

JEE ADVANCED BOOSTER TEST-3

JEE 2024

Date	Timing	Maximum Marks	Duration
4th September, 2023	4:00 PM - 7:00 PM	177	3 Hours

General Instructions

- The question paper consists of 3 Subjects (Subject I: **Physics**, Subject II: **Chemistry**, Subject III: **Mathematics**). Each Subject has **two** sections (Section 1 & Section 2).
- Section 1** contains **3 types** of questions [**Type A, Type B and Type C**].
Type A contains **Five (05) Multiple Choice Questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.
Type B contains **Five (05) Multiple Correct Answers Type Questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE THAN ONE CHOICE** is correct.
Type C contains **ONE (01) paragraph**. Based on the paragraph, there are **TWO (02)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- Section 2** contains **6 Numerical Value Type Questions**. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places. If the answer is an Integer value, then do not add zero in the decimal places. *In the OMR, do not bubble the $\oplus$ sign for positive values. However, for negative values, $\ominus$ sign should be bubbled.* (Example: 6, 81, 1.50, 3.25, 0.08)
- For answering a question, an ANSWER SHEET (OMR SHEET) is provided separately. Please fill your **Test Code, Roll No.** and **Group** properly in the space given in the ANSWER SHEET.

Syllabus:

Physics : DC Circuits, Capacitors, Magnetic Effects of Current, Energy and Momentum, Rotation
Chemistry : Electrochemistry, Surface Chemistry, Organic Halides, States of Matter, Chemical Bonding
Mathematics : DC - I, DC - II, Conic Section

Name of the Candidate (In CAPITALS) :

Roll Number :

OMR Bar Code Number :

Candidate's Signature : Invigilator's Signature

MARKING SCHEME

SECTION-1 | Type A

- This section contains **Five (05)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the answer. For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

SECTION-1 | Type B

- This section contains **Five (05)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks : 0 If unanswered;
Negative Marks : -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
choosing **ONLY** (A), (B) and (D) will get +4 marks; choosing **ONLY** (A) and (D) will get +2 marks;
choosing **ONLY** (A) will get +1 mark;
choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
choosing any other option(s) will get -2 marks.

SECTION-1 | Type C

- This section contains **ONE paragraphs**. Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme;
Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

SECTION - 2

- This section contains **6 Integer Type Questions**. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places. If the answer is an Integer value, then do not add zero in the decimal places. *In the OMR, do not bubble the $\oplus$ sign for positive values. However, for negative values, $\ominus$ sign should be bubbled.* (Example: 6, 81, 1.50, 3.25, 0.08)
 - Answer to each question will be evaluated according to the following marking scheme:
Full Marks: +3 If **ONLY** the correct Integer value is entered. There is **NO negative marking**.
Zero Marks: 0 In all other cases.
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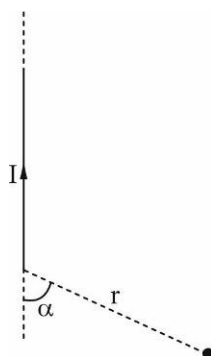
SECTION-1 | Type A

This section consists of 5 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

1. A metal sphere of radius ' a ' is surrounded by a concentric metal sphere of radius ' b ' ($b > a$). The space between the spheres is filled with a material whose electrical conductivity σ varies with the electric field strength E according to relation $\sigma = kE$, where k is a constant. A potential difference V is maintained between two spheres. The current between the spheres is:

(A) $\left[\frac{4\pi k V^2}{\{\ell \ln(b/a)\}^2} \right]$ (B) $\left[\frac{2\pi k V^2}{\{\ell \ln(b/a)\}^2} \right]$ (C) $\left[\frac{\pi k V^2}{4\{\ell \ln(b/a)\}^2} \right]$ (D) $\left[\frac{8\pi k V^2}{\{\ell \ln(b/a)\}^2} \right]$

2. For the semi-infinite wire shown, a point P is taken at a distance r from its finite end as shown. Assume that the current in the wire is I , the field at P is:

