5. WORK, ENERGY AND POWER

1. INTRODUCTION

This chapter explains the concepts of work and energy and how these quantities are related to each other. The law of conservation of energy is an important tool in physics, for the analysis of motion of a system of particles or bodies, and in understanding various phenomena in nature. When the nature of forces involved in a process are not exactly known, or when we want to avoid complicated calculations, then the law of conservation of energy proves to be an indispensable tool in solving many problems. The importance of energy cannot be explained in words. The progress of science and civilization is based on finding new ways to efficiently use the energy available in nature in various forms. Energy is required by a person to perform his/her daily activities, as well as to run our automobiles and machines. Depletion of natural energy resources is a major concern these days. The efficiency of energy utilization processes and quantity of energy sources harnessed by a country determines the pace of its economic development.

2. WORK

2.1 Work

In physics, a force is said to do work only when it acts on a body, and if there is a consequential displacement of the point of application in the direction of the force.

For example, say if a constant force F displaces a body through displacement s then the work done, W, is given by

$W = Fs\cos\theta = \vec{F}.\vec{s}$

where s is magnitude of displacement and θ is angle between force and displacement. The SI unit of work is Joule or Newton-metre.

Sign Convention of Work

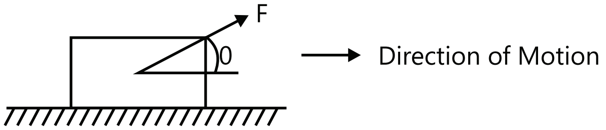

Figure 5.1: Motion of block in directon of applied force

We now define the sign convention of work as follows:

When $0 < \theta < 90^0$,

then $W = Fs\cos\theta$ is positive

i.e., when the force constantly supports the motion of a body,work done by that force is said to be positive.

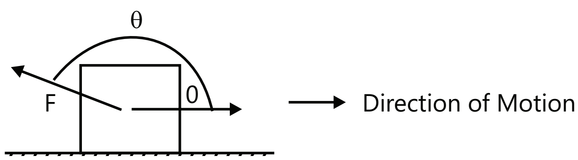

Figure 5.2: Motion of block

then $W = Fs\cos\theta = -ve$

i.e., in this case force is not truly supporting the motion of the body and hence the work done by that force is said to be negative.

2.2 Nature of Work

Work done is signified by the equation: $\vec{F}.\vec{S}$

Based on this equation, three possible situations are possible regarding the nature or sign of the work done as listed here under:

(a) To begin with, the work done is said to be positive if the angle between the force and the displacement vectors is an acute angle.

E.g., when a horse pulls a cart on a level road, the work done by the horse is positive.

(b) Second, the work done is zero if the force and the displacement vectors are perpendicular to each other.

E.g., when a body is moved along a circular path by a string, then the work done due to the string is zero.

(c) The last possible situation is that the work done is said to be negative if the angle between the force and the displacement vectors is an obtuse angle.

E.g., when a body slides over a rough surface, the resultant work done due to the frictional force is negative. (It is pertinent here to remember the fact that the angle between the force and the displacement is 180 degrees.)

PLANCESS CONCEPTS

Students should be able to deduce that by positive work, force is actually doing what it is meant for, i.e. force wants to move a body in certain direction and if it moves in that direction then it's positive work.

Anurag Saraf (JEE 2011, AIR 226)

Illustration 1: Assume that a body is displaced from $\overrightarrow{r_A} = (2m, 4m, -6m)$ to $\overrightarrow{r_B} = (6i - 4j + 2k)m$ under a constant force $\vec{F} = (2i + 3j - k)N$. Now, calculate the total work done. **(JEE MAIN)**

Sol: The work done by the constant force $\vec{F}$ during displacement $\vec{S}$ of a particle is scalar product of force and displacement and is given by $W = \vec{F} \bullet \vec{S}$

$$\overrightarrow{r_A} = \left(2\hat{i} + 4\hat{j} - 6\hat{k}\right)m \quad \vec{S} = \overrightarrow{r_B} - \overrightarrow{r_A} = \left(6\hat{i} - 4\hat{j} + 2\hat{k}\right) - \left(2\hat{i} + 4\hat{j} - 6\hat{k}\right) = 4\hat{i} - 8\hat{j} + 8\hat{k}$$

$$W = \vec{F}.\vec{S} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right).\left(4\hat{i} - 8\hat{j} + 8\hat{k}\right) = 8 - 24 - 8 = (-24\hat{j})$$

Illustration 2: A block of total mass 5 kg is being raised vertically upwards with the help of a string attached to it and it rises with an acceleration of 2 m/s^2. Find the work done due to the tension in the string if the block rises by 2.5 m. Also, calculate the work done due to the gravity and the net work done. **(JEE ADVANCED)**

Sol: The tension in the string is acting vertically upwards and the block is also moving vertically upwards, so the work done by the tension will be positive. The force of gravity is acting vertically downwards so the work done by gravity will be negative.

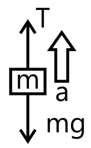

Figure 5.3

Let us first calculate the tension T.

From the force diagram T-mg = 5a; T = 5(9.8 + 2) = 59 N.

As it is clear that both T and displacement S are in the same direction (upwards), then work done by the tension T is W based on which we calculate that $W = Ts = 59(2.5) = 147.5\,J$.

Now, work done due to gravity = –mgs = –5(9.8) (2.5) = –122.5 J

Therefore, net work done on the block = work done by T + work done by mg = 147.5+ (–122.5) = 25 J.

PLANCESS CONCEPTS

Point of application of force also plays a major role.

Zero work is done by a force in following cases: -If the point of application of force is not changed in space but the body moves. If body doesn't move but the point of application of force moves.

Nivvedan (JEE 2009, AIR 113)

3. WORK DONE BY A VARIABLE FORCE

We need to be aware of the fact that when the force is an arbitrary function of position, then we need the principles of calculus to evaluate the work done by it. The Fig. 5.4 given here under shows F (x) as some function x. We now begin our evaluation in this regard by replacing the actual variation of the force by a series of small steps. In the Fig. 5.4 provided, the area under each segment of the curve is approximately equal to the area of a rectangle. Based on the height of the rectangle, the amount of work done is given by the relation, $\Delta W_n = F_n \Delta X_n$. Therefore, the total work done is approximately given by the summation of the areas of both the recangles: $W \approx \sum F_n \Delta X_n$. As the number of the steps is reduced, the tops portions of the rectangle more closely resemble the actual curve shown in the Fig.5.4. In limit $\Delta x \to 0$, which is equivalent to letting the number of steps to be infinite, the discrete sum is replaced by a continuous integral. $W = \int_{x_1}^{x_2} F(x)\,dx$ = area under the F – x curve and the x – axis

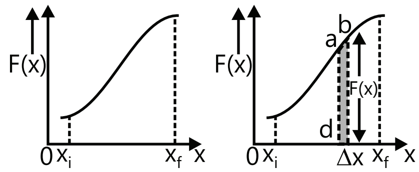

Figure 5.4: Work done on particle by variable force

Illustration 3: A force $F = (10 + 0.50X)$ is observed to act on a particle in the x direction, where F is in newton and x in meter. Find the actual work done by this force during a displacement from x=0 to x=2.0 m. **(JEE MAIN)**

Sol: If a particle is being displaced under action of variable force, the work done by this force is calculated as $W = \int_{s_1}^{s_2} \vec{F} \cdot d\vec{s}$.

As we know that the force is a variable quantity, we shall find the work done in a small displacement from x to x +

dx and then integrate the resultant value to calcuate the total work done. The work done in this small displacement is calculated as

$dW = \vec{F}.\vec{dx} = \left(10 + 0.50x\right)dx$. Thus, $W = \int_0^{2.0} \left(10 + 0.50x\right)dx \quad = \left[10x + 0.50\frac{x^2}{2}\right]_0^{2.0} = 21$ J.

4. CONSERVATIVE AND NON-CONSERVATIVE FORCES

A force is said to be of the conservative category if the work done by it in moving a particle from one point to another does not depend upon the path taken but depends only upon the initial and final positions. The work done by a conservative force around a closed path calculated to be zero. Gravitational force, electric force, spring force, etc. are some of the examples of this category. Basically, all central forces are conservative forces. In contrast, if the work done by a force in moving a body from one point to another depends upon the path followed, then the force is said to be of the nonconservative category. The work done by such a force around a closed path cannot be zero. For example, both the frictional and viscous forces work in an irreversible manner and hence a definite part of energy is lost in overcoming these frictional forces. (Mechanical energy is converted to other energy forms such as heat, sound, etc.). Therefore, these forces are of the nonconservative category.

5. WORK DONE AGAINST FRICTION

We know that the frictional force always acts opposite to the direction of motion (and hence direction of the displacement); therefore, the work done by the frictional force is always on the negative side. Further, the work done by the frictional force is invariably lost in the form of heat and sound energy and thus it is a nonconservative force.

PLANCESS CONCEPTS

The work done by the frictional force is either negative or zero, but never positive. The frictional force always resists the attempted work done along a horizontal surface. Work done along a horizontal surface is given by: $-\mu mgl$, where

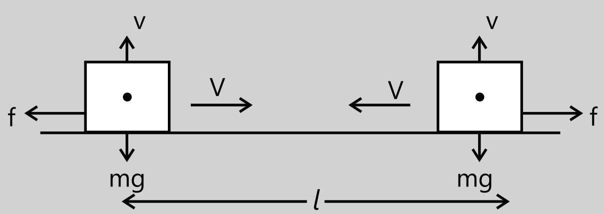

Figure 5.5

m is the mass of the object ;

μ is the coefficient of friction

g is the acceleration due to gravity ($9.8 m/s^2$)

l is the distance traveled by the block along the rough surface

Similarly, work done along an inclined surface with an angle θ from horizontal is given by $-\mu mgl\cos\theta$

Nitin Chandrol (JEE 2012, AIR 134)

Illustration 4: It is observed that a block of mass 4 kg slides down a plane inclined at 37° with the horizontal. The length of the plane is calculated to be of 3 m. The value of the coefficient of sliding friction between the block and the plane is 0.2. Based on the above, find the work done due to the gravity, the frictional force, and the normal reaction between the block and the plane. **(JEE MAIN)**

Sol: Normal reaction is always perpendicular to the inclined plane hence it is perpendicular to the displacement and thus the work done by it is zero. Whereas the frictional force is in opposite direction to the displacement and hence the work done by the firctional force is negative. The work done by the component of gravitational force along the inclined plane will be positive.

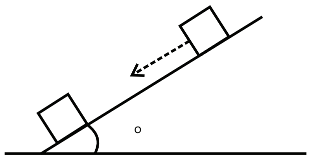

Figure 5.6

Total force acting on the block moving on inclined plane constitutes frictional force, normal reaction due to ground and gravitational force acting on wire. The work done on block is given as $W = Fs\cos\theta$

As the normal reaction is perpendicular to the point of displacement, work done by the normal reaction R = R s cos 90° = 0. The magnitude of displacement s = 3 m and the angle between force of gravity (mg) and displacement is equal to (90°–37°).

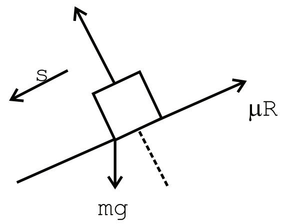

Figure 5.7

Therefore, work done by gravity = mgs cos (90°–37°)

= mgs sin $37^0 = 4 \times 9.8 \times 3 \times 3/5 = 70.56$ J

Work done by friction = –(μR)s = –(μ mg cos 37°)s = –0.2 × 4 × 9.8 × 4 / 5 × 3 = –18.816 J.

6. POWER

Power is defined as the rate at which the actual work is done. If an amount of work ΔW is done in time Δt, then average power, $P_n = \frac{\Delta W}{\Delta t}$ and instantaneous power, $P = \lim_{\Delta t \to 0}\left(\frac{\Delta W}{\Delta t}\right) = \frac{dW}{dt}$.

It is a well-known fact that work done by a force F on an object that has infinitesimally small displacement ds is dw=F.ds. Then, instantaneous power, $P = \frac{dW}{dt} = \frac{\vec{F}d\vec{s}}{dt} = \vec{F}.\vec{v}$.

The S I unit of power is Watt (W) or Joule/second (J/s) and it is a scalar quantity. Dimensions of power is $M^1L^2T^{-3}$.

Illustration 5: A block of mass m is allowed to slide down a fixed smooth inclined plane of angle θ and length ℓ. Calculate the magnitude of power developed by the gravitational force when the block reaches the bottom.

(JEE ADVANCED)

Sol: The power dlivered by the force $\vec{F}$ is the scalar product of the force and velocity i.e. $P = \vec{F}.\vec{v}$

When body reaches bottom of the inclined plane the velocity of of body is $v = \sqrt{2gh} = \sqrt{2g \bullet \ell\sin\theta}$ and the angle between velocity and vertical will be $(90-\theta)^o$. $P = \vec{F}.\vec{V} = mg\sin\theta\sqrt{2g\ell\sin\theta} = \sqrt{2m^2g^3\ell\sin^3\theta}$.

Illustration 6: A particle of mass m is moving in a circular path of constant radius r such that its centripetal accelecration a_c is varying with time t as $a_c = k^2rt^2$, where k is a constant. The power delivered to the particle by the force acting on it is

(JEE MAIN)

(A) $2\pi mk^2r^2$ (B) mk^2r^2t (C) $\frac{(mk^4r^2t^5)}{3}$ (D) Zero

Sol: (B) As the centripetal force is perpendicular to the direction of the velocity, the work done and power delivered by the centripetal force will be zero, whereas the tangential force is in the direction of the velocity so the power delivered to the particle of mass m is $P = F_t \bullet v$

Here $a_c = k^2rt^2$ or $\frac{v^2}{r} = k^2rt^2$ or $v = krt$

Therefore, tangential acceleration, $a_t = \frac{dv}{dt} = kr$ or tangential force, $F_t = m\,a_t = m\,kr$

However, only tangential force does work. Power = F_tv = (mkr)(krt) or Power = mk^2r^2t

7. ENERGY

Generally, the energy of a body is signified by the body's capacity to do work. It is a scalar quantity and shares the same unit as that of work (Joule in SI unit). In mechanics, both kinetic and potential energies are involved with dynamics of the body.

7.1 Potential Energy

7.1.1. Potential Energy

Potential energy of a body is the energy possessed by virtue of its position or due to its state. It is independent of the way in which the body is transformed to this state. Although it is a relative parameter, it depends upon its value at reference level. We can define the change in potential energy as the negative of work done by the conservative force in operation in carrying a body from a reference position to the position under consideration.

7.1.2 Definition

$\Delta U = -W_{AB}$ where A is the initial state, B is the final state, and W_{AB} is the total work done by conservative forces. We know that potential energy depends upon the work done by conservative force only. Hence, it cannot be defined for the nonconservative force (s). This is because of the proven fact that in this type work done depends upon the path followed alone.

7.1.3 Gravitational Potential Energy (GPE)

Suppose if we lift a block through some height (h) from A to B, then the work is done defying the gravity. The work done in such a case is stored normally in the form of gravitational potential energy of the block-energy system. Therefore, we can write that work done in raising the block = (mg)h. This is exactly equal to the increase in gravitational potential energy (GPE) of the block.

If the center of a body of mass m is raised by a height h, then increase in GPE = mgh

If the center of a body of mass m is lowered by a distance h, decrease in GPE = mgh

7.1.4 Elastic Potential Energy

Suppose when a spring is elongated (or compressed), then work is done against the restoring force of the spring. This resultant work done is stored in the spring in the form of elastic potential energy.

7.1.5 Nature of Restoring Force

Suppose if a spring is extended or compressed by a distance x, the spring then exerts a restoring force so as to oppose this change.

PLANCESS CONCEPTS

GPE is always thought of as only of block. But to be more specific it is the energy of block-earth system. Potential energy never comes in context of a single particle. It is always for a configuration. In the case of GPE, writers however generally skip writing "Earth" each time.

Chinmay S Purandare (JEE 2012, AIR 698)

7.1.6 Spring

In case of a spring, natural length of the spring is assumed to be the reference point and correspondingly is always assigned zero potential energy (This is a universal assumption.). However, in gravity, we can choose any point as our reference and hence assign it any value of potential energy.

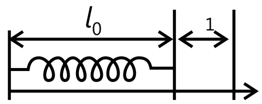

Figure 5.8: Energy stored in stretched spring

For Stretching

$$U_f - U_i = -\int_i^f \vec{F}.d\vec{S}\ ; U_f - 0 = -\int_0^{x_i} kx(-i)(dx)i\ ; U = \frac{1}{2}kx_1^2$$

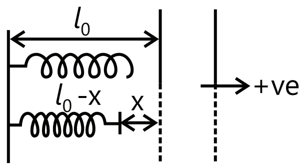

Figure 5.9: Energy stored in compressed spring

For Compression

$$U_f - U_i = -\int_i^f \vec{F}.d\vec{S} = -\int_0^{x_i} kxi(dx)(-i) = U = \frac{1}{2}kx^2$$

Thus, if the spring is either stretched or compressed from natural length by x the corresponding potential energy is $1/2kx^2$

7.1.7 Relationship between Force and Potential Energy

Now, let us discuss the relationship between force and potential energy.

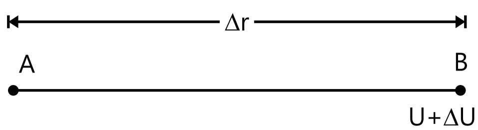

Figure 5.10

Let us assume that a body is taken from A to B in such away that there is no net change in its kinetic engery. Then

$\Rightarrow$ Work done = −change in P.E. ; $F\Delta r = U - (U + \Delta U) = -\Delta U$

$$\Rightarrow \quad F_{avg} = -\left(\frac{\Delta U}{\Delta r}\right) \text{ if } \Delta r \to 0; \quad F = -\lim_{\Delta r \to 0}\frac{\Delta u}{\Delta r} = -\frac{\partial U}{\partial r}$$

7.2 Kinetic Energy

Kinetic energy (KE) is the energy of a body possessed by virtue of its motion alone. Therefore, a body of mass m and moving with a velocity v has a kinetic energy $E_k = \frac{1}{2}mv^2$.

We already know that velocity is a relative parameter; therefore, KE is also a relative parameter.

We provide a detailed account on kinetic energy after presenting the concept of conservation of mechanical energy.

8. EQUILIBRIUM

We have already studied in the chapter on "Laws of Motion" that a body is said to be in translatory equilibrium only if net force acting on the body is zero, i.e., $\vec{F}_{net} = 0$

However, if the forces are conservative, then $F = -\frac{dU}{dr}$; for equilibrium, then

F = 0; Thus, $-\frac{dU}{dr}=0$, or $\frac{dU}{dr}=0$

i.e., exactly at the equilibrium position the slope of U-r graph is zero or the potential energy is optimum (maximum or minimum or constant). Equilibria are of three types, i.e., stable equilibrium, unstable equilibrium, and neutral equilibrium. Further, the situations where F = 0 and dU/dr = 0 can be obtained only under three conditions as specified hereunder.

(a) If $\frac{d^2U}{dr^2} > 0$, then it is stable equilibrium;

(b) If $\frac{d^2U}{dr^2} < 0$, then it is unstable equilibrium; and

(c) If $\frac{d^2U}{dr^2} = 0$, then it is neutral equilibrium.

PLANCESS CONCEPTS

A system always wants to minimize its energy. The above equilibriums are categorized only on this basis. Stable indicates that if system is disturbed slightly, from these configuration, it would try to come back to its original state (position of energy minima). For unstable equilibrium, a slight disturbance would cause the system to find some other suitable configuration (position of energy maxima). A neutral equilibrium is generally found when U becomes constant and each position is a state of equilibrium. A slight disturbance has no after reactions and the new state is also an equilibrium position.

Anurag Saraf (JEE 2011, AIR 226)

Illustration 7: The potential energy of a particle of mass 5 kg, moving in xy plane, is given by U = (–7x + 24y)J where x and y being in meters. Initially (at t=0), the particle is at the origin and has velocity $\vec{v}=\left(14.4\hat{i}+4.2\hat{j}\right)m/s$.

Then Calculate (a) the acceleration of the particle and (b) the direction of acceleration of the particle. (c) The speed of the particle at t = 4 s. **(JEE MAIN)**

Sol: If particle has potential energy U then corresponding conservative force, is $F=-\frac{dU}{dr}$ and according to the Newton's second law of motion $\vec{F}=m\vec{a}$. The direction of acceleration is calculated as $\tan\theta=\frac{a_y}{a_x}$.

(a) Acceleration,

$$F_x=\frac{\delta U}{\delta x}, F_y=-\frac{\delta U}{\delta y} \Rightarrow F_x=7N, \quad F_y=-24N; \quad \Rightarrow a_x=7/5,\ a_y=-24/5$$

(b) Direction of acceleration $\theta=\tan^{-1}\left(\frac{a_y}{a_x}\right)$;

(c) $\vec{v}=\vec{u}+\vec{a}t$; $v_x=14.4+\frac{7}{5}\times 4=20$; $v_y=4.2-\frac{24}{5}\times 4=(-15)$

Illustration 8: The potential energy of a particle in a certain field has the form U= a/r² – b/r, where a and b are positive constants and r is the distance from the center of the field. Find the value of r_0 corresponding to equilibrium position of the particles and hence examine whether this position is stable. **(JEE ADVANCED)**

Sol: Conservative force acting on the particle is $F=-\frac{dU}{dr}$. Under stable equilibrium particle has minimum potential energy while potential energy is maximum in case of unstable equilibrium.

$U(r) = a/r^2 - b/r$

$\text{Force} = F = -\frac{dU}{dr} = -\left(\frac{-2a}{r^3} + \frac{b}{r^2}\right)$; $\qquad F = -\frac{(br - 2a)}{r^3}$

At equilibrium, then $F = \frac{dU}{dr} = 0$

Hence, br – 2a = 0 at equilibrium.

Further, $r = r_0 = 2a/b$ corresponds to equilibrium.

At stable equilibrium, the potential energy of a particle is at its minimum, whereas at unstable equilibrium, it is the maximum. From the principles of calculus, we know that for minimum value around a point $r = r_0$, the first derivative should be zero and the second derivative should be invariably positive.

For minimum potential energy, the applicable conditions are

$$\frac{dU}{dr} = 0 \quad \text{and} \quad \frac{d^2U}{dr^2} > 0 \quad \text{at} \quad r = r_0$$

However, we have already used dU/dr = 0 to obtain $r = r_0 = 2a/b$.

Now, in a similar way let us investigate the second derivative.

$$\frac{d^2U}{dr^2} = \frac{d}{dr}\left(\frac{dU}{dr}\right) = \frac{d}{dr}\left(-\frac{2a}{r^3} + \frac{b}{r^2}\right) = \frac{6a}{r^4} - \frac{2b}{r^3}$$

$$\text{At } r = r_0 = 2a/b, \ \frac{d^2U}{dr^2} = \frac{6a - 2br_0}{r_0^4} = \frac{2a}{r_0^4} > 0.$$

Based on our calculations, the potential energy function U(r) has a minimum value only when $r_0 = 2a/b$. Therefore, we conclude that the system has stable equilibrium only at the minimum potential energy state.

9. WORK ENERGY THEOREM

Suppose that a particle is acted upon by various forces and consequently undergoes a displacement. Then there is a change in its kinetic energy by an amount equal to the total (net) work (W_{net}) done on the particle by all the forces.

i.e., $W_{net} = K_f - K_i = \Delta K$ ... (i)

We call the above expression as the work-energy theorem.

Expression (i) is valid irrespective of the fact that whether the forces are constant or varying and whether the path followed by the particle is straight or curved.

We further elaborate expression (i) as follows:

$W_c + W_{NC} + W_{Oth} = \Delta K$ ... (ii)

where W_c is the work done by conservative forces

W_{sc} is the work done by nonconservative forces

W_{oth} is the work done by all other forces which are not included in the category of conservative, nonconservative, and pseudo forces.

'Since $W_c = \Delta U$' (based on definition of potential energy), therefore, expression (ii) can be accordingly modified as

$W_{NC} + W_{oth} = \Delta K + \Delta U = \Delta(K + U) = \Delta E$ ... (iii)

In expression (iii), the term K + U = E is known as the mechanical energy of the system.

Illustration 9: Find how much will mass "m" rise if 4 m falls away. Block are at rest and in equilibrium **(JEE MAIN)**

Sol: Initially the block is at rest. When the block rises to the maximum height, it again comes to rest momentarily. So, by work energy theorem the total work done on the block by force of gravity and spring force is zero.

Applying work energy theorem (WET) on a block of mass m

$W_g + W_{sp} = K.E._f - K.E._i$

Let the final displacement of the block from the initial equilibrium is x. Then

$$-mg\left(\frac{5mg}{k}+X\right)+\frac{1}{2}k\left(\frac{25m^2g^2}{k^2}\right)-\frac{1}{2}kx^2=0;\ \frac{1}{2}kx^2+mgx-\frac{15m^2g^2}{2k}=0\ ; \qquad x=\frac{3mg}{k}$$

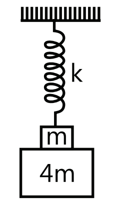

Figure 5.11

PLANCESS CONCEPTS

Whenever there is frictional force, energy is dissipated which is equal to work done by frictional force and the dissipated energy converts into heat. Practically, machine handlers do a lot of things to minimize friction and reduce energy losses by applying lubricants and rollers in their parts.

Yashwanth Sandupatla (JEE 2012, AIR 821)

Illustration 10: A body of mass m was slowly hauled up the hill as shown in the Fig. 5.12 provided by a force F which at each point was directed along a tangent to the trajectory. Find the work done due to this force if the height of the hill is h, the length of its base is l, and the coefficient of friction is μ. **(JEE ADVANCED)**

Sol: As block hauls slowly, the kinetic energy will not change throughout the motion. And the sum of the work done by applied force, gravitational force, normal reaction and frictional force will be zero as per work energy theorem.

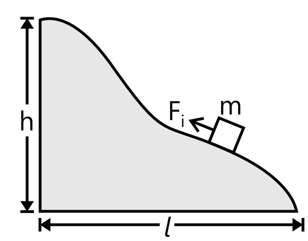

Figure 5.12

The four forces that are acting on the body are listed hereunder.

(a) Weight (mg),

(b) Normal reaction (N),

(c) Friction (f), and

(d) The applied force (F)

According to the principle of work-energy theorem

$W_{net} = \Delta$ KE or $W_{mg}+W_N+W_f+W_F = 0$ ... (i)

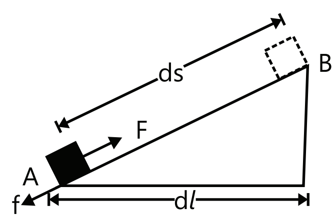

Figure 5.13

Here, $\Delta KE=0$, because $K_i=0=K_f$ $\therefore W_{mg} = -mgh$; $W_N = 0$

(This is because the normal reaction is perpendicular to displacement at all the points.)

W_f can be calculated as $f=\mu mg\cos\theta$

$$\therefore \quad \left(dW_{AB}\right)_f = -f\,ds = -\left(\mu\, mg\cos\theta\right)ds = -\mu\, mg(dl)\ \ (\text{as } ds\cos\theta = dl)$$

$$\therefore \quad f = -\mu\, mg\sum dl = -\mu\, mgl$$

Substituting these values in Eq. (i), we obtain the expression $W_F = mgh+\mu mgl$.

Note: Here again, if we desire to solve this problem without using the concept of work-energy theorem, then we will first evaluate magnitude of applied force $\vec{F}$ at different locations following which we will then integrate $\left(=\vec{F}.\vec{d}r\right)$ with proper limits.

