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Complex Numbers and Quadratic Equations

TOPIC 1 Integral Powers of Iota, Algebraic Operations of Complex Numbers, Conjugate, Modulus and Argument or Amplitude of a Complex Number

1. If $\frac{3+i\sin\theta}{4-i\cos\theta}, \theta \in [0, 2\pi]$, is *a* real number, then an argument of $\sin\theta + i\cos\theta$ is: **[Jan. 7, 2020 (II)]**

(a) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$ (b) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$

(c) $-\tan^{-1}\left(\frac{3}{4}\right)$ (d) $\tan^{-1}\left(\frac{4}{3}\right)$

2. If the four complex numbers $z, \bar{z}, \bar{z} - 2\text{Re}(\bar{z})$ and $z - 2\text{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then $|z|$ is equal to : **[Sep. 05, 2020 (I)]**

(a) $4\sqrt{2}$ (b) 4 (c) $2\sqrt{2}$ (d) 2

3. The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is : **[Sep. 05, 2020 (II)]**

(a) -2^{15} (b) $2^{15}i$ (c) $-2^{15}i$ (d) 6^5

4. If $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1, (m, n \in \mathbf{N})$, then the greatest common divisor of the least values of m and n is _________. **[Sep. 03, 2020 (I)]**

5. If z_1, z_2 are complex numbers such that $\text{Re}(z_1) = |z_1 - 1|$, $\text{Re}(z_2) = |z_2 - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $\text{Im}(z_1 + z_2)$ is equal to : **[Sep. 03, 2020 (II)]**

(a) $\frac{2}{\sqrt{3}}$ (b) $2\sqrt{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

6. Let z be a complex number such that $\left|\frac{z-i}{z+2i}\right| = 1$ and $|z| = \frac{5}{2}$. Then the value of $|z + 3i|$ is : **[Jan. 9, 2020 (I)]**

(a) $\sqrt{10}$ (b) $\frac{7}{2}$ (c) $\frac{15}{4}$ (d) $2\sqrt{3}$

7. If z be a complex number satisfying $|\text{Re}(z)| + |\text{Im}(z)| = 4$, then $|z|$ cannot be: **[Jan. 9, 2020 (II)]**

(a) $\sqrt{\frac{17}{2}}$ (b) $\sqrt{10}$ (c) $\sqrt{7}$ (d) $\sqrt{8}$

8. Let $z \in C$ with $\text{Im}(z) = 10$ and it satisfies $\frac{2z-n}{2z+n} = 2i - 1$ for some natural number n. Then : **[April 12, 2019 (II)]**

(a) $n = 20$ and $\text{Re}(z) = -10$
(b) $n = 40$ and $\text{Re}(z) = 10$
(c) $n = 40$ and $\text{Re}(z) = -10$
(d) $n = 20$ and $\text{Re}(z) = 10$

9. The equation $|z - i| = |z - 1|, i = \sqrt{-1}$, represents: **[April 12, 2019 (I)]**

(a) a circle of radius $\frac{1}{2}$.
(b) the line through the origin with slope 1.
(c) a circle of radius 1.
(d) the line through the origin with slope – 1.

10. If $a > 0$ and $z = \frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to : **[April 10, 2019 (I)]**

(a) $-\frac{1}{5} - \frac{3}{5}i$ (b) $-\frac{3}{5} - \frac{1}{5}i$

(c) $\frac{1}{5} - \frac{3}{5}i$ (d) $-\frac{1}{5} + \frac{3}{5}i$

11. If z and ω are two complex numbers such that $|z\omega|=1$ and $\arg(z)-\arg(\omega)=\frac{\pi}{2}$, then: **[April 10, 2019 (II)]**

(a) $\bar{z}\omega=i$ (b) $z\bar{\omega}=\frac{-1+i}{\sqrt{2}}$

(c) $\bar{z}\omega=-i$ (d) $z\bar{\omega}=\frac{1-i}{\sqrt{2}}$

12. Let $z \in C$ be such that $|z|<1$. If $\omega=\frac{5+3z}{5(1-z)}$, then : **[April 09, 2019 (II)]**

(a) $5\,\text{Re}(\omega)>4$ (b) $4\,\text{Im}(\omega)>5$

(c) $5\,\text{Re}(\omega)>1$ (d) $5\,\text{Im}(\omega)<1$

13. If $\frac{z-\alpha}{z+\alpha}(\alpha \in R)$ is a purely imaginary number and $|z|=2$, then a value of α is : **[Jan. 12, 2019 (I)]**

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\sqrt{2}$

14. Let z_1 and z_2 be two complex numbers satisfying $|z_1|=9$ and $|z_2|-|3|-|4i||=|4$. Then the minimum value of $|z_1-z_2|$ is : **[Jan. 12, 2019 (II)]**

(a) 0 (b) $\sqrt{2}$ (c) 1 (d) 2

15. Let z be a complex number such that $|z|+z=3+i$ $\left(\text{where } i=\sqrt{-1}\right)$.

Then $|z|$ is equal to : **[Jan. 11, 2019 (II)]**

(a) $\frac{\sqrt{34}}{3}$ (b) $\frac{5}{3}$ (c) $\frac{\sqrt{41}}{4}$ (d) $\frac{5}{4}$

16. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1|=4|z_2|$. If $z=\frac{3z_1}{2z_2}+\frac{2z_2}{3z_1}$ then: **[Jan. 10 2019 (II)]**

(a) $\text{Re}(z)=0$ (b) $|z|=\sqrt{\frac{5}{2}}$

(c) $|z|=\frac{1}{2}\sqrt{\frac{17}{2}}$ (d) $\text{Im}(z)=0$

17. Let $A=\left\{\theta \in \left(-\frac{\pi}{2},\pi\right): \frac{3+2i\sin\theta}{1-2i\sin\theta} \text{ is purely imaginary}\right\}$.

Then the sum of the elements in A is: **[Jan. 9 2019 (I)]**

(a) $\frac{5\pi}{6}$ (b) π (c) $\frac{3\pi}{4}$ (d) $\frac{2\pi}{3}$

18. The set of all $\alpha \in R$, for which $w=\frac{1+(1-8\alpha)z}{1-z}$ is a purely imaginary number, for all $z \in C$ satisfying $|z|=1$ and $Re\, z \neq 1$, is **[Online April 15, 2018]**

(a) {0} (b) an empty set

(c) $\left\{0, \frac{1}{4}, -\frac{1}{4}\right\}$ (d) equal to R

19. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is: **[2016]**

(a) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

20. If z is a non-real complex number, then the minimum value of $\frac{lmz^5}{(lmz)^5}$ is : **[Online April 11, 2015]**

(a) –1 (b) –4 (c) –2 (d) –5

21. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z+\frac{1}{2}\right|$: **[2014]**

(a) is strictly greater than $\frac{5}{2}$

(b) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

(c) is equal to $\frac{5}{2}$

(d) lie in the interval (1, 2)

22. For all complex numbers z of the form $1+i\alpha$, $\alpha \in R$, if $z^2=x+iy$, then **[Online April 19, 2014]**

(a) $y^2-4x+2=0$ (b) $y^2+4x-4=0$

(c) $y^2-4x-4=0$ (d) $y^2+4x+2=0$

23. Let $z \neq -i$ be any complex number such that $\frac{z-i}{z+i}$ is a purely imaginary number.

Then $z+\frac{1}{z}$ is: **[Online April 12, 2014]**

(a) zero

(b) any non-zero real number other than 1.

(c) any non-zero real number.

(d) a purely imaginary number.

24. If z_1, z_2 and z_3, z_4 are 2 pairs of complex conjugate numbers, then

$\arg\left(\frac{z_1}{z_4}\right)+\arg\left(\frac{z_2}{z_3}\right)$ equals: **[Online April 11, 2014]**

(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) π

25. Let w (Im w $\neq$ 0) be a complex number. Then the set of all complex number z satisfying the equation

$w-\overline{w}z=k(1-z)$, for some real number k, is **[Online April 9, 2014]**

(a) $\{z:|z|=1\}$ (b) $\{z:z=\overline{z}\}$

(c) $\{z:z\neq 1\}$ (d) $\{z:|z|=1, z\neq 1\}$

26. If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1+z}{1+\overline{z}}\right)$ equals: **[2013]**

(a) $-\theta$ (b) $\frac{\pi}{2}-\theta$ (c) θ (d) $\pi-\theta$

27. Let z satisfy $|z|=1$ and $z=1-\overline{z}$.
Statement 1 : z is a real number.

Statement 2 : Principal argument of z is $\frac{\pi}{3}$

[Online April 25, 2013]

(a) Statement 1 is true Statement 2 is true; Statement 2 is a correct explanation for Statement 1.
(b) Statement 1 is false; Statement 2 is true
(c) Statement 1 is true, Statement 2 is false.
(d) Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

28. Let $a=\text{Im}\left(\frac{1+z^2}{2iz}\right)$, where z is any non-zero complex number. **[Online April 23, 2013]**

The set $A=\{a:|z|=1 \text{ and } z\neq\pm 1\}$ is equal to:

(a) $(-1, 1)$ (b) $[-1, 1]$ (c) $[0, 1)$ (d) $(-1, 0]$

29. If $Z_1 \neq 0$ and Z_2 be two complex numbers such that $\frac{Z_2}{Z_1}$ is a purely imaginary number, then $\left|\frac{2Z_1+3Z_2}{2Z_1-3Z_2}\right|$ is equal to:

[Online April 9, 2013]

(a) 2 (b) 5 (c) 3 (d) 1

30. $|z_1+z_2|^2+|z_1-z_2|^2$ is equal to **[Online May 26, 2012]**

(a) $2(|z_1|+|z_2|)$ (b) $2(|z_1|^2+|z_2|^2)$

(c) $|z_1||z_2|$ (d) $|z_1|^2+|z_2|^2$

31. Let Z and W be complex numbers such that $|Z|=|W|$, and arg Z denotes the principal argument of Z.

[Online May 19, 2012]

Statement 1: If arg Z + arg $W = \pi$, then $Z=-\overline{W}$.

Statement 2: $|Z|=|W|$, implies arg $Z-$ arg $\overline{W}=\pi$.

(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.
(d) Statement 1 is false, Statement 2 is true.

32. Let Z_1 and Z_2 be any two complex number.

Statement 1: $|Z_1-Z_2|\geq|Z_1|-|Z_2|$

Statement 2: $|Z_1+Z_2|\leq|Z_1|+|Z_2|$ **[Online May 7, 2012]**

(a) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation of Statement 1.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.
(c) Statement 1 is true, Statement 2 is false.
(d) Statement 1 is false, Statement 2 is true.

33. The number of complex numbers z such that $|z-1|=|z+1|=|z-i|$ equals **[2010]**

(a) 1 (b) 2 (c) ∞ (d) 0

34. The conjugate of a complex number is $\frac{1}{i-1}$ then that complex number is **[2008]**

(a) $\frac{-1}{i-1}$ (b) $\frac{1}{i+1}$ (c) $\frac{-1}{i+1}$ (d) $\frac{1}{i-1}$

35. If $z=x-iy$ and $z^{\frac{1}{3}}=p+iq$, then $\left(\frac{x}{p}+\frac{y}{q}\right)\Big/(p^2+q^2)$ is equal to **[2004]**

(a) –2 (b) –1 (c) 2 (d) 1

36. Let z and w be complex numbers such that $\overline{z}+i\overline{w}=0$ and arg $zw=\pi$. Then arg z equals **[2004]**

(a) $\frac{5\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{4}$

37. If $\left(\frac{1+i}{1-i}\right)^x=1$ then **[2003]**

(a) $x=2n+1$, where n is any positive integer
(b) $x=4n$, where n is any positive integer
(c) $x=2n$, where n is any positive integer
(d) $x=4n+1$, where n is any positive integer.

38. If z and ω are two non-zero complex numbers such that $|z\omega|=1$ and $Arg(z)-Arg(\omega)=\frac{\pi}{2}$, then $\bar{z}\omega$ is equal to **[2003]**

(a) -1 (b) 1 (c) $-i$ (d) i

39. If $|z-4|<|z-2|$, its solution is given by **[2002]**

(a) $Re(z)>0$ (b) $Re(z)<0$

(c) $Re(z)>3$ (d) $Re(z)>2$

40. z and w are two non zero complex numbers such that $|z|=|w|$ and $\text{Arg } z+\text{Arg } w=\pi$ then z equals **[2002]**

(a) $\overline{\omega}$ (b) $-\overline{\omega}$ (c) ω (d) $-\omega$

TOPIC 2 Rotational Theorem, Square Root of a Complex Number, Cube Roots of Unity, Geometry of Complex Numbers, De-moiver's Theorem, Powers of Complex Numbers

41. Let $z = x + iy$ be a non-zero complex number such that $z^2 = i|z|^2$, where $i=\sqrt{-1}$, then z lies on the: **[Sep. 06, 2020 (II)]**

(a) line, $y=-x$ (b) imaginary axis

(c) line, $y=x$ (d) real axis

42. If a and b are real numbers such that $(2+\alpha)^4 = a+b\alpha$, where $\alpha=\frac{-1+i\sqrt{3}}{2}$, then $a + b$ is equal to : **[Sep. 04, 2020 (II)]**

(a) 9 (b) 24 (c) 33 (d) 57

43. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is : **[Sep. 02, 2020 (I)]**

(a) $\frac{1}{2}(1-i\sqrt{3})$ (b) $\frac{1}{2}(\sqrt{3}-i)$

(c) $-\frac{1}{2}(\sqrt{3}-i)$ (d) $-\frac{1}{2}(1-i\sqrt{3})$

44. The imaginary part of $(3+2\sqrt{-54})^{1/2}-(3-2\sqrt{-54})^{1/2}$ can be : **[Sep. 02, 2020 (II)]**

(a) $-\sqrt{6}$ (b) $-2\sqrt{6}$ (c) 6 (d) $\sqrt{6}$

45. Let $\alpha=\frac{-1+i\sqrt{3}}{2}$. If $a=(1+\alpha)\sum_{k=0}^{100}\alpha^{2k}$ and $b=\sum_{k=0}^{100}\alpha^{3k}$, then a and b are the roots of the quadratic equation: **[Jan. 8, 2020 (II)]**

(a) $x^2+101x+100=0$ (b) $x^2-102x+101=0$

(c) $x^2-101x+100=0$ (d) $x^2+102x+101=0$

46. If $\text{Re}\left(\frac{z-1}{2z+i}\right)=1$, where $z=x+iy$, then the point (x, y) lies on a: **[Jan. 7, 2020 (I)]**

(a) circle whose centre is at $\left(-\frac{1}{2},-\frac{3}{2}\right)$.

(b) straight line whose slope is $-\frac{2}{3}$.

(c) straight line whose slope is $\frac{3}{2}$.

(d) circle whose diameter is $\frac{\sqrt{5}}{2}$.

47. If $z=\frac{\sqrt{3}}{2}+\frac{i}{2}(i=\sqrt{-1})$, then $(1+iz+z^5+iz^8)^9$ is equal to: **[April 08, 2019 (II)]**

(a) 0 (b) 1

(c) $(-1+2i)^9$ (d) -1

48. Let $\left(-2-\frac{1}{3}i\right)^3=\frac{x+iy}{27}\left(i=\sqrt{-1}\right)$, where x and y are real numbers then y – x equals : **[Jan. 11, 2019 (I)]**

(a) 91 (b) -85 (c) 85 (d) -91

49. Let $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$. If R($z$) and I($z$) respectively denote the real and imaginary parts of z, then: **[Jan. 10, 2019 (II)]**

(a) $I(z)=0$ (b) $R(z)>0$ and $I(z)>0$

(c) $R(z)<0$ and $I(z)>0$ (d) R(z) = – (c)

50. The least positive integer n for which $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n=1$, is **[Online April 16, 2018]**

(a) 2 (b) 6 (c) 5 (d) 3

51. The point represented by 2 + i in the Argand plane moves 1 unit eastwards, then 2 units northwards and finally from there $2\sqrt{2}$ units in the south–westwards direction. Then its new position in the Argand plane is at the point represented by : **[Online April 9, 2016]**

(a) 1 + i (b) 2 + 2i (c) –2 – 2i (d) –1 – i

52. A complex number z is said to be unimodular if |z| = 1. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1-2z_2}{2-z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a: **[2015]**

(a) circle of radius 2.
(b) circle of radius $\sqrt{2}$.
(c) straight line parallel to x-axis
(d) straight line parallel to y-axis.

53. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies : **[2012]**
(a) either on the real axis or on a circle passing through the origin.
(b) on a circle with centre at the origin
(c) either on the real axis or on a circle not passing through the origin.
(d) on the imaginary axis.

54. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals **[2011]**
(a) (1, 1) (b) (1, 0) (c) (–1, 1) (d) (0, 1)

55. If $|z + 4| \leq 3$, then the maximum value of $|z+1|$ is **[2007]**
(a) 6 (b) 0 (c) 4 (d) 10

56. If $\omega = \frac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, then z lies on **[2005]**
(a) an ellipse (b) a circle
(c) a straight line (d) a parabola

57. If z_1 and z_2 are two non- zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then arg z_1 – arg z_2 is equal to **[2005]**
(a) $\frac{\pi}{2}$ (b) $-\pi$ (c) 0 (d) $\frac{-\pi}{2}$

58. If the cube roots of unity are **1**, ω, ω^2 then the roots of the equation $(x-1)^3 + 8 = 0$, are **[2005]**
(a) $-1, -1+2\omega, -1-2\omega^2$
(b) $-1, -1, -1$
(c) $-1, 1-2\omega, 1-2\omega^2$
(d) $-1, 1+2\omega, 1+2\omega^2$

59. If $|z^2 - 1| = |z|^2 + 1$, then z lies on **[2004]**
(a) an ellipse (b) the imaginary axis
(c) a circle (d) the real axis

60. The locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 & z_2 are complex numbers) will be **[2002]**
(a) an ellipse (b) a hyperbola
(c) a circle (d) none of these

TOPIC 3 Solutions of Quadratic Equations, Sum and Product of Roots, Nature of Roots, Relation Between Roots and Co-efficients, Formation of an Equation with Given Roots.

61. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is: **[Sep. 06, 2020 (I)]**
(a) 2 (b) 3 (c) 1 (d) 4

62. If α and β are the roots of the equation $2x(2x+1) = 1$, then β is equal to: **[Sep. 06, 2020 (II)]**
(a) $2\alpha(\alpha+1)$ (b) $-2\alpha(\alpha+1)$
(c) $2\alpha(\alpha-1)$ (d) $2\alpha^2$

63. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is : **[Sep. 05, 2020 (I)]**
(a) $\frac{5}{9}$ (b) $\frac{25}{81}$ (c) $\frac{5}{27}$ (d) $\frac{25}{9}$

64. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, the the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to : **[Sep. 05, 2020 (II)]**
(a) $\frac{27}{32}$ (b) $\frac{1}{24}$ (c) $\frac{3}{8}$ (d) $\frac{27}{16}$

65. Let $u = \frac{2z+i}{z-ki}$, $z = x+iy$ and $k > 0$. If the curve represented by Re(u) + Im(u) = 1 intersects the y-axis at the points P and Q where $PQ = 5$, then the value of k is : **[Sep. 04, 2020 (I)]**
(a) 3/2 (b) 1/2 (c) 4 (d) 2

66. Let $\lambda \neq 0$ be in **R**. If α and β are roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to : **[Sep. 04, 2020 (II)]**
(a) 27 (b) 18 (c) 9 (d) 36

67. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to : **[Sep. 03, 2020 (I)]**

(a) $\frac{9}{4}(9+q^2)$ (b) $\frac{9}{4}(9-q^2)$

(c) $\frac{9}{4}(9+p^2)$ (d) $\frac{9}{4}(9-p^2)$

68. The set of all real values of λ for which the quadratic equations, $(\lambda^2+1)x^2-4\lambda x+2=0$ always have exactly one root in the interval (0, 1) is : **[Sep. 03, 2020 (II)]**
(a) (0,2) (b) (2,4] (c) (1,3] (d) (–3,–1)

69. Let α and β be the roots of the equation, $5x^2+6x-2=0$. If $S_n=\alpha^n+\beta^n$, $n=1,2,3,...,$ then : **[Sep. 02, 2020 (I)]**
(a) $6S_6+5S_5=2S_4$ (b) $6S_6+5S_5+2S_4=0$
(c) $5S_6+6S_5=2S_4$ (d) $5S_6+6S_5+2S_4=0$

70. The number of real roots of the equation, $e^{4x}+e^{3x}-4e^{2x}+e^x+1=0$ is: **[Jan. 9, 2020 (I)]**
(a) 1 (b) 3 (c) 2 (d) 4

71. The least positive value of 'a' for which the equation, $2x^2+(a-10)x+\frac{33}{2}=2a$ has real roots is ________. **[Jan. 8, 2020 (I)]**

72. If the equation, $x^2+bx+45=0$ $(b\in R)$ has conjugate complex roots and they satisfy $|z+1|=2\sqrt{10}$, then: **[Jan. 8, 2020 (I)]**
(a) $b^2-b=30$ (b) $b^2+b=72$
(c) $b^2-b=42$ (d) $b^2+b=12$

73. Let α and β be the roots of the equation $x^2-x-1=0$. If $p_k=(\alpha)^k+(\beta)^k$, $k\ge 1$, then which one of the following statements is not true ? **[Jan. 7, 2020 (II)]**
(a) $p_3=p_5-p_4$
(b) $P_5=11$
(c) $(p_1+p_2+p_3+p_4+p_5)=26$
(d) $p_5=p_2\cdot p_3$

74. Let α and β be two real roots of the equation $(k+1)\tan^2x-\sqrt{2}.\lambda\tan x=(1-k)$, where $k(\ne-1)$ and λ are real numbers. If $\tan^2(\alpha+\beta)=50$, then a value of λ is: **[Jan. 7, 2020 (I)]**
(a) $10\sqrt{2}$ (b) 10 (c) 5 (d) $5\sqrt{2}$

75. If α and β are the roots of the quadratic equation, $x^2+x\sin\theta-2\sin\theta=0$ $0,\theta\in\left(0,\frac{\pi}{2}\right)$, then $\frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})(\alpha-\beta)^{24}}$ is equal to : **[April 10, 2019 (I)]**
(a) $\frac{2^{12}}{(\sin\theta-4)^{12}}$ (b) $\frac{2^{12}}{(\sin\theta+8)^{12}}$
(c) $\frac{2^{12}}{(\sin\theta-8)^{6}}$ (d) $\frac{2^{6}}{(\sin\theta+8)^{12}}$

76. The number of real roots of the equation $5+|2^x-1|=2^x(2^x-2)$ is: **[April 10, 2019 (II)]**
(a) 3 (b) 2 (c) 4 (d) 1

77. Let p, q $\in$ R. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2+px+q=0$, then: **[April 9, 2019 (I)]**
(a) $p^2-4q+12=0$ (b) $q^2-4p-16=0$
(c) $q^2+4p+14=0$ (d) $p^2-4q-12=0$

78. If m is chosen in the quadratic equation $(m^2+1)x^2-3x+(m^2+1)^2=0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is: **[April 09, 2019 (II)]**
(a) $10\sqrt{5}$ (b) $8\sqrt{3}$ (c) $8\sqrt{5}$ (d) $4\sqrt{3}$

79. The sum of the solutions of the equation $|\sqrt{x}-2|+\sqrt{x}(\sqrt{x}-4)+2=0,(x>0)$ is equal to: **[April 8, 2019 (I)]**
(a) 9 (b) 12 (c) 4 (d) 10

80. If α and β be the roots of the equation $x^2-2x+2=0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n=1$ is : **[April 8, 2019 (I)]**
(a) 2 (b) 5 (c) 4 (d) 3

81. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2+m(m-4)x+2=0$, then the least value of m for which $\lambda+\frac{1}{\lambda}=1$, is : **[Jan. 12, 2019 (I)]**
(a) $2-\sqrt{3}$ (b) $4-3\sqrt{2}$
(c) $-2+\sqrt{2}$ (d) $4-2\sqrt{3}$

82. If one real root of the quadratic equation $81x^2+kx+256=0$ is cube of the other root, then a value of k is : **[Jan. 11, 2019 (I)]**
(a) –81 (b) 100 (c) 144 (d) –300

83. Consider the quadratic equation $(c-5)x^2-2cx+(c-4)=0$, $c\ne5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0, 2) and its other root lies in the interval (2, 3). Then the number of elements in S is: **[Jan. 10, 2019 (I)]**
(a) 18 (b) 12 (c) 10 (d) 11

84. The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2+(3-\lambda)x+2=\lambda$ has the least value is: **[Jan. 10, 2019 (II)]**
(a) $\frac{15}{8}$ (b) 1 (c) $\frac{4}{9}$ (d) 2

85. Let α and β be two roots of the equation $x^2+2x+2=0$, then $\alpha^{15}+\beta^{15}$ is equal to: **[Jan. 9, 2019 (I)]**
(a) –256 (b) 512 (c) –512 (d) 256

86. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2-11x+\alpha=0$ are rational numbers is: **[Jan. 09, 2019 (II)]**
(a) 3 (b) 2 (c) 4 (d) 5

87. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1, 5], then m lies in the interval: **[Jan. 09, 2019 (II)]**
(a) $(-5, -4)$ (b) $(4, 5)$
(c) $(5, 6)$ (d) $(3, 4)$

88. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6i\,z_0^{81} - 3i\,z_0^{93}$, then arg z is equal to: **[Jan. 09, 2019 (II)]**
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) 0

89. Let p, q and r be real numbers ($p \neq q$, $r \neq 0$), such that the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to. **[Online April 16, 2018]**
(a) $p^2 + q^2 + r^2$ (b) $p^2 + q^2$
(c) $2(p^2 + q^2)$ (d) $\frac{p^2 + q^2}{2}$

90. If an angle A of a ΔABC satisfies $5\cos A + 3 = 0$, then the roots of the quadratic equation, $9x^2 + 27x + 20 = 0$ are. **[Online April 16, 2018]**
(a) $\sin A, \sec A$ (b) $\sec A, \tan A$
(c) $\tan A, \cos A$ (d) $\sec A, \cot A$

91. If $\tan A$ and $\tan B$ are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$ then the value of $3\sin^2(A+B) - 10\sin(A+B).\cos(A+B) - 25\cos^2(A+B)$ is **[Online April 15, 2018]**
(a) 25 (b) -25 (c) -10 (d) 10

92. If $f(x)$ is a quadratic expression such that $f(a) + f(b) = 0$, and -1 is a root of $f(x) = 0$, then the other root of $f(x) = 0$ is **[Online April 15, 2018]**
(a) $-\frac{5}{8}$ (b) $-\frac{8}{5}$ (c) $\frac{5}{8}$ (d) $\frac{8}{5}$

93. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to: **[2018]**
(a) 0 (b) 1 (c) 2 (d) -1

94. If, for a positive integer n, the quadratic equation,
$x(x+1) + (x+1)(x+2) + ... + (x+\overline{n-1})(x+n) = 10n$
has two consecutive integral solutions, then n is equal to: **[2017]**
(a) 11 (b) 12 (c) 9 (d) 10

95. The sum of all the real values of x satisfying the equation $2^{(x-1)(x^2+5x-50)} = 1$ is: **[Online April 9, 2017]**
(a) 16 (b) 14 (c) -4 (d) -5

96. Let p(x) be a quadratic polynomial such that p(0)=1. If p(x) leaves remainder 4 when divided by x–1 and it leaves remainder 6 when divided by x+1; then : **[Online April 8, 2017]**
(a) $p(b) = 11$ (b) $p(b) = 19$
(c) $p(-2) = 19$ (d) $p(-2) = 11$

97. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$ is : **[2016]**
(a) 6 (b) 5 (c) 3 (d) -4

98. If x is a solution of the equation, $\sqrt{2x+1} - \sqrt{2x-1} = 1, \left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2 - 1}$ is equal to : **[Online April 10, 2016]**
(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $2\sqrt{2}$ (d) 2

99. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to: **[2015]**
(a) 3 (b) -3 (c) 6 (d) -6

100. If the two roots of the equation, $(a-1)(x^4 + x^2 + 1) + (a+1)(x^2 + x + 1)^2 = 0$ are real and distinct, then the set of all values of 'a' is : **[Online April 11, 2015]**
(a) $\left(0, \frac{1}{2}\right)$ (b) $\left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$
(c) $\left(-\frac{1}{2}, 0\right)$ (d) $(-\infty, -2) \cup (2, \infty)$

101. If $2 + 3i$ is one of the roots of the equation $2x^3 - 9x^2 + kx - 13 = 0$, $k \in R$, then the real root of this equation : **[Online April 10, 2015]**
(a) exists and is equal to $-\frac{1}{2}$.
(b) exists and is equal to $\frac{1}{2}$.
(c) exists and is equal to 1.
(d) does not exist.

102. If $a \in R$ and the equation
$-3(x-[x])^2 + 2(x-[x]) + a^2 = 0$
(where $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval: **[2014]**
(a) $(-2, -1)$ (b) $(-\infty, -2) \cup (2, \infty)$
(c) $(-1, 0) \cup (0, 1)$ (d) $(1, 2)$

103. The equation $\sqrt{3x^2 + x + 5} = x - 3$, where x is real, has; **[Online April 19, 2014]**
(a) no solution (b) exactly one solution
(c) exactly two solution (d) exactly four solution

104. The sum of the roots of the equation, $x^2 + |2x-3| - 4 = 0$, is: **[Online April 12, 2014]**

(a) 2 (b) -2 (c) $\sqrt{2}$ (d) $-\sqrt{2}$

105. If α and β are roots of the equation,

$x^2 - 4\sqrt{2}kx + 2e^{4\ln k} - 1 = 0$ for some k, and $\alpha^2 + \beta^2 = 66$, then $\alpha^3 + \beta^3$ is equal to: **[Online April 11, 2014]**

(a) $248\sqrt{2}$ (b) $280\sqrt{2}$ (c) $-32\sqrt{2}$ (d) $-280\sqrt{2}$

106. If $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ are the roots of the equation,

$ax^2 + bx + 1 = 0$ ($a \neq 0$, a, b, $\in$ R), then the equation, $x\left(x+b^3\right)+\left(a^3-3abx\right)=0$ as roots :

[Online April 9, 2014]

(a) $\alpha^{3/2}$ and $\beta^{3/2}$ (b) $\alpha\beta^{1/2}$ and $\alpha^{1/2}\beta$

(c) $\sqrt{\alpha\beta}$ and $\alpha\beta$ (d) $\alpha^{-\frac{3}{2}}$ and $\beta^{-\frac{3}{2}}$

107. If p and q are non-zero real numbers and $\alpha^3+\beta^3=-p, \alpha\beta=q$, then a quadratic equation whose roots are $\frac{\alpha^2}{\beta}, \frac{\beta^2}{\alpha}$ is : **[Online April 25, 2013]**

(a) $px^2 - qx + p^2 = 0$ (b) $qx^2 + px + q^2 = 0$
(c) $px^2 + qx + p^2 = 0$ (d) $qx^2 - px + q^2 = 0$

108. If α and β are roots of the equation $x^2 + px + \frac{3p}{4} = 0$, such that $|\alpha - \beta| = \sqrt{10}$, then p belongs to the set : **[Online April 22, 2013]**

(a) $\{2, -5\}$ (b) $\{-3, 2\}$ (c) $\{-2, 5\}$ (d) $\{3, -5\}$

109. If a complex number z statisfies the equation $z + \sqrt{2}\,|z+1| + i = 0$, then $|z|$ is equal to : **[Online April 22, 2013]**

(a) 2 (b) $\sqrt{3}$ (c) $\sqrt{5}$ (d) 1

110. Let $p, q, r \in R$ and $r > p > 0$. If the quadratic equation $px^2 + qx + r = 0$ has two complex roots α and β, then $|\alpha| + |\beta|$ is **[Online May 19, 2012]**

(a) equal to1
(b) less than 2 but not equal to 1
(c) greater than 2
(d) equal to 2

111. If the sum of the square of the roots of the equation $x^2 - (\sin\alpha - 2)x - (1 + \sin\alpha) = 0$ is least, then α is equal to **[Online May 12, 2012]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

112. The value of k for which the equation $(k-2)x^2 + 8x + k + 4 = 0$ has both roots real, distinct and negative is **[Online May 7, 2012]**

(a) 6 (b) 3 (c) 4 (d) 1

113. Let for $a \neq a_1 \neq 0$,

$f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$

and $p(x) = f(x) - g(x)$.

If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of p (b) is : **[2011 RS]**

(a) 3 (b) 9 (c) 6 (d) 18

114. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4,3). Rahul made a mistake in writing down coefficient of x to get roots (3,2). The correct roots of equation are : **[2011 RS]**

(a) 6, 1 (b) 4, 3 (c) $-6, -1$ (d) $-4, -3$

115. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re $z = 1$, then it is necessary that : **[2011]**

(a) $\beta \in (-1, 0)$ (b) $|\beta| = 1$

(c) $\beta \in (1, \infty)$ (d) $\beta \in (0, 1)$

116. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$ **[2010]**

(a) -1 (b) 1 (c) 2 (d) -2

117. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is : **[2009]**

(a) less than $4ab$ (b) greater than $-4ab$
(c) less than $-4ab$ (d) greater than $4ab$

118. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is **[2007]**

(a) $(3, \infty)$ (b) $(-\infty, -3)$ (c) $(-3, 3)$ (d) $(-3, \infty)$

119. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4, lie in the interval **[2006]**

(a) $-2 < m < 0$ (b) $m > 3$
(c) $-1 < m < 3$ (d) $1 < m < 4$

120. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30°$ and $\tan 15°$, respectively, then the value of $2 + q - p$ is **[2006]**

(a) 2 (b) 3 (c) 0 (d) 1

121. If $z^2 + z + 1 = 0$, where z is complex number, then the value of $\left(z+\frac{1}{z}\right)^2 + \left(z^2+\frac{1}{z^2}\right)^2 + \left(z^3+\frac{1}{z^3}\right)^2 + \ldots\ldots\ldots + \left(z^6+\frac{1}{z^6}\right)^2$ is **[2006]**

(a) 18 (b) 54 (c) 6 (d) 12

122. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $-\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0, a \neq 0$ then **[2005]**

(a) $a = b + c$ (b) $c = a + b$
(c) $b = c$ (d) $b = a + c$

123. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals **[2005]**

(a) -2 (b) 3 (c) 2 (d) 1

124. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is **[2004]**

(a) 4 (b) 12 (c) 3 (d) $\frac{49}{4}$

125. If $(1-p)$ is a root of quadratic equation $x^2 + px + (1-p) = 0$ then its root are **[2004]**

(a) $-1, 2$ (b) $-1, 1$ (c) $0, -1$ (d) $0, 1$

126. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is **[2003]**

(a) 3 (b) 2 (c) 4 (d) 1

127. The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is **[2003]**

(a) $-\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $-\frac{2}{3}$ (d) $\frac{1}{3}$

128. Let Z_1 and Z_2 be two roots of the equation $Z^2 + aZ + b = 0$, Z being complex. Further, assume that the origin, Z_1 and Z_2 form an equilateral triangle. Then **[2003]**

(a) $a^2 = 4b$ (b) $a^2 = b$
(c) $a^2 = 2b$ (d) $a^2 = 3b$

129. If p and q are the roots of the equation $x^2 + px + q = 0$, then **[2002]**

(a) $p = 1, q = -2$ (b) $p = 0, q = 1$
(c) $p = -2, q = 0$ (d) $p = -2, q = 1$

130. Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$ **[2002]**

(a) is always positive (b) is always negative
(c) does not exist (d) none of these

131. Difference between the corresponding roots of $x^2+ax+b=0$ and $x^2+bx+a=0$ is same and $a \neq b$, then **[2002]**

(a) $a + b + 4 = 0$ (b) $a + b - 4 = 0$
(c) $a - b - 4 = 0$ (d) $a - b + 4 = 0$

132. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α/β and β/α as its roots is **[2002]**

(a) $3x^2 - 19x + 3 = 0$ (b) $3x^2 + 19x - 3 = 0$
(c) $3x^2 - 19x - 3 = 0$ (d) $x^2 - 5x + 3 = 0$.

