(a) remains a constant because the electric field is uniform
(b) increases because the charge moves along the electric field
(c) decreases because the charge moves along the electric field
(d) decreases because the charge moves opposite to the electric field

3. Figure shows some equipotential lines distributed in space. A charged object is moved from point A to point B.

(a) The work done in Fig. (i) is the greatest
(b) The work done in Fig. (ii) is least
(c) The work done is the same in Fig. (i), Fig.(ii) and Fig. (iii)
(d) The work done in Fig. (iii) is greater than Fig. (ii) but equal to that in

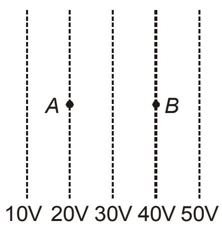


Fig. (i)

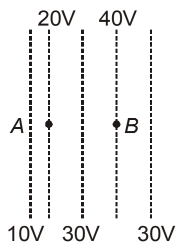


Fig. (ii)

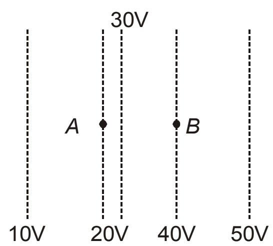


Fig. (iii)

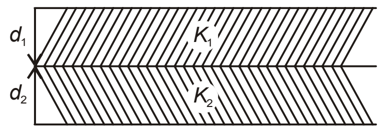
4. The electrostatic potential on the surface of a charged conducting sphere is 100V. Two statements are made in this regard S_1 at any point inside the sphere, electric intensity is zero. S_2 at any point inside the sphere, the electrostatic potential is 100V. Which of the following is a correct statement?

(a) S_1 is true but S_2 is false
(b) Both S_1 and S_2 are false
(c) S_1 is true, S_2 is also true and S_1 is the cause of S_2
(d) S_1 is true, S_2 is also true but the statements are independant

5. Equipotentials at a great distance from a collection of charges whose total sum is not zero are approximately

(a) spheres (b) planes
(c) paraboloids (d) ellipsoids

6. A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant K_1 and the other has thickness d_2 and dielectric constant K_2 as shown in figure. This arrangement can be thought as a dielectric slab of thickness d (= $d_1 + d_2$) and effective dielectric constant K. The K is

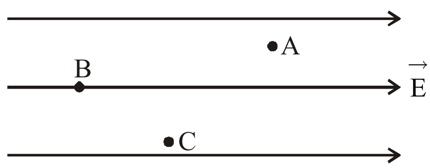


(a) $\dfrac{K_1d_1 + K_2d_2}{d_1 + d_2}$ (b) $\dfrac{K_1d_1 + K_2d_2}{K_1 + K_2}$

(c) $\dfrac{K_1K_2(d_1 + d_2)}{(K_1d_1 + K_2d_2)}$ (d) $\dfrac{2K_1K_2}{K_1 + K_2}$

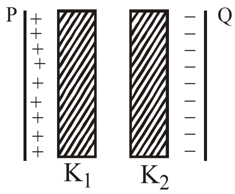
NEET/AIPMT (2013-2017) Questions

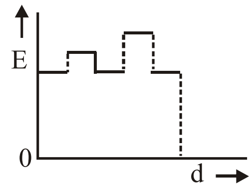
7. A, B and C are three points in a uniform electric field. The electric potential is ***[2013]***

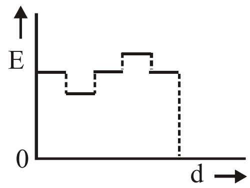


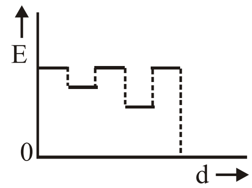
(a) maximum at B
(b) maximum at C
(c) same at all the three points A, B and C
(d) maximum at A

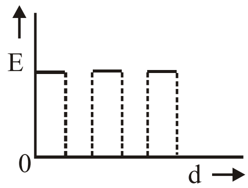
8. Two thin dielectric slabs of dielectric constants K_1 and K_2 ($K_1 < K_2$) are inserted between plates of a parallel plate capacitor, as shown in the figure. The variation of electric field 'E' between the plates with distance 'd' as measured from plate P is correctly shown by : ***[2014]***



(a) 

(b) 

(c) 

(d) 

9. A conducting sphere of radius R is given a charge Q. The electric potential and the electric field at the centre of the sphere respectively are: ***[2014]***

(a) Zero and $\dfrac{Q}{4\pi\varepsilon_0 R^2}$

(b) $\dfrac{Q}{4\pi\varepsilon_0 R}$ and Zero

(c) $\dfrac{Q}{4\pi\varepsilon_0 R}$ and $\dfrac{Q}{4\pi\varepsilon_0 R^2}$

(d) Both are zero

10. In a region, the potential is represented by $V(x, y, z) = 6x - 8xy - 8y + 6yz$, where V is in volts and x, y, z are in metres. The electric force experienced by a charge of 2 coulomb situated at point (1, 1, 1) is : ***[2014]***

(a) $6\sqrt{5}$ N (b) 30 N

(c) 24 N (d) $4\sqrt{35}$ N

11. A parallel plate air capacitor of capacitance C is connected to a cell of emf V and then disconnected from it. A dielectric slab of dielectric constant K, which can just fill the air gap of the capacitor, is now inserted in it. Which of the following is incorrect ? ***[2015]***

(a) The energy stored in the capacitor decreases K times.

(b) The chance in energy stored is $\frac{1}{2}CV^2\left(\frac{1}{K}-1\right)$

(c) The charge on the capacitor is not conserved.

(d) The potential difference between the plates decreases K times.

12. If potential (in volts) in a region is expressed as $V(x, y, z) = 6xy - y + 2yz$, the electric field (in N/C) at point (1, 1, 0) is : ***[2015 RS]***

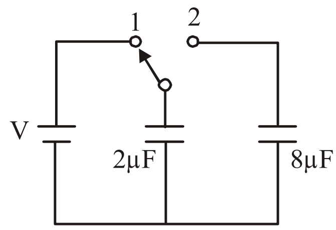
(a) $-(6\hat{i}+5\hat{j}+2\hat{k})$ (b) $-(2\hat{i}+3\hat{j}+\hat{k})$

(c) $-(6\hat{i}+9\hat{j}+\hat{k})$ (d) $-(3\hat{i}+5\hat{j}+3\hat{k})$

13. A parallel plate air capacitor has capacity 'C' distance of separation between plates is 'd' and potential difference 'V' is applied between the plates. Force of attraction between the plates of the parallel plate air capacitor is : ***[2015 RS]***

(a) $\dfrac{CV^2}{2d}$ (b) $\dfrac{CV^2}{d}$ (c) $\dfrac{C^2V^2}{2d^2}$ (d) $\dfrac{C^2V^2}{2d^2}$

14.



