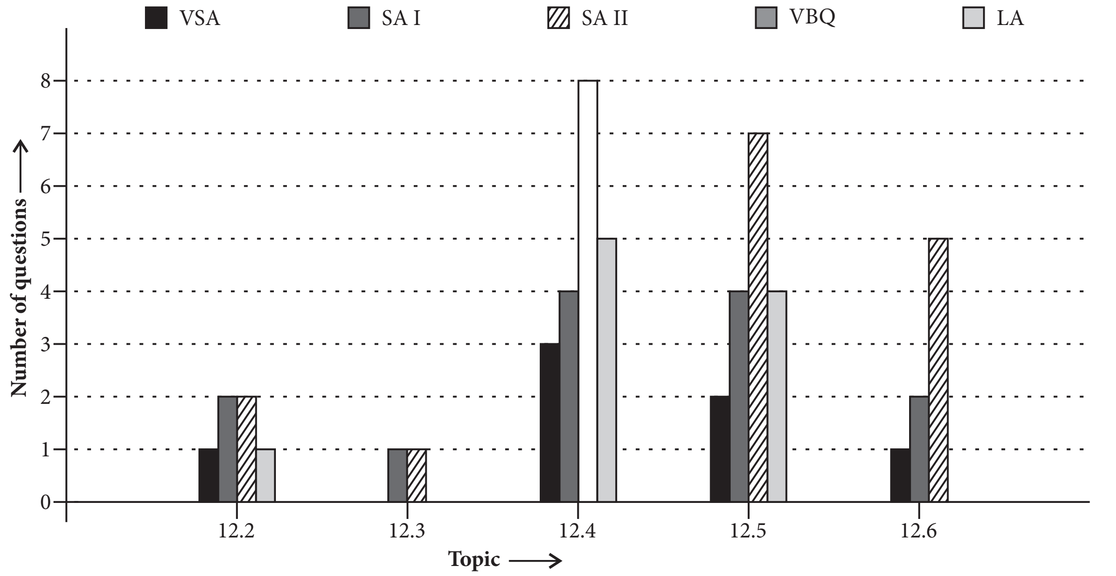
12 Atoms

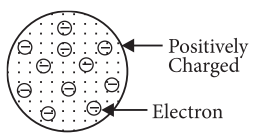
Topicwise Analysis of Last 10 Years' CBSE Board Questions



- Maximum weightage is of *Bohr Model of Hydrogen Atom.*
- Maximum VSA, SAI, SAII and LA type questions were asked from *Bohr Model of Hydrogen Atom.*
- No VBQ type questions were asked till now.

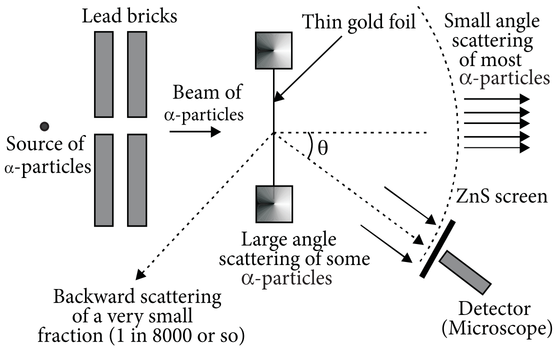
QUICK RECAP

- **Thomson's model of atom :** It was proposed by J. J. Thomson in 1898. According to this model, the positive charge of the atom is uniformly distributed throughout the volume of the atom and the negatively charged electrons are embedded in it like seeds in a watermelon.

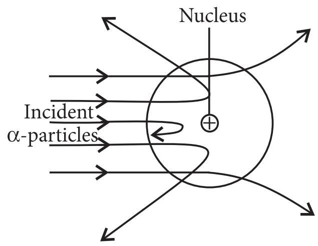


Rutherford's α-scattering experiment

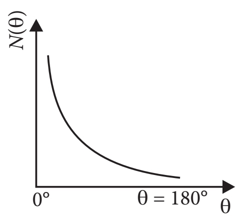
- Rutherford and his two associates, Geiger and Marsden, studies the scattering of the α-particles from a thin gold foil in order to investigate the structure of the atom.



Schematic arrangement of the Geiger-Marsden experiment



- **Rutherford's observations and results :**
  - Most of the α-particles pass through the gold foil without any deflection. This shows that most of the space in an atom is empty.
  - Few α-particles got scattered, deflecting at various angles from 0 to π. This shows that atom has a small positively charged core called 'nucleus' at centre of atom, which deflects the positively charged α-particles at different angles depending on their distance from centre of nucleus.
  - Very few α-particles (1 in 8000) suffers deflection of 180°. This shows that size of nucleus is very small, nearly 1/8000 times the size of atom.



This graph shows deflection of number of particles with angle of deflection θ.

Rutherford's α-scattering formulae

- Number of α particles scattered per unit area, $N(\theta)$ at scattering angle θ varies inversely as $\sin^4(\theta/2)$, *i.e.*,

$$N(\theta) \propto \frac{1}{\sin^4(\theta/2)}$$

- **Impact parameter :** It is defined as the perpendicular distance of the initial velocity vector of the alpha particle from the centre of the nucleus, when the particle is far away from the nucleus of the atom.
  - The scattering angle θ of the α particle and impact parameter b are related as

$$b = \frac{Ze^2 \cot(\theta/2)}{4\pi\varepsilon_0 K}$$

  where K is the kinetic energy of the α-particle and Z is the atomic number of the nucleus.
  - Smaller the impact parameter, larger the angle of scattering θ.
- **Distance of closest approach :** At the distance of closest approach whole kinetic energy of the alpha particles is converted into potential energy.
  - Distance of closest approach

$$r_0 = \frac{2Ze^2}{4\pi\varepsilon_0 K}$$

Rutherford's nuclear model of the atom :

According to this the entire positive charge and most of the mass of the atom is concentrated in a small volume known as the nucleus with electrons revolving around it just as planets revolve around the sun.

Bohr's model :

Bohr combined classical and early quantum concepts and gave his theory of hydrogen and hydrogen-like atoms which have only one orbital electron. His postulates are

- An electron can revolve around the nucleus only in certain allowed circular orbits of definite energy and in these orbits it does not radiate. These orbits are known as stationary orbits.
- Angular momentum of the electron in a stationary orbit is an integral multiple of $h/2\pi$.

$$i.e.,\ L = \frac{nh}{2\pi} \quad \text{or,} \quad mvr = \frac{nh}{2\pi}$$

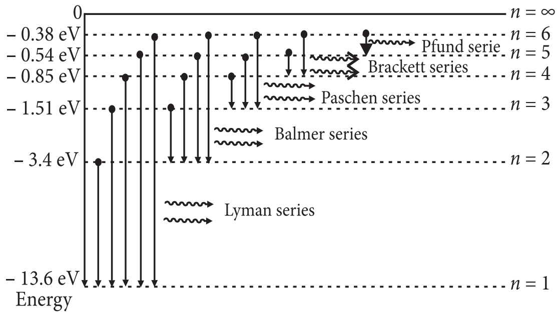
This is known as Bohr's quantisation condition.

- The emission of radiation takes place when an electron makes a transition from a higher to a lower orbit. The frequency of the radiation is given by
$$\upsilon = \frac{E_2 - E_1}{h}$$
where E_2 and E_1 are the energies of the electron in the higher and lower orbits respectively.

Bohr's formulae

- Radius of n^{th} orbit
$$r_n = \frac{4\pi\varepsilon_0 n^2 h^2}{4\pi^2 m Z e^2}; \quad r_n = \frac{0.53 n^2}{Z}\ \text{Å}$$
- Velocity of the electron in the n^{th} orbit
$$v_n = \frac{1}{4\pi\varepsilon_0}\frac{2\pi Z e^2}{nh} = \frac{2.2\times 10^6\ Z}{n}\ \text{m/s}$$
The kinetic energy of the electron in the n^{th} orbit
$$K_n = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r_n} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{2\pi^2 m e^4 Z^2}{n^2 h^2}$$
$$= \frac{13.6 Z^2}{n^2}\ \text{eV}$$
The potential energy of the electron in the n^{th} orbit
$$U_n = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n} = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{4\pi^2 m e^4 Z^2}{n^2 h^2}$$
$$= \frac{-27.2 Z^2}{n^2}\ \text{eV}$$
- Total energy of electron in the n^{th} orbit
$$E_n = U_n + K_n = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{2\pi^2 m e^4 Z^2}{n^2 h^2}$$
$$= -\frac{13.6 Z^2}{n^2}\ \text{eV}$$
$K_n = -E_n$, $U_n = 2E_n = -2K_n$
- Frequency of the electron in the n^{th} orbit
$$\upsilon_n = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{4\pi^2 Z^2 e^4 m}{n^3 h^3} = \frac{6.62\times 10^{15} Z^2}{n^3}$$
- Wavelength of radiation in the transition from $n_2 \to n_1$ is given by $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$
where R is called Rydberg's constant.
$$R = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2\pi^2 m e^4}{ch^3} = 1.097\times 10^7\ \text{m}^{-1}$$