TOPIC 4 Condition for Common Roots, Maximum and Minimum value of Quadratic Equation, Quadratic Expression in two Variables, Solution of Quadratic Inequalities.

133. If 5, 5r, 5r^2 are the lengths of the sides of a triangle, then r cannot be equal to: **[Jan. 10, 2019 (I)]**

(a) $\frac{3}{4}$ (b) $\frac{5}{4}$ (c) $\frac{7}{4}$ (d) $\frac{3}{2}$

134. Let $a, b \in R$, $a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to : **[Jan. 9, 2020 (II)]**

(a) 25 (b) 26 (c) 28 (d) 24

135. If $\lambda \in R$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2-\lambda)x + (10-\lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is **[Online April 15, 2018]**

(a) 20 (b) $2\sqrt{5}$ (c) $2\sqrt{7}$ (d) $4\sqrt{2}$

136. If $|z - 3 + 2i| \leq 4$ then the difference between the greatest value and the least value of $|z|$ is **[Online April 15, 2018]**

(a) $\sqrt{13}$ (b) $2\sqrt{13}$ (c) 8 (d) $4 + \sqrt{13}$

137. If the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have a common root different from -1, then $|b|$ is equal to : **[Online April 9, 2016]**

(a) 2 (b) 3 (c) $\sqrt{3}$ (d) $\sqrt{2}$

138. If non-zero real numbers b and c are such that min f(x) > max g(x), where $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ $(x \in R)$;

then $\left|\frac{c}{b}\right|$ lies in the interval: **[Online April 19, 2014]**

(a) $\left(0, \frac{1}{2}\right)$ (b) $\left[\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

(c) $\left[\frac{1}{\sqrt{2}}, \sqrt{2}\right]$ (d) $\left(\sqrt{2}, \infty\right)$

139. If equations $ax^2 + bx + c = 0$ $(a, b, c \in R, a \neq 0)$ and $2x^2 + 3x + 4 = 0$ have a common root, then a : b : c equals: **[Online April 9, 2014]**

(a) $1:2:3$ (b) $2:3:4$ (c) $4:3:2$ (d) $3:2:1$

140. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a,b,c \in R$, have a common root, then $a : b : c$ is **[2013]**

(a) $1:2:3$ (b) $3:2:1$ (c) $1:3:2$ (d) $3:1:2$

141. The least integral value α of x such that $\frac{x-5}{x^2+5x-14} > 0$, satisfies : **[Online April 23, 2013]**

(a) $\alpha^2 + 3\alpha - 4 = 0$ (b) $\alpha^2 - 5\alpha + 4 = 0$

(c) $\alpha^2 - 7\alpha + 6 = 0$ (d) $\alpha^2 + 5\alpha - 6 = 0$

142. The values of 'a' for which one root of the equation $x^2 - (a+1)x + a^2 + a - 8 = 0$ exceeds 2 and the other is lesser than 2, are given by : **[Online April 9, 2013]**

(a) $3 < a < 10$ (b) $a \geq 10$

(c) $-2 < a < 3$ (d) $a \leq -2$

143. If $\left|z - \frac{4}{z}\right| = 2$, then the maximum value of $|z|$ is equal to : **[2009]**

(a) $\sqrt{5}+1$ (b) 2 (c) $2+\sqrt{2}$ (d) $\sqrt{3}+1$

144. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is **[2009]**

(a) 1 (b) 4 (c) 3 (d) 2

145. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is **[2006]**

(a) $\frac{1}{4}$ (b) 41 (c) 1 (d) $\frac{17}{7}$

146. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval **[2005]**

(a) $(5, 6]$ (b) $(6, \infty)$ (c) $(-\infty, 4)$ (d) $[4, 5]$

147. The value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assume the least value is **[2005]**

(a) 1 (b) 0 (c) 3 (d) 2

Hints & Solutions

1. **(b)** Let $z=\dfrac{3+i\sin\theta}{4-i\cos\theta}$, after rationalising

$$z=\frac{(3+i\sin\theta)}{(4-i\cos\theta)}\times\frac{(4+i\cos\theta)}{(4+i\cos\theta)}$$

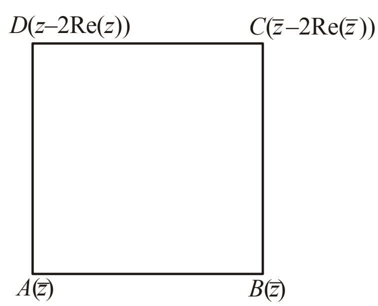
As z is purely real

$$\Rightarrow\quad 3\cos\theta+4\sin\theta=0\quad\Rightarrow\ \tan\theta=-\frac{3}{4}$$

$$arg(\sin\theta+i\cos\theta)=\pi+\tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$$

$$=\pi+\tan^{-1}\left(-\frac{4}{3}\right)=\pi-\tan^{-1}\left(\frac{4}{3}\right)$$

2. **(c)**



Let $z=x+iy$

$\because$ Length of side of square = 4 units

Then, $|z-\bar{z}|=4\Rightarrow|2iy|=4\Rightarrow|y|=2$

Also, $|z-(z-2\,\text{Re}(z))|=4$

$\Rightarrow|2\,\text{Re}(z)|=4\ \Rightarrow|2x|=4\Rightarrow|x|=2$

$\therefore|z|=\sqrt{x^2+y^2}=\sqrt{4+4}=2\sqrt{2}$

3. **(c)** $\because\ -1+\sqrt{3}i=2\cdot e^{\frac{2\pi}{3}i}$ and $1-i=\sqrt{2}\cdot e^{-\frac{i\pi}{4}}$

$$\therefore\left(\frac{-1+\sqrt{3}i}{1-i}\right)^{30}=\left(\sqrt{2}e^{\left(\frac{2\pi}{3}+\frac{\pi}{4}\right)i}\right)^{30}$$

$$=2^{15}\cdot e^{-\frac{\pi}{2}i}=-2^{15}\cdot i.$$

4. **(4)**

Given that $\left(\dfrac{1+i}{1-i}\right)^{m/2}=\left(\dfrac{1+i}{i-1}\right)^{n/3}=1$

$$\Rightarrow\left(\frac{(1+i)^2}{2}\right)^{m/2}=\left(\frac{(1+i)^2}{-2}\right)^{n/3}=1$$

$\Rightarrow i^{m/2}=(-i)^{n/3}=1$

m (least) = 8, n (least) = 12

GCD (8, 12) = 4.

5. **(b)** Let $z_1=x_1+iy_1$ and $z_2=x_2+iy_2$

$\because|z_1-1|=\text{Re}(z_1)$

$\Rightarrow(x_1-1)^2+y_1^2=x_1^2$

$\Rightarrow y_1^2-2x_1+1=0$ ...(i)

$|z_2-1|=\text{Re}(z_2)\ \Rightarrow(x_2-1)^2+y_2^2=x_2^2$

$\Rightarrow y_2^2-2x_2+1=0$ ...(ii)

From eqn. (i) – (ii),

$y_1^2-y_2^2-2(x_1-x_2)=0$

$\Rightarrow y_1+y_2=2\left(\dfrac{x_1-x_2}{y_1-y_2}\right)$ ...(iii)

$\because\arg(z_1-z_2)=\dfrac{\pi}{6}$

$\Rightarrow\tan^{-1}\left(\dfrac{y_1-y_2}{x_1-x_2}\right)=\dfrac{\pi}{6}$

$\Rightarrow\dfrac{y_1-y_2}{x_1-x_2}=\dfrac{1}{\sqrt{3}}$

$\Rightarrow\dfrac{2}{y_1+y_2}=\dfrac{1}{\sqrt{3}}\qquad\left[\text{From, }\dfrac{y_1-y_2}{x_1-x_2}=\dfrac{2}{y_1+y_2}\right]$

$\therefore y_1+y_2=2\sqrt{3}\Rightarrow\text{Im}(z_1+z_2)=2\sqrt{3}$

6. **(b)** Let $z=x+iy$

Then, $\left|\dfrac{z-i}{z+2i}\right|=1\ \Rightarrow x^2+(y-1)^2$

$=x^2+(y+2)^2\Rightarrow-2y+1=4y+4$

$\Rightarrow\quad 6y=-3\ \Rightarrow y=-\dfrac{1}{2}$

$\because\quad|z|=\dfrac{5}{2}\ \Rightarrow x^2+y^2=\dfrac{25}{4}$

$\Rightarrow \quad x^2 = \frac{24}{4} = 6$

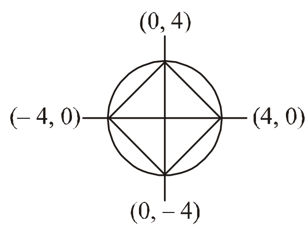
$\therefore \quad z = x + iy \qquad \Rightarrow z = \pm\sqrt{6} - \frac{i}{2}$

$|z + 3i| = \sqrt{6 + \frac{25}{4}} = \sqrt{\frac{49}{4}}$

$\Rightarrow \quad |z + 3i| = \frac{7}{2}$

7. **(c)** $z = x + iy$
$|x| + |y| = 4$

$|z| = \sqrt{x^2 + y^2}$
Minimum value of
$|z| = 2\sqrt{2}$
Maximum value of
$|z| = 4$



$|z| \in \left[\sqrt{8}, \sqrt{16}\right]$

So, |z| can't be $\sqrt{7}$.

8. **(c)** Let Re $(z) = x$ i.e., $z = x + 10i$
$2z - n = (2i - 1)(2z + n)$
$(2x - n) + 20i = (2i - 1)((2x + n) + 20i)$
On comparing real and imaginary parts,
$-(2x + n) - 40 = 2x - n$ and $20 = 4x + 2n - 20$
$\Rightarrow 4x = -40$ and $40 = -40 + 2n$
$\Rightarrow x = -10$ and $n = 40$
Hence, Re$(z) = -10$

9. **(b)** Given equation is, $|z - 1| = |z - i|$
$\Rightarrow \quad (x - 1)^2 + y^2 = x^2 + (y - 1)^2$ [Here, $z = x + iy$]
$\Rightarrow \quad 1 - 2x = 1 - 2y \Rightarrow x - y = 0$
Hence, locus is straight line with slope 1.

10. **(a)** $z = \frac{(1+i)^2}{a-i} \times \frac{a+i}{a+i}$

$z = \frac{(1 - 1 + 2i)(a + i)}{a^2 + 1} = \frac{2ai - 2}{a^2 + 1}$

$|z| = \sqrt{\left(\frac{-2}{a^2+1}\right)^2 + \left(\frac{2a}{a^2+1}\right)^2} = \sqrt{\frac{4 + 4a^2}{(a^2+1)^2}}$

$\Rightarrow |z| = \sqrt{\frac{4(1+a^2)}{(1+a^2)^2}} = \frac{2}{\sqrt{1+a^2}}$ ...(i)

Since, it is given that $|z| = \sqrt{\frac{2}{5}}$

Then, from equation (i),

$\sqrt{\frac{2}{5}} = \frac{2}{\sqrt{1+a^2}}$

Now, square on both side; we get
$\Rightarrow 1 + a^2 = 10 \Rightarrow a = \pm 3$
Since, it is given that $a > 0 \Rightarrow a = 3$

Then, $z = \frac{(1+i)^2}{a-i} = \frac{1 + i^2 + 2i}{3 - i} = \frac{2i}{3-i}$

$= \frac{2i(3+i)}{10} = \frac{-1+3i}{5}$

Hence, $\bar{z} = \frac{-1}{5} - \frac{3}{5}i$

11. **(c)** Given $|z\omega| = 1$ ...(i)

and $\arg\left(\frac{z}{\omega}\right) = \frac{\pi}{2}$ ...(ii)

$\therefore \frac{z}{\omega} + \frac{\bar{z}}{\bar{\omega}} = 0 \qquad \left[\because \text{Re}\left(\frac{z}{\omega}\right) = 0\right]$

$\Rightarrow z\bar{\omega} = -\bar{z}\bar{\omega}$

from equation (i), $z\bar{z}\omega\bar{\omega} = 1$ [using $z\bar{z} = |z|^2$]

$(\bar{z}\omega)^2 = -1 \Rightarrow \bar{z}\omega = \pm i$

from equation (ii), $-\arg(\bar{z}) - \arg\omega = \frac{\pi}{2}$ $-\arg(\bar{z}\omega) = \frac{-\pi}{2}$

Hence, $\bar{z}w = -i$

12. **(c)** $\omega = \frac{5+3z}{5-5z} \Rightarrow 5\omega - 5\omega z = 5 + 3z$

$\Rightarrow 5\omega - 5 = z(3 + 5\omega) \Rightarrow z = \frac{5(\omega - 1)}{3 + 5\omega}$

$\because |z| < 1, \therefore 5|\omega - 1| < |3 + 5\omega|$

$\Rightarrow 25(\omega\bar{\omega} - \omega - \bar{\omega} + 1) < 9 + 25\omega\bar{\omega} + 15\omega + 15\bar{\omega}$

$\left(\because |z|^2 = z\,\bar{z}\right)$

$\Rightarrow 16 < 40\omega + 40\bar{\omega} \Rightarrow \omega + \bar{\omega} > \frac{2}{5} \Rightarrow 2\text{Re}(\omega) > \frac{2}{5}$

$\Rightarrow \text{Re}(\omega) > \frac{1}{5}$

13. (a) Let $t = \dfrac{z-\alpha}{z+\alpha}$

$\because$ t is purely imaginary number.

$\therefore$ $t + \bar{t} = 0$

$\Rightarrow \dfrac{z-\alpha}{z+\alpha} + \dfrac{\bar{z}-\alpha}{\bar{z}+\alpha} = 0$

$\Rightarrow (z-\alpha)(\bar{z}+\alpha) + (\bar{z}-\alpha)(z+\alpha) = 0$

$\Rightarrow z\bar{z} - \alpha^2 + z\bar{z} - \alpha^2 = 0$

$\Rightarrow z\bar{z} - \alpha^2 = 0$

$\Rightarrow |z|^2 - \alpha^2 = 0$

$\Rightarrow \alpha^2 = 4$

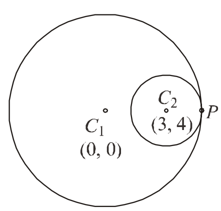
$\Rightarrow \alpha = \pm 2$

14. (a) $|z_1| = 9, |z_2 - 3 - 4i| = 4$

z_1 lies on a circle with centre $C_1(0, 0)$ and radius $r_1 = 9$

z_2 lies on a circle with centre $C_2(3, 4)$ and radius $r_2 = 4$

So, minimum value of $|z_1 - z_2|$ is zero at point of contact (i.e. A)



15. (b) Since, $|z| + z = 3 + i$

Let $z = a + ib$, then

$|z| + z = 3 + i \Rightarrow \sqrt{a^2+b^2} + a + ib = 3 + i$

Compare real and imaginary coefficients on both sides

$b = 1, \sqrt{a^2+b^2} + a = 3$

$\sqrt{a^2+1} = 3 - a$

$a^2 + 1 = a^2 + 9 - 6a$

$6a = 8 \Rightarrow a = \dfrac{4}{3}$

Then,

$|z| = \sqrt{\left(\dfrac{4}{3}\right)^2 + 1} = \sqrt{\dfrac{16}{9} + 1} = \dfrac{5}{3}$

16. (none) Let $z_1 = r_1 e^{i\theta}$ and $z_2 = r_2 e^{i\phi}$

$3|z_1| = 4|z_2| \Rightarrow 3r_1 = 4r_2$

$z = \dfrac{3z_1}{2z_2} + \dfrac{2z_2}{3z_1} = \dfrac{3r_1}{2r_2} e^{i(\theta-\phi)} + \dfrac{2}{3}\dfrac{r_2}{r_1} e^{i(\phi-\theta)}$

$= \dfrac{3}{2} \times \dfrac{4}{3} (\cos(\theta-\phi) + i\sin(\theta-\phi)) +$

$\dfrac{2}{3} \times \dfrac{3}{4} [\cos(\theta-\phi) - i\sin(\theta-\phi)]$

$z = \left(2 + \dfrac{1}{2}\right)\cos(\theta-\phi) + i\left(2 - \dfrac{1}{2}\right)\sin(\theta-\phi)$

$\therefore \quad |z| = \sqrt{\dfrac{25}{4}\cos^2(\theta-\phi) + \dfrac{9}{4}\sin^2(\theta-\phi)}$

$= \sqrt{\dfrac{16}{4}\cos^2(\theta-\phi) + \dfrac{9}{4}} \Rightarrow \dfrac{3}{2} \le |z| \le \dfrac{5}{2}$

