3. Axioms, Postulates and Theorems

Exercise 3.1

1. Question

What are undefined objects in Euclid's geometry?

Answer

The undefined objects in Euclid's geometry are point, line and plane.

2. Question

What is the difference between an axiom and a postulate?

Answer

difference between an axiom and a postulate:

(1) Axiom is generally true for any field of science, while postulates can be specific on a particular field.

(2) It is impossible to prove from other axioms, while postulates are provable to axioms.

3. Question

Give an example for the following axioms from your experience:

(a) If equals are added to equals, the wholes are equals.

(b) The whole is greater than the part.

Answer

(a) let the number of apples in basket A = 10

And number of bananas in basket B = 10

Add 5 number of mangoes in each basket we get equal number of fruits in both of these baskets that is equal to 15.

(b) let we have a baskets of 10 apples.

If we remove 4 apples from the basket which were earlier part of that basket. The apples removed are less in number than the total apples earlier present.

Hence, we can say the whole is greater than the part.

4. Question

What is the need of introducing axioms?

Answer

There are certain elementary statement, which are self evident and which are accepted without any question in such statments we need axioms. Also axioms are needed because these statements are also applicable to other areas of mathematics and science.

5. Question

You have seen earlier that the set of all natural numbers is closed under addition (closure property). Is this an axiom or something you can prove?

Answer

Closure Property of natural numbers: Let there be two natural numbers be x and y

Then according to closure property of natural numbers under addition

If a is a natural number and b is a natural number then a + b is also a natural number.

Now this is something that can be proved by giving examples.

A natural number is set of whole numbers excluding zero, so all the positive integers are Natural numbers.

And when positive natural number is added to another positive natural number we will have a positive integer only.

Let a = 2 and b = 99

Then, a + b = 101 which is also a natural number

You can take any two natural numbers and repeat the above process, addition of those numbers will always be a natural number.

Exercise 3.2

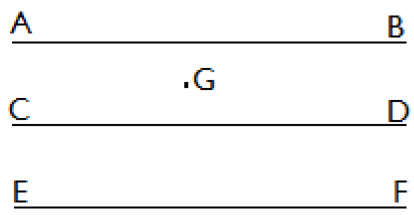
1 A. Question

Draw diagrams illustrating each of the following situation:

Three straight lines which do not pass through a fixed point.

Answer

Three straight lines which do not pass through a fixed point can be drawn as:



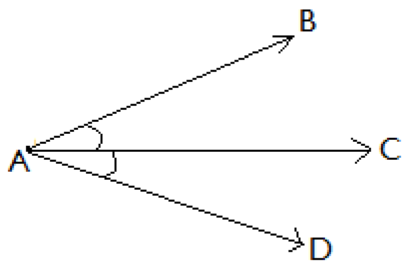
Where AB, CD and EF are three straight lines. And G is a fix point. All the three lines do not pass through the given fixed point.

1 B. Question

Draw diagrams illustrating each of the following situation:

A point and rays emanating from that point such that the angle between any two adjacent rays is an acute angle.

Answer



Where A is a point and AB, AC and AD are rays emanating from point A.

Angle between any two adjacent rays is acute i.e. $< 90°$.

$\angle BAC < 90°$

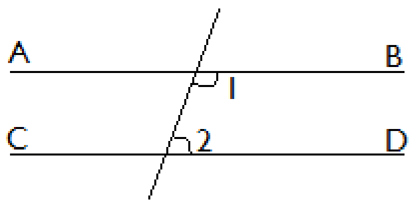
And $\angle DAC < 90°$

1 C. Question

Draw diagrams illustrating each of the following situation:

Two angles which are not adjacent angles, but still supplementary.

Answer



Here, $\angle 1$ and $\angle 2$ are not adjacent angles. But they are supplementary angles because they two interior angles between two parallel lines.

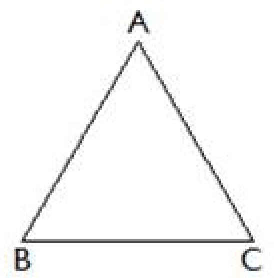
i.e. ∠1 + ∠2 = 180°

1 D. Question

Draw diagrams illustrating each of the following situation:

Three points in the plane which are equidistant from each other.

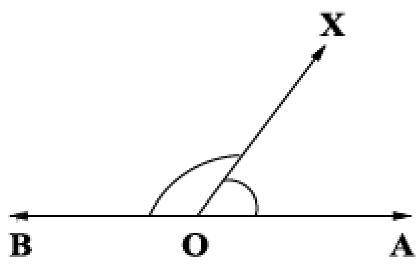
Answer



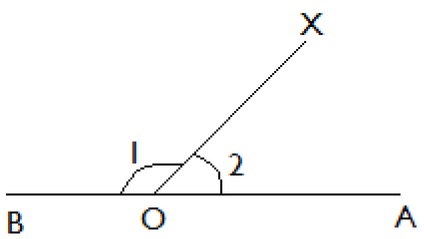
It is an equilateral triangle where all the three points A, B and C are equidistant from each other.