10. KINETIC ENERGY

Now, let us attempt to develop a relationship between the work done and the change in speed of a particle. Based on the Fig. 5.14 provided, we observe that the particle moves from point P_1 to P_2 under the action of a net force $\vec{F}$

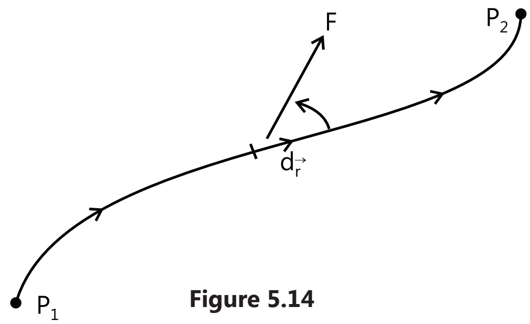

Figure 5.14

$$W = \int_{P_1}^{P_2} \vec{F}.d\vec{r}\ ;\ \vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}\ ;\ d\vec{r} = dx\,\hat{i} + d_y\,\hat{j} + dz\,\hat{k}$$

$$W = \int_{P_2}^{P_2} \left(F_x dx + F_Y d_y + F_z d_z\right)$$

It is very clear for us now that a particle moves along a curved path from point P_1 to P_2, only when acted upon by a force F that varies in both magnitude and direction. $F_x = ma_x = \frac{md\nu_x}{dt}$; $\int_{P_1}^{P_2} F_x dx = \int_{P_1}^{P_2} m\frac{dV_x}{dt}dx$

Treating now ν_x as a function of position, we obtain:

$$\frac{d\nu_x}{dt} = \frac{d\nu_x}{dx}\left(\frac{dx}{dt}\right) = \frac{d\nu_x}{dx}.\nu_x = \nu_x\frac{d\nu_x}{dx};\ \therefore \int_{P_1}^{P_2} F_x d_x = \int_{P_1}^{P_2} m\frac{d\nu_x}{dt}dx = \int_{P_1}^{P_2} m\nu_x\frac{d\nu_x}{dx}dx = \int_{P_1}^{P_2} m\nu_x d\nu_x = \frac{1}{2}m\nu_x^2\Big|_{\nu_{x1}}^{\nu_{x2}} = \frac{1}{2}m\left(\nu^2_{x2} - \nu^2_{x1}\right)$$

ν_{x1} = velocity in x-direction at P_1; ν_{x2} = velocity in x-direction at P_2.

We now apply the same principle for terms in y and z.

$$W = \frac{1}{2}M\left[\nu_{x2}^2 + \nu_{y2}^2 + \nu_{z2}^2 - \left(\nu_{x1}^2 + \nu_{y1}^2 + \nu_{z1}^2\right)\right] = \frac{1}{2}M\left(\nu_2^2 - \nu_1^2\right);\quad W = \frac{1}{2}m\nu_2^2 - \frac{1}{2}m\nu_1^2$$

Define: $K = \frac{1}{2}m\nu^2 \equiv$ Kinetic energy of particle

KE: Potential of a particle to do work by virtue of its velocity.

We know that the work done on the particle by the net force equals the change in KE of the particle.

$W = K_2 - K_1$ or $\Rightarrow W = \Delta K$ Work–Energy Theorem.

For a particle $\vec{P} = M\vec{v}$ (linear momentum); $\therefore K = \frac{1}{2m}P^2$

Regarding KE, the following two points are very significant.

(a) Since, both m and v^2 are always positive, KE is always positive and hence does not depend on the directional parameter of motion of the body.

(b) KE depends on the frame of reference. For example, the KE of a person of mass m in a train moving with speed v is zero in the frame of train, whereas in the frame of earth the KE is $\frac{1}{2}mv^2$ for the same person.

PLANCESS CONCEPTS

Energy can never be negative.

No! Only kinetic energy can't be negative. If anyone generally speaks about energy, it means the sum of potential and kinetic energies. However, we can always choose such a reference in which this sum is negative. Hence, total energy can be negative.

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Illustration 11: A uniform chain of length ℓ and mass m overhangs a smooth table with its two-third parts lying on the table. Find the kinetic energy of the chain as it completely slips off the table. **(JEE MAIN)**

Sol: The initial kinetic energy of the chain is zero. When chain start slipping off table the loss in its potential energy is equal to the gain in its kinetic energy.

Let us take the potential energy at the table as zero. Now, consider a part dx of the chain at a depth x below the surface of the table. The mass of this part is $dm = \frac{m}{\ell}dx$ and hence its potential energy is $-(m/\ell\ dx)gx$.

The potential energy of the one-third of the chain that overhangs is given by $U_1 = \int_0^{\ell/3} -\frac{m}{\ell}gx\,dx$

$$= -\left[\frac{m}{\ell}g\left(\frac{x^2}{2}\right)\right]_0^{\ell/3} = -\frac{1}{18}mg\ell$$

However, this is also the potential energy of the full chain in the initial position; this is because the part lying on the table has zero potential energy. Now, we can calculate the potential energy of the chain when it completely slips off the table as

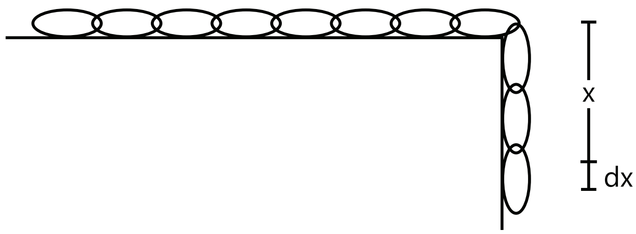


Figure 5.15

$U_2 = \int_0^{\ell} -\frac{m}{\ell}gx\,dx = -\frac{1}{2}mg\ell$ The loss in potential energy is $= \left(-\frac{1}{18}mg\ell\right) - \left(-\frac{1}{2}mg\ell\right) = \frac{4}{9}mg\ell$.

Basically, this should be equal to the gain in the KE in this case. However, the initial KE is zero. Hence, the KE of the chain as it completely slips off the table is $\frac{4}{9}mg\ell$.

Illustration 12: A block of mass m is pushed against a spring of spring constant k fixed at one end to a wall. The block can slide on a frictionless table as shown in the Fig. 5.16. The natural length of the spring is taken as L_0 and it is compressed to half its natural length when the block is released. Now, based on the above find the velocity of the block as a function of its distance x from the wall. **(JEE ADVANCED)**

Sol: The block will move under action of restoring force of spring when spring is released. The block will have constant kinetic energy when it looses contact with the spring. In this process the energy of system will be conserved as there are no external forces acting on the system. (Spring + block system)

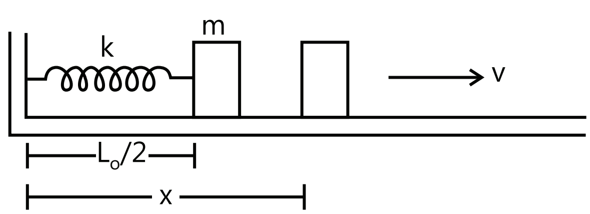


Figure 5.16

When the block is released, naturally the spring pushes it toward right. The velocity of the block keep on inreasing till the block loses contact with the spring and thereafter moves with constant velocity.

Initially, the compression of the spring is $L_0/2$. But when the distance of the block from the wall becomes x, where $x < L_0$, the compression is $(L_0 - x)$. Applying the principle of conservation of energy

$$\frac{1}{2}k\left(\frac{L_0}{2}\right)^2 = \frac{1}{2}k(L_0 - x)^2 + \frac{1}{2}mv^2 \text{. Solving this, } v = \sqrt{\frac{k}{m}}\left[\frac{L_0^2}{4} - (L_0 - x)^2\right]^{1/2}$$

Thus, when the spring acquires its natural length, then $x = L_0$ and $v = \sqrt{\frac{k}{m}}\frac{L_0}{2}$. Thereafter, the velocity of the block remains constant.

11. MOTION IN A VERTICAL CIRCLE

Let us consider a particle of mass m attached to one end of a string and rotated in a vertical circle of radius r with centre O. The speed of the particle will decrease as the particle travels from the lowest point to the highest point but increases in the reverse direction due to acceleration due to gravity.

Thus, if the particle is moving with velocity v at any instant at A, (where the string is subtending an angle θ with the vertical), then the forces acting on the particle are tension T in the string directed toward AO and weight mg acting downward.

Further, the net force T–mg cos θ is directed toward the cenetr and hence provides the centripetal force

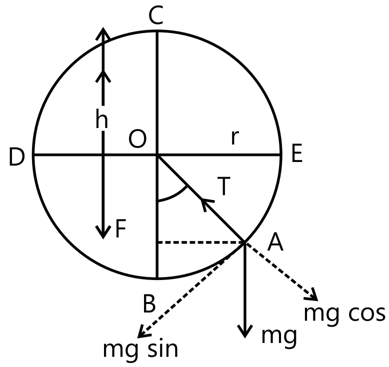


Figure 5.17: Motion in vertical circle

$$T - mg\cos\theta = \frac{mv^2}{r}; T = m\left(g\cos\theta + \frac{v^2}{r}\right)$$

If v_0 is the speed of the particle at the highest point, then the velocity increases as the particle falls through any height h. However, if it falls from C to A, then the vertical distance h is given by

$$h = CF = CO + OF = CO + OA\cos\theta = r + r\cos\theta; h = r(1+\cos\theta)$$

$v^2 = v_0^2 + 2gh = v_0^2 + 2gr(1+\cos\theta)$ (Because there is no actual work done due to the influence of tension)

(i) At the highest point C, $\theta = 180^0$

Tension at $C = T_c = m\left[\frac{v_0^2}{r} + g\cos(180)\right] = m\left[\frac{v_0^2}{r} - g\right]$ ... (i)

The particle will now fall because the string will slacken if T_c is negative. Therefore, the minimum velocity at the highest point is corresponding to the situation where T_c is just zero, i.e., when $m\left[\frac{v_0^2}{r} - g\right] = 0$, or $v_0 = \sqrt{rg}$

(ii) At the lowest point B, $\theta = 0$, tension T_B is given by $T_B = m\left[\frac{v_B^2}{r} + g\right]$

where v_B is velocity at B. $v_B^2 = v_0^2 + 4rg = rg + 4rg = 5rg$; $(\text{using } v^2 = u^2 + 2gh)$; $v_B = \sqrt{5rg}$ ... (ii)

Minimum tension at B when the particle completes the circle is given by $T_B = m\left[\frac{5rg}{r} + g\right] = 6mg$

At the point E, when $\theta = 90^0$, $T_E = \frac{mv_E^2}{r}$

Where velocity at E is given by $V_E = V_c^2 + 2rg = rg + 2rg = 3rg$; $V_E = \sqrt{3rg}$

Tension at E corresponding to speed V_E is $T_E = m\left(\frac{3rg}{r}\right) = 3mg$

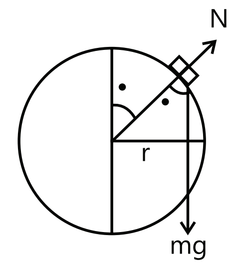


Figure 5.18

(iii) In another case the particle of mass m is not tied to the string but is moving along a circular track of radius r and has normal reaction N. However, it is moving with a velocity v and its radius vector is subtending an angle θ with the vertical, then $mg\cos\theta - N = \frac{mv^2}{r}$.
At the highest point, $mg - N = \frac{mv^2}{r}$; when ... (iii)

$N = 0$, $V = \sqrt{rg}$ Therefore, $V = \sqrt{rg}$ is the minimum speed with which the particle can move at the highest point without losing contact.

Condition of Looping the Loop $(u \geq \sqrt{5gR})$

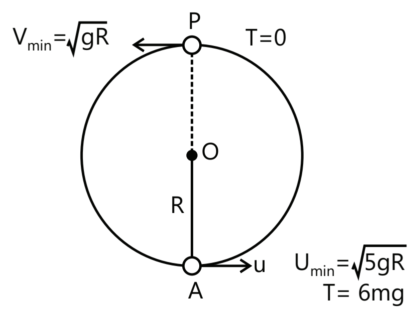


Figure 5.19

The particle will complete the circle only if the string does not slack even at the highest point ($\theta=\pi$). Thus, tension in the string should be obviously greater than or equal to zero ($T \geq 0$) at $\theta=\pi$. In the critical case, however, by substituting T = 0 and $\theta=\pi$ in Eq. (iii), we obtain

$mg = \frac{mv_{min}^2}{R}$ or $v_{min}^2 = gR$ or $v_{min} = \sqrt{gR}$ (at the highest point)

Further, by substituting $\theta = \pi$ in Eq. (i), h = 2R

Therefore, from Eq. (ii) $u_{min}^2 = v_{min}^2 + 2gh$ or $u_{min}^2 = gR + 2g(2R)$ or $u_{min} = \sqrt{5gR}$

Thus, if $u \geq \sqrt{5gR}$, then the particle will complete the circle.

At $u = \sqrt{5gR}$, the velocity at the highest point is $v = \sqrt{gR}$ and the tension in the string is zero.

By substituting $\theta = 0°$ and $v = \sqrt{5gR}$ in Eq. (iii), we get T = 6mg or in the critical condition tension in the string at the lowest position is 6mg as shown in the Fig. 5.19. If $u < \sqrt{5gR}$, then the following two cases are possible.

Condition of Leaving the Circle $(\sqrt{2gR} < u < \sqrt{5gR})$

If $u < \sqrt{5gR}$, then the tension in the string will be zero before reaching the highest point. From Eq. (iii), tension in the string is zero (T=0) where, $\cos\theta = \frac{-v^2}{Rg}$ or $\cos\theta = \frac{2gh - u^2}{Rg}$

Now, by substituting, this value of $\cos\theta$ in Eq. (i), we obtain $\frac{2gh - u^2}{Rg} = 1 - \frac{h}{R}$ or $h = \frac{u^2 + Rg}{3g} = h_1$ (say) ... (iv)

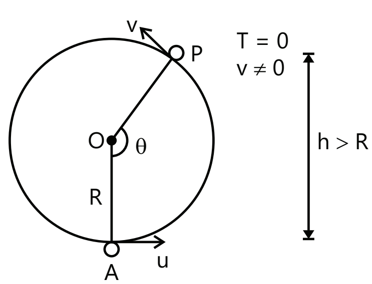


Figure 5.20

Or, in other words, we can say that at height h_1 tension in the string becomes zero. Further, if $u < \sqrt{5gR}$, then the velocity of the particle becomes zero when $0 = u^2 - 2gh$ or $h = \frac{u^2}{2g} = h_2$ (say) ... (v)

i.e., at height h_2 velocity of the particle becomes zero. Now, the particle will move out from the circle if tension alone in the string becomes zero but not the velocity or T=0 but $v \neq 0$. This is possible only when $h_1 < h_2$ or $\frac{u^2 + Rg}{3g} < \frac{u^2}{2g}$ or $2u^2 + 2Rg < 3u^2$ or $u^2 > 2Rg$ or $u > \sqrt{2Rg}$.

Therefore, if $\sqrt{2gR} < u < \sqrt{5gR}$, the particle moves out from the circle.

From Eq.(iv), we observe that h >R if $u^2 > 2Rg$. Thus, the particle, will move out of the circle when h>R or $90° < \theta < 180°$. This situation is shown in the Fig. 4.75.

$\sqrt{2gR} < u < \sqrt{5gR}$ or $90° < \theta < 180°$

Note, however, that after leaving the circle, the particle will follow a parabolic path.

Condition of Oscillation $(0 < u < \sqrt{2gR})$

The particle will oscillate, however, only if velocity of the particle becomes zero but not tension in the string. Or, in other words, v = 0, but $T \neq 0$. This is possible only when $h_2 < h_1$.

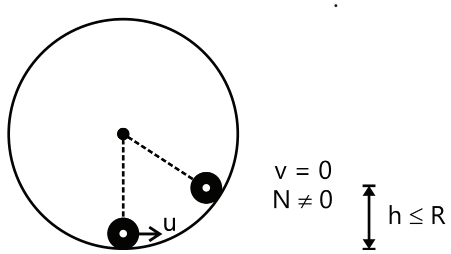


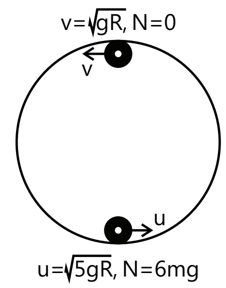
Figure 5.21

Or $\frac{u^2}{2g} < \frac{u^2 + Rg}{3g}$ or $3u^2 < 2u^2 + 2Rg$ or $u^2 < 2Rg$ or $u < \sqrt{2Rg}$

Moreover, if $h_1 = h_2$, $u = \sqrt{2Rg}$ then both tension and velocity becomes zero simultaneously.

Further, from Eq (iv), we observe that $h \leq R$ if $u \leq \sqrt{2Rg}$. Thus, for $0 < u \leq \sqrt{2gR}$, the particle oscillates in the lower half of the circle $(0° < \theta \leq 90°)$. This situation is shown in the Fig. 5.21. $(0 < u < \sqrt{2gR})$ or $(0° < \theta \leq 90°)$

Note: The above three conditions have been derived for a particle that is moving only in a vertical circle and attached to a string. The same conditions apply, however, if a particle moves inside a smooth spherical shell also of radius R. The only difference here is that the tension is replaced by the normal reaction N.

Condition of Looping the Loop is $(u \geq \sqrt{5gR})$	 **Figure 5.22**

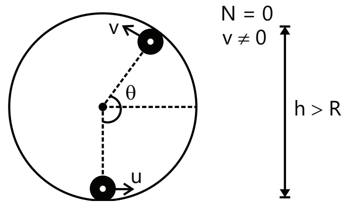
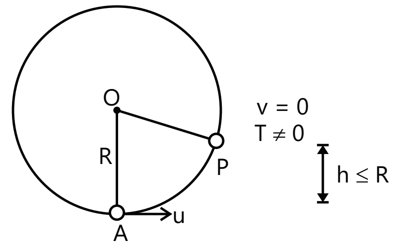
Condition of Leaving the Circle $(\sqrt{2gR} < u < \sqrt{5gR})$	 **Figure 5.23**
Condition of Oscillation $(0 < u < \sqrt{2gR})$	 **Figure 5.24**

Illustration 31: A heavy particle hanging from a fixed point by a light inextensible string of length l is projected horizontally with speed $\sqrt{gl}$. Now, find the speed of the particle and the inclination of the string to the vertical at the instant of the motion when the tension in the string is equal to the weight of the particle. **(JEE ADVANCED)**

Sol: Loss in the kinetic energy of the particle is equal to the gain in the potential energy. Apply Newton's second law along the direction of the string.

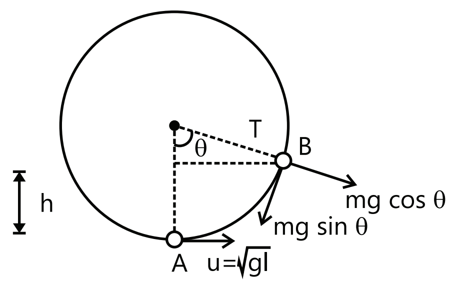


Figure 5.25

Let T = mg at angle θ as shown in the Fig. 5.25.

$H = l\,(1 - \cos\theta)$ ... (i)

Applying the principle of conservation of mechanical energy between points A and B, we obtain $\frac{1}{2}m(u^2 - v^2) = mgh$

Here, $u^2 = gl$ ... (ii)

and v = speed of particle in position B $\therefore \quad v^2 = u^2 - 2gh$ ... (iii)

Further, $T - mg\cos\theta = \frac{mv^2}{l}$ or $mg - mg\cos\theta = \frac{mv^2}{l}$ (T = mg)

Or $\quad v^2 = gl(1 - \cos\theta)$ ... (iv)

Now, by substituting the values of v^2, u^2 and h from Eqs. (iv), (ii) and (i) in Eq. (iii), we obtain

$gl(1 - \cos\theta) = gl - 2gl(1 - \cos\theta)$ or $\cos\theta = \frac{2}{3}$ or $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

Further, by substituting $\cos\theta = \frac{2}{3}$ in Eq. (iv), we obtain $v = \sqrt{\frac{gl}{3}}$

PLANCESS CONCEPTS

If a particle of mass m is connected to a light rod and whirled in a vertical circle of radius R, then to complete the circle, the minimum velocity of the particle at the bottommost point is not $\sqrt{5gR}$. Because, in this case, velocity of the particle at the topmost point can be zero also. Using conservation of mechanical energy between points A and B as shown in Fig. 5.26(a) we get

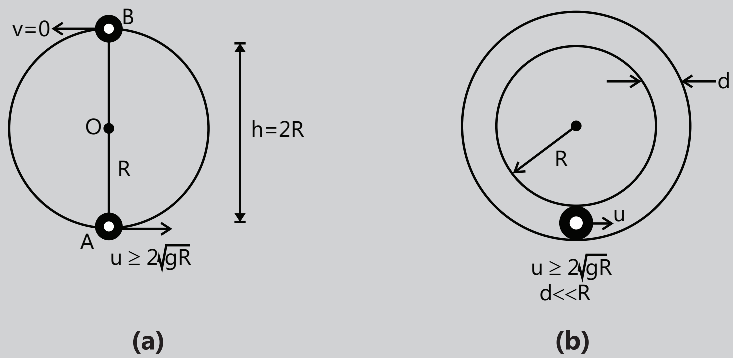


Figure 5.26

$\frac{1}{2}m(u^2 - v^2) = mgh$ or $\frac{1}{2}mu^2 = mg(2R)$ (as v = 0) $\therefore$ $u = 2\sqrt{gR}$

Therefore, the minimum value of u in the case is $2\sqrt{gR}$.

Same is the case when a particle is compelled to move inside a smooth vertical tube as shown in Fig 5.26(b).

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12. A BODY MOVING INSIDE A HOLLOW TUBE

Our discussion above holds good in this case too, but instead of tension in the string we have the normal reaction of the surface. If we take N is the normal reaction at the lowest point, then $N - mg = \frac{mv_1^2}{r}$; $N = m\left(\frac{v_1^2}{r} + g\right)$ However, at the highest point of the circle, $N + mg = \frac{mv_2^2}{r}$

$N = m\left(\frac{v_2^2}{r} - g\right)$; $N \geq 0$ $\Rightarrow$ Implies the condition $V_1 \geq \sqrt{5rg}$

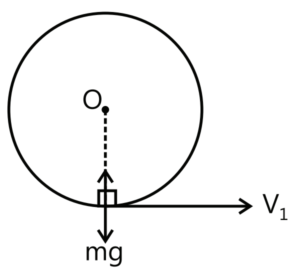


Figure 5.27: Block moving inside hollow sphere

In the same way as shown above, all the other equations similarly can be obtained by just replacing tension T by reaction N.

13. BODY MOVING ON A SPHERICAL SURFACE

Consider that the small body of mass m is placed on top of a smooth sphere whose radius is r.

Now, if the body slides down the surface, at what point does it fly off the surface?

Consider the point C where the mass is, at a certain instant. Now, the acting forces are the normal reaction R and the weight mg. Further, the radial component of the weight is mg $\cos\theta$ acting toward the center. The centripetal force in this case is taken as $mg\cos\theta - R = \frac{mv^2}{r}$

where v is the velocity of the body at O.

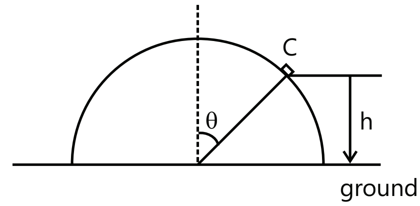


Figure 5.28: Motion of body on spherical surface

$$R = m\left(g\cos\theta - \frac{v^2}{r}\right) \quad \text{... (i)}$$

Now, it is clear that the body flies off the surface at the point where R becomes zero.

$$\text{i.e.,}\ g\cos\phi - R = \frac{mv^2}{r} \quad \text{... (ii)}$$

To find v, we apply the principle of conservation of energy

$$\text{i.e.,}\ \frac{1}{2}mv^2 = mg(BN) = mg\,(OB - ON) = mgr\,(1-\cos\phi)$$

$$v^2 = 2rg\,(1-\cos\phi); \qquad 2(1-\cos\phi) = \frac{v^2}{rg} \quad \text{... (iii)}$$

From equations (ii) and (iii), we obtain

$$\cos\phi = 2 - 2\cos\phi; \qquad 3\cos\phi = 2$$

$$\cos\phi = \frac{2}{3}; \qquad \phi = \cos^{-1}\left(\frac{2}{3}\right) \quad \text{... (iv)}$$

This exactly denotes the angle at which the body goes off the surface. The height from the ground of that point is

$$= AN = r(1+\cos\phi) = r\left(1+\frac{2}{3}\right) = \frac{5}{3}\ r$$

Illustration 32: A point mass m starts from rest and slides down the surface of a frinctionless solid sphere of radius R as shown in the Fig. 5.29 provided. At what angle will this body break off the surface of the sphere? Also, find the velocity with which it will break off. **(JEE MAIN)**

Sol: As the block slides down, the loss in potential energy is equal to gain in kinetic energy and at time of break off, the normal reaction from the sphere on block is zero.

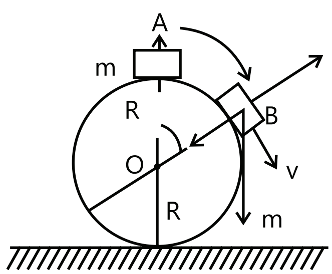


Figure 5.29

Applying principle of conservation of energy (COE), at the points A and B

$$mgR\,(1-\cos\theta) = \frac{1}{2}mv^2 \quad \text{... (i)}$$

Force equation in this equation is $mg\cos\theta - N = mv^2/R$ ... (ii)

N = 0 for break off.

$$\therefore v = \sqrt{gR\cos\theta} \quad \text{... (iii)}$$

Replacing this value in (i)

We get $\cos\theta = 2/3$ Putting this in (iii) we get $v = \sqrt{\frac{2}{3}gR}$.

Illustration 33: A heavy particle is suspended by a string of length ℓ. The horizontal velocity of the particle is v_0. However, the string becomes slack at some angle and the particle proceeds on a parabolic path. Find the value of v_0 if the particle passes through the point of suspension. **(JEE ADVANCED)**

Sol: While particle moves in vertical circle, the tension in the string provides the necessary centripetal force. The loss in kinetic energy is equal to the gain in potential energy. At point the string become slack the tension in the string is zero.

Let us suppose the string becomes slack when the particle reaches the point P. We now assume that the string OP makes an angle θ with the upward vertical. Further, the only force acting on the particle at the point P is its weight mg. Further, the radial component of the force is $mg\cos\theta$. Now, as the particle moves along the circle upto P,

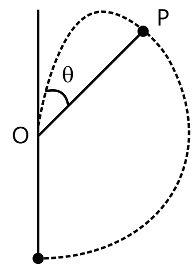


Figure 5.30

$$mg\cos\theta = m\left(\frac{v^2}{\ell}\right) \Rightarrow v^2 = g\ell\cos\theta \qquad \text{... (i)}$$

where v is its speed at the point P. Now, applying the principle of conservation of energy

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mg\ell(1+\cos\theta) \text{ or } v^2 = v_0^2 - 2g\ell(1+\cos\theta) \qquad \text{... (ii)}$$

From (i) and (ii), $v_0^2 = 2g\ell(1+\cos\theta) = g\ell\cos\theta$ or $v_0^2 = g\ell(2+3\cos\theta)$ ... (iii)

From hereon, the particle follows a parabolic path due to acceleration due to gravity. Then as it passes through the point of suspension O, the equations for horizontal and vertical motion give

$$\ell\sin\theta = (v\cos\theta)t \quad \text{and} \quad -\ell\cos\theta = (v\sin\theta)t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad -\cos\theta = (v\sin\theta)\left(\frac{\ell\sin\theta}{v\cos\theta}\right) - \frac{1}{2}g\left(\frac{\ell\sin\theta}{v\cos\theta}\right)^2$$

or, $\quad -\cos^2\theta = \sin^2\theta - \frac{1}{2}g\frac{\ell\sin^2\theta}{v^2\cos\theta}$

or, $\quad -\cos^2\theta = 1 - \cos^2\theta - \frac{1}{2}\frac{g\ell\sin^2\theta}{g\ell\cos^2\theta}$ [From(i)]

or, $\quad 1 = \frac{1}{2}\tan^2\theta$ or, $\tan\theta = \sqrt{2}$

From (iii), $v_0 = \left[g\ell(2+\sqrt{3})\right]^{1/2}$

14. VARIOUS FORMS OF ENERGY: THE LAW OF CONSERVATION OF ENERGY

Conservation of Energy

We observe that in many processes the sum of both the kinetic and potential energies does not remain a constant. This may be due to the influence of dissipative forces such as friction.