17. (d) Suppose $z = \dfrac{3 + 2i\sin\theta}{1 - 2i\sin\theta}$

Since, z is purely imaginary, then $z + \bar{z} = 0$

$\Rightarrow \dfrac{3+2i\sin\theta}{1-2i\sin\theta} + \dfrac{3-2i\sin\theta}{1+2i\sin\theta} = 0$

$\Rightarrow \dfrac{(3+2i\sin\theta)(1+2i\sin\theta) + (3-2i\sin\theta)(1-2i\sin\theta)}{1+4\sin^2\theta}$

$= 0$

$\Rightarrow \sin^2\theta = \dfrac{3}{4} \Rightarrow \sin\theta = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \theta = -\dfrac{\pi}{3}, \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

Now, the sum of elements in $A = -\dfrac{\pi}{3} + \dfrac{\pi}{3} + \dfrac{2\pi}{3} = \dfrac{2\pi}{3}$

18. (a) $\because |z| = 1$ & $Re\, z \ne 1$

Suppose $z = x + iy \Rightarrow x^2 + y^2 = 1$ .....(i)

Now, $w = \dfrac{1 + (1-8\alpha)z}{1-z}$

$\Rightarrow w = \dfrac{1 + (1-8\alpha)(x+iy)}{1-(x+iy)}$

$\Rightarrow w = \dfrac{1 + (1-8\alpha)(x+iy))((1-x)+iy)}{1-(x+iy))((1-x)+iy)}$

$\Rightarrow w = \dfrac{[(1 + x(1-8\alpha)(1-x) - (1-8\alpha)y^2]}{(1-x)^2 + y^2}$

$+ i\dfrac{[(1 + x(1-8\alpha))y - (1-8\alpha)y(1-x)]}{(1-x)^2 + y^2}$

As, w is purely imaginary. So,

$$Re\, w = \frac{\left[(1+x(1-8\alpha))(1-x)-(1-8\alpha)y^2\right]}{(1-x)^2+y^2} = 0$$

$\Rightarrow (1-x)+x(1-8\alpha)(1-x) = (1-8\alpha)y^2$

$\Rightarrow (1-x)+x(1-8\alpha)-x^2(1-8\alpha) = (1-8x)y^2$

$\Rightarrow (1-x)+x(1-8\alpha) = 1-8\alpha$ [From (i), $x^2+y^2=1$]

$\Rightarrow 1-8\alpha = 1$

$\Rightarrow \alpha = 0$

$\therefore \alpha \in \{0\}$

19. **(b)** Rationalizing the given expression

$$\frac{(2+3i\sin\theta)(1+2i\sin\theta)}{1+4\sin^2\theta}$$

For the given expression to be purely imaginary, real part of the above expression should be equal to zero.

$$\Rightarrow \frac{2-6\sin^2\theta}{1+4\sin^2\theta} = 0 \quad \Rightarrow \sin^2\theta = \frac{1}{3}$$

$$\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{3}} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

20. **(b)** Let $z = re^{i\theta}$

Consider $\dfrac{\text{Im}z^5}{(\text{Im}z)^5} = \dfrac{r^5(\sin 5\theta)}{r^5(\sin\theta)^5}$

$(\because e^{i\theta} = \cos\theta + i\sin\theta)$

$$= \frac{\sin 5\theta}{\sin^5\theta} = \frac{16\sin^5\theta - 20\sin^3\theta + 5\sin\theta}{\sin^5\theta}$$

$$= \frac{16\sin^5\theta}{\sin^5\theta} - \frac{20\sin^3\theta}{\sin^5\theta} + \frac{5\sin\theta}{\sin^5\theta}$$

$= 5\,\text{cosec}^4\theta - 20\,\text{cosec}^2\theta + 16$

minimum value of $\dfrac{\text{Im}z^5}{(\text{Im}z)^5}$ is -4.

21. **(d)** We know minimum value of $|Z_1+Z_2|$ is $||Z_1|-|Z_2||$. Thus minimum value of $\left|Z+\frac{1}{2}\right|$ is $\left||Z|-\frac{1}{2}\right|$

$$\le \left|Z+\frac{1}{2}\right| \le |Z| + \frac{1}{2}$$

Since, $|Z| \ge 2$ therefore

$$2-\frac{1}{2} < \left|Z+\frac{1}{2}\right| < 2+\frac{1}{2}$$

$$\Rightarrow \frac{3}{2} < \left|Z+\frac{1}{2}\right| < \frac{5}{2}$$

22. **(b)** Let $z = 1 + i\alpha,\ \alpha \in R$

$z^2 = (1+i\alpha)(1+i\alpha)$

$x + iy = (1 + 2i\alpha - \alpha^2)$

On comparing real and imaginary parts, we get

$x = 1 - \alpha^2,\ y = 2\alpha$

Now, consider option (b), which is

$y^2 + 4x - 4 = 0$

LHS : $y^2+4x-4 = (2\alpha)^2 + 4(1-\alpha^2) - 4$

$= 4\alpha^2 + 4 - 4\alpha^2 - 4$

$= 0 =$ R.H.S.

Hence, $y^2 + 4x - 4 = 0$

23. **(c)** Let $z = x + iy$

$\dfrac{z-i}{z+i}$ is purely imaginary means its real part is zero.

$$\frac{x+iy-i}{x+iy+i} = \frac{x+i(y-1)}{x+i(y+1)} \times \frac{x-i(y+1)}{x-i(y+1)}$$

$$= \frac{x^2 - 2ix(y+1) + xi(y-1) + y^2 - 1}{x^2+(y+1)^2}$$

$$= \frac{x^2+y^2-1}{x^2+(y+1)^2} - \frac{2xi}{x^2+(y+1)^2}$$

for pure imaginary, we have

$$\frac{x^2+y^2-1}{x^2+(y+1)^2} = 0$$

$\Rightarrow x^2 + y^2 = 1$

$\Rightarrow (x+iy)(x-iy) = 1$

$$\Rightarrow x + iy = \frac{1}{x-iy} = z$$

and $\dfrac{1}{z} = x - iy$

$$z + \frac{1}{z} = (x+iy)+(x-iy) = 2x$$

$\left(z+\dfrac{1}{z}\right)$ is any non-zero real number

24. **(a)** Consider $\arg\left(\dfrac{z_1}{z_4}\right) + \arg\left(\dfrac{z_2}{z_3}\right)$

$= \arg(z_1) - \arg(z_4) + \arg(z_2) - \arg(z_3)$

$= (\arg(z_1)+\arg(z_2)) - (\arg(z_3)+\arg(z_4))$

given $\begin{pmatrix} z_2 = \bar{z}_1\ \& \\ z_4 = \bar{z}_3 \end{pmatrix}$

$= (\arg(z_1)+\arg(\bar{z}_1)) - (\arg(z_3)+\arg(\bar{z}_3))$

$\left\{\begin{array}{l}\text{also}(\arg(\bar{z}_1) = -\arg(z_1))\\ \arg(\bar{z}_3) = -\arg(z_3)\end{array}\right\}$

$= (\arg(z_1)-\arg(z_1)) - (\arg(z_3)-\arg(z_3))$

$= 0 - 0 = 0$

25. **(d)** Consider the equation

$w - \bar{w}z = k(1-z), k \in R$

Clearly $z \neq 1$ and $\frac{w-\bar{w}z}{1-z}$ is purely real

$\therefore \overline{\frac{w-\bar{w}z}{1-z}} = \frac{w-\bar{w}z}{1-z}$

$\Rightarrow \frac{\bar{w}-w\bar{z}}{1-\bar{z}} = \frac{w-\bar{w}z}{1-z}$

$\Rightarrow \bar{w} - \bar{w}z - w\bar{z} + wz\bar{z} = w - w\bar{z} - \bar{w}z + \bar{w}z\bar{z}$

$\Rightarrow \bar{w} + w|z|^2 = w + \bar{w}|z|^2$

$\Rightarrow (w-\bar{w})(|z|^2) = w - \bar{w}$

$\Rightarrow |z|^2 = 1 \quad (\because \text{Im}\, w \neq 0)$

$\Rightarrow |z| = 1$ and $z \neq 1$

$\therefore$ The required set is $\{z : |z| = 1, z \neq 1\}$

26. **(c)** Given $|z| = 1, \arg z = \theta$

$\Rightarrow \bar{z} = \frac{1}{z}$

$\therefore \arg\left(\frac{1+z}{1+\bar{z}}\right) = \arg\left(\frac{1+z}{1+\frac{1}{z}}\right) = \arg(z) = \theta.$

27. **(b)** Let $z = x + iy, \bar{z} = x - iy$

Now, $z = 1 - \bar{z}$

$\Rightarrow x + iy = 1 - (x - iy)$

$\Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

Now, $|z| = 1 \Rightarrow x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$

$\Rightarrow y = \pm\frac{\sqrt{3}}{2}$

Now, $\tan\theta = \frac{y}{x}$ (θ is the argument)

$= \frac{\sqrt{3}}{2} \div \frac{1}{2}$ (+ve since only principal argument)

$= \sqrt{3}$

$\Rightarrow \theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

Hence, z is not a real number

So, statement-1 is false and 2 is true.

28. **(a)** Let $z = x + iy \Rightarrow z^2 = x^2 - y^2 + 2ixy$

Now, $\frac{1+z^2}{2iz} = \frac{1+x^2-y^2+2ixy}{2i(x+iy)} = \frac{(x^2-y^2+1)+2ixy}{2ix-2y}$

$= \frac{(x^2-y^2+1)+2ixy}{-2y+2ix} \times \frac{-2y-2ix}{-2y-2ix}$

$= \frac{y(x^2+y^2-1)+x(x^2+y^2+1)i}{2(x^2+y^2)}$

$a = \frac{x(x^2+y^2+1)}{2(x^2+y^2)}$

Since, $|z| = 1 \Rightarrow \sqrt{x^2+y^2} = 1$

$\Rightarrow x^2 + y^2 = 1$

$\therefore a = \frac{x(1+1)}{2\times 1} = x$

Also $z \neq 1 \Rightarrow x + iy \neq 1$

$\therefore A = (-1, 1)$

29. **(d)** Let $z_1 = 1 + i$ and $z_2 = 1 - i$

$\frac{z_2}{z_1} = \frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = -i$

$\frac{2z_1+3z_2}{2z_1-3z_2} = \frac{2+3\left(\frac{z_2}{z_1}\right)}{2-3\left(\frac{z_2}{z_1}\right)} = \frac{2-3i}{2+3i}$

$\left|\frac{2z_1+3z_2}{2z_1-3z_2}\right| = \left|\frac{2-3i}{2+3i}\right| = \frac{|2-3i|}{|2+3i|} \quad \left[\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$

$= \frac{\sqrt{4+9}}{\sqrt{4+9}} = 1$

30. **(b)** $|z_1+z_2|^2 + |z_1-z_2|^2$

$= |z_1|^2 + |z_2|^2 + 2|z_1||z_2| + |z_1|^2 + |z_2|^2 - 2|z_1||z_2|$

$= 2|z_1|^2 + 2|z_2|^2 = 2\left[|z_1|^2 + |z_2|^2\right]$

31. **(a)** Let $|Z| = |W| = r$

$\Rightarrow Z = re^{i\theta}, W = re^{i\phi}$

where $\theta + \phi = \pi$

$\therefore \overline{W} = re^{-i\phi}$

Now, $Z = re^{i(\pi-\phi)} = re^{i\pi} \times e^{-i\phi} = -re^{-i\phi}$

$= -\overline{W}$

Thus, statement-1 is true but statement-2 is false.

32. **(b)** Statement - 1 and 2 both are true.

It is fundamental property.

But Statement - 2 is not correct explanation for Statement - 1.

33. **(a)** Let $z = x + iy$

$|z-1| = |z+1| \Rightarrow (x-1)^2 + y^2 = (x+1)^2 + y^2$

$\Rightarrow x = 0 \quad \Rightarrow \text{Re}\, z = 0$

$|z-1| = |z-i| \Rightarrow (x-1)^2 + y^2 = x^2 + (y-1)^2$

$\Rightarrow x = y$

$|z+1| = |z-i| \Rightarrow (x+1)^2 + y^2 = x^2 + (y-1)^2$

$\Rightarrow x = -y$

Only (0, 0) will satisfy all conditions.

$\Rightarrow$ Number of complex number $z = 1$

34. **(c)** $\overline{\left(\frac{1}{i-1}\right)} = \frac{1}{\overline{(i-1)}} = \frac{1}{-i-1} = \frac{-1}{i+1}$

35. **(a)** Given that $z^{\frac{1}{3}} = p + iq$

$\Rightarrow \bar{z} = p^3 + (iq)^3 + 3p(iq)(p + iq)$

$\Rightarrow x - iy = p^3 - 3pq^2 + i(3p^2q - q^3)$

Comparing both side, we get

$\therefore x = p^3 - 3pq^2 \Rightarrow \frac{x}{p} = p^2 - 3q^2 \quad ...(i)$

and $y = q^3 - 3p^2q \Rightarrow \frac{y}{q} = q^2 - 3p^2 \quad ...(ii)$

Adding (i) and (ii), we get

$\therefore \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2 \quad \therefore \left(\frac{x}{p} + \frac{y}{q}\right) \Big/ (p^2 + q^2) = -2$

36. **(c)** Given that arg $zw = \pi$

$\Rightarrow \arg z + \arg w = \pi \quad ...(i)$

$\bar{z} + i\bar{w} = 0 \Rightarrow \bar{z} = -i\bar{w}$

Replace i by $-i$, we get

$\therefore \; z = iw \Rightarrow \arg z = \frac{\pi}{2} + \arg w$

$\Rightarrow \arg z = \frac{\pi}{2} + \pi - \arg z \quad$ (from (i))

$\therefore \arg z = \frac{3\pi}{4}$

37. **(b)** Given that

$\left(\frac{1+i}{1-i}\right)^x = 1 \quad \Rightarrow \left[\frac{(1+i)^2}{1-i^2}\right]^x = 1$

$\left(\frac{1+i^2+2i}{1+1}\right)^x = 1 \Rightarrow (i)^x = 1; \; \therefore x = 4n; \;\; n \in I^+$

38. **(a)** $|\bar{z}\omega| = |\bar{z}||\omega| = |z||\omega| = |z\omega| = 1 \; [\because |\bar{z}| = |z|]$

$\text{Arg}(\bar{z}\omega) = \arg(\bar{z}) + \arg(\omega)$

$= -\arg(z) + \arg\omega = -\frac{\pi}{2}$

$[\because \arg(\bar{z}) = -\arg(z)]$

$\therefore \bar{z}\omega = -1$

39. **(c)** Given that $|z-4| < |z-2|$

Let $z = x + iy$

$\Rightarrow |(x-4) + iy)| < |(x-2) + iy|$

$\Rightarrow (x-4)^2 + y^2 < (x-2)^2 + y^2$

$\Rightarrow x^2 - 8x + 16 < x^2 - 4x + 4 \Rightarrow 12 < 4x$

$\Rightarrow x > 3 \Rightarrow \text{Re}(z) > 3$

40. **(b)** Let $|z| = |\omega| = r$

$\therefore z = re^{i\theta}, \; \omega = re^{i\phi} \quad$ where $\theta + \phi = \pi$.

$\therefore z = re^{i(\pi-\phi)} = re^{i\pi} \,.\, e^{-i\phi} = -re^{-i\phi} = -\bar{\omega}$.

$[\because e^{i\pi} = -1$ and $\bar{\omega} = re^{-i\phi}]$

41. **(c)** Let $z = x + iy$

$\because z^2 = i|z|^2$

$\therefore x^2 - y^2 + 2ixy = i(x^2 + y^2)$

$\Rightarrow x^2 - y^2 = 0$ and $2xy = x^2 + y^2$

$\Rightarrow (x-y)(x+y) = 0$ and $(x-y)^2 = 0$

$\Rightarrow x = y$

42. **(a)** Given that, $\alpha = \frac{-1+\sqrt{3}i}{2} = \omega$

$\therefore (2+\omega)^4 = a + b\omega \Rightarrow (4 + \omega^2 + 4\omega)^2 = a + b\omega$

$\Rightarrow (\omega^2 + 4(1+\omega))^2 = a + b\omega$

$\Rightarrow (\omega^2 - 4\omega^2)^2 = a + b\omega$

$[\because 1 + \omega = -\omega^2]$

$\Rightarrow (-3\omega^2)^2 = a + b\omega \Rightarrow 9\omega^4 = a + b\omega$

$\Rightarrow 9\omega = a + b\omega \quad (\because \omega^3 = 1)$

On comparing, $a = 0, b = 9$

$\Rightarrow a + b = 0 + 9 = 9$.