2 A. Question

Recognise the type of angles in the following figures:



Answer



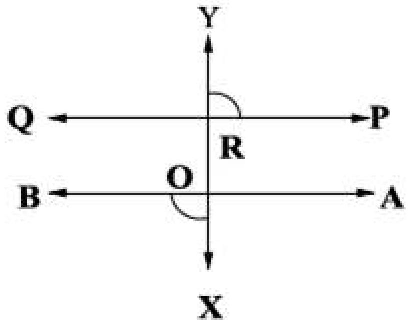
in this figure, ∠1 and ∠2 are making a linear pair.

i.e. ∠1 + ∠2 = 180°

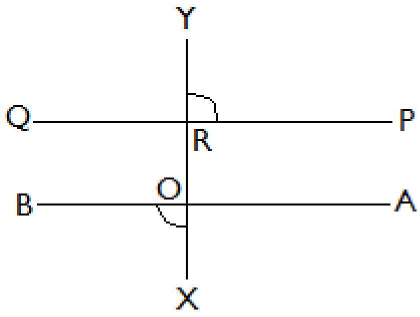
hence, ∠1 and ∠2 are straight angles.

2 B. Question

Recognise the type of angles in the following figures:



Answer



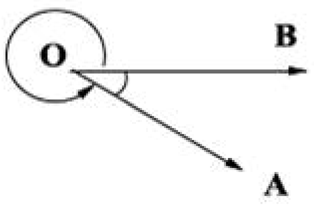
As we see, in the given figure the lines are perpendicular to each other.

Hence, ∠YRP = ∠XOB = 90°

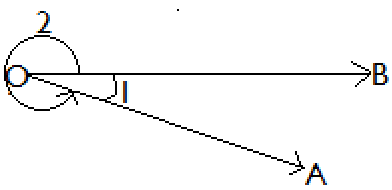
So, the given angles, ∠YRP and ∠XOB are right angles.

2 C. Question

Recognise the type of angles in the following figures:



Answer



Here, in the given figure,

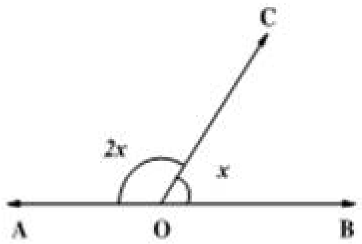
∠BOA = ∠1 is acute angle.

And the angle left after ∠BOA is known as reflex angle.

Hence, ∠2 is reflex angle.

3 A. Question

Find the value of x in each of the following diagrams:



Answer

Given: ∠AOC = 2x and ∠BOC = x

We know by proposition 1,

∠AOC + ∠BOC = 180°

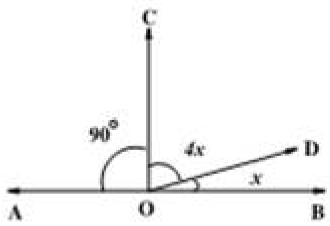
x + 2x = 180°

3x = 180°

x = 60°

3 B. Question

Find the value of x in each of the following diagrams:



Answer

Given: ∠AOC = 90°, ∠COD = 4x and ∠BOD = x

We know by proposition 1,

∠AOC + ∠COD + ∠BOD = 180°

90° + 4x + x = 180°

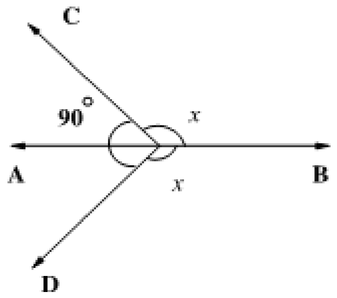
5x = 180° - 90°

5x = 90°

x = 18°

3 C. Question

Find the value of x in each of the following diagrams:



Answer

Given: ∠COD = 90°, ∠COB = x and ∠BOD = x

As we know, sum of all angles around a point is 360°.

∠COD + ∠COB + ∠BOD = 360°

90° + x + x = 360°

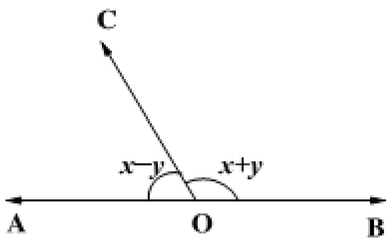
2x = 360° - 90°

2x = 270°

x = 135°

3 D. Question

Find the value of x in each of the following diagrams:



Answer

Given: ∠AOC = x-y and ∠BOC = x + y

We know by proposition 1,

∠AOC + ∠BOC = 180°

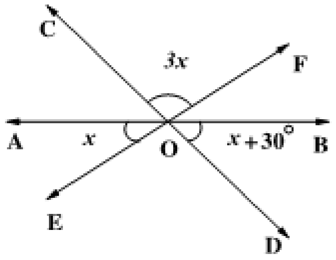
x - y + x + y = 180°

2x = 180°

x = 90°

3 E. Question

Find the value of x in each of the following diagrams:



Answer

Given: ∠COF = 3x, ∠AOE = x and ∠BOD = x + 30

We know by proposition 4,

∠AOE = ∠BOF = x (vertically opposite angles)

We know by proposition 1,

∠COF + ∠BOF + ∠BOD = 180°

3x + x + x + 30° = 180°

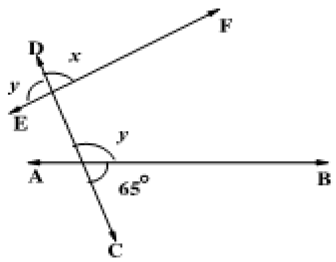
5x = 180° - 30°

5x = 150°

x = 30°

3 F. Question

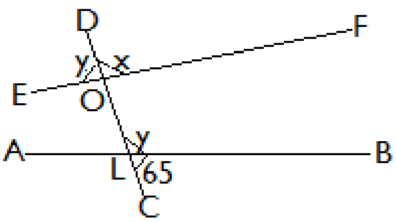
Find the value of x in each of the following diagrams:



