CHAPTER 1

Trigonometric Functions & Equations

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. Suppose $\sin^3 x \sin 3x = \sum_{m=0}^{n} C_m \cos mx$ is an identity in x, where $C_0, C_1, ...C_n$ are constants, and $C_n \neq 0$. then the value of n is _________ *(1981 - 2 Marks)*

2. The solution set of the system of equations $x + y = \frac{2\pi}{3}$, $\cos x + \cos y = \frac{3}{2}$, where x and y are real, is _________ . *(1987 - 2 Marks)*

3. The set of all x in the interval $[0, \pi]$ for which $2\sin^2 x - 3\sin x + 1 \geq 0$, is _________ . *(1987 - 2 Marks)*

4. The sides of a triangle inscribed in a given circle subtend angles α, β and γ at the centre. The minimum value of the arithmetic mean of $\cos\left(\alpha + \frac{\pi}{2}\right)$, $\cos\left(\beta + \frac{\pi}{2}\right)$ and $\cos\left(\gamma + \frac{\pi}{2}\right)$ is equal to _________ *(1987 - 2 Marks)*

5. The value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$ is equal to _________ *(1991 - 2 Marks)*

6. If $K = \sin(\pi/18)\sin(5\pi/18)\sin(7\pi/18)$, then the numerical value of K is _________ . *(1993 - 2 Marks)*

7. If $A > 0, B > 0$ and $A + B = \pi/3$, then the maximum value of $\tan A \tan B$ is_________ . *(1993 - 2 Marks)*

8. General value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is_________ . *(1996 - 1 Mark)*

9. The real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ are ..., ..., and_________ . *(1997 - 2 Marks)*

B True / False

1. If $\tan A = (1 - \cos B)/\sin B$, then $\tan 2A = \tan B$. *(1983 - 1 Mark)*

2. There exists a value of θ between 0 and 2π that satisfies the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$. *(1984 - 1 Mark)*

C MCQs with One Correct Answer

1. If $\tan\theta = -\frac{4}{3}$, then $\sin\theta$ is *(1979)*
   (a) $-\frac{4}{5}$ but not $\frac{4}{5}$
   (b) $-\frac{4}{5}$ or $\frac{4}{5}$
   (c) $\frac{4}{5}$ but not $-\frac{4}{5}$
   (d) None of these.

2. If $\alpha + \beta + \gamma = 2\pi$, then *(1979)*
   (a) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
   (b) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$
   (c) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
   (d) None of these.

3. Given $A = \sin^2\theta + \cos^4\theta$ then for all real values of θ *(1980)*
   (a) $1 \leq A \leq 2$
   (b) $\frac{3}{4} \leq A \leq 1$
   (c) $\frac{13}{16} \leq A \leq 1$
   (d) $\frac{3}{4} \leq A \leq \frac{13}{16}$

4. The equation $2\cos^2\frac{x}{2}\sin^2 x = x^2 + x^{-2}$; $0 < x \leq \frac{\pi}{2}$ has
   (a) no real solution
   (b) one real solution
   (c) more than one solution
   (d) none of these *(1980)*

5. The general solution of the trigonometric equation $\sin x + \cos x = 1$ is given by : *(1981 - 2 Marks)*
   (a) $x = 2n\pi$; $n = 0, \pm1, \pm2 ...$
   (b) $x = 2n\pi + \pi/2$; $n = 0, \pm1, \pm2...$
   (c) $x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$; $n = 0, \pm1, \pm2...$
   (d) none of these

6. The value of the expression $\sqrt{3}\ \text{cosec}\ 20° - \sec 20°$ is equal to *(1988 - 2 Marks)*
(a) 2 (b) 2 sin 20°/sin 40°
(c) 4 (d) 4 sin 20°/sin 40°

7. The general solution of
$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$ is *(1989 - 2 Marks)*
(a) $n\pi + \frac{\pi}{8}$ (b) $\frac{n\pi}{2} + \frac{\pi}{8}$
(c) $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$ (d) $2n\pi + \cos^{-1}\frac{3}{2}$

8. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$
In the variable x, has real roots. Then p can take any value in the interval *(1990 - 2 Marks)*
(a) $(0, 2\pi)$ (b) $(-\pi, 0)$ (c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $(0, \pi)$

9. Number of solutions of the equation *(1993 - 1 Mark)*
$\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is :
(a) 0 (b) 1 (c) 2 (d) 3

10. Let $0 < x < \frac{\pi}{4}$ then $(\sec 2x - \tan 2x)$ equals *(1994)*
(a) $\tan\left(x - \frac{\pi}{4}\right)$ (b) $\tan\left(\frac{\pi}{4} - x\right)$
(c) $\tan\left(x + \frac{\pi}{4}\right)$ (d) $\tan^2\left(x + \frac{\pi}{4}\right)$

11. Let n be a positive integer such that
$\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$. Then *(1994)*
(a) $6 \le n \le 8$ (b) $4 < n \le 8$
(c) $4 \le n \le 8$ (d) $4 < n < 8$

12. If ω is an imaginary cube root of unity then the value of
$\sin\left\{(\omega^{10} + \omega^{23})\pi - \frac{\pi}{4}\right\}$ is *(1994)*
(a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

13. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) =$ *(1995S)*
(a) 11 (b) 12 (c) 13 (d) 14

14. The general values of θ satisfying the equation
$2\sin^2\theta - 3\sin\theta - 2 = 0$ is *(1995S)*
(a) $n\pi + (-1)^n \pi/6$ (b) $n\pi + (-1)^n \pi/2$
(c) $n\pi + (-1)^n 5\pi/6$ (d) $n\pi + (-1)^n 7\pi/6$

15. $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if *(1996 - 1 Mark)*
(a) $x + y \ne 0$ (b) $x = y, x \ne 0$
(c) $x = y$ (d) $x \ne 0, y \ne 0$

16. In a triangle PQR, $\angle R = \pi/2$. If $\tan(P/2)$ and $\tan(Q/2)$ are the roots of the equation $ax^2 + bx + c = 0\ (a \ne 0)$ then. *(1999 - 2 Marks)*
(a) $a + b = c$ (b) $b + c = a$
(c) $a + c = b$ (d) $b = c$

17. Let $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$. Then $f(\theta)$ is *(2000S)*
(a) ≥ 0 only when $\theta \ge 0$ (b) ≤ 0 for all real θ
(c) ≥ 0 for all real θ (d) ≤ 0 only when $\theta \le 0$

18. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}$
$= 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is *(2001S)*
(a) 0 (b) 2 (c) 1 (d) 3

19. The maximum value of $(\cos\alpha_1).(\cos\alpha_2)\ldots(\cos\alpha_n)$, under the restrictions
$0 \le \alpha_1, \alpha_2, \ldots, \alpha_n \le \frac{\pi}{2}$ and $(\cot\alpha_1).(\cot\alpha_2)\ldots(\cot\alpha_n) = 1$ is *(2001S)*
(a) $1/2^{n/2}$ (b) $1/2^n$ (c) $1/2n$ (d) 1

20. If $\alpha + \beta = \pi/2$ and $\beta + \gamma = \alpha$, then $\tan\alpha$ equals *(2001S)*
(a) $2(\tan\beta + \tan\gamma)$ (b) $\tan\beta + \tan\gamma$
(c) $\tan\beta + 2\tan\gamma$ (d) $2\tan\beta + \tan\gamma$

21. The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution is *(2002S)*
(a) 4 (b) 8 (c) 10 (d) 12

22. Given both θ and ϕ are acute angles and $\sin\theta = \frac{1}{2}$, $\cos\phi = \frac{1}{3}$, then the value of $\theta + \phi$ belongs to *(2004S)*
(a) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ (b) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$
(c) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right]$ (d) $\left(\frac{5\pi}{6}, \pi\right]$

23. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$ where $\alpha, \beta \in [-\pi, \pi]$. Pairs of α, β which satisfy both the equations is/are *(2005S)*
(a) 0 (b) 1 (c) 2 (d) 4

24. The values of $\theta \in (0, 2\pi)$ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, are *(2006 - 3M, –1)*
(a) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (b) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$
(c) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (d) $\left(\frac{41\pi}{48}, \pi\right)$

25. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then *(2006 - 3M, –1)*
(a) $t_1 > t_2 > t_3 > t_4$ (b) $t_4 > t_3 > t_1 > t_2$
(c) $t_3 > t_1 > t_2 > t_4$ (d) $t_2 > t_3 > t_1 > t_4$

26. The number of solutions of the pair of equations
$$2\sin^2\theta - \cos 2\theta = 0$$
$$2\cos^2\theta - 3\sin\theta = 0$$
in the interval $[0, 2\pi]$ is *(2007 - 3 Marks)*
(a) zero (b) one (c) two (d) four

27. For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has *(JEE Adv. 2014)*
(a) infinitely many solutions
(b) three solutions
(c) one solution
(d) no solution

28. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to *(JEE Adv. 2016)*
(a) $-\frac{7\pi}{9}$ (b) $-\frac{2\pi}{9}$
(c) 0 (d) $\frac{5\pi}{9}$

29. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is equal to *(JEE Adv. 2016)*
(a) $3 - \sqrt{3}$ (b) $2(3 - \sqrt{3})$
(c) $2(\sqrt{3} - 1)$ (d) $2(2 - \sqrt{3})$

D MCQs with One or More than One Correct

1. $\left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 + \cos\frac{5\pi}{8}\right)\left(1 + \cos\frac{7\pi}{8}\right)$ is equal to *(1984 - 3 Marks)*
(a) $\frac{1}{2}$ (b) $\cos\frac{\pi}{8}$
(c) $\frac{1}{8}$ (d) $\frac{1 + \sqrt{2}}{2\sqrt{2}}$

2. The expression $3\left[\sin^4\left(\frac{3\pi}{2} - \alpha\right) + \sin^4(3\pi + \alpha)\right] - 2\left[\sin^6\left(\frac{\pi}{2} + \alpha\right) + \sin^6(5\pi - \alpha)\right]$ is equal to *(1986 - 2 Marks)*
(a) 0 (b) 1
(c) 3 (d) $\sin 4\alpha + \cos 6\alpha$
(e) none of these

3. The number of all possible triplets (a_1, a_2, a_3) such that $a_1 + a_2\cos(2x) + a_3\sin^2(x) = 0$ for all x is *(1987 - 2 Marks)*
(a) zero (b) one (c) three
(d) infinite (e) none

4. The values of θ lying between $\theta = 0$ and $\theta = \pi/2$ and satisfying the equation *(1988 - 2 Marks)*
$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0 \text{ are}$$
(a) $7\pi/24$ (b) $5\pi/24$ (c) $11\pi/24$ (d) $\pi/24$.

5. Let $2\sin^2 x + 3\sin x - 2 > 0$ and $x^2 - x - 2 < 0$ (x is measured in radians). Then x lies in the interval *(1994)*
(a) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (b) $\left(-1, \frac{5\pi}{6}\right)$
(c) $(-1, 2)$ (d) $\left(\frac{\pi}{6}, 2\right)$

6. The minimum value of the expression $\sin\alpha + \sin\beta + \sin\gamma$, where α, β, γ are real numbers satisfying $\alpha + \beta + \gamma = \pi$ is *(1995)*
(a) positive (b) zero
(c) negative (d) –3

7. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is *(1998 - 2 Marks)*
(a) 0 (b) 5 (c) 6 (d) 10

8. Which of the following number(s) is/are rational? *(1998 - 2 Marks)*
(a) sin 15° (b) cos 15°
(c) sin 15° cos 15° (d) sin 15° cos 75°

9. For a positive integer n, let *(1999 - 3 Marks)*
$$f_n(\theta) = \left(\tan\frac{\theta}{2}\right)(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)....(1+\sec 2^n\theta).$$
Then
(a) $f_2\left(\frac{\pi}{16}\right) = 1$ (b) $f_3\left(\frac{\pi}{32}\right) = 1$
(c) $f_4\left(\frac{\pi}{64}\right) = 1$ (d) $f_5\left(\frac{\pi}{128}\right) = 1$

10. If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then *(2009)*
(a) $\tan^2 x = \frac{2}{3}$ (b) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$
(c) $\tan^2 x = \frac{1}{3}$ (d) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

11. For $0<\theta<\frac{\pi}{2}$, the solution (s) of

$$\sum_{m=1}^{6} \operatorname{cosec}\left(\theta+\frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta+\frac{m\pi}{4}\right)=4\sqrt{2}$$

is (are) *(2009)*

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{12}$ (d) $\frac{5\pi}{12}$

12. Let $\theta, \varphi \in [0, 2\pi]$ be such that $2\cos\theta(1-\sin\varphi)=\sin^2\theta$ $\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\varphi-1, \tan(2\pi-\theta)>0$ and $-1<\sin\theta<-\frac{\sqrt{3}}{2}$, then φ cannot satisfy *(2012)*

(a) $0<\varphi<\frac{\pi}{2}$ (b) $\frac{\pi}{2}<\varphi<\frac{4\pi}{3}$

(c) $\frac{4\pi}{3}<\varphi<\frac{3\pi}{2}$ (d) $\frac{3\pi}{2}<\varphi<2\pi$

13. The number of points in $(-\infty\ \infty)$, for which $x^2-x\sin x-\cos x=0$, is *(JEE Adv. 2013)*

(a) 6 (b) 4 (c) 2 (d) 0

14. Let $f(x)=x\sin\pi x, x>0$. Then for all natural numbers n, $f'(x)$ vanishes at *(JEE Adv. 2013)*

(a) A unique point in the interval $\left(n, n+\frac{1}{2}\right)$

(b) A unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$

(c) A unique point in the interval $(n, n+1)$

(d) Two points in the interval $(n, n+1)$

E Subjective Problems

1. If $\tan\alpha=\frac{m}{m+1}$ and $\tan\beta=\frac{1}{2m+1}$, find the possible values of $(\alpha+\beta)$. *(1978)*

2. (a) Draw the graph of $y=\frac{1}{\sqrt{2}}(\sin x+\cos x)$ from $x=-\frac{\pi}{2}$ to $x=\frac{\pi}{2}$.