(a) The more general form of law of conservation of energy was established by taking into account other forms of energy such as thermal, electrical, chemical, nuclear, etc.

(b) The charges in all forms of energy is given by: $\Delta KE + \Delta U + \Delta(\text{all other forms of energy}) \equiv 0$

This is what we mean by the law of conservation of energy and it is one of the most important principles of physics.

"The total energy is neither increased nor decreased in any process. Energy can be transformed from one form to another, and transferred from one body to another, but the total amount remains constant."

PROBLEM-SOLVING TACTICS

(a) One should always isolate the known and unknown quantities and write equations and solve them.

(b) The next step would be to find out a way from unknown to known quantities and write equations and solve them.

(c) One should always be very careful in doing so to avoid silly mistakes such as unit change of parameter.

(d) Energy is scalar in nature. However, get a clear idea of what is being gained or lost by which entity.

(e) Physical visualization of any problem will always help in increaseaing confidence in solving equations pertaining to the same.

(f) Further, problems involving integration would be easy to understand if you go event by event and then solve.

(g) Special cases and boundary conditions of circular motion are definitely recommended to be mastered because many problems break down to these special cases just after few manipulations.

FORMULAE SHEET

S. NO.	DESCRIPTION	FORMULA
1	Kinetic energy of the particle	$K(v) = \frac{1}{2}mv^2 = \frac{1}{2}m\vec{v}.\vec{v}$
2	Work done by force F	$W = \vec{F}.\vec{r}$ (here $\vec{r}$ is total displacement)
3	Work done by variable force	$w = \int \vec{F}.d\vec{r}$
4	Power generated by force F acting on body	$P = \frac{dW}{dt} = \vec{F}.\frac{d\vec{r}}{dt} = \vec{F}.\vec{v}$
5	Increase in Kinetic Energy = Decrease in Potential Energy	$KE = -\Delta U$
6	Energy conservation principle	$\Delta K + \Delta U = 0;\ \frac{1}{2}mv^2 = mgh$ or, $v = \sqrt{2gh}$
7	For a Spring work done W	$W = \int_{x_1}^{x_2} -kx\,dx = \frac{1}{2}k\left(x_1^2 - x_2^2\right)$
8	Work-Energy principle	$W_{net} = \Delta KE = K_f - K_i$
9	Work done by variable forces in short range	For $\vec{F} = \vec{F}_1 + \vec{F}_2 + ...$ $W = \int \vec{F}.d\vec{r} = \int\left(\vec{F}_1 + \vec{F}_2 + ...\right).\,d\vec{r}$
10	For conservative forces, change in potential energy	$U_f - U_i = -\int_{r_i}^{r_f} \vec{F}.d\vec{r}$
11	Elastic Potential Energy	$U = \frac{1}{2}kx^2$

Solved Examples

JEE Main/Boards

Example 1: An object of mass 5 kg falls from rest through a vertical distance of 20 m and attains a velocity of 10 m/s. How much work is done by the resistance of the air on the object? $\left(g=10 m/s^2\right)$

Sol: According to work energy theorem, the total work done by force of gravity and force of air resistance on object is equal to the change in kinetic energy.

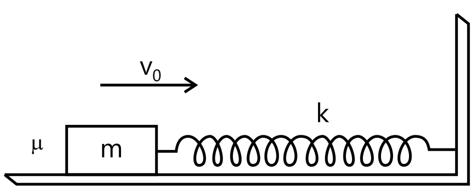
Work done by all forces = Change in KE

$$W_{air}+W_{gravity}=\Delta K.E.;\ W_{air}+mgh=\frac{1}{2}mv^2$$

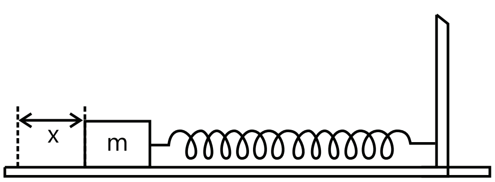
$$W_{air}=\frac{1}{2}mv^2-mgh;\ =\frac{1}{2}x5x10x10-5x10x20$$

$$W_{air}=-750J$$

Example 2: A block is projected horizontally on a rough horizontal floor. The coefficient of friction between the block and the floor is μ. The block strikes a light spring of stiffness k with a velocity v_0. Find the maximum compression of the spring.

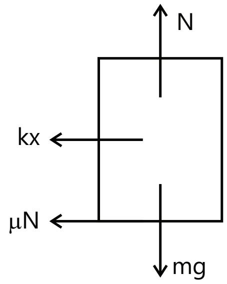


Sol: At the instant of maximum compression the block will come to rest momentarily. By applying work energy theorem the sum of work done by the force of friction, and spring force will be equal to change in kinetic energy. Work done by normal reaction and gravitational force will be zero.



Since the block slides and the spring is compressed through a distance x the net retarding force acting on it

$$=F=-\left(kx+\mu N\right)=-\left(\mu mg+kx\right)$$



$\Rightarrow$ work done by net force for the displacement

$$\Rightarrow\ W=\int_0^x F\,dx;\ \Rightarrow\ \Delta KE=-\int_0^x\left(\mu mg+kx\right)dx$$

$$\Rightarrow\ \left(0-\frac{1}{2}mv_0^2\right)=-\left(\mu mgx+\frac{kx^2}{2}\right)$$

Example 3: Two smooth balls of mass m_1 and m_2 connected by a light inextensible string are at the opposite points of horizontal diameter of a smooth semicylindrical surface of radius R. if m_1 is released, find its speed at any angular distance θ moved by m_2.

Sol: Loss in potential energy of system comprising masses m_1 and m_2, is equal to gain in kinetic energy of the system.

Let the ball m_2 moves through an angle θ, the mass m will fall through a distance $h_1 = R\theta$.

The ball m_2 rises through a height h_2 as,

$h_2 = R\sin\theta$

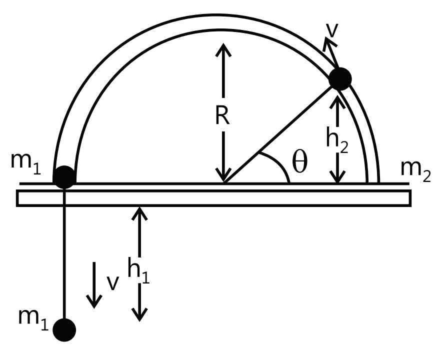
The change in gravitational potential energy of m1 is

$$\Delta PE_1=-m_1gh_1=m_1gR\theta$$

(Since m1 loses its potential energy as it falls down). The change in gravitional potential energy of m2 is

$$\Delta PE_2=-m_2gh_2=m_2gR\sin\theta$$

(Since m_2 gains potential energy as it rises up)



$\Rightarrow$ The total change in gravitational potential energy

$= \Delta PE = -m_1 gR\theta + m_2 gR\sin\theta$

$= gR(m_2 \sin\theta - m_1 \theta)$ ... (i)

$= \Delta KE = \frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 = \frac{(m_1 + m_2)v^2}{2}$ ... (ii)

where v = speed of m1 and m2 at the position as shown in the Fig.5.26 provided. From the principle of conservation of energy, we obtain

$= \Delta KE + \Delta PE = 0$ ... (iii)

Using (i)–(iii), we obtain

$$\frac{1}{2}(m_1 + m_2)v^2 - gR(m_1\theta - m_2 \sin\theta) = 0$$

$$\Rightarrow v = \sqrt{\frac{2gR(m_1\theta - m_2 \sin\theta)}{(m_1 + m_2)}}$$

Example 4: A locomotive of mass m starts moving so that its velocity varies according to the law $v = \alpha\sqrt{s}$, where α is a constant and s is the distance covered. Find the total work done by all the forces acting on the locomotive during the first second after the beginning of motion.

Sol: Velocity is given as the function of distance covered so we can find the acceleration and by second law of motion we can find the force. As force comes out to be constant the work done by force is product of force and displacement.

Given $v = \alpha\sqrt{s}$

Differentiating w.r.t. 't', we get

$$\frac{dv}{dt} = \frac{1}{2}\alpha s^{-1/2}\frac{ds}{dt} = \frac{\alpha}{2\sqrt{s}}v = \frac{\alpha}{2\sqrt{s}} x \alpha\sqrt{s} = \frac{\alpha^2}{2}$$

$\therefore$ Acceleration $a = \frac{\alpha^2}{2}$

Now, force acting on the locomotive is

$F = ma = m\frac{\alpha^2}{2}$; Here, $u = 0$

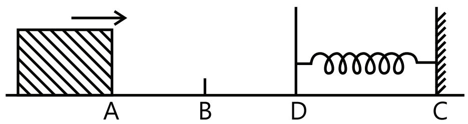
Now, using $s = ut + \frac{1}{2}at^2$, we have

$$s = 0 + \frac{1}{2}\frac{\alpha^2}{2}t^2 = \frac{\alpha^2 t^2}{4}$$

Thus total work done on locomotive is when t=1 s is

$$W = Fs = \frac{m\alpha^2}{2} \times \frac{\alpha^2 t^2}{4} = \frac{m\alpha^4}{8} J$$

Example 5: A 0.5 kg block slides from the point A on a horizontal track with an initial speed of 3 m/s toward a weightless horizontal spring of length 1 m and force constant 2 N/m. The part AB of the track is frictionless and the part BC has the coefficients of static and kinetic friction as 0.22 and 0.2, respectively. If the distances AB and BD are 2 m and 2.14 m, respectively find the total distance through which the block moves before it comes to rest completely (Take $g = 10 m/s^2$).



Sol: The sum of work done by force of friction and spring force is equal to change in kinetic energy of the block.

Suppose the block comes to rest at the point E, i.e., let DE = x. The kinetic energy of the block is spent in overcoming friction and compressing the spring through a distance DE = x.

Kinetic energy of the block

$= \frac{1}{2}mv^2 = \frac{1}{2} x 0.5 x 3^2 = 2.25 J$ ... (i)

As the part AB of the track is frictionless, work done in moving from A to B is zero.

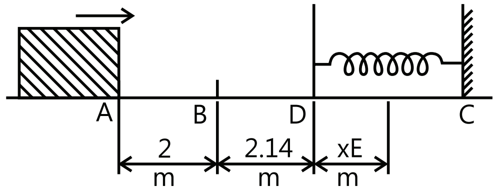
Let normal reaction of the block = mg.

C oefficient of friction = μ

Force due to friction along the track

$BC = \mu mg = 0.2 x 0.5 x 10 = 1N$

Distance through which the block moves against the frictional force = 2.14 + x m



Work done by block against friction before it comes to rest

$= \mu$ mg $(2.14 + x)$; $= (2.14 + x) J$ ... (ii)

Let the spring constant = k

$\therefore$ Work done by the block in compressing the spring through distance X

$= \frac{1}{2}kx^2$; $= \frac{1}{2}2x^2 = x^2 J$ ... (iii)

Adding (ii) and (iii) and equating it to (i), we get

$2.14 + x + x^2 = 2.25$; or $x^2 + x - 0.11 = 0$

or $100x^2 + 100x - 11 = 0$

or $(10x + 11)(10x - 1) = 0$

$\therefore\ x = -\frac{11}{10}$ or $x = \frac{1}{10}$; Since $x \neq -\frac{1}{10}$.

$\therefore\ x = \frac{1}{10} = 0.1m$

Restoring force of the spring

$= kx = 2x0.1 = 0.2N$ ... (iv)

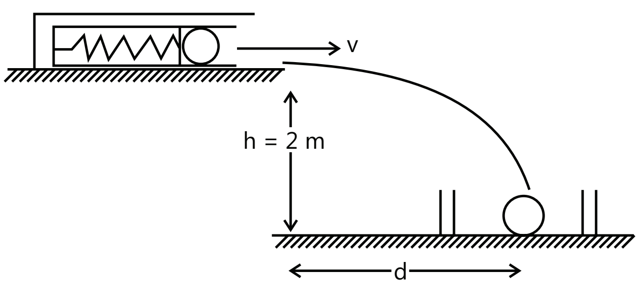
Static frictional force of the block

$\mu_{static} mg = 0.22x0.5x10 = 1.1N$ ... (v)

From (iv) and (v) it is clear that the static frictional force is greater than the restoring force of the spring. Therefore, the block will not move in the backward direction. Hence the total distance through which the block moves before it comes to rest completely is

2.00 + 2.14 + 0.10 = 4.24 m

Example 6: In a spring gun having spring constant 100 N/m, a small ball of mass 0.1 kg is put in its barrel by compressing the spring through 0.05 m as shown in the Fig. 5.29. Find,



(a) The velocity of the ball when the spring is released.

(b) Where a box should be placed on the ground so that ball falls in it, if the ball leaves the gun horizontally at a height of 2 m above the ground? $(g = 10m/s^2)$.

Sol: As the spring expands the potential energy stored in the spring is converted in the kinetic energy of ball. The horizontal distance moved by the ball will depend on time taken by ball to fall vertical height h

(a) When the spring is released its elastic potential energy is converted into kinectic energy

$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}kx^2$; $\Rightarrow v = \sqrt{\frac{5}{2}} m/s$

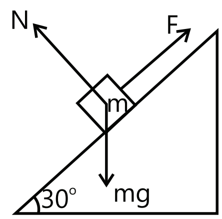
(b) As vertical component of velocity of the ball is zero, the time taken by the ball to reach the ground is

$h = \frac{1}{2}gt^2$; $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2}{5}}$ seconds

Therefore, the horizontal distance traveled by the ball in this time is

$d = v.t = \sqrt{\frac{5}{2}} x \sqrt{\frac{2}{5}} = 1m$

Example 7: A block of mass 2 kg is pulled up on a smooth incline of angle 30° with horizontal. If the block moves with an acceleration of $1m/s^2$, find the power delivered by the pulling force at a time t = 4 s after motion starts. What is the average power delivered during these four seconds after the motion starts?



Sol: Apply newton's second law of motion along the direction of incline to find the applied force. As acceleration is constant and initial velocity is zero, the final velocity at time t is v=at along incline and power is $P = F \bullet v$.

The forces acting on the block are shown in the Fig. 5.30 provided.

Resolving forces parallel to incline

$F - mg\sin\theta = ma$; $\Rightarrow F = mg\sin\theta + ma$

$= 2x9.8x\sin 30^0 + 2x1 = 11.8N$

The velocity after t = 4 s is v=u+at

$= 0 + 1x4 = 4m/s$

Power delivered by force at t = 4 s

$= Force\,x\,Velocity = 11.8x4 = 47.2W$

The displacement during t = 4 s is given by the formula

$v^2=u^2+2as$; $v^2=0+2\times1\times S$

$\therefore\ S = 8m$

Work done in t = 4 s is W = Force × distance

$= 11.8x8 = 94.4J$

$\therefore$ average power delivered

$= \frac{workdone}{time} = \frac{94.4}{4} = 23.6W$

Example 8: A chain of mass m = 0.80 kg and length l = 1.5 m rests on a rough-surfaced table so that one of its ends hangs over the edge. The chain starts sliding off the table all by itself provided the overhanging part equals n = 1/3 of the chain length. What will be the total work performed by the friction forces acting on the chain by the moment it slides completely off the table?

Sol: When chain starts sliding off the table, the friction is limiting. So from this we can find the coefficient of friction. As the chain slides off, the force of friction will be variable in nature as length of chain on table is decreasing. So the work done by the force of friction for infinitesimal displacement ds is $dW = \vec{F}.d\vec{s}$

Let μ be the coefficient of friction between chain and table.

Weight of hanging part $= \mu$

(weight of horizontal part) $nmg = \mu\ (l-n)mg$

$$\mu = \frac{n}{l-n}$$

Let x be the length of the hanging part at some time instant.

Frictional force f (x) =N (normal reaction)

$-\frac{\mu(\ell - x)mg}{\ell}$ The work done by the frictional force if the hanging part increases to (x + dx) is

$dW = f(x)dx$

$$W = \int dW = -\int_{\ell}^{1} \frac{\mu(\ell - x)mg}{\ell}; w = -\frac{\mu mg}{\ell}\left[\ell x - \frac{x^2}{2}\right]_{\ell}^{i}$$

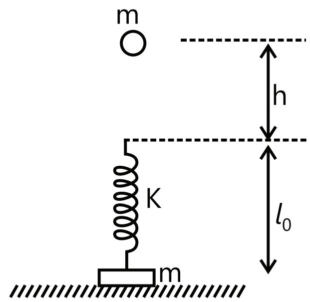
$$W = -\mu mg\left[\ell(1-n) - \frac{\ell}{2}(1-n^2)\right]:$$

Substituting the value of μ from (I), we get

$$w = -\frac{n(1-n)mg\ell}{2} \quad = \frac{1}{3} \times \frac{2}{3} \times \frac{mgl}{2} = \frac{mgl}{9}$$

JEE Advanced/Boards

Example 1: In the Fig. 5.31 shown a massless spring of stiffness k and natural length l_0 is rigidly attached to a block of mass m and is in vertical position. A wooden ball of mass m is released from rest to fall under gravity. Having fallen a height h the ball strikes the spring and gets stuck up in the spring at the top. What should be the minimum value of h so that the lower block will just loose contact with the ground later on? Find also the correspoinding maximum compression in the spring. Assume that $l_o >> \frac{4mg}{k}$. Neglect any loss of energy.



Sol: When ball falls from height h, the loss in its potential energy is equal to the gain in its kinetic energy. At the point of the maximum compression of the spring the ball comes to rest momentarily. After this the ball will again start moving up till the spring is again elongated to the point where the lower block looses contact with ground. For minimum value of h the ball will again come to rest at this point. So the total loss in gravitational potential energy will be equal to gain in the elastic potential energy.

The minimum force needed to lift the lower block is equal to its weight. During upward motion the spring will get elongated. If elongation in the spring for just lifting the block is x_0 then.

$$kx_0 = mg; \qquad \Rightarrow x_0 = \frac{mg}{k} \qquad \text{...... (i)}$$

From COE

$$mg(l_0 + h) = mg(l_0 + x_0) + \frac{1}{2}kx_0^2;$$

$$\Rightarrow mgh = mgx_0 + \frac{1}{2}kx_0^2$$

$$\Rightarrow mgh = \frac{(mg)^2}{k} + \frac{1}{2}\frac{m^2g^2}{k}; \quad \Rightarrow h = \frac{3mg}{2k}$$

During downward motion, suppose maximum compression in the spring is x. From COE

$$mg(l_0 + h) = mg(l_0 - x) + \frac{1}{2}kx^2$$

$$\Rightarrow mgh = -mgx + \frac{1}{2}kx^2$$

$$\Rightarrow mg = \frac{3mg}{2k} = -mgx + \frac{1}{2}kx^2$$

$$\Rightarrow 3(mg)^2 = -2mg\,kx + k^2x^2$$

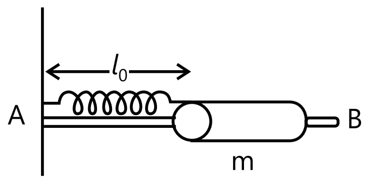
$$\Rightarrow k^2x^2 - 2mgkx - 3(mg)^2 = 0$$

$$\Rightarrow x = \frac{2mgk \pm \sqrt{4(mgk)^2 + 12k^2(mg)^2}}{2k^2}$$

$$= \frac{2mgk \pm 4mgk}{2k^2} \Rightarrow x = \frac{3mg}{k}$$

Example 2: A smooth, light horizontal rod AB can rotate about a vertical axis passing through its end A. The rod is fitted with a small sleeve of mass m attached to the end A by a weightless spring of length ℓ_0 and stiffness k. What work must be performed to slowly get this system going and reach the angular velocity ω?

Sol: When system starts moving about a point A, the spring force provides the necessary centripetal force to the sleeve of mass m to move with angular speed ω. The work done by external agent will be equal to the kinetic energy of the spring and elastic potential energy of the spring.



The mass m rotates in a circle of radius ℓ, which is the extended length of the spring. Centripetal force on $m = k(\ell - \ell_0) = m\omega^2\ell$

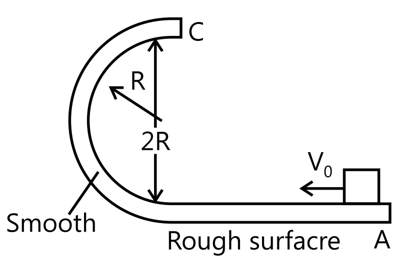
or, $\ell = \frac{\ell_0}{1-n}$ where $n = \frac{m\omega^2}{k}$

W = Change in KE of m + energy stored in the spring

$$= \frac{1}{2}m\omega^2 l^2 + \frac{1}{2}k(\ell - \ell_0)^2 = \frac{1}{2}m\frac{l_0^2\,\omega^2}{(1-n)^2} + \frac{1}{2}k\left[\frac{l_0}{1-n} - l_0\right]^2$$

$$W = \frac{1}{2}\frac{k\ell_0^2}{(1-n)^2}\left[\frac{m\omega^2}{k} + n^2\right]$$

Example 3: A small block is projected with a speed V_o on a horizontal track which turns into a semicircle (vertical) of radius R. Find the minimum value of v_0 so that the body will hit the point A after leaving the track at its highest point. The arrangement is shown in the figure, given that the straight part is rough and the curved part is smooth. The coefficient of friction is μ.



Sol: While block travels on the frictional surface AB, the work done by the frictional force is equal to the change in kinetic energy of the block. The horizontal distance moved by the block after leaving track at point C, will depend on time taken by disc to fall vertical height 2R. At point C, for minimum velocity, normal force on the block is zero.

Let the block escape the point at C with a velocity V horizontally. Since it hits the initial spot A after falling thorugh a height 2R we can write $(2R) = (1/2)gt^2$

where t = time of its fall

$\Rightarrow t = 2\sqrt{R/g}$

$\therefore$ the distance $AB = 2v\sqrt{R/g}$

$\Rightarrow d = 2v\sqrt{R/g}$ ... (i)

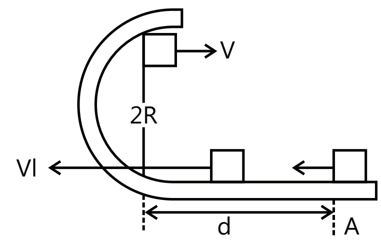
Work–energy theorem applied to the motion of the body from A to B leads

$\Delta KE = W_F$

$\Rightarrow \frac{1}{2}mv_0^2 - \frac{1}{2}mv_1^2 = \mu mgd$

$\Rightarrow v_0 = \sqrt{v_1^2 + 2\mu gd}$ ... (ii)

Energy conservation between B and C yields



$\Rightarrow \frac{1}{2}mv_0^2 - \frac{1}{2}mv_1^2 = mg(2R)$

$\Rightarrow v_1 = \sqrt{v^2 + 4gR}$ ... (iii)

When the disc escapes C, its minimum speed v can be given as

$\frac{mv^2}{R} = mg$ ($\therefore$ the normal contact force $= 0$)

$\Rightarrow v = \sqrt{gR}$ ... (iv)

By using (iii) and (iv), we obtain

$v_1 = \sqrt{5gR}$ ... (v)

Using (i) and (iv), we obtain $d = (\sqrt{gR})2\left(\sqrt{\frac{R}{g}}\right) = 2R$ ... (vi)

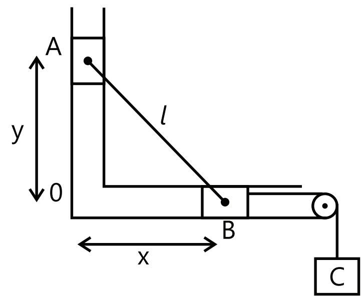
Putting the values of v_1 and d in (ii), we obtain

$v_0 = \sqrt{5gR + 2\mu g(2R)}$

$\Rightarrow v_0 = \sqrt{(5 + 4\mu)gR}$

Example 4: Two bodies A and B connected by a light rigid bar of 10 m long move in two frictionless guides as shown in the Figure. If B starts from rest when it is vertically below A, find the velocity of B when X = 6 m.

Sol: As the blocks A and C fall vertically downwards, the loss in its potential energy is equal to gain in kinetic energy of blocks A, B and C.



Assume

$m_A = m_B = 200\text{kg}$

and $m_c = 100\,\text{kg}$

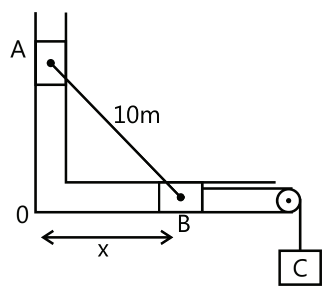
At the instant, when the bar is as shown in the Figure

$$x^2 + y^2 = l^2;\ \therefore\ 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \quad \text{... (i)}$$

$$\therefore\quad x\frac{dx}{dt} = -y\frac{dy}{dt} \quad \text{... (ii)}$$

where $\frac{dx}{dt}$ = velocity of B and $\frac{dy}{dt}$ = velocity of A

Applying the law of conservation of energy, loss of potential energy of A, if it is going down when the rod is vertical to the position as shown in the Fig.



$= m_A g(10-8) = 2 \times 200 \times 9.8$

C moves down 6 m since B moves 6 m along x-axis.

Total loss of potential energy

$= 200 \times 9.8 \times 2 - 100 \times 9.8 \times 6 = 100 \times 9.8 \times 10 = 9800\,\text{J}.$

This must be equal to kinetic energy gained

Kinetic energy gained

$$= \frac{1}{2}m_A\left(v_A\right)^2 + \frac{1}{2}m_B\left(v_B\right)^2 + \frac{1}{2}m_C\left(v_C\right)^2$$

$$= \frac{1}{2}\times 200\left(\frac{dy}{dt}\right)^2 + \frac{1}{2}\times 200\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}\times 100\left(\frac{dx}{dt}\right)^2$$

$$= 100\left(\frac{dy}{dt}\right)^2 + 150\left(\frac{dx}{dt}\right)^2$$

$$= 100\left[\frac{x}{y}\frac{dx}{dt}\right]^2 + 150\left(\frac{dx}{dt}\right)^2 \quad \text{from (ii)}$$

$$= 100\left[\frac{6}{8}\frac{dx}{dt}\right]^2 + 150\left(\frac{dx}{dt}\right)^2$$

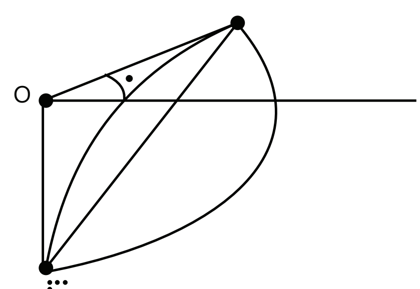
$$= \left[100 \times \frac{9}{16} + 150\right]\left(\frac{dx}{dt}\right)^2 = \frac{3300}{16}v_B^{\,2}$$

$$\therefore\ \frac{3300}{16}v_B^2 = 9800$$

$$\therefore\ v_B = \sqrt{\frac{98 \times 16}{33}} = 7 \times 4\sqrt{\frac{2}{33}} = 6.9\text{ms}^{-1}$$

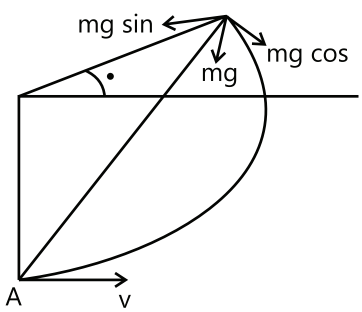
$\therefore$ Veloctiy of B at the required moment is = 6.9ms^{-1}

Example 5: A particle is suspended by a string of length 'l '. It is projected with such a velocity v along the horizontal such that after the string becomes slack it flies through its initial position. Find v.



Sol: As the string becomes slack, the tension in the string becomes zero. Apply the Newton's second law of motion along the direction of string at the instant of slacking. The loss in kinetic energy is equal to gain in potential energy as the particle moves in vertical plane.

Let the velocity be v' at B where the string become slack and the string makes angle θ with horizontal by the law of conservation of energy.