Spectral series of hydrogen atom : When the electron in a H-atom jumps from higher energy level to lower energy level, the difference of energies of the two energy levels is emitted as radiation of particular wavelength, known as spectral line. Spectral lines of different wavelengths are obtained for transition of electron between two different energy levels, which are found to fall in a number of spectral series given by



- **Lyman series**
  - Emission spectral lines corresponding to the transition of electron from higher energy levels (n_2 = 2, 3, ...,∞) to first energy level (n_1 = 1) constitute Lyman series.
$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right]$$
where n_2 = 2, 3, 4, ...,∞
  - Series limit line (shortest wavelength) of Lyman series is given by
$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = R \quad \text{or} \quad \lambda = \frac{1}{R}$$
  - The first line (longest wavelength) of the Lyman series is given by
$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3R}{4} \quad \text{or} \quad \lambda = \frac{4}{3R}$$
  - Lyman series lie in the ultraviolet region of electromagnetic spectrum.
  - Lyman series is obtained in emission as well as in absorption spectrum.
- **Balmer series**
  - Emission spectral lines corresponding to the transition of electron from higher energy levels (n_2 = 3, 4, ...∞) to second energy level (n_1 = 2) constitute Balmer series.

$$\frac{1}{\lambda}=R\left[\frac{1}{2^2}-\frac{1}{n_2^2}\right]$$

where $n_2 = 3, 4, 5..........,\infty$

- Series limit line (shortest wavelength) of Balmer series is given by

$$\frac{1}{\lambda}=R\left[\frac{1}{2^2}-\frac{1}{\infty^2}\right]=\frac{R}{4} \text{ or } \lambda=\frac{4}{R}$$

- The first line (longest wavelength) of the Balmer series is given by

$$\frac{1}{\lambda}=R\left[\frac{1}{2^2}-\frac{1}{3^2}\right]=\frac{5R}{36} \text{ or } \lambda=\frac{36}{5R}$$

- Balmer series lie in the visible region of electromagnetic spectrum.
- This series is obtained only in emission spectrum.

▶ **Paschen series**

- Emission spectral lines corresponding to the transition of electron from higher energy levels ($n_2 = 4, 5, ...,\infty$) to third energy level ($n_1 = 3$) constitute Paschen series.

$$\frac{1}{\lambda}=R\left[\frac{1}{3^2}-\frac{1}{n_2^2}\right]$$

where $n_2 = 4, 5, 6.........,\infty$

- Series limit line (shortest wavelength) of the Paschen series is given by

$$\frac{1}{\lambda}=R\left[\frac{1}{3^2}-\frac{1}{\infty^2}\right]=\frac{R}{9} \text{ or } \lambda=\frac{9}{R}$$

The first line (longest wavelength) of the Paschen series is given by

$$\frac{1}{\lambda}=R\left[\frac{1}{3^2}-\frac{1}{4^2}\right]=\frac{7R}{144} \text{ or } \lambda=\frac{144}{7R}$$

- Paschen series lie in the infrared region of the electromagnetic spectrum.
- This series is obtained only in the emission spectrum.

▶ **Brackett Series**

- Emission spectral lines corresponding to the transition of electron from higher energy levels ($n_2 = 5, 6, 7, ...,\infty$) to fourth energy level ($n_1 = 4$) constitute Brackett series.

$$\frac{1}{\lambda}=R\left[\frac{1}{4^2}-\frac{1}{n_2^2}\right]$$

where $n_2 = 5, 6, 7..........,\infty$

Series limit line (shortest wavelength) of Brackett series is given by

$$\frac{1}{\lambda}=R\left[\frac{1}{4^2}-\frac{1}{\infty^2}\right]=\frac{R}{16} \text{ or } \lambda=\frac{16}{R}$$

- The first line (longest wavelength) of Brackett series is given by

$$\frac{1}{\lambda}=R\left[\frac{1}{4^2}-\frac{1}{5^2}\right]=\frac{9R}{400} \text{ or } \lambda=\frac{400}{9R}$$

- Brackett series lie in the infrared region of the electromagnetic spectrum.
- This series is obtained only in the emission spectrum.

▶ **Pfund series**

- Emission spectral lines corresponding to the transition of electron from higher energy levels ($n_2 = 6, 7, 8,.......,\infty$) to fifth energy level ($n_1 = 5$) constitute Pfund series.

$$\frac{1}{\lambda}=R\left[\frac{1}{5^2}-\frac{1}{n_2^2}\right]$$

where $n_2 = 6, 7,...........,\infty$

- Series limit line (shortest wavelength) of Pfund series is given by

$$\frac{1}{\lambda}=R\left[\frac{1}{5^2}-\frac{1}{\infty^2}\right]=\frac{R}{25} \text{ or } \lambda=\frac{25}{R}$$

- The first line (longest wavelength) of the Pfund series is given by

$$\frac{1}{\lambda}=R\left[\frac{1}{5^2}-\frac{1}{6^2}\right]=\frac{11R}{900} \text{ or } \lambda=\frac{900}{11R}$$

- Pfund series also lie in the infrared region of electromagnetic spectrum.
- This series is obtained only in the emission spectrum.

▶ Number of spectral lines due to transition of electron from n^{th} orbit to lower orbit is

$$N=\frac{n(n-1)}{2}$$

Ionization energy and ionization potential

▶ Ionisation : The process of knocking an electron out of the atom is called ionisation. ionisation energy $=\frac{13.6}{n^2}$ eV

▶ Ionisation energy : The energy required, to knock an electron completely out of the atom.

▶ Ionisation potential $=\frac{13.6Z^2}{n^2}$ V

Previous Years' CBSE Board Questions

12.2 Alpha Particle scattering and Rutherford's Nuclear Model of Atom

VSA (1 mark)

1. Why is the classical (Rutherford) model for an atom of electron orbiting around the nucleus not able to explain the atomic structure? *(Delhi 2012C)*

SA I (2 marks)

2. Using Rutherford's model of the atom, derive the expression for the total energy of the electron in hydrogen atom. What is the significance of total negative energy possessed by the electron? *(AI 2014)*

3. In an experiment on α–particle scattering by a thin foil of gold, draw a plot showing the number of particles scattered versus the scattering angle θ. Why is it that a very small fraction of the particles are scattered at $\theta > 90°$?
Write two important conclusions that can be drawn regarding the structure of the atom from the study of this experiment. *(Foreign 2013)*

SA II (3 marks)

4. Draw a schematic arrangement of the Geiger – Marsden experiment for studying α-particle scattering by a thin foil of gold. Describe briefly, by drawing trajectories of the scattered α-particles, how this study can be used to estimate the size of the nucleus. *(Foreign 2010, AI 2009)*

5. State the basic assumption of the Rutherford model of the atom. Explain, in brief, why this model cannot account for the stability of an atom. *(Delhi 2010C)*

LA (5 marks)

6. In Rutherford scattering experiment, draw the trajectory traced by α-particles in the coulomb field of target nucleus and explain how this led to estimate the size of the nucleus. *(3/5, AI 2015C)*

12.3 Atomic Spectra

SA I (2 marks)

7. Calculate the shortest wavelength in the Balmer series of hydrogen atom. In which region (infrared, visible, ultraviolet) of hydrogen spectrum does this wavelength lie? *(AI 2015)*

SA II (3 marks)

8. The second member of Lyman series in hydrogen spectrum has wavelength 5400 Å. Find the wavelength of the first member. *(Delhi 2008)*

12.4 Bohr Model of Hydrogen Atom

VSA (1 mark)

9. What is the ratio of radii of the orbits corresponding to first excited state and ground state in a hydrogen atom? *(Delhi 2010)*

10. Define ionisation energy. What is its value for a hydrogen atom? *(AI 2010)*

11. State Bohr's quantisation condition for defining stationary orbits. *(Foreign 2010)*

SA I (2 marks)

12. State Bohr postulate of hydrogen atom that gives the relationship for the frequency of emitted photon in a transition. *(Foreign 2016)*

13. Show that the radius of the orbit in hydrogen atom varies as n^2, where n is the principal quantum number of the atom. *(Delhi 2015)*

14. Using Bohr's postulates of the atomic model, derive the expression for the radius of n^{th} electron orbit. Hence obtain the expression for Bohr's radius. *(AI 2014)*

15. In the ground state of hydrogen atom, its Bohr radius is given as 5.3×10^{-11} m. The atom is excited such that the radius becomes 21.2×10^{-11} m. Find (i) the value of the principal quantum number and (ii) the total energy of the atom in this excited state. *(Delhi 2013C)*

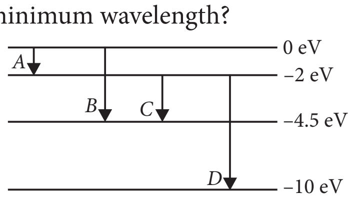
SA II (3 marks)

16. (a) The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. Calculate its radius in $n = 3$ orbit.
(b) The total energy of an electron in the first excited state of the hydrogen atom is 3.4 eV. Find out its (i) kinetic energy and (ii) potential energy in this state. *(Delhi 2014C)*

17. Using Bohr's postulates, obtain the expression for the total energy of the electron in the stationary states of the hydrogen atom. Hence draw the energy level diagram showing how the line spectra corresponding to Balmer series occur due to transition between energy levels. *(Delhi 2013)*

18. Using Bohr's postulates for hydrogen atom, show that the total energy (E) of the electron in the stationary states can be expressed as the sum of kinetic energy (K) and potential energy (U), where $K = -2U$. Hence deduce the expression for the total energy in the n^{th} energy level of hydrogen atom. *(Foreign 2012)*

19. The energy levels of a hypothetical atom are shown below. Which of the shown transitions will result in the emission of a photon of wavelength 275 nm?
Which of these transitions correspond to emission of radiation of (i) maximum and (ii) minimum wavelength?