(b) If $\cos(\alpha+\beta)=\frac{4}{5}$, $\sin(\alpha-\beta)=\frac{5}{13}$, and α, β lies between 0 and $\frac{\pi}{4}$, find $\tan 2\alpha$. *(1979)*

3. Given $\alpha+\beta-\gamma=\pi$, prove that $\sin^2\alpha+\sin^2\beta-\sin^2\gamma=2\sin\alpha\sin\beta\cos\gamma$ *(1980)*

4. Given $A=\left\{x:\frac{\pi}{6}\le x\le\frac{\pi}{3}\right\}$ and $f(x)=\cos x-x(1+x)$; find $f(A)$. *(1980)*

5. For all θ in $[0, \pi/2]$ show that, $\cos(\sin\theta)\ge\sin(\cos\theta)$. *(1981 - 4 Marks)*

6. Without using tables, prove that $(\sin 12°)(\sin 48°)(\sin 54°)=\frac{1}{8}$. *(1982 - 2 Marks)*

7. Show that $16\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{8\pi}{15}\right)\cos\left(\frac{16\pi}{15}\right)=1$ *(1983 - 2 Marks)*

8. Find all the solution of $4\cos^2 x\sin x-2\sin^2 x=3\sin x$ *(1983 - 2 Marks)*

9. Find the values of $x\in(-\pi, +\pi)$ which satisfy the equation $8^{(1+|\cos x|+|\cos^2 x|+|\cos^3 x|+......)}=4^3$ *(1984 - 2 Marks)*

10. Prove that $\tan\alpha+2\tan 2\alpha+4\tan 4\alpha+8\cot 8\alpha=\cot\alpha$ *(1988 - 2 Marks)*

11. ABC is a triangle such that

$$\sin(2A+B)=\sin(C-A)=-\sin(B+2C)=\frac{1}{2}.$$

If A, B and C are in arithmetic progression, determine the values of A, B and C. *(1990 - 5 Marks)*

12. If $\exp\{(\sin^2x+\sin^4x+\sin^6x+ \ldots\ldots\ldots\ \infty)\ \ln 2\}$ satisfies the equation $x^2-9x+8=0$, find the value of $\frac{\cos x}{\cos x+\sin x}, 0<x<\frac{\pi}{2}$. *(1991 - 4 Marks)*

13. Show that the value of $\frac{\tan x}{\tan 3x}$, wherever defined never lies between $\frac{1}{3}$ and 3. *(1992 - 4 Marks)*

14. Determine the smallest positive value of x (in degrees) for which
$\tan(x+100°)=\tan(x+50°)\tan(x)\tan(x-50°)$. *(1993 - 5 Marks)*

15. Find the smallest positive number p for which the equation $\cos(p\sin x)=\sin(p\cos x)$ has a solution $x\in[0,2\pi]$. *(1995 - 5 Marks)*

16. Find all values of θ in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ satisfying the equation $(1-\tan\theta)(1+\tan\theta)\sec^2\theta+2^{\tan^2\theta}=0$. *(1996 - 2 Marks)*

17. Prove that the values of the function $\frac{\sin x\cos 3x}{\sin 3x\cos x}$ do not lie between $\frac{1}{3}$ and 3 for any real x. *(1997 - 5 Marks)*

18. Prove that $\sum_{k=1}^{n-1}(n-k)\cos\frac{2k\pi}{n}=-\frac{n}{2}$, where $n\ge 3$ is an integer. *(1997 - 5 Marks)*

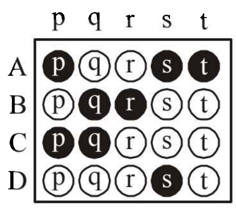
19. In any triangle ABC, prove that *(2000 - 3 Marks)*

$$\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}.$$

20. Find the range of values of t for which $2\sin t=\frac{1-2x+5x^2}{3x^2-2x-1}$, $t\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. *(2005 - 2 Marks)*

F Match the Following

DIRECTIONS (Q. 1): *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*
If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



1. In this questions there are entries in columns 1 and 2. Each entry in column 1 is related to exactly one entry in column 2. Write the correct letter from column 2 against the entry number in column 1 in your answer book.

$\dfrac{\sin 3\alpha}{\cos 2\alpha}$ is *(1992 - 2 Marks)*

Column I	Column II
(A) positive	(p) $\left(\dfrac{13\pi}{48}, \dfrac{14\pi}{48}\right)$
(B) negative	(q) $\left(\dfrac{14\pi}{48}, \dfrac{18\pi}{48}\right)$
	(r) $\left(\dfrac{18\pi}{48}, \dfrac{23\pi}{48}\right)$
	(s) $\left(0, \dfrac{\pi}{2}\right)$

I Integer Value Correct Type

1. The number of all possible values of θ where $0 < \theta < \pi$, for which the system of equations

$(y + z)\cos 3\theta = (xyz)\sin 3\theta$

$x\sin 3\theta = \dfrac{2\cos 3\theta}{y} + \dfrac{2\sin 3\theta}{z}$

$(xyz)\sin 3\theta = (y + 2z)\cos 3\theta + y\sin 3\theta$

have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is *(2010)*

2. The number of values of θ in the interval, $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ such that $\theta \neq \dfrac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is *(2010)*

3. The maximum value of the expression

$\dfrac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is *(2010)*

4. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend at the center, angles of $\dfrac{\pi}{k}$ and $\dfrac{2\pi}{k}$, where $k > 0$, then the value of $[k]$ is *(2010)*

[**Note** : $[k]$ denotes the largest integer less than or equal to k]

5. The positive integer value of $n > 3$ satisfying the equation

$\dfrac{1}{\sin\left(\dfrac{\pi}{n}\right)} = \dfrac{1}{\sin\left(\dfrac{2\pi}{n}\right)} + \dfrac{1}{\sin\left(\dfrac{3\pi}{n}\right)}$ is *(2011)*

6. The number of distinct solutions of the equation

$\dfrac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$

in the interval $[0, 2\pi]$ is *(JEE Adv. 2015)*

GP_3480

Section-B JEE Main / AIEEE

1. The period of $\sin^2\theta$ is **[2002]**
(a) π^2 (b) π (c) 2π (d) $\pi/2$

2. The number of solution of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is **[2002]**
(a) 2 (b) 3 (c) 0 (d) 1

3. Which one is not periodic **[2002]**
(a) $|\sin 3x| + \sin^2 x$ (b) $\cos\sqrt{x} + \cos^2 x$
(c) $\cos 4x + \tan^2 x$ (d) $\cos 2x + \sin x$

4. Let α, β be such that $\pi < \alpha - \beta < 3\pi$.
If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ **[2004]**
(a) $\frac{-6}{65}$ (b) $\frac{3}{\sqrt{130}}$
(c) $\frac{6}{65}$ (d) $-\frac{3}{\sqrt{130}}$

5. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$ then the difference between the maximum and minimum values of u^2 is given by **[2004]**
(a) $(a-b)^2$ (b) $2\sqrt{a^2+b^2}$
(c) $(a+b)^2$ (d) $2(a^2+b^2)$

6. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals **[2004]**
(a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{3}$

7. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is **[2006]**
(a) 4 (b) 6 (c) 1 (d) 2

8. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is **[2006]**
(a) $\frac{(1-\sqrt{7})}{4}$ (b) $\frac{(4-\sqrt{7})}{3}$
(c) $-\frac{(4+\sqrt{7})}{3}$ (d) $\frac{(1+\sqrt{7})}{4}$

9. Let **A** and **B** denote the statements
A : $\cos\alpha + \cos\beta + \cos\gamma = 0$
B : $\sin\alpha + \sin\beta + \sin\gamma = 0$
If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$, then : **[2009]**
(a) **A** is false and **B** is true (b) both **A** and **B** are true
(c) both **A** and **B** are false (d) **A** is true and **B** is false

10. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha$ = **[2010]**
(a) $\frac{56}{33}$ (b) $\frac{19}{12}$ (c) $\frac{20}{7}$ (d) $\frac{25}{16}$

11. If $A = \sin^2 x + \cos^4 x$, then for all real x : **[2011]**
(a) $\frac{13}{16} \le A \le 1$ (b) $1 \le A \le 2$
(c) $\frac{3}{4} \le A \le \frac{13}{16}$ (d) $\frac{3}{4} \le A \le 1$

12. In a ΔPQR, If $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to : **[2012]**
(a) $\frac{5\pi}{6}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

13. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to : **[JEE M 2013]**
(a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ (b) $\frac{p^2+q^2\cos\theta}{p\cos\theta + q\sin\theta}$
(c) $\frac{p^2+q^2}{p^2\cos\theta + q^2\sin\theta}$ (d) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

14. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as : **[JEE M 2013]**
(a) $\sin A\cos A + 1$ (b) $\sec A\,\mathrm{cosec} A + 1$
(c) $\tan A + \cot A$ (d) $\sec A + \mathrm{cosec} A$

15. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ where $x \in R$ and $k \ge 1$. Then $f_4(x) - f_6(x)$ equals **[JEE M 2014]**
(a) $\frac{1}{4}$ (b) $\frac{1}{12}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

16. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is: **[JEE M 2016]**
(a) 7 (b) 9
(c) 3 (d) 5

Solutions & Explanations

1 Trigonometric Functions & Equations

Section-A : JEE Advanced/ IIT-JEE

A 1. 6 2. ϕ 3. $\left[0, \frac{\pi}{6}\right] \cup \left\{\frac{\pi}{2}\right\} \cup \left[\frac{5\pi}{6}, \pi\right]$ 4. $\frac{-\sqrt{3}}{2}$

5. $\frac{1}{64}$ 6. $\frac{1}{8}$ 7. $\frac{1}{3}$ 8. $n\pi,\ n\pi \pm \frac{\pi}{3}$ 9. $-\frac{\pi}{2}, \frac{\pi}{2}, 0$

B 1. T 2. F

C 1. (b) 2. (a) 3. (b) 4. (a) 5. (c) 6. (c) 7. (b) 8. (d)
9. (c) 10. (b) 11. (d) 12. (c) 13. (c) 14. (d) 15. (b) 16. (a)
17. (c) 18. (c) 19. (a) 20. (c) 21. (b) 22. (b) 23. (d) 24. (a)
25. (b) 26. (c) 27. (d) 28. (c) 29. (c)

D 1. (c) 2. (b) 3. (d) 4. (a, c) 5. (d) 6. (c) 7. (c) 8. (c)
9. (a,b,c,d) 10. (a, b) 11. (c, d) 12. (a, c, d) 13. (c) 14. (b, c)

E 1. $n\pi + \frac{\pi}{4}$ 2. (b) $\frac{56}{33}$ 4. $\left[\frac{1}{2} - \frac{\pi}{3}\left(1 + \frac{\pi}{3}\right), \frac{\sqrt{3}}{2} - \frac{\pi}{6}\left(1 + \frac{\pi}{6}\right)\right]$

8. $n\pi, n\pi + (-1)^n \frac{\pi}{10}, n\pi + (-1)^n \left(\frac{-3\pi}{10}\right)$ 9. $\pm\frac{\pi}{3}, \pm\frac{2\pi}{3}$ 11. $45°, 60°, 75°$

12. $\frac{\sqrt{3}-1}{2}$ 14. $30°$ 15. $\frac{\pi\sqrt{2}}{4}$ 16. $\pm\frac{\pi}{3}$

20. $\left[-\frac{\pi}{2}, \frac{-\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$

F 1. (A) - r, (B) - p

I 1. 3 2. 3 3. 2 4. 3 5. 7 6. 8

Section-B : JEE Main/ AIEEE

1. (b) 2. (b) 3. (b) 4. (d) 5. (a) 6. (c) 7. (a) 8. (c)
9. (b) 10. (a) 11. (d) 12. (a) 13. (b) 14. (b) 15. (b) 16. (a)

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. Given $\sin^3 x . \sin 3x = \sum_{m=0}^{n} C_m \cos mx$

$$\sin^3 x \sin 3x = \frac{1}{4}[3\sin x - \sin 3x]\sin 3x$$

$$= \frac{1}{4}\left[\frac{3}{2}.2\sin x.\sin 3x - \sin^2 3x\right]$$

$$= \frac{1}{4}\left[\frac{3}{2}(\cos 2x - \cos x) - \frac{1}{2}(1 - \cos 6x)\right]$$

$$= \frac{1}{8}\left[\cos 6x + 3\cos 2x - 3\cos x - 1\right]$$

We observe that on LHS 6 is the max value of m.

$\therefore \quad n = 6$

2. The equations are $x + y = 2\pi/3$ ... (i)

$\cos x + \cos y = 3/2$ ...(ii)

From eq. (ii) $2\cos\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{3}{2}$

$\Rightarrow \quad 2\cos\frac{\pi}{3}\cos\frac{x-y}{2} = \frac{3}{2}$ [Using eq. (i)]

$\Rightarrow \quad 2.\frac{1}{2}\cos\frac{x-y}{2} = \frac{3}{2} \Rightarrow \cos\frac{x-y}{2} = \frac{3}{2} > 1$

Which has no solution.

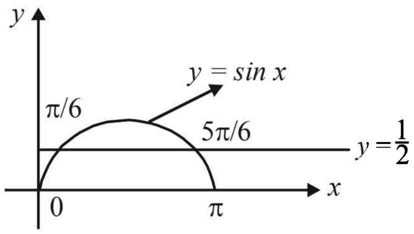
$\therefore$ The solution of given equations is ϕ.