$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + mg\ell(1+\sin\theta) \quad \text{... (i)}$$

$$\text{or } v'^2 = v^2 - 2g\ell(1+\sin\theta) \quad \text{... (ii)}$$

By the dynamics of circular motion

$$mg\sin\theta = \frac{mv'^2}{\ell};\Rightarrow v'^2 = g\ell\sin\theta \quad \text{... (iii)}$$

From equations (ii) and (iii), we get

$$\therefore g\ell\sin\theta = v^2 - 2g\ell(1+\sin\theta) \quad \text{... (iv)}$$

At B the particle becomes a projectile of velocity v' at $90-\theta$ with the horizontal.

Here $u_x = v'\sin\theta \,\&\, u_y = v'\cos\theta$

$a_x = 0 \;\&\; a_y = -g$

$$\therefore \ell\cos\theta - v'\sin\theta t \quad \text{... (v)}$$

$$\therefore t = \frac{\ell\cos\theta}{v'\sin\theta} \;\&\; -\ell(1+\sin\theta) = v'\cos\theta$$

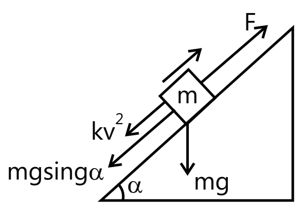
$$\frac{l\cos\theta}{v'\sin\theta} - \frac{1}{2}g\frac{I^2\cos^2\theta}{v'^2\sin^2\theta} \quad ; \quad \Rightarrow 2\sin^3\theta + 3\sin^2\theta - 1 = 0$$

$\therefore \sin\theta = \frac{1}{2}$ is the acceptable solution

$$\therefore v^2 = 2g\ell + 3g\ell\, x\frac{1}{2} = \frac{7g\ell}{2} \Rightarrow v = \sqrt{\frac{7g\ell}{2}}$$

[from equation (iv)]

Example 6: A motorcar of mass 1000kg attains a speed of 64 km/hr when running down an inclination of 1 in 20 with the engine shut off. It can attain a speed of 48 km/hr up the same incline when the engine is switched on. Assuming that the resistance varies as the square of the velocity, find the power developed by engine.



Sol: While moving down the incline plane car attains constant speed of 64 km/hr and while moving up the incline plane it attains the constant speed of 48 km/hr.While moving down the plane force of gravity is balanced by force of friction. While moving up on incline plane, the force developed by the engine of the car is balanced by the frictional force and force of gravity. The power developed by the engine is given by $P = \vec{F}\cdot\vec{V}$

When the motor car is moving down the plane there is force Mg sin α down the plane. This is opposed by the resistance, which is proportional to square of the velocity. That is $Mg\sin\alpha \propto V^2$

$Mg\, x\frac{1}{20} = kV^2$ where k is a constant.

$$\therefore \frac{1000\,x\,g}{20} = k\left(64x\frac{5}{18}\right)^2$$

$$k = \frac{1000\,x\,g}{20}x\left(\frac{18}{64\,x5}\right)^2 \quad \text{... (i)}$$

When the engine is on, let the tractive force (force exerted by engine) be F. This is used to overcome the force due to incline and the resistance offered.

$$\therefore \quad F = k(48x5/18)^2 + \frac{Mg}{20};$$

$$F = k(48x5/18)^2 + \frac{1000\,x\,g}{20}$$

$$F = \frac{1000\,x\,g}{20}x\left(\frac{18}{64\,x5}\right)^2 x\left(48\,x\frac{5}{18}\right)^2 + \frac{1000\,x\,g}{20}$$

$$= \frac{1000\,x\,9.8}{20}\left[\frac{9}{16}-1\right]$$

$$= \frac{50\,x\,9.8\,x\,2.5}{16} = 765.6\text{N}$$

Power developed = Force x Velocity

$$= 765.6 \text{ x } 48 \text{ x } 5/18 = 10208\,\text{W} = 10.2\,\text{W}$$

JEE Main/Boards

Exercise 1

Q.1 What is meant by zero work? State the conditions under which a force does no work. Give any one example.

Q.2 Two bodies of unequal masses have same linear momentum. Which one has greater K.E.?

Q.3 Two bodies of unequal masses have same K.E. Which one has greater linear momentum?

Q.4 How do potential energy and K.E. of a spring vary with displacement? Is this variation different from variation in potential energy and K.E. of a body in free fall?

Q.5 Explain what is meant by work. Obtain an expression for work done by a constant force.

Q.6 Discuss the absolute and gravitational units of work on m.k.s. and c.g.s systems.

Q.7 What is meant by positive work, negative work and zero work? Illustrate your answer with two example of each type.

Q.8 Obtain graphically and mathemativally work done by a variable force.

Q.9 What are conservative and non-conservative forces, explain with examples. Mention some of their properties.

Q.10 What is meant by power and energy? Give their units.

Q.11 Explain the meaning of K.E. with examples. Obtain an expression for K.E. of a body moving uniformly?

Q.12 State and explain work energy principle.

Q.13 What do you mean by potential energy? Give any two examples of potential energy other than that of the gravitational potential energy.

Q.14 Obtain an expression for gravitational potential energy of a body.

Q.15 Explain what is meant by potential energy of a spring? Obtain an expression for it and discuss the nature of its variation.

Q.16 Mention some of the different forms of energy and discuss them briefly.

Q.17 A particle moves along the x-axis from x=0 to x=5m under the influence of a force given by $f = 7 - 2x + 3x2$. Calculate the work done.

Q.18 The relation between the displacement x and the time t for a body of mass 2kg moving under the action of a force is given by $x = t^3/3$, where x is the metre and t is in second. Calculate work done by the body in first 2 seconds.

Q.19 A woman pushes a trunk on a railway platform which has a rough surface. She applies a force of 100N over a distance of 10m. Thereafter, she gets progressively tired and her applied force reduces linearly with distance to 50N. The total distance through which trunk has been moved is 20m. Plot the force applied by the woman and frictional force which is 50N against the distance. Calculate the work done by the two forces over 20m.

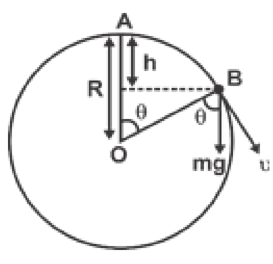
Q.20 A body of mass 50kg has a momentum of 1000kg ms^{-1}. Calculate its K.E.

Q.21 A bullet of mass 50g moving with a velocity of 400ms-1 strikes a wall and goes out from the other side with a velocity of $100ms^{-1}$. Calculate work done in passing through the wall.

Q.22 A body dropped from a height H reaches the ground with a speed of $1.2\sqrt{gH}$. Calculate the work done by air-friction.

Q.23 A bullet weighting 10g is fired with a velocity of $800ms^{-1}$. After passing thorugh a mud wall 1m thick, its velocity decreases to 100 m/s. Find the average resistance offered by the mud wall.

Q.24 A particle originally at rest at the highest point of a smooth vertical circle of radius R, is slightly displaced. Find the vertical distance below the highest point where the particle will leave the circle.



Exercise 2

Single Correct Choice Type

Q.1 A force $\vec{F} = -k\left(y\hat{i} + x\hat{j}\right)$, where k is a positive constant, acts on a particle moving in the xy plane.

Starting from the origin, the particle is taken along the positive x-axis to the point (a, 0), and the parallel to the y-axis to the point (a, a). The total work done by the force on the particle is:

(A) $2ka^2$ (B) $2ka^2$ (C) $-ka^2$ (D) ka^2

Q.2 Supposing that the earth of mass m moves around the sun in a circular orbit of radius 'R', the work done in half revolution is:

(A) $\frac{mv^2}{R} x \pi R$ (B) $\frac{mv^2}{R} x 2R$

(C) Zero (D) None of these

Q.3 A string of mass 'm' and length 'I' rests over a frictionless table with 1/4th of its length hanging from a side. The work done in bringing the hanging part back on the table is:

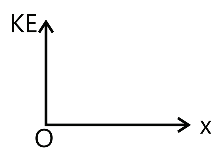
(A) $mgl/4$ (B) $mgl/32$

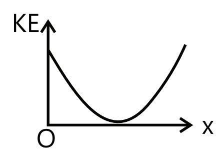
(C) $mgl/16$ (D) None of these

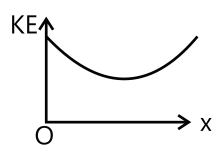
Q.4 A weight mg is suspended from a spring. If the elongation in the spring is x_o, the elastic energy stored in it is:

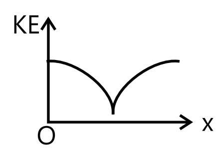
(A) $\frac{1}{2}mgx_o$ (B) $2mgx_0$ (C) mgx_0 (D) $\frac{1}{4}mgx_0$

Q.5 A ball is thrown up with a certain velocity at angle θ to the horizontal. The kinetic energy KE of the horizontal. The kinetic energy KE of the ball varies with horizontal displacement x as:

(A)


(B)


(C)


(D)


Q.6 A body m_1 is projected upwards with velocity v_1 another body m_2 of same mass is projected at an angle of 45°. Both reach the same height. What is the ratio of their kinectic energies at the point of projection:

(A) 1 (B) 1/2 (C) 1/3 (D) 1/4

Q.7 A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, then the work required to pull the hanging part onto the table is:

(A) MgL (B) $\frac{MgL}{3}$ (C) $\frac{4MgL}{9}$ (D) $\frac{MgL}{18}$

Q.8 A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to:

(A) $t^{1/2}$ (B) $t^{3/4}$ (C) $t^{3/2}$ (D) t^2

Q.9 An alpha particle of energy 4 MeV is scattered through 180° by a fixed uranium nucleus. The distance of the closest approach is of the order of

(A) $1\mathring{A}$ (B) 10^{-10} cm

(C) 10^{-12} cm (D) 10^{-15} cm

Q.10 A simple pendulum has a string of length and bob of mass m. When the bob is at its lowest position, it is given the minimum horizontal speed necessary for it to move in a circular path about the point of suspension. The tension in the string at the lowest position of the bob is:

(A) 3mg (B) 4mg (C) 5mg (D) 6mg

Q.11 A horse pulls a wagon with a force of 360N at an angle of 60^0 with the horizontal at a speed of 10Km/hr. The power of the horse is:

(A) 1000 W (B) 2000 W

(C) 500 W (D) 750 W

Q.12 A man pulls a bucket of water from a well of depth H. If the mass of the rope and that of the bucket full of water are m and M respectively, then the work done by the man is:

(A) $(m+M)gh$ (B) $\left(\frac{m}{2}+M\right)gh$

(C) $\left(\frac{m+M}{2}\right)gh$ (D) $\left(m+\frac{M}{2}\right)gh$

Q13 A small block of mass m is kept on a rough inclined surface of inclination θ fixed in a elevator. The elevator goes up with a uniform velocity v and the block does not slide on the wedge. The work done by the force of friction on block in time t will be-

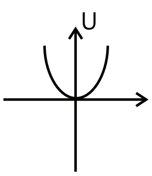
(A) Zero (B) mgvtcosθ

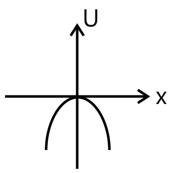
(C) mgvtsinθ (D) mgvtsin2θ

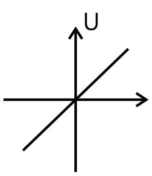
Q.14 Two equal masses are attached to the two ends of a spring of spring constant k. the masses are pulled out symmetrically k. the masses are pulled out symmetrically to stretch the spring by a length x over its natural length. The work done by the spring on each mass is-

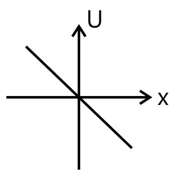
(A) $\frac{1}{2}kx^2$ (B) $-\frac{1}{2}kx^2$ (C) $\frac{1}{4}kx^2$ (D) $-\frac{1}{4}kx^2$

Q.15 A particle is acted by a force F-kx, where k is a +ve constant. Its potential energy at x-0 is zero. Which curve correctly represents the variation of potential energy of the block with respect to x?

(A)


(B)


(C)


(D)


Q.16 If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along there different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3.

(A) $W_1 > W_2 > W_3$ (B) $W_1 = W_2 = W_3$

(C) $W_1 < W_2 < W_3$ (D) $W_2 > W_1 > W_3$

Q.17 An ideal spring with spring-constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is,

(A) $k = \frac{(2+\sqrt{3})mg}{\sqrt{3}R}$ (B) $\frac{2Mg}{k}$

(C) $\frac{Mg}{k}$ (D) $\frac{4Mg}{2k}$

Q.18 The total work done on a particle is equal to the change in its kinetic energy:

(A) Always

(B) Only if the forces acting on it are conservative

(C) Only if gravitational force alone acts on it

(D) Only if elastic force alone acts on it

Previous Years Questions

Q.1 Two masses of 1g and 4g are moving with equal kinetic energies. The ratio of the magnitudes of their momenta is: ***(1980)***

(A) 4:1 (B) $\sqrt{2}:1$ (C) 1:2 (D) 1:16

Q.2 A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position, and has a speed μ. The magnitude of the change in its velocity as it reaches a position, where the string is horizontal, is ***(1998)***

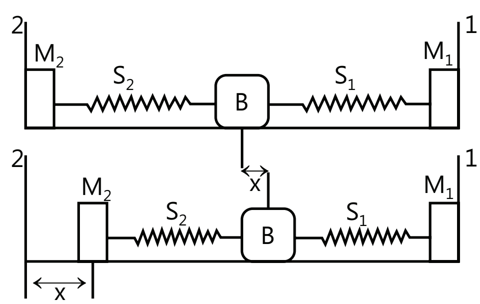
(A) $\sqrt{u^2 - 2gL}$ (B) $\sqrt{2gL}$

(C) $\sqrt{u^2 - gL}$ (D) $\sqrt{2(u^2 - gL)}$

Q.3 A wind-powered generator converts wind energy into electric energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to ***(2000)***

(A) v (B) v^2 (C) v^3 (D) v^4

Q.4 An ideal spring with spring constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is ***(2002)***



(A) $\frac{4Mg}{k}$ (B) $\frac{2Mg}{k}$ (C) $\frac{Mg}{k}$ (D) $\frac{Mg}{2k}$

Q.5 A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and 4k, respectively. The other ends are attached to supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall I by a small distance x and released. The block returns and moves a maximum distance y towards wall 2. Displacement x and y are measured with respect to the equilibrium position of the block B.

The ratio $\frac{y}{x}$ is ***(2008)***

(A) 4 (B) 2 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Q.6 This question has Statement-I and Statement-II. Of the four choices given after the statements, choose the one that best describes the two statements.

If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2. ***(2012)***

Statement-I: If stretched by the same amount, work done on S_1, will be more than that on S_2

Statement-II: $k_1 < k_2$

(A) Statement-I is false, Statement-II is true

(B) Statement-I is true, Statement-II is false

(C) Statement-I is true, Statement-II is the correct explanation for Statement-I

(D) Statement-I is true, Statement-II is true, and Statement-II is not the correct explanation for Statement-I.

Q.7 A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20 % efficiency rate. Take g = 9.8 ms^{-2}: ***(2016)***

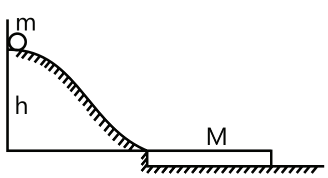
(A) 6.45×10^{-3} kg (B) 9.89×10^{-3} kg

(C) 12.89×10^{-3} kg (D) 2.45×10^{-3} kg

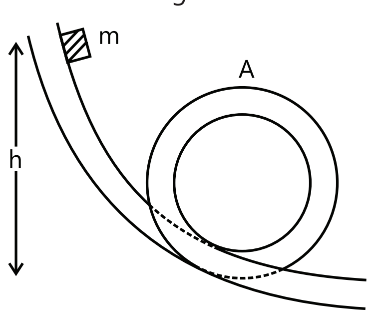
JEE Advanced/Boards

Exercise 1

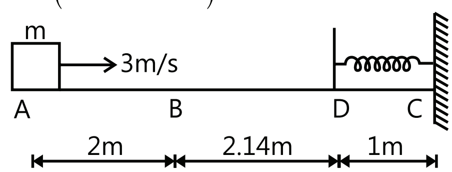
Q.1 A small disc of mass m slides down a smooth hill of height h without initial velocity and gets onto a plank of mass M lying on the horizontal plane at the base of hill as shown in the Figure. Due to friction between the disc and the plank, disc slows down and beginning with a certain moment, moves in one piece with the plank. Find out total work performed by the frictional forces in this process.



Q.2 A block of mass m starts from rest to slide along a smooth frictionless track of the shape shown in the Figure. What should be height h so that when the mass reaches point A on the track, it pushes the track with a force equal to thrice it weight?

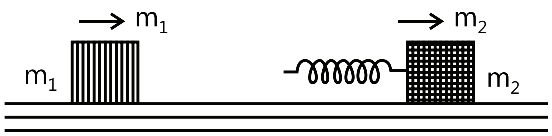


Q.3 A 0.5kg block slides from point A on a horizontal track with a initial speed of 3m/s towards a weightless spring of length I m and having a force constant 2 N/m. The part AB of the track is frictionless and the part BC has coefficent of static and kinetic friction as 0.22 and 0.20 respectively. If the distacnces AB and BD are 2m and 2.14m respectively, find the total distance through which the block moves before it comes to rest completely. $\left(g = 10 m/s^2\right)$



Q.4 A particle is suspended from a fixed point by a string of length 5m. It is projected horizontally from the equilibrium position with such a speed that the string slackens after the particle has reached a height of 8m above the lowest point. Find the speed of the particle just before the string slackens and the height to which the particle will rise further.

Q.5 Two blocks of masses $m_1 = 2kg$ and $m_2 = 5kg$ are moving in the same direction along a fricionless surface with speeds 10m/s and 3m/s respectively, m_2 being ahead of m_1. An ideal spring with k=1120N/m is attached to the back side of m_2. Find the maximum compression of the spring when the blocks collide.

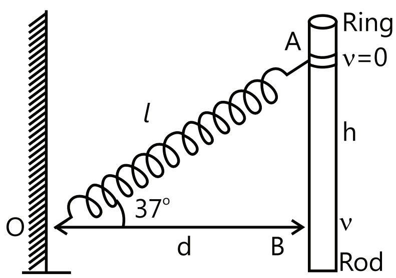


Q.6 An automobile of mass m accelerates, starting from rest, while the engine supplies constant power P; show that:

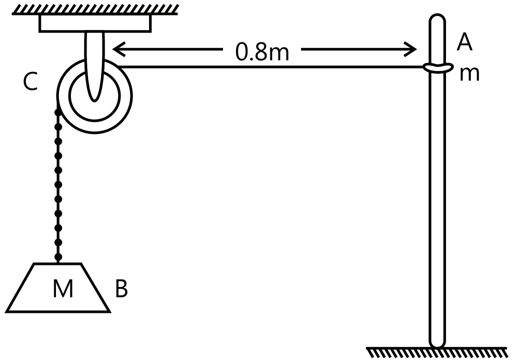
(a) The velocity is given as a function of time by $v = (2Pt/m)^{1/2}$

(b) The position is given as a function of time by $s = (8P/9m)^{1/2} t^{3/2}$

Q.7 One end of a light spring of natural length d and spring constant k is fixed on a rigid wall and the other end is fixed to a smooth ring of mass m which can slide without friction on a vertical rod fixed at a distance d from the wall. Initially the spring makes an angle of 37° with the horizontal as shown in the Figure. If the system is released from rest, find the speed of the ring when the spring becomes horizontal $[\sin 37^0 = 3/5]$



Q.8 A ring of mass m=0.3kg slides over a smooth vertical rod A. Attached to the ring is a light stirng passing over a smooth fixed pulley at a distance of 0.8m from the rod as shown in the figure. At the other end of the string there is a mass M=0.5kg. The ring is held in level with the pully and then released.

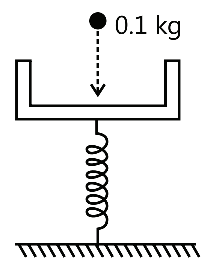


(a) Determine the distance by which the mass m moves down before coming to rest for the first time.

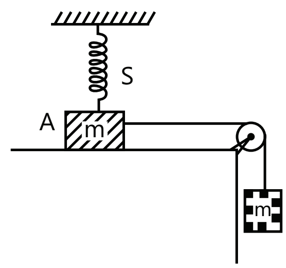
(b) How far below the initial position of m is the equilibrium position of m located?

Q.9 A string, with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2m from the wall, has a point mass M=2kg attached to it at a distance of 1m from the wall. A mass m=0.5kg attached at the free end is held at rest so that the string is horizontal between the wall and the pulley and vertical beyond the pulley. Find the speed with which the mass M will hit the wall when the mass m is released?

Q.10 A massless platform is kept on a light elastic spring, as shown in the Figure. When a sand particle of mass 0.1kg is dropped on the pan from a height of 0.2m, the particle strikes the pan and sticks of 0.2m, the particle strikes the pan and sticks to it while the spring compresses by 0.01m. From what height should be particle be dropped to cause a compression of 0.04m?

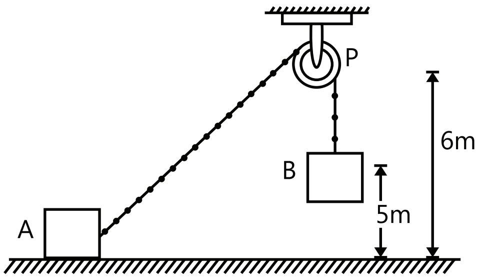


Q.11 Two blocks A and B each having mass of 0.32kg are connected by a light string passing over a smooth pully as shown in the Figure.



The horizontal surface on which the block A slides is smooth. The block A is attached to a spring of force constant 40N/m whose other end is fixed to a support 0.40m above the horizontal surface. Initially, when the system is releasesd to move, the spring is vertical and unstretched. Find the velocity of the block A at the instant it breaks off the surface below it. $[g = 10m/s^2]$

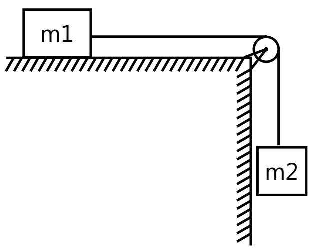
Q.12 A block of mass m is held at rest on a smooth horizontal floor. A light frictionless, small pulley is fixed at a height of 6m from the floor. A light inextensible string of length 16m, connected with A passes over the pulley and another identical block B is hung from the string. Initial height of B is 5m from the floor as shown in the figure. When the system is released from rest, B starts to move vertically downwards and A slides on the floor towards right.



(a) If at an instant the string makes an angle θ with the horizontal, calculate relation between velocity u of A velocity v of B.

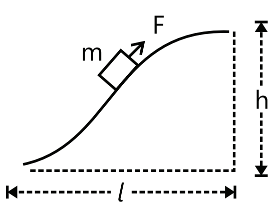
(b) Calculate v when B strikes the floor. $\left(g = 10m/s^2\right)$

Q.13 Two blocks are connected by a string as shown in the Figure. They are released from rest. Show that after they have moved a distance L, their common speed is given by $\sqrt{\frac{2(m_2 - \mu ml)gl}{(m_1 - m_2)}}$, where μ is the coeffiecient of friction between the floor and the blocks.



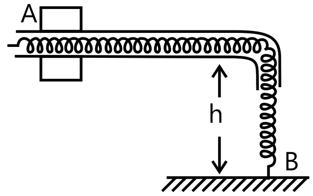
Q.14 A particle of mass m is moving in a circular path of constant radius r such that is centripetal acceleration α_c is varying with time t as $\alpha_c = k^2rt^2$ when k is a constant. what is the power delivered to the particle by the forces acting on it?

Q.15 A body of mass m was slowly pulled up the hill as shown in the Figure. by a force F which at each point was directed along a tangent to the trajectory.

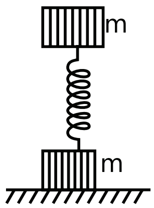


Find the work performed by this force, if the height of the hill is h, the length of its base l, and the co-effiecient of frinction between m and the hill is.

Q.16 A chain A B of length l is loaded in a smooth horizontal tube so that a part of its length h hangs freely and touches the surface of the table with its end B. At a certain moment, the end A of the chain is set free, with what velocity will this end of the chain slip out of the tube?



Q.17 A system consists of two identical cubes, each of mass m, linked together by the compressed weightless spring constant k. The cubes are also connected by a thread which is burned through at a certain moment. Find:



(a) At what values of Δl the initial compression of the spring, the lower cube will bounce up after the thread has been burned through:

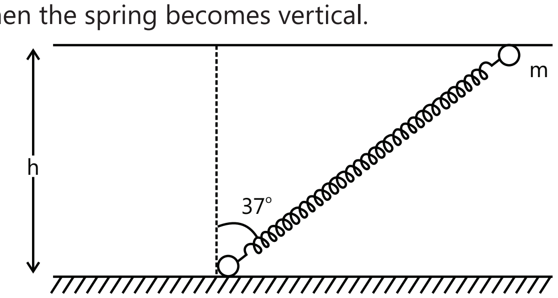
(b) To what height h the centre of gravity of this system will rise if the initial compression of the spring $\Delta l = 7mg/k$

Q.18 A stone with weight w is thrown vertically upward into the air with initial speed v_0. If a constant force f due to air drag acts on the stone throughout its flight:

(a) Show that the maximum height reached by the stone is $h = \frac{v_0^2}{2g\left[1 + (f/w)\right]}$.

(b) Show that the speed of the stone upon impact with the ground is $v = v_0\left(\frac{w - f}{w + f}\right)^{1/2}$

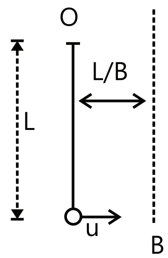
Q.19 One end of spring of natural length h is fixed at the ground and the other end is fitted with a smooth ring of mass m which is allowed to slide on a horizontal rod fixed at a height h as shown in Figure. Initially, the spring makes an angle of 37° with the vertical when the system is released from rest. Find the speed of the ring when the spring becomes vertical.



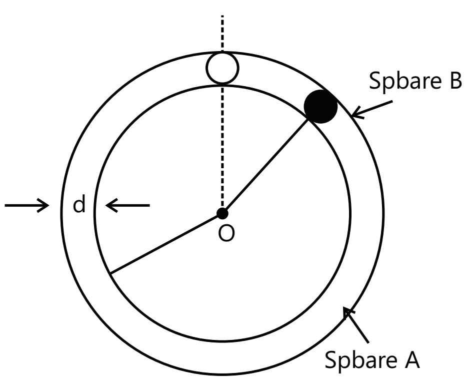
Q.20 A nail is located at a certain distance verticaly below the point of suspension of a simple pendulum.

The pendulum bob is released from the position where the string makes an angle of 60° with the downward vertical. Find the distance of the nail from the point of suspension such that the bob will just perform a complete revolution with the nail as centre. The length of the pendulum is 1m.

Q.21 A partical is suspended vertically from a point O by an inextensible massless string of length L.A vertical line A B is at a distance of L/8 form O as shown in the Figure. The particle is given a horizontal velocity u. At some point, its motion cases to be circular and eventually the object passes through the line A B. At the instant of crossing A B, its velocity is horizontal. Find u.



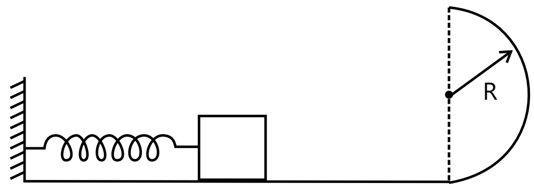
Q.22 A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B. The smaller sphere A has a radius R and the space between the two spheres has a width d. the ball has a diameter very slightly less that d. All surfaces are frictionless. The ball is given a gental push (towards the right in the Figure). The angle made by the radius vector of the ball with the upwards vertical is denoted by (see Figure).



(a) Express the total normal reaction force exerted by the spheres on the ball as a function of angle θ

(b) Let N_A and N_B denote the magnitudes of the normal reaction forces on the ball exerted by the spheres A and B, respectively. Sketch the variations of N_A and N_B as fuctions of cos θ in the range $0 \leq \theta \leq \pi$ by drawing two separate graphs in your answer book, taking $\cos\theta$ on the horizontal axes.

Q.23 Figure shows a smooth track, a part of which is a circle of radius R. A block of mass m is pushed against a spring of spring constant k fixed at the left end and is then released. Find the initial compression of the spring so that the block presses the track with a force mg when it reaches the point P, where the radius of the track is horizontal.