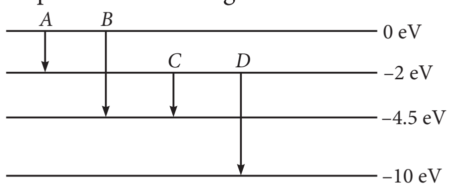
(Delhi 2011)

20. Using the postulates of Bohr's model of hydrogen atom, obtain an expression for the frequency of radiation emitted when atom make a transition from the higher energy state with quantum number n_i to the lower energy state with quantum number n_f ($n_f < n_i$). *(Foreign 2011)*

21. Using the relevant Bohr's postulates, derive the expressions for the
(a) speed of the electron in the n^{th} orbit,
(b) radius of the n^{th} orbit of the electron, in hydrogen atom. *(Delhi 2010C)*

22. State any two postulates of Bohr's theory of hydrogen atom.
What is the maximum possible number of spectral lines observed when the hydrogen atom is in its second excited state? Justify your answer. Calculate the ratio of the maximum and minimum wavelengths of the radiations emitted in this process. *(AI 2010C)*

23. (a) The energy levels of an atom are as shown below. Which of them will result in the transition of a photon of wavelength 275 nm?



(b) Which transition corresponds to emission of radiation of maximum wavelength? *(Delhi 2009)*

LA (5 marks)

24. (a) Write two important limitations of Rutherford model which could not explain the observed features of atomic spectra. How were these explained in Bohr's model of hydrogen atom?
(b) Using Bohr's postulates, obtain the expression for the radius of the n^{th} orbit in hydrogen atom. *(4/5, Delhi 2015C)*

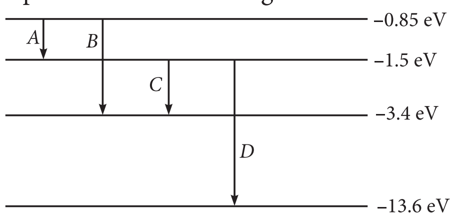
25. Using Bohr's postulates, derive the expression for the total energy of the electron in the stationary states of the hydrogen atom. *(3/5, Foreign 2014)*

26. (a) Using Bohr's theory of hydrogen atom, derive the expression for the total energy of the electron in the stationary states of the atom.
(b) If electron in the atom is replaced by a particle (muon) having the same charge but mass about 200 times as that of the electron to form a muonic atom, how would (i) the radius and (ii) the ground state energy of this be affected? *(3/5, Delhi 2012C)*

27. (a) Using postulates of Bohr's theory of hydrogen atom, show that
(i) the radii of orbits increase as n^2, and

(ii) the total energy of the electron increase as $1/n^2$, where n is the principal quantum number of the atom. *(3/5, AI 2011C)*

28. The energy level diagram of an element is given below. Identify, by doing necessary calculations, which transition corresponds to the emission of a spectral line of wavelength 102.7 nm.



(Delhi 2008)

12.5 The Line Spectra of the Hydrogen Atom

VSA (1 mark)

29. When is H_α line of the Balmer series in the emission spectrum of hydrogen atom obtained? *(Delhi 2013C)*

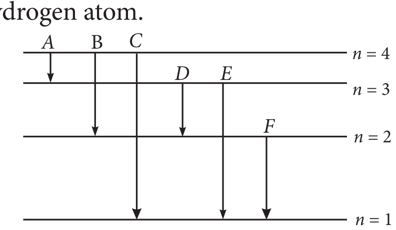
30. What is the maximum number of spectral lines emitted by a hydrogen atom when it is in the third excited state? *(AI 2013C)*

SA I (2 marks)

31. Define ionization energy.
How would the ionization energy change when electron in hydrogen atom is replaced by a particle of mass 200 times that of the electron but having the same charge? *(AI 2016)*

32. An electron jumps from fourth to first orbit in an atom. How many maximum number of spectral lines can be emitted by the atom? To which series these lines correspond? *(Foreign 2016)*

33. The figure shows energy level diagram of hydrogen atom.



(a) Find out the transition which results in the emission of a photon of wavelength 496 nm.

(b) Which transition corresponds to the emission of radiation of maximum wavelength? Justify your answer. *(AI 2015C)*

34. (i) In hydrogen atom, an electron undergoes transition from 2nd excited state to the first excited state and then to the ground state. Identify the spectral series to which these transitions belong.

(ii) Find out the ratio of the wavelengths of the emitted radiations in the two cases. *(AI 2012C)*

SA II (3 marks)

35. Using Rydberg formula, calculate the longest wavelength belonging to Lyman and Balmer series of hydrogen spectrum. In which region these transitions lie? *(3/5, Foreign 2015)*

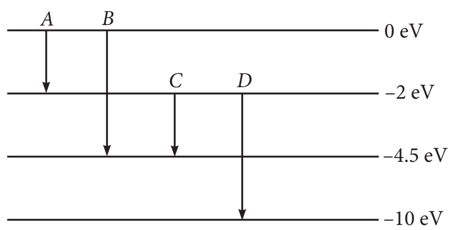
36. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. Upto which energy level the hydrogen atoms would be excited?
Calculate the wavelengths of the first member of Lyman and first member of Balmer series. *(Delhi 2014)*

37. The value of ground state energy of hydrogen atom is −13.6 eV.

(i) Find the energy required to move an electron from the ground state to the first excited state of the atom.

(ii) Determine (a) the kinetic energy and (b) orbital radius in the first excited state of the atom.
(Given the value of Bohr radius = 0.53 Å). *(AI 2014C)*

38. (a) The energy levels of a hypothetical hydrogen-like atom are shown in the figure. Find out the transition, from the ones shown in the figure, which will result in the emission of a photon of wavelength 275 nm.



(b) Which of these transitions corresponds to the emission of radiation of (i) maximum and (ii) minimum wavelength? *(Foreign 2013)*

39. The electron in hydrogen atom is initially in the third excited state. What is the maximum number of spectral lines which can be emitted when it finally moves to the ground state? *(1/3, Delhi 2012)*

40. The ground state energy of hydrogen atom is –13.6 eV. If an electron makes a transition from an energy level – 0.85 eV to –3.4 eV, calculate the wavelength of the spectral line emitted. To which series of hydrogen spectrum does this wavelength belong? *(AI 2012)*

41. The electron in a given Bohr orbit has a total energy of –1.5 eV. Calculate its
(i) kinetic energy.
(ii) potential energy.
(iii) wavelength of radiation emitted, when this electron makes a transition to the ground state.
[Given : Energy in the ground state = –13.6 eV and Rybderg's constant = 1.09×10^7 m^{-1}] *(Delhi 2011C)*

LA (5 marks)

42. Using Rydberg formula, calculate the wave-lengths of the spectral lines of the first member of the Lyman series and of the Balmer series. *(2/5, Foreign 2014)*

43. Using Bohr's postulates, derive the expression for the frequency of radiation emitted when electron in hydrogen atom undergoes transition from higher energy state (quantum number n_i) to the lower state, (n_f). When electron in hydrogen atom jumps from energy state $n_i = 4$ to $n_f = 3, 2, 1$. Identify the spectral series to which the emission lines belong. *(AI 2013)*

44. Calculate the wavelength of the first spectral line in the corresponding Lyman series of this atom. *(2/5, Delhi 2012C)*

45. Calculate the wavlength of H_α line in Balmer series of hydrogen atom, given Rydberg constant $R = 1.097 \times 10^7$ m^{-1}. *(2/5, AI 2011C)*

12.6 de Broglie's Explanation of Bohr's Second Postulate of Quantisation

VSA (1 mark)

46. State de-Broglie hypothesis. *(Delhi 2012)*

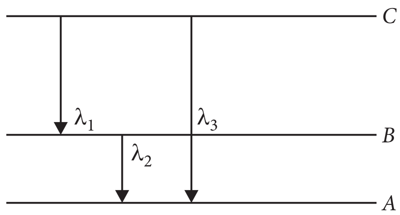
SA I (2 marks)

47. Calculate the de-Broglie wavelength of the electron orbiting in the $n = 2$ state of hydrogen atom. *(AI 2016)*

48. Use de-Broglie's hypothesis to write the relation for the n^{th} radius of Bohr orbit in terms of Bohr's quantization condition of orbital angular momentum. *(Foreign 2016)*

SA II (3 marks)

49. (i) State Bohr's quantization condition for defining stationary orbits. How does de Broglie hypothesis explain the stationary orbits ?
(ii) Find the relation between the three wavelengths λ_1, λ_2 and λ_3 from the energy level diagram shown below.