A capacitor of 2μF is charged as shown in the diagram. When the switch S is turned to position 2, the percentage of its stored energy dissipated is : ***[2016]***

(a) 0% (b) 20%

(c) 75% (d) 80%

15. A capacitor is charged by a battery. The battery is removed and another identical uncharged capacitor is connected in parallel. The total electrostatic energy of resulting system:

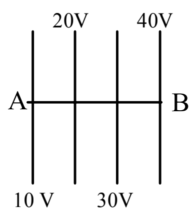
(a) decreases by a factor of 2 ***[2017]***

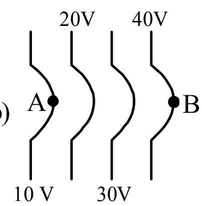
(b) remains the same

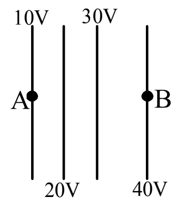
(c) increases by a factor of 2

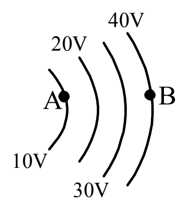
(d) increases by a factor of 4

16. The diagrams below show regions of equipotentials. ***[2017]***

(a) 

(b) 

(c) 

(d) 

A positive charge is moved from A to B in each diagram.

(a) In all the four cases the work done is the same

(b) Minimum work is required to move q in figure (a)

(c) Maximum work is required to move q in figure (b)

(d) Maximum work is required to move q in figure (c)

Hints & Solutions

EXERCISE - 1

1. (d) 2. (c) 3. (a) 4. (d) 5. (c)

6. (b) Potential at the centre of the triangle,

$$V = \frac{\Sigma q}{4\pi\varepsilon_0 r} = \frac{2q - q - q}{4\pi\varepsilon_0 r} = 0$$

Obviously, $E \neq 0$

7. (b) As $V = \frac{Q}{4\pi\varepsilon_0 r}$ i.e. $V \propto \frac{1}{r}$

$$\therefore \frac{V_1}{V_2} = \frac{r_2}{r_1}$$

8. (c) Due to small dipole, $V \propto \frac{1}{r^2}$.

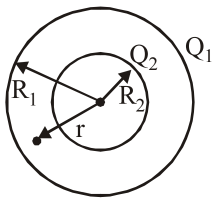
9. (a) K.E. = Work done = eV

$$\frac{1}{2}mv^2 = eV \quad \therefore \quad v = \sqrt{\frac{2eV}{m}}$$

10. (b) $W_{BA} = q(V_A - V_B)$

$$= q\left[\frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}\right] = \frac{qQ}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

11. (c)

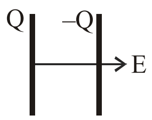


$$V_r = \frac{Q_2}{4\pi\varepsilon_0 r} + \frac{Q_1}{4\pi\varepsilon_0 R_1}$$

$$V_r = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_2}{r} + \frac{Q_1}{R_1}\right)$$

12. (b) The magnitude of electric field by any one plate is

$$\frac{\sigma}{2\varepsilon_o} \text{ or } \frac{Q}{2A\varepsilon_o}$$



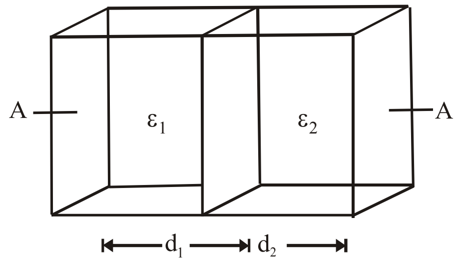
Now force magnitude is $|Q||E|$ i.e. $|F| = \frac{Q^2}{2A\varepsilon_o}$

13. (b) Charge on α particle, $q = 2e$.
K.E. = work done = $q \times V = 2e \times 10^6 V = 2$ MeV.

14. (b) In parallel, potential is same, say V

$$\frac{Q_1}{Q_2} = \frac{C_1 V}{C_2 V} = \frac{C_1}{C_2}$$

15. (a)



$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \frac{1}{C_s} = \frac{1}{\frac{\varepsilon_1 A}{d_1} + \frac{\varepsilon_2 A}{d_2}}$$

$$\Rightarrow \frac{1}{C_s} = \frac{1}{A}\left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right) \Rightarrow C_s = \frac{A\varepsilon_1\varepsilon_2}{\varepsilon_2 d_1 + \varepsilon_1 d_2}$$

16. (d) Common potential, $V = \frac{Q}{4\pi\varepsilon_0 r} + \frac{0}{4\pi\varepsilon_0 r}$

$$= \frac{Q + 0}{4\pi\varepsilon_0 (r + r')}$$

$\therefore$ charge on smaller sphere of radius r' is

$$4\pi\varepsilon_0 r' \times V = \frac{Qr'}{r + r'}$$

17. (d) $C = \frac{\epsilon_0 A}{d}$

$A \rightarrow$ common area, Here $A = A_1$

18. (a) In air $F_{air} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

In medium $F_m = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{Kr^2}$

$\therefore \frac{F_m}{F_{air}} = \frac{1}{K} \Rightarrow F_m = \frac{F_{air}}{K}$ (decreases K-times)

19. (b) Energy will be lost during transfer of charge (heating effect).

20. (c) $E \propto \frac{\lambda}{2\pi\varepsilon_0 r}$ hence $E \propto \frac{1}{r}$

21. (d) When oil is placed between space of plates

$$C = \frac{2A\varepsilon_0}{d} \quad ...(1)$$

$$\left[\because C = \frac{KA\varepsilon_0}{d}, \text{ where } K = 2\right]$$

When oil is removed $C' = \frac{A\varepsilon_0}{d}$ .... (2)

on comparing both equation, weget $C' = C/2$

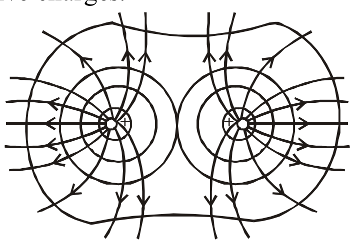
22. (c) When a battery across the plates of capacitor is disconnected and dielectric slab is placed in between the plates, then
(i) capacity C increases
(ii) charge q remains unchanged
(iii) potential V decreases
(iv) energy E decreases

23. (d) Electric field $E = \frac{\sigma}{\varepsilon} = \frac{Q}{A\varepsilon}$
ε of kerosine oil is more than that of air.
As ε increases, E decreases.

24. (d) $U = \frac{1}{2}CV^2$

$$U = \frac{1}{2}\left(\frac{A \epsilon_0}{d}\right)(Ed)^2 = \frac{1}{2}A \epsilon_0 E^2 d$$

25. (c) Equipotential surfaces are normal to the electric field lines. The following figure shows the equipotential surfaces along with electric field lines for a system of two positive charges.