43. **(c)** $\left(\frac{1+\cos\frac{5\pi}{18} + i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18} - i\sin\frac{5\pi}{18}}\right)^3$

$= \left(\frac{2\cos^2\frac{5\pi}{36} + i2\sin\frac{5\pi}{36}\cdot\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36} - i2\sin\frac{5\pi}{36}\cdot\cos\frac{5\pi}{36}}\right)^3$

$$=\left(\frac{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}}\right)^3=\left(\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}\right)^6$$

$$=\cos\left(6\times\frac{5\pi}{36}\right)+i\sin\left(6\times\frac{5\pi}{36}\right)=\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}$$

$$=-\frac{\sqrt{3}}{2}+i\frac{1}{2}=-\frac{1}{2}(\sqrt{3}-i)$$

44. (b) $3+2\sqrt{-54}=3+6\sqrt{6}i$

Let $\sqrt{3+6\sqrt{6}i}=a+ib$

$\Rightarrow a^2-b^2=3$ and $ab=3\sqrt{6}$

$\Rightarrow a^2+b^2=\sqrt{(a^2-b^2)^2+4a^2b^2}=15$

So, $a=\pm3$ and $b=\pm\sqrt{6}$

$\sqrt{3+6\sqrt{6}\,i}=\pm(3+\sqrt{6}\,i)$

Similarly, $\sqrt{3-6\sqrt{6}\,i}=\pm(3-\sqrt{6}\,i)$

lm $(\sqrt{3+6\sqrt{6}i}-\sqrt{3-6\sqrt{6}i})=\pm2\sqrt{6}$

45. (b) Let $\alpha=\omega$, $b=1+\omega^3+\omega^6+.....=101$

$a=(1+\omega)(1+\omega^2+\omega^4+.....\omega^{198}+\omega^{200})$

$$=(1+\omega)\frac{\left(1-(\omega^2)^{101}\right)}{1-\omega^2}=\frac{(\omega+1)(\omega^{202}-1)}{(\omega^2-1)}$$

$$\Rightarrow a=\frac{(1+\omega)(1-\omega)}{1-\omega^2}=1$$

Required equation $=x^2-(101+1)x+(101)\times1=0$

$\Rightarrow x^2-102x+101=0$

46. (d) $\because$ $z=x+iy$

$$\left(\frac{z-1}{2z+i}\right)=\frac{(x-1)+iy}{2(x+iy)+i}$$

$$=\frac{(x-1)+iy}{2x+(2y+1)i}\times\frac{2x-(2y+1)i}{2x-(2y+1)i}$$

$$\text{Re}\left(\frac{z+1}{2z+i}\right)=\frac{2x(x-1)+y(2y+1)}{(2x)^2+(2y+1)^2}=1$$

$$\Rightarrow\left(x+\frac{1}{2}\right)^2+\left(y+\frac{3}{4}\right)^2=\left(\frac{\sqrt{5}}{4}\right)^2.$$

47. (d) $\frac{\sqrt{3}}{2}+\frac{i}{2}=-i\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=-i\omega$

where ω is imaginary cube root of unity.

Now, $(1+iz+z^5+iz^8)^9$

$=(1+\omega-i\omega^2+i\omega^2)^9=(1+\omega)^9$

$=(-\omega^2)^9=-\omega^{18}=-1$ $(\because 1+\omega+\omega^2=0)$

48. (a) $-(6+i)^3=x+iy$

$\Rightarrow$ $-[216+i^3+18i(6+i)]=x+iy$

$\Rightarrow$ $-[216-i+108i-18]=x+iy$

$\Rightarrow$ $-216+i-108i+18=x+iy$

$\Rightarrow$ $-198-107i=x+iy$

$\Rightarrow$ $x=-198, y=-107$

$\Rightarrow$ $y-x=-107+198=91$

49. (a) $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$

$$=\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^5+\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)^5$$

$$=\left(e^{i\frac{\pi}{6}}\right)^5+\left(e^{-i\frac{\pi}{6}}\right)^5=2\cos\frac{\pi}{6}=\sqrt{3}$$

$\Rightarrow$ $I(z)=0$, $\text{Re}(z)=\sqrt{3}$

50. (d) Let $l=\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)$.

$$\therefore l=\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{1+i\sqrt{3}}{1+i\sqrt{3}}\right)$$

$$=\left(\frac{-2+i2\sqrt{3}}{4}\right)=\left(\frac{1-i\sqrt{3}}{-2}\right)$$

$$\text{Also, } l=\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{1-i\sqrt{3}}{1-i\sqrt{3}}\right)$$

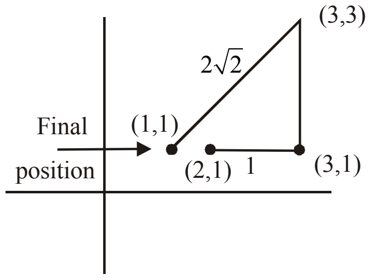
$$=\left(\frac{4}{-2-i2\sqrt{3}}\right)=\left(\frac{-2}{1+i\sqrt{3}}\right)$$

$$\text{Now, }\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^3=\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)$$

$$=\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{-2}{1+i\sqrt{3}}\right)\times\left(\frac{1-i\sqrt{3}}{-2}\right)=1$$

$\therefore$ least positive integer n is 3.

51. (a)



So new position is at the point $1 + i$

52. (a) $\left|\frac{z_1 - 2z_2}{2 - z_1\overline{z}_2}\right| = 1$

$\Rightarrow |z_1 - 2z_2|^2 = |2 - z_1\overline{z}_2|^2$

$\Rightarrow (z_1 - 2z_2)\overline{(z_1 - 2z_2)} = (2 - z_1\overline{z}_2)\overline{(2 - z_1\overline{z}_2)}$

$\Rightarrow (z_1 - 2z_2)(\overline{z}_1 - 2\overline{z}_2) = (2 - z_1\overline{z}_2)(2 - \overline{z}_1 z_2)$

$\Rightarrow (z_1\overline{z}_1) - 2z_1\overline{z}_2 - 2\overline{z}_1 z_2 + 4z_2\overline{z}_2$

$= 4 - 2\overline{z}_1 z_2 - 2z_1\overline{z}_2 + z_1\overline{z}_1 z_2\overline{z}_2$

$\Rightarrow |z_1|^2 + 4|z_2|^2 = 4 + |z_1|^2|z_2|^2$

$\Rightarrow |z_1|^2 + 4|z_2|^2 - 4 - |z_1|^2|z_2|^2 = 0$

$(|z_1|^2 - 4)(1 - |z_2|^2) = 0$

$\because |z_2| \neq 1$

$\therefore |z_1|^2 = 4$

$\Rightarrow |z_1| = 2$

$\Rightarrow$ Point z_1 lies on circle of radius 2.

53. (a) $\frac{z^2}{z-1} = \frac{\overline{z}^2}{\overline{z}-1}$ $\left[\because \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z}_1}{\overline{z}_2}\right]$

$\Rightarrow z\overline{z}z - z^2 = z.\overline{z}.\overline{z} - \overline{z}^2$

$\Rightarrow |z|^2.z - z^2 = |z|^2.\overline{z} - \overline{z}^2$

$\Rightarrow |z|^2(z - \overline{z}) - (z - \overline{z})(z + \overline{z}) = 0$

$\Rightarrow (z - \overline{z})(|z|^2 - (z + \overline{z})) = 0$

Either $z - \overline{z} = 0$ or $|z|^2 - (z + \overline{z}) = 0$

Either $z = \overline{z} \Rightarrow$ real axis

or $|z|^2 = z + \overline{z} \Rightarrow z\overline{z} - z - \overline{z} = 0$

represents a circle passing through origin.

54. (a) $(1+\omega)^7 = A + B\omega$

$(-\omega^2)^7 = A + B\omega$ $(\because \omega^{14} = \omega^{12}.\omega^2 = \omega^2)$

$-\omega^2 = A + B\omega$

$1 + \omega = A + B\omega$

$\Rightarrow A = 1, B = 1.$

55. (a) $|z+1| = |z+4-3| \leq |z+4| + |-3| \leq |3| + |-3|$

$\Rightarrow |z+1| \leq 6 \Rightarrow |z+1|_{max} = 6$

56. (c) Given that $w = \frac{z}{z - \frac{1}{3}i}$

$\Rightarrow |w| = \frac{|z|}{|z - \frac{1}{3}i|} = 1$ $\left[\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$

$\Rightarrow |z| = \left|z - \frac{1}{3}i\right|$

$\Rightarrow$ distance of z from origin and point $\left(0, \frac{1}{3}\right)$ is same

hence z lies on bisector of the line joining points (0, 0) and (0, 1/3).

Hence z lies on a straight line.

57. (c) $|z_1 + z_2| = |z_1| + |z_2| \Rightarrow z_1$ and z_2 are collinear and are to the same side of origin; hence arg z_1 – arg $z_2 = 0$.

58. (c) $\because (x-1)^3 + 8 = 0 \Rightarrow (x-1) = (-2)\,(1)^{1/3}$

$\Rightarrow x - 1 = -2$ or -2ω or $-2\omega^2$

or $x = -1$ or $1 - 2\omega$ or $1 - 2\omega^2$.

59. (b) Given that $|z^2 - 1| = |z|^2 + 1 \Rightarrow |z^2 - 1|^2 = (z\overline{z} + 1)^2$

$[\because |z|^2 = z\overline{z}]$

$\Rightarrow (z^2 - 1)(\overline{z}^2 - 1) = (z\overline{z} + 1)^2 (\because \overline{z_1 - z_2} = \overline{z}_1 - \overline{z}_2)$

$\Rightarrow z^2\overline{z}^2 - z^2 - \overline{z}^2 + 1 = z^2\overline{z}^2 + 2z\overline{z} + 1$

$\Rightarrow z^2 + 2z\overline{z} + \overline{z}^2 = 0$

$\Rightarrow (z + \overline{z})^2 = 0 \Rightarrow z = -\overline{z}$

$\Rightarrow z$ is purely imaginary

60. (b) Let the circle be $|z - z_0| = r$. Then according to given conditions $|z_0 - z_1| = r + a$ ...(i)

$|z_0 - z_2| = r + b$ ...(ii)

Subtract (ii) from (i)

we get $|z_0 - z_1| - |z_0 - z_2| = a - b$.

$\therefore$ Locus of centre z_0 is $|z - z_1| - |z - z_2|$ $= a - b$, which represents a hyperbola.

61. **(a)** $\because \alpha+\beta = 64, \alpha\beta = 256$

$$\frac{\alpha^{3/8}}{\beta^{5/8}}+\frac{\beta^{3/8}}{\alpha^{5/8}}=\frac{\alpha+\beta}{(\alpha\beta)^{5/8}}=\frac{64}{(2^8)^{5/8}}=\frac{64}{32}=2$$

62. **(b)** Let α and β be the roots of the given quadratic equation,

$2x^2+2x-1=0$ ...(i)

Then, $\alpha+\beta=-\frac{1}{2}\Rightarrow -1=2\alpha+2\beta$

and $4\alpha^2+2\alpha-1=0$ [$\because \alpha$ is root of eq. (i)]

$\Rightarrow 4\alpha^2+2\alpha+2\alpha+2\beta=0\Rightarrow \beta=-2\alpha(\alpha+1)$

63. **(b)** Let $|x|=y$ then

$9y^2-18y+5=0$

$\Rightarrow 9y^2-15y-3y+5=0$

$\Rightarrow (3y-1)(3y-5)=0$

$\Rightarrow y=\frac{1}{3}$ or $\frac{5}{3}\Rightarrow |x|=\frac{1}{3}$ or $\frac{5}{3}$

Roots are $\pm\frac{1}{3}$ and $\pm\frac{5}{3}$

$\therefore$ Product $=\frac{25}{81}$

64. **(d)** Let α and β be the roots of the quadratic equation

$7x^2-3x-2=0$

$\therefore \alpha+\beta=\frac{3}{7}, \alpha\beta=\frac{-2}{7}$

Now, $\frac{\alpha}{1-\alpha^2}+\frac{\beta}{1-\beta^2}$

$$=\frac{\alpha-\alpha\beta(\alpha+\beta)+\beta}{1-(\alpha^2+\beta^2)+(\alpha\beta)^2}$$

$$=\frac{(\alpha+\beta)-\alpha\beta(\alpha+\beta)}{1-(\alpha+\beta)^2+2\alpha\beta+(\alpha\beta)^2}$$

$$=\frac{\frac{3}{7}+\frac{2}{7}\times\frac{3}{7}}{1-\frac{9}{49}+2\times\frac{-2}{7}+\frac{4}{49}}=\frac{27}{16}$$

65. **(d)** $u=\frac{2(x+iy)+i}{(x+iy)-ki}=\frac{2x+i(2y+1)}{x+i(y-k)}$

Real part of $u=\text{Re}(u)=\frac{2x^2+(y-K)(2y+1)}{x^2+(y-K)^2}$

Imaginary part of u

$$=\text{Im}(u)=\frac{-2x(y-K)+x(2y+1)}{x^2+(y-K)^2}$$

$\because \text{Re}(u)+\text{Im}(u)=1$

$\Rightarrow 2x^2+2y^2-2Ky+y-K-2xy+2Kx+2xy+x$

$=x^2+y^2+K^2-2Ky$

Since, the curve intersect at y-axis

$\therefore x=0$

$\Rightarrow y^2+y-K(K+1)=0$

Let y_1 and y_2 are roots of equations if $x=0$

$\because y_1+y_2=-1$

$y_1y_2=-(K^2+K)$

$\therefore (y_1-y_2)^2=(1+4K^2+4K)$

Given $PQ=5\Rightarrow |y_1-y_2|=5$

$\Rightarrow 4K^2+4K-24=0\Rightarrow K=2$ or -3

as $K>0$, $\therefore K=2$

66. **(b)** Since α is common root of $x^2-x+2\lambda=0$ and $3x^2-10x+27\lambda=0$

$\therefore 3\alpha^2-10\alpha+27\lambda=0$ ...(i)

$3\alpha^2-3\alpha+6\lambda=0$ ...(ii)

$\therefore$ On subtract, we get $\alpha=3\lambda$

Now, $\alpha\beta=2\lambda\Rightarrow 3\lambda\cdot\beta=2\lambda\Rightarrow \beta=\frac{2}{3}$

$\Rightarrow \alpha+\beta=1\Rightarrow 3\lambda+\frac{2}{3}=1\Rightarrow \lambda=\frac{1}{9}$ and

$\alpha\gamma=9\lambda\Rightarrow 3\lambda\cdot\gamma=9\lambda\Rightarrow \gamma=3$

$\therefore \frac{\beta\gamma}{\lambda}=18$

67. **(d)** $\alpha\cdot\beta=2$ and $\alpha+\beta=-p$ also $\frac{1}{\alpha}+\frac{1}{\beta}=-q$

$\Rightarrow p=2q$

Now $\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$

$$=\left[\alpha\beta+\frac{1}{\alpha\beta}-\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right]\left[\alpha\beta+\frac{1}{\alpha\beta}+2\right]$$

$= \frac{9}{2}\left[\frac{5}{2} - \frac{\alpha^2+\beta^2}{2}\right] = \frac{9}{4}[5-(p^2-4)]$

$= \frac{9}{4}(9-p^2)$ $\quad [\because \alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta]$

68. (c) The given quadratic equation is

$(\lambda^2+1)x^2 - 4\lambda x + 2 = 0$

$\because$ One root is in the interval (0, 1)

$\therefore f(0)f(1) \le 0$

$\Rightarrow 2(\lambda^2+1-4\lambda+2) \le 0$

$\Rightarrow 2(\lambda^2-4\lambda+3) \le 0$

$(\lambda-1)(\lambda-3) \le 0 \Rightarrow \lambda \in [1, 3]$

But at $\lambda = 1$, both roots are 1 so $\lambda \ne 1$

$\therefore \lambda \in (1, 3]$

69. (c) Since, α and β are the roots of the equaton $5x^2+6x-2=0$

Then, $5\alpha^2+6\alpha-2=0$, $5\beta^2+6\beta-2=0$

$5\alpha^2+6\alpha=2$

$5S_6+6S_5 = 5(\alpha^6+\beta^6)+6(\alpha^5+\beta^5)$

$= (5\alpha^4+6\alpha^5)+(5\beta^6+6\beta^5)$

$= \alpha^4(5\alpha^2+6\alpha)+\beta^4(5\beta^2+6\beta)$

$= 2(\alpha^4+\beta^4) = 2S_4$

70. (a) Let $e^x = t \in (0, \infty)$

Given equation

$t^4+t^3-4t^2+t+1=0$

$\Rightarrow t^2+t-4+\frac{1}{t}+\frac{1}{t^2}=0$

$\Rightarrow \left(t^2+\frac{1}{t^2}\right)+\left(t+\frac{1}{t}\right)-4=0$

Let $t+\frac{1}{t}=y$

$(y^2-2)+y-4=0 \Rightarrow y^2+y-6=0$

$y^2+y-6=0 \Rightarrow y=-3, 2$

$\Rightarrow y=2 \Rightarrow t+\frac{1}{t}=2$

$\Rightarrow e^x+e^{-x}=2$

$x = 0$, is the only solution of the equation

Hence, there only one solution of the given equation.