Q.24 A particle of mass 100g is suspended from one end of a weightless string of length 100cm and is allowed to swing in a vertical plane. The speed of the mass is 200cm/s when the string makes an angle of 60° with the vertical.

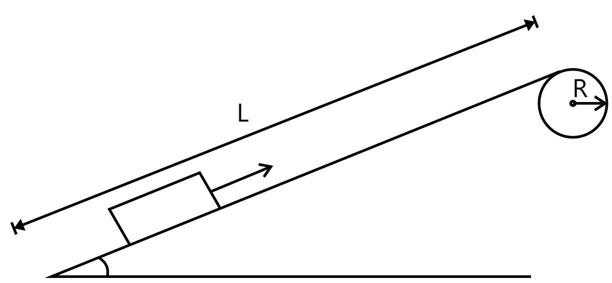
Determine

(a) The tension in the string at 60° and

(b) The speed of the particle when it is at the lowest position. (Take g=980 cm/s^2)

Q.25 A smooth horizontal rod AB can rotate about a vertical axis passing thorugh its end A. The rod is fitted with a small sleeve of mass m attached ot the end A by a weightless spring of length l and spring constant k. what work must be performed to slowly get this system going and reaching to slowly get this system going and reaching the angular velocity ω

Q.26 Figure shows a smooth track which consists of a straight inclined part of length L joining smoothly with the circular part. A particle of mass m is projected up the inclined from its bottom.

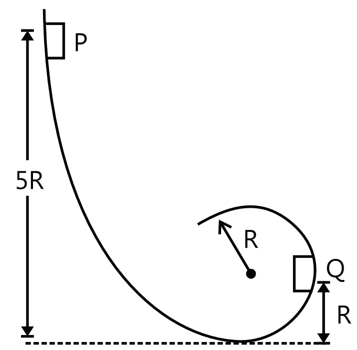


(a) Find the minimum projection speed u_0 for which the particle reaches the top of track.

(b) Assuming that the projection speed is 2 u_0 and that the block does't lose contact with the track before reaching its top. Find the force acting on it when it reaches the top.

(c) Assuming that the projection speed is only slightly grater than v_0, where will the block lose contact with the track?

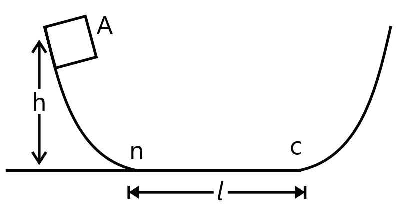
Q.27 A small block of mass m slides along a smooth frictional track as shown in the Figure.



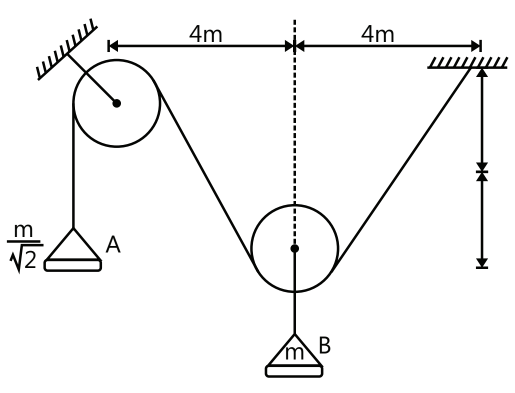
(a) If it starts from rest at P, what is the resultant force acting on it at Q?

(b) At what height above the bottom of the loop should the block be released so that the force it exerts against the track at the top of the loop equals its weight?

Q.28 A particle slides along a track with elevated ends and a flat central part as shown in the Figure. The flat central part as shown in the Figure. The flat part has a length l=3.0m. The curved portions of the track are frictionless. For the flat part the coefficient of kinectic friction is $\mu_k = 0.20$, the partical is released at point A which is at a height h=1.5m above the flat part of the track. Where does the particle finally come to rest?



Q.29 The system of mass A and B shown in the Figure is released from rest with x=0, determine



(a) The velocity of mass B when x=3m.

(b) The maximum displacement of mass B.

Exercise 2

Single Correct Choice Type

Q.1 When water falls from the top of a water fall 100m high:

(A) It freezes

(B) It warms up slightly

(C) It evaporates

(D) There is no change in temperature.

Q.2 A 2kg block is dropped from a height of 0.4m on a spring of force constant 2000N/m. the maximum compression of the spring is:

(A) 0.1m (B) 0.2m (C) 0.01m (D) 0.02m

Q.3 A partical of mass M is moving in a horizontal circle of radius 'R' under the centripetal force equal to K/R^2, where K is constant. The potential energy of the particle is

(A) K/2R (B) –K/2R (C) K/R (D) –K/R

Q.4 A linear harmonic oscillator of force constant 2×10^6N and amplitude 0.01 m has a total mechanical energy of 160 J. Its

(A) Maximum potential energy is 100 J

(B) Maximum kinetic energy is 100 J

(C) Maximum potential energy is 160 J

(D) Maximum potential energy is zero.

Q.5 The potential energy of particle varies with position x according to the relation

$U(x) = 2x_4 - 27x$ the point $x = 3/2$ is point of

(A) Unstable equilibrium (B) Stable equilibrium

(C) Neutral equilibrium (D) None of these

Q.6 A particle of mass m is fixed to one end of a light rigid rod of length l and rotated in a vertical circular path about is other end. The minimum speed of the particle at its hightest point must be

(A) Zero (B) $\sqrt{g\ell}$ (C) $\sqrt{1.5g\ell}$ (D) $\sqrt{2g\ell}$

Q.7 A particle of mass m is fixed to one end of a light spring of force constant k and unstretched length l. The system is rotated about the other end of the spring with an angular velocity ω, in gravity free space. The increase in length of the spring will be:

(A) $\sqrt{1.5g\ell}$ (B) $\sqrt{g\ell}$ (C) $\sqrt{2g\ell}$ (D) None

Q.8 A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time as $a_c = k^2rt^2$ where k is a constant. The power delivered to the particle by the forces acting on it is:

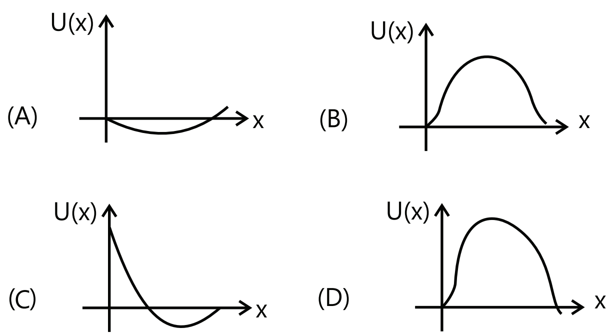
(A) $2\pi mk^2r^2t$ (B) mk^2r^2t

(C) $\frac{1}{3}mk^4r^2t^5$ (D) 0

Q.9 A simple pendulum having a bob of mass m is suspended from the ceiling of a car used in a stunt film shooting. The car moves up along an inclined cliff at a speed v and makes a jump to leave the cliff and lands at some distance. Let R be the maximum height of the car from the top of the cliff. The tension in the string when the car is in air is

(A) mg (B) $mg\frac{mv^2}{R}$ (C) $mg\frac{mv^2}{R}$ (D) Zero

Q.10 A particle, which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of particle is



Q.11 A wind-powered generator converts wind energy into electric energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electric energy. For wind speed V, the electrical power output will be proportional to

(A) V (B) v^2 (C) v^3 (D) v^4

Q.12 A block of mass M is hanging over a smooth and light pulley thorugh a light string. The other end of the string is pulled by a constant force F. The string energy of the block increases by 20 J in 1s.

(A) The tension in the string is Mg.

(B) The tension in the string is F.

(C) The work done by the tension on the block is 20J in the above 1s.

(D) The work done by the force of gravity is -20J in the above 1s.

Q.13 Consider two observers moving with respect to each other at a speed v along a straight line. They observe a block of mass m moving a distance ℓ on a rough surface. The following quantities will be same as observed by the two observers.

(A) Kinetic energy of the block at time t

(B) Work done by friction.

(C) Total work done on the block

(D) Acceleration of the block.

Multiple Correct Choice Type

Q.14 A particle of mass m is attached to a light string of length ℓ, the other end of which is fixed. Initially the string is kept horizontal and the particle is given an upward velocity v, the particle is just able to complete a circle.

(A) The string becomes slack when the particle reaches its highest point

(B) The velocity of the particle becomes zero

(C) The kinetic energy of the ball in initial position was $\frac{1}{2}mv^2 = mg\ell$.

(D) The particle again passes through the initial position.

Q.15 The kinetic energy of a particle continuously increases with time.

(A) The resultant force on the particle must be parallel to the velocity at all instants

(B) The resultant force on the particle must be at an angle less than 90^0 all the time

(C) Its height above the ground level must continuously decreases.

(D) The magnitude of its linear momentum is increasing continuously.

Q.16 One end of a light spring of constant k is fixed to a wall and the other end is tied to block placed on a smooth horizontal surface. In a displacement, the work done by the spring is $\frac{1}{2}kx^2$. The possible cases are.

(A) The spring was initially compressed by a distance x and was finally in its natural length.

(B) It was initially stretched by a distance x and finally was in its natural length.

(C) It was initially in its natural length and finally in a compressed position.

(D) It was initially in its natural length and finally in a stretched position.

Q.17 No work is done by a force on an object if,

(A) The force is always perpendicular to its velocity

(B) The force is always perpendicular to its acceleration

(C) The object is stationary but the point of application of the force moves on the object

(D) The object moves in such a way that the point of application of the force remains fixed

Q.18 A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that,

(A) Its velocity is constant

(B) It acceleration is constant

(C) Its kinetic energy is constant

(D) It moves in a circular path

Q.19 A heavy stone is thrown from a cliff of height h in a given direction. The speed with which it hits the ground:

(A) Must depend on the speed of projection

(B) Must be larger then the speed of projection

(C) Must be independent of the speed of projection

(D) May be smaller than the speed of projection

Q.20 You lift a suitcase from the floor and keep it on a table. The work done by you on the suitcase does not depend on:

(A) The path taken by the suitcase

(B) The time taken by you in doing so

(C) The weight of the suitcase

(D) Your weight

Assertion Reasoning Type

(A) Both assertion and reason are true and reason is the correct explanation of Assertion.

(B) Both assertion and reason are true and reason is not the correct explanation of assertion.

(C) Assertion is true but reason is false

(D) Assertion is false but reason is true.

Q.21 Assertion: For stable equilibrium force has to be zero and potential energy should be minimum.

Reason: For equilibrium, it is not necessary that the force is not zero.

Q.22 Assertion: The work done in pushing a block is more than the work done in pulling the block is more than the work done in pulling the block on a rough surface.

Reason: In the pushing condition normal reaction is more

Q.23 Assertion: Potential energy is defined for only conservation forces Reason $\vec{F} = -\frac{d\hat{Un}}{dr}$

Q.24 Assertion: An object of mass m is initially at rest. A constant force F acts on it. Then the velocity gained by the object in a fixed displacement is proportional to $\frac{1}{\sqrt{m}}$

Reason: For a given force and displacement velocity is always inversely proportional to root of mass.

Comprehension Type

Paragraph 1

The work done by external forces on a body is equal to change of kinetic energy of the body. This is true for both constant and variable force (variable in both magnitude and direction). For a particle $W = \Delta k$. For a system,

$W_{net} = W_{cal} + W_{pseudo} = \Delta Kcm$ or

$W_{ext} + W_{nonconservative} = \Delta K + \Delta U$.

In the absence of external and non conservative forces, total mechanical energy of the system remain conserved.

Q.25 I-work done in raising a box onto a platform depends on how fast it it raised II-work is an inter convertible form of energy.

(A) 1-False II-true (B) 1-False II-False

(C) 1-True II-False (D) 1-True II-true

Q.26 Consider a case of rigid body rolling without sliding over a rough horizontal surface

(A) There will be a non-zero conservative force acting on the body and work done by non-conservative force will be positive.

(B) There will be non-zero non-conservative force acting on body and work done by non-conservative force will be negative.

(C) There will be no non-conservative force acting on the body but totoal mechanical energy will not be conserved.

(D) There will be no non-conservative force acting on the body and total mechanical energy will be conserved.

Q.27 Now consider a case of rigid body rolling with sliding along rough horizontal plane and Vcm is linear. Velocity by $\omega = V_{cm} / 2R$, R is radius of body at (t=0)

(A) There is no non-conservative force acting on body.

(B) There is a non-conservative force acting on body and direction of force is opposite to direction of velocity.

(C) There is a non-conservative force acting on body and direction of the force along the direction of velocity.

(D) None of these.

Q.28 In the above problem if W=3Vcm/R where Vcm velocity of centre of mass at t=0

(A) There is non-conservative force acting on body.

(B) There is non-conservative force acting on body the direction of velocity of centre of mass.

(C) There is a non-conservative force acting on body opposite to the direction of velocity

(D) None of these

Paragraph 2

Two idedtical beads are attached to free ends of two identical springs of spring constant $k = \frac{\left(2+\sqrt{3}\right)mg}{\sqrt{3}R}$. Initially both springs make an angle of 60^0 at the fixed point normal length of each spring is 2R. Where R is the radius of smooth ring over which bead is sliding. Ring is placed on vertical plane and beads are at symmetry with respect to vertical line as diameter.

Q.29 Normal reaction on one of the bead at initial moment due to ring is

(A) $mg / 2$ (B) $\sqrt{3}mg / 2$

(C) mg (D) Insufficient data

Q.30 Relative acceleration between two beads at the initial moment:

(A) g/2 vertically away from each other

(B) g/2 horizontally towards each other

(C) $2g / \sqrt{3}$ Vertically away from each other

(D) $2g / \sqrt{3}$ Horizontally towards each other

Q.31 The speed of bead when spring is at normal length

(A) $\sqrt{\frac{\left(2-\sqrt{3}\right)gR}{\sqrt{3}}}$ (B) $\sqrt{\frac{\left(2+\sqrt{3}\right)gR}{\sqrt{3}}}$

(C) $\sqrt{\frac{2gR}{\sqrt{3}}}$ (D) $\sqrt{3gR}$

Q.32 Choose the correct statement

(A) Maximum angle made by spring after collision is same as that at initial moment.

(B) If the collision is perfectly inelastic, the total energy is conserved.

(C) If the collision is perfectly elastic, each bead undergoes SHM.

(D) Both linear momentum and angular momentum with respect to centre of smooth ring are conserved only at the instant of collision.

Match the Columns

Q.33 A single conservative force acts on a body of mass 1kg that moves along the x-axis. The potential energy U(x) is given by $U(x) = 20 + \left(x-2\right)^2$ where x is the meters. At x=5.0m the particle has a kinetic energy of 20 J then:

Column-I		**Column-II**	
(A)	Minimum value of x in meters	(p)	29
(B)	Maximum value of x in meters	(q)	7.38
(C)	Maximum potential energy in joules	(r)	49
(V)	Maximum kinetic Energy in joules	(s)	-3.38

Q.34 A body of mass 75 kg is lifted by 15m with an acceleration of g/10 by an ideal string. If work done by tension in string is W_1, magnitude of work done by gravitational force is W_2, kinetic energy when it has lifted is K and speed is W_1, magnitude of work done by gravitational force is W_2, kinetic energy when it has lifted is K and speed of mass when it has lifted is v then: (data in column is given in SI units) (g=10 m/s^2)

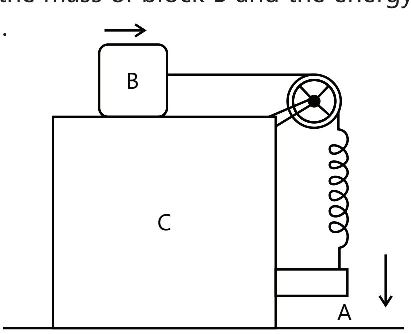
Column I		Column II	
(A)	W_1	(p)	10800
(B)	W_2	(q)	1080
(C)	K	(r)	11880
(D)	v	(s)	5.47

Previous Years' Questions

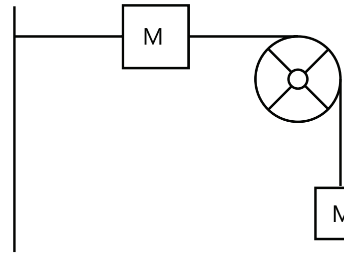
Q.1 The displacement x of a particle moving in one dimension, under the action of a constant force is related to the time t by the equation $t = \sqrt{x} + 3$. Where x is in metre and t in second, Find: (a) The displacement of the particle when its velocity is zero, and (b) The work done by the force in the first 6s. ***(1980)***

Q.2 A body of mass 2kg is being dragged with a uniform velocity of 2m/s on a rough horizontal plane. The coefficient of friction between the body and the surface is 0.20, J=4.2 J/cal and g=9.8m/s^2. Calculate the amount of heat generated in 5s. ***(1980)***

Q.3 Two blocks A and B are connected to each other by a string and a spring; the string passes over a frictionless pulley as shown in the Figure. Black B slides over the horizontal top surface of a stationary block C and the block A slides along the vertical side of C, both with the same uniform speed. The coefficient of friction between the surfaces of block is 0.2. Force constant of the spring is 1960 N/m. If mass of block A is 2kg. Calculate the mass of block B and the energy stored in the spring. ***(1982)***

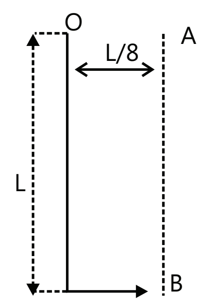


Q.4 A string, with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2m from the wall has a point mass M=2kg attached to it at a distance of 1m from the wall. A mass m=0.5kg attached at the free end is held at rest so that the string is horizontal between the wall the pulley and vertical beyond the pulley. What will be the speed with which the mass M will hit the wall when the mass the m is released? (Take g=9.8 m/s^2) ***(1985)***



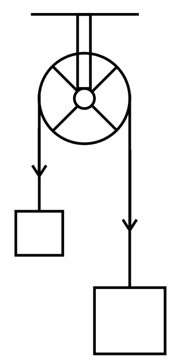
Q.5 A bullet of mass M is fired with a velocity 50m/s at an angle θ with the horizontal. At the highest point of its trajectory, it collides head-on with a bob of mass 3M suspended by a massless string of length 10/3m gets embedded in the bob. After the collision the string moves through an angle of 120^o. Find (a) The angle θ (b) The vertical and horizontal coordinates of the initial position of the bob with respect to the point of firing of the bullet (take g = 10 m/s^2) ***(1988)***

Q.6 A particle is suspended vertically from a point O by an inextensible massless string of length L. A vertical line AB is at a distance L/8 from O as shown in Figure. The object is given a horizontal velocity u.

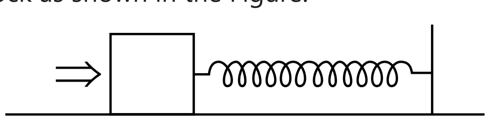


At some point, its motion ceases to be circular and eventually the object passes through the line AB. At the instant of crossing AB, its velocity is horizontal. Find u. ***(1999)***

Q.7 A light inextensible string that goes over a smooth fixed pulley as shown in the Figure connects two blocks of masses 0.36 kg and 0.72kg. taking $g = 10\ ms^{-2}$, find the work done (in Joule) by string on the block of mass 0.36kg during the first second after the system is released from rest ***(2009)***



Q.8 A block of mass 0.18kg is attached to a spring of force constant 2N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is unstretched. An impulse is given to the block as shown in the Figure.



The block slides a distance of 0.06m and comes to rest for the first time. The initial velocity of the block in m/s is $v = \frac{N}{10}$. Then N is. ***(2011)***

Q.9 The work done on a particle of mass m by a force $K\left[\frac{x}{\left(x^2+y^2\right)^{3/2}}\hat{i}+\frac{y}{\left(x^2+y^2\right)^{3/2}}\hat{j}\right]$ (K being a constant of appropriate dimensions, when the particle is taken from the point (a, 0) to the point (0, a) along a circular path of radius a about the origin in the x-y plane is ***(2013)***

(A) $\frac{2K\pi}{a}$ (B) $\frac{K\pi}{a}$ (C) $\frac{K\pi}{2a}$ (D) 0

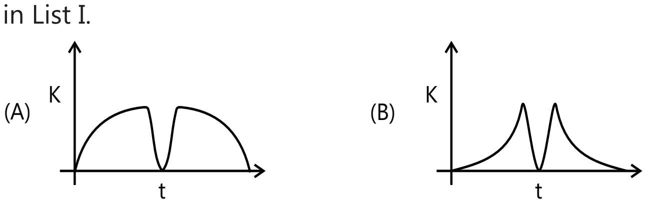
Q.10 A bob of mass m, suspended by a string of length l1 is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length l2, which is initially at rest. Both the strings are mass-less and inextensible.

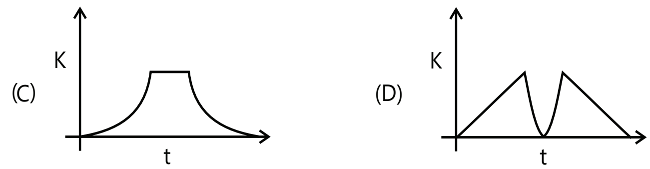
If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio l_1/l_2 is ***(2013)***

Q.11 A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in m/s) of the particle is zero, the speed (in m/s) after 5 s is ***(2013)***

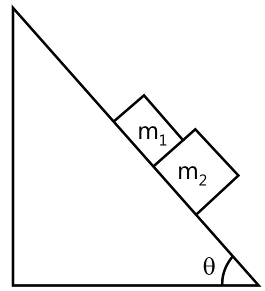
Q.12 A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figures are only illustrative and not to the scale. ***(2014)***

Q.13 A block of mass m_1 = 1 kg another mass m_2 = 2kg, are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in List I.





The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to μ = 0.3. In List II expressions for the friction on the block m_2 are given. Match the correct expression of the friction in List II with the angles given in List I, and choose the correct option.



The acceleration due to gravity is denoted by g.

[Useful information: tan (5.5°) ≈ 0.1; tan (11.5°) ≈ 0.2; tan (16.5°) ≈ 0.3] ***(2014)***

	List I		List II
I	θ = 5°	p.	$m_2 g \sin\theta$
II	θ = 10°	q.	$(m_1+m_2) g \sin\theta$
III	θ = 15°	r.	$\mu m_2 g \cos\theta$
IV	θ = 20°	s.	$\mu(m_1 + m_2) g \cos\theta$

Code:

(A) P-1, Q-1, R-1, S-3

(B) P-2, Q-2, R-2, S-3

(C) P-2, Q-2, R-2, S-4

(D) P-2, Q-2, R-3, S-3

Q.14 Consider two different metallic strips (1 and 2) of same dimensions (lengths ℓ, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option(s) is(are) ***(2015)***

(A) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$

(B) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$

(C) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$

(D) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

PlancEssential Questions

JEE Main/Boards

Exercise 1

Q. 18 Q.19 Q.22 Q.23

Exercise 2

Q.1 Q.5 Q.10 Q.13
Q.15 Q.16 Q.17

Previous Years' Questions

Q.3 Q.4 Q.8

JEE Advanced/ Boards

Exercise 1

Q.3 Q.5 Q.7 Q.11
Q.13 Q.19 Q.21 Q.30

Exercise 2

Q.4 Q.10 Q.12 Q.33
Q.34

Previous Years' Questions

Q.3 Q.4 Q.6 Q.8

Answer Key

JEE Main/Boards

Exercise 1

Q.1 Zero **Q.18** 16 J **Q.19** 1750 J; -1000 J

Q.20 10^4 J **Q.21** 3.75×10^3 J **Q.22** -0.28 mgH

Q.23 3150N

Exercise 2

Single Correct Choice Type

Q.1 C **Q.2** C **Q.3** B **Q.4** A **Q.5** C

Q.6 B **Q.7** C **Q.8** C **Q.9** C **Q.10** D

Q.11 C **Q.12** B **Q.13** C **Q.14** D **Q.15** B

Q.16 B **Q.17** B **Q.18** A

Previous Years' Questions

Q.1 C **Q.2** D **Q.3** C **Q.4** B **Q.5** C

Q.6 A **Q.7** C

JEE Advanced/Boards

Exercise 1

Single Correct option

Q.1 $mgh\left(\frac{M}{M+m}\right)$ **Q.2** 4 R **Q.3** 4.24 M

Q.4 $5.42M/S^{1/2}, 0.97M$ **Q.5** 0.25 M **Q.6** $s=\left(\frac{8P}{9m}\right)^{1/2}t^{3/2}$

Q.7 $d\sqrt{\frac{3g}{2d}+\frac{k}{16m}}$ **Q.8** (a) 1.5m (b) 0.6 m **Q.9** 3.3 m/s

Q.10 3.32 m **Q.11** 154 m/s **Q.12** $\frac{40}{\sqrt{41}}$ m/s

Q.14 mk^2r^2t **Q.15** $mgh+\mu mgl$ **Q.16** $\sqrt{2hg\,\ell n\left(\frac{\ell}{h}\right)}$

Q.17 (a) $\Delta\ell > \frac{3mg}{k}$ (b) $h=\frac{8mg}{k}$ **Q.18** $v_0\left(\frac{w-f}{w+f}\right)^{1/2}$ **Q.19** $\frac{h}{4}\sqrt{\frac{k}{m}}$

Q.20 0.8m **Q.21** $2.14\sqrt{gL}$

Q.22 (a) $N=mg(\cos\theta-2)$; (b) for $\theta\le\cos^{-1}(2/3), N_B=0, N_A=mg(3\cos\theta-2)$

and for $\theta\ge\cos^{-1}(2/3) N_A=0, N_B=mg(2-3\cos\theta)$

Q.23 $\sqrt{\frac{3mgR}{k}}$ **Q.24** 2.12 m/s **Q.25** $\frac{k\ell^2x'(1+x')}{2(1-x')^2}$

Q.26 (a) $\sqrt{2g[R(1-\cos\theta)+L\sin\theta]}$; (b) $6mg\left(1-\cos\theta+\frac{L}{R}\sin\theta\right)$

(c) The radius through the particle makes an angle $\cos^{-1}\left(\frac{2}{3}\right)$ with the vertical.

Q.27 3 R **Q.28** Third trip **Q.29** $8\sqrt{2}$ m

Exercise 2

Single Correct Choice Type

Q.1 B **Q.2** A **Q.3** C **Q.4** C **Q.5** B

Q.6 A **Q.7** B **Q.8** B **Q. 9** D **Q.10** D

Q.11 C **Q.12** B **Q.13** D

Multiple Correct Choice Type

Q.14 A, D **Q.15** B, D **Q.16** A, B **Q.17** A, C, D **Q.18** C, D

Q.19 A, B **Q.20** A, B, D

Assertion Reasoning Type

Q.21 B **Q.22** A **Q.23** A **Q.24** B

Comprehension Type

Paragraph 1

Q.25 A **Q.26** D **Q.27** B **Q.28** B

Paragraph 2

Q.29 C **Q.30** D **Q.31** C **Q.32** D

Match the Columns

Q.33 $A \to s; B \to q; C \to r; D \to p$

Q.34 $A \to r; B \to p; C \to q; D \to s$

Previous Years' Questions

Q.1 0,0

Q.2 9.33 cal

Q.3 0.098J

Q.4 3.29m/s

Q.5 30^0, (108.25m, 31.25m)

Q.6 $u = \sqrt{gL\left(2 + \frac{3\sqrt{3}}{2}\right)}$

Q.7 8 J

Q.8 D

Q.9 D

Q.10 5

Q.11 5

Q.12 B

Q.13 D

Q.14 A, C

Solutions

JEE Main/Boards

Exercise 1

Sol 1: If the vector product $\vec{F}.\vec{s} = 0$, then work done is zero when

$\vec{F}$ = force acting on the body

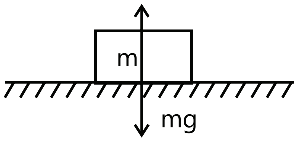
$\vec{s}$ = displacement of the body

Force does no work under given condition

(i) Direction of force and displacement is perpendicular

(ii) Displacement is zero

Example:



Work done by gravity and normal force is zero.