(Delhi 2016)

50. The kinetic energy of the electron orbiting in the first excited state of hydrogen atom is 3.4 eV. Determine the de Broglie wavelength associated with it. *(Foreign 2015)*

51. An electron is revolving around the nucleus with a constant speed of 2.2×10^8 m/s. Find the de Broglie wavelength associated with it. *(AI 2014C)*

52. Using Bohr's second postulate of quantization of orbital angular momentum show that the circumference of the electron in the n^{th} orbital state in hydrogen atom is n times the de Broglie wavelength associated with it. *(2/3, Delhi 2012)*

53. (a) Using de Broglie's hypothesis, explain with the help of a suitable diagram, Bohr's second postulate of quantization of energy levels in a hydrogen atom.
(b) The ground state energy of hydrogen atom is –13.6 eV. What are the kinetic and potential energies of the electron in this state? *(AI 2011)*

Detailed Solutions

1. According to electromagnetic theory, electron revolving around the nucleus are continuously accelerated. Since an accelerated charge emits energy, the radius of the circular path of a revolving electron should go on decreasing and ultimately it should fall into the nucleus. So, it could not explain the structure of the atom. As matter is stable, we cannot expect the atoms to collapse.

2. An electron revolving in an orbit of H-atom, has both kinetic energy and electrostatic potential energy. Kinetic energy of the electron revolving in a circular orbit of radius r is $E_K = \frac{1}{2}mv^2$

Since, $\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$

$$\therefore \quad E_K = \frac{1}{2} \times \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \quad \text{or} \quad E_K = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r} \qquad \text{... (i)}$$

Electrostatic potential energy of electron of charge $-e$ revolving around the nucleus of charge $+e$ in an orbit of radius r is

$$E_P = \frac{1}{4\pi\varepsilon_0}\frac{+e \times -e}{r} \quad \text{or} \quad E_P = \frac{-1}{4\pi\varepsilon_0}\frac{e^2}{r} \qquad \text{... (ii)}$$

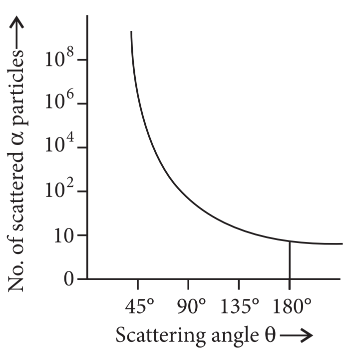
So, total energy of electron in orbit of radius r is

$$E = E_K + E_P \quad \text{or} \quad E = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r} - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\text{or} \quad E = \frac{-1}{4\pi\varepsilon_0}\frac{e^2}{2r}$$

The –ve sign of the energy of electron indicates that the electron and nucleus together form a bound system *i.e.*, electron is bound to the nucleus.

3.



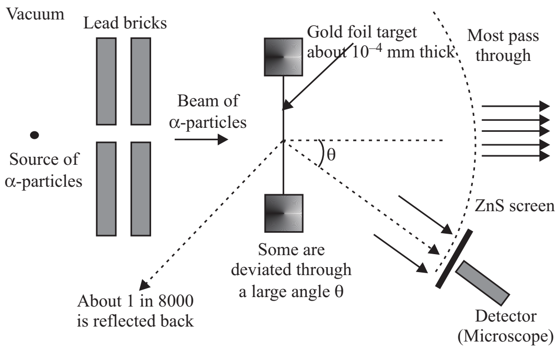
A very small fraction of α-particles are scattered at $\theta > 90°$ because the size of nucleus is very small nearly 1/8000 times the size of atom. So, a few α-particles experience a strong repulsive force and turn back.

Conclusions :

(i) Entire positive charge and most of the mass of the atom is concentrated in the nucleus with the electrons some distance away.

(ii) Size of the nucleus is about 10^{-15} m to 10^{-14} m, while size of the atom is 10^{-10} m, so the electrons are at distance 10^4 m to 10^5 m from the nucleus, and being large empty space in the atom, most α particles go through the empty space.

4.



Only a small fraction of the number of incident α-particles (1 in 8000) rebound back. This shows that the number of α-particles undergoing head on collision is small. This implies that the entire positive charge of the atom is concentrated in a small volume.

So, this experiment is an important way to determine an upper limit on the size of nucleus.

5. Assumptions of Rutherford's atomic model :

(i) Every atom consists of a tiny central core called the atomic nucleus, in which the entire positive charge and atmost entire mass of the atom are concentrated.

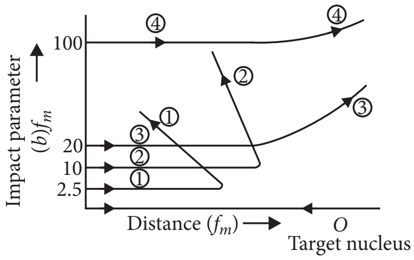
(ii) The size of nucleus is of the order of 10^{-15} m, which is very small as compared to the size of the atom which is of the order of 10^{-10} m.

(iii) The atomic nucleus is surrounded by certain number of electrons. As atom on the whole is electrically neutral, the total negative charge of electrons surrounding the nucleus is equal to total positive charge on the nucleus.

(iv) The electrons revolve around the nucleus in various circular orbits.

Refer to answer 1.

6.



The size of the nucleus can be obtained by finding impact parameter b using trajectories of α-particle. The impact parameter is the perpendicular distance of the initial velocity vector of α-particle from the central line of nucleus, when it is far away from the atom. Rutherford calculated impact parameter as

$$b = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2 \cot(\theta/2)}{E}$$

where, $E = KE$ of α-particle

θ = scattering angle

Z = atomic number of atom

The size of the nucleus is smaller than the impact parameter.

7. Wavelength (λ) of Balmer series is given by

$$\frac{1}{\lambda} = R_H\left[\frac{1}{2^2} - \frac{1}{n_i^2}\right] \text{ where } n_i = 3, 4, 5, ...$$

For shortest wavelength, when transition of electrons take place from $n_i = \infty$ to $n_f = 2$ orbit, wavelength of emitted photon is shortest.

$$\frac{1}{\lambda_{min}} = R_H\left[\frac{1}{2^2} - \frac{1}{\infty}\right] = \frac{1.097 \times 10^7}{4}$$

$\therefore \quad \lambda_{min} = 3.646 \times 10^{-7}$ m = 3646 Å

This wavelength lies in visible region of electromagnetic spectrum.

8. For Lyman series,

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \qquad [\therefore n = 2, 3, 4, ...]$$

Let λ_1 and λ_2 be the wavelength of the first and second line respectively, then

$$\frac{1}{\lambda_1} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = R\left(1 - \frac{1}{4}\right) = \frac{3}{4}R \qquad ...(i)$$

and $$\frac{1}{\lambda_2} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = R\left(1 - \frac{1}{9}\right) = \frac{8}{9}R \qquad ...(ii)$$

Dividing (ii) by (i), we get

$$\frac{\frac{1}{\lambda_2}}{\frac{1}{\lambda_1}} = \frac{\frac{8}{9}R}{\frac{3}{4}R} \Rightarrow \frac{1}{\lambda_2} \times \frac{\lambda_1}{1} = \frac{8}{9} \times \frac{4}{3}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{32}{27}$$

As $\lambda_2 = 5400$

$$\therefore \quad \lambda_1 = \frac{32}{27} \times \lambda_2 = \frac{32}{27} \times 5400 = 6400 \text{ Å}$$

9. Since $r \propto n^2$.

For ground state, $n_1 = 1$

For 1st excited state, $n_2 = 2$

$\therefore$ Required ratio of radii of the orbits

$$\frac{r_2}{r_1} = \frac{n_2^2}{n_1^2} = \frac{2^2}{1} = 4:1$$

10. Ionisation energy for an atom is defined as the energy required to remove an electron completely from the outermost shell of the atom.

For hydrogen atom,

$E = E_\infty - E_1 = 0 - (-13.6)$ eV = 13.6 eV

11. Quantum condition : The stationary orbits are those in which angular momentum of electron is an integral multiple of $\frac{h}{2\pi}$ *i.e.*,

$$mvr = n\frac{h}{2\pi}, \; n = 1, 2, 3, ... \qquad ...(ii)$$

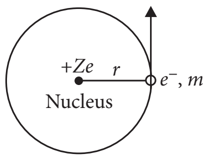
Integer n is called the principal quantum number. This equation is called Bohr's quantum condition.

12. Frequency condition : An atom can emit or absorb radiation in the form of discrete energy photons only when an electron jumps from a higher to a lower orbit or from a lower to a higher orbit, respectively.

$h\upsilon = E_i - E_f$

where υ is frequency of radiation emitted, E_i and E_f are the energies associated with stationary orbits of principal quantum number n_i and n_f respectively (where $n_i > n_f$).