3. We have $2\sin^2 x - 3\sin x + 1 \geq 0$

$\Rightarrow \quad (2\sin x - 1)(\sin x - 1) \geq 0$

$\Rightarrow \quad \left(\sin x - \frac{1}{2}\right)(\sin x - 1) \geq 0 \Rightarrow \sin x \leq \frac{1}{2}$ or $\sin x \geq 1$

But we know that $\sin x \leq 1$ and $\sin x \geq 0$ for $x \in [0, \pi]$



$\Rightarrow$ either $\sin x = 1$ or $0 \leq \sin x \leq \frac{1}{2}$

$\Rightarrow$ either $x = \pi/2$ or $x \in [0, \pi/6] \cup [5\pi/6, \pi]$

Combining, we get $x \in [0, \pi/6] \cup \{\pi/2\} \cup [5\pi/6, \pi]$

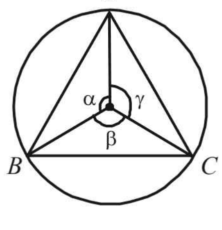
4. We know that A.M. $\geq$ G.M.

$\Rightarrow$ Min value of AM. is obtained when AM = GM

$\Rightarrow$ The quantities whose AM is being taken are equal.

i.e., $\cos\left(\alpha + \frac{\pi}{2}\right) = \cos\left(\beta + \frac{\pi}{2}\right)$

$= \cos\left(\gamma + \frac{\pi}{2}\right)$



$\Rightarrow \sin\alpha = \sin\beta = \sin\gamma$

Also $\alpha + \beta + \gamma = 360° \Rightarrow \alpha = \beta = \gamma = 120° = 2\pi/3$

$\therefore$ Min value of A.M.

$$= \frac{\cos\left(\frac{2\pi}{3} + \frac{\pi}{2}\right) + \cos\left(\frac{2\pi}{3} + \frac{\pi}{2}\right) + \cos\left(\cos\frac{2\pi}{3} + \frac{\pi}{2}\right)}{3}$$

$$= \frac{-\sin\frac{2\pi}{3}}{3} = -\frac{\sqrt{3}}{2}$$

5. $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$

$$= \left(\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\right)^2$$

$$= \left[\cos\left(\frac{\pi}{2} - \frac{\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{3\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{5\pi}{14}\right)\right]^2$$

$$= \left[\cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7}\right]^2 = \left[\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\right]^2$$

$$= \left(\frac{1}{8\sin\pi/7}\sin\frac{8\pi}{7}\right)^2 = \left(\frac{\sin(\pi + \pi/7)}{8\sin\pi/7}\right)^2$$

$$= \left(\frac{-\sin\pi/7}{8\sin\pi/7}\right)^2 = \left(\frac{1}{8}\right)^2 = \frac{1}{64}$$

6. $K = \sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}$

$$= \cos\left(\frac{\pi}{2} - \frac{\pi}{18}\right)\cos\left(\frac{\pi}{2} - \frac{5\pi}{18}\right)\cos\left(\frac{\pi}{2} - \frac{7\pi}{18}\right)$$

$$= \cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9} = \frac{1}{2^3\sin\frac{\pi}{9}}.\sin\frac{8\pi}{9}$$

[Using $\cos\alpha\cos 2\alpha\cos 2^2\alpha \ldots\ldots\ldots \cos 2^{n-1}\alpha$

$= \frac{1}{2^n\sin\alpha}.\sin(2^n\alpha)$]

$$= \frac{1}{8\sin\pi/9}.\sin\pi/9 = \frac{1}{8}$$

7. $A + B = \pi/3 \Rightarrow \tan(A + B) = \sqrt{3}$

$\Rightarrow \quad \frac{\tan A + \tan B}{1 - \tan A\tan B} = \sqrt{3} \Rightarrow \frac{\tan A + \frac{y}{\tan A}}{1 - y} = \sqrt{3}$

[where $y = \tan A\tan B$]

$\Rightarrow \tan^2 A + \sqrt{3}(y - 1)\tan A + y = 0$

For real value of $\tan A$, $3(y - 1)^2 - 4y \geq 0$

$\Rightarrow 3y^2 - 10y + 3 \geq 0 \Rightarrow (y - 3)(y - \frac{1}{3}) \geq 0$

$\Rightarrow\ y \le \frac{1}{3}$ or ≥ 3

But $A, B > 0$ and $A + B = \pi/3 \Rightarrow A, B < \pi/3$
$\Rightarrow\ \tan A \tan B < 3$

$\therefore\ y \le \frac{1}{3}$ i.e., max. value of y is 1/3.

8. $\tan^2\theta + \sec 2\theta = 1$

$t^2 + \frac{1+t^2}{1-t^2} = 1$ where $t = \tan\theta$

$\therefore\ t^2(t^2-3) = 0 \ \therefore\ \tan\theta = 0, \pm\sqrt{3}$ etc.

which means $\theta = n\pi$ *and* $\theta = n\pi \pm \pi/3$

9. $\cos^7 x = 1 - \sin^4 x = (1-\sin^2 x)(1+\sin^2 x)$
$= \cos^2 x\,(1+\sin^2 x)$
$\therefore \cos x = 0$ or $x = \pi/2, -\pi/2$
or $\cos^5 x = 1 + \sin^2 x$ or $\cos^5 x - \sin^2 x = 1$
Now maximum value of each $\cos x$ or $\sin x$ is 1.
Hence the above equation will hold when
$\cos x = 1$ and $\sin x = 0$. Both these imply $x = 0$

Hence $x = -\frac{\pi}{2}, \frac{\pi}{2}, 0$

B. True/False

1. $\tan A = \frac{1-\cos B}{\sin B} = \frac{2\sin^2 B/2}{2\sin B/2\cos/2} = \tan B/2$

Hence $\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\tan B/2}{1-\tan^2 B/2} = \tan B$

$\therefore$ Statement is true.

2. Given equation is $\sin^4\theta - 2\sin^2\theta - 1 = 0$
Here, $D = 4 + 4 = 8$

$\therefore\ \sin^2\theta = \frac{2\pm\sqrt{8}}{2} = 1\pm\sqrt{2}.$

But $\sin^2\theta$ can not be –ve $\quad \therefore \sin^2\theta = \sqrt{2}+1$

But as $-1 \le \sin\theta \le 1 \ \therefore\ \sin^2\theta \ne \sqrt{2}+1$

Thus there is no value of θ which satisfy the given equation.
$\therefore$ Statement is false.

C. MCQs with ONE Correct Answer

1. **(b)** $\tan\theta = \frac{-4}{3} \Rightarrow \theta \in$ II quad or IV quad .

$\therefore 0 < \sin\theta < 1$ or $-1 < \sin\theta < 0$

$\therefore \sin\theta$ may be $\frac{4}{5}$ or $-\frac{4}{5}$

2. **(a)** $\alpha+\beta+\gamma = 2\pi \Rightarrow \frac{\alpha}{2}+\frac{\beta}{2}+\frac{\gamma}{2} = \pi$

$\therefore\ \tan\left(\frac{\alpha}{2}+\frac{\beta}{2}\right) = \tan\left(\pi - \frac{\gamma}{2}\right) = -\tan\frac{\gamma}{2}$

$\Rightarrow\ \frac{\tan\alpha/2 + \tan\beta/2}{1-\tan\alpha/2\tan\beta/2} = -\tan\gamma/2$

$\Rightarrow\ \tan\alpha/2 + \tan\beta/2 + \tan\gamma/2$
$= \tan\alpha/2 \tan\beta/2 \tan\gamma/2$

3. **(b)** $A = \sin^2\theta + \cos^4\theta = \sin^2\theta + (1-\sin^2\theta)^2$

$= \sin^4\theta - \sin^2\theta + 1 \ \Rightarrow\ A = \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4}$

But $0 \le \left(\sin^2\theta - \frac{1}{2}\right)^2 \le \frac{1}{4}$

$\therefore\ \frac{3}{4} \le \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4} \le 1$ or $\frac{3}{4} \le A \le 1$

4. **(a)** The given equation is

$2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + \frac{1}{x^2}$ where $0 < x \le \frac{\pi}{2}$

LHS $= 2\cos^2\frac{x}{2}\sin^2 x = (1+\cos x)\sin^2 x$

$\because\ 1 + \cos x < 2$ and $\sin^2 x \le 1$ for $0 < x \le \frac{\pi}{2}$

$\therefore (1+\cos x)\sin^2 x < 2$

And R.H.S. $= x^2 + \frac{1}{x^2} \ge 2 \quad \therefore$ For $0 < x \le \frac{\pi}{2}$,

given equation is not possible for any real value of x.

5. **(c)** $\sin x + \cos x = 1$

$\Rightarrow\ \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos = \frac{1}{\sqrt{2}}$

$\Rightarrow\ \sin x\cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4} = \sin\frac{\pi}{4}$

$\Rightarrow \sin(x+\pi/4) = \sin\pi/4$
$\Rightarrow x + \pi/4 = n\pi + (-1)^n\pi/4, n \in Z$ (the set of integers)
$\Rightarrow x = n\pi + (-1)^n \pi/4 - \pi/4$
where n = 0, ± 1, ± 2, ...

6. **(c)** The given expression is $\sqrt{3}\,\mathrm{cosec}\,20° - \sec 20°$

$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°} = \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20°\cos 20°}$

$$=4\left[\frac{\frac{\sqrt{3}}{2}\cos 20^\circ-\frac{1}{2}\sin 20^\circ}{2\sin 20^\circ\cos 20^\circ}\right]$$

$$=4\left[\frac{\sin 60^\circ\cos 20^\circ-\cos 60^\circ\sin 20^\circ}{\sin(2\times 20^\circ)}\right]$$

$$=\frac{4\sin(60^\circ-20^\circ)}{\sin 40^\circ}=\frac{4\sin 40^\circ}{\sin 40^\circ}=4$$

7. **(b)** The given equation is

$\sin x-3\sin 2x+\sin 3x=\cos x-3\cos 2x+\cos 3x$

$\Rightarrow 2\sin 2x\cos x-3\sin 2x=2\cos 2x\cos x-3\cos 2x$

$\Rightarrow \sin 2x(2\cos x-3)=\cos 2x(2\cos x-3)$

$\Rightarrow \sin 2x=\cos 2x$ (as $\cos x\neq 3/2$)

$\Rightarrow \tan 2x=1\Rightarrow 2x=n\pi+\pi/4\Rightarrow x=\frac{n\pi}{2}+\frac{\pi}{8}$

8. **(d)** The given equation is

$(\cos p-1)x^2+(\cos p)x+\sin p=0$

For this equation to have real roots $D\geq 0$

$\Rightarrow \cos^2 p-4\sin p(\cos p-1)\geq 0$

$\Rightarrow \cos^2 p-4\sin p\cos p+4\sin^2 p+4\sin p-4\sin^2 p\geq 0$

$\Rightarrow (\cos p-2\sin p)^2+4\sin p(1-\sin p)\geq 0$

For every real value of p $(\cos p-2\sin p)^2\geq 0$ and $1-\sin p\geq 0$ $\therefore D\geq 0,\forall p\in(0,\pi)$

9. **(c)** The given eq is, $\tan x+\sec x=2\cos x$

$\Rightarrow \frac{\sin x}{\cos x}+\frac{1}{\cos x}=2\cos x$

$\Rightarrow \sin x+1=2\cos^2 x\Rightarrow 2\sin^2 x+\sin x-1=0$

$\Rightarrow (2\sin x-1)(\sin x+1)=0\Rightarrow \sin x=\frac{1}{2},-1$

$\Rightarrow x=\frac{\pi}{6},\frac{5\pi}{6},\frac{3\pi}{2}\in[0,2\pi]$

But for $x=\frac{3\pi}{2}$, given eq. is not defined,

$\therefore$ only 2 solutions.

10. **(b)** $\sec 2x-\tan 2x=\frac{1-\sin 2x}{\cos 2x}=\frac{1-\cos 2\left(\frac{\pi}{4}-x\right)}{\sin 2\left(\frac{\pi}{4}-x\right)}$

$$=\frac{2\sin^2\left(\frac{\pi}{4}-x\right)}{2\sin\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-x\right)}=\tan\left(\frac{\pi}{4}-x\right)$$

11. **(d)** $\sin\frac{\pi}{2n}+\cos\frac{\pi}{2n}=\frac{\sqrt{n}}{2}$

$\Rightarrow \sin^2\frac{\pi}{2n}+\cos^2\frac{\pi}{2n}+2\sin\frac{\pi}{2n}\cos\frac{\pi}{2n}=\frac{n}{4}$

$\Rightarrow 1+\sin\frac{\pi}{n}=\frac{n}{4}\Rightarrow \sin\frac{\pi}{n}=\frac{n-4}{4}$

For $n=2$ the given equation is not satisfied.

Considering $n>1$ and $n\neq 2$

$0<\sin\frac{\pi}{n}<1\Rightarrow 0<\frac{n-4}{4}<1\Rightarrow 4<n<8.$

12. **(c)** $\sin\left\{(\omega^{10}+\omega^{23})\pi-\frac{\pi}{4}\right\}=\sin\left\{(\omega+\omega^2)\pi-\frac{\pi}{4}\right\}$

$=\sin\left(-\pi-\frac{\pi}{4}\right)=-\sin\left(\pi+\frac{\pi}{4}\right)=\sin\pi/4=1/\sqrt{2}$

13. **(c)** $3(\sin x-\cos x)^4+6(\sin x+\cos x)^2+4(\sin^6 x+\cos^6 x)$

$=3(1-\sin 2x)^2+6(1+\sin 2x)$

$+4[(\sin^2 x+\cos^2 x)^3-3\sin^2 x\cos^2 x(\sin^2 x+\cos^2 x)]$

$=3-6\sin 2x+3\sin^2 2x+6+6\sin 2x+4\left[1-\frac{3}{4}\sin^2 2x\right]$

$=13+3\sin^2 2x-3\sin^2 2x=13$

14. **(d)** The given equation is $2\sin^2\theta-3\sin\theta-2=0$

$\Rightarrow (2\sin\theta+1)(\sin\theta-2)=0$

$\Rightarrow \sin\theta=-\frac{1}{2}$ [$\because \sin\theta-2=0$ is not possible]

$\Rightarrow \sin\theta=\sin(-\pi/6)=\sin(7\pi/6)$

$\Rightarrow \theta=n\pi+(-1)^n(-\pi/6)=n\pi+(-1)^n 7\pi/6$

$\Rightarrow$ Thus, $\theta=n\pi+(-1)^n 7\pi/6$

15. **(b)** We have $\sec^2\theta=\frac{4xy}{(x+y)^2}$

But $\sec^2\theta\geq 1\Rightarrow \frac{4xy}{(x+y)^2}\geq 1$

$\Rightarrow 4xy\geq x^2+y^2+2xy$

$\Rightarrow x^2+y^2-2xy\leq 0\Rightarrow (x-y)^2\leq 0$

$\Rightarrow x-y=0$ [as perfect square of real number can never be negative]

Also then $x\neq 0$ as then $\sec^2\theta$ will become indeterminate.