Answer

Given: ∠BLC = 65° and ∠BLO = y

∠EOD = y and ∠FOD = x



We know by proposition 1,

∠BLC + ∠BLO = 180°

65° + y = 180°

y = 180°-65°

y = 115°

We know by proposition 1,

∠EOD + ∠FOD = 180°

y + x = 180°

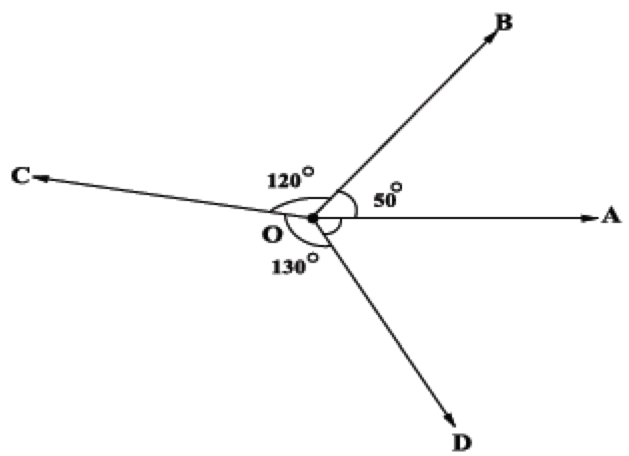
115° + x = 180°

x = 180° - 115°

x = 65°

4. Question

Which pair of angles are supplementary in the following diagram? Are they supplementary rays?



Answer

As we know, sum of all angles around a point is 360°. ∠BOC + ∠AOB + ∠AOD + ∠COD = 360°

120° + 50° + ∠AOD + 130° = 360°

∠AOD + 300° = 360°

∠AOD = 360° - 300°

∠AOD = 60°

In this,

∠AOD + ∠BOC = 60° + 120° = 180°

And ∠AOB + ∠COD = 50° + 130° = 180°

These are the pair of supplementary angles. because sum of angles is 180° hence these are supplementary rays.

5. Question

Suppose two adjacent angles are supplementary. Show that if one of them is an obtuse angle, then the other angle must be acute.

Answer

As we know, in case of supplementary angles the sum of angles must be 180°

Let two adjacent angles are x and y.

For these angles to be supplement,

x + y = 180° ...(1)

let x = 120° (obtuse angle)

put it in (1)

120° + y = 180°

⇒ y = 180° - 120°

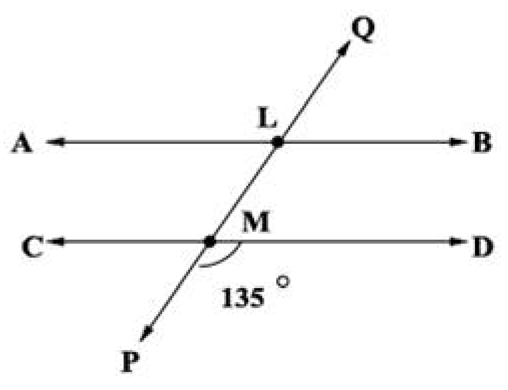
y = 60° (i.e. acute angle)

hence, to have a pair of supplementary angle if one angle is obtuse then other must be acute angle.

Exercise 3.3

1. Question

Find all the angles in the following figure.



Answer

Given: ∠DMP = 135°

We know by proposition 4,

∠LMC = ∠DMP = 135° (vertically opposite angles (V.O.A))

We know by proposition 1,

∠DMP + ∠DML = 180°

135° + ∠DML = 180°

∠DML = 180° - 135°

∠DML = 45°

∠LMC = ∠DML = 45° (vertically opposite angles (V.O.A))

∠BLQ = ∠DML = 45° (Corresponding angles)

∠ALM = ∠BLQ = 45° (vertically opposite angles (V.O.A))

We know by proposition 1,

∠ALM + ∠ALQ = 180°

45° + ∠ALQ = 180°

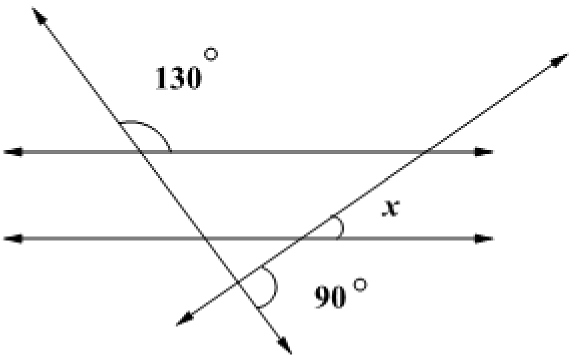
∠ALQ = 180° - 45°

∠ALQ = 135°

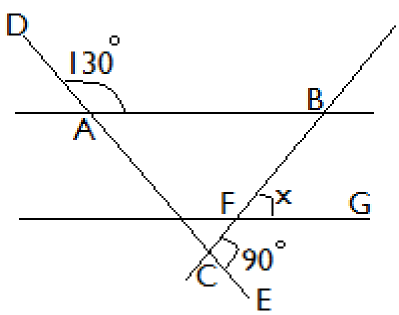
∠BLM = ∠ALQ = 135° (vertically opposite angles (V.O.A))

2. Question

Find the value of x in the diagram below.