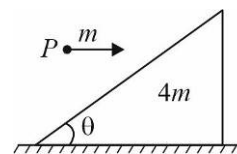


(A) $\frac{\mu_0 I}{4\pi r}$ (B) $\frac{\mu_0 I}{4\pi r} \sin \alpha$ (C) $\frac{\mu_0 I}{4\pi r} \cot\left(\frac{\alpha}{2}\right)$ (D) $\frac{\mu_0 I}{4\pi r} \tan\left(\frac{\alpha}{2}\right)$

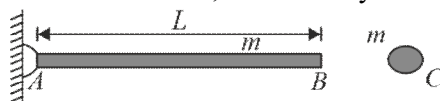
SPACE FOR ROUGH WORK

3. In the figure shown, a particle P of mass m strikes the inclined smooth plane of wedge of mass $4m$ horizontally and rebounds vertically. The wedge is on a smooth horizontal plane. Assume no toppling of the wedge. If the angle θ is 60° , then the co-efficient of restitution is:

- (A) $\frac{7}{12}$ (B) $\frac{1}{\sqrt{3}}$
 (C) $\frac{7}{4\sqrt{3}}$ (D) $\frac{1}{3}$



4. A uniform bar AB of mass m and a ball of the same mass are released from rest from the same horizontal position. The bar is hinged at end A . There is gravity downwards. What is the distance of the point from point B that has the same acceleration as the ball, immediately after release?



- (A) $\frac{2L}{3}$ (B) $\frac{L}{3}$ (C) $\frac{L}{2}$ (D) $\frac{3L}{4}$

5. There exists a uniform magnetic and electric field each of magnitude 1 T and 1 V/m respectively along positive y-axis. A charged particle of mass 1 kg and of charge 1 C is having velocity 1 m/sec along x-axis and is at origin at $t = 0$. Then the co-ordinate of particle at time π seconds will be:

- (A) $(0, 1, 2)$ (B) $(0, -\pi^2 / 2, -2)$
 (C) $(2, \pi^2 / 2, 2)$ (D) $(0, \pi^2 / 2, 2)$

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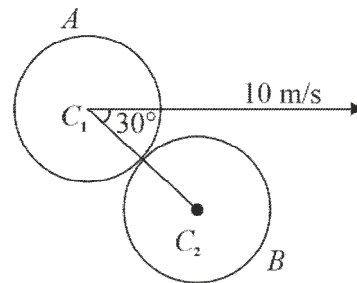
SECTION-1 | Type B

This section consists of 5 Multiple Correct Answers Type Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE THAN ONE CHOICE** is correct.

6. A parallel plate capacitor of capacitance C_0 is charged to a voltage V and then the battery is disconnected. A dielectric covering one-third area of each plate is now inserted as shown in the figure. If charges on the capacitor plates get redistributed such that the portions covered with dielectric and not covered with the dielectric share equal amounts of charge, which of the following statements is (are) true?



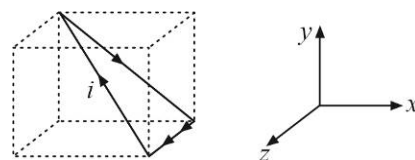
- (A) Dielectric constant of the dielectric is 2.0
(B) Charge appearing due to polarization on the surface of the dielectric is $0.25C_0V$
(C) Force of electrostatic interaction between portions of the plates covered with dielectric is equal to that between uncovered portions
(D) Force of electrostatics interaction between the plates after insertion of the dielectric becomes $9/8$ times of its value before insertion of dielectric
7. A smooth ball collides elastically with another identical ball B with velocity 10 m/s at an angle of 30° from the line joining their centers C_1 and C_2 , then which of the following statements is (are) true?



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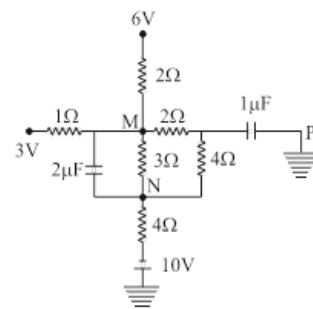
8. A current loop with current i is in the form of a triangle whose geometry is as follows. Dotted (imaginary) lines form a cube of side a . Then, regarding this loop, which of the following statements is(are) true?