Sol 2: Two bodies have same linear momentum so

$m_1v_1 = m_2v_2$

m_1 = mass of first object

v_1 = velocity of first object

m_2 = mass of second object

v_2 = velocity of second object

Case – I : $m_1 > m_2$

$m_1v_1 = m_2v_2 \Rightarrow v_1 < v_2$

kinetic energy KE. = $\frac{1}{2} m_1v^2$

$KE_1 = \frac{1}{2} m_1v_1^2$; $K_2 = \frac{1}{2} m_2v_2^2 \quad \Rightarrow K_2 > K_1$

Case – II : $m_2 > m_1$

$m_1v_1 = m_2v_2$

$\Rightarrow v_2 < v_1 \Rightarrow KE_1 > KE_2$

Sol 3: K.E. of object 1 is $KE_1 = \frac{1}{2} m_1u_1^2$

K.E. of object 2 is $KE_2 = \frac{1}{2} m_2u_2^2$

$KE_1 = KE_2$

$\Rightarrow \frac{1}{2} m_1u_1^2 = \frac{1}{2} m_2u_2^2$

$\Rightarrow m_1u_1^2 = m_2u_2^2 ; \frac{(m_1u_1)^2}{m_1} = \frac{(m_2u_2)^2}{m_2}$

Suppose $m_1 > m_2 \Rightarrow m_1u_1 > m_2u_2$

$\Rightarrow$ Linear momentum of 1st object is more than 2nd object.

Sol 4: Springs are generally taken as massless and spring transfers its potential energy to the kinetic and energy of the body to which it is attached whereas during a free fall, PE of a body is converted into its KE

In other words, for a spring all the energy is stored as potential energy whereas for body, potential energy is converted into kinetic energy.

Sol 5: work is the vector product $\vec{F}.\vec{s}$

$w = \vec{F}.\vec{s}$

$\vec{F}$ = force acting on the object

$\vec{s}$ = displacement of the object

If a constant force F displaces a body through displacements then the work done, w is given by

$w = Fs \cos\theta$

s = net displacement

θ = angle between force and displacement

Sol 6: Absolute unit of work on m.k.s is Joule (J)

Absolute unit of work on c.g.s. is erg

Gravitational unit of work on m.k.s. is kg-m

Gravitational unit of work on c.g.s is gm-cm

Sol 7: Work done $w = \vec{F}.\vec{s}$

$w = |\vec{F}|\ |\vec{s}| \cos\theta$

θ = angle between force and displacement when $0 < \theta, \pi/2$

$w = |F||s| \cos\theta$ is positive.

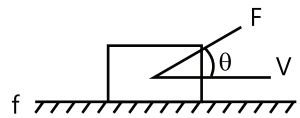
when $\theta = \pi/2 \Rightarrow W = 0$

when $\pi/2 < \theta < \pi$

$W = |F||s| \cos\theta$ is negative

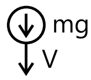
Examples:

(1) Positive work



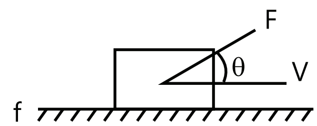
Work done by F is positive

Free falling object (Positive work)



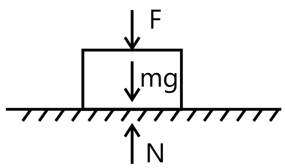
Work done by gravity is negative

(2) Negative work

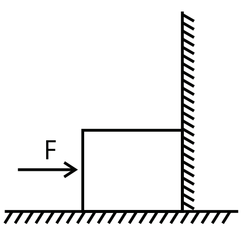


Work done by friction is negative

Zero work

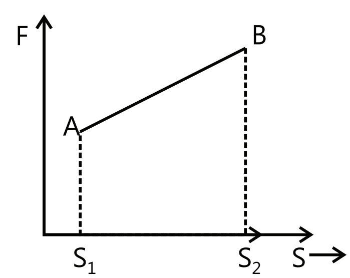


Work done by F is zero



Work done by F is zero

Sol 8: Graphically work is the area under the force displacement graph



$$W_{A\to B} = \int_{s_1}^{s_2} \vec{F}.\vec{ds}$$

Mathematically, work is the integral dot product of force vector and infinitesimal displacement vector

$$W = \int_{A}^{B} \vec{F}.\vec{ds}$$

Sol 9: Conservative force: It is a force with the property that the work done in moving a particle between two points is independent of the taken path. Example gravity

Non-conservative force – it is a force with the property that the work done in moving a particle between two points is dependent on the path taken. Example friction

Properties of conservative force

(i) Work from one point to another point on any path is same

(ii) $\frac{dF_x}{dy} = \frac{dF_y}{dx}; \frac{dF_x}{dz} = \frac{dF_2}{dx}; \frac{dF_y}{dz} = \frac{dF_2}{dy}$

Where $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_2\hat{k}$

Properties of non-conservative force

(i) Work done from one point to another is dependent on path taken.

(ii) Work cannot be recovered back

Sol 10: Power: It is defined as the rate at which the work is done SI unit of power is (J/s)

Energy: Energy of a body is the capacity of the body to do work. SI unit of energy is J.

Sol 11: It is the energy possessed by a body by virtue of its motion. A body of mass m moving with a velocity v has a kinetic energy.

$E_k = \frac{1}{2}mv^2$

$KE = w = \int \vec{F}.d\vec{s} = \int m\vec{a}ds = m\int v\frac{dv}{ds}ds = \frac{1}{2}mv^2$

Sol 12: When a particle is acted upon by various forces and undergoes a displacement, then its kinetic energy changes. By an amount equal to the total work done w_{net} on the particle by all the forces

$w_{net} = sk$

$w_{net} = w_c + w_{Nc} + w_{oth}$

w_C = work done by conservative force

w_{Nc} = work done by non-conservative force

w_{oth} = work done by other forces which are not included in above category

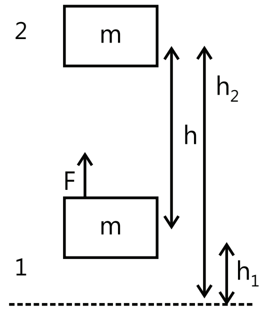
Sol 13: Potential energy – It is the energy of a body possessed by virtue of its position on the energy possessed by the body due to its state

Example:

(i) Energy stored in the compressed spring

(ii) When a rubber band is stretched potential energy is stored in it.

Sol 14:



Suppose a mass m is raised from point 1 to point 2 and suppose change in kinetic energy is negligible. Potential energy is negative of the work done

$\Delta U = -W$

$W = \int_{h_1}^{h_2} mg\,ds$

$W = -mg(h_2 - h_1)$

$W = -mgh \Rightarrow \Delta U = mgh$

Sol 15: Potential energy of spring is the energy stored in the spring when compressed or stretched relative to its natural length.

Suppose a force F is acting on the spring of spring constant k

To keep the spring compressed in this position, the applied force should be same as kx

i.e. $F_s = kx$

Work done in compressing by a distance x is given by

$w = \int_0^x kx'dx' \Rightarrow w = \frac{1}{2}kx^2$

Work done is equal to change in potential energy of a spring

$w = U$

$U = \frac{1}{2}kx^2$

Potential energy is zero at spring's natural length and is proportional to the square of the distance from mean position

Sol 16: Different forms of energy are

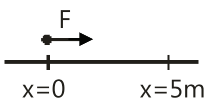
(i) Kinetic Energy – It is the energy possessed by a body by virtue of its motion.

(ii) Potential energy – It is the energy of a object on a system due to the position of the body and the arrangement of the particles of the system.

Example: Gravitational potential energy –

(iii) Mechanical energy – it is the sum of potential energy and kinetic energy

Sol 17: Force $f = 7 - 2x + 3x^2$



Work done

$$w = \int F.dx = \int_0^5 (7 - 2x + 3x^2)dx$$

$$= \left[7x - x^2 + x^3\right]_0^5 = 7(5 - 0) - [5^2 - 0^2] + [5^3 - 0^3]$$

W = 35 – 25 + 125 = 135 N-m

So, work done is 135 N-m

Sol 18: Mass = 2 kg

$x = t^3/3$; $dx = t^2\,dt$

We need to find the force by first finding the acceleration of the body

$$v = \frac{dx}{dt} = \frac{d(t^2/3)}{dt} = t^2$$

$$a = \frac{dv}{dt} = 2t$$

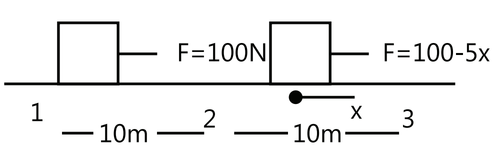
F = ma = 2 × 2t = 4t

Work done

$$w = \int F.dx = \int_0^2 4t.t^2dt = \int_0^2 4t^3dt = \left[t^4\right]_0^2 = 2^4 = 16\text{ J}$$

w = 16 J

Sol 19:



Work done by friction $w_f = \int Fdx$

Since friction is constant all over the motion so displacement = 20 m

$w_f = + f\,\Delta x = -50 \times 20$

$w_f = -1000$ J

[∵ Direction of friction force is opposite to displacement.

So force is – 50]

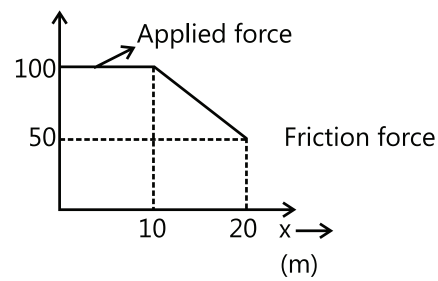
Work done by force f is

$$w_f = w_{f1\to2} + w_{f2\to3} = \int_0^{10} F_1dx + \int_0^{10} F_2dx$$

$$= 100 \times 10 + \int_0^{10}(100 - 5x)dx = 1000 + \left[100x - \frac{5x^2}{2}\right]_0^{10}$$

$w_f = 1000 + [100(10\text{-}0) - 5/2\,(10^2\text{-}0)] = 1000 + 1000 - 250$

$w_f = 1750$ J



Sol 20: Mass = 50 kg = m

Momentum = 1000 kg m/s = mv

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m} = \frac{1}{2}\frac{(1000)^2}{50} = 10000\text{ J}$$

Sol 21: Mass = 50 g = 0.05 kg

Initial velocity before collision = v_i = 400 m/Δ

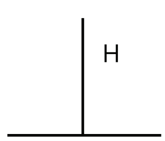
Final velocity after passing through the wall=v_f=100m/s

By work energy theorem, Work done by the bullet is equal to the negative change in kinetic energy of the bullet

$$w = \Delta KE = -KE_f + KE_i = \frac{-1}{2} \times 0.05\,[(100)^2 - (400)^2]$$

$$w = -\frac{5}{2}[1 - 16] \times 100 = 3.75 \times 10^3\text{ J}$$

Sol 22: By work energy theorem



$w_{net} = \Delta KE$

$w_{gravity} + w_{friction} = \Delta KE$

$$mgH + w_{friction} = \frac{1}{2}m\,(1.44\,gH - 0)$$

$mgH + w_{friction} = 0.72$ mgH

$w_{friction} = (0.72 - 1)$ mgH = -0.28 mgH

Sol 23: Mass of bullet = 10 g = 0.01 kg

Initial velocity of bullet = 800 m/s

Thickness of mud wall = 1 m

Final velocity of bullet = 100 m/s

By work energy theorem

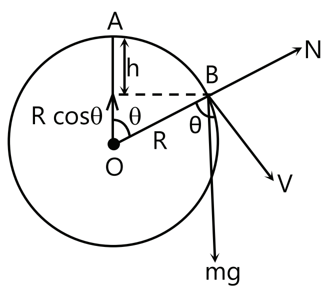
$W_{net} = \Delta KE = \frac{1}{2} m [v_f^2 - v_i^2]F$

$w_r = \frac{1}{2} (0.01) [(100)^2 - (800)^2] = \frac{100}{2} [1^2 - 8^2]$

w = –50 × 63 = –3150 N

So average resistance offered is 3150 N

Sol 24:



This is a very standard problem for a JEE aspirant.

Let us say at point B, the particle loses its contact. At point B say the particle has velocity v.

$mg \cos\theta = N + \frac{mv^2}{R}$

$N = mg \cos\theta - \frac{mv^2}{R}$ ... (i)

Now when the particle is about to lose contact, the normal reaction between the particle and surface becomes zero.

$\therefore N = 0$

$\Rightarrow mg \cos\theta = \frac{mv^2}{R}$ ... (ii)

Now energy at point A, taking O as reference;

$E_A = 0 + mg R$

$E_B = \frac{1}{2} mv^2 + mg R \cos\theta$

Using Energy conservation

$E_A = E_B$

$\Rightarrow Mg R = \frac{1}{2} mv^2 + mg R \cos\theta$

$2 mg R (1 - \cos\theta) = mv^2$

$2mg (1 - \cos\theta) = \frac{mv^2}{R}$ ... (iii)

Putting this value of $\frac{mv^2}{R}$ in eq^n ... (iv)

$mg \cos\theta = 2mg (1 - \cos\theta)$

$3 \cos \theta = 2$

$\cos\theta = \frac{2}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$

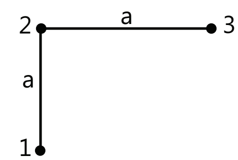
And now $h = R (1 - \cos\theta) = R\left(1 - \frac{2}{3}\right)$

$h = \frac{R}{3}$ m.

Exercise 2

Single Correct Choice Type

Sol 1: (C) Work done = $\int Fx.dx + \int Fy.dy$



$F = Fx\ \hat{i} + Fy\ \hat{j}$

Fx = –ky; Fy = –kx

$w = w_{1\text{-}2} + w_{2\text{-}3}$

$\int_0^a Fydy + \int_0^a Fxdx$

$-\int_0^a K \times 0 \times dy + \int_0^a -kadx = 0 + -ka (a - 0) = -ka^2$

Sol 2: (C) Work done is zero as force and displacement are perpendicular to each other so $\vec{F}.\vec{ds} = 0$

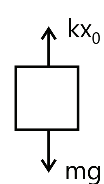
Sol 3: (B) By work energy theorem

$w_{net} = \Delta KE$[assuming negligible K.E.]

$w_{gravity} + w_{force} = 0$

$w_{force} = -\left(-\frac{m}{4} g\left(\frac{l}{8}\right)\right) = \frac{mgl}{32}$

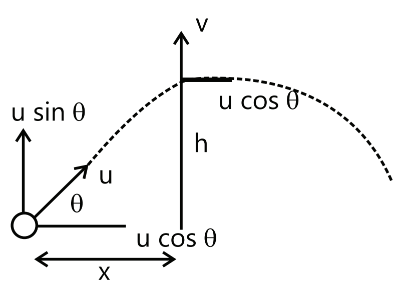
Sol 4: (A) by force equilibrium



$kx_0 = \frac{mg}{x_0}$

Elastic energy $= \frac{1}{2} kx_0^2 = \frac{1}{2} \frac{mg}{x_0} x_0^2 = \frac{mgx_0}{2}$

Sol 5: (C)



Vertical velocity at height h is v_y

$v_y = u \sin\theta - gt$

$t = \frac{x}{u\cos\theta}$

$v_y = u \sin\theta - \frac{gx}{u\cos\theta}$

$KE = \frac{1}{2} m[v_y^2 + v_x^2] = \frac{1}{2} m \left[u^2 + \frac{g^2x^2}{u^2\cos^2\theta} - 2\tan\theta gx\right]$

so graph (c) is correct

Sol 6: (B)

Maximum height $H = \frac{u^2\sin^2\theta}{2g}$

$H_1 = \frac{v_1^2}{2g}$; $H_2 = \frac{v_2^2\left(\frac{1}{\sqrt{2}}\right)^2}{2g} = \frac{v_2^2}{ug}$

$H_1 = H_2$

$\Rightarrow \frac{v_1^2}{2} = \frac{v_2^2}{4}$

$v_2^2 = 2v_1^2$

$\frac{KE_1}{KE_2} = \frac{1/2m_1v_1^2}{1/2m_2v_2^2} = \frac{v_1^2}{v_2^2} = \frac{1}{2}$

Sol 7: (C) By work energy theorem

$w_{net} = \Delta KE$ [Assuming negligible K.E.]

$w_{gravity} + w_{force} = 0$

$W_{net} = \left(-\frac{1}{18}mg\ell\right) - \left(-\frac{1}{2}mg\ell\right) = \frac{4}{9}mg\ell$

Sol 8: (C) P = F.V

P = ma.V

$P = \left[\frac{mdV}{dt}V\right]$

$\int_0^t \frac{Pdt}{m} = \int_0^v vdv \Rightarrow \frac{p}{m} = \frac{v^2}{2}$

$v = \sqrt{\frac{2Pt}{m}} \Rightarrow x \propto t^{\frac{1}{2}+1}$

$x \propto t^{3/2}$

Sol 9: (C) Mechanical energy conservation

$KE_{alpa} + PE_i + KE = PE_{electrostatic}$

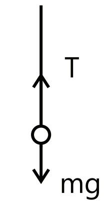
$4 \times 10^6 \times 10^{-19} = \frac{9\times10^{10}\times10^{-19}\times100\times10^{-9}}{r}$

$r \sim 10^{-14}$ m

$r \sim 10^{-12}$ cm

Sol 10: (D) Minimum velocity required is $v = \sqrt{5gR}$

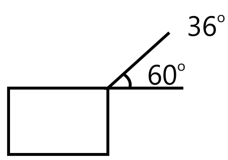
by Newton's second law



$T - mg = \frac{mv^2}{R}$

$T = mg + \frac{m5gR}{R} = 6mg$

Sol 11: (C) F = 360 N



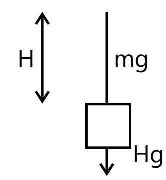
Work done is 1 hr

is w = 360 cos 60° 10 × 100

Work done per sec= $\frac{3600\times10^3\cos60°}{60\times60}$

$= 1000\ \frac{1}{2} = 500$ w

Sol 12: (B) By work energy theorem



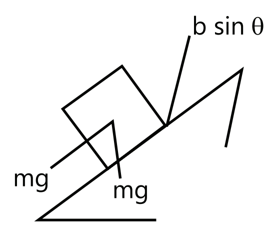
$w_{net} = \Delta KE = 0$

$w_{man} + w_{gravity} = 0$

$w_{man} - MgH - mg\frac{H}{2} = 0$

$w_{man} = \left(\frac{m}{2} + M\right) gH$

Sol 13: (C)



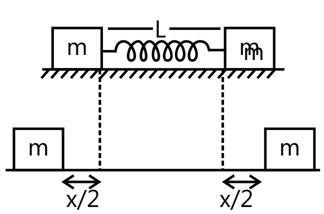
By force equilibrium

$f = mg \sin\theta$

Work done by friction f is equal to $f \sin\theta$ vt

$w = mg\ vt \sin\theta$

Sol 14: (D)



Potential energy of spring is ½ kx^2

By work energy theorem

$w_{net} = \Delta KE$

$w_{force} + w_{spring} = 0$

$w_{spring} = \frac{-1}{2} kx^2$

Since displacements of both the masses are same so work done by spring on both masses is same.

So work done on each mass = $\frac{-1}{4} kx^2$

Sol 15: (B) F = kx

$U = -\int F dx = -\int kx dx \Rightarrow U = -\frac{kx^2}{2} + c$

$U(x = 0) = 0 \Rightarrow 0 = 0 + c \Rightarrow c = 0$

$U = \frac{-kx^2}{2}$

Sol 16: (B) $w_1 = w_2 = w_3$ as gravitational force is conservative and work done by conservative forces is independent of path taken.

Sol 17: (B) By work energy theorem

$w_{net} = \Delta K$

h is the maximum extension of the spring

$w_{gravity} + w_{spring} = 0$

$+mgh - \frac{1}{2} kh^2 = 0$

$h = \frac{2mg}{k}$

Sol 18: (A) Work energy theorem includes all the forces. Conservation as well as non-conservative. This theorem is always true.

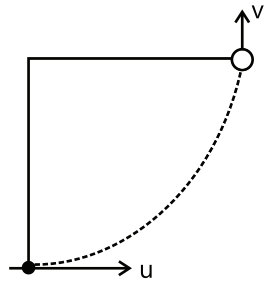
Previous Years' Questions

Sol 1: (C) $p = \sqrt{2Km}$

or $p \propto \sqrt{m}$, $\frac{m_1}{m_2} = \frac{1}{4}$

$\therefore \frac{p_1}{p_2} = \frac{1}{2}$

Sol 2: (D)



From energy conservation $v^2 = u^2 - 2gL$

Now, since the rwo velocity vectors shown in figure are mutually perpendicular, hence the magnitude of change of velocity will be given by

$|\Delta \vec{v}| = \sqrt{u^2 + v^2}$

Substituting value of v^2 from eq. (i)

$|\Delta \vec{v}| = \sqrt{u^2 + u^2 - 2gL} = \sqrt{2(u^2 - gL)}$

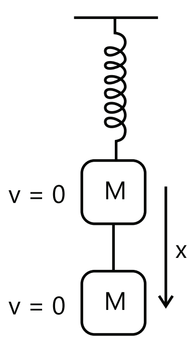
Sol 3: (C) Power = $\vec{F} . \vec{v}$ = Fv

$F = v\left(\frac{dm}{dt}\right) = v\left\{\frac{d(\rho \times \text{volume})}{dt}\right\}$

$= \rho v\left\{\frac{d(\text{volume})}{dt}\right\} \rho v(Av) = \rho Av^2$

$\therefore$ power $P = \rho Av^3$ or $P \propto v^3$

Sol 4: (B) Let x be the maximum extension of the spring. From conservation of mechanical energy



Decrease in gravitational potential energy = increase in elastic potential energy

$\therefore Mg\,x = \frac{1}{2} kx^2$

or $x = \frac{2Mg}{k}$

Sol 5: (C) From energy conservation,

$\frac{1}{2} kx^2 = \frac{1}{2}(4k)y^2$

$\frac{y}{x} = \frac{1}{2}$

Sol 6: (A) $F = k_1 S_1 = k_2 S_2$

$W_1 = FS_1, W_2 = FS_2$

$k_1S_1^2 > k_2S_2^2$

$S_1 > S_2$

$k_1 < k_2$

$W \propto k$

$W_1 < W_2$

Sol 7: (C) Let fat used be 'x' kg

$\Rightarrow$ Mechanical energy available = $x \times 3.8 \times 10^7 \times \frac{20}{100}$

Work done in lifting up = $10 \times 9.8 \times 1000$

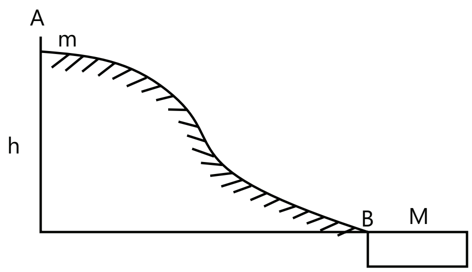
$\Rightarrow x \times 3.8 \times 10^7 \times \frac{20}{100} = 9.8 \times 10^4$

$\Rightarrow x \approx 12.89 \times 10^{-3}$ kg.

JEE Advanced/Boards

Exercise 1

Sol 1: By work energy theorem for point A to point B



$w_{net} = \Delta KE$

$w_g = KE_f - KE_i$

$mgh = \frac{1}{2} mv^2$

$v = \sqrt{2gh}$

Now m, M both move together so by conservation of linear momentum

$mv = (M + m) v'$

$v' = \frac{m\sqrt{2gh}}{M+m}$

v' is the combined velocity of (m + M) system.

Applying work energy theorem for the whole process

$w_{net} = \Delta KE$

$w_{gravity} + w_{friction} = kE_f - kE_i \; (kE_i = 0)$

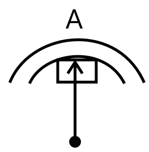
$mgh + w_{friction} = \frac{1}{2} (M + m) \left(\frac{m\sqrt{2gh}}{M+m}\right)^2$

$mgh + w_{friction} = \frac{m^2gh}{M+m}$

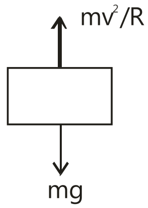
$w_{friction} = \frac{m^2gh}{M+m} - mgh$

$w_{friction} = -\frac{Mmgh}{(M+m)}$

Sol 2:



Drawing FBD at point A



So net upward force exerted by the mass is $\frac{mv^2}{R} - mg$

Which is equal to 3 mg $\Rightarrow \frac{mv^2}{R}$ - mg = 3 mg

$\frac{mv^2}{R} = 4$ mg

$v = \sqrt{4gR} = 2\sqrt{gR}$

Now applying work-energy theorem

$w_{net} = \Delta KE$

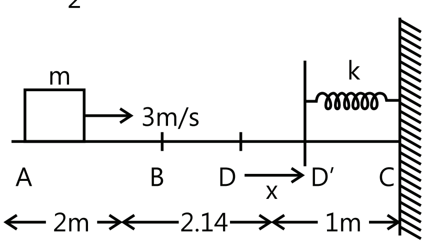
$w_{gravity} = K_f$

$mg(h-2R) = \frac{1}{2} m (4gR)$

$h - 2R = 2R$

$h = 4R$

Sol 3: Mass = $\frac{1}{2}$ kg



Let us assume that block stops at point D' which is at distance x m from D.

By applying work energy theorem

$w_{net} = \Delta KE$

$w_{friction} + w_{spring} = K_f - K_i$ ... (i)

$(K_f = 0)$

$w_{friction} = -\mu\, mg\,(BD')$

$w_{friction} = -(0.20)\, mg\,(2.14 + x)$ ... (ii)

$w_{spring} = -\frac{1}{2}kx^2$ ... (iii)

[∵ block is in motion from point B to D′ so we will take kinetic friction]

Substituting (i), (ii) and (iii) we get

$(0.2)\, mg\,(-2.14 - x) - \frac{1}{2}kx^2 = -\frac{1}{2}m\,(g)$

Substituting value of m we get

$2.14 + x + \frac{1}{2} \times 2x^2 = +\frac{9}{4}$

$x^2 + x + 2.14 - 2.25 = 0$

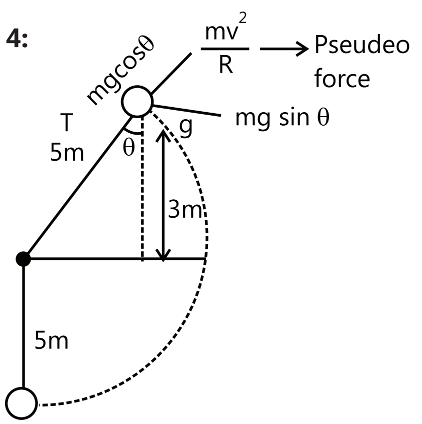
$x^2 + x - 0.11 = 0$

$(x - 0.1)(x + 1.1) = 0$

$x = 0.1$ m

So, total distance thought which block moves is 2 + 2.14 + 0.1 m $\Rightarrow$ 4.24 m

Sol 4:



$\cos\theta = \frac{3}{5}$

When particle is at 8m height from lowest point tension is just zero. So balancing force in the direction of string

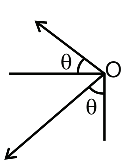
We get $mg\cos\theta - \frac{mv^2}{R} = 0$

$gR\cos\theta = v^2$

$v = \sqrt{gR\cos\theta}$

Substituting values we get

$v = \sqrt{9.8 \times 5 \times 0.6} \Rightarrow 5.42$ m/s



Maximum height attained = $\frac{u^2 \sin^2\theta}{2g}$

$= \frac{9.8 \times 5 \times 0.6 \times (0.8)^2}{2 \times 9.8} = 0.97$ m

Sol 5: At maximum compression, velocity of both blocks will be equal so, let v be the final velocity of both the blocks x be the compression in the spring. Applying moment conservation, we get

$2 \times 10 + 5 \times 3 = (2 + 5)\,V$

$\Rightarrow v = \frac{35}{7} = 5$m/s

Applying work energy theorem

$w_{net} = \Delta KE = KE_f - KE_i$

$-\frac{1}{2}kx^2 = \frac{1}{2}(5+2)(5)^2 - \left[\frac{1}{2}(5)(3)^2 + \frac{1}{2}(2)(10)^2\right]$

$\Rightarrow \frac{1}{2}kx^2 = 100 + \frac{45}{2} - \frac{25 \times 7}{2}$

$x^2 = 0.0625$

$x = 0.25$ m

Sol 6: P = Power = F.V = constant

(a) $F = ma = \frac{mdv}{dt}$

$P = \frac{mdv}{dt}.v$

$\int vdv = \int \frac{P}{m}\,dt$

$\int_0^v vdv = \frac{P}{m}\int_0^t dt$

[∵ p and m are constant]

$\frac{v^2}{2} = \frac{pt}{m}$

$v = \sqrt{\frac{2pt}{m}}$

(b) $F = ma = \frac{mvdv}{dx} \Rightarrow P = \frac{mvdv}{dx}v$

$\int_0^v v^2dv = \int_0^s \frac{Pdx}{m}$

$$\frac{v^3}{3} = \frac{ps}{m}$$

$$s = \frac{mv^3}{3P} = \frac{m}{3P}\left(\frac{2Pt}{m}\right)^{3/2}$$

$$s = \left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$$

Sol 7: Length of the spring at point A = $\frac{d}{\cos 37°} = \frac{5d}{3}$

By work energy theorem

$w_{net} = \Delta KE$

$$\underbrace{mgh}_{w_{gravity}} + \frac{1}{2}k(x^2 - 0^2) = \frac{1}{2}mv^2$$

$[x = l - d = \frac{d}{4}]$

$$h = l \sin\theta = \frac{5d}{4} \cdot \frac{3}{5} = \frac{3d}{4}$$

$$v = d\sqrt{\frac{3g}{2d} + \frac{k}{16m}}$$

Sol 8: (a) When mass m comes to rest for the first time kinetic energy of both the masses is zero.