Answer



We know by proposition 1,

∠DAB + ∠BAC = 180°

130° + ∠BAC = 180°

∠BAC = 180° - 130°

∠BAC = 50°

And again We know by proposition 1,

∠BCE + ∠BCA = 180°

90° + ∠BCA = 180°

∠BCA = 180° - 90°

∠BCA = 90°

In triangle ABC,

∠ACB + ∠BAC + ∠CBA = 180°

90° + 50° + ∠CBA = 180°

∠CBA = 180° - 140°

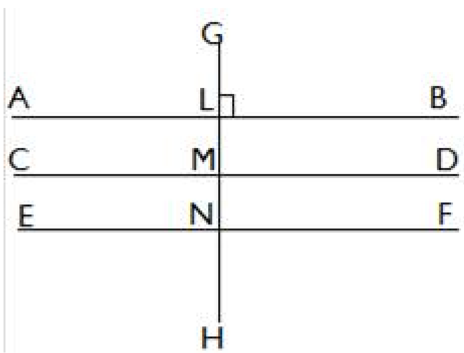
∠CBA = 40°

∠BFG = x = ∠CBA = 40° (alteranate angles)

3. Question

Show that if a straight line is perpendicular to one of the two or more parallel lines, then it is also perpendicular to the remaining lines.

Answer

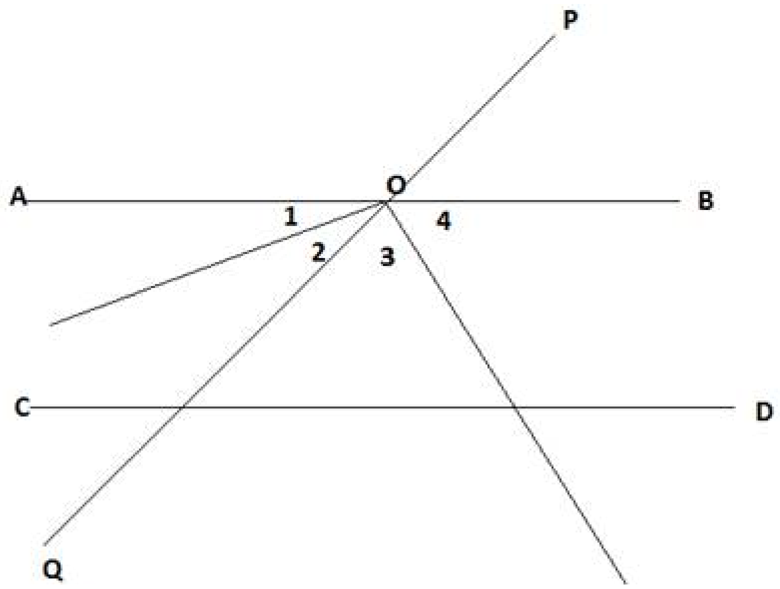


Let GH intersect AB and CD at L and M respectively. Since GH is perpendicular to AB. We have ∠GLA = 90°. GH is also perpendicular to CD. Thus ∠GMC = 90° . we get ∠GLA = ∠GMC = 90°. Thus the lines are parallel. And angles are corresponding angle. Since CD is also parallel with EF. Hence ∠GNE = 90°. Hence it is perpendicular to all other lines in the same way.

4. Question

Let $\overline{AB}$ and $\overline{CD}$ be two parallel lines and $\overline{PQ}$ be a transversal. Show that the angle bisectors of a pair of two internal angles on the same side of the transversal are perpendicular to each other.

Answer



To Prove: ∠ 2 + ∠ 4 = 90°

The situation is shown in the diagram above

AB is parallel to CD and PQ is a traversal

From the diagram we can see that (∠1 + ∠ 2) is an interior angle and similarly ∠3 + ∠ 4 is another interior angle made on the same side

Now ∠1 = ∠ 2 (As they are angular bisector of ∠ AOQ

And similarly, ∠3 = ∠ 4 (As they are bisector of ∠ BOQ)

From the straight line AB we know that angle made on straight line makes 180°

Therefore, ∠ 1 + ∠ 2 + ∠ 3 + ∠ 4 = 180°

2 (∠ 2 + ∠ 4) = 180°

∠ 2 + ∠ 4 = 90°

Hence, Proved.

Additional Problems 3

1 A. Question

If a = 60 and b = a, then b = 60 by ________

A. Axiom 1

B. Axiom 2

C. Axiom 3

D. Axiom 4

Answer

By axiom 1 we get,

Things which are equal to the same thing are equal to one another.

Here, 60 and b both are equal to a, hence b = 60.

1 B. Question

Given a point on the plane, one can draw ______ lines through that point.

A. unique

B. two

C. finite number

D. infinitely many

Answer

One can draw an infinite number of lines through a given point on a plane.

1 C. Question

Given two points in a plane, the number of lines which can be drawn to pass through these two points is ________

A. zero

B. exactly one

C. at most one

D. more than one

Answer

Exactly one line can be drawn passing through two given points in a plane.

1 D. Question

If two angles are supplementary, then their sum is ______

A. 90°

B. 180°

C. 270°

D. 360°

Answer

If two angles are supplementary, then their sum is = 180°.

Let, ∠x and ∠y are two supplementary angles.