- (A) Magnetic field at centre of cube is $-\frac{6\mu_0 i}{4\sqrt{3}\pi a}(\hat{i} + \hat{j})$
- (B) Magnetic field at centre of cube is $-\frac{\sqrt{3}\mu_0 i}{6\pi a}(\hat{i} + \hat{j})$
- (C) Magnetic field at centre of cube is $-\frac{9\mu_0 i}{2\pi a}(\hat{i} + \hat{j})$
- (D) Magnetic moment of the loop has no z -component



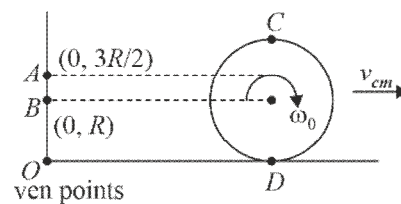
9. Figure shows a network of capacitors and resistances in steady state. Potentials of some of the points are given. Then which of the following statements is (are) true?

- (A) Potential at $M = 2.6V$
- (B) Potential at $N = -1.6V$
- (C) Charge across $2\mu F$ capacitors is $8.4\mu C$
- (D) Charge across $2\mu F$ capacitor is $4.2\mu C$



10. A cylinder is rolling without sliding on a smooth horizontal surface as shown. O is origin, B and A are two points on y -axis C and D are topmost and bottom most point of cylinder at the given instant. Mark the correct statements.

- (A) Angular momentum of body about C and D will be same in magnitude
- (B) Angular momentum of cylinder will be minimum about A among the given points
- (C) Angular momentum will be minimum about B
- (D) Angular momentum about O and C will be same in magnitude



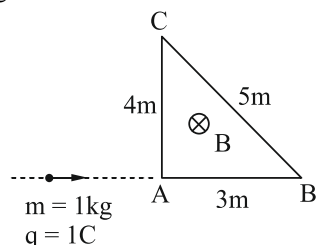
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SECTION-1 | Type C

This section consists of **ONE (01) paragraph**. Based on each paragraph, there are **TWO (02) questions**. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

PARAGRAPH FOR Q-11 & 12

A small particle of mass $m = 1 \text{ kg}$ and charge of $q = 1 \text{ C}$ enter perpendicularly in a triangular region of uniform magnetic field of strength 2 T as shown in figure.



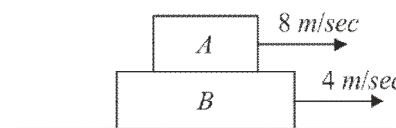
11. Calculate maximum velocity of the particle with which it should enter so that it completes a half – circle in magnetic region.
- (A) 2 m/s (B) 2.5 m/s (C) 3 m/s (D) 4 m/s
12. In previous question, if particle enters perpendicularly with velocity 48 m/s in magnetic region. Then, how much time will it spend in magnetic region.
- (A) $\frac{11\pi}{360} \text{ sec}$ (B) $\frac{7\pi}{360} \text{ sec}$ (C) $\frac{13\pi}{360} \text{ sec}$ (D) $\frac{17\pi}{360} \text{ sec}$

SPACE FOR ROUGH WORK

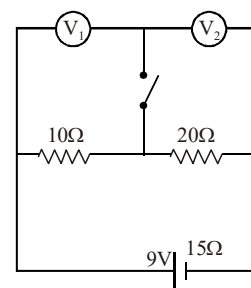
SECTION-2

This section consists of 6 Numerical Value Type Questions. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places. If the answer is an Integer value, then do not add zero in the decimal places. *In the OMR, do not bubble the $\oplus$ sign for positive values. However, for negative values, $\ominus$ sign should be bubbled.* (Example: 6, 81, 1.50, 3.25, 0.08)

1. Block A of mass 1 kg is placed on the rough surface of block B of mass 3 kg . Block B is placed on smooth horizontal surface. Blocks are given the velocities as shown. Find net work done (on the system in J) by the frictional forces acting on the blocks afterwards. Assume B to be sufficiently long so that A doesn't fall off.

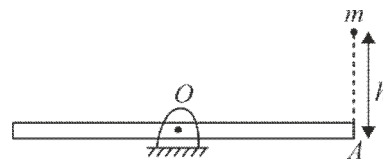


2. In the circuit shown in the figure, the electromotive force of the battery is 9 V and its internal resistance is 15Ω . The two identical voltmeters can be considered ideal. Let V_1 and V_1' be the reading of 1st voltmeter when switch is open and closed respectively. Similarly, V_2 and V_2' be the reading of 2nd voltmeter when switch is open and closed respectively. Find $\frac{V_2' - V_2}{V_1 - V_1'}$.