EXERCISE - 2

1. (d) When negative terminal is grounded, positive terminal of battery is at +12 V. When positive terminal is grounded, the negative terminal will be at –12 V.
2. (b) Potential at any point inside the sphere = potential at the surface of the sphere = 10V.
3. (a) $$\frac{Vr^2}{k\cos\theta} = qd = \frac{(2.0\times10^{-5}\,V)(0.1m)^2}{\left\{9.0\times10^9\,\frac{m}{\left(\frac{C}{V}\right)}\right\}\left(\frac{\sqrt{3}}{2}\right)} = 2.57\times10^{-17}\,Cm.$$

Note that the units cancel to leave units appropriate for a dipole moment.
4. (d) When S_1 is closed and S_2 is opened, the capacitor will get charged to a potential difference of V volts.
5. (b) $V = \frac{q}{4\pi\varepsilon_o r} = \frac{(Ze)}{4\pi \epsilon_0 r}$

$$9\times10^9\,\frac{(50\times1.6\times10^{-19})}{9\times10^{-15}} = 8\times10^6\,V$$

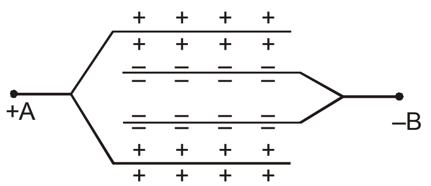
6. (d) $U = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q^2}{a} - \frac{8q^2}{a} - \frac{4q^2}{a}\right)$

$$\Rightarrow U = \frac{1}{4\pi\varepsilon_0}\left(-\frac{10q^2}{a}\right)$$

$$\Rightarrow U = -\frac{9\times10^9\times10\times(0.1\times10^{-6})^2}{\left(\frac{10}{100}\right)}$$

$\Rightarrow U = -9\times10^{-3}J$

7. (b) Charge on each plate of each capacitor
$Q = \pm CV = \pm 25\times10^6 \times 200 = \pm5\times10^{-3}\,C$
8. (c) $q_1 = C_1V = 10\times12 = 120\,\mu C$
$q_2 = C_2V = KC_1\times V = 5\times10\times12 = 600\,\mu C$
Additional charge that flows
$= q_2 - q_1 = 600 - 120 = 480\,\mu C.$
9. (c) As battery is disconnected, total charge Q is shared equally by two capacitors.
Energy of each capacitor $= \frac{(Q/2)^2}{2C} = \frac{1}{4}\frac{Q^2}{2C} = \frac{1}{4}U.$
10. (c) $C_s = \frac{C_1C_2}{C_1+C_2} = 3$
$C_p = C_1 + C_2 = 16 \;\therefore\; C_1C_2 = 48$
$C_1 - C_2 = \sqrt{(C_1+C_2)^2 - 4C_1C_2}$
$= \sqrt{16^2 - 4\times48} = \sqrt{64} = 8$
$C_1 + C_2 = 16\mu F$
$C_1 - C_2 = 8\mu F$
$\Rightarrow 2C_1 = 24\mu F \Rightarrow C_1 = 12\mu F$
$\therefore C_2 = \frac{48}{12} = 4\mu F$
11. (c) C = 10 μF ; d = 8 cm
C' = ? ; d' = 4 cm
$C = \frac{A\epsilon_0}{d} \Rightarrow C \alpha \frac{1}{d}$
If d is halved then C will be doubled.
Hence C' = 2C = 2 × 10 μF = 20 μF
12. (a) In series $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ and charge on each capacitor is same.
13. (b) It consists of two capacitors in parallel, therefore, the total capacitance is $= \frac{2\epsilon_0 A}{d}$



(The plates of B, having negative charge do not constitute a capacitor).
14. (c) All the charge given to inner sphere will pass on to the outer one. So capacitance that of outer one is $4\pi\epsilon_0 b$.
15. (c) As work is done by the field, K.E. of the body increases by

$K.E. = W = q(V_A - V_B)$

$= 10^{-8}(600-0) = 6\times10^{-6}$ J

16. (c) Consider the potential at D be 'V'.
Potential drop across C_1 is $(V - V_1)$ and C_2 is $(V_2 - V)$

$\therefore q_1 = C_1(V - V_1),\ q_2 = C_2(V_2 - V)$

As $q_1 = q_2$ [capacitors are in series]

$\therefore C_1(V - V_1) = C_2(V_2 - V)$

$$v = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

17. (c) $V_0 = \dfrac{q}{C_0}$

$$V = \frac{q}{C} \Rightarrow \frac{V}{V_0} = \frac{C_0}{C}$$

$$\Rightarrow \frac{C_0}{C} = \frac{500}{75} = \frac{20}{3}$$

By definition, $C = kC_0 \Rightarrow k = \dfrac{20}{3}$

18. (a) Here, $A = 90\text{ cm}^2 = 90\times10^{-4}\text{ m}^2$;
$d = 2.5\text{ mm} = 2.5\times10^{-3}$ m; V = 400 volt

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.854\times10^{-12}\times90\times10^{-4}}{2.5\times10^{-3}}$$

$= 3.187\times10^{-11}$ F

$$W = \frac{1}{2}CV^2 = \frac{1}{2}\times3.187\times10^{-11}\times(400)^2$$

$= 2.55\times10^{-6}$ J

19. (d) Let 'n' such capacitors are in series and such 'm' such branch are in parallel.

$\therefore 250\times n = 1000 \quad \therefore n = 4$ ...(i)

Also $\dfrac{8}{n}\times m = 16$

$m = \dfrac{16\times n}{8} = 8$ ...(ii)

$\therefore$ No. of capacitor $= 8\times4 = 32$

20. (d) For a parallel plate capacitor $C = \dfrac{\varepsilon_0 A}{d}$

$$\therefore A = \frac{Cd}{\varepsilon_0} = \frac{1\times10^{-3}}{8.85\times10^{-12}} = 1.13\times10^8\text{ m}^2$$

This corresponds to area of square of side 10.6 km which shows that one farad is very large unit of capacitance.

21. (a) $E = \dfrac{1}{2}CV^2 = \dfrac{1}{2}\times1\times10^{-6}\times(4000)^2 = 8$ J.

22. (a) $C_s = \dfrac{2\times8}{2+8} = 1.6\mu F$

Since, $Q = C_sV = 1.6\times10^{-6}\times300$

$Q = 4.8\times10^{-4}$ C

$$V_1 = \frac{4.8\times10^{-4}}{2\times10^{-6}} = 240V$$

$$V_2 = \frac{4.8\times10^{-4}}{8\times10^{-6}} = 60V$$

$$U = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2}$$

$$\Rightarrow U = \frac{\left(4.8\times10^{-4}\right)^2}{2}\left(\frac{1}{1.6\times10^{-6}}\right)$$

$\Rightarrow U = 3\times2.4\times10^{-2}$ J

$\Rightarrow U = 7.2\times10^{-2}$ J

23. (b) In steady state, both the capacitors are at the same potential,

i.e., $\dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2}$ or $\dfrac{Q_1}{C} = \dfrac{Q_2}{2C}$ or $Q_2 = 2Q_1$

Also $Q_1 + Q_2 = Q$

$\therefore Q_1 = \dfrac{Q}{3}, Q_2 = \dfrac{2Q}{3}$

24. (a)

25. (a) Here, $V(x) = \dfrac{20}{x^2-4}$ volt

We know that $E = -\dfrac{dV}{dx} = -\dfrac{d}{dx}\left(\dfrac{20}{x^2-4}\right)$

or, $E = +\dfrac{40x}{(x^2-4)^2}$

At $x = 4\ \mu m$,

$$E = +\frac{40\times4}{(4^2-4)^2} = +\frac{160}{144} = +\frac{10}{9}\text{ volt}/\mu m.$$

Positive sign indicates that $\vec{E}$ is in +ve x-direction.