71. (8) Since, $2x^2+(a-10)x+\frac{33}{2}=2a$ has real roots,

$\therefore D \ge 0$

$\Rightarrow (a-10)^2-4(2)\left(\frac{33}{2}-2a\right) \ge 0$

$\Rightarrow (a-10)^2-4(33-4a) \ge 0$

$\Rightarrow a^2-4a-32 \ge 0$

$\Rightarrow (a-8)(a+4) \ge 0$

$\Rightarrow a \le -4 \cup a \ge 8$

$\Rightarrow a \in (-\infty, -4] \cup [8, \infty)$

72. (a) Let $z = \alpha \pm i\beta$ be the complex roots of the equation

So, sum of roots $= 2\alpha = -b$ and

Product of roots $= \alpha^2+\beta^2 = 45$

$(\alpha+1)^2+\beta^2 = 40$

Given, $|z+1| = 2\sqrt{10}$

$\Rightarrow (\alpha+1)^2-\alpha^2=-5$ $\quad [\because \beta^2 = 45-\alpha^2]$

$\Rightarrow 2\alpha+1=-5 \Rightarrow 2\alpha=-6$

Hence, $b = 6$ and $b^2-b=30$

73. (d) $\alpha^5 = 5\alpha+3$

$\beta^5 = 5\beta+3$

$p_5 = 5(\alpha+\beta)+6 = 5(1)+6$

$[\because \text{from } x^2-x-1=0, \alpha+\beta = \frac{-b}{a} = 1]$

$p_5 = 11$ and $p_5 = \alpha^2+\beta^2 = \alpha+1+\beta+1$

$p_2 = 3$ and $p_3 = \alpha^3+\beta^3 = 2\alpha+1+2\beta+1$

$= 2(1)+2 = 4$

$p_2 \times p_3 = 12$ and $p_5 = 11 \Rightarrow p_5 \ne p_2 \times p_3$

74. (b) $(k+1)\tan^2 x - \sqrt{2}\lambda \tan x + (k-1) = 0$

$\tan\alpha + \tan\beta = \frac{\sqrt{2}\lambda}{k+1}$ [Sum of roots]

$\tan\alpha \cdot \tan\beta = \frac{k-1}{k+1}$ [Product of roots]

$\therefore \tan(\alpha+\beta) = \frac{\frac{\sqrt{2}\lambda}{k+1}}{1-\frac{k-1}{k+1}} = \frac{\sqrt{2}\lambda}{2} = \frac{\lambda}{\sqrt{2}}$

$\tan^2(\alpha+\beta) = \frac{\lambda^2}{2} = 50$

$\lambda = 10.$

75. (b) Given equation is, $x^2+x\sin\theta-2\sin\theta=0$

$\alpha+\beta = -\sin\theta$ and $\alpha\beta = -2\sin\theta$

$$\frac{(\alpha^{12}+\beta^{12})\alpha^{12}\beta^{12}}{(\alpha^{12}+\beta^{12})(\alpha-\beta)^{24}}=\frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$$

$\therefore\ |\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\sqrt{\sin^2\theta+8\sin\theta}$

$\therefore\ \frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}=\frac{(2\sin\theta)^{12}}{\sin^{12}\theta(\sin\theta+8)^{12}}=\frac{2^{12}}{(\sin\theta+8)^{12}}$

76. **(d)** Let $2^x-1=t$

$5+|t|=(t+1)(t-1)\Rightarrow|t|=t^2-6$

When $t>0$, $t^2-t-6=0\Rightarrow t=3$ or -2

$t=-2$ (rejected)

When $t<0$, $t^2+t-6=0\Rightarrow t=-3$ or 2 (both rejected)

$\therefore\ 2^x-1=3\Rightarrow 2^x=4\Rightarrow x=2$

77. **(d)** Since $2-\sqrt{3}$ is a root of the quadratic equation $x^2+px+q=0$

$\therefore 2+\sqrt{3}$ is the other root

$\Rightarrow x^2+px+q=[x-(2-\sqrt{3})][x-(2+\sqrt{3})]$

$=x^2-(2+\sqrt{3})x-(2-\sqrt{3})x+(2^2-(\sqrt{3})^2)$

$=x^2-4x+1$

Now, by comparing $p=-4$, $q=1$

$\Rightarrow p^2-4q-12=16-4-12=0$

78. **(c)** Sum of roots $=\frac{3}{m^2+1}$

$\because$ sum of roots is greatest. $\therefore\ m=0$

Hence equation becomes $x^2-3x+1=0$

Now, $\alpha+\beta=3,\ \alpha\beta=1\Rightarrow|-\alpha-\beta|=\sqrt{5}$

$\left|\alpha^3-\beta^3\right|=\left|(\alpha-\beta)\left(\alpha^2+\beta^2+\alpha\beta\right)\right|=\sqrt{5}(9-1)=8\sqrt{5}$

79. **(d)** Let $\sqrt{x}=a$

$\therefore$ given equation will become:

$|a-2|+a(a-4)+2=0$

$\Rightarrow\ |a-2|+a^2-4a+4-2=0$

$\Rightarrow\ |a-2|+(a-2)^2-2=0$

Let $|a-2|=y$ (Clearly $y\geq 0$)

$\Rightarrow\ y+y^2-2=0$

$\Rightarrow\ y=1$ or -2 (rejected)

$\Rightarrow\ |a-2|=1\Rightarrow a=1,3$

When $\sqrt{x}=1\Rightarrow x=1$

When $\sqrt{x}=3\Rightarrow x=9$

Hence, the required sum of solutions of the equation $=10$

80. **(c)** The given quadratic equation is $x^2-2x+2=0$

Then, the roots of the this equation are $\frac{2\pm\sqrt{-4}}{2}=1\pm i$

Now, $\frac{\alpha}{\beta}=\frac{1-i}{1+i}=\frac{(1-i)^2}{1-i^2}=i$

or $\frac{\alpha}{\beta}=\frac{1-i}{1+i}=\frac{(1-i)^2}{1-i^2}=-i$ So, $\frac{\alpha}{\beta}=\pm i$

Now, $\left(\frac{\alpha}{\beta}\right)^n=1\Rightarrow(\pm i)^n=1$

$\Rightarrow$ n must be a multiple of 4.

Hence, the required least value of $n=4$.

81. **(b)** Let roots of the quadratic equation are α, β.

Given, $\lambda=\frac{\alpha}{\beta}$ and $\lambda+\frac{1}{\lambda}=1\Rightarrow\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=1$

$\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=1$...(i)

The quadratic equation is, $3m^2x^2+m(m-4)x+2=0$

$\therefore\ \alpha+\beta=\frac{m(4-m)}{3m^2}=\frac{4-m}{3m}$ and $\alpha\beta=\frac{2}{3m^2}$

Put these values in eq (1),

$$\frac{\left(\frac{4-m}{3m}\right)^2}{\frac{2}{3m^2}}=3$$

$\Rightarrow\ (m-4)^2=18\Rightarrow m=4\pm\sqrt{18}$

Therefore, least value is $4-\sqrt{18}=4-3\sqrt{2}$

82. **(d)** Let α and β be the roots of the equation, $81x^2+kx+256=0$

Given $(\alpha)^{\frac{1}{3}}=\beta\Rightarrow\alpha=\beta^3$

$\because$ Product of the roots $=\frac{256}{81}$

$\therefore\ (\alpha)(\beta)=\frac{256}{81}$

$\Rightarrow\ \beta^4=\left(\frac{4}{3}\right)^4\Rightarrow\beta=\frac{4}{3}\Rightarrow\alpha=\frac{64}{27}$

$\because$ Sum of the roots $=-\frac{k}{81}$

$\therefore \quad \alpha+\beta=-\frac{k}{81} \Rightarrow \frac{4}{3}+\frac{64}{27}=-\frac{k}{81}$

$\Rightarrow \quad k=-300$

83. **(d)** Consider the quadratic equation

$(c-5)x^2-2cx+(c-4)=0$

Now, $f(0).f(3)>0$ and $f(0).f(2)<0$

$\Rightarrow \quad (c-4)(4c-49)>0$ and $(c-4)(c-24)<0$

$\Rightarrow \quad c \in (-\infty, 4) \cup \left(\frac{49}{4}, \infty\right)$ and $c \in (4, 24)$

$\Rightarrow \quad c \in \left(\frac{49}{4}, 24\right)$

Integral values in the interval $\left(\frac{49}{4}, 24\right)$ are 13, 14, ..., 23.

$\therefore \quad S=\{13, 14, ..., 23\}$

84. **(d)** The given quadratic equation is

$x^2+(3-\lambda)x+2=\lambda$

Sum of roots $=\alpha+\beta=\lambda-3$

Product of roots $=\alpha\beta=2-\lambda$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(\lambda-3)^2-2(2-\lambda)$

$=\lambda^2-4\lambda+5$

$=(\lambda-2)^2+1$

For least $(\alpha^2+\beta^2)$ $\lambda=2$.

85. **(a)** Consider the equation

$x^2+2x+2=0$

$x=\frac{-2\pm\sqrt{4-8}}{2}=-1\pm i$

Let $\alpha=-1+i, \beta=-1-i$

$\alpha^{15}+\beta^{15}=(-1+i)^{15}+(-1-i)^{15}$

$=\left(\sqrt{2}e^{i\frac{3\pi}{4}}\right)^{15}+\left(\sqrt{2}e^{-i\frac{3\pi}{4}}\right)^{15}$

$=\left(\sqrt{2}\right)^{15}\left[e^{\frac{i45\pi}{4}}+e^{\frac{-i45\pi}{4}}\right]$

$=\left(\sqrt{2}\right)^{15}.2\cos\frac{45\pi}{4}=\left(\sqrt{2}\right)^{15}.2\cos\frac{3\pi}{4}$

$=\frac{-2}{\sqrt{2}}\left(\sqrt{2}\right)^{15}$

$=-2\left(\sqrt{2}\right)^{14}=-256$

86. **(a)** The roots of $6x^2-11x+\alpha=0$ are rational numbers.

$\therefore$ Discriminant D must be perfect square number.

$D=(-11)^2-4\cdot 6\cdot\alpha$

$=121-24\alpha$ must be a perfect square

Hence, possible values for α are

$\alpha=3, 4, 5.$

$\therefore$ 3 positive integral values are possible.

87. **(b)** Given quadratic equation is: $x^2-mx+4=0$

Both the roots are real and distinct.

So, discriminant $B^2-4AC>0$.

$\therefore \quad m^2-4\cdot 1\cdot 4>0$

$\therefore \quad (m-4)(m+4)>0$

$\therefore \quad m\in(-\infty,-4)\cup(4,\infty) \quad$ ...(i)

Since, both roots lies in [1, 5]

$\therefore \quad -\frac{-m}{2}\in(1,5)$

$\Rightarrow \quad m\in(2,10) \quad$ ...(ii)

And $1\cdot(1-m+4)>0 \Rightarrow m<5$

$\therefore \quad m\in(-\infty,5) \quad$ ...(iii)

And $1\cdot(25-5m+4)>0 \Rightarrow m<\frac{29}{5}$

$\therefore \quad m\in\left(-\infty\frac{29}{5}\right) \quad$ ...(iv)

From (i), (ii), (iii) and (iv), $m\in(4,5)$

88. **(a)** $\because z_0$ is a root of quadratic equation

$x^2+x+1=0$

$\therefore \quad z_0=\omega$ or $\omega^2 \Rightarrow z_0{}^3=1$

$\therefore \quad z=3+6i\,z_0{}^{81}-3i\,z_0{}^{93}$

$=3+6i((z_0)^3)^{27}-3i((z_0)^3)^{31}$

$=3+6i-3i$

$=3+3i$

$\therefore \quad \arg(z)\tan^{-1}\left(\frac{3}{3}\right)=\frac{\pi}{4}$

89. **(b)** $\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r}$

$\frac{x+p+x+q}{(x+p)(x+q)}=\frac{1}{r}$

$(2x+p+q)r=x^2+px+qx+pq$

$x^2+(p+q-2r)x+pq-pr-qr=0$

Let α and β be the roots.

$\therefore\ \alpha+\beta=-(p+q-2r)$ .... (i)

& $\alpha\beta=pq-pr-qr$ .... (ii)

$\because\ \alpha=-\beta$ (given)

$\therefore$ in eq. (1), we get

$\Rightarrow\ -(p+q-2r)=0$ ... (iii)

Now, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(-(p+q-2r))^2-2(pq-pr-qr)$ .... (from (i) and (ii))

$=p^2+q^2+4r^2+2pq-4pr-4qr-2pq+2pr+2qr$

$=p^2+q^2+4r^2-2pr-2qr$

$=p^2+q^2+2r(2r-p-q)$ ... (from (iii))

$=p^2+q^2+0$

$=p^2+q^2$

90. **(b)** Here, $9x^2+27x+20=0$

$$\therefore\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{-27\pm\sqrt{27^2-4\times9\times20}}{2\times9}$$

$$\Rightarrow x=-\frac{4}{3},-\frac{5}{3}$$

Given, $\cos A=-\frac{3}{5}$

$$\therefore\ \sec A=\frac{1}{\cos A}=-\frac{5}{3}$$

Here, A is an obtuse angle.

$$\therefore\ \tan A=-\sqrt{\sec^2 A-1}=-\frac{4}{3}.$$

Hence, roots of the equation are $\sec A$ and $\tan A$.

91. **(b)** As $\tan A$ and $\tan B$ are the roots of $3x^2-10x-25=0$,

$$\text{So, } \tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\frac{10}{3}}{1+\frac{25}{3}}=\frac{10/3}{28/3}=\frac{5}{14}$$

Now, $\cos 2(A+B)=-1+2\cos^2(A+B)$

$$=\frac{1-\tan^2(A+B)}{1+\tan^2(A+B)}\Rightarrow\cos^2(A+B)=\frac{196}{221}$$

$\therefore\ 3\sin^2(A+B)-10\sin(A+B)\cos(A+B)-25\cos^2(A+B)$

$=\cos^2(A+B)[3\tan^2(A+B)-10\tan(A+B)-25]$

$$=\frac{75-700-4900}{196}\times\frac{196}{221}=-\frac{5525}{196}\times\frac{196}{221}=-25$$

92. **(d)** If a and -1 are the roots of the polynomial, then we get

$f(x)=x^2+(1-a)x-a.$

$\therefore\ \ f(1)=2-2a$

and $f(2)=6-3a$

As, $f(1)+f(2)=0\ \Rightarrow\ 2-2a+6-3a=0\ \Rightarrow\ a=\frac{8}{5}$

Therefore, the other root is $\frac{8}{5}$

93. **(b)** α,β are roots of $x^2-x+1=0$

$\therefore\ \ \alpha=-\omega$ and $\beta=-\omega^2$

where ω is cube root of unity

$\therefore\ \ \alpha^{101}+\beta^{107}=(-\omega)^{101}+(-\omega)^{107}$

$=-[\omega^2+\omega]=-[-1]=1$

94. **(a)** We have, $\sum_{r=1}^{n}(x+r-1)(x+r)=10n$

$$\sum_{r=1}^{n}(x^2+xr+(r-1)x+r^2-r)=10n$$

$$\Rightarrow\ \sum_{r=1}^{n}(x^2+(2r-1)x+r(r-1))=10n$$

$\Rightarrow\ nx^2+\{1+3+5+....+(2n-1)\}x+\{1.2+2.3+....+(n-1)n\}=10n$

$$\Rightarrow\ nx^2+n^2x+\frac{(n-1)n(n+1)}{3}=10n$$

$$\Rightarrow\ x^2+nx+\frac{n^2-31}{3}=0$$

Let α and $\alpha+1$ be its two solutions

($\because$ it has two consequtive integral solutions)

$\Rightarrow\ \alpha+(\alpha+1)=-n$

$$\Rightarrow\ \alpha=\frac{-n-1}{2}\quad ...(i)$$

$$\text{Also } \alpha(\alpha+1)=\frac{n^2-31}{3}\quad ...(ii)$$

Putting value of (i) in (ii), we get

$$-\left(\frac{n+1}{2}\right)\left(\frac{1-n}{2}\right)=\frac{n^2-31}{3}$$

$\Rightarrow\ n^2=121\ \Rightarrow n=11$

95. **(c)** $(x-1)(x^2+5x-50)=0$

$\Rightarrow\ (x-1)(x+10)(x-5)=0$

$\Rightarrow\ x=1,5,-10$

Sum $=-4$

96. **(c)** Let $p(x)=ax^2+bx+c$

$\because\ \ p(0)=1\Rightarrow c=1$

Also, $p(1)=4$ & $p(-1)=6$

$\Rightarrow\ a+b+1=4$ & $a-b+1=6$

$\Rightarrow\ a+b=3$ & $a-b=5$

$\Rightarrow\ a=4$ & $b=-1$

$p(x)=4x^2-x+1$

$p(b)=16-2+1=15$

$p(-2)=16+2+1=19$

97. **(c)** $(x^2-5x+5)^{x^2+4x-60}=1$

Case I

$x^2-5x+5=1$ and $x^2+4x-60$ can be any real number

$\Rightarrow x=1,4$

Case II

$x^2-5x+5=-1$ and $x^2+4x-60$ has to be an even number

$\Rightarrow x=2,3$

where 3 is rejected because for $x=3$,

$x^2+4x-60$ is odd.