13 According to Bohr's theory, a hydrogen atom consists of a nucleus with a positive charge Ze, and a single electron of charge $-e$, which revolves around it in a circular orbit of radius r.



Here Z is the atomic number and for hydrogen Z =1. The electrostatic force of attraction between the hydrogen nucleus and the electron is

$$F = \frac{k\,e.e}{r^2} = \frac{k\,e^2}{r^2} \quad \left[\text{where } k = \frac{1}{4\pi\varepsilon_0}\right]$$

To keep the electron in its orbit, the centripetal force on the electron must be equal to the electrostatic attraction. Therefore,

$$\frac{mv^2}{r} = \frac{ke^2}{r^2}$$

or $\quad mv^2 = \dfrac{ke^2}{r}$ ...(i)

or $\quad r = \dfrac{ke^2}{mv^2}$ ...(ii)

where m is the mass of the electron, and v, its speed in an orbit of radius r.

Bohr's quantisation condition for angular momentum is

$L = mvr = \dfrac{nh}{2\pi}$ or $r = \dfrac{nh}{2\pi mv}$ ...(iii)

From equation (ii) and (iii), we get

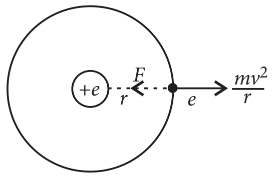
$\dfrac{ke^2}{mv^2} = \dfrac{nh}{2\pi mv}$ or $v = \dfrac{2\pi ke^2}{nh}$ ...(iv)

Substituting this value of v in equation (iii), we get

$$r = \frac{nh}{2\pi m}\cdot\frac{nh}{2\pi ke^2}$$

$\Rightarrow \quad r = \dfrac{n^2h^2}{4\pi^2 mke^2} \quad \therefore\ r \propto n^2$

14. Radius of n^{th} orbit of hydrogen atom : In H-atom, an electron having charge $-e$ revolves around the nucleus of charge $+e$ in a circular orbit of radius r, such that necessary centripetal force is provided by the electrostatic force of attraction between the electron and nucleus.



i.e., $\dfrac{mv^2}{r} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e.e}{r^2}$ or $mv^2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r}$ ...(i)

From Bohr's quantization condition

$mvr = \dfrac{nh}{2\pi}$ or $v = \dfrac{nh}{2\pi mr}$ ...(ii)

Using equation (ii) in (i), we get

$$m\cdot\left(\frac{nh}{2\pi mr}\right)^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \quad \text{or} \quad \frac{m\cdot n^2h^2}{4\pi^2m^2r^2} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

or $\quad r = \dfrac{n^2h^2\varepsilon_0}{\pi me^2}$ ...(iii)

where n = 1, 2, 3, ... is principal quantum number. Equation (iii), gives the radius of n^{th} orbit of H-atom. So the radii of the orbits increase proportionally with n^2 *i.e.*, $[r \propto n^2]$. Radius of first orbit of H-atom is called Bohr radius a_0 and is given by

$a_0 = \dfrac{h^2\varepsilon_0}{\pi me^2}$ for n = 1 or a_0 = 0.529 Å

So, radius of n^{th} orbit of H-atom then becomes $r = n^2 \times 0.529$ Å

15. (i) Since, $r \propto n^2$; $\dfrac{r_n}{r_g} = \dfrac{n^2}{1^2}$

$$\frac{21.2\times10^{-11}}{5.3\times10^{-11}} = \frac{n^2}{1}$$

$\dfrac{212}{53} = n^2 \Rightarrow n^2 = 4$ or $n = \sqrt{4} = 2$

(ii) We know that $E = \dfrac{-13.6}{n^2} = \dfrac{-13.6}{4} = -3.4$ eV

16. (a) Radius of n^{th} orbit, $r_n \propto n^2$

$\Rightarrow \quad \dfrac{r_3}{r_1} = \dfrac{3^2}{1^2} = 9$

or $\quad r_3 = 9r_1 = 9 \times 5.3 \times 10^{-11}$

$= 47.7 \times 10^{-11}$ m $= 4.77 \times 10^{-10}$ m

(b) (i) Kinetic Energy,

$E_k = -E = -(-3.4) \times 1 = 3.4$ eV

(ii) Potential Energy,

$E_p = 2E = -3.4 \times 2 = -6.8$ eV

17. (i) According to Bohr's postulates, in a hydrogen atom, as single electron revolves around a nucleus of charge $+e$. For an electron moving with a uniform speed in a circular orbit of a given radius, the centripetal force is provided by coulomb force of attraction between the electron and the nucleus. The gravitational attraction may be neglected as the mass of electron and proton is very small.

So, $\frac{mv^2}{r} = \frac{ke^2}{r^2}$ $\left(\text{Where, } k = \frac{1}{4\pi\varepsilon_0}\right)$

or $mv^2 = \frac{ke^2}{r}$ ...(i)

Where, m = mass of electron
r = radius of electronic orbit
v = velocity of electron
Again, by Bohr's second postulates

$$mvr = \frac{nh}{2\pi}$$

Where, n = 1, 2, 3......

or $v = \frac{nh}{2\pi mr}$

Putting the value of v in eq. (i)

$$m\left(\frac{nh}{2\pi mr}\right)^2 = \frac{ke^2}{r} \Rightarrow r = \frac{n^2h^2}{4\pi^2kme^2} \quad \text{...(ii)}$$

Kinetic energy of electron,

$$E_k = \frac{1}{2}mv^2 = \frac{ke^2}{2r} \quad \left(\because \frac{mv^2}{r} = \frac{ke^2}{r^2}\right)$$

Using eq. (ii) we get

$$E_k = \frac{ke^2}{2}\frac{4\pi^2kme^2}{n^2h^2} = \frac{2\pi^2k^2me^4}{n^2h^2}$$

Potential energy of electron,

$$E_p = -\frac{k(e)\times(e)}{r} = -\frac{ke^2}{r}$$

Using eq. (ii), we get

$$E_p = -ke^2 \times \frac{4\pi^2kme^2}{n^2h^2} = -\frac{4\pi k^2me^4}{n^2h^2}$$

Hence, total energy of the electron in the n^{th} orbit
$E = E_p + E_k$

$$= -\frac{4\pi^2k^2me^4}{n^2h^2} + \frac{2\pi^2k^2me^4}{n^2h^2}$$

$$= -\frac{2\pi^2k^2me^4}{n^2h^2} = -\frac{13.6}{n^2}\text{ eV}$$

When the electron in a hydrogen atom jumps from higher energy level to the lower energy level, the difference of energies of the two energy levels is emitted as a radiation of particular wavelength. It is called a spectral line.

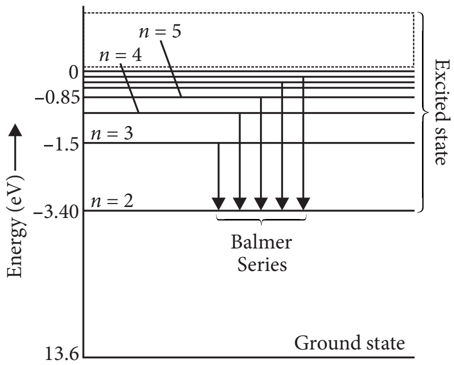
(ii) In H-atom, when an electron jumps form the orbit n_i to orbit n_f, the wavelength of the emitted radiation is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]; \; R = 1.09 \times 10^7 \text{ m}^{-1}$$

For Balmar series, n_f = 2 and n_i = 3, 4, 5, ...

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$$

Where, n_i = 3, 4, 5, ...
These spectral lines lie on the visible region.



18. According to Bohr's postulates for hydrogen atom, electron revolves in a circular orbit around the heavy positively charged nucleus. These are the stationary (orbits) states of the atom.
For a particular orbit, electron moves there, so it has kinetic energy.
Also, there is potential energy due to charge on electron and heavy positively charged nucleus.
Hence, total energy (E) of atom is sum of kinetic energy (K) and potential energy (U).
i.e., $E = K + U$
Let us assume that the nucleus has positive charge Ze.
An electron moving with a constant speed v along a circle of radius r with centre at the nucleus.
Force acting on electron due to nucleus is given by

$$F = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

The acceleration of electron $= \frac{v^2}{r}$ (towards the centre).
If m = mass of an electron, then from Newton's second law

$$F = m\left(\frac{v^2}{r}\right)$$

$$\Rightarrow \frac{Ze^2}{4\pi\varepsilon_0 r^2} = m\left(\frac{v^2}{r}\right) \Rightarrow r = \frac{Ze^2}{4\pi\varepsilon_0 mv^2} \quad ...(i)$$

From Bohr's quantisation rules,

$$mvr = n\frac{h}{2\pi} \quad ...(ii)$$

Where, n is a positive integer
Substituting the value of r from eq. (i), we get

$$mv.\frac{Ze^2}{4\pi\varepsilon_0(mv^2)} = n\frac{h}{2\pi}$$

$$v = \frac{Ze^2}{2\varepsilon_0 hn} \quad ...(iii)$$

So, kinetic energy, $K = \frac{1}{2}mv^2 = \frac{Z^2e^4}{8\varepsilon_0^2h^2n^2}$ ...(iv)