26. (a) Capacitance of spherical conductor $= 4\pi\varepsilon_0 a$
where a is radius of conductor.

Therefore, $C = \dfrac{1}{9\times10^9}\times1 = \dfrac{1}{9}\times10^{-9}$

$= 0.11\times10^{-9}$ F $= 1.1\times10^{-10}$ F

27. (d) In equilibrium, $F = qE = (ne)\dfrac{V}{d} = mg$

$$n = \frac{mg\,d}{eV} = \frac{1.96\times10^{-15}\times9.8\times0.02}{1.6\times10^{-19}\times800} = 3$$

28. (c) Length of body diagonal $= \sqrt{3}\,b$

$\therefore$ Distance of centre of cube from each corner,

$$r = \frac{\sqrt{3}}{2}b$$

Total P.E. of charge + q at the centre

$$= \frac{8q(-q)}{4\pi\varepsilon_o r} = \frac{-8q^2}{4\pi\varepsilon_o(\sqrt{3}\,b/2)} = \frac{-4q^2}{\pi\varepsilon_o\sqrt{3}\,b}$$

29. (d) Potential at origin

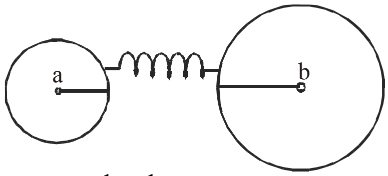
$= (V_1+V_3+V_5+.....)-(V_2+V_4+V_6+.....)$

$$\Rightarrow \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{x_0}-\frac{1}{2x_0}+\frac{1}{3x_0}.....\infty\right]$$

$$\Rightarrow \frac{q}{4\pi\varepsilon_0 x_0}\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}.....\infty\right]$$

$$\Rightarrow \frac{q}{4\pi\varepsilon_0 x_0}\log_e(1+1) \Rightarrow \frac{q}{4\pi\varepsilon_0 x_0}\log_e 2$$

30. (c)



Let charge on each sphere $= q$

when they are connected together their potential will be equal .

Now let charge on $a = q_1$ and on $b = 2q - q_1$

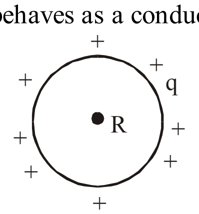
$$\Rightarrow V_a = V_b \text{ or } \frac{1}{4\pi\varepsilon_o}\frac{q_1}{a} = \frac{1}{4\pi\varepsilon_o}\frac{2q-q_1}{b}$$

$$\Rightarrow \frac{q_1}{2q-q_1} = \frac{a}{b}$$

$$\frac{E_a}{E_b} = \frac{\frac{1.}{4\pi\varepsilon_o}\frac{q_1}{a^2}}{\frac{1}{4\pi\varepsilon_o}\frac{q_2}{b^2}} = \left(\frac{q_1}{2q-q_1}\right)\frac{b^2}{a^2} = \frac{a}{b}.\frac{b^2}{a^2} = \frac{b}{a}$$

$= b : a$

31. (b) In shell, q charge is uniformly distributed over its surface, it behaves as a conductor.



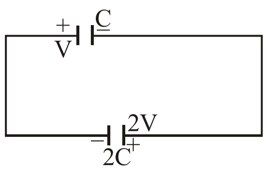
V= potential at surface $= \dfrac{q}{4\pi\varepsilon_0 R}$ and inside

$$V = \frac{q}{4\pi\varepsilon_0 R}$$

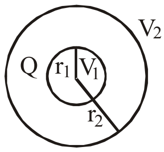
Because of this it behaves as an equipotential surface.

32. (b) Energy stored,

$$U = \frac{1}{2}C_{aq}V^2_{net} = \frac{1}{2}(3C)V^2 = \frac{3}{2}CV^2$$



33. (a)



Situation 1 :

$$V_1 - V_2 = V = K\frac{Q}{r_1} - K\frac{Q}{r_2} = KQ\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$$

Situation 2 :

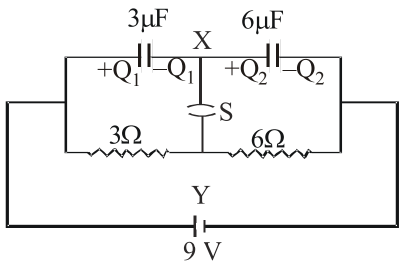
$$V'_1 - V'_2 = V' = \left[\frac{KQ}{r_1}-\frac{3KQ}{r_2}\right]-\left[\frac{KQ}{r_2}-\frac{3KQ}{r_2}\right]$$

$$= KQ\left[\frac{1}{r_1}-\frac{1}{r_2}\right] = V$$

34. (c) Since, all A, B, C, D and E lie on an equipotential surface so,

$W = 0$

35. (c) When steady state is reached, the current I coming from the battery is given by



$9 = I(3+9) \Rightarrow I = 1A$

$\Rightarrow$ potential difference across 3 Ω resistance = 3V and potential difference across 6 Ω resistance =6V

$\Rightarrow$ p.d. across 3 μF capacitor = 3V

and p.d. across 6μF capacitor = 6V

$\therefore$ Charge on 3 μF capacitor, $Q_1 = 3 \times 3 = 9\ \mu C$

Charge on 6μF capacitor, $Q_2 = 6 \times 6 = 36\ \mu C$

$\Rightarrow$ Charge $(-Q_1)$ is shifted from the positive plate of 6μF capacitor. The remaining charge on the positive plate of 6 μF capacitor is shifted through the switch.

$\therefore$ Charge passing through the switch

$= 36 - 9 = 27\ \mu C$

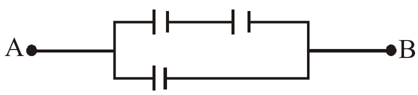
36. (a) As $x = t\left(1-\frac{1}{K}\right)$, where x is the addition distance of plate, to restore the capacity of original value.

$$\therefore\ 3.5\times10^{-5} = 4\times10^{-5}\left(1-\frac{1}{K}\right).$$

Solving, we get, K = 8.