Case III

x^2-5x+5 can be any real number and

$x^2+4x-60=0$

$\Rightarrow x=-10,6$

$\Rightarrow$ Sum of all values of x

$=-10+6+2+1+4=3$

98. **(a)** $\sqrt{2x+1}-\sqrt{2x-1}=1$ .....(i)

$\Rightarrow\ 2x+1+2x-1-2\sqrt{4x^2-1}=1$

$\Rightarrow\ 4x-1=2\sqrt{4x^2-1}$

$\Rightarrow\ 16x^2-8x+1=16x^2-4$

$\Rightarrow\ 8x=5$

$\Rightarrow\ x=\frac{5}{8}$ which satisfies equation (i)

So, $\sqrt{4x^2-1}=\frac{3}{4}$

99. **(a)** $\alpha,\beta=\frac{6\pm\sqrt{36+8}}{2}=3\pm\sqrt{11}$

$\alpha=3+\sqrt{11}$, $\beta=3-\sqrt{11}$

$\therefore a_n=\left(3+\sqrt{11}\right)^n-\left(3-\sqrt{11}\right)^n\quad \frac{a_{10}-2a_8}{2a_9}$

$$=\frac{\left(3+\sqrt{11}\right)^{10}-\left(3-\sqrt{11}\right)^{10}-2\left(3+\sqrt{11}\right)^8+2\left(3-\sqrt{11}\right)^8}{2\left[\left(3+\sqrt{11}\right)^9-\left(3-\sqrt{11}\right)^9\right]}$$

$$=\frac{\left(3+\sqrt{11}\right)^8\left[\left(3+\sqrt{11}\right)^2-2\right]+\left(3-\sqrt{11}\right)^8\left[2-\left(3-\sqrt{11}\right)^2\right]}{2\left[\left(3+\sqrt{11}\right)^9-\left(3-\sqrt{11}\right)^9\right]}$$

$$=\frac{\left(3+\sqrt{11}\right)^8\left(9+11+6\sqrt{11}-2\right)+\left(3-\sqrt{11}\right)^8\left(2-9-11+6\sqrt{11}\right)}{2\left[\left(3+\sqrt{11}\right)^9-\left(3-\sqrt{11}\right)^9\right]}$$

$$=\frac{6\left(3+\sqrt{11}\right)^9-6\left(3-\sqrt{11}\right)^9}{2\left[\left(3+\sqrt{11}\right)^9-\left(3-\sqrt{11}\right)^9\right]}=\frac{6}{2}=3$$

100. **(b)** $(a-1)(x^4+x^2+1)+(a+1)(x^2+x+1)^2=0$

$\Rightarrow (a-1)(x^2+x+1)(x^2-x+1)+(a+1)(x^2+x+1)^2=0$

$\Rightarrow (x^2+x+1)[(a-1)(x^2-x+1)+(a+1)(x^2+x+1)]=0$

$\Rightarrow (x^2+x+1)(ax^2+x+a)=0$

For roots to be distinct and real, $a\neq 0$ and $1-4a^2>0$

$\Rightarrow a\neq 0$ and $a^2<\frac{1}{4}$

$\Rightarrow a\in\left(-\frac{1}{2},0\right)\cup\left(0,\frac{1}{2}\right)$

101. **(b)** $\alpha=2+3i;\ \beta=2-3i,\ \gamma=?$

$\alpha\beta\gamma=\frac{13}{2}\quad\left[\text{since product of roots}=\frac{d}{a}\right]$

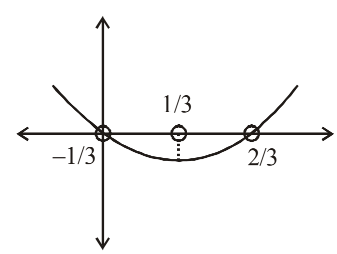
$\Rightarrow (4+9)\gamma=\frac{13}{2}\Rightarrow\gamma=\frac{1}{2}$

102. **(c)** Consider $-3(x-[x])^2+2[x-[x]]+a^2=0$

$\Rightarrow\ 3\{x\}^2-2\{x\}-a^2=0\qquad(\because x-[x]=\{x\})$

$\Rightarrow\ 3\left(\{x\}^2-\frac{2}{3}\{x\}\right)=a^2, a\neq 0$

$\Rightarrow\ a^2=3\{x\}\left(\{x\}-\frac{2}{3}\right)$



Now, $\{x\}\in(0,1)$ and $\frac{-2}{3}\leq a^2<1$ (by graph)

Since , x is not an integer

$\therefore\ a\in(-1,1)-\{0\}$

$\Rightarrow\ a\in(-1,0)\cup(0,1)$

103. **(a)** Consider $\sqrt{3x^2+x+5}=x-3$

Squaring both the sides, we get

$3x^2+x+5=(x-3)^2$

$\Rightarrow\ 3x^2+x+5=x^2+9-6x$

$\Rightarrow\ 2x^2+7x-4=0$

$\Rightarrow\ 2x^2+8x-x-4=0$

$\Rightarrow\ 2x(x+4)-1(x+4)=0$

$\Rightarrow x = \frac{1}{2}$ or $x = -4$

For $x = \frac{1}{2}$ and $x = -4$

L.H.S. $\neq$ R.H.S. of equation, $\sqrt{3x^2+x+5} = x-3$

Also, for every $x \in R$, LHS $\neq$ RHS of the given equation.

$\therefore$ Given equation has no solution.

104. (c) $x^2 + |2x-3| - 4 = 0$

$$|2x-3| = \begin{cases} (2x-3) & \text{if } x > \frac{3}{2} \\ -(2x-3) & \text{if } x < \frac{3}{2} \end{cases}$$

for $x > \frac{3}{2}$, $x^2 + 2x - 3 - 4 = 0$

$x^2 + 2x - 7 = 0$

$$x = \frac{-2 \pm \sqrt{4+28}}{2} = \frac{-2 \pm 4\sqrt{2}}{2} = -1 \pm 2\sqrt{2}$$

Here $x = 2\sqrt{2} - 1$ $\left\{2\sqrt{2} - 1 < \frac{3}{2}\right\}$

for $x < \frac{3}{2}$

$x^2 - 2x + 3 - 4 = 0$

$\Rightarrow x^2 - 2x - 1 = 0$

$$\Rightarrow x = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

Here $x = 1 - \sqrt{2}$ $\left\{(1-\sqrt{2}) < \frac{3}{2}\right\}$

Sum of roots : $(2\sqrt{2} - 1) + (1 - \sqrt{2}) = \sqrt{2}$

105. (d) $x^2 - 4\sqrt{2}kx + 2e^{4\ln k} - 1 = 0$

or, $x^2 - 4\sqrt{2}kx + 2k^4 - 1 = 0$

$\alpha + \beta = 4\sqrt{2}k$ and $\alpha.\beta = 2k^4 - 1$

Squaring both sides, we get

$(\alpha + \beta)^2 = (4\sqrt{2}k)^2 \Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 32k^2$

$66 + 2\alpha\beta = 32k^2$

$66 + 2(2k^4 - 1) = 32k^2$

$66 + 4k^4 - 2 = 32k^2 \Rightarrow 4k^4 - 32k^2 + 64 = 0$

or, $k^4 - 8k^2 + 16 = 0 \Rightarrow (k^2)^2 - 8k^2 + 16 = 0$

$\Rightarrow (k^2 - 4)(k^2 - 4) = 0 \Rightarrow k^2 = 4, k^2 = 4$

$\Rightarrow k = \pm 2$

Now, $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$\therefore \alpha^3 + \beta^3 = (4\sqrt{2}k)\,[66 - (2k^4 - 1)]$

Putting $k = -2$, ($k = +2$ cannot be taken because it does not satisfy the above equation)

$\therefore \alpha^3 + \beta^3 = (4\sqrt{2}(-2))[66 - 2(-2)^4 - 1]$

$\alpha^3 + \beta^3 = (-8\sqrt{2})\,(66 - 32 + 1) = (-8\sqrt{2})\,(35)$

$\therefore \alpha^3 + \beta^3 = -280\sqrt{2}$

106. (a) Let $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ be the roots of $ax^2 + bx + 1 = 0$

$$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \left(\frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}}\right) = -\frac{b}{a}$$

$$\frac{1}{\sqrt{\alpha}\sqrt{\beta}} = \frac{1}{a} \Rightarrow a = \sqrt{\alpha\beta}$$

$$b = -\left(\sqrt{\alpha} + \sqrt{\beta}\right)$$

$$x(x + b^3) + (a^3 - 3abx) = 0$$

$$\Rightarrow x^2 + (b^3 - 3ab)x + a^3 = 0$$

Putting values of a and b, we get

$$x^2 + \left[\left(-\sqrt{\alpha} + \sqrt{\beta}\right)^3 + 3\left(\sqrt{\alpha\beta}\right)\left(\sqrt{\alpha} + \sqrt{\beta}\right)\right] + (\alpha\beta)^{3/2} = 0$$

$$\Rightarrow x^2 - \left[\alpha^{3/2} + \beta^{3/2} + 3\sqrt{\alpha\beta}\left(\sqrt{\alpha} + \sqrt{\beta}\right) - 3\sqrt{\alpha\beta}(\sqrt{\alpha} + \sqrt{\beta})\right]x + (\alpha\beta)^{3/2} = 0$$

$$\Rightarrow x^2 - (\alpha^{3/2} + \beta^{3/2})x + \alpha^{3/2}\beta^{3/2} = 0$$

Roots of this equation are $\alpha^{3/2}, \beta^{3/2}$

107. (b) Given $\alpha^3 + \beta^3 = -p$ and $\alpha\beta = q$

Let $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ be the root of required quadratic equation.

So, $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{-p}{q}$

and $\frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta = q$

Hence, required quadratic equation is

$$x^2 - \left(\frac{-p}{q}\right)x + q = 0$$

$$\Rightarrow x^2 + \frac{p}{q}x + q = 0 \;\Rightarrow\; qx^2 + px + q^2 = 0$$

108. (c) Given quadratic eqn. is

$$x^2 + px + \frac{3p}{4} = 0$$

So, $\alpha+\beta=-p, \alpha\beta=\frac{3p}{4}$

Now, given $|\alpha-\beta|=\sqrt{10}$

$\Rightarrow \alpha-\beta=\pm\sqrt{10}$
$\Rightarrow (\alpha-\beta)^2=10 \Rightarrow \alpha^2+\beta^2-2\alpha\beta=10$
$\Rightarrow (\alpha+\beta)^2-4\alpha\beta=10$

$\Rightarrow p^2-4\times\frac{3p}{4}=10 \Rightarrow p^2-3p-10=0$

$\Rightarrow p=-2, 5 \Rightarrow p\in\{-2,5\}$

109. (c) Given equation is

$z+\sqrt{2}|z+1|+i=0$

put $z=x+iy$ in the given equation.

$(x+iy)+\sqrt{2}\,|x+iy+1|+i=0$

$\Rightarrow x+iy+\sqrt{2}\left[\sqrt{(x+1)^2+y^2}\right]+i=0$

Now, equating real and imaginary part, we get

$x+\sqrt{2}\sqrt{(x+1)^2+y^2}=0$ and

$y+1=0 \Rightarrow y=-1$

$\Rightarrow x+\sqrt{2}\sqrt{(x+1)^2+(-1)^2}=0 \quad (\because y=-1)$

$\Rightarrow \sqrt{2}\sqrt{(x+1)^2+1}=-x$

$\Rightarrow 2[(x+1)^2+1]=x^2$
$\Rightarrow x^2+4x+4=0$
$\Rightarrow x=-2$

Thus, $z=-2+i(-1) \Rightarrow |z|=\sqrt{5}$

110. (c) Given quadratic equation is
$px^2+qx+r=0$ ...(i)
$D=q^2-4pr$
Since α and β are two complex root
$\therefore \beta=\bar{\alpha} \Rightarrow |\beta|=|\bar{\alpha}| \Rightarrow |\beta|=|\alpha| \quad (\because|\bar{\alpha}|=|\alpha|)$
Consider
$|\alpha|+|\beta|=|\alpha|+|\alpha| \quad (\because|\beta|=|\alpha|)$
$=2|\alpha|>2.1=2 \quad (\because|\alpha|>1)$
Hence, $|\alpha|+|\beta|$ is greater than 2.

111. (d) Given equation is
$x^2-(\sin\alpha-2)x-(1+\sin\alpha)=0$
Let x_1 and x_2 be two roots of quadratic equation.
$\therefore x_1+x_2=\sin\alpha-2$ and $x_1x_2=-(1+\sin\alpha)$
$(x_1+x_2)^2=(\sin\alpha-2)^2=\sin^2\alpha+4-4\sin\alpha$

$\Rightarrow x_1^2+x_2^2=\sin^2\alpha+4-4\sin\alpha-2x_1x_2$
$=\sin^2\alpha+4-4\sin\alpha+2(1+\sin\alpha)$
$=\sin^2\alpha-2\sin\alpha+6$ ...(i)

Now, By putting

$\alpha=\frac{\pi}{6}, \alpha=\frac{\pi}{4}, \alpha=\frac{\pi}{3}$ and $\alpha=\frac{\pi}{2}$ in (i) one by one

We get least value of $x_1^2+x_2^2$ at $\frac{\pi}{2}$

Hence, $\alpha=\frac{\pi}{2}$

112. (b) $(k-2)x^2+8x+k+4=0$
If real roots then,
$8^2-4(k-2)(k+4)>0$
$\Rightarrow k^2+2k-8<16$
$\Rightarrow k^2+6k-4k-24<0$
$\Rightarrow (k+6)(k-4)<0$
$\Rightarrow -6<k<4$
If both roots are negative
then $\alpha\beta$ is +ve

$\Rightarrow \frac{k+4}{k-2}>0 \Rightarrow k>-4$

Also, $\frac{k-2}{k+4}>0 \Rightarrow k>2$

Roots are real so, $-6<k<4$
So, 6 and 4 are not correct.
Since, $k>2$, so 1 is also not correct value of k.
$\therefore k=3$

113. (d) $p(x)=0$

$\Rightarrow f(x)=g(x)$

$\Rightarrow ax^2+bx+c=a_1x^2+b_1x+c_1$

$\Rightarrow (a-a_1)x^2+(b-b_1)x+(c-c_1)=0.$

It has only one solution, $x=-1$

$\Rightarrow b-b_1=a-a_1+c-c_1$ ...(i)

Sum of roots $\frac{-(b-b_1)}{(a-a_1)}=-1-1$

$\Rightarrow \frac{b-b_1}{2(a-a_1)}=1$

$\Rightarrow b-b_1=2(a-a_1)$ ...(ii)

Now $p(-2)=2$
$\Rightarrow f(-2)-g(-2)=2$
$\Rightarrow 4a-2b+c-4a_1+2b_1-c_1=2$
$\Rightarrow 4(a-a_1)-2(b-b_1)+(c-c_1)=2$ ...(iii)
From equations, (i), (ii) and (iii)

$a-a_1=c-c_1=\frac{1}{2}(b-b_1)=2$

Now, $p(2) = f(2) - g(2)$

$= 4(a - a_1) + 2(b - b_1) + (c - c_1)$

$= 8 + 8 + 2 = 18$

114. (a) Let the correct equation be

$ax^2 + bx + c = 0$

Now, Sachin's equation

$ax^2 + bx + c' = 0$

Given that, roots found by Sachin's are 4 and 3

$\Rightarrow -\frac{b}{a} = 7$ ....(i)

Rahul's equation, $ax^2 + b'x + c = 0$

Given that roots found by Rahul's are 3 and 2

$\Rightarrow \frac{c}{a} = 6$ ...(ii)

From (i) and (ii), roots of the correct equation $x^2 - 7x + 6 = 0$ are 6 and 1.

115. (c) Since both the roots of given quadratic equation lie in the line $Re\, z = 1$ i.e., $x = 1$, hence real part of both the roots are 1.

Let both roots be $1 + i\alpha$ and $1 - i\alpha$

Product of the roots, $1 + \alpha^2 = \beta$

$\because \alpha^2 + 1 \geq 1$

$\therefore \beta \geq 1 \Rightarrow \because \beta \in (1, \infty)$

116. (b) $x^2 - x + 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{1-4}}{2}$

$x = \frac{1 \pm \sqrt{3}\,i}{2}$

$\alpha = \frac{1}{2} + i\frac{\sqrt{3}}{2} = -\omega^2$

$\beta = \frac{1}{2} - \frac{i\sqrt{3}}{2} = -\omega$

$\alpha^{2009} + \beta^{2009} = (-\omega^2)^{2009} + (-\omega)^{2009}$

$= -\omega^2 - \omega = 1$ $[\because \omega^3 = 1]$

117. (b) Given that roots of the equation $bx^2 + cx + a = 0$ are imaginary

$\therefore \quad c^2 - 4ab < 0$ ....(i)

Let $y = 3b^2x^2 + 6\,bc\,x + 2c^2$

$\Rightarrow 3b^2x^2 + 6\,bc\,x + 2c^2 - y = 0$

As x is real, $D \geq 0$

$\Rightarrow 36\,b^2c^2 - 12\,b^2\,(2c^2 - y) \geq 0$

$\Rightarrow 12\,b^2\,(3\,c^2 - 2\,c^2 + y) \geq 0 \quad [\because b^2 \geq 0]$

$\Rightarrow c^2 + y \geq 0 \quad \Rightarrow y \geq -c^2$

But from eqn. (i), $c^2 < 4ab$ or $-c^2 > -4ab$

$\therefore$ we get $y \geq -c^2 > -4ab$

$\Rightarrow y > -4\,ab$

118. (c) Let α and β are roots of the equation $x^2 + ax + 1 = 0$

$\alpha + \beta = -a$ and $\alpha\beta = 1$

Given that $|\alpha - \beta| < \sqrt{5}$

$\Rightarrow \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} < \sqrt{5}$

$\left(\because (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta\right)$

$\Rightarrow \sqrt{a^2 - 4} < \sqrt{5} \Rightarrow a^2 - 4 < 5$

$\Rightarrow a^2 - 9 < 0 \Rightarrow a^2 < 9 \Rightarrow -3 < a < 3$

$\Rightarrow a \in (-3, 3)$

119. (c) Given equation is $x^2 - 2mx + m^2 - 1 = 0$

$\Rightarrow (x - m)^2 - 1 = 0$

$\Rightarrow (x - m + 1)(x - m - 1) = 0$

$\Rightarrow x = m - 1, m + 1$

$m - 1 > -2$ and $m + 1 < 4$

$\Rightarrow m > -1$ and $m < 3 \Rightarrow -1 < m < 3$

120. (b) Given that $x^2 + px + q = 0$

Sum of roots $= \tan 30° + \tan 15° = -p$

Product of roots $= \tan 30° . \tan 15° = q$

$\tan 45° = \frac{\tan 30° + \tan 15°}{1 - \tan 30°.\tan 15°} \Rightarrow \frac{-p}{1-q} = 1$

$\Rightarrow -p = 1 - q \Rightarrow q - p = 1$

$\therefore 2 + q - p = 3$

121. (d) $z^2 + z + 1 = 0 \Rightarrow z = \omega$ or ω^2

So, $z + \frac{1}{z} = \omega + \omega^2 = -1$

$\left[\because \frac{1}{z} = \omega^2 \text{ and } 1 + \omega + \omega^2 = 0\right]$

$z^2 + \frac{1}{z^2} = \omega^2 + \omega = -1,$ $[\because \omega^3 = 1]$

$z^3 + \frac{1}{z^3} = \omega^3 + \omega^3 = 2$

$z^4+\frac{1}{z^4}=-1,\ z^5+\frac{1}{z^5}=-1$

and $z^6+\frac{1}{z^6}=2$

$\therefore$ The given sum = 1+1 + 4 + 1 + 1 + 4 = 12

122. (b) $\tan\left(\frac{P}{2}\right),\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2+bx+c=0$

$$\tan\left(\frac{P}{2}\right)+\tan\left(\frac{Q}{2}\right)=-\frac{b}{a}$$

$$\tan\left(\frac{P}{2}\right)\cdot\tan\left(\frac{Q}{2}\right)=\frac{c}{a}$$

$$\frac{\tan\left(\frac{P}{2}\right)+\tan\left(\frac{Q}{2}\right)}{1-\tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)}=\tan\left(\frac{P}{2}+\frac{Q}{2}\right)=1$$

$\left[\because P+Q=\frac{\pi}{2}\right]$

$$\Rightarrow \frac{-\frac{b}{a}}{1-\frac{c}{a}}=1 \Rightarrow -\frac{b}{a}=\frac{a-c}{a}$$

$\Rightarrow -b=a-c \Rightarrow c=a+b$

123. (d) Let α, $\alpha+1$ be roots
Then $\alpha+\alpha+1=b=$ sum of roots
$\alpha(\alpha+1)=c=$ product of roots
$\therefore\ b^2-4c=(2\alpha+1)^2-4\alpha(\alpha+1)=1$.