Potential energy of the atom,

$$U = -\frac{Ze^2}{4\pi\varepsilon_0 r} \quad ...(v)$$

Using eq. (iii) in eq. (i), we get

$$r = \frac{Ze^2}{4\pi\varepsilon_0 m\frac{(Ze^2)^2}{(2\varepsilon_0 hn)^2}} = \frac{4\varepsilon_0^2h^2n^2}{(4\pi\varepsilon_0)mZe^2}$$

$$r = \frac{\varepsilon_0 h^2n^2}{\pi mZe^2}$$

Using value of r in eq. (v), we get

$$U = \frac{-Ze^2}{4\pi\varepsilon_0\left(\frac{\varepsilon_0 h^2n^2}{\pi mZe^2}\right)} = \frac{-Z^2e^4m}{4\varepsilon_0^2h^2n^2}$$

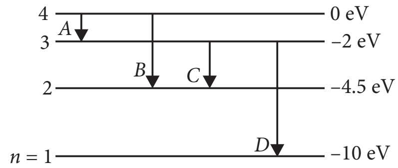
So, the total energy,
$E = K + U$

$$= +\frac{mZ^2e^4}{8\varepsilon_0^2h^2n^2} - \frac{mZ^2e^4}{4\varepsilon_0^2h^2n^2} = -\frac{Z^2e^4m}{8\varepsilon_0^2h^2n^2}$$

For H-atom $Z = 1$, so the total energy of the n^{th} energy level of H-atom.

$$E_n = -\frac{me^4}{8n^2\varepsilon_0^2h^2}$$

19.



The wavelength of emitted radiation from state (n = 4) to the state (n = 2) is

$$\lambda = \frac{hc}{(E_4 - E_2)} = \frac{6.6\times10^{-34}\times3\times10^8}{[0-(-4.5)]\times1.6\times10^{-19}}$$

$= 2.75 \times 10^{-7}$ m $= 275 \times 10^{-9}$ m = 275 nm
Hence, transition shown by arrow B corresponds to emission of wavelength = 275 nm.
(i) The maximum wavelength of emitted radiation from state n = 4 to n = 3 is

$$\lambda = \frac{hc}{[0-(-2)]\,\text{eV}} \Rightarrow \lambda = \frac{6.6\times10^{-34}\times3\times10^8}{2\times1.6\times10^{-19}}$$

$= 6.18 \times 10^{-7}$ m $= 618 \times 10^{-9}$ m = 618 nm
Hence transition A corresponds to maximum wavelength.
(ii) The minimum wavelength of emitted radiation from state n = 3 to n = 1 is

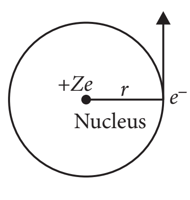
$$\lambda = \frac{6.6\times10^{-34}\times3\times10^8}{[-2\text{ eV}-(-10\text{ eV})]} = \frac{6.6\times10^{-34}\times3\times10^8}{8\times1.6\times10^{-19}}$$

$\lambda = 1.55 \times 10^{-7}$ m = 155 nm
Hence transition D corresponds to minimum wavelength.

20. Suppose m be the mass of an electron and v be its speed in n^{th} orbit of radius r. The centripetal force for revolution is produced by electrostatic attraction between electron and nucleus.

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2} \quad ...(i)$$

or, $mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$



So, kinetic energy $K = \frac{1}{2}mv^2$

$$K = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r}$$

Potential energy $= \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(-e)}{r} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$

Total energy, E = K.E. + P.E.

$$= \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r} + \left(-\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}\right)$$

$$E = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r}$$

For n^{th} orbit, E can be written as

$$E_n = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r}$$

Again from Bohr's postulate for quantization of angular momentum.

$$mvr = \frac{nh}{2\pi}$$

$$v = \frac{nh}{2\pi\, mr}$$

Substituting this value of v in equation (i), we get

$$\frac{m}{r}\left[\frac{nh}{2\pi mr}\right]^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$

$$\text{or, } r = \frac{\varepsilon_0 h^2 n^2}{\pi m Ze^2} \text{ or, } r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m Ze^2} \quad \text{...(ii)}$$

Substituting value of r_n in equation (ii), we get

$$E_n = -\frac{1}{2\times 4\pi\varepsilon_0}\frac{Ze^2}{\left(\frac{\varepsilon_0 h^2 n^2}{\pi m Ze^2}\right)} = \frac{mZ^2e^4 \times ch}{8\varepsilon_0^2 ch^3 n^2}$$

$$\text{or, } E_n = -\frac{Z^2 Rhc}{n^2}, \quad \text{where } R = \frac{me^4}{8\varepsilon_0^2 ch^3}$$

R is called Rydberg constant. For hydrogen atom $Z = 1$.

$$E_n = \frac{-Rch}{n^2}$$

If n_i and n_f are the quantum numbers of initial and final states and E_i and E_f are energies of electron in H-atom in initial and final state, we have

$$E_i = \frac{-Rch}{n_i^2} \quad \text{and} \quad E_f = \frac{-Rch}{n_f^2} \quad \text{...(i)}$$

If υ is the frequency of emitted radiation.

we get $\upsilon = \frac{E_i - E_f}{h}$

from eq. (i)

$$\upsilon = \frac{1}{h}\left(\frac{-Rch}{n_i^2} + \frac{Rch}{n_f^2}\right)$$

$$\upsilon = Rc\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

21. (a) Speed of the electron in the n^{th} orbit : The centripetal force required for revolution is provided by the electrostatic force of attraction between the electron and the nucleus.

$$\therefore \quad \frac{mv^2}{r} = \frac{KZe^2}{r^2} \qquad \left[\text{where, } K = \frac{1}{4\pi\varepsilon_0}\right]$$

$$\Rightarrow \quad r = \frac{KZe^2}{mv^2} \quad \text{...(i)}$$

The angular momentum for any permitted (stationary) orbit is

$$mvr = \frac{nh}{2\pi}$$

where n is any positive integer.

$$r = \frac{nh}{2\pi mv} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{KZe^2}{mv^2} = \frac{nh}{2\pi mv} \quad \therefore \quad v = \frac{2\pi KZe^2}{nh}$$

For hydrogen atom, $Z = 1$

$$\therefore \quad v = \frac{2\pi Ke^2}{nh}$$

(b) *Refer to answer 14.*

22. Bohr's postulates of atomic model : Bohr introduced three postulates and laid the foundations of quantum mechanics.

(i) In a hydrogen atom, an electron revolves in certain stable orbits called stationary orbits without the emission of radiant energy.

(ii) The angular momentum in the stationary orbits is an integral multiple of $\frac{h}{2\pi}$.

$$\therefore \quad L = mvr = \frac{nh}{2\pi}$$

where n is an integer called a quantum number.

In second excited state *i.e.*, $n = 3$, three spectral lines can be obtained corresponding to transition of electron from $n = 3$ to $n = 1$, $n = 3$ to $n = 2$ and $n = 2$ to $n = 1$.

For Lyman series, $n = 3$ to $n = 1$, for minimum wavelength, $\frac{1}{\lambda_{min}} = R\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = \frac{8R}{9}$ ...(i)

For Balmer series, $n = 3$ to $n = 2$, for maximum wavelength, $\frac{1}{\lambda_{max}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36}R$ ...(ii)

Dividing eq. (i) by (ii), we get

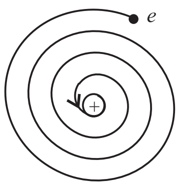
$$\frac{\lambda_{max}}{\lambda_{min}} = \frac{8R/9}{5R/36} = \frac{32}{5}$$

$\lambda_{max} : \lambda_{min} = 32 : 5$

23. *Refer to answer 19.*

24. (a) Limitation of Rutherford's model :

(i) Rutherford's atomic model is inconsistent with classical physics. According to electromagnetic theory, an electron is a charged particle moving in the circular orbit around the nucleus and



is accelerated, so it should emit radiation continuously and thereby loose energy. Due to this, radius of the electron would decrease continuously and also the atom should then produce continuous spectrum, and ultimately electron will fall into the nucleus and atom will collapse in 10^{-8} s. But the atom is fairly stable and it emits line spectrum.

(ii) Rutherford's model is not able to explain the spectrum of even most simplest H-spectrum.

Bohr's postulates to resolve observed features of atomic spectrum :

(i) Quantum condition: Of all the possible circular orbits allowed by the classical theory, the electrons are permitted to circulate only in those orbits in which the angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}, h$ being Planck's constant. Therefore, for any permitted orbit,

$$L = mvr = \frac{nh}{2\pi}, \ n = 1, 2, 3 \ldots.,$$

where n is called the principal quantum number, and this equation is called Bohr's quantisation condition.

(ii) Stationary orbits: While resolving in the permissible orbits, an electron does not radiate energy. These non-radiating orbits are called stationary orbits.