37. (d) For series $C' = \dfrac{C_1 \times C_2}{C_1+C_2} = \dfrac{4\times4}{4+4} = 2\mu F$

For parallel $C_{eq} = C' + C_2 = 2 + 4 = 6\mu F$



38. (b) $C = \dfrac{K\varepsilon_0 A}{d}$

$1 \times 10^{-12} = \dfrac{1 \times \epsilon_0 A}{d}$ .... (1)

$2 \times 10^{-12} = \dfrac{K \times \epsilon_0 A}{2d}$ .... (2)

$\dfrac{(2)}{(1)} \Rightarrow \dfrac{K}{2} = 2 \quad \text{or} \quad K = 4$

39. (d) The charge Q given to outer sphere distributes as Q_1 outside and Q_2 inside which induces charge $-Q_2$ on outside of inner sphere, $+Q_2$ on inside of inner sphere which is earthed.

The inside of outer and the inner sphere constitute a spherical condenser having capacitance $4\pi \epsilon_0 \dfrac{ab}{b-a}$ and the outside of the outer constitutes an isolated sphere of capacitance $4\pi \epsilon_0 b$.

$\therefore$ the effective capacitance is

$4\pi \epsilon_0 \dfrac{ab}{b-a} + 4\pi \epsilon_0 b$

$= 4\pi \epsilon_0 b\left[\dfrac{a}{b-a} + 1\right] = 4\pi \epsilon_0 b\left[\dfrac{a+b-a}{b-a}\right]$

$C = 4\pi \epsilon_0 \dfrac{b^2}{b-a}$

40. (a) $C_a = \dfrac{\epsilon_0 A}{d}$ and $C_b = \dfrac{\epsilon_0 A}{\dfrac{d}{2} + \dfrac{d}{2K}} = \dfrac{2\epsilon_0 A(1+K)}{d}$

and $C_c = \dfrac{\epsilon_0 \dfrac{A}{2}}{d} + \dfrac{\epsilon_0 \dfrac{A}{2} K}{d} = \dfrac{\epsilon_0 A}{2d}(1+K)$

or $C_b = \dfrac{\epsilon_0 A}{d} 2(1+K) > C_a$

or $C_c = \dfrac{\epsilon_0 A}{d} \dfrac{1+K}{2} > C_a \qquad \therefore C_b$ and $C_c > C_a$.

41. (d) When S_1 and S_3 is closed V_1 = 30 V and potential drop across C_2 becomes 20 V.

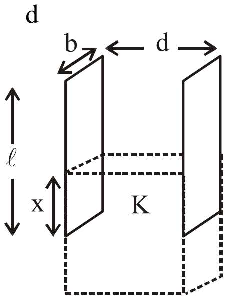
42. (a) The molecules of liquid will convert into induced dipole, get oriented along the electric field produced between the plates and rise due to force of attraction.

43. (d) Charge given to a hollow conductor resides only on the outer surface.

44. (b) $C = C_1 + KC_2 = \dfrac{(\ell - x)b\epsilon_0}{d} + \dfrac{Kxb\epsilon_0}{d}$

$= \dfrac{b\epsilon_0}{d}[\ell - x + Kx]$

$C = \dfrac{b\epsilon_0}{d}[\ell + x(K-1)]$



$V = \dfrac{q}{C} = \dfrac{qd}{b\epsilon_0[\ell + x(K-1)]}$

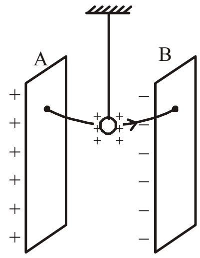
as x decreases, V increases.

45. (c) C_2 and C_3 are parallel so $V_2 = V_3$
C_1 and combination of C_2 & C_3 is in series.
So, $V = V_2 + V_1$ or $V = V_3 + V_1$
and also $Q_1 = Q_2 + Q_3$

46. (c) Since, potential difference across C_2 is greater than C_1.

$\Rightarrow C_1 > C_2 \left[\because V = \dfrac{q}{C} \text{ and q is same in series}\right]$

47. (c) The ball on touching plate A will get positively charged. It will be repelled by A and get attracted towards B. After touching B it will get negatively charged. It will now be repelled by B and get attracted towards A. Thus it will remain oscillating and at the extreme position touch the plates.



48. (a) Initial charge on first capacitor is $CV = Q_1$.
Initial charge on second capacitor is $2CV = Q_2$.
Final capacitance of first capacitor is KC
If V' is the common potential then

$$V' = \frac{Q_1 + Q_2}{C_1' + C_2} \Rightarrow V' = \frac{CV + 2CV}{KC + 2C} = \frac{3V}{2+K}$$

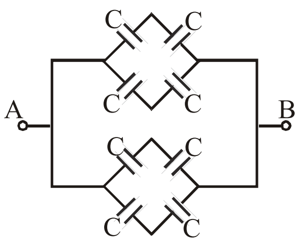
49. (a) Since battery is still in connection, so,
$V = V_0$
$\Rightarrow Q_0 = C_0V_0$ and $Q = kC_0V_0$
$\Rightarrow Q = kQ_0$
Since, $k > 1$
$\Rightarrow Q > Q_0$

Also, $U_0 = \dfrac{1}{2}Q_0V_0$ and

$U = \dfrac{1}{2}QV = kU_0 \qquad \{\therefore Q = kQ_0 \text{ and } V = V_0\}$

Hence, $U > U_0$

50. (b)



The figure shows two independent balanced wheatstone Brides connected in parallel each having a capacitance C. So,

$$C_{net} = C_{AB} = 2C$$

51. (b) Ket C_0 be the capacitance initially and C be the capacitance finally. The $C_0 = \dfrac{\varepsilon_0 A}{d}$

Since, $Q = C_0 V \Rightarrow Q = \dfrac{\varepsilon_0 AV}{d}$

Further, $E_0 = \dfrac{V}{d}$ and $E = \dfrac{E_0}{K} \Rightarrow E = \dfrac{V}{Kd}$

Also, if U_i is the initial energy, then $U_i = \dfrac{1}{2}C_0V^2$

After the introduction of slab if U_f be the final energy, then

$$U_f = \frac{1}{2}CV_{slab}^2 = \frac{1}{2}(KC_0)\left(\frac{V}{K}\right)^2$$

$$\Rightarrow U_f = \frac{1}{2}\frac{C_0V^2}{K} \Rightarrow \Delta U = U_2 - U_1$$

$$\Rightarrow \Delta U = \frac{1}{2}C_0V^2\left(\frac{1}{K} - 1\right)$$

Since, work done = Decrease in Potential Energy

$\Rightarrow W = -\Delta U$

$$\Rightarrow W = \frac{1}{2}\frac{\varepsilon_0 AV^2}{d}\left(1 - \frac{1}{K}\right)$$

52. (b) Potential difference across the branch de is 6 V. Net capacitance of de branch is 2.1 μF
So, $q = CV$
$\Rightarrow q = 2.1 \times 6\ \mu C$
$\Rightarrow q = 12.6\ \mu C$
Potential across 3 μF capacitance is

$$V = \frac{12.6}{3} = 4.2 \text{ volt}$$

Potential across 2 and 5 combination in parallel is $6 - 4.2 = 1.8$ V
So, $q' = (1.8)(5) = 9\ \mu C$

53. (a) Whenever a charge (+50 nC) is kept inside a hollow metallic spherical shell, it induces an equal and opposite charge on the inner surface and an equal and same type of charges on the outer surface.
∴ Inside, induced charge is – 50 nC and outside, +50 nC – 150 nC already present.