16. **(a)** Given that in ΔPQR, $\angle R=\pi/2$

$\Rightarrow \angle P+\angle Q=\pi/2\Rightarrow \frac{\angle P}{2}+\frac{\angle Q}{2}=\frac{\pi}{4}$

Also tan $P/2$ and tan $Q/2$ are roots of the equation $ax^2 + bx + c = 0\ (a \neq 0)$

$\therefore$ tan $P/2$ + tan $Q/2 = -\frac{b}{a}$; $\tan P/2 \tan Q/2 = c/a$

Now consider, $\tan\left(\frac{P+Q}{2}\right) = \frac{\tan P/2 + \tan Q/2}{1-\tan P/2 \tan Q/2}$

$\Rightarrow \tan\frac{\pi}{4} = \frac{-b/a}{1-c/a} \Rightarrow 1-\frac{c}{a} = -\frac{b}{a}$

$\Rightarrow a - c = -b \Rightarrow a + b = c$

17. **(c)** $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$

$= (\sin\theta + 3\sin\theta - 4\sin^3\theta).\sin\theta$

$= (4\sin\theta - 4\sin^3\theta)\sin\theta = \sin^2\theta\,(4-4\sin^2\theta)$

$= 4\sin^2\theta\,(1-\sin^2\theta)$

$= 4\sin^2\theta\cos^2\theta = (2\sin\theta\cos\theta)^2 = (\sin 2\theta)^2 \geq 0$

which is true for all θ.

18. **(c)** To simplify the det. Let $\sin x = a$; $\cos x = b$ the equation becomes

$$\begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix} = 0$$ Operating $C_2 - C_1$; $C_3 - C_2$ we get

$$\begin{vmatrix} a & b-a & 0 \\ b & a-b & b-a \\ b & 0 & a-b \end{vmatrix} = 0$$

$\Rightarrow a(a-b)^2 - (b-a)[b(a-b) - b(b-a)] = 0$

$\Rightarrow a(a-b)^2 - 2b(b-a)(a-b) = 0$

$\Rightarrow (a-b)^2(a-2b) = 0 \Rightarrow (a = b)$ or $a = 2b$

$\Rightarrow \frac{a}{b} = 1$ or $\frac{a}{b} = 2$

$\Rightarrow \tan x = 1$ or $\tan x = 2$. But we have $-\pi/4 \leq x \leq \pi/4$

$\Rightarrow \tan(-\pi/4) \leq \tan x \leq \tan(\pi/4) \Rightarrow -1 \leq \tan x \leq 1$

$\therefore \tan x = 1 \Rightarrow x = \pi/4$ $\therefore$ Only one real root is there.

19. **(a)** We are given that

$(\cot\alpha_1).(\cot\alpha_2)\ ....\ (\cot\alpha_n) = 1$

$\Rightarrow (\cos\alpha_1)(\cos\alpha_2)\ ....\ (\cos\alpha_n)$

$= (\sin\alpha_1)(\sin\alpha_2)....(\sin\alpha_n)$ ....(1)

Let $y = (\cos\alpha_1)(\cos\alpha_2)\ ....\ (\cos\alpha_n)$ (to be max.)

Squaring both sides, we get

$y^2 = (\cos^2\alpha_1)(\cos^2\alpha_2)\ ....\ (\cos^2\alpha_n)$

$= \cos\alpha_1\sin\alpha_1\cos\alpha_2\sin\alpha_2\ ....\ \cos\alpha_n\sin\alpha_n$ (Using (1))

$= \frac{1}{2^n}[\sin 2\alpha_1 \sin 2\alpha_2\ ....\ \sin 2\alpha_n]$

As $0 \leq \alpha_1, \alpha_2, ...\alpha_n \leq \pi/2$

$\therefore 0 \leq 2\alpha_1, 2\alpha_2, ...2\alpha_n \leq \pi$

$\Rightarrow 0 \leq \sin 2\alpha_1, \sin 2\alpha_2, ...\sin 2\alpha_n \leq 1$

$\therefore y^2 \leq \frac{1}{2^n}.1 \Rightarrow y \leq \frac{1}{2^{n/2}}$

$\therefore$ Max. value of y is $1/2^{n/2}$.

20. **(c)** Given that $\alpha + \beta = \pi/2 \Rightarrow \alpha = \pi/2 - \beta$

$\Rightarrow \tan\alpha = \tan(\pi/2 - \beta) = \cot\beta = \frac{1}{\tan\beta}$

$\Rightarrow \tan\alpha\tan\beta = 1 \Rightarrow 1 + \tan\alpha\tan\beta = 2.$

$\therefore \tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} \Rightarrow \tan\gamma = \frac{\tan\alpha - \tan\beta}{2}$

$\Rightarrow 2\tan\gamma = \tan\alpha - \tan\beta \Rightarrow \tan\alpha = 2\tan\gamma + \tan\beta$

21. **(b)** We know that

$\Rightarrow -\sqrt{a^2+b^2} \leq a\cos\theta + b\sin\theta \leq \sqrt{a^2+b^2}$

NOTE THIS STEP

$\Rightarrow -\sqrt{74} \leq 7\cos x + 5\sin x \leq \sqrt{74}$

$\Rightarrow -\sqrt{74} \leq 2k+1 \leq \sqrt{74} \Rightarrow -8.6 \leq 2k+1 \leq 8.6$

$\Rightarrow -4.8 \leq k \leq 3.8$

(considering only integral values)

$\Rightarrow k$ can take 8 integral values.

22. **(b)** Given that $\sin\theta = 1/2$ and $\cos\phi = 1/3$ and θ and ϕ both are acute angles

$\therefore \theta = \pi/6$ and $0 < \frac{1}{3} < \frac{1}{2}$

or $\cos\pi/2 < \cos\phi < \cos\pi/3$ or $\pi/3 < \phi < \pi/2$

$\therefore \frac{\pi}{3} + \frac{\pi}{6} < \theta + \phi < \frac{\pi}{2} + \frac{\pi}{6}$ or $\frac{\pi}{2} < \theta + \phi < \frac{2\pi}{3}$

$\Rightarrow \theta + \phi \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

23. **(d)** Given that $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$ where $\alpha, \beta \in [-\pi, \pi]$

Now, $\cos(\alpha - \beta) = 1 \Rightarrow \alpha - \beta = 0 \Rightarrow \alpha = \beta$

Now, $\cos(\alpha + \beta) = 1/e \Rightarrow \cos 2\alpha = 1/e$

$\because 0 < 1/e < 1$ and $2\alpha \in [-2\pi, 2\pi]$

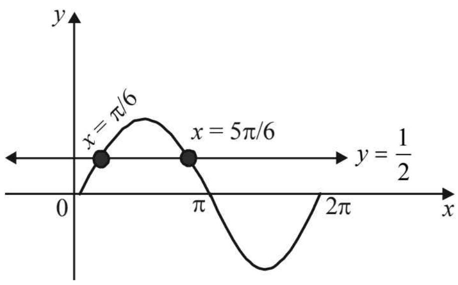
$\Rightarrow$ There will be two values of 2α satisfying $\cos 2\alpha = 1/e$ in $[0, 2\pi]$ and two in $[-2\pi, 0]$.

$\Rightarrow$ There will be four values of α in $[-\pi, \pi]$ and correspondingly four values of β. Hence there are four sets of (α, β).

24. **(a)** $2\sin^2\theta - 5\sin\theta + 2 > 0$

$\Rightarrow (\sin\theta - 2)(2\sin\theta - 1) > 0$

$\Rightarrow \sin\theta < \frac{1}{2}$ $\quad [\because -1 \le \sin\theta \le 1]$



From graph, we get $x \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

25. **(b)** $\because \theta \in \left(0, \frac{\pi}{4}\right) \Rightarrow \tan\theta < 1$ and $\cot\theta > 1$

Let $\tan\theta = 1 - x$ and $\cot\theta = 1 + y$

Where $x, y > 0$ and are very small, then

$\therefore t_1 = (1-x)^{1-x}, t_2 = (1-x)^{1+y}, t_3 = (1+y)^{1-x}, t_4 = (1+y)^{1+y}$

Clearly, $t_4 > t_3$ and $t_1 > t_2$ also, $t_3 > t_1$ **NOTE THIS STEP**

Thus $t_4 > t_3 > t_1 > t_2$.

26. **(c)** $2\sin^2\theta - \cos 2\theta = 0 \Rightarrow 1 - 2\cos 2\theta = 0$

$\Rightarrow \cos 2\theta = \frac{1}{2} \Rightarrow 2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ ....(1)

where $\theta \in [0, 2\pi]$

Also $2\cos^2\theta - 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta + 3\sin\theta - 2 = 0$

$\Rightarrow (2\sin\theta - 1)(\sin\theta + 2) = 0 \Rightarrow \sin\theta = \frac{1}{2}$

$[\because \sin\theta \ne -2]$

$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$ .... (2), where $\theta \in [0, 2\pi]$

Combining (1) and (2), we get $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore$ Two solutions are there.

27. **(d)** $\sin x + 2\sin 2x - \sin 3x = 3$

$\Rightarrow \sin x + 4\sin x\cos x - 3\sin x + 4\sin^3 x = 3$

$\Rightarrow \sin x(-2 + 2\cos x + 4\sin^2 x) = 3$

$\Rightarrow \sin x(-2 + 2\cos x + 4 - 4\cos^2 x) = 3$

$\Rightarrow 2 + 2\cos x - 4\cos^2 x = \frac{3}{\sin x}$

$\Rightarrow 2 - \left(4\cos^2 x - 2.2\cos x.\frac{1}{2} + \frac{1}{4}\right) + \frac{1}{4} = \frac{3}{\sin x}$

$\Rightarrow \frac{9}{4} - \left(2\cos x - \frac{1}{2}\right)^2 = \frac{3}{\sin x}$

LHS $\le \frac{9}{4}$ and RHS ≥ 3

$\therefore$ Equation has no solution.

28. **(c)** $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$

$\Rightarrow \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \cos^2 x - \sin^2 x$

$\Rightarrow \cos\left(x - \frac{\pi}{3}\right) = \cos 2x \Rightarrow x - \frac{\pi}{3} = 2n\pi \pm 2x$

$\Rightarrow x = \frac{2n\pi}{3} + \frac{\pi}{9}$ or $x = -2n\pi - \frac{\pi}{3}$

For $x \in S, n = 0 \Rightarrow x = \frac{\pi}{9}, -\frac{\pi}{3}$

$n = 1 \Rightarrow x = \frac{7\pi}{9}$; $\quad n = -1 \Rightarrow x = \frac{-5\pi}{9}$

$\therefore$ Sum of all values of $x = \frac{\pi}{9} - \frac{\pi}{3} + \frac{7\pi}{9} - \frac{5\pi}{9} = 0$

29. **(c)** $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$

$$= \sum_{k=1}^{13} \frac{1}{\sin\frac{\pi}{6}}\left[\frac{\sin\left\{\frac{\pi}{4} + \frac{k\pi}{6} - \left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right)\right\}}{\sin\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}\right]$$

$$\sum_{k=1}^{13} 2\left[\cot\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)\right]$$

$$= 2\left[\left\{\cot\frac{\pi}{4} - \cot\left(\frac{\pi}{4} + \frac{\pi}{6}\right)\right\} + \left\{\cot\left(\frac{\pi}{4} + \frac{\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{2\pi}{6}\right)\right\} + \dots\dots + \left\{\cot\left(\frac{\pi}{4} + \frac{12\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{13\pi}{6}\right)\right\}\right]$$

$$= 2\left[\cot\frac{\pi}{4} - \cot\left(\frac{\pi}{4} + \frac{13\pi}{6}\right)\right] = 2\left[1 - \cot\frac{5\pi}{12}\right]$$

$$= 2\left[1 - \frac{\sqrt{3}-1}{\sqrt{3}+1}\right] = 2\left[1 - \left(2 - \sqrt{3}\right)\right] = 2\left(\sqrt{3} - 1\right)$$

D. MCQs with ONE or MORE THAN ONE Correct

1. **(c)** We have,

$(1+\cos\pi/8)\ (1+\cos 3\pi/8)(1+\cos 5\pi/8)\ (1+\cos 7\pi/8)$

$= (1+\cos\pi/8)\ (1+\cos 3\pi/8)(1+\cos(\pi-3\pi/8))\ (1+\cos(\pi-\pi/8))$

$= (1+\cos\pi/8)\ (1+\cos 3\pi/8)(1-\cos 3\pi/8)\ (1-\cos\pi/8)$

$= (1-\cos^2\pi/8)\ (1-\cos^2 3\pi/8) = \sin^2\pi/8\ \sin^2 3\pi/8$

$= \frac{1}{4}.[2\sin\pi/8\ \sin\ (\pi/2-\pi/8)]^2$

$= \frac{1}{4}.[2\sin\pi/8\ \cos\pi/8)]^2 = \frac{1}{4}.\sin^2\pi/4 = \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$

$\therefore$ (c) is the correct answer.