(iii) Frequency condition: An atom can emit or absorb radiation in the form of discrete energy photons only when an electron jumps from a higher to a lower orbit or from a lower to a higher orbit, respectively.

$$h\upsilon = E_i - E_f$$

where ν is frequency of radiation emitted, E_i and E_f are the energies associated with stationary orbits of principal quantum number n_i and n_f respectively (where $n_i > n_f$).

(b) *Refer to answer 13.*

25. *Refer to answer 17(i).*

26. (a) *Refer to answer 17(i)*

(b) $\because$ Radius $r = \frac{n^2h^2\varepsilon_0}{\pi me^2}$ or $r \propto \frac{1}{m}$

$\therefore$ when we increase the mass 200 times, the radius reduces to 200 times.

Similarly, ground state energy for hydrogen,

$$E = \frac{-me^4}{8\varepsilon_0^2 n^2 h^2}$$

i.e. $E \propto m$

$\therefore$ when we increase the mass 200 times, the ground state energy also increases by a factor 200.

27. *Refer to answer 17(i).*

28. For element D

Ground state energy, $E_1 = -13.6$ eV

Excited state energy, $E_2 = -1.5$ eV

Energy of photon emitted, $E = E_2 - E_1$

$= -1.5 - (-13.6) = -1.5 + 13.6 = 12.1$ eV

$\therefore$ Wavelength of photon emitted,

$$\lambda = \frac{hc}{E} = \frac{6.62\times10^{-34}\times3\times10^{8}}{12.1\times1.6\times10^{-19}} = \frac{19.86\times10^{-7}}{19.36}$$

$= 1.027 \times 10^{-7} = 102.7$ nm

For element C

$E_1 = -3.4$ eV, $E_2 = -1.5$ eV

$\therefore \ E = -1.5 - (-3.4) = -1.5 + 3.4 = 1.9$ eV

$$\therefore \ \lambda = \frac{6.62\times10^{-34}\times3\times10^{8}}{1.9\times1.6\times10^{-19}} = \frac{19.86\times10^{-7}}{3.04}$$

$= 6.539 \times 10^{-7}$ m $= 653.9$ nm

For element B

$E_1 = -3.4$ eV, $E_2 = -0.85$ eV

$\therefore \ E = -0.85 - (-3.4) = -0.85 + 3.4 = 2.55$ eV

$$\therefore \ \lambda = \frac{6.62\times10^{-34}\times3\times10^{8}}{2.55\times1.6\times10^{-19}} = \frac{19.86\times10^{-7}}{4.08}$$

$= 4.867 \times 10^{-7}$ m $= 486.7$ nm

For element A

$E_1 = -1.5$ eV, $E_2 = -0.85$ eV

$\therefore \ E = -0.85 - (-1.5) = -0.85 + 1.5 = 0.65$

$$\therefore \quad \lambda = \frac{6.62\times10^{-34}\times3\times10^{8}}{0.65\times1.6\times10^{-19}} = \frac{19.86\times10^{-7}}{1.04}$$

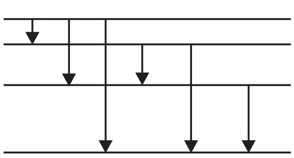
$= 19.096 \times 10^{-7}$ m = 1909.6 nm

The element D corresponds to a spectral line of wavelength 102.7 nm.

29. H_{α} line of the Balmer series in the emission spectrum of hydrogen atoms obtained when the transition occurs from $n = 3$ to $n = 2$ state.

30. Number of spectral lines obtained due to transition of electron from $n = 4$ (3rd exited state) to $n = 1$ (ground state) is

$$N = \frac{(4)(4-1)}{2} = 6$$

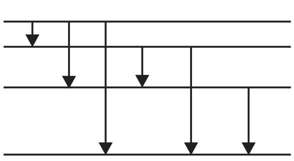


31. The minimum energy, required to free the electron from the ground state of the hydrogen atom, is known as ionization energy of that atom.

$E_0 = \dfrac{me^4}{8\varepsilon_0^2 h^2}$ *i.e,.* $E_0 \propto m$, so when electron in hydrogen atom is replaced by a particle of mass 200 times that of the electron, ionization energy increases by 200 times.

32. Number of spectral lines obtained due to transition of electron from $n = 4$ (3rd exited state) to $n = 1$ (ground state) is

$$N = \frac{(4)(4-1)}{2} = 6$$



These lines correspond to Lyman series.

33. (a) $\lambda = 496$ nm $= 496 \times 10^{-9}$ m

$$E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{496\times10^{-9}} \text{ J}$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{496\times10^{-9}\times1.6\times10^{-19}} = 2.5 \text{ eV}$$

This energy corresponds to the transition $A(n = 4$ to $n = 3)$ for which the energy change = 2 eV

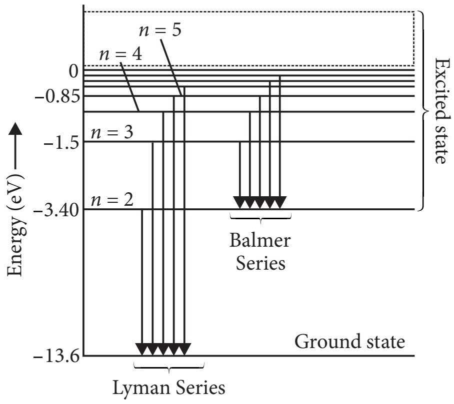
(b) Energy of emitted photon is given by,

$$E = \frac{hc}{\lambda} \quad \therefore \quad \lambda_{max} \propto \frac{1}{E_{min}}$$

Transition A, for which the energy emission is minimum, corresponds to the emission of radiation of maximum wavelength.

34. (i) An electron undergoes transition from 2nd excited state to the first excited state is Balmer series and then to the ground state is Lyman series.

(ii) The wavelength of the emitted radiations in the two cases.



For $n_2 \xrightarrow{\lambda} n_1$

$\Delta E = (-3.40 + 13.6) = 10.20$ eV

$$\lambda_2 = \frac{6.626\times10^{-34}\times3\times10^{8}}{10.2\times1.6\times10^{-19}}$$

$$\lambda_2 = \frac{19.878\times10^{-7}}{10.2\times1.6} = 1.218\times10^{-7} \text{ m} = 1218 \text{ Å}$$

For $n_3 \rightarrow n_2$

$\Delta E = (-1.5 + 3.4) = 1.9$ eV

$$\lambda_1 = \frac{19.878\times10^{-7}}{1.9\times1.6} = 6.538\times10^{-7} \text{ m} = 6538 \text{ Å}$$

The ratio $\dfrac{\lambda_1}{\lambda_2} = \dfrac{6538}{1218} = 5.36$

35. For longest wavelength of Lyman series $n_i = 2$

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$$

$$\lambda_{max} = \frac{4}{3R} = \frac{4}{3\times1.097\times10^{7}} = 1.215\times10^{-7} \text{ m}$$

$\lambda_{max} = 1215$ Å

The lines of the Lyman series are found in ultraviolet region.

(ii) For longest wavelength of Balmer series $n_i = 3$

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5}{36}R$$

$$\lambda_{max} = \frac{36}{5R} = \frac{36}{5 \times 1.097 \times 10^7} = 6.563 \times 10^{-7} \text{ m} = 6563 \text{ Å}$$

Balmer series lie in the visible region of electromagnetic spectrum.

36. Here, $\Delta E = 12.5$ eV

Energy of an electron in n^{th} orbit of hydrogen atom is,

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

In ground state, $n = 1$

$E_1 = -13.6$ eV

Energy of an electron in the excited state after absorbing a photon of 12.5 eV energy will be

$E_n = -13.6 + 12.5 = -1.1$ eV

$$\therefore \quad n^2 = \frac{-13.6}{E_n} = \frac{-13.6}{-1.1} = 12.36 \quad \Rightarrow \quad n = 3.5$$

Here, state of electron cannot be fraction.

So, $n = 3$ (2nd exited state).

The wavelength λ of the first member of Lyman series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4}R$$

$$\Rightarrow \lambda = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^7} \quad \Rightarrow \quad \lambda = 1.215 \times 10^{-7} \text{ m}$$

$\Rightarrow \lambda = 121 \times 10^{-9}$ m $\Rightarrow \lambda = 121$ nm

The wavelength λ' of the first member of the Balmer series is given by

$$\frac{1}{\lambda'} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36}R$$

$$\Rightarrow \lambda' = \frac{36}{5R} = \frac{36}{5 \times (1.097 \times 10^7)}$$

$= 6.56 \times 10^{-7}$ m $= 656 \times 10^{-9}$ m $= 656$ nm

37. (i) $\because \; E_n = \dfrac{-13.6}{n^2}$ eV

Energy of the photon emitted during a transition of the electron from the first excited state to its ground state is,

$\Delta E = E_2 - E_1$

$$= \frac{-13.6}{2^2} - \left(\frac{-13.6}{1^2}\right) = \frac{-13.6}{4} + \frac{13.6}{1} = -3.40 + 13.6$$

$= 10.2$ eV

This transition lies in the region of Lyman series.