54. (a) Required ratio

$$= \frac{\text{Energy stored in capacitor}}{\text{Workdone by the battery}} = \frac{\frac{1}{2}CV^2}{Ce^2},$$

where C = Capacitance of capacitor
V = Potential difference,
e = emf of battery

$$= \frac{\frac{1}{2}Ce^2}{Ce^2}. \quad (\because V = e)$$

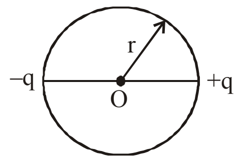
$$= \frac{1}{2}$$

55. (b) $U_i = k\left[\dfrac{(-3q)\,q}{L} \times 3 + \dfrac{(q)\times(q)}{L} \times 3\right] = \dfrac{-6kq^2}{L}$

$U_f = 0$
Work done by electric field = – Change in potential energy

$$= U_i - U_f = \frac{-6kq^2}{L}$$

56. (c)



$$\frac{mv^2}{r} = \frac{kq^2}{(2r)^2}\ ;\ mv^2 = \frac{kq^2}{4r}$$

Kinetic energy of each particle

$$= \frac{1}{2}mv^2 = \frac{kq^2}{8r}$$

57. (a) $V = V_1 + V_2 + V_3 = \dfrac{1}{4\pi \in_0}.\dfrac{Q}{R} + \dfrac{1}{4\pi \in_0}\left(\dfrac{-2Q}{R}\right)$

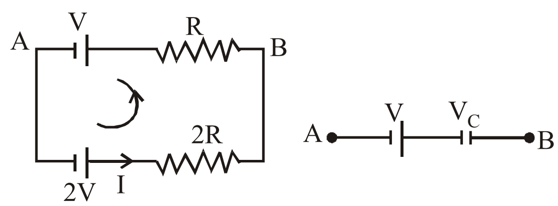
$$+ \frac{1}{4\pi \in_0}\left(\frac{3Q}{R}\right) = \frac{1}{4\pi \in_0}\left(\frac{2Q}{R}\right)$$

58. (c) Capacitance of capacitor (C) = 6 μF = 6×10^{-6} F; Initial potential $(V_1) = 10$ V and final potential $(V_2) = 20$ V. The increase in energy (ΔU)

$$= \frac{1}{2}C(V_2^2 - V_1^2) = \frac{1}{2}\times(6\times10^{-6})\times[(20)^2 - (10)^2]$$

$$= (3\times10^{-6})\times 300 = 9\times10^{-4}\ J.$$

59. (b) As the capacitors offer infinite resistance to steady current so, the equivalent circuit is



Using ohm's law, current in circuit is

$$2V - V = I(2R + R) \Rightarrow I = \frac{V}{3R}$$

The voltage drop across

$$V_{AB} = 2V - \frac{V}{3R}\times 2R = \frac{4}{3}V$$

$V_{AB} = \frac{4}{3}V = V + V_C$

$\Rightarrow$ Voltage drop across C = $\frac{1}{3}$V.

60. (a) The electrostatic pressure at a point on the surface of a uniformly charged sphere = $\frac{\sigma^2}{2\in_0}$

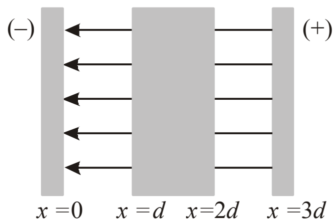
$\therefore$ The force on a hemispherical shell = $\frac{\sigma^2}{2\in_0}\times\pi R^2$

61. (c) Even after introduction of dielectric slab, direction of electric field will be perpendicular to the plates and directed from positive plate to negative plate.

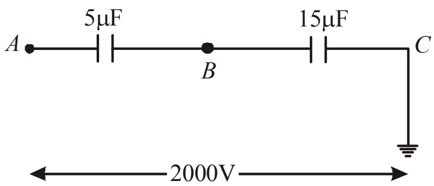
Further, magnitude of electric field in air $= \frac{\sigma}{\varepsilon_0}$

Magnitude of electric field in dielectric $= \frac{\sigma}{K\varepsilon_0}$

Similarly electric lines always flows from higher to lower potential, therefore, electric potential inceases continuously as we move from x = 0 to x = 3d.



62. (c) The given circuit can be redrawn as follows



$(V_A - V_B) = \left(\frac{15}{5+15}\right)\times 2000 \Rightarrow V_A - V_B = 1500V$

$\Rightarrow 2000 - V_B = 1500V \Rightarrow V_B = 500V$

63. (a) $V_R = \frac{V_0}{4} = V_0 e^{-\frac{t}{RC}} \Rightarrow \frac{1}{4} = e^{\frac{t}{10}}$

$\Rightarrow 4 = e^{\frac{t}{10}} \Rightarrow \log_e 4 = \frac{t}{10} \Rightarrow t = 10\log 4 = 13.86s$

$(RC = 2.5\times 10^6 \times 4\times 10^{-6} = 10)$

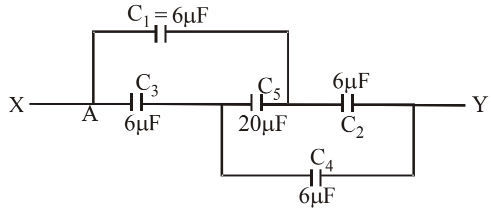
64. (c) $\frac{1}{2}C_1V_1^2 = \frac{1}{2}C_2V_2^2$

because total energy is transferred (given).

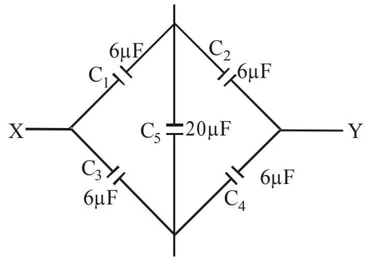
$\therefore \frac{1}{2}\times 900\times 10^{-6}\times 100^2 = \frac{1}{2}\times 100\times 10^{-6}\times V^2$

$\therefore V^2 = 90000 \Rightarrow V = 300V.$

65. (d)

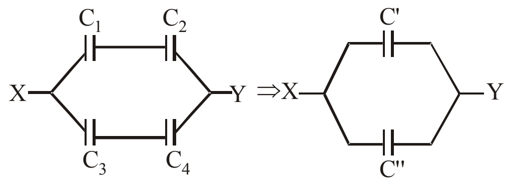


Equivalent circuit



As $\frac{C_1}{C_3} = \frac{C_2}{C_4}$

Hence no charge will flow through 20μF



C_1 and C_2 are in series, also C_3 and C_4 are in series.
Hence C' = 3 μF, C'' = 3 μF
C' and C'' are in parallel hence net capacitance
= C' + C'' = 3 + 3 = 6 μF