124. (d) Given that 4 is a root of $x^2+px+12=0$

$\Rightarrow 16+4p+12=0 \Rightarrow p=-7$

Now, the equation $x^2+px+q=0$

has equal roots.
$\therefore$ D = 0

$$\Rightarrow p^2-4q=0 \Rightarrow q=\frac{p^2}{4}=\frac{49}{4}$$

125. (c) Let the second root be α.

Then $\alpha+(1-p)=-p \Rightarrow \alpha=-1$

Also $\alpha.(1-p)=1-p$

$\Rightarrow (\alpha-1)(1-p)=0 \Rightarrow p=1\ [\because \alpha=-1]$

$\therefore$ Roots are $\alpha=-1$ and $1-p=0$

126. (c) Given that

$x^2-3|x|+2=0 \Rightarrow |x|^2-3|x|+2=0$

$\Rightarrow (|x|-2)(|x|-1)=0$

$\Rightarrow |x|=1,2 \Rightarrow x=\pm1,\pm2$

$\therefore$ No. of solution = 4

127. (b) Let one roots of given equation be α
$\therefore$ Second roots be 2α then

$$\alpha+2\alpha=3\alpha=\frac{1-3a}{a^2-5a+3}$$

$$\Rightarrow \alpha=\frac{1-3a}{3(a^2-5a+3)} \quad ....(i)$$

and $\alpha.2\alpha=2\alpha^2=\frac{2}{a^2-5a+3}$

$$\therefore\ 2\left[\frac{1}{9}\frac{(1-3a)^2}{(a^2-5a+3)^2}\right]=\frac{2}{a^2-5a+3}$$

[from (i)]

$$\frac{(1-3a)^2}{(a^2-5a+3)}=9$$

$\Rightarrow 9a^2-6a+1=9a^2-45a+27$

$\Rightarrow 39a=26 \Rightarrow a=\frac{2}{3}$

128. (d) Given that $Z^2+aZ+b=0$;

$Z_1+Z_2=-a\ \&\ Z_1Z_2=b$

$0, Z_1, Z_2$ form an equilateral triangle

$\therefore\ 0^2+Z_1{}^2+Z_2{}^2=0.Z_1+Z_1.Z_2+Z_2.0$

(for an equilateral triangle,

$Z_1{}^2+Z_2{}^2+Z_3{}^2=Z_1Z_2+Z_2Z_3+Z_3Z_1$)

$\Rightarrow Z_1{}^2+Z_2{}^2=Z_1Z_2$

$\Rightarrow (Z_1+Z_2)^2=3Z_1Z_2$

$\therefore\ a^2=3b$

129. (a) $p+q=-p \Rightarrow q=2p$
and $pq=q \Rightarrow q(p-1)=0$
$\Rightarrow q=0$ or $p=1$.
If $q=0$, then $p=0$.
or $p=1$, then $q=-2$.

130. (a) Product of real roots $=\frac{c}{a}=\frac{9}{t^2}>0,\ \forall\ t\in R$

$\therefore$ Product of real roots is always positive.

131. (a) Let α and β are roots of the equation $x^2 + ax + b = 0$ and γ and δ be the roots of the equation $x^2 + bx + a = 0$ respectively.

$\therefore \alpha + \beta = -a, \alpha\beta = b$ and $\gamma + \delta = -b, \gamma\delta = a$.

Given $|\alpha - \beta| = |\gamma - \delta| \Rightarrow (\alpha - \beta)^2 = (\gamma - \delta)^2$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$

$\Rightarrow a^2 - 4b = b^2 - 4a$

$\Rightarrow (a^2 - b^2) + 4(a - b) = 0$

$\Rightarrow a + b + 4 = 0 \qquad (\because a \neq b)$

132. (a) Given that $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$;

$\Rightarrow$ α & β are roots of equation, $x^2 = 5x - 3$

or $x^2 - 5x + 3 = 0$

$\therefore \alpha + \beta = 5$ and $\alpha\beta = 3$

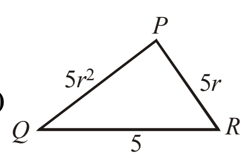
Thus, the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

$$x^2 - x\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{\alpha\beta}{\alpha\beta} = 0$$

$$\Rightarrow x^2 - x\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right) + 1 = 0$$

or $3x^2 - 19x + 3 = 0$

133. (c)



ΔPQR is possible if

$5 + 5r > 5r^2$

$\Rightarrow 1 + r > r^2$

$\Rightarrow r^2 - r - 1 < 0$

$$\Rightarrow \left(r - \frac{1}{2} + \frac{\sqrt{5}}{2}\right)\left(r - \frac{1}{2} - \frac{\sqrt{5}}{2}\right) < 0$$

$$\Rightarrow r \in \left(\frac{-\sqrt{5}+1}{2}, \frac{\sqrt{5}+1}{2}\right)$$

$$\because \frac{7}{4} \notin \left(\frac{-\sqrt{5}+1}{2}, \frac{\sqrt{5}+1}{2}\right) \quad \therefore r \neq \frac{7}{4}$$

134. (a) $ax^2 - 2bx + 5 = 0$,

If α and α are roots of equations, then sum of roots

$$2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$$

and product of roots $= \alpha^2 = \frac{5}{a} \Rightarrow \frac{b^2}{a^2} = \frac{5}{a}$

$\Rightarrow b^2 = 5a \qquad (a \neq 0) \qquad$ ...(i)

For $x^2 - 2bx - 10 = 0$

$\alpha + \beta = 2b$ ...(ii)

and $\alpha\beta = -10$ ...(iii)

$\alpha = \frac{b}{a}$ is also root of $x^2 - 2bx - 10 = 0$

$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$

By eqn. (i) $\Rightarrow 5a - 10a^2 - 10a^2 = 0$

$\Rightarrow 20a^2 = 5a$

$\Rightarrow a = \frac{1}{4}$ and $b^2 = \frac{5}{4}$

$\alpha^2 = 20$ and $\beta^2 = 5$

Now, $\alpha^2 + \beta^2 = 5 + 20 = 25$

135. (b) Let, the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ are α and β.

Also roots of the given equation are

$$\frac{\lambda - 2 \pm \sqrt{4 - 4\lambda + \lambda^2 - 40 + 4\lambda}}{2} = \frac{\lambda - 2 \pm \sqrt{\lambda^2 - 36}}{2}$$

The magnitude of the difference of the roots is $\left|\sqrt{\lambda^2 - 36}\right|$

$$\text{So, } \alpha^3 + \beta^3 = \frac{(\lambda - 2)^3}{4} + \frac{3(\lambda - 2)(\lambda^2 - 36)}{4}$$

$$= \frac{(\lambda - 2)(4\lambda^2 - 4\lambda - 104)}{4} = (\lambda - 2)(\lambda^2 - \lambda - 26) = f(\lambda)$$

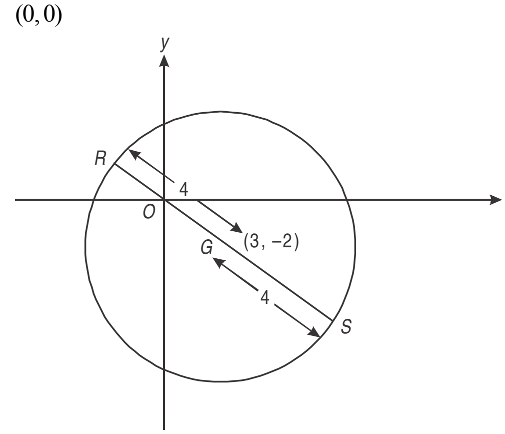
As $f(\lambda)$ attains its minimum value at $\lambda = 4$.

Therefore, the magintude of the difference of the roots is

$|i\sqrt{20}| = 2\sqrt{5}$

136. (b) $|z - (3 - 2i)| \leq 4$ represents a circle whose centre is $(3, -2)$ and radius $= 4$.

$|z| = |z - 0|$ represents the distance of point 'z' from origin $(0, 0)$



Suppose RS is the normal of the circle passing through origin 'O' and G is its center $(3, -2)$.

Here, OR is the least distance

and OS is the greatest distance

$OR = RG - OG$ and $OS = OG + GS \qquad$ ...(i)

As, $RG = GS = 4$

$OG = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$

From (i), $OR = 4 - \sqrt{13}$ and $OS = 4 + \sqrt{13}$

So, required difference $= \left(4+\sqrt{13}\right) - \left(4-\sqrt{13}\right)$

$= \sqrt{13} + \sqrt{13} = 2\sqrt{13}$

137. (c) $x^2 + bx - 1 = 0$ common root

$x^2 + x + b = 0$

$- \quad - \quad -$

$x = \dfrac{b+1}{b-1}$

Put $x = \dfrac{b+1}{b-1}$ in equation

$\left(\dfrac{b+1}{b-1}\right)^2 + \left(\dfrac{b+1}{b-1}\right) + b = 0$

$(b+1)^2 + (b+1)(b-1) + b(b-1)^2 = 0$

$b^2 + 1 + 2b + b^2 - 1 + b(b^2 - 2b + 1) = 0$

$2b^2 + 2b + b^3 - 2b^2 + b = 0$

$b^3 + 3b = 0$

$b(b^2 + 3) = 0$

$b^2 = -3$

$b = \pm\sqrt{3}i$

$|b| = \sqrt{3}$

138. (d) We have

$f(x) = x^2 + 2bx + 2c^2$

and $g(x) = -x^2 - 2cx + b^2, (x \in R)$

$\Rightarrow f(x) = (x+b)^2 + 2c^2 - b^2$

and $g(x) = -(x+c)^2 + b^2 + c^2$

Now, $f_{\min} = 2c^2 - b^2$ and $g_{\max} = b^2 + c^2$

Given : $\min f(x) > \max g(x)$

$\Rightarrow 2c^2 - b^2 > b^2 + c^2$

$\Rightarrow c^2 > 2b^2$

$\Rightarrow |c| > |b|\sqrt{2}$

$\Rightarrow \dfrac{|c|}{|b|} > \sqrt{2} \Rightarrow \left|\dfrac{c}{b}\right| > \sqrt{2}$

$\Rightarrow \left|\dfrac{c}{b}\right| \in (\sqrt{2}, \infty)$.

139. (b) Let α, β be the common roots of both the equations.

For first equation $ax^2 + bx + c = 0$,
we have

$\alpha + \beta = \dfrac{-b}{a}$ ...(i)

$\alpha . \beta = \dfrac{c}{a}$ ...(ii)

For second equation $2x^2 + 3x + 4 = 0$,
we have

$\alpha + \beta = \dfrac{-3}{2}$ ...(iii)

$\alpha . \beta = \dfrac{2}{1}$ ...(iv)

Now, from (i) & (iii) & from (ii) & (iv)

$\dfrac{-b}{a} = \dfrac{-3}{2}$ $\qquad$ $\dfrac{c}{a} = \dfrac{2}{1}$

$\dfrac{b}{a} = \dfrac{3/2}{1}$

Therefore on comparing we get $a = 1$, $b = \dfrac{3}{2}$ & $c = 2$

putting these values in first equation, we get

$x^2 + \dfrac{3}{2}x + 2 = 0$ or $2x^2 + 3x + 4 = 0$

from this, we get $a = 2$, $b = 3$; $c = 4$

or $a : b : c = 2 : 3 : 4$

140. (a) Given equations are

$x^2 + 2x + 3 = 0$ ...(i)

$ax^2 + bx + c = 0$ ...(ii)

Roots of equation (i) are imaginary roots in order pair. According to the question (ii) will also have both roots same as (i). Thus

$\dfrac{a}{1} = \dfrac{b}{2} = \dfrac{c}{3} = \lambda$ (say)

$\Rightarrow a = \lambda, b = 2\lambda, c = 3\lambda$

Hence, required ratio is 1 : 2 : 3

141. (a) $\dfrac{x-5}{x^2 + 5x - 14} > 0 \Rightarrow x^2 + 5x - 14 < x - 5$

$\Rightarrow x^2 + 4x - 9 < 0$

$\Rightarrow \alpha = -5, -4, -3, -2, -1, 0, 1$

$\alpha = -5$ does not satisfy any of the options

$\alpha = -4$ satisfy the option (a) $\alpha^2 + 3\alpha - 4 = 0$

142. (c) $x^2-(a+1)x+a^2+a-8=0$

Since roots are different, therefore $D>0$

$\Rightarrow (a+1)^2-4(a^2+a-8)>0$

$\Rightarrow (a-3)(3a+1)<0$

There are two cases arises.

Case I. $a-3>0$ and $3a+1<0$

$\Rightarrow a>3$ and $a<-\frac{11}{3}$

Hence, no solution in this case

Case II : $a-3<0$ and $3a+11>0$

$\Rightarrow a<3$ and $a>-\frac{11}{3}$

$\therefore -\frac{11}{3}<a<3 \Rightarrow -2<a<3$

143. (a) Given that $\left|z-\frac{4}{z}\right|=2$

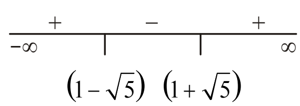
$$|z| = \left|z-\frac{4}{z}+\frac{4}{z}\right| \le \left|z-\frac{4}{z}\right|+\frac{4}{|z|}$$

$$\Rightarrow |z| \le 2+\frac{4}{|z|}$$

$$\Rightarrow |z|^2-2|z|-4\le 0$$

$$\Rightarrow \left(|z|-\frac{2+\sqrt{20}}{2}\right)\left(|z|-\frac{2-\sqrt{20}}{2}\right)\le 0$$

$$\Rightarrow \left(|z|-(1+\sqrt{5})\right)\left(|z|-(1-\sqrt{5})\right)\le 0$$



$$\Rightarrow \left(-\sqrt{5}+1\right) \le |z| \le \left(\sqrt{5}+1\right)$$

$$\Rightarrow |z|_{max}=\sqrt{5}+1$$

144. (d) Let the roots of equation $x^2-6x+a=0$ be α and 4β and that of the equation $x^2-cx+6=0$ be α and 3β. Then

$\alpha+4\beta=6$ ...(i) $4\alpha\beta=a$ ...(ii)

and $\alpha+3\beta=c$ ...(iii) $3\alpha\beta=6$ ...(iv)

$\Rightarrow a=8$ (from (ii) and (iv))

$\therefore$ The equation becomes $x^2-6x+8=0$

$\Rightarrow (x-2)(x-4)=0$

$\Rightarrow$ roots are 2 and 4

$\Rightarrow \alpha=2, \beta=1 \therefore$ Common root is 2.

145. (b) $y=\frac{3x^2+9x+17}{3x^2+9x+7}$

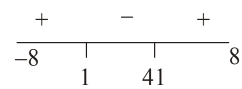
$3x^2(y-1)+9x(y-1)+7y-17=0$

$D\ge 0 \quad \because$ x is real

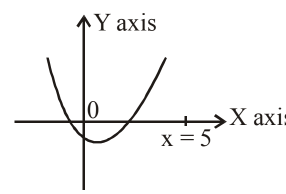
$81(y-1)^2-4\times 3(y-1)(7y-17)\ge 0$

$\Rightarrow (y-1)(y-41)\le 0 \Rightarrow 1\le y\le 41$

$\therefore$ Max value of y is 41



146. (c) Given that both roots of quadratic equation are less than 5 then (i)



Discriminant ≥ 0

$4k^2-4(k^2+k-5)\ge 0$

$4k^2-4k^2-4k+20\ge 0$

$4k\le 20 \Rightarrow k\le 5$

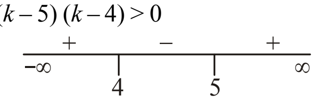
(ii) $p(5)>0$

$\Rightarrow f(5)>0$; $25-10k+k^2+k-5>0$

$\Rightarrow k^2-9k+20>0$

$\Rightarrow k(k-4)-5(k-4)>0$

$\Rightarrow (k-5)(k-4)>0$



$\Rightarrow k\in(-\infty,4)\cup(5,\infty)$

(iii) $\frac{\text{Sum of roots}}{2}<5$

$\Rightarrow -\frac{b}{2a}=\frac{2k}{2}<5$

$\Rightarrow k<5$

The intersection of (i), (ii) & (iii) gives $k\in(-\infty,4)$.

147. (a) Given equation is $x^2-(a-2)x-a-1=0$

$\Rightarrow \alpha+\beta=a-2$; $\alpha\beta=-(a+1)$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=a^2-2a+6=(a-1)^2+5$

For min. value of $\alpha^2+\beta^2$, $a-1=0$

$\Rightarrow a=1$.