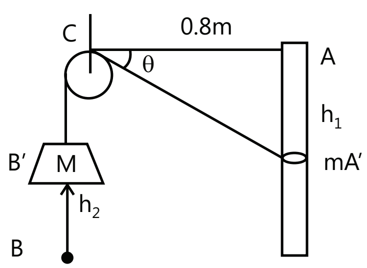
work energy theorem

$w_{net} = \Delta KE = KE_i - KE_i$

$-Mgh_2 + mgh = 0 \Rightarrow Mh_2 = mh_1$

$\Rightarrow h_1 = \frac{5}{2}h_2$ ... (i)

Length of the string is constant, so



BC + Ac = A′C + B′C

BC – B′C = A′C – AC

$h_2 + 0.8 = A'C$

$(0.8)^2 + h_1^2$

$= (h_2 + 0.8)^2$

$h_1^2 = h_2^2 + 1.6\, h_2$

By (i)

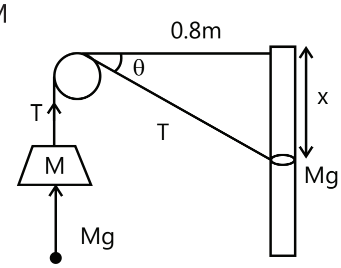
$$\frac{25}{9}h_2^2 = h_2^2 + 1.6\, h_2$$

$$\frac{16h_2^2}{9} = 1.6\, h_2$$

$$h_2 = \frac{9}{10}$$

$$h_1 = \frac{9}{10} \times \frac{5}{3} = 1.5 \text{ m}$$

(b) Force equilibrium on M



T = Mg

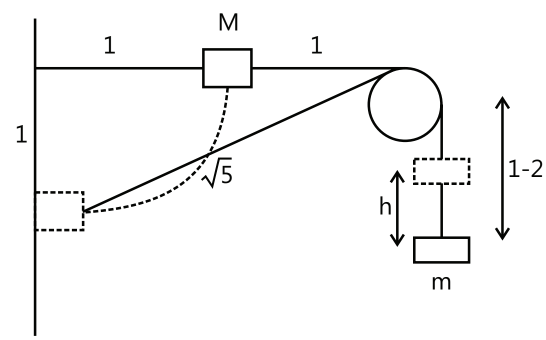
Force equilibrium on m

$T \sin\theta = mg$

$Mg \sin\theta = mg$

$$\sin\theta = \frac{M}{m} = \frac{3}{5}$$

Sol 9: M "falls" and loses potential energy. This loss of potential energy is converted to gain in potential energy of m and gain in kinetic of energy for m and M both.



Let the total length of the string be l. So, the length of the hanging part in the beginning = l-2.

Since, total mechanical energy is conserved.

Loss in M.E. = Gain in M.E.

$$Mg1 = \frac{1}{2}mv^2 + mgh + \frac{1}{2}MV^2 \text{........} *$$

h can be obtained from the conservation of the length of the string.

$$h = l - 2 - (l - \sqrt{5} - 1) = \sqrt{5} - 1$$

We want V, v can be obtained in terms of V.

As M "falls", it moves in circular path with its velocity along the tangent. The velocity along the tangent can be resolved into two components, one along the length of the string and the other perpendicular to the length of the string. The component along the length of the string is same as the velocity of m as m always moves along the length of the string.

$$V\cos\theta = v$$

$$\cos\theta = \frac{2}{\sqrt{5}}$$

From,

$$*, 2\times 9.8\times 1=\frac{1}{2}\times 0.5\times V^2\cos^2\theta+0.5\times 9.8\times\left(\sqrt{5}-1\right)+\frac{1}{2}\times 2V^2$$

V can be obtained.

Sol 10: Work energy theorem

$w_{net} = \Delta KE$

Since initial and final velocity of sand particles are zero so $\Delta KE = 0$

$w_{gravity} = mg\ (0.2 + 0.01)$

$w_{net} = 0$

$w_{gravity} + w_{spring} = 0$

$(0.1) \times (10) \times (0.21) - \frac{1}{2} k (0.1)^2 = 0$

$k = 0.42 \times 10^4 = 4200$ N/m

Now if compression is 0.04 m

$w_{gravity} + w_{spring} = 0$

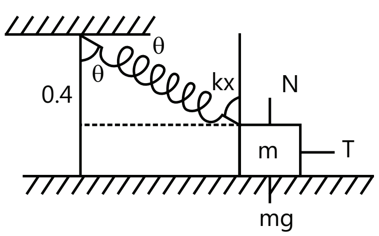
$\Rightarrow (0.1)\times(10)(h + 0.04) - \frac{1}{2} \times 4200\ (0.04)^2 = 0$

$h + 0.04 = 2100 \times 16 \times 10^{-4}$

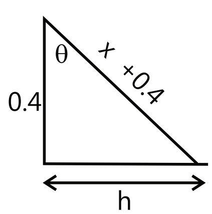
$h + 0.04 = 0.21 \times 16$

$h = 3.36 - 0.04 = 3.32$ m

Sol 11: Let the extension kx



By Newton's second law



h = 0.3 m

$N - mg + kx \cos\theta = 0$

$N = 0$

$kx \cos\theta = mg$ ... (i)

By geometry $(x + 0.4) \cos\theta = 0.4$

By (i) $x \cos\theta = \frac{0.32\times 10}{40} = \frac{3.2}{40} = 0.08$

$(x + 0.4)\ \frac{0.08}{x} = 0.4$

$x + 0.4 = 5x$

$x = 0.1$ m

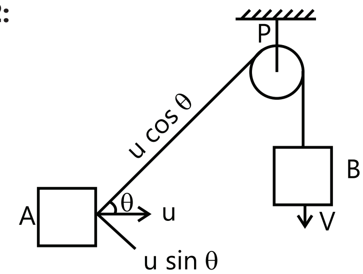
By work energy theorem

$w = \frac{1}{2} mv^2 + \frac{1}{2} mv^2$

$-\frac{1}{2} \times 40\ (0.1)^2 + 0.32\times 10\times 0.3 = 0.32\ v^2$

$v = 1.54$ m/s

Sol 12:



(a) Velocity of A along the string is equal velocity of B along the string

Length of string is constant

$\Rightarrow AP + BP =$ constant

Differentiate w.r.t

$\frac{d(AP)}{dt} + \frac{d(BP)}{dt} = 0$

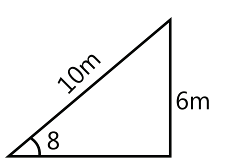
$-u \cos\phi + v = 0$

$v = u \cos\phi$

(b) When B strikes ground length

BP = 6 cm

So length of AP = 16 – 6 = 10 m



$\sin\phi = \frac{6}{10} = \frac{3}{5}$

$\cos\phi = \frac{4}{5} \Rightarrow u = \frac{v}{4/5} = \frac{5v}{4}$

By work energy theorem

$w_{net} = \Delta KE$

$w_{gravity} + w_{string} = \frac{1}{2} mv^2 + \frac{1}{2} mu^2 - 0$

$mg \times 5 = \frac{1}{2} mv^2 + \frac{1}{2} m \left(\frac{25v^2}{16}\right)$

$10\ g = v^2 + \frac{25}{16} v^2$

$$v^2 = \frac{10g \times 16}{41}$$

$$v = \frac{\sqrt{10g \times 16}}{41} = \frac{40}{\sqrt{41}} \text{ m/s}$$

Sol 13: By work energy theorem [Velocity of both blocks is same]

$w_{net} = \Delta KE$

$$w_{friction} + w_{gravity} = \frac{1}{2} m_1 u^2 + \frac{1}{2} m_2 u^2$$

$$-\mu m_1 g L + m_2 gL = \frac{1}{2}(m_1 + m_2) u^2$$

$$u = \sqrt{\frac{2(m_2 - \mu m_1)gL}{m_1 + m_2}}$$

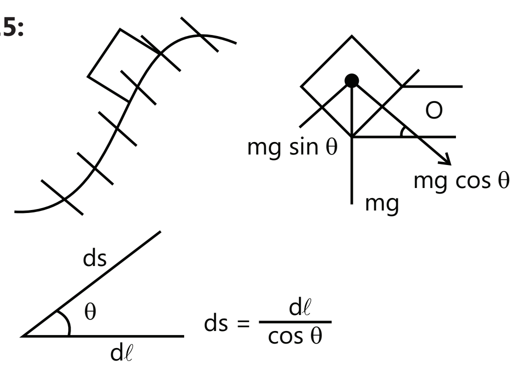
Sol 14: Since $\alpha_c = k^2rt^2$

$$\frac{v^2}{r} = k^2rt^2 \Rightarrow v^2 = k^2r^2t^2$$

$$\Rightarrow v = krt \Rightarrow F = m\frac{dv}{dt} = mkr$$

Power = F.v = (mkr)×(krt) = mk^2r^2t

Sol 15:



At any point on the path frictional force f = μ mg cos θ

θ = angle between path and the horizontal surface at some point

$$w_{friction} = -\int(\mu mg\cos\theta)\, ds$$

$$= -\int \mu mg\cos\theta \frac{d\ell}{\cos\theta} = -\int^{\ell} \mu mg d\ell = -\mu mg\ell$$

$w_{gravity} = -mgh$

By work energy theorem

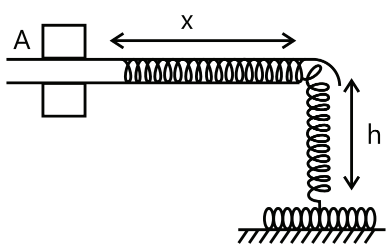
$w_{net} = \Delta KE$

$w_f + w_{gravity} + w_{frictoin} = 0$

$w_f - \mu mgl - mgh = 0$

$w_f = \mu\, mgl + mgh$

Sol 16: At any point of time, let the length of chain remaining in tube be x



m′ = mass of chain above ground = $\frac{m}{\ell}(h + x)$

Now by Newton's second law on chain of length (x + h). As length of chain which has fallen on ground has no effect on the upward chain.

F = m′a

$$\left(\frac{m}{\ell}h\right)g = \frac{m(h+x)}{\ell} a$$

$$\frac{mhg}{\ell} = \frac{m(h+x)}{\ell}\left(-\frac{vdv}{dx}\right)$$

[∵ x is decreasing with increase in length]

$$\int_{\ell-h}^{0} -\frac{hg}{h+x} dx = \int_0^{v'} vdv$$

[v′ is the velocity of the end]

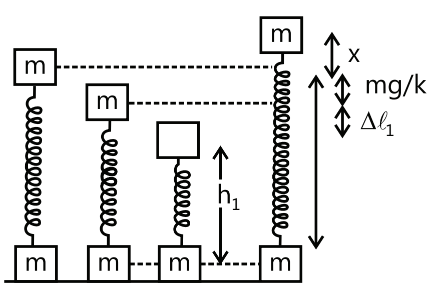
$$-\left[hg\log(h+x)\right]_{\ell-h}^{0} = \left.\frac{v^2}{2}\right]_0^{v'}$$

$$-hg\log\left(\frac{h}{\ell}\right) = \frac{v'^2}{2}$$

$$v'^2 = 2hg\log\left(\frac{\ell}{h}\right)$$

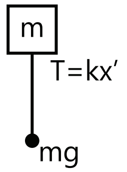
$$v' = \sqrt{2hg\log\left(\frac{\ell}{h}\right)}$$

Sol 17:



Let the natural length of spring be λ_1

Initially there is same compression x in spring in equilibrium



$mg = kx' \Rightarrow x' = \frac{mg}{k}$

Now it is further compressed by $\Delta\lambda_1$ by thread

Now if thread is burnt it will go at upward extreme which is x distance above natural length of spring. Spring will just lift the lower block so by newton 2nd law; T = kx = mg

$x = \frac{mg}{k}$

By mechanical energy conservation

$PE_{spring} + PE_{gravity} = PE'_{gravity} + PE'_{spring}$

$$\frac{1}{2}k\left(\Delta\ell_1 + \frac{mg}{k}\right)^2 + mgh_1$$

$$= mg\left(\frac{2mg}{k} + \Delta\ell_1 + h_1\right) + \frac{1}{2}k\left(\frac{mg}{k}\right)^2$$

$$\frac{1}{2}k\left(\Delta\ell_1^2 + \left(\frac{mg}{k}\right)^2 + 2\frac{mg}{k}\Delta\ell_1\right)$$

$$= \frac{2(mg)^2}{k} + mg\Delta\lambda_1 + \frac{1}{2}k\left(\frac{mg}{k}\right)^2$$

$$\frac{1}{2}k\Delta\lambda_1^2 = \frac{2(mg)^2}{k}$$

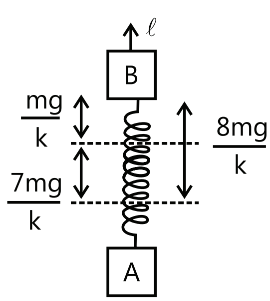
$$\Delta\lambda_1 = \frac{2mg}{k}$$

To lift block of mass m

$$\Delta\ell > \Delta\lambda_1 \frac{mg}{k}$$

$$\Delta\ell > \frac{3mg}{k}$$

(b)



We will find the velocity of block B when block A will just lift upwards

$$\frac{1}{2}k\left(\frac{7mg}{k}\right)^2 - \frac{1}{2}k\left(\frac{mg}{k}\right)^2 - \frac{8mg}{k}mg = \frac{1}{2}mv^2$$

$$v^2 = \frac{32m^2g^2}{k}$$

Now block A and B together form a system with acceleration – g, $V_{cm} = \frac{v}{2}$

So,

$v^2 = u^2 + 2as$

$O = v_{cm}^2 - 2gs$

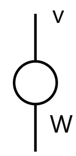
$v^2 = 8gs$

$$s = \frac{v^2}{8g} = \frac{4mg}{k}$$

Movement of centre of gravity

$$= \left(\frac{8mg}{k}\right)\frac{m}{2m} + \frac{4mg}{k} = \frac{8mg}{k} \text{ upwards}$$

Sol 18: (a)



By Newton's second law

$$(w + f) = \left(\frac{w}{g}\right)a$$

$$a = -g\left(1 + \frac{f}{w}\right) = \frac{vdv}{dx}$$

$$\int_{v_0}^{0} vdv = -\int_0^s g\left(1 + \frac{f}{w}\right)dx$$

$$0 - \frac{v_0^2}{2} = -g\left(1 + \frac{f}{w}\right)s$$

$$s = \frac{v_0^2}{2g\left(1 + \frac{f}{w}\right)}$$

Final velocity = v

(b) By work energy theorem

$w_{friction} + w_{gravity} = \Delta KE$

$$-2fs + 0 = \frac{1}{2}m(v^2 - v_0^2);\ v^2 - v_0^2 = \frac{-4fs}{m}$$

$$v^2 = \frac{-4b}{m} \frac{v_0^{\,2}}{2g\left(1+\frac{f}{w}\right)} + v_0^{\,2}$$

$$\Rightarrow v^2 = v_0^{\,2}\frac{[-4f+2f+2w]}{2(w+f)} \quad [mg = w] = v^2 = v_0^{\,2}\frac{[w-f]}{[w+f]}$$

$$v = v_0\left(\frac{w-f}{w+f}\right)^{1/2}$$

Sol 19: Let the speed of ring is v

Length of spring = $\frac{h}{\cos 37°} = \frac{5h}{4}$

By mechanical energy conservation

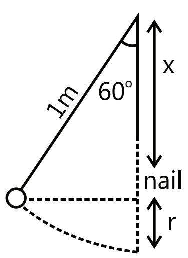
$PE_1 + KE_1 = PE_2 + KE_2$

$$\frac{1}{2}K\left(\frac{5h}{4}-h\right)^2 + 0 = 0 + \frac{1}{2}mv^2$$

$$K.\frac{1}{2}.\frac{h^2}{16} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{k}{m}}\frac{h}{4}$$

Sol 20:



x + r = 1

For velocity of pendulum when string becomes vertical is v

Work energy theorem

$w_{gravity} = KE$

m/g (1-cos60°) = ½ mv^2

$$v = \sqrt{2g\times\frac{1}{2}}$$

$$v = \sqrt{g} \text{ m/s} \qquad ...(i)$$

For circular motion to be just completed

$$v = \sqrt{5gr} \qquad ...(ii)$$

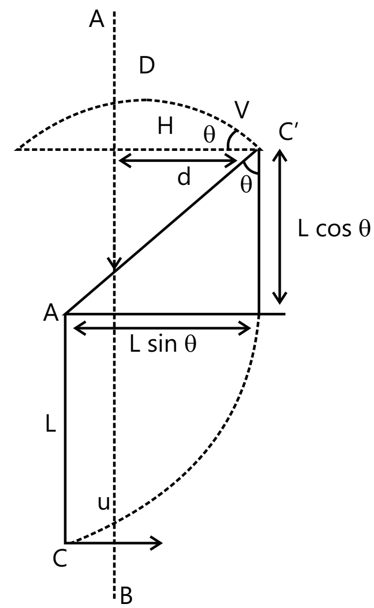
By (i) and (ii)

$$\sqrt{g} = \sqrt{5gr}$$

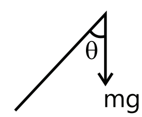
$$r = \frac{1}{5} \text{ m}$$

$$x = (1 - \frac{1}{5}) \text{ m} = 0.8 \text{ m}$$

Sol 21:



$$H = \frac{v^2\sin^2\theta}{2g}$$



By Newton's second law

$$mg\cos\theta = \frac{mv^2}{R}$$

$$v^2 = g\ell\cos\theta \qquad ... (i)$$

Let us assume that pendulum leaves circular motion at point C with velocity v making an angle θ with horizontal

Applying work energy theorem from point S to C′

$w_{net} = \Delta K$

$$w_{gravity} = \frac{1}{2}m(v^2 - a^2)$$

$$-mgr(1+\cos\theta) = \frac{1}{2}m(v^2-u^2)$$

$$v^2 = u^2 - 2g\ell(1+\cos\theta) \qquad ... (ii)$$

From point C′ to D it will follow parabolic path and velocity at line AB is horizontal

$$L\sin\theta - \frac{L}{4} = \frac{v^2\sin 2\theta}{2g} \qquad ... (iii)$$

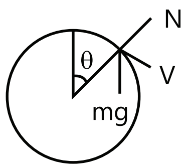
By (i) and (ii)

$u^2 = g\ell\,(2 + 3\cos\theta)$

By (ii) and (iii)

$u = 2.14\sqrt{g\ell}$

Sol 22:



By work energy conservation

$w_1 = \Delta KE$

$mg\,R\,(1 - \cos\theta) = \frac{1}{2}\,mv^2$

$2mg\,(1 - \cos\theta) = \frac{mv^2}{R}$

By newton's 2nd law

$Mg\cos\theta - N = 2\,mg\,(1 - \cos\theta)$

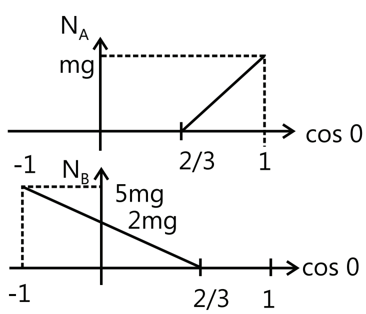
$N = mg\,(3\cos\theta - 2)$

For $\theta \le \cos^{-1}(2/3)$; $N_B = 0$,

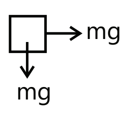
$N_A = mg\,(3\cos\theta - 2)$

For $\theta \ge \cos^{-1}(2/3)$ $N_A = 0$,

$N_B = mg\,(2 - 3\cos\theta)$



Sol 23: Let the initial compression be x



By Newton's second law F = ma

$mg = \frac{mv^2}{R}$

By work energy theorem

$w_{net} = \Delta K$

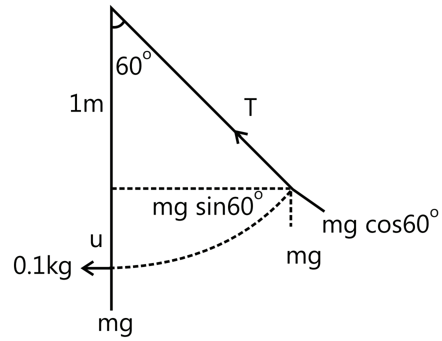
$w_{spring} + w_{gravity} = \frac{1}{2}\,mv^2 - 0$

$\frac{1}{2}\,kx^2 - mgR = \frac{1}{2}\,mgR$

$\frac{1}{2}kx^2 = \frac{3mgR}{2x}$

$x = \sqrt{\frac{3mgR}{k}}$

Sol 24:



(a) By newton's second law

$T - mg\cos 60° = \frac{mv^2}{R}$

$T = \frac{mg}{2} + \frac{m}{R}\,(4) = (4.9 + 4)\,m$

$= 8.9 \times 0.1 = 0.89\text{ N} = 8.9 \times 10^4$ dyne

(b) By work energy theorem

$w_{net} = \Delta kE$

$mg\ell\,(1 - \cos 60°) = u^2 - v^2$

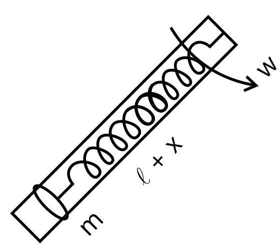
$u^2 = v^2 + mg\ell\,(1 - \cos 60°)$

$= 4 + 0.1 \times 9.8 \times 1 \times \frac{1}{2} = 4 + 0.49$

$u^2 = 4.49$

$u = 2.12$ m/s

Sol 25:



By Newton's second law

$Kx = m\omega^2\,(\ell + x)$

$x = \frac{m\omega^2\ell}{k - mw^2}$

Let $x' = \frac{m\omega^2}{k}$

$x = \frac{x'\ell}{1 - x'}$

By work energy theorem

$w_{net} = \Delta KE$

$w_{spring} + w_{force} = \frac{1}{2}\, m\, \omega^2 (\ell + x)^2$

$\frac{-1}{2} kx^2 + w_{force} = \frac{1}{2} m\omega^2 (\ell + x)^2$

$w_{force} = \frac{1}{2} kx^2 + \frac{1}{2} m\omega^2 (\ell + x)^2$

$= \frac{1}{2} k \frac{x'^2 \ell^2}{(1-x')^2} + \frac{1}{2} m\omega^2 \left(\ell + \frac{x'\ell}{1-x'}\right)^2$

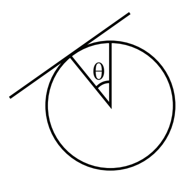
$= \frac{k\ell^2}{2(1-x')^2} \left[x^2 + \left(\frac{m\omega^2}{k}\right)\right]$

$w_{force} = \frac{k\ell^2}{2(1-x')^2} [x'^2 + x']$

$= \frac{k\ell^2 x'(1+x')}{2(1-x')^2}$ where $x' = \frac{m\omega^2}{k}$

Sol 26: (a) Minimum speed is required so in the limiting case velocity of block at highest point is zero

By work energy theorem



$w_{net} = \Delta KE$

$w_{gravity} = 0 - \frac{1}{2} mu_0^2$

$- mg [L \sin\theta + R (1 - \cos\theta)] = -\frac{1}{2} mu_0^2$

$u_0 = \sqrt{2g[L\sin\theta + R(1-\cos\theta)]}$

(b) Let the final velocity be v at top point

$w_{gravity} = \frac{1}{2} mv [v^2 - 4 u_0^2]$

$-mg [L \sin\theta + R (1 - \cos\theta)] = \frac{1}{2}m[v^2 - 4u_0^2]$

$v^2 = 3u_0^2$

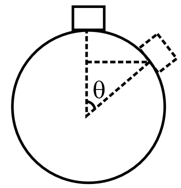
By Newton's second law

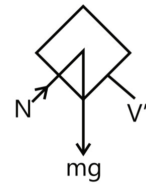
$\text{Force} = ma = \frac{mv^2}{R} = \frac{m(3u_0^2)}{R}$

$= \frac{3m}{R} [2G(R(1\text{-}\cos\theta) + L \sin\theta]$

$= 6\, mg [1 - \cos\theta + \frac{L}{R} \sin\theta]$

(c) If the projection speed is slightly greater than u_0, then speed at top most point is just than zero.





Particle will lose contact when normal just becomes zero.

So by Newton's second law

$mg \cos\theta = \frac{mV'^2}{R}$

$v'^2 = g R \cos\theta$

By work energy theorem, $w_{net} = \Delta KE$

$mg R (1 - \cos\theta) = \frac{1}{2} mv'^2 = \frac{1}{2} mg R \cos\theta$

$2(1 - \cos\theta) = \cos\theta$

$2 - \cos\theta = - \cos\theta$

$\cos\theta = \frac{2}{3}$

So it will lose contact when particle makes an angle $\cos^{-1}\left(\frac{2}{3}\right)$ with vertical.

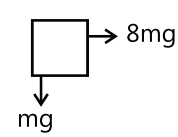
Sol 27: (a) By work energy theorem

$w_{net} = \Delta KE$

$mg (5R - R) = \frac{1}{2} mv^2$

$v^2 = 8gR$

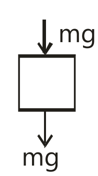
By Newton's second law force exerted in horizontal direction $= \frac{mv^2}{R} = \frac{8mgR}{R} = 8mg$



Net force $= \sqrt{8^2 + 1} = \sqrt{65}$ mg

By Netwons second law

$\text{Force} = \frac{mv^2}{R}$



$mg + mg = \frac{mv^2}{R} \Rightarrow v^2 = 2gR$

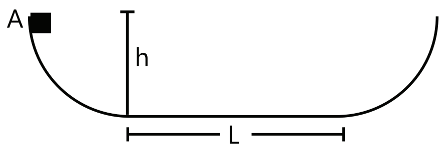
Let the height be h

By work energy theorem

$mg(h - 2R) = \frac{1}{2} m\, 2gR$

$h - 2R = R \quad \Rightarrow h = 3R$

Sol 28: (Coming to a stop) A particle can slide along a track with elevated ends and a flat central part, as shown in the figure below. The flat part has length L = 40 cm. The curved portions of the track are frictionless, but for the flat part the coefficient of kinetic friction is μ_k = 0.2. The particle is released from rest at point A, which is at height h = L/2. How far from the left edge of the flat part does the particle finally stop?