66. (c) Energy stored per unit volume

$= \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0\frac{V^2}{d^2} \quad \left(\because E = \frac{V}{d}\right)$

67. (a) Charge $Q = C_1V$
Total capacity of combination (parallel) $C = C_1 + C_2$

$P.D. = \frac{Q}{C} = \frac{C_1V}{C_1 + C_2}$

68. (a) We know that potential energy of two charge system is given by $U = \frac{1}{4\pi\in_0}\frac{q_1q_2}{r}$

According to question,

$U_A = \frac{1}{4\pi\in_0}\frac{(+q)(-Q)}{a} = -\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a}$

and $U_B = \frac{1}{4\pi\in_0}\frac{(+q)(-Q)}{a} = -\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a}$

$\Delta U = U_B - U_A = 0$
When known that for conservative force,
$W = -\Delta U = 0$

69. (c) We know that potential energy of discrete system of charges is given by

$$U = \frac{1}{4\pi \epsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{31}}\right)$$

According to question,

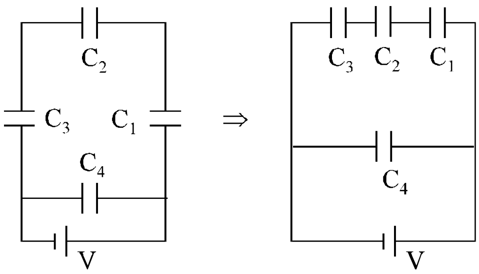
$$U_{initial} = \frac{1}{4\pi \epsilon_0}\left(\frac{q_1q_2}{0.3} + \frac{q_2q_3}{0.5} + \frac{q_3q_1}{0.4}\right)$$

$$U_{final} = \frac{1}{4\pi \epsilon_0}\left(\frac{q_1q_2}{0.3} + \frac{q_2q_3}{0.1} + \frac{q_3q_1}{0.4}\right)$$

$$U_{final} - U_{initial} = \frac{1}{4\pi \epsilon_0}\left(\frac{q_2q_3}{0.1} - \frac{q_2q_3}{0.5}\right)$$

$$= \frac{1}{4\pi \epsilon_0}\left[10q_2q_3 - 2q_2q_3\right] = \frac{q_3}{4\pi \epsilon_0}(8q_2)$$

70. (b)



Equivalent capacitance for three capacitors (C_1, C_2 & C_3) in series is given by

$$\frac{1}{C_{eq.}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2C_3 + C_3C_1 + C_1C_2}{C_1C_2C_3}$$

$$\Rightarrow \quad C_{eq.} = \frac{C_1C_2C_3}{C_1C_2 + C_2C_3 + C_3C_1}$$

$$\Rightarrow \quad C_{eq.} = \frac{C(2C)(3C)}{C(2C) + (2C)(3C) + (3C)C} = \frac{6}{11}C$$

$\Rightarrow$ Charge on capacitors (C_1, C_2 & C_3) in series

$$= C_{eq}V = \frac{6C}{11}V$$

Charge on capacitor $C_4 = C_4V = 4C\,V$

$$\frac{\text{Charge on } C_2}{\text{Charge on } C_4} = \frac{\frac{6C}{11}V}{4CV} = \frac{6}{11} \times \frac{1}{4} = \frac{3}{22}$$

71. (d) In series, $C_{eff} = \dfrac{C_1}{n_1}$

$\therefore$ Energy stored,

$$E_S = \frac{1}{2}C_{eff}V_S^2 = \frac{1}{2}\frac{C_1}{n_1}16V^2 = 8V^2\frac{C_1}{n_1}$$

In parallel, $C_{eff} = n_2C_2$

$\therefore$ Energy stored, $E_p = \dfrac{1}{2}n_2C_2V^2$

$$\therefore \quad \frac{8V^2C_1}{n_1} = \frac{1}{2}n_2C_2V^2 \Rightarrow C_2 = \frac{16C_1}{n_1n_2}$$

72. (a) $q = CV_1 \cos wt$

$$\Rightarrow i = \frac{dq}{dt} = -\omega Cv_1 \sin \omega t$$

Also, $\omega^2 = \dfrac{1}{LC}$ and $V = V_1 \cos \omega t$

At $t = t_1$, $V = V_2$ and $i = -\omega CV_1 \sin \omega t_1$

$\therefore \cos \omega t_1 = \dfrac{V_2}{V_1}$ (–ve sign gives direction)

Hence, $i = V_1\sqrt{\dfrac{C}{L}}\left(1 - \dfrac{V_2^2}{V_1^2}\right)^{1/2} = \left(\dfrac{C(V_1^2 - V_2^2)}{L}\right)^{1/2}$

73. (d) The charge on each of two facing surfaces will be zero.
74. (d) Potential and potential energy are different quantities and cannot be equated.
75. (d) Two equipotential surfaces are not necessarily parallel to each other.

EXERCISE - 3

Exemplar Questions

1. (d) As capacitor offers infinite resistance in dc-circuit. So, current flows through 2Ω resistance from left to right, given by

$$I = \frac{V}{R + r} = \frac{2.5V}{2 + 0.5} = \frac{2.5}{2.5} = 1\text{ A}$$

So, the potential difference across 2Ω resistance
$V = IR = 1 \times 2 = 2$ volt.
Since, capacitor is in parallel with 2Ω resistance, so it also has 2V potential difference across it.
As current does not flow through capacitor branch so no potential drop will be accross 10Ω resistance.
The charge on capacitor
$q = CV = (4\,\mu F) \times 2V = 8\,\mu C$

2. (c) The direction of electric field is always perpendicular to the direction of electric field and equipotential surface maintained at high electrostatic potential to other equipotential surface maintained at low electrostatic potential.
The positively charged particle experiences the electrostatic force in the direction of electric field i.e., from high electrostatic potential to low electrostatic potential. Thus, the work done by the electric field on the positive charge, so electrostatic potential energy of the positive charge decreases because speed of charged particle moves in the direction of field due to force $q\vec{E}$.

3. (c) The work done (in displacing a charge particle) by a electric force is given by $W_{12} = q(V_2 - V_1)$. Here initial and final potentials are same in all three cases are equal (20V) and same charge is moving from A to B, so work done is (ΔVq) same in all three cases.

4. (c) As we know that the relation between electric field intensity E and electric potential V is

$$E = -\frac{dV}{dr}$$

Electric field intensity E = 0 then $\frac{dV}{dr} = 0$

This imply that V = constant

Thus, E = 0 inside the charged conducting sphere then the constant electrostatic potential 100V at every where inside the sphere and it verifies the shielding effect also.

5. (a) Here we have to findout the shape of equipotential surface, these surface are perpendicular to the field lines, so there must be electric field which can not be without charge.