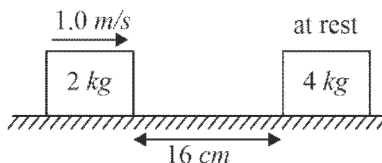


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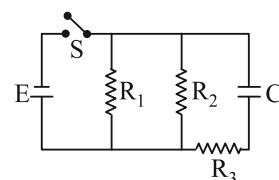
3. A uniform rod of length 12 metre and mass m is hinged at midpoint and lies horizontally at rest. Rod is free to rotate in vertical plane about hinge O . A particle of mass m is released from height h as shown in figure. Particle collides with end A and sticks to it. Calculate minimum height h (in metre) so that system can complete vertical circular motion.



4. The friction coefficient between the horizontal surface and each of the blocks shown in the figure is 0.2. The collision between the blocks is perfectly elastic. Find the separation between them (in cm) when they come to rest. (Take $g = 10 \text{ m/s}^2$)



5. The capacitor is charged by closing the switch S . The switch is then opened and the capacitor is allowed to discharge. Let τ_1 and τ_2 be the time constant of the circuit during the charging and discharging of the capacitor respectively. Take $R_1 = R_2 = R_3 = R$, and the battery to be ideal with negligible resistance.



Ratio $\frac{\tau_2}{\tau_1}$ is _____.

6. A thin conducting wire in the shape of a 'figure of eight' is situated with its circular loops in two planes making an angle of 120° with each other. If the current in the loop is i and the radius is a , determine the magnetic field (in tesla) at a point of intersection of the two axes of the loops.

(Given: $\frac{a}{i} = \left(\frac{\pi}{16}\right) \times 10^{-7} \text{ m / Amp}$)

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SECTION-1 | Type A

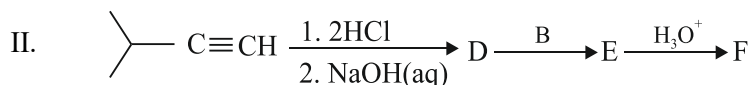
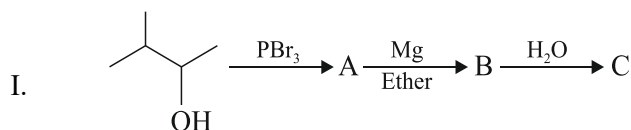
This section consists of 5 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

1. Two beakers A and B contains AgNO_3 solutions of concentrations 0.1 M and 0.02 M respectively. Pure silver electrodes are now lowered in each beaker. Now NH_3 is added in each beaker till concentration of NH_3 in beaker A and B become 0.1M and 0.2 M respectively. If two beakers are now connected in galvanic cell arrangement. What is the EMF of resulting cell at 298 K?

(Use $\log_{10} 2 = 0.3$, $\frac{RT}{F} (2.303) = 0.06$ at 298K). **Given :** $E^\circ_{\text{Ag}^+/\text{Ag}} = 0.8 \text{ V}$, $K_f[\text{Ag}(\text{NH}_3)_2]^+ = 10^8$

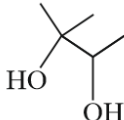
- (A) $7.8 \times 10^{-2} \text{ V}$ (B) $9.6 \times 10^{-2} \text{ V}$ (C) $5.2 \times 10^{-2} \text{ V}$ (D) $6.5 \times 10^{-2} \text{ V}$

2. Consider reaction scheme I and II.



Choose the correct statements from the following:

(I) C is Isopentane

(II) D is 

(III) F is 2, 3, 4, 5 - tetramethylhexan-3-ol

(IV) D gives yellow precipitate with I_2 / NaOH .