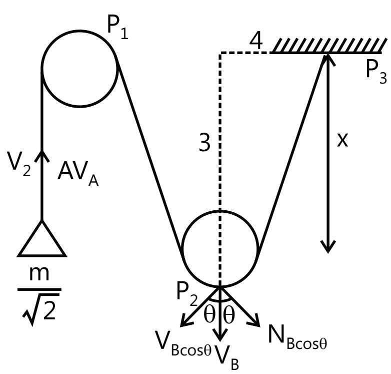


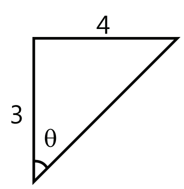
The initial energy is mgh = mgL/2. On the level ground, the particle experiences a constant friction force f = μ_k N = $\mu_k$mg. It will stop once the work W = -fs done by friction has dissipated all the initial energy:

$$mgL/2 = fs = \mu_k mgs \quad \Rightarrow \quad s = \frac{L}{2\mu_k} = 100\,\text{cm}$$

So the particle will make two full passes (one moving right, one moving left) over the flat area, then stop halfway across (20 cm from the left edge) on its third trip.

Sol 29:





$$\cos\theta = \frac{3}{5}$$

$$\sin\theta = \frac{4}{5}$$

(a) Length of string is constant

$$\Rightarrow AP_1 + P_1P_2 + P_2P_3 = 0$$

$$\frac{d(AP_1)}{dt} \; \frac{+d(P_1P_2)}{dt} + \frac{d(P_2P_3)}{dt} = 0$$

$$-V_A + V_B + V_B\cos\theta = 0$$

$$V_A = V_B(\cos\theta + \cos\theta)$$

$$V_A = \frac{6}{5}\,v_B$$

Now, by work energy theorem

$$w_{net} = \Delta KE$$

$$w_{gravityA} + w_{gravityB} = \Delta KE \qquad .....(i)$$

Initially length of string between P_1 and P_3 is 8m

Finally length of string between P_1 and P_3 is 10 m so A has moved (10 – 8) upward

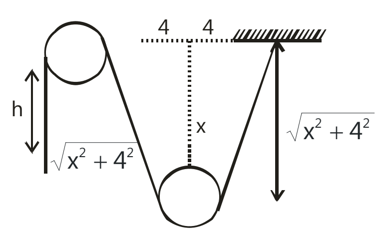
By (i)

$$-\frac{m}{\sqrt{2}}\,g \times 2 + mg \times 3 = \frac{1}{2}\,mv_B^{\,2} + \frac{1}{2}\,\frac{m}{\sqrt{2}}\,v_A^2$$

$$mg\,[3 - \sqrt{2}\,] = \frac{1}{2}\,m\left[V_B^2 + \frac{36}{25}\frac{V_B^{\,2}}{\sqrt{2}}\right]$$

$$V_B = \sqrt{\frac{(3-\sqrt{2})g\times 2}{\left(1+\dfrac{36}{25\sqrt{2}}\right)}} \approx 4\text{m/s}$$

(b) Velocity of A and B is zero at maximum displacement



$$h = \sqrt{x^2+4^2} - 8$$

By work energy theorem

$$-\frac{m}{\sqrt{2}}\,gh + mg\,x = 0$$

$$h = \sqrt{2}\,x$$

$$2\sqrt{x^2+h^2} - 8 = \sqrt{2}\,x$$

$$x = 8\sqrt{2}\,\text{m}$$

Exercise 2

Single Correct Choice Type

Sol 1: (B) Potential of water after falling down will convert in heat and sound. So temperature will increase slightly.

Sol 2: (A) By work energy theorem

$w_{net} = \Delta KE$

$w_{gravity} + w_{spring} = 0$

$mg\,(0.4 + x) + -\frac{1}{2}\,kx^2 = 0$

$20 \times 0.4 + 20\,x - 1000\,x^2 = 0$

$1000\,x^2 - 20\,x - 8 = 0$

$$x = \frac{20 \pm \sqrt{400 + 32000}}{2000}$$

$$x = \frac{20 \pm 180}{2000} = \frac{200}{2000} = \frac{1}{10} = 0.1\text{ m}$$

Sol 3: (C) $F - \frac{-dv}{dR}$

$$U = -\int FdR = -\left[\frac{k}{R^2}dR\right] = -\left[\frac{-k}{R}\right] = \frac{k}{R}$$

Sol 4: (C) Mechanical energy is ME = KE + PE

Maximum potential energy is 160 J when

Kinetic energy is zero i.e. at end points.

Sol 5: (B) $u = 2x^4 - 27\,x$

$$F = \frac{-dU}{dx}$$

$F = -[8x^3 - 27]$

at x = 3/2

Force is zero

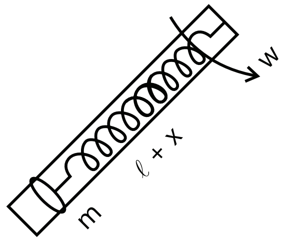
$$f\left(\frac{3}{2}^{+}\right] = \text{-ve}$$

$$F\left(\frac{3}{2}^{-}\right] = \text{+ve}$$

So this is stable equilibrium

Sol 6: (A) Minimum speed must be zero as it is connected to rod so it will not leave the circular motion at any point in the path

Sol 7: (B)



By Newton's second law

$Kx = m\omega^2\,(\ell + x)$

$$x = \frac{m\omega^2\ell}{k - m\omega^2}$$

$$\text{Let } x' = \frac{m\omega^2}{k}$$

$$x = \frac{x'\ell}{1 - x'}$$

By work energy theorem

$w_{net} = \Delta KE$

$$w_{spring} + w_{force} = \frac{1}{2}\,m\,\omega^2\,(\ell + \lambda)^2$$

$$\frac{-1}{2}\,kx^2 + w_{force} = \frac{1}{2}\,m\omega^2\,(\ell + x)^2$$

$$w_{force} = \frac{1}{2}\,kx^2 + \frac{1}{2}\,m\omega^2\,(\ell + x)^2$$

$$= \frac{1}{2}\,k\,\frac{x'^2\,\ell^2}{(1 - x')^2} + \frac{1}{2}\,m\omega^2\left(\ell + \frac{x'\ell}{1 - x'}\right)^2$$

$$= \frac{k\ell^2}{2(1 - x')^2}\left[x^2 + \left(\frac{m\omega^2}{k}\right)\right]$$

$$w_{force} = \frac{k\ell^2}{2(1 - x')^2}\,[x'^2 + x']$$

$$= \frac{k\ell^2 x'(1 + x')}{2(1 - x')^2} \text{ where } x' = \frac{m\omega^2}{k}$$

Sol 8: (B) $a_c = k^2\,rt^2 = \frac{v^2}{R}$

$v = krt$

$$a_t = \frac{dv}{dt} = kr$$

Power $P = F.V = ma_t.v = mkr.\,Krt = m\,k^2\,r^2\,t$

Sol 9: (D) Tension is zero as can and pendulum are falling freely under gravity

Sol 10: (D)

$f(x) = -kx + ax^2$

$U(x) = \int -f(x)\,dx$

$U(x) = -\left[-\frac{kx^2}{2} + \frac{ax^4}{4}\right] + c = \frac{kx^2}{2}\ \frac{-ax^4}{4} + c$

It corresponds to graph (D) for c = 0

Sol 11: (C) Power = F.v

Force = rate of change of linear momentum of wind

$\frac{dm}{dt} = \rho AV$ where ρ= density

A = area of blades

V = velocity

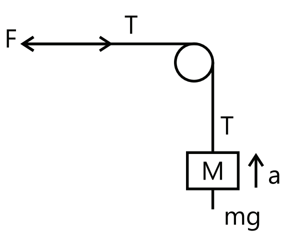
$F = \frac{d(m.v)}{dt} = v\,\frac{dm}{dt} = v\,\rho AV$

$F = \rho AV^2$

Power = ρAV^3

$P \propto v^2$

Sol 12: (B)



By force equilibrium

F = T

By Newton's second law

T – mg = ma

T = m (g + a)

By work energy theorem

$w_{net} = \Delta KE$

$w_{tension} + w_{gravity} = 20J$

$w_{tension} = 120 - w_{gravity}$

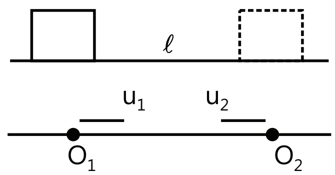
$w_{gravity} = -Mgh = -Mg\,v \times 1$

$= -Mg\sqrt{\frac{40}{M}} = -g\sqrt{40M}$

$KE = 20\ J = \frac{1}{2}Mv^2$

$v = \sqrt{\frac{40}{M}}$

Sol 13: (D)



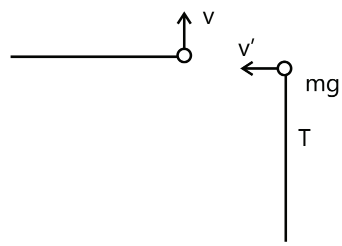
$u_1 + u_2 = V$

Acceleration will be same for both as acceleration of both observers is zero.

Kinetic energy will be different so by work energy theorem work done will also depend on kinetic energy.

Multiple Correct Choice Type

Sol 14: (A, D)



By Newton's second law

$T + mg = \frac{mv^2}{\ell}$

T = 0 at highest point

So string becomes slack at the highest point.

Sol 15: (B, D)

KE is increasing

⇒ Velocity is increasing

⇒ Resultant force must be at an angle less than 90° so that a component of force in the direction of velocity will increase its velocity

⇒ Linear momentum is increasing

Sol 16: (A, B)

In (C) and (D) work done by spring is $\frac{-1}{2}kx^2$ but in (A) and (B) work done is $\frac{+1}{2}kx^2$

Sol 17: (A, C, D) Work done = $\vec{F}.\vec{ds}$

(A) If force is always perpendicular to velocity then,

$\vec{F}.\vec{ds} = 0$

(B) If there is some initial velocity in the direction of force then, work done can be non-zero

(C), (D) Work done depends only on the displacement of point of application of force.

Sol 18: (C, D) Work done = 0 so kinetic energy is constant

Since in velocity and acceleration, direction is changing so they are not constant.

Sol 19: (A, B) By work energy theorem

$w_{net} = \Delta KE$

$mgh = \frac{1}{2} m (v_b^2 - v_i^2)$

So final velocity is larger than initial and will depend on speed of projection.

Sol 20: (A, B, D) By work energy theorem

$w_{net} = \Delta KE = 0$

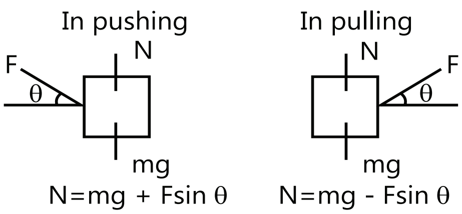
$w_{you} + w_{gravity} = 0$

$w_{you} = -w_{gravity} = + mgh$

Assertion Reasoning Type

Sol 21: (B) Force has to be zero

Sol 22: (A)



Sol 23: (A) $U = -\int \vec{F}.\overrightarrow{dr}$

Assume a closed loop in space

Since $\vec{F}$ is a conservative force, the line integral is zero. Thus U is a state function.

So potential energy is defined only for conservative force.

Sol 24: (B) By work energy theorem $w_{net} = \Delta KE$

$F.\Delta S = \frac{1}{2} mv^2 \text{ ; } v \propto \frac{1}{\sqrt{m}}$

Comprehension Type

Sol 25: (A) Work done in raising box = $-w_{gravity}$

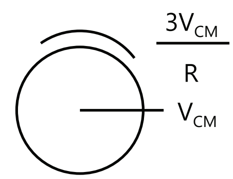
$= -(-mgh) = mgh$

1 – false

Sol 26: (D) There is no friction and non-conservative force so mechanical energy is conserved.

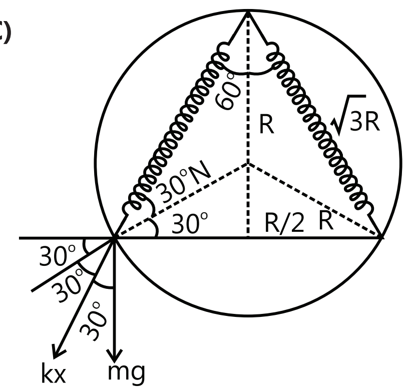
Sol 27: (B) As there is sliding at t =0 so friction will act opposite to the direction of velocity

Sol 28: (B) $\omega > \frac{v_{cm}}{R}$



So friction will act in the direction of velocity to increase the velocity and decrease the angular acceleration

Sol 29: (C)



Since acceleration of bead in the normal direction is zero. So by Newton's second law

$N - kx \cos 30° = mg \cos 60°$

$N = \frac{mg}{2} + \frac{kx\sqrt{3}}{2} = \frac{mg}{2} + \frac{\sqrt{3}}{2} \frac{(2+\sqrt{3})mg}{\sqrt{3}R} (2-\sqrt{3})R$

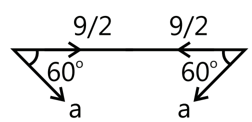
$= \frac{mg}{2} + \frac{mg}{2} = mg$

Sol 30: (D) Newton's second law in direction perpendicular to normal

(i) $mg \cos 30° + kx \cos 60° = ma$

$\frac{mg\sqrt{3}}{2} + \frac{(2+\sqrt{3})mg}{\sqrt{3}R}(2-\sqrt{3}) \times \frac{1}{2} = ma$

$\frac{mg\sqrt{3}}{2} + \frac{mg}{2\sqrt{3}} = ma;$



$\frac{mg}{2}\left[\sqrt{3} + \frac{1}{\sqrt{3}}\right] = ma$

$a = \frac{2g}{\sqrt{3}}$

From figure relative acceleration is $\frac{a}{2} + \frac{a}{2} = a = \frac{2g}{\sqrt{3}}$

Sol 31: (C) By mechanical energy conservation

$PE_i + KE_i = PE_f + KE_f$

$\frac{mgR}{x} + \frac{1}{2}kx^2 + 0 = 0 + \frac{1}{2}mv^2$

$\frac{mgR}{2} + \frac{(2+\sqrt{3})mg}{\sqrt{3}R}(2-\sqrt{3})^2 R^2 = mv^2$

$v^2 = \frac{gR}{2} + \left(\frac{2}{\sqrt{3}} - 1\right) gR$

$v = \sqrt{\frac{2gR}{\sqrt{3}}}$

Sol 32: (D) (A) Wrong, collision can be inelastic

(B) In perfectly inelastic collision energy is not conserved

(C) For SHM, θ should be small.

(D) At the instant of collision, they are at the bottom

$\Rightarrow \Sigma F = 0$ and $\Sigma M = 0$

$\Rightarrow$ Momentum conserved

Match the Columns

Sol 33: A→s; B→q; C→r; D→p

Total mechanical energy is conserved

U(x) + KE = constant

At x = 5

U(x) = 29

KE = 20

Total ME at any x = 49

maximum P.E. = 49

maximum K.E. = 49 – U_{min} = 49 – 20 = 29

When U(x) is maximum then x will take extreme values.

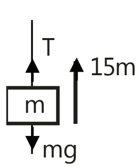
$20 + (x-2)^2 = 49$

$(x-2)^2 = 29$

$x - 2 = +\sqrt{29}$; $x - 2 = -\sqrt{29}$

x = 7.38 ; x = –3.38

Sol 34: (A→r; B-p, C-q, D-s)



Work done by gravity

w = +mgh = 0.72 × 10 × 15

w_2 = 10800

By Newton's second law

T – mg = ma; T = m(g + a)

$T = 72\left(R + \frac{g}{10}\right) = \frac{11g}{10} \times 72 = 72 \times 11 = 792$

Work done w_1 = 792 × 15 = 11880

By work energy theorem

$w_{string} + w_{gravity} = \frac{1}{2}mv^2$

11880 – 10800 = KE

KE = 1080

KE = ½ mv² = 1080

v = 5.47 m/s

Previous Years' Questions

Sol 1: Given $t = \sqrt{x} + 3$

or $\sqrt{x} = (t-3)$ ...... (i)

$\therefore x = (t-3)^2 = t^2 - 6t + 9$ ....(ii)

Differentiating this equation with respect to time, we get

Velocity $v = \frac{dx}{dt} = 2t - 6$ .... (iii)

(a) v = 0 when 2t – 6 = 0 or t = 3s

Substituting in Eq. (i), we get

$\sqrt{x} = 0$ or x = 0

i.e., displacement of particle when velocity is zero is also zero.

(b) From eq. (iii) speed of particle

At t = 0 is $v_i = |v| = 6$ m/s

At t = 6 s is $v_f = |v| = 6$ m/s

From work energy theorem,

Work done = change in kinetic energy

$= \frac{1}{2}m[v_f^2 - v_i^2] = \frac{1}{2}m[(6)^2 - (6)^2] = 0$

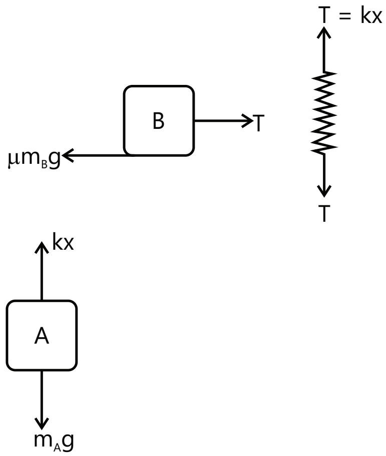
Sol 2: s = vt = 2 × 5 = 10 m

Q = work done against friction

= μmgs = 0.2 × 2 × 9.8 × 10 = 39.2 J = 9.33 cal

Sol 3: Normal reaction between blocks A and C will be zero. Therefore, there will be no friction between them.

Both A and B are moving with uniform speed. Therefore net force on them should be zero.



For equilibrium of A

$m_A g = kx$

$\therefore x = \frac{m_A g}{k} = \frac{(2)(9.8)}{1960} = 0.01$ m

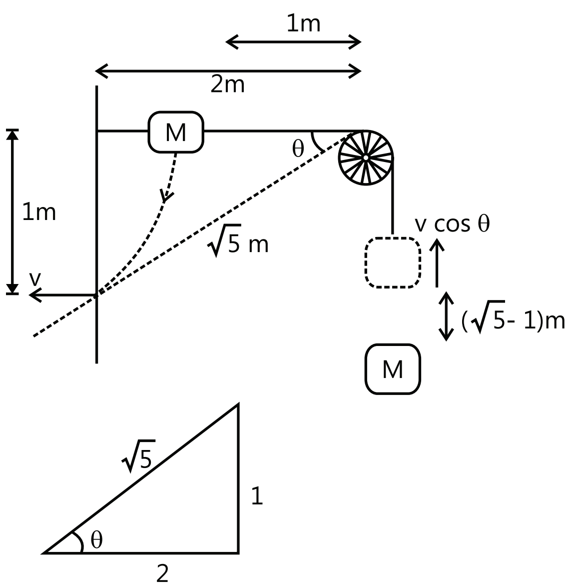
For equilibrium of B $\mu m_B g = T = kx = m_A g$

$m_B = \frac{m_A}{\mu} = \frac{2}{0.2} = 10$ kg

Energy stored in spring

$U = \frac{1}{2}kx^2 = \frac{1}{2}(1960)(0.01)^2 = 0.098$ J

Sol 4: Let M strikes with speed v. Then, velocity of m at this instant will be v cos θ or $\frac{2}{\sqrt{5}}$ v. Further M will fall a distance of 1 m while m will rise up by ($\sqrt{5}-1$) m. From energy conservation: decrease in potential energy of M = increase in potential energy of m+ increase in kinetic energy of both the blocks.

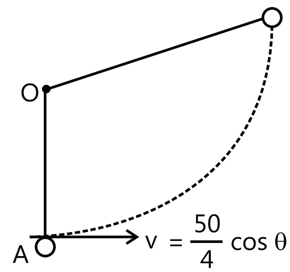


or $(2)(9.8)(1) = (0.5)(9.8)(\sqrt{5}-1)$

$+\frac{1}{2}\times 2\times v^2 + \frac{1}{2}\times 0.5\times\left(\frac{2v}{\sqrt{5}}\right)^2$

Solving this equation, we get v = 3.29 m/s

Sol 5: (a) At the highest point, velocity of bullet is 50cosθ. So, by conservation of linear momentum



$M(50\cos\theta) = 4\,Mv$

$\therefore v = \left(\frac{50}{4}\right)\cos\theta$ ... (i)

At point B. T = 0 but $v \neq 0$

Hence, $4\,Mg\cos 60° = \frac{(4M)v^2}{\ell}$

or $v^2 = \frac{g}{2}\ell = \frac{50}{3}$ ... (ii)

$\left(\text{as } \ell = \frac{10}{3}\text{m amd } g = 10\text{m}/\text{s}^2\right)$

Also, $v^2 = u^2 - 2gh = u^2 - 2g\left(\frac{3}{2}\ell\right) = u^2 - 3(10)\left(\frac{10}{3}\right)$

or $v^2 = u^2 - 100$

or solving eqs. (i), (ii) and (iii), we get

cos θ = 0.86 or θ = 30°

(b) $x = \frac{\text{Range}}{2} = \frac{1}{2}\left(\frac{u^2\sin 2\theta}{g}\right)$

$= \frac{50\times 50\times\sqrt{3}}{2\times 10\times 2} = 108.25$ m

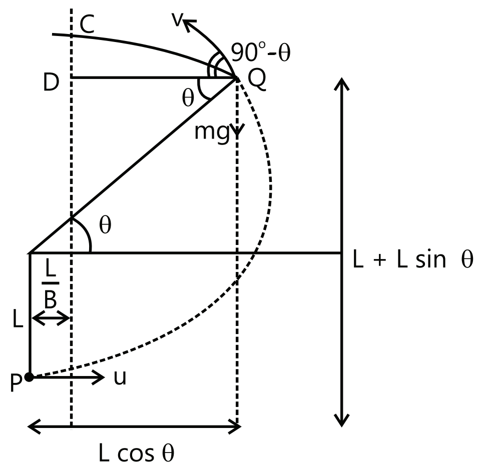
$y = H = \frac{u^2\sin^2\theta}{2g} = \frac{50\times 50\times 1}{2\times 10\times 4} = 31.25$ m

Hence, the desired coordinates are (108.25 m, 31.25 m).

Sol 6: Let the string slack s at point Q as shown in figure. From P to Q path is circular and beyond Q path is parabolic. At point C, velocity of particle becomes horizontal therefore. QD = half the range of the projectile

Now, we have following equations

(1) $T_Q = 0$. Therefore, $mg\sin\theta = \frac{mv^2}{L}$ ... (i)



(2) $v^2 = u^2 - 2gh = u^2 - 2gL(1 + \sin\theta)$ ... (ii)

(3) QD = $\frac{1}{2}$(Range)

$$\Rightarrow \left(L\cos\theta - \frac{L}{g}\right) = \frac{v^2 \sin 2(90^\circ - \theta)}{2g} = \frac{v^2 \sin 2\theta}{2g} \quad \text{... (iii)}$$

Eq. (iii) can be written as

$$\left(\cos\theta - \frac{1}{g}\right) = \left(\frac{v^2}{gL}\right) \sin\theta \cos\theta$$

Substituting value of $\left(\frac{v^2}{gL}\right)$ = sinθ from Eq. (i), we get

$$\left(\cos\theta - \frac{1}{8}\right) = \sin^2\theta \cos\theta = (1 - \cos^2\theta)\cos\theta$$

or $\cos\theta - \frac{1}{8} = \cos\theta - \cos^3\theta$

$\therefore \cos^3\theta = \frac{1}{8}$ or $\cos\theta = \frac{1}{2}$ or $\theta = 60°$

∴ From Eq. (i) $v^2 = gL\sin\theta = gL\sin 60°$

or $v^2 = \frac{\sqrt{3}}{2} gL$

∴ Substituting this value of v^2 in eq. (ii)

$u^2 = v^2 + 2gL(1 + \sin\theta)$

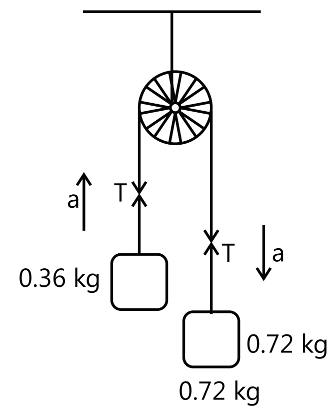
$$= \frac{\sqrt{3}}{2} gL + 2gL\left(1 + \frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{2} gL + 2gL$$

$$= gL\left(2 + \frac{3\sqrt{3}}{2}\right)$$

$$u = \sqrt{gL\left(2 + \frac{3\sqrt{3}}{2}\right)}$$

Sol 7: $a = \frac{\text{Net pulling force}}{\text{Total mass}} = \frac{0.72g - 0.36g}{0.72 + 0.36} = \frac{g}{3}$

$$S = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{g}{3}\right)(1)^2 = \frac{g}{6}$$



$T - 0.36\,g = 0.36\,a = 0.36\,\frac{g}{3}$

$\therefore T = 0.48\,g$

Now, $W_T = TS\cos 0°$ (on 0.36 kg mass)

$$= (0.48\,g)\left(\frac{g}{6}\right)(1) = 0.08(g^2) = 0.08(10)^2 = 8\text{ J}$$

Sol 8: (D) Decrease in mechanical energy

= work done against friction

$\therefore \frac{1}{2}mv^2 - \frac{1}{2}kx^2 = \mu mgx$

or $v = \sqrt{\frac{2\mu mgx + kx^2}{m}}$

Substituting the values, we get

$v = 0.4\text{ m/s} = \left(\frac{4}{10}\right)\text{m/s}$

∴ Answer is D

Sol 9: (D)

$$dw = F.dr = F.(dx\,\hat{i} + dy\hat{j})$$

$$= K\int \frac{xdx}{\left(x^2 + y^2\right)^{3/2}} + \frac{ydy}{\left(x^2 + y^2\right)^{3/2}}$$

$$x^2 + y^2 = a^2$$

$$w = \frac{K}{a^3}\int_a^0 xdx + \int_0^a ydy = \frac{K}{a^3}\left(\frac{-a^2}{2} + \frac{a^2}{2}\right) = 0$$

Sol 10: (5) The initial speed of 1st bob (suspended by a string of length l_1) is $\sqrt{5gl_1}$.

The speed of this bob at highest point will be $\sqrt{gl_1}$.

When this bob collides with the other bob there speeds will be interchanged.

$$\sqrt{gl_1} = \sqrt{5gl_2} \Rightarrow \frac{l_1}{l_2} = 5$$

Sol 11: (5) Power $= \frac{dW}{dt} \Rightarrow W = 0.5 \times 5 = 2.5 = KE_f - KE_i$

$$2.5 = \frac{M}{2}\left(v_f^2 - v_i^2\right) \Rightarrow v_f = 5$$

Sol 12: (B) $\frac{d(KE)}{dt} = mv\frac{dv}{dt}$

Sol 13: (D) Condition for not sliding,

$f_{max} > (m_1 + m_2)\, g \sin\theta$

$\mu N > (m_1 + m_2)\, g \sin\theta$

$0.3\, m_2\, g \cos\theta \geq 30 \sin\theta$

$6 \geq 30 \tan\theta$

$1/5 \geq \tan\theta$

$0.2 \geq \tan\theta$

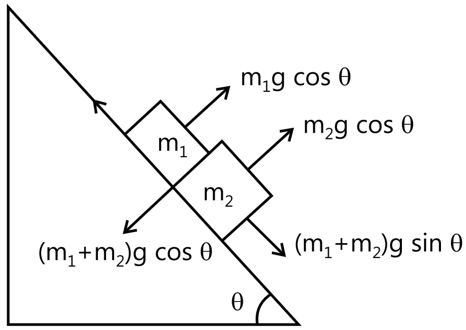
$\therefore$ for P, Q

$f = (m_1 + m_2)\, g \sin\theta$

For R and S

$F = f_{max} = \mu m_2 g \sin\theta$

Sol 14: (A, C)



As $I_1 = I_2$

$n_1 w_1 d_1 v_1 = n_2 w_2 d_2 v_2$

Now, $\frac{V_2}{V_1} = \frac{B_2 v_2 w_2}{B_2 v_1 w_1} = \left(\frac{B_2 w_2}{B_1 w_1}\right)\left(\frac{n_1 w_1 d_1}{n_2 w_2 d_2}\right) = \frac{B_2 n_1}{B_1 n_2}$