So, the collection of charges, whose total sum is not zero, with regard to great distance can be considered as a point charge. The equipotentials due to point charge are spherical in shape as electric potential due to point charge q is given by

$$V = K_e \frac{q}{r}$$

This suggest that electric potentials due to point charge is same for all equidistant points. The locus of these equidistant points which are at same potential, form spherical surface.

The lines of field from point charge are radial. So the equipotential surface perpendicular to field lines from a sphere.

6. (c) The capacitance of parallel plate capacitor filled with dielectric of thickness d_1 and dielectric constant K_1 is

$$C_1 = \frac{K_1\varepsilon_o A}{d_1}$$

Similarly, capacitance of parallel plate capacitor filled with dielectric of thickness d_2 and dielectric constant K_2 is

$$C_2 = \frac{K_2\varepsilon_o A}{d_2}$$

Since both capacitors are in series combination, then the equivalent capacitance is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

or $$C = \frac{C_1C_2}{C_1 + C_2} = \frac{\frac{K_1\varepsilon_0 A}{d_1}\frac{K_2\varepsilon_0 A}{d_2}}{\frac{K_1\varepsilon_0 A}{d_1} + \frac{K_2\varepsilon_0 A}{d_2}}$$

$$C = \frac{K_1K_2\varepsilon_0 A}{K_1d_2 + K_2d_1} \quad \text{... (i)}$$

So multiply the numerator and denominator of equation (i) with $(d_1 + d_2)$

$$C = \frac{K_1K_2\varepsilon_0 A}{(K_1d_2 + K_2d_1)} \times \frac{(d_1 + d_2)}{(d_1 + d_2)}$$

$$= \frac{K_1K_2(d_1 + d_2)}{(K_1d_2 + K_2d_1)} \times \frac{\varepsilon_0 A}{(d_1 + d_2)} \quad \text{... (ii)}$$

So, the equivalent capacitances is

$$C = \frac{K\varepsilon_0 A}{(d_1 + d_2)} \quad \text{... (iii)}$$

Comparing, (ii) and (iii), the dielectric constant of new capacitor

$$K = \frac{K_1K_2(d_1 + d_2)}{K_1d_2 + K_2d_1}$$

NEET/AIPMT (2013-2017) Questions

7. (a) Potential at B, V_B is maximum

$$V_B > V_C > V_A$$

As in the direction of electric field potential decreases.

8. (c) Electric field, $E \propto \frac{1}{K}$

As $K_1 < K_2$ so $E_1 > E_2$

Hence graph (c) correctly dipicts the variation of electric field E with distance d.

9. (b) Due to conducting sphere

At centre, electric field E = 0

And electric potential $V = \frac{Q}{4\pi \epsilon_0 R}$

10. (d) $$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

$$= -[(6 - 8y)\hat{i} + (-8x - 8 + 6z)\hat{j} + (6y)\hat{k}]$$

At (1, 1, 1), $\vec{E} = 2\hat{i} + 10\hat{j} - 6\hat{k}$

$$\Rightarrow (\vec{E}) = \sqrt{2^2 + 10^2 + 6^2} = \sqrt{140} = 2\sqrt{35}$$

$$\therefore F = q\vec{E} = 2 \times 2\sqrt{35} = 4\sqrt{35}$$

11. (c) Capacitance of the capacitor, $C = \frac{Q}{V}$

After inserting the dielectric, new capacitance

$C^1 = K.C$

New potential difference

$$V^1 = \frac{V}{K}$$

$$u_i = \frac{1}{2}cv^2 = \frac{Q^2}{2C} \quad (\because Q = cv)$$

$$u_f = \frac{Q^2}{2f} = \frac{Q^2}{2kc} = \frac{C^2V^2}{2KC} = \left(\frac{u_i}{k}\right)$$

$$\Delta u = u_f - u_i = \frac{1}{2}cv^2\left\{\frac{1}{k} - 1\right\}$$

As the capacitor is isolated, so change will remain conserved p.d. between two plates of the capacitor

$$L = \frac{Q}{KC} = \frac{V}{K}$$

12. (a) Potential in a region

V = 6xy − y + 2yz

As we know the relation between electric potential and electric field is $\vec{E} = \frac{-dV}{dx}$

$$\vec{E} = \left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

$$\vec{E} = \left[(6y\hat{i} + (6x - 1 + 2z)\hat{j} + (2y)\hat{k}\right]$$

$$\vec{E}_{(1,1,0)} = -(6\hat{i} + 5\hat{j} + 2\hat{k})$$

13. (a) Force of attraction between the plates,

$F = qE$

$$= q \times \frac{\sigma}{2\epsilon_0} = q\frac{q}{2A\epsilon_0}$$

$$= \frac{q^2}{2\left(\frac{\epsilon_0 A}{d}\right) \times d} = \frac{c^2v^2}{2cd} = \frac{cv^2}{2d}$$

Here, $c = \frac{\epsilon_0 A}{d}$, $q = cv$, A = area

14. (d) **When S and 1 are connected**

The $2\mu F$ capacitor gets charged. The potential difference across its plates will be V.

The potential energy stored in $2\ \mu F$ capacitor

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2} \times 2 \times V^2 = V^2$$

When S and 2 are connected

The $8\mu F$ capacitor also gets charged. During this charging process current flows in the wire and some amount of energy is dissipated as heat. The energy loss is

$$\Delta U = \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2}(V_1 - V_2)^2$$

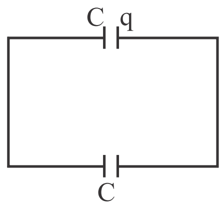
Here, $C_1 = 2\mu F$, $C_2\ 8\ \mu F$, $V_1 = V$, $V_2 = 0$

$$\therefore\ \Delta U = \frac{1}{2} \times \frac{2 \times 8}{2 + 8}(V - 0)^2 = \frac{4}{5}V^2$$

The percentage of the energy dissipated

$$= \frac{\Delta U}{Ui} \times 100 = \frac{\frac{4}{5}V^2}{V^2} \times 100 = 80\%$$

15. (a) When battery is replaced by another uncharged capacitor



As uncharged capacitor is connected parallel

So, $C' = 2C$

and $$V_c = \frac{q_1 + q_2}{C_1 + C_2}$$

$$V_c = \frac{q + 0}{C + C} \Rightarrow V_c = \frac{V}{2}$$

Initial Energy of system, $U_i = \frac{1}{2}CV^2$ ...(i)

Final energy of system, $U_f = \frac{1}{2}(2C)\left(\frac{V}{2}\right)^2$

$$= \frac{1}{2}CV^2\left(\frac{1}{2}\right) \quad \text{...(ii)}$$

From equation (i) and (ii)

$$U_f = \frac{1}{2}U_i$$

i.e., Total electrostatic energy of resulting system decreases by a factor of 2

16. (a) As the regions are of equipotentials, so Work done $W = q\Delta V$

ΔV is same in all the cases hence work - done will also be same in all the cases.