2. **(b)** The given expression is

$= 3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]$

$-2\ [\sin^6(\pi/2+\alpha)+\sin^6(5\pi-\alpha)]$

$= 3\left[\cos^4\alpha+\sin^4\alpha\right]-2\left[\cos^6\alpha+\sin^6\alpha\right]$

$= 3\left[\left(\cos^2\alpha+\sin^2\alpha\right)^2-2\sin^2\alpha\cos^2\alpha\right]$

$-\ 2\left[(\cos^2\alpha+\sin^2\alpha)^3-3\cos^2\alpha\sin^2\alpha\ (\cos^2\alpha+\sin^2\alpha)\right]$

$= 3\left[1-2\sin^2\alpha\cos^2\alpha\right]-2\left[1-3\cos^2\alpha\sin^2\alpha\right]$

$= 3-6\sin^2\alpha\cos^2\alpha-2+6\sin^2\alpha\cos^2\alpha = 1$

3. **(d)** Since $a_1 + a_2\cos 2x + a_3\sin^2 x = 0$ for all x

Putting $x = 0$ and $x = \pi/2$, we get $a_1 + a_2 = 0$ ....(1)

and $a_1 - a_2 + a_3 = 0$ ....(2)

$\Rightarrow\ a_2 = -a_1$ and $a_3 = -2a_1$

$\therefore$ The given equation becomes

$a_1 - a_1\cos 2x - 2a_1\sin^2 x = 0,\ \forall x$

$\Rightarrow\ a_1\ (1-\cos 2x - 2\sin^2 x) = 0, \forall x$

$\Rightarrow\ a_1\ (2\sin^2 x - 2\sin^2 x) = 0,\ \forall x$

The above is satisfied for all values of a_1.

Hence infinite number of triplets $(a_1, -a_1, -2a_1)$ are possible.

4. **(a,c)** We have

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Operating $C_1 \to C_1 + C_2$

$$\begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 2 & 1+\cos^2\theta & 4\sin 4\theta \\ 1 & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Operating $R_1 \to R_1 - R_2$; $R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Expanding along R_1 we get $[1+4\sin\ 4\theta+1] = 0$

$\Rightarrow\ 2(1+2\sin 4\theta) = 0 \Rightarrow\ \sin\ 4\theta = -\frac{1}{2}$

$\Rightarrow\ 4\theta = \pi+\pi/6$ or $2\pi-\pi/6$

$\Rightarrow\ 4\theta = 7\pi/6$ or $11\pi/6$

$\Rightarrow\ \theta = 7\pi/24$ or $11\pi/24$

5. **(d)** $2\sin^2 x + 3\sin x - 2 > 0$

$(2\sin x - 1)(\sin x + 2) > 0$

$\Rightarrow\ 2\sin x - 1 > 0\ \ (\because -1 \le \sin x \le 1)$

$\Rightarrow\ \sin x > 1/2 \Rightarrow x \in (\pi/6, 5\pi/6)$ ....(1)

Also $x^2 - x - 2 < 0$

$\Rightarrow\ (x-2)\ (x+1) < 0 \Rightarrow -1 < x < 2$ ....(2)

Combining (1) and (2) $x \in (\pi/6, 2)$.

6. **(c)** $\sin\alpha + \sin\beta + \sin\gamma$

$= 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + 2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}$

$= 2\sin\left(\frac{\pi}{2}-\frac{\gamma}{2}\right)\cos\frac{\alpha-\beta}{2} + 2\sin\left(\frac{\pi}{2}-\frac{\alpha+\beta}{2}\right)\cos\frac{\gamma}{2}$

$= 2\cos\frac{\gamma}{2}\left[\cos\frac{\alpha-\beta}{2}+\cos\frac{\alpha+\beta}{2}\right]$

$= 2\cos\alpha/2\ \cos\beta/2\cos\gamma/2$

$\therefore$ Each $\cos\alpha/2,\ \cos\beta/2,\ \cos\gamma/2$ lies between -1 and 1, therefore $-1 \le \cos\alpha/2,\ \cos\beta/2, \cos\gamma/2 \le 1$

$\Rightarrow\ -2 \le 2\cos\alpha/2,\ \cos\beta/2, \cos\gamma/2 \le 2$

$\Rightarrow\ -2 \le \cos\alpha + \cos\beta + \cos\gamma \le 2$

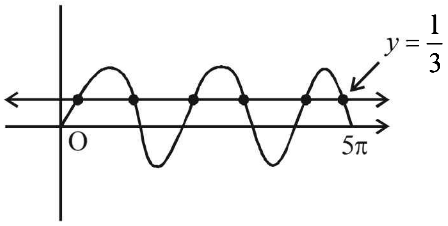
$\therefore$ min value $= -2$.

7. **(c)** $3\sin^2 x - 7\sin x + 2 = 0$, put $\sin x = s$

$\Rightarrow\ (s-2)\ (3s-1) = 0\ \Rightarrow\ s = 1/3$

($s = 2$ is not possible)

Number of solutions of $\sin x = \frac{1}{3}$ from the following graph is 6 between $[0, 5\pi]$



8. **(c)** We know that $\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$ (irrational)

$\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$ (irrational)

$\sin 15° . \cos 15° = \frac{1}{2}$ (2 sin 15° cos 15°)

$= \frac{1}{2}\sin 30° = \frac{1}{4}$ (rational)

$\sin 15° \cos 75° = \sin 15° \cos (90-15°)$

$= \sin 15° \sin 15° = \sin^2 15° = \frac{1}{2}(1+\cos 30°)$

$= \frac{1}{2}\left(1-\frac{\sqrt{3}}{2}\right)$ (irrational)

9. **(a, b, c, d)** $E = f_n(\theta) = \frac{\sin(\theta/2)}{\cos(\theta/2)}$

$$\left[\frac{2\cos^2(\theta/2)}{\cos\theta}.\frac{2\cos^2\theta}{\cos 2\theta}.\frac{2\cos^2 2\theta}{\cos 4\theta}.....\frac{2\cos^2 2^{n-1}\theta}{\cos 2^n\theta}\right]$$

$$= \frac{\sin\theta}{\cos\theta}\left[\frac{2\cos^2\theta}{\cos 2\theta}.\frac{2\cos^2 2\theta}{\cos 4\theta}....\frac{2\cos^2 2^{n-1}\theta}{\cos 2^n\theta}\right] = \tan 2^n\theta$$

$n = 2, \theta = \frac{\pi}{16}$

$f_2\left(\frac{\pi}{16}\right) = \tan 4.\frac{\pi}{16} = \tan\frac{\pi}{4} = 1.$

Similarly, $f_3\left(\frac{\pi}{32}\right)$, $f_4\left(\frac{\pi}{64}\right)$ *and* $f_5\left(\frac{\pi}{128}\right)$ *is* $\tan\frac{\pi}{4} = 1$.

10. **(a, b)** Given that

$\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5} \Rightarrow 3\sin^4 x + 2\cos^4 x = \frac{6}{5}$

$\Rightarrow \sin^4 x + 2[\sin^4 x + \cos^4 x] = \frac{6}{5}$

$\Rightarrow \sin^4 x + 2[1 - 2\sin^2 x \cos^2 x] = \frac{6}{5}$

$\Rightarrow \sin^4 x + 2 - 4\sin^2 x(1-\sin^2 x) = \frac{6}{5}$

$\Rightarrow 5\sin^4 x - 4\sin^2 x + 2 - \frac{6}{5} = 0$

$\Rightarrow 25\sin^4 x - 20\sin^2 x + 4 = 0$

$\Rightarrow (5\sin^2 x - 2)^2 = 0 \Rightarrow \sin^2 x = \frac{2}{5}$

$\Rightarrow \cos^2 x = \frac{3}{5}$ and $\tan^2 x = \frac{2}{3}$

Also $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{625} + \frac{3}{625} = \frac{5}{625} = \frac{1}{125}$

11. **(c, d)** We have

$$\sum_{m=1}^{6}\text{cosec}\left[\theta + \frac{(m-1)\pi}{4}\right]\text{cosec}\left[\theta + \frac{m\pi}{4}\right] = 4\sqrt{2}$$

$$\Rightarrow \sum_{m=1}^{6}\frac{\sin\frac{\pi}{4}}{\sin\left[\theta + \frac{(m-1)\pi}{4}\right]\sin\left[\theta + \frac{m\pi}{4}\right]} = 4$$

$$\Rightarrow \sum_{m=1}^{6}\frac{\sin\left[\left(\theta + \frac{m\pi}{4}\right) - \left(\theta + \frac{(m-1)\pi}{4}\right)\right]}{\sin\left(\theta + \frac{(m-1)\pi}{4}\right)\sin\left(\theta + \frac{m\pi}{4}\right)} = 4$$

$$\Rightarrow \sum_{m=1}^{6}\frac{\left[\begin{array}{l}\sin\left(\theta + \frac{m\pi}{4}\right)\cos\left(\theta + \frac{(m-1)\pi}{4}\right)\\ -\cos\left(\theta + \frac{m\pi}{4}\right)\sin\left(\theta + \frac{(m-1)\pi}{4}\right)\end{array}\right]}{\sin\left(\theta + \frac{(m-1)\pi}{4}\right)\sin\left(\theta + \frac{m\pi}{4}\right)} = 4$$

$$\Rightarrow \sum_{m=1}^{6}\left[\cot\left(\theta + \frac{(m-1)\pi}{4}\right) - \cot\left(\theta + \frac{m\pi}{4}\right)\right] = 4$$

$$\Rightarrow \left[\cot\theta - \cot\left(\theta + \frac{\pi}{4}\right)\right] + \left[\cot\left(\theta + \frac{\pi}{4}\right) - \cot\left(\theta + \frac{2\pi}{4}\right)\right]$$

$$+ ... + \left[\cot\left(\theta + \frac{5\pi}{4}\right) - \cot\left(\theta + \frac{6\pi}{4}\right)\right] = 4$$

$\Rightarrow \cot\theta - \cot\left(\theta + \frac{3\pi}{2}\right) = 4 \Rightarrow \cot\theta + \tan\theta = 4$

$\Rightarrow \cos^2\theta + \sin^2\theta = 4\sin\theta\cos\theta$

$\Rightarrow \sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6} \Rightarrow \theta = \frac{\pi}{12}$ or $\frac{5\pi}{12}$

12. (a, c, d)
As $\tan(2\pi - \theta) > 0 \Rightarrow -\tan\theta > 0 \Rightarrow \tan\theta < 0$
$\therefore \quad \theta \in$ II or IV quadrant ...(1)

And $-1 < \sin\theta < \frac{-\sqrt{3}}{2}$
$\Rightarrow \theta \in$ III or IV quadrant ...(2)
Also $\theta \in [0, 2\pi]$ ...(3)
Combining above three equations (1), (2) and (3); $\theta \in$ IV quadrant and more precisely $\frac{3\pi}{2} < \theta < \frac{5\pi}{3}$

$(\because -1 < \sin\theta < -\frac{\sqrt{3}}{2})$

Now,
$2\cos\theta(1 - \sin\phi) = \sin^2\theta(\tan\theta/2 + \cot\theta/2)\cos\phi - 1$

$$\Rightarrow 2\cos\theta(1 - \sin\phi) = \sin^2\theta \times \frac{1}{\sin\theta/2\ \cos\theta/2}.\cos\phi - 1$$

$\Rightarrow 2\cos\theta(1 - \sin\phi) = 2\sin\theta\cos\phi - 1$
$\Rightarrow 2\cos\theta + 1 = 2\sin(\theta + \phi)$

$\because \quad \theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right) \qquad \therefore 2\cos\theta + 1 \in (1, 2)$

$\Rightarrow \quad 1 < 2\sin(\theta + \phi) < 2$

$\Rightarrow \frac{1}{2} < \sin(\theta + \phi) < 1 \Rightarrow \theta + \phi \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

or $\theta + \phi \in \left(2\pi + \frac{\pi}{6}, 2\pi + \frac{5\pi}{6}\right)$ or $\left(\frac{13\pi}{6}, \frac{17\pi}{6}\right)$

But $\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)$ (as $\theta + \phi \in [0, 4\pi]$)

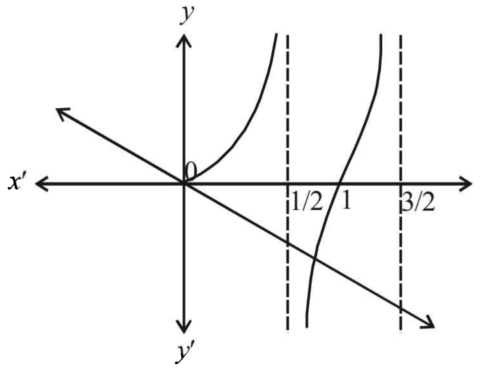
$$\Rightarrow \frac{13\pi}{6} < \theta + \phi < \frac{17\pi}{6} \Rightarrow \frac{13\pi}{6} - \theta < \phi < \frac{17\pi}{6} - \theta$$

$$\Rightarrow \frac{13\pi}{6} - \theta_{max} < \phi < \frac{17\pi}{6} - \theta_{min}$$

$$\Rightarrow \frac{13\pi}{6} - \frac{5\pi}{3} < \phi < \frac{17\pi}{6} - \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \phi < \frac{4\pi}{3}$$

13. (c) Let $f(x) = x^2 - x\sin x - \cos x$
$\therefore \quad f'(x) = 2x - x\cos x = x(2 - \cos x)$
$\therefore \quad f$ is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$
Also $\lim_{x \to \infty} f(x) = \infty$, $\lim_{x \to -\infty} f(x) = \infty$ and $f(0) = -1$
$\therefore \quad y = f(x)$ meets x-axis twice.
i.e., $f(x) = 0$ has two points in $(-\infty, \infty)$.