∴ ∠x + ∠y = 180°

1 E. Question

The measure of an angle which is 5 times its supplement is ________

A. 30°

B. 60°

C. 120°

D. 150°

Answer

Let, the angle = 5x and the supplement = x

∴5x + x = 180°

⇒ 6x = 180°

⇒ x = 30°

∴ Measure of the angle = 5 × 30°= 150°

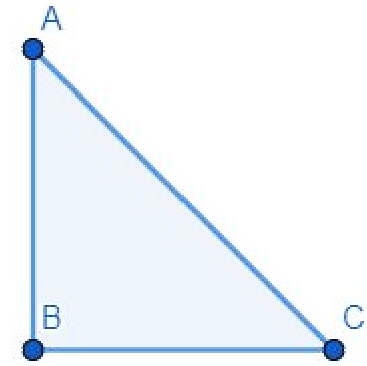
2. Question

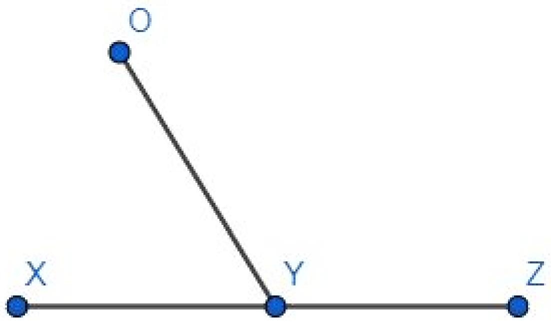
What is the difference between a pair of supplementary angles and a pair of complementary angles?

Answer

A pair of angles is complementary if the sum of their measures adds up to 90°s.

A pair of angles supplementary if the sum of their measures adds up to 180°s.





Δ ABC is a right angle. Triangle (where ∠ABC = 90°)

Here, ∠BAC + ∠ACB =90°

∴ ∠BAC and ∠ACB are complementary angle.

In the 2nd figure,

∠XYO + ∠OYZ = 180°

∴ ∠XYO and ∠OYZ are supplementary angles.

3. Question

What is the least number of non-collinear points required to determine a plane?

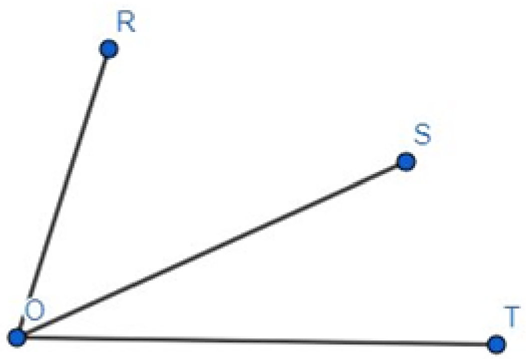
Answer

A plane is determined by 3 non-collinear points.

4. Question

When do you say two angles are adjacent?

Answer



Two angles are adjacent where they have a common side and a common vertex and they do not overlap.

Here, ∠ROS and ∠SOT are adjacent to each other.

They have a common side SO and a common vertex O.

5. Question

Let $\overline{AB}$ be a segment with C and D between them such that the order of points on the segment is A, C, D, B. Suppose AD = BC. Prove that AC = DB.

Answer

According to the problem,

⇒ AD = BC

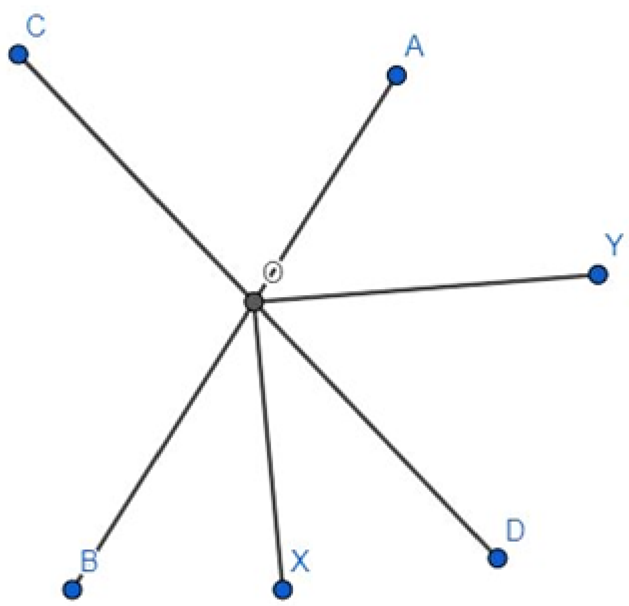
⇒ AB – BD = AB – AC

⇒ AC = BD

6. Question

Let $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ be two straight lines intersecting at O. Let $\overrightarrow{OX}$ be the bisector of ∠BOD. Draw $\overrightarrow{OY}$ between $\overrightarrow{OD}$ and $\overrightarrow{OA}$ such that $\overrightarrow{OY} \perp \overrightarrow{OX}$. Prove that $\overrightarrow{OY}$ bisects ∠DOA.