(ii) (a) The energy levels of H-atom are given by

$$E_n = -\frac{Rhc}{n^2} = -\frac{13.6}{n^2} \text{ eV}$$

For first excited state $n = 2$

$$E_2 = -\frac{13.6}{(2)^2} \text{ eV} = -3.4 \text{ eV}$$

Kinetic energy of electron in ($n = 2$) state is

$K_2 = -E_2 = +3.4$ eV

(b) Radius in the first excited state

$r_1 = (2)^2 (0.53)$ Å

$r_1 = 2.12$ Å

38. *Refer to answer 19.*

39. *Refer to answer 32.*

40. $hv = \dfrac{hc}{\lambda} = (E_2 - E_1)$ or $\lambda = \dfrac{hc}{(E_2 - E_1)}$

$$\therefore \quad \lambda = \left[\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{[-0.85 - (-3.4)] \times 1.6 \times 10^{-19}}\right] \text{m}$$

$= 4.875 \times 10^{-7}$ m $= 4875$ Å

Balmer series

41. (i) The kinetic energy (E_k) of the electron in an orbit is equal to negative of its total energy (E)

$E_k = -E = -(-1.5) = 1.5$ eV

(ii) The potential energy (E_p) of the electron in an orbit is equal to twice of its total energy (E)

$E_p = 2E = -1.5 \times 2 = -3.0$ eV

(iii) Here, ground state energy of the H-atom $= -13.6$ eV

When the electron goes from the excited state to the ground state, energy emitted is given by

$E = -1.5 - (-13.6) = 12.1$ eV $= 12.1 \times 1.6 \times 10^{-19}$ J

Now, $\quad E = \dfrac{hc}{\lambda}$

$$\lambda = \frac{hc}{E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{12.1 \times 1.6 \times 10^{-19}}$$

$\lambda = 1.025 \times 10^{-7}$

$\lambda = 1025$ Å

42. *Refer to answer 35.*

43. $\because \; \dfrac{mv^2}{r_n} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r_n^2}$ ...(i)

and $mvr_n = \dfrac{nh}{2\pi}$ ...(ii)

From eqn. (i) and (ii)

$\therefore \quad r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m e^2}$

Total energy

$$E_n = \frac{1}{2}mv_n^2 - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} = \frac{1}{8\pi\varepsilon_0}\frac{e^2}{r_n} - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n}$$

$$E_n = -\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r_n} = -\frac{1}{8\varepsilon_0^2}\frac{me^4}{h^2n^2}$$

$$E_n = \frac{-Rhc}{n^2}$$

where Rydberg constant $R = \frac{me^4}{8\varepsilon_0^2 h^3 c}$

Energy emitted $\Delta E = E_i - E_f$

$$\Delta E = Rhc\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

But $\Delta E = h\upsilon$

$$\upsilon = Rc\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right] \quad \text{or} \quad \upsilon = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

When electron in hydrogen atom jumps from energy state $n_i = 4$ to $n_f = 3, 2, 1$, the Paschen, Balmer and Lyman spectral series are found.

44. *Refer to answer 35.*

45. *Refer to answer 35.*

46. de-Broglie hypothesis : It states that a moving particle sometimes acts as a wave and sometimes as a particle or a wave is associated with moving particle which controls the particle in every respect. The wave associated with moving particle is called matter wave or de-Broglie wave whose wavelength is given by

$$\lambda = \frac{h}{mv}$$

where m and v are the mass and velocity of the particle and h is Planck's constant.

47. Kinetic energy of the electron in the second state of hydrogen atom

$$E_K = \frac{13.6 \text{ eV}}{n^2} = \frac{13.6 \text{ eV}}{4} = 3.4 \times 1.6 \times 10^{-19} \text{ J}$$

de Broglie wavelength $\lambda = \frac{h}{\sqrt{2mE_K}}$

$$= \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times3.4\times1.6\times10^{-19}}} = 0.67 \text{ nm}$$

48. According to Bohr's postulates,

$$mvr = \frac{nh}{2\pi} \quad \text{... (i)}$$

(where mvr = angular momentum of an electron and n is an integer).

Thus, the centripetal force, $\frac{mv^2}{r}$ (experienced by the electron) is due to the electrostatic attraction, $\frac{kZe^2}{r^2}$ Where, Z = Atomic number

Therefore, $\frac{mv^2}{r} = \frac{kZe^2}{r^2}$

Substituting the value of v^2 from (i), we obtain:

$$\frac{m}{r}\frac{n^2h^2}{4\pi^2m^2r^2} = \frac{kZe^2}{r^2} \quad \therefore \quad r = \frac{n^2h^2}{4\pi^2 mkZe^2}$$

The relation for the n^{th} radius of Bohr orbit in terms of Bohr's quantization condition of orbital angular momentum $= \frac{n^2h^2}{4\pi^2 mkZe^2}$.

49. (i) Bohr's quantization condition : The electron revolves around the nucleus only in those orbits for which the angular momentum is an integral multiple of $h/2\pi$.

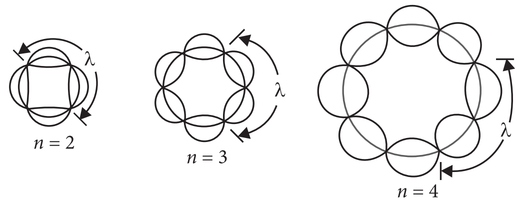
i.e., $L = mvr = n\frac{h}{2\pi}$; $n = 1, 2, 3, ...$

de Broglie hypothesis may be used to derive Bohr's formula by considering the electron to be a wave spread over the entire orbit, rather than as a particle which at any instant is located at a point in its orbit. The stable orbits in an atom are those which are standing waves. Formation of standing waves require that the circumference of the orbit is equal in length to an integral multiple of the wavelength. Thus, if r is the radius of the orbit

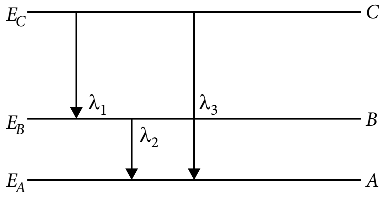
$$2\pi r = n\lambda = \frac{nh}{p} \quad \left(\because \lambda = \frac{h}{p}\right)$$

which gives the angular momentum quantization

$$L = pr = n\frac{h}{2\pi}$$



(ii)



Clearly, from energy level diagram,

$E_C - E_A = (E_C - E_B) + (E_B - E_A)$

(On the basis of energy of emitted photon).

$$\frac{hc}{\lambda_3} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\Rightarrow \quad \frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \Rightarrow \lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

which is the required relation between the three given wavelengths.

50. Kinetic energy in the first excited state of hydrogen atom

$E_K = 3.4 \text{ eV} = 3.4 \times 1.6 \times 10^{-19}$ J

De Broglie wavelength, $\lambda = \dfrac{h}{\sqrt{2m\,E_K}}$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}}} = 0.67 \text{ nm}$$

51. $v = 2.2 \times 10^8$ m/s

$\lambda = ?$

$\lambda = \dfrac{h}{P} \quad \therefore \quad P = mv$

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 2.2 \times 10^8}$$

$= 0.331 \times 10^{-11} = 3.3 \times 10^{-12}$ m

52. According to Bohr's second postulate quantization of angular momentum

$$mv_n r_n = n\frac{h}{2\pi}$$

or $r_n = \dfrac{nh}{2\pi m v_n}$ ...(i)

where h is the Planck's constant

Circumference of the electron in the n^{th} orbital state in hydrogen atom,

$2\pi r_n = 2\pi \dfrac{nh}{2\pi m v_n}$ (Using (i))

$2\pi r_n = n\dfrac{h}{m v_n}$ ...(ii)

But de Broglie wavelength of the electron

$\lambda = \dfrac{h}{m v_n}$ ...(iii)

From (ii) and (iii), we get

$2\pi r_n = n\lambda$

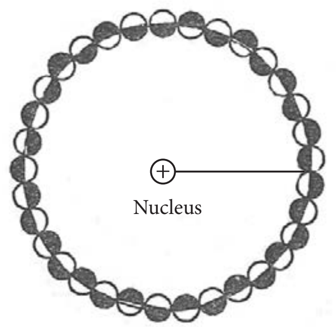
53. (a) According to de-Broglie, a stationary orbit is that which contains an integral number of de-Broglie waves associated with the revolving electron.

For an electron revolving in n^{th} circular orbit of radius r_n,

Total distance covered = Circumference of the orbit $= 2\pi r_n$

$\therefore$ For the permissible orbit, $2\pi r_n = n\lambda$

According to de-Broglie,



$\lambda = \dfrac{h}{m v_n}$

where v_n is speed of electron revolving in n^{th} orbit.

$\therefore \quad 2\pi r_n = \dfrac{nh}{m v_n}$

or $\quad m v_n r_n = \dfrac{nh}{2\pi} = n\left(\dfrac{h}{2\pi}\right)$

(b) For ground state, $n = 1$

$$E = \frac{-13.6}{n^2} = \frac{-13.6}{1^2} = -13.6 \text{ eV}$$

$\therefore \quad K.E. = -E = -(-13.6) = 13.6$ eV

$\because \quad P.E. = 2E$

$\therefore \quad P.E. = 2(-13.6) = -27.2$ eV