14. (b, c) We have $f(x) = x\sin\pi x, x > 0$
$\Rightarrow f'(x) = \sin\pi x + x\pi\cos\pi x$
$f'(x) = 0 \Rightarrow \tan\pi x = -\pi x$



We observe, from graph of $y = \tan\pi x$ and $y = -\pi x$ that they intersect at unique point in the intervals

$$(n, n+1) \text{ and } \left(n + \frac{1}{2}, n+1\right)$$

E. Subjective Problems

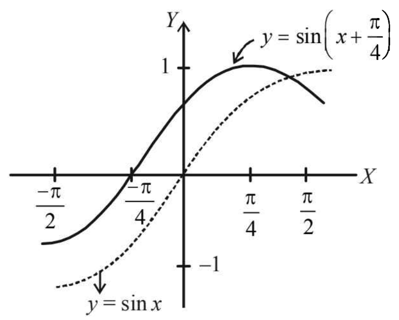
1. We know $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\ \tan\beta}$

$$\Rightarrow \tan(\alpha + \beta) = \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \frac{m}{m+1}.\frac{1}{2m+1}} = \frac{2m^2 + 2m + 1}{2m^2 + 2m + 1} = 1$$

$\Rightarrow \alpha + \beta = n\pi + \pi/4$ where $n \in Z$.

2. (a) Given: $y = \frac{1}{\sqrt{2}}(\sin x + \cos x) = \sin\left(x + \frac{\pi}{4}\right)$ ...(1)

Now, to draw the graph of $y = \sin\left(x + \frac{\pi}{4}\right)$, we first draw the graph of $y = \sin x$ and then on shifting it by $-\frac{\pi}{4}$, we will obtain the required graph as shown in figure given below.



(b) $\cos(\alpha + \beta) = \frac{4}{5}$

$$\Rightarrow \tan(\alpha + \beta) = \frac{3}{4}, 0 < \alpha, \beta < \frac{\pi}{4}$$

$$\sin(\alpha-\beta)=\frac{5}{13}\Rightarrow\tan(\alpha-\beta)=\frac{5}{12}$$

$$\therefore\ \tan 2\alpha=\tan[(\alpha+\beta)+(\alpha-\beta)]$$

$$=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}}=\frac{9+5}{12}\times\frac{16}{11}=\frac{56}{33}$$

3. Given $\alpha+\beta-\gamma=\pi$ and to prove that
$\sin^2\alpha+\sin^2\beta-\sin^2\gamma=2\sin\alpha\sin\beta\cos\gamma$
L.H.S. $=\sin^2\alpha+\sin^2\beta-\sin^2\gamma$
[Using $\sin^2 A-\sin^2 B=\sin(A+B)\sin(A-B)$]
$=\sin^2\alpha+\sin(\beta+\gamma)\sin(\beta-\gamma)$
$=\sin^2\alpha+\sin(\beta+\gamma)\sin(\pi-\alpha)$ $(\because\ \alpha+\beta-\gamma=\pi)$
$=\sin^2\alpha+\sin(\beta+\gamma)\sin\alpha$
$=\sin\alpha(\sin\alpha+\sin(\beta+\gamma))$
$=\sin\alpha[\sin[\pi-(\beta-\gamma)]+\sin(\beta+\gamma)]$
$=\sin\alpha[\sin(\beta-\gamma)+\sin(\beta+\gamma)]$
$=\sin\alpha[2\sin\beta\cos\gamma]=2\sin\alpha\sin\beta\cos\gamma=$ R.H.S.

4. $A=\left\{x:\frac{\pi}{6}\le x\le\frac{\pi}{3}\right\}$

$f(x)=\cos x-x(1+x)$

$f'(x)=-\sin x-1-2x<0,\ \forall x\in A$

$\therefore f$ is a decreasing function.

$$\therefore \text{ as } \frac{\pi}{6}\le x\le\frac{\pi}{3}\Rightarrow f\left(\frac{\pi}{3}\right)\le f(x)\le\left(\frac{\pi}{6}\right)$$

$$\Rightarrow\ \cos\frac{\pi}{3}-\frac{\pi}{3}\left(1+\frac{\pi}{3}\right)\le f(x)\le\cos\frac{\pi}{6}-\frac{\pi}{6}\left(1+\frac{\pi}{6}\right)$$

$$\therefore\ f(A)=\left[\frac{1}{2}-\frac{\pi}{3}\left(1+\frac{\pi}{3}\right),\frac{\sqrt{3}}{2}-\frac{\pi}{6}\left(1+\frac{\pi}{6}\right)\right]$$

5. We have

$$\cos\theta+\sin\theta=\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta\right]$$

$$=\sqrt{2}\sin(\pi/4+\theta)$$

$$\therefore\ \cos\theta+\sin\theta\le\sqrt{2}<\pi/2\ \left(\begin{matrix}\because\sqrt{2}=1.414\\ \pi/2=1.57\end{matrix}\right)$$

$\therefore\ \cos\theta+\sin\theta<\pi/2\ \Rightarrow\ \cos\theta<\pi/2-\sin\theta$ ...(1)

As $\theta\in[0,\pi/2]$ in which $\sin\theta$ increases.

$\therefore$ Taking sin on both sides of eq. (1), we get

$\sin(\cos\theta)<\sin(\pi/2-\sin\theta)$

$\sin(\cos\theta)<\cos(\sin\theta)$

$\Rightarrow\ \cos(\sin\theta)>\sin(\cos\theta)$ ....(1)

Hence the result.

6. L.H.S. $=\sin12^\circ\sin48^\circ\sin54^\circ=\frac{1}{2}[2\sin12^\circ\cos42^\circ]\sin54^\circ$

$$=\frac{1}{2}\sin^2 54^\circ-\frac{1}{2}\sin54^\circ=\frac{1}{4}\left[2\sin^2 54^\circ-\sin54^\circ\right]$$

Now we know that $\sin 54^\circ=\frac{1+\sqrt{5}}{4}$

$$\therefore\ \text{We get,}=\frac{1}{4}\left[2\left(\frac{1+\sqrt{5}}{4}\right)^2-\left(\frac{1+\sqrt{5}}{4}\right)\right]$$

$$=\frac{1}{4}\left[2\left(\frac{1+5+2\sqrt{5}}{16}\right)-\left(\frac{1+\sqrt{5}}{4}\right)\right]$$

$$=\frac{1}{4}\times\frac{1}{8}\left[6+2\sqrt{5}-2-2\sqrt{5}\right]$$

$$=\frac{1}{32}\times4=\frac{1}{8}=\text{R.H.S.}$$

7. We know that,

$$\cos A\cos2A\cos4A\ \ldots\cos2^n A=\frac{1}{2^{n+1}\sin A}\sin.(2^{n+1}A)$$

$$\therefore\ 16\cos\frac{2\pi}{15}\cos2\left(\frac{2\pi}{15}\right)\cos2^2\left(\frac{2\pi}{15}\right)\cos2^3\left(\frac{2\pi}{15}\right)$$

$$=16\,.\,\frac{\sin(2^4A)}{2^4\sin A}\qquad(\text{where } A=2\pi/15)$$

$$=16\,.\,\frac{\sin(32\pi/15)}{16\sin2\pi/15}=\frac{\sin(32\pi/15)}{\sin(2\pi+2\pi/15)}=\frac{\sin(32\pi/15)}{\sin(32\pi/15)}=1$$

8. Given eq. is,

$4\cos^2 x\sin x-2\sin^2 x=3\sin x$

$\Rightarrow\ 4\cos^2x\sin x-2\sin^2x-3\sin x=0$

$\Rightarrow\ 4(1-\sin^2x)\sin x-2\sin^2x-3\sin x=0$

$\Rightarrow\ \sin x[4\sin^2x+2\sin x-1]=0$

$\Rightarrow$ either $\sin x=0$ or $4\sin^2x+2\sin x-1=0$

If $\sin x=0\Rightarrow x=n\pi$

$\Rightarrow$ If $4\sin^2x+2\sin x-1=0\Rightarrow\sin x=\frac{-1\pm\sqrt{5}}{4}$

If $\sin x=\frac{-1\pm\sqrt{5}}{4}=\sin18^\circ=\sin\frac{\pi}{10}$

then $x=nx+(-1)^4\frac{\pi}{10}$

If $\sin x=-\left(\frac{\sqrt{5}+1}{4}\right)=\sin(-54^\circ)=\sin\left(\frac{-3\pi}{10}\right)$

then $x=n\pi+(-1)^n\left(\frac{-3\pi}{10}\right)$

Hence, $x = n\pi,\ n\pi + (-1)^n \frac{\pi}{10}$ or $n\pi + (-1)^n\left(\frac{-3\pi}{10}\right)$

where n is some integer

9. The given equation is

$$8^{(1+|\cos x|+|\cos^2 x|+|\cos^3 x|+....)} = 4^3$$

$\Rightarrow 2^{3(1+|\cos x|+|\cos^2 x|+|\cos^3 x|+....)} = 2^6$

$\Rightarrow 3\,(1+|\cos x|+|\cos^2 x|+|\cos^3 x|+...) = 6$

$\Rightarrow 1+|\cos x|+|\cos^2 x|+|\cos^3 x|+.... = 2$

$\Rightarrow \frac{1}{1-|\cos\ x|} = 2$ **NOTE THIS STEP**

$\Rightarrow 1-\cos x = 1/2 \Rightarrow |\cos x| = \frac{1}{2}$

$\Rightarrow x = \pi/3, -\pi/3, 2\pi/3, -2\pi/3, ...$

The values of $x \in (-\pi, \pi)$ are $\pm\ \pi/3,\ \pm\ 2\pi/3$.

10. We know that $\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

$\Rightarrow \frac{1-\tan^2\alpha}{\tan\alpha} = 2\cot 2\alpha \Rightarrow \cot\alpha - \tan\alpha = 2\cot 2\alpha$

Now we have to prove

$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$

LHS

$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 4\,(2\cot 2\,.\,4\alpha)$

$= \tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 4\,(\cot 4\alpha - \tan 4\alpha)$ [Using (1)]

$= \tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 4\cot 4\alpha - 4\tan 4\alpha$

$= \tan\alpha + 2\tan 2\alpha + 2\,(2\cot 2.2\ \alpha)$

$= \tan\alpha + 2\tan 2\alpha + 2\,(\cot\alpha - \tan 2\alpha)$

$= \tan\alpha + 2\tan 2\alpha + 2\,(2\cot 2\alpha - \tan 2\alpha)$ [Using (1)]

$= \tan\alpha + 2\cot 2\alpha$

$= \tan\alpha + (\cot\alpha - \tan\alpha)$ [Using (1)]

$= \cot\alpha =$ RHS.

11. Given that in ΔABC, A, B and C are in A.P.

$\therefore A + C = 2B$

also $A + B + C = 180° \Rightarrow B + 2B = 180° \Rightarrow B = 60°$

Also given that, $\sin(2A+B) = \sin(C-A) = -\sin(B+2C) = \frac{1}{2}$

$\Rightarrow \sin(2A+60°) = \sin(C-A) = -\sin(60+2C) = \frac{1}{2}$ ...(1)

From eq. (1), we have

$\sin(2A+60°) = \frac{1}{2} \Rightarrow 2A + 60° = 30°,\ 150°$

but A can not be –ve

$\therefore\ 2A + 60° = 150° \Rightarrow 2A = 90° \Rightarrow A = 45°$

Again from (1) $\sin(60° + 2C) = -\frac{1}{2}$

$\Rightarrow 60° + 2C = 210°$ or $330°$

$\Rightarrow C = 75°$ or $135°$

Also from (1) $\sin(C-A) = \frac{1}{2} \Rightarrow C - A = 30°, 150°$

For $A = 45°, C = 75°$ or $195°$ (not possible) $\therefore\ C = 75°$

Hence we have $A = 45°$, $B = 60°$, $C = 75°$.

12. Let $y = \exp\,[\sin^2 x + \sin^4 x + \sin^6 x + ... \infty]\ln 2$

$= e^{In\,\cdot\,2^{\sin^2 x+\sin^4 x+\sin^6 x+....\infty}}$

$= 2^{\sin^2 x+\sin^4 x+\sin^6 x+....\infty} = 2^{\frac{\sin^2 x}{1-\sin^2 x}} = 2^{\tan^2 x}$

As y satisfies the eq.

$x^2 - 9x + 8 = 0 \qquad \therefore y^2 - 9y + 8 = 0$

$\Rightarrow (y-1)\,(y-8) = 0 \Rightarrow y = 1, 8$

$\Rightarrow 2^{\tan^2 x} = 1$ or $2^{\tan^2 x} = 8$

$\Rightarrow \tan^2 x = 0$ or $\tan^2 x = 3$

$\Rightarrow \tan x = 0$ or $\tan x = \sqrt{3}, -\sqrt{3}$

$\Rightarrow x = 0$ or $x = \pi/3,\ 2\pi/3$

But given that $0 < x < \pi/2 \Rightarrow x = \pi/3$

Hence $\frac{\cos x}{\cos x + \sin x} = \frac{1}{1+\tan x} = \frac{1}{1+\sqrt{3}} = \frac{\sqrt{3}-1}{2}$

13. Let $y = \frac{\tan x}{\tan 3x} \Rightarrow y = \frac{\tan x(1-3\tan^2 x)}{3\tan x - \tan^3 x}$

$\Rightarrow 3y - 3\tan^2 x = 1 - 3\tan^2 x$

$\Rightarrow (y-3)\tan^2 x = 3y - 1 \Rightarrow \tan^2 x = \frac{3y-1}{y-3}$

$\Rightarrow \frac{3y-1}{y-3} > 0$ (L.H.S. being a prefect square)

$\Rightarrow \frac{(3y-1)\,(y-3)}{(y-3)^2} > 0 \Rightarrow (3y-1)\,(y-3) > 0$

$\Rightarrow$ +ve | –ve | +ve ($-\infty$, 1/3, 3) $\Rightarrow y < \frac{1}{3}$ *or* $y > 3$

Thus y never lies between $\frac{1}{3}$ and 3.