Answer



Let, ∠BOX= ∠DOX = k°

According the figure,

∠BOD + ∠AOD = 180°

According the problem,

∠XOY =90° ⇒ ∠XOD + ∠YOD = 90° ⇒ ∠YOD = 90° -k°

∴ ∠BOX + ∠AOY = 90°

∠AOY = 90°- k°

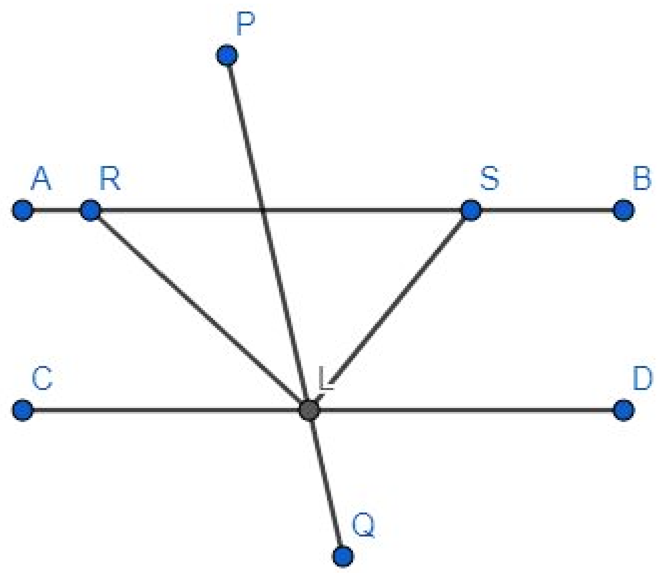
∴ ∠AOY = ∠YOD

OY bisects ∠AOD

7. Question

Let $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ be two parallel lines and $\overleftrightarrow{PQ}$ be a transversal. Let $\overleftrightarrow{PQ}$ intersect $\overleftrightarrow{AB}$ in L. Suppose the bisector of ∠ALP intersect $\overleftrightarrow{CD}$ in R and the bisector of ∠PLB intersect $\overleftrightarrow{CD}$ in S. Prove that ∠LRS + ∠RSL = 90°.

Answer



We know, ∠ALP + ∠BLP = 180°

According to problem,

∠ALR = ∠RLP = ∠ALD/2

Similarly,

∠BLS = ∠SLP = ∠BLP/2

∴ ∠RLP + ∠SLP = ∠ALD/2 + ∠BLD/2

⇒ ∠RLS = (∠ALP + ∠BLP)/2 = 180° = 90°

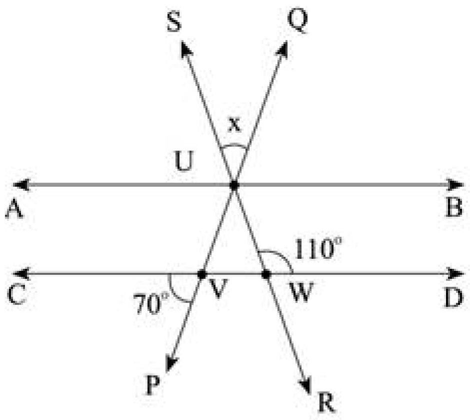
∴ ∠LRS + ∠RSL = 180° - ∠RLS

∠LRS + ∠RSL =180° - 90°

=90°

8. Question

In the adjoining figure, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel lines. The transversals $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ intersect at U on the line $\overleftrightarrow{AB}$. Given that ∠DWU = 110° and ∠CVP = 70°, find the measure of ∠QUS.



Answer

We know, ∠CVP = 70°,

∴∠UVW = 70° [∵ Vertically opposite angles]

Again,

⇒ ∠UWV + ∠DWU = 180°

⇒ ∠UWV = 180° - 110°

⇒ ∠UWV = 70°

In ΔUVW,

⇒ ∠UVW + ∠UWV + ∠VUW = 180°

⇒ 70° + 70° + ∠VUW = 180°

⇒ ∠VUW = 180° - 140° = 40°

∴ ∠QUS = ∠VUW = 40° [Vertically opposite angle]

9. Question

What is the angle between the hour's hand and minute's hand of a clock at (i) 1.40 hours, (ii) 2.15 hours? (Use 1° = 60 minutes.)

Answer

Hour's hand rotates 360° in = 12 hour = 720 min

Minute's hand rotates 360° in = 1 hour = 60 min

i) At, 1.40hrs

Time past after 0.00 hrs = 1 hr + 40 min = 100 min

In 100 min hour's hand rotate = $\frac{100\times360^\circ}{720} = 50^\circ$

In 100 minminute's hand rotate = $\frac{100\times360^\circ}{60}$

∴ Difference between the hands,

= 240° - 50°

= 190°

ii) At 2.15 hrs,

Time past after 0.00 hrs = 2 hr + 15 min = 135 min

In 135 min hour's hand rotate = $\frac{135\times360^\circ}{720} = 67.5^\circ$

In 135 min minute's hand rotate = $\frac{135\times360^\circ}{60} = 810^\circ = 810^\circ - 720^\circ = 90^\circ$

∴ Difference between the hands,

= 90° - 67.5°

= 22.5°

10. Question

How much would hour's hand have moved from its position at 12 noon when the time is 4.24 p.m.?

Answer

At 4.24 hrs,

Time past after 12.00 hrs = 4 hr + 24 min = 264 min

In 264 min hour's hand rotate = $\frac{264\times360^\circ}{720} = 132^\circ$

In 264 min minute's hand rotate = $\frac{264\times360^\circ}{60} = 1584^\circ = 1584^\circ - 1440^\circ = 144^\circ$

∴ Difference between the hands,

= 144° - 132°

= 12°

11. Question

Let $\overline{AB}$ be a line segment and let C be the midpoint of $\overline{AB}$. Extend $\overline{AB}$ to D such that B lies between A and D. Prove that AD + BD = 2CD.