(A) I, II, III

(B) I, II, III, IV

(C) I, III

(D) I, III, IV

SPACE FOR ROUGH WORK

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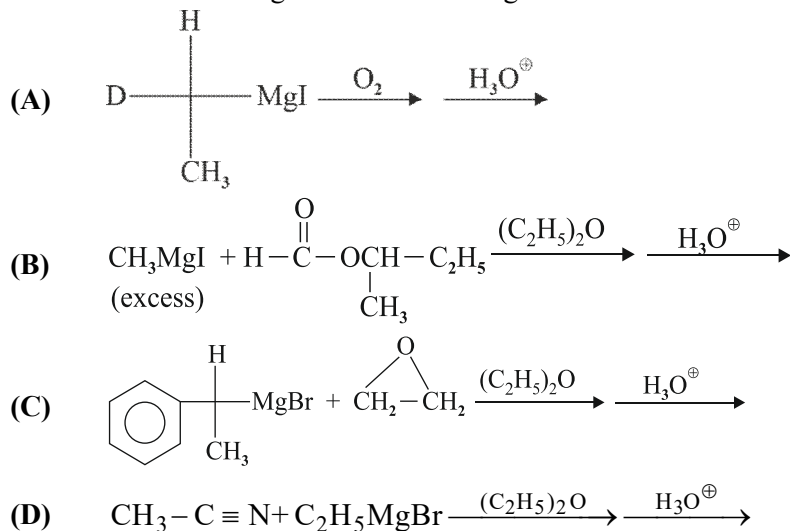
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5. The correct statements are :
- (a) If there is a nodal plane perpendicular to the internuclear axis and lying between the nuclei of bonded atoms then corresponding orbital is antibonding molecular orbital
 - (b) If a nodal plane lies in the inter-nuclear axis, then corresponding orbital is pi (π) bonding molecular orbital.
 - (c) The σ -bonding molecular orbital does not contain nodal planes containing the internuclear axis.
 - (d) When N_2 is converted to N_2^+ , N–N bond strength increases.
- (A) Only (a), (b) & (c) (B) Only (a) & (b)
(C) Only (a) (D) (a), (b), (c) & (d)
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SPACE FOR ROUGH WORK

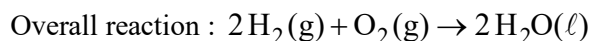
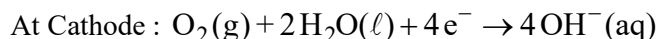
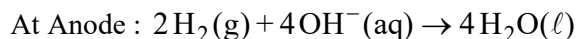
SECTION-1 | Type B

This section consists of 5 Multiple Correct Answers Type Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE THAN ONE CHOICE** is correct.

6. Which of the following reactions will not give 2° chiral alcohol as one or more of major organic products?



7. In hydrogen – oxygen fuel cell, the following reactions take place.



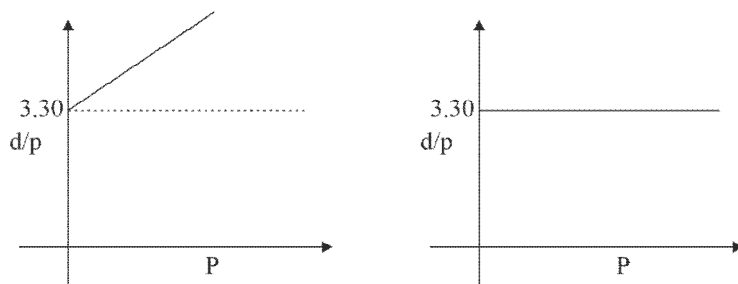
$\Delta H_f^\circ \text{H}_2\text{O}(\ell) = -285 \text{ kJ/mol}$, $\Delta G_f^\circ \text{H}_2\text{O}(\ell) = -240 \text{ kJ/mol}$, $1\text{F} = 96500 \text{ C}$

Choose the correct statements from the following:

- (A) The standard EMF of the cell is 0.62 V
- (B) The Theoretical efficiency of the fuel cell is nearly 84%.
- (C) The number of moles of H_2 , when reacted with excess of O_2 in the fuel cell at 25°C and 1 bar, needed to produce 4.8 kJ of useful work is 0.02.
- (D) The standard EMF of the cell is 1.24 V