14. Given that,

$\tan(x+100°) = \tan(x+50°)\tan x\tan(x-50°)$

$\Rightarrow \frac{\tan(x+100°)}{\tan x} = \tan(x+50°)\tan(x-50°)$

$\Rightarrow \frac{\sin(x+100°)\cos x}{\cos(x+100°)\sin x} = \frac{\sin(x+50°)\sin(x-50°)}{\cos(x+50°)\cos(x-50°)}$

$\Rightarrow \frac{\sin(2x+100°)+\sin 100°}{\sin(2x+100°)-\sin 100°} = \frac{\cos 100° - \cos 2x}{\cos 100° + \cos 2x}$

Applying componendo and dividendo, we get

$\Rightarrow \dfrac{2\sin(2x+100°)}{2\sin 100°}=\dfrac{2\cos 100°}{-2\cos 2x}$

$\Rightarrow$ $2\sin(2x+100°)\cos 2x=-2\sin 100°\cos 100°$

$\Rightarrow$ $\sin(4x+100°)+\sin 100°=-\sin 200°$

$\Rightarrow$ $\sin(4x+10°+90°)+\sin(90°+10°)=-\sin(180+20°)$

$\Rightarrow$ $\cos(4x+10°)+\cos 10°=\sin 20°$

$\Rightarrow$ $\cos(4x+10°)=\sin 20°-\cos 10°$

$\Rightarrow$ $\cos(4x+10°)=\sin 20°-\sin 80°$

$=-2\cos 50°\sin 30°=-2\cos 50°.\dfrac{1}{2}=-\cos 50°=\cos 130°$

$\Rightarrow$ $4x+10°=130° \Rightarrow x=30°$

15. Given that $\cos\theta=\sin\phi$

where $\theta=p\sin x,\ \phi=p\cos x$

Above is possible when both $\theta=\phi=\dfrac{\pi}{4}$ or $\theta=\phi=\dfrac{5\pi}{4}$

$\therefore$ $p\sin x=\dfrac{\pi}{4}$ or $p\sin x=\dfrac{5\pi}{4}$

and $p\cos x=\dfrac{\pi}{4}$ or $p\cos x=\dfrac{5\pi}{4}$

Squaring and adding, $p^2=\dfrac{\pi^2}{16}.2$ or $\dfrac{25\pi^2}{16}.2$

$\therefore$ $p=\dfrac{\pi}{4}\sqrt{2}$ only for least positive value or $p=\dfrac{\pi}{4}\sqrt{2}$

16. **Given :** $(1-\tan\theta)(1+\tan\theta)\sec^2\theta+2^{\tan^2\theta}=0$

or $(1-\tan^2\theta)(1+\tan^2\theta)+2^{\tan^2\theta}=0$

Let us put $\tan^2\theta=t$

$\therefore$ $(1-t)(1+t)+2^t=0$ or $1-t^2+2^t=0$

It is clearly satisfied by $t=3$.

as $-8+8=0$ $\therefore$ $\tan^2\theta=3$

$\therefore$ $p=\pm\pi/3$ in the given interval.

17. Let $y=\dfrac{\sin x\cos 3x}{\sin 3x\cos x}=\dfrac{\tan x}{\tan 3x}$

We have $y=\dfrac{\tan x}{\tan 3x}=\dfrac{\tan x(1-3\tan^2 x)}{3\tan x-\tan^3 x}=\dfrac{1-3\tan^2 x}{3-\tan^2 x}$

(the expression is not defined if $\tan x=0$)

$\Rightarrow$ $3y-(\tan^2 x)y=1-3\tan^2 x \Rightarrow 3y-1=(y-3)\tan^2 x$

$\Rightarrow$ $\tan^2 x=\dfrac{3y-1}{y-3}=\dfrac{(3y-1)(y-3)}{(y-3)^2}$

Since $\tan^2 x>0$, we get $(3y-1)(y-3)>0$

$\Rightarrow$ $\left(y-\dfrac{1}{3}\right)(y-3)>0 \Rightarrow y<\dfrac{1}{3}$ or $y>3$

This shows that y cannot lie between $\dfrac{1}{3}$ and 3.

18. Expanding the sigma on putting $k=1, 2, 3, ..., n$

$S=(n-1)\cos\dfrac{2\pi}{n}+(n-2)\cos 2.\dfrac{2\pi}{n}+......$

$+1.\cos(n-1)\cos\dfrac{2\pi}{n}$ ...(1)

We know that $\cos\theta=\cos(2\pi-\theta)$

Replacing each angle θ by $2\pi-\theta$ in (1), we get

$S=(n-1)\cos(n-1)\dfrac{2\pi}{n}+(n-2)\cos(n-2)\dfrac{2\pi}{n}+......$

$+1.\cos\dfrac{2\pi}{n}$ by (1) .....(2)

Add terms in (1) and (2) having the same angle and take n common

$\therefore 2S=n\left[\cos\dfrac{2\pi}{n}+\cos\dfrac{4\pi}{n}+\cos\dfrac{6\pi}{n}+....+\cos(n-1)\dfrac{2\pi}{n}\right]$

Angles are in A.P. of $d=\dfrac{2\pi}{n}$

$2S=n\left[\dfrac{\sin(n-1)\frac{\pi}{n}}{\sin\frac{\pi}{n}}\cos\dfrac{\frac{2\pi}{n}+(n-1)\frac{2\pi}{n}}{2}\right]$ **NOTE THIS STEP**

$=n.1\cos\pi=-n$ $\because \sin(\pi-\theta)=\sin\theta$ $\therefore$ $S=-n/2$

19. We have, $A+B+C=\pi$

$\Rightarrow$ $\dfrac{A}{2}+\dfrac{B}{2}+\dfrac{C}{2}=\dfrac{\pi}{2} \Rightarrow \dfrac{A}{2}+\dfrac{B}{2}=\dfrac{\pi}{2}-\dfrac{C}{2}$

or $\cot\left(\dfrac{A}{2}+\dfrac{B}{2}\right)=\cot\left(\dfrac{\pi}{2}-\dfrac{C}{2}\right)$

$\Rightarrow$ $\dfrac{\cot\frac{A}{2}.\cot\frac{B}{2}-1}{\cot\frac{A}{2}+\cot\frac{B}{2}}=\tan\dfrac{C}{2}$

$\Rightarrow$ $\cot\dfrac{A}{2}+\cot\dfrac{B}{2}+\cot\dfrac{C}{2}=\cot\dfrac{A}{2}.\cot\dfrac{B}{2}.\cot\dfrac{C}{2}$

20. Given that, $2\sin t=\dfrac{1-2x+5x^2}{3x^2-2x-1}$, $t\in[-\pi/2,\pi/2]$

This can be written as

$(6\sin t-5)x^2+2(1-2\sin t)x-(1+2\sin t)=0$

For given equation to hold, x should be some real number, therefore above equation should have real roots i.e., $D\ge 0$

$\Rightarrow$ $4(1-2\sin t)^2+4(6\sin t-5)(1+2\sin t)\ge 0$

$\Rightarrow$ $16\sin^2 t-8\sin t-4\ge 0 \Rightarrow (4\sin^2 t-2\sin t-1)\ge 0$

$\Rightarrow$ $4\left(\sin t-\dfrac{\sqrt{5}+1}{4}\right)\left(\sin t+\dfrac{\sqrt{5}-1}{4}\right)\ge 0$

$\Rightarrow \quad \sin t \le -\left(\frac{\sqrt{5}-1}{4}\right)$ or $\sin t \ge \frac{\sqrt{5}+1}{4}$

$\Rightarrow \quad \sin t \le \sin(-\pi/10)$ or $\sin t \ge \sin(3\pi/10)$

$\Rightarrow \quad t \le -\pi/10$ or $t \ge 3\pi/10$

(Note that $\sin x$ is an increasing function from $-\pi/2$ to $\pi/2$)

$\therefore$ range of t is $[-\pi/2, -\pi/10] \cup [3\pi/10, \pi/2]$.

F. Match the Following

1. (i) If $\frac{13\pi}{48} < \alpha < \frac{14\pi}{48} \Rightarrow \frac{13\pi}{16} < 3\alpha < \frac{14\pi}{16}$

and $\frac{13\pi}{24} < 2\alpha < \frac{14\pi}{24}$

$\Rightarrow 3\alpha \in II$ quad and $2\alpha \in II$ quad $\Rightarrow \sin 3\alpha = +$ ve

$\cos 2\alpha = -$ ve $\quad \therefore \frac{\sin 3\alpha}{\cos 2\alpha} = -ve$

$\therefore$ (B) corresponds to (p).

If $\alpha \in \left(\frac{14\pi}{48}, \frac{18\pi}{48}\right) \Rightarrow \frac{14\pi}{16} < 3\alpha < \frac{18\pi}{16}$

and $\frac{14\pi}{24} < 2\alpha < \frac{18\pi}{24}$

$\Rightarrow 3\alpha \in II$ or III quad and $2\alpha \in II$ quad

$\Rightarrow$ Nothing can be said about the sign of $\frac{\sin 3\alpha}{\cos 2\alpha}$ over this interval.

If $\alpha \in \left(\frac{18\pi}{48}, \frac{23\pi}{48}\right)$ then $\frac{18\pi}{16} < 3\alpha < \frac{23\pi}{16}$

and $\frac{18\pi}{24} < 2\alpha < \frac{23\pi}{24}$

$\Rightarrow 3\alpha \in III$ quad and $2\alpha \in II$ quad

$\Rightarrow \sin 3\alpha = -$ ve, $\cos 2a = -$ ve $\quad \therefore \frac{\sin 3\alpha}{\cos 2\alpha} = +ve$

$\therefore$ (A) corresponds to (r)

If $\alpha \in (0, \pi/2)$

$\Rightarrow \quad 0 < 3\alpha < 3\pi/2$ and $2 < 2\alpha < \pi$

$\Rightarrow$ Nothing can be said about the sign of $\frac{\sin 3\alpha}{\cos 2\alpha}$ over the given interval.

I. Integer Value Correct Type

1. **(3)** The given equations are

$xyz \sin 3\theta = (y+z)\cos 3\theta$ —(1)

$xyz \sin 3\theta = 2z \cos 3\theta + 2y \sin 3\theta$ —(2)

$xyz \sin 3\theta = (y+2z)\cos 3\theta + y \sin 3\theta$ —(3)

Operating (1) – (2) and (3) – (1), we get

$(\cos 3\theta - 2\sin\theta)y - (\cos 3\theta)z = 0$

and $\sin 3\theta\, y + (\cos 3\theta)z = 0$

which is homogeneous system of linear equation. But

$y \ne 0, z \ne 0$

$\therefore \frac{\cos 3\theta - 2\sin 3\theta}{\sin 3\theta} = -\frac{\cos 3\theta}{\cos 3\theta} \Rightarrow \cos 3\theta = \sin 3\theta$

$\Rightarrow \tan 3\theta = 1 \Rightarrow 3\theta = n\pi + \frac{\pi}{4} \Rightarrow \theta = (4n+1)\frac{\pi}{12}, n \in Z$

For $\theta \in (0, \pi) \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$

$\therefore$ Three such solutions are possible.

2. **(3)** $\tan\theta = \cot 5\theta, \theta \ne \frac{n\pi}{5}$

$\Rightarrow \cos\theta\cos 5\theta - \sin 5\theta \sin\theta = 0 \Rightarrow \cos 6\theta = 0$

$\Rightarrow 6\theta = \frac{-5\pi}{2}, \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$\Rightarrow \theta = \frac{-5\pi}{12}, \frac{-\pi}{4}, \frac{-\pi}{12}, \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}$

Again $\sin 2\theta = \cos 4\theta = 1 - 2\sin^2 2\theta$

$\Rightarrow 2\sin^2 2\theta + \sin 2\theta - 1 = 0 \Rightarrow \sin 2\theta = -1, \frac{1}{2}$

$\Rightarrow 2\theta = \frac{-\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow \theta = \frac{-\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$

So common solutions are $\theta = \frac{-\pi}{4}, \frac{\pi}{12}$ and $\frac{5\pi}{12}$

$\therefore$ Number of solutions = 3.

3. **(2)** Let $f(\theta) = \frac{1}{g(\theta)}$

where $g(\theta) = \sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta$

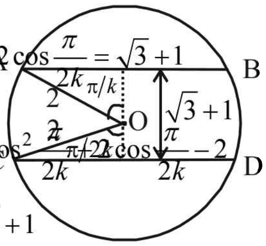
Clearly f is maximum when g is minimum

Now $g(\theta) = \frac{1-\cos 2\theta}{2} + \frac{3}{2}\sin 2\theta + \frac{5}{2}(1+\cos 2\theta)$

$= 3 + 2\cos 2\theta + \frac{3}{2}\sin 2\theta \ge 3 + \left(-\sqrt{4+\frac{9}{4}}\right)$

$\therefore g_{\min} = 3 - \frac{5}{2} = \frac{1}{2} \Rightarrow f_{\max} = 2.$

4. **(3)** From figure, we get



$$2\cos\frac{\pi}{k}+2\cos\frac{\pi}{2k}=\sqrt{3}+1$$

$$\Rightarrow 2\times 2\cos^2\frac{\pi}{2k}+2\cos\frac{\pi}{2k}-2=\sqrt{3}+1$$

$$\Rightarrow 4\cos^2\frac{\pi}{2k}+2\cos\frac{\pi}{2k}-\left(3+\sqrt{3}\right)=0$$

$$\Rightarrow \cos\frac{\pi}{2k}=\frac{-2\pm\sqrt{4+16\left(3+\sqrt{3}\right)}}{8}=\frac{-1\pm\sqrt{13+4\sqrt{3}}}{4}$$

$$=\frac{-1\pm\left(2\sqrt{3}+1\right)}{4}=\frac{\sqrt{3}}{2}\text{ or }-\left(\frac{\sqrt{3}+1}{2}\right)$$

As $\frac{\pi}{2k}$ is an acute angle, $\cos\frac{\pi}{2k}=\frac{\sqrt{3}}{2}=\cos\frac{\pi}{6}\Rightarrow k=3$

5. **(7)** We have, $\frac{1}{\sin\frac{\pi}{n}}-\frac{1}{\sin\frac{3\pi}{n}}=\frac{1}{\sin\frac{2\pi}{n}}$

$$\Rightarrow \frac{\sin\frac{3\pi}{n}-\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}}=\frac{1}{\sin\frac{2\pi}{n}}\Rightarrow\frac{2\cos\frac{2\pi}{n}\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}}=\frac{1}{\sin\frac{2\pi}{n}}$$

$$\Rightarrow 2\sin\frac{2\pi}{n}\cos\frac{2\pi}{n}=\sin\frac{3\pi}{n}\Rightarrow\sin\frac{4\pi}{n}-\sin\frac{3\pi}{n}=0$$

$$\Rightarrow 2\cos\frac{7\pi}{2n}\sin\frac{\pi}{2n}=0\Rightarrow\cos\frac{7\pi}{2n}=0\text{ or }\sin\frac{\pi}{2n}=0$$

$$\Rightarrow\frac{7\pi}{2n}=(2k+1)\frac{\pi}{2}\text{ or }\frac{\pi}{2n}=2k\pi\text{ where }k\in Z$$

$$\Rightarrow n=\frac{7}{2k+1}\text{ or n}=\frac{1}{4k}$$

($n=\frac{1}{4k}$ not possible for any integral value of k)

As $n>3$, for $k=0$, we get $n=7$.