Answer

AD + BD

⇒ (AB + BD) + BD

⇒ AB + 2BD

⇒ 2BC + 2BD

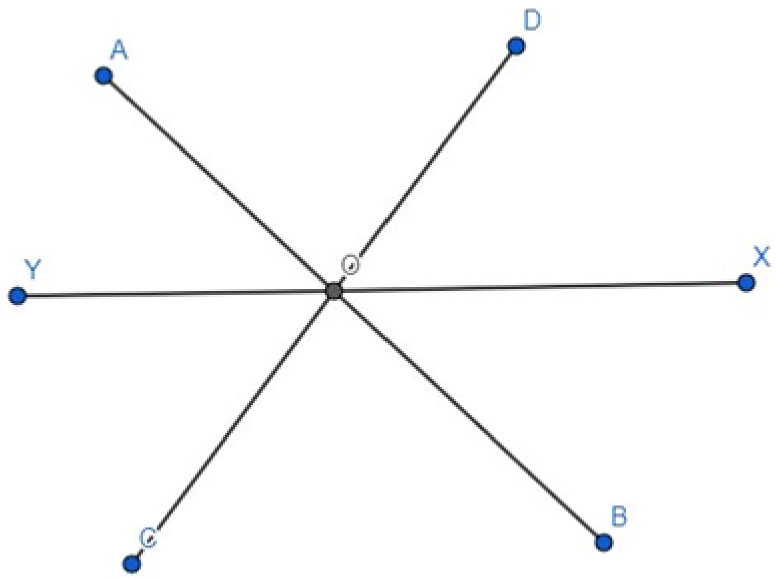
⇒ 2(BC + BD)

⇒ 2CD

12. Question

Let $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ be two lines intersecting at a point O. Let $\overrightarrow{OX}$ be a ray bisecting ∠BOD. Prove that the extension of $\overrightarrow{OX}$ to the left of O bisects ∠AOC.

Answer



OX is extended to OY

OX is bisector of ∠BOD

∴ ∠DOX = ∠BOX

From figure,

∠DOX =∠COY

And ∠BOX = ∠AOY

∴ ∠COY = ∠AOY

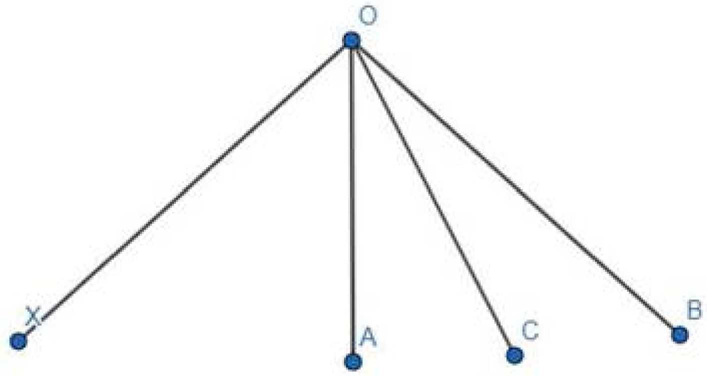
∴OY intersect ∠AOC

13. Question

Let $\overrightarrow{OX}$ be a ray and let $\overrightarrow{OA}$ and $\overrightarrow{OB}$ be two rays on the same side of $\overrightarrow{OX}$, with $\overrightarrow{OA}$ between $\overleftrightarrow{OX}$ and $\overrightarrow{OB}$. Let $\overrightarrow{OC}$ be the bisector of ∠AOB. Prove that

∠XOA + ∠XOB = 2∠XOC

Answer



⇒ ∠XOA + ∠XOB

⇒ ∠XOA (∠XOA + ∠AOB)

⇒ 2∠XOA + 2∠AOC

⇒ 2(∠XOA + ∠AOC)

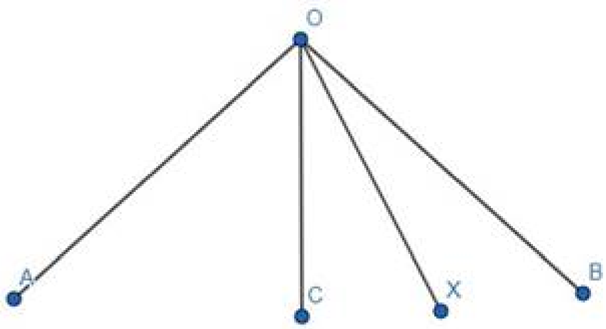
⇒ 2∠XOC

14. Question

Let $\overrightarrow{OA}$ and $\overrightarrow{OB}$ be two rays and let $\overrightarrow{OX}$ be a ray between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ such that ∠AOX >∠XOB. Let OC be the bisector of ∠AOB.

∠AOX – ∠XOB = 2∠COX

Answer



⇒ ∠AOX - ∠XOB

⇒ (∠AOC + ∠COX) – (∠BOC - ∠COX)

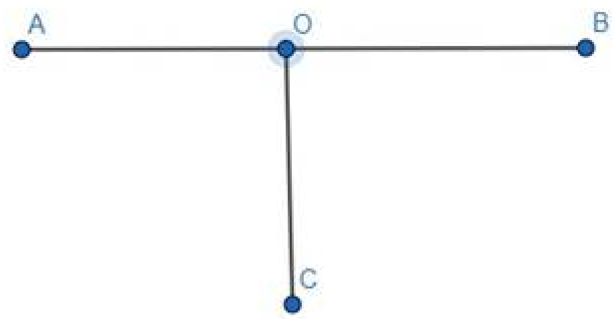
⇒ ∠AOC - ∠BOC + 2∠COX

⇒ 2∠COX (∵ ∠AOC = ∠BOC)

15. Question

Let $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ be three rays such that $\overrightarrow{OC}$ lies between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Suppose the bisectors of ∠AOC and ∠COB are perpendicular to each other. Prove that B, O, A are collinear.