6. **(8)** $\frac{5}{4}\cos^2 2x+\cos^4 x+\sin^4 x+\cos^6 x+\sin^6 x=2$

$$\Rightarrow \frac{5}{4}\cos^2 2x+1-\frac{1}{2}\sin^2 2x+1-\frac{3}{4}\sin^2 2x=2$$

$$\Rightarrow \frac{5}{4}(\cos^2 2x-\sin^2 2x)=0\Rightarrow\cos 4x=0$$

$$\Rightarrow 4x=(2n+1)\frac{\pi}{2}\text{ or }\quad x=(2n+1)\frac{\pi}{8}$$

For $x\in[0, 2\pi]$, n can take values 0 to 7

$\therefore$ 8 solutions.

Section-B JEE Main/ AIEEE

1. **(b)** $\sin^2\theta=\frac{1-\cos 2\theta}{2}$; Period $=\frac{2\pi}{2}=\pi$

2. **(b)** The given equation is tanx + secx = 2 cos x;

$\Rightarrow \sin x+1=2\cos^2 x\Rightarrow\sin x+1=2(1-\sin^2 x)$;

$\Rightarrow 2\sin^2 x+\sin x-1=0$;

$\Rightarrow (2\sin x-1)(\sin x+1)=0\Rightarrow\sin x=\frac{1}{2},-1.$;

$\Rightarrow x=30°, 150°, 270°.$

3. **(b)** $\because \cos\sqrt{x}$ is non periodic

$\therefore \cos\sqrt{x}+\cos^2 x$ can not be periodic.

4. **(d)** $\pi<\alpha-\beta<3\pi$

$$\Rightarrow\frac{\pi}{2}<\frac{\alpha-\beta}{2}<\frac{3\pi}{2}\Rightarrow\cos\frac{\alpha-\beta}{2}<0$$

$$\sin\alpha+\sin\beta=-\frac{21}{65}$$

$$\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=-\frac{21}{65}\quad .....(1)$$

$$\cos\alpha+\cos\beta=-\frac{27}{65}$$

$$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=-\frac{27}{65}\quad .....(2)$$

Square and add (1) and (2)

$$4\cos^2\frac{\alpha-\beta}{2}=\frac{(21)^2+(27)^2}{(65)^2}=\frac{1170}{65\times 65}$$

$$\therefore\cos^2\frac{\alpha-\beta}{2}=\frac{9}{130}\Rightarrow\cos\frac{\alpha-\beta}{2}=-\frac{3}{\sqrt{130}}$$

5. **(a)** $u^2=a^2+b^2+2\sqrt{(a^4+b^4)\cos^2\theta\sin^2\theta+a^2b^2(\cos^4\theta+\sin^4\theta)}\ \ldots(1)$

Now $(a^4+b^4)\cos^2\theta\sin^2\theta+a^2b^2(\cos^4\theta+\sin^4\theta)$

$= (a^4+b^4)\cos^2\theta\sin^2\theta+a^2b^2(1-2\cos^2\theta\sin^2\theta)$

$= (a^4+b^4-2a^2b^2)\cos^2\theta\sin^2\theta+a^2b^2$

$= (a^2-b^2)^2.\dfrac{\sin^2 2\theta}{4}+a^2b^2 \quad \ldots(2)$

$\because 0 \le \sin^2 2\theta \le 1$

$\Rightarrow 0 \le (a^2-b^2)^2\dfrac{\sin^2 2\theta}{4} \le \dfrac{(a^2-b^2)^2}{4}$

$\Rightarrow a^2b^2 \le (a^2-b^2)^2\dfrac{\sin^2 2\theta}{4}+a^2b^2$

$\le (a^2-b^2)^2.\dfrac{1}{4}+a^2b^2 \quad \ldots.(3)$

$\therefore$ from (1), (2) and (3)

Minimum value of $u^2 = a^2+b^2+2\sqrt{a^2b^2} = (a+b)^2$

Maximum value of u^2

$= a^2+b^2+2\sqrt{\left(a^2-b^2\right)^2.\dfrac{1}{4}+a^2b^2}$

$= a^2+b^2+\dfrac{2}{2}\sqrt{(a^2+b^2)^2} = 2(a^2+b^2)$

$\therefore$ Max value - Min value

$= 2(a^2+b^2)-(a+b^2) = (a-b)^2$

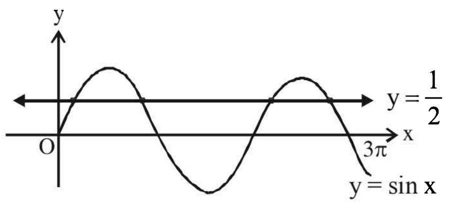
6. **(c)** The direction cosines of the line are $\cos\theta, \cos\beta, \cos\theta$

$\therefore \cos^2\theta+\cos^2\beta+\cos^2\theta = 1$

$\Rightarrow 2\cos^2\theta = \sin^2\beta = 3\sin^2\theta$ (given)

$\Rightarrow 2\cos^2\theta = 3-3\cos^2\theta \qquad \therefore \cos^2\theta = \dfrac{3}{5}$

7. **(a)**



$2\sin^2 x+5\sin x-3 = 0$

$\Rightarrow (\sin x+3)(2\sin x-1) = 0$

$\Rightarrow \sin x = \dfrac{1}{2}$ and $\sin x \ne -3$

$\therefore$ In $[0,3\pi]$, x has 4 values.

8. **(c)** $\cos x+\sin x = \dfrac{1}{2} \Rightarrow 1+\sin 2x = \dfrac{1}{4}$

$\Rightarrow \sin 2x = -\dfrac{3}{4}$, so x is obtuse and

$\dfrac{2\tan x}{1+\tan^2 x} = -\dfrac{3}{4} \Rightarrow 3\tan^2 x+8\tan x+3 = 0$

$\therefore \tan x = \dfrac{-8\pm\sqrt{64-36}}{6} = -\dfrac{-4\pm\sqrt{7}}{3}$

as $\tan x < 0 \quad \therefore \tan x = \dfrac{-4-\sqrt{7}}{3}$

9. **(b)** We have

$\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta) = -\dfrac{3}{2}$

$\Rightarrow 2[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)]+3 = 0$

$\Rightarrow 2[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)]$
$+\sin^2\alpha+\cos^2\alpha+\sin^2\beta+\cos^2\beta+\sin^2\gamma+\cos^2\gamma = 0$

$\Rightarrow [\sin^2\alpha+\sin^2\beta+\sin^2\gamma+2\sin\alpha\sin\beta+2\sin\beta\sin\gamma$
$+2\sin\gamma\sin\alpha]+[\cos^2\alpha+\cos^2\beta+\cos^2\gamma+2\cos\alpha\cos\beta$
$+2\cos\beta\cos\gamma+2\cos\gamma\cos\alpha] = 0$

$\Rightarrow [\sin\alpha+\sin\beta+\sin\gamma]^2+(\cos\alpha+\cos\beta+\cos\gamma)^2 = 0$

$\Rightarrow \sin\alpha+\sin\beta+\sin\gamma = 0$ and $\cos\alpha+\cos\beta+\cos\gamma = 0$

$\therefore$ A and B both are true.

10. **(a)** $\cos(\alpha+\beta) = \dfrac{4}{5} \Rightarrow \tan(\alpha+\beta) = \dfrac{3}{4}$

$\sin(\alpha-\beta) = \dfrac{5}{13} \Rightarrow \tan(\alpha-\beta) = \dfrac{5}{12}$

$\tan 2\alpha = \tan[(\alpha+\beta)+(\alpha-\beta)] = \dfrac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}.\frac{5}{12}} = \dfrac{56}{33}$

11. **(d)** $A = \sin^2 x+\cos^4 x = \sin^2 x+\cos^2 x(1-\sin^2 x)$

$= \sin^2 x+\cos^2 x-\dfrac{1}{4}(2\sin x.\cos x)^2 = 1-\dfrac{1}{4}\sin^2(2x)$

Now $0 \le \sin^2(2x) \le 1 \Rightarrow 0 \ge -\dfrac{1}{4}\sin^2(2x) \ge -\dfrac{1}{4}$

$\Rightarrow 1 \ge 1-\dfrac{1}{4}\sin^2(2x) \ge 1-\dfrac{1}{4} \Rightarrow 1 \ge A \ge \dfrac{3}{4}$

12. **(b)** Given $3\sin P+4\cos Q = 6 \quad \ldots(i)$

$4\sin Q+3\cos P = 1 \quad \ldots(ii)$

Squaring and adding (i) & (ii) we get

$9\sin^2 P+16\cos^2 Q+24\sin P\cos Q$
$+16\sin^2 Q+9\cos^2 P+24\sin Q\cos P = 36+1 = 37$

$\Rightarrow$ $9(\sin^2 P+\cos^2 P)+16(\sin^2 Q+\cos^2 Q)$
$+24(\sin P\cos Q+\cos P\sin Q)=37$

$\Rightarrow$ $9+16+24\sin(P+Q)=37$

[$\because \sin^2\theta+\cos^2\theta=1$ and $\sin A\cos B+\cos A\sin B = \sin(A+B)$]

$\Rightarrow$ $\sin(P+Q)=\dfrac{1}{2}$ $\Rightarrow$ $P+Q=\dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$

$\Rightarrow$ $R=\dfrac{5\pi}{6}$ or $\dfrac{\pi}{6}$ $(\because P+Q+R=\pi)$

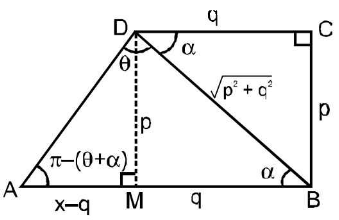
If $R=\dfrac{5\pi}{6}$ then $0<P,Q<\dfrac{\pi}{6}$

$\Rightarrow$ $\cos Q<1$ and $\sin P<\dfrac{1}{2}$

$\Rightarrow$ $3\sin P+4\cos Q<\dfrac{11}{2}$ which is not true.

So $R=\dfrac{\pi}{6}$

13. (a) From Sine Rule



$$\frac{AB}{\sin\theta}=\frac{\sqrt{p^2+q^2}}{\sin(\pi-(\theta+\alpha))}$$

$$AB=\frac{\sqrt{p^2+q^2}\,\sin\theta}{\sin\theta\cos\alpha+\cos\theta\sin\alpha}=\frac{(p^2+q^2)\sin\theta}{q\sin\theta+p\cos\theta}$$

$$\left(\because \cos\alpha=\frac{q}{\sqrt{p^2+q^2}} \text{ and } \sin\alpha=\frac{p}{\sqrt{p^2+q^2}}\right)$$

14. (b) Given expression can be written as

$$\frac{\sin A}{\cos A}\times\frac{\sin A}{\sin A-\cos A}+\frac{\cos A}{\sin A}\times\frac{\cos A}{\cos A-\sin A}$$

$$\left(\because \tan A=\frac{\sin A}{\cos A} \text{ and } \cot A=\frac{\cos A}{\sin A}\right)$$

$$=\frac{1}{\sin A-\cos A}\left\{\frac{\sin^3 A-\cos^3 A}{\cos A\sin A}\right\}$$

$$=\frac{\sin^2 A+\sin A\cos A+\cos^2 A}{\sin A\cos A}=1+\sec A\operatorname{cosec} A$$

15. (b) Let $f_k(x)=\dfrac{1}{k}(\sin^k x+\cos^k x)$

Consider

$$f_4(x)-f_6(x)=\frac{1}{4}(\sin^4 x+\cos^4 x)-\frac{1}{6}(\sin^6 x+\cos^6 x)$$

$$=\frac{1}{4}[1-2\sin^2 x\cos^2 x]-\frac{1}{6}[1-3\sin^2 x\cos^2 x]$$

$$=\frac{1}{4}-\frac{1}{6}=\frac{1}{12}$$

16. (a) $\cos x+\cos 2x+\cos 3x+\cos 4x=0$

$\Rightarrow 2\cos 2x\cos x+2\cos 3x\cos x=0$

$$\Rightarrow 2\cos x\left(2\cos\frac{5x}{2}\cos\frac{x}{2}\right)=0$$

$$\cos x=0,\ \cos\frac{5x}{2}=0,\ \cos\frac{x}{2}=0$$

$$x=\pi,\frac{\pi}{2},\frac{3\pi}{2},\frac{\pi}{5},\frac{3\pi}{5},\frac{7\pi}{5},\frac{9\pi}{5}$$