Answer



∠AOC and ∠COB are perpendicular to each other.

∴ ∠AOC = 90° and ∠BOC = 90°

∴ ∠AOB = ∠AOC + ∠BOC = 90° + 90° = 180°

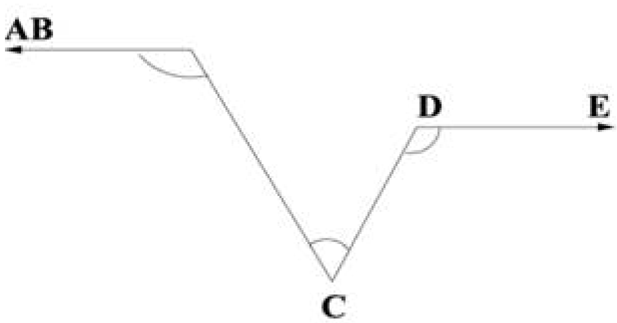
∴ O ,A and B are Collinear.

16. Question

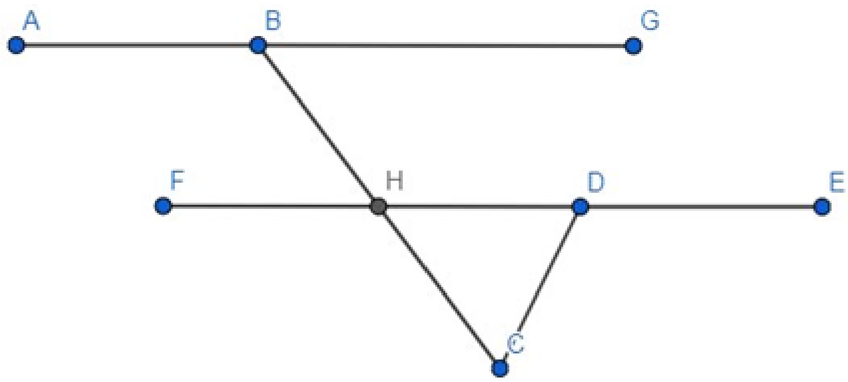
In the adjoining figure, $\overleftrightarrow{AB} \parallel \overleftrightarrow{DE}$.

Prove that

∠ABC – ∠DCB + ∠CDE = 180°.



Answer



⇒ ∠ABC =180° - ∠GBC =180° - ∠DHC …… (1)

And

⇒ ∠CDE = 180°- ∠HDC …… (2)

From (1) + (2) we get,

⇒ ∠ABC + ∠CDE = 180° - ∠DHC + 180°- ∠HDC

⇒ ∠ABC + ∠CDE = 360° - (∠DHC + ∠HDC)

⇒ ∠ABC + ∠CDE = 360° - (180° - ∠HCD) [from ΔDHC]

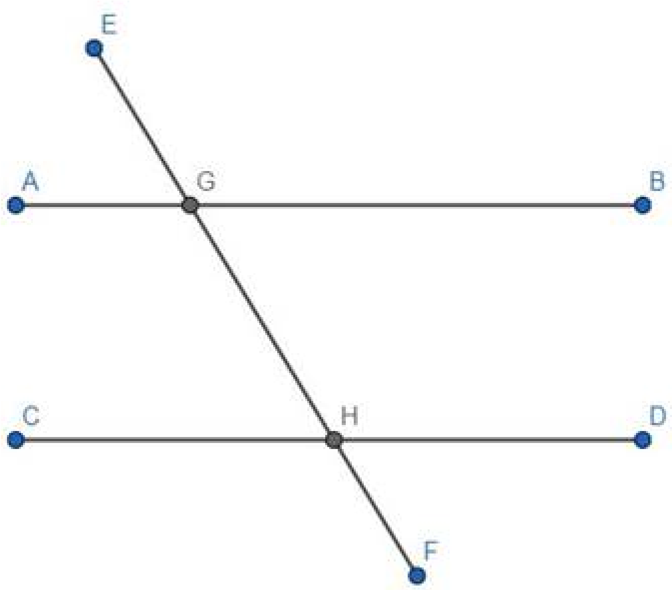
⇒ ∠ABC + ∠CDE = 180° + ∠DCB

⇒ ∠ABC – ∠DCB + ∠CDE = 180°

17. Question

Consider two parallel lines and a transversal. Among the measures of 8 angles formed, how many distinct numbers are there?

Answer



AB and CD are two parallel lines.

EF is the transversal, which cuts AB at G and CD at H.

Here, ∠AGE = ∠BGH [Vertically opposite angle]

∠AGE = ∠GHC [Corresponding angle]

∠GHC = ∠FHD [Vertically opposite angle]

∴ ∠AGE = ∠BGH = ∠GHC = ∠FHD

Similarly, ∠BGE = ∠AGH [Vertically opposite angle]

∠BGE = ∠GHD [Corresponding angle]

∠GHD = ∠FHC [Vertically opposite angle]

∴ ∠BGE = ∠AGH = ∠GHD = ∠FHC

So, we have only 2 distinct values.