SPACE FOR ROUGH WORK

8. Which of the following statements are correct?
- (A) Chloroform is used as a paint remover, propellant in aerosols, and a metal cleaning and finishing solvent.
 - (B) The antiseptic properties of iodoform are due to liberation of free iodine.
 - (C) Freons are reactive and highly toxic.
 - (D) Chloroform is slowly oxidized by air in presence of light to an extremely poisonous gas.
9. Which of the following are correct statements?
- (A) Micelle formation is an Entropy driven process
 - (B) Formation of micelles takes place when temperature is below Kraft Temperature (T_k) and concentration is above critical micelle concentration (CMC)
 - (C) A colloid of $\text{Fe}(\text{OH})_3$ is prepared by adding a little excess (required to completely precipitate Fe^{3+} ions as $\text{Fe}(\text{OH})_3$) of hot water in FeCl_3 solution. The particles of this sol will move towards cathode during electrophoresis.
 - (D) According to Hardy-Schulze rules the coagulation (flocculating) value of Fe^{3+} ion will be more than Ba^{2+} or Na^+ .
10. For gases A & B plots between $\frac{d}{P}$ [d = density of gas (g/L), P = pressure (atm)] and P at 300 K temp, was obtained as:



Choose the correct statements. (Given $R = 0.082 \text{ L atm K}^{-1} \text{ mol}^{-1}$)

- (A) Gas A shows non-ideal behaviour
- (B) Molar mass of gas B is 81.18 g mol^{-1}
- (C) The plot of $\left(\frac{d}{P}\right)$ vs Temperature for ideal gas should have a positive slope.
- (D) Gas B shows ideal behaviour.

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SECTION-1 | Type C

This section consists of **ONE (01) paragraph**. Based on each paragraph, there are **TWO (02) questions**. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

PARAGRAPH FOR Q-11 & 12

The molar conductance of NaCl varies with concentration as shown in the following table and all values follow the equation.

$$\lambda_m^c = \lambda_m^\infty - b\sqrt{c}$$

Where λ_m^c = molar conductance

λ_m^∞ = molar conductance at infinite dilution

c = molar concentration

Molar Concentration of NaCl	Molar Conductance in $\text{ohm}^{-1} \text{cm}^2 \text{mol}^{-1}$
4×10^{-4}	107
9×10^{-4}	97

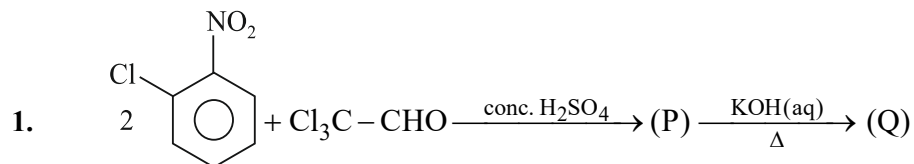
When a certain conductivity cell was filled with $25 \times 10^{-4} \text{ M}$ NaCl solution, the resistance of the cell was found to be 1000 ohm. At infinite dilution, molar conductance of Cl^- and SO_4^{2-} are $80 \text{ ohm}^{-1} \text{cm}^2 \text{mol}^{-1}$ and $160 \text{ ohm}^{-1} \text{cm}^2 \text{mol}^{-1}$ respectively.

11. What is the molar conductance of NaCl at infinite dilution?
- (A) $147 \text{ ohm}^{-1} \text{cm}^2 \text{mol}^{-1}$ (B) $107 \text{ ohm}^{-1} \text{cm}^2 \text{mol}^{-1}$
(C) $127 \text{ ohm}^{-1} \text{cm}^2 \text{mol}^{-1}$ (D) $157 \text{ ohm}^{-1} \text{cm}^2 \text{mol}^{-1}$
12. What is the cell constant of the conductivity cell?
- (A) 0.385 cm^{-1} (B) 3.85 cm^{-1} (C) 38.5 cm^{-1} (D) 0.1925 cm^{-1}

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SECTION-2

This section consists of 6 Numerical Value Type Questions. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places. If the answer is an Integer value, then do not add zero in the decimal places. *In the OMR, do not bubble the $\oplus$ sign for positive values. However, for negative values, $\ominus$ sign should be bubbled.* (Example: 6, 81, 1.50, 3.25, 0.08)



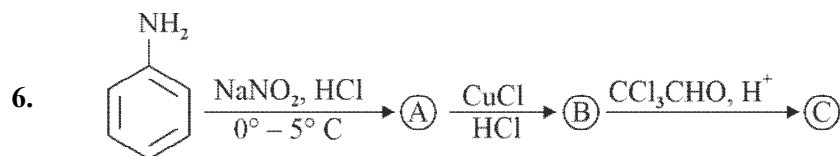
Find the number of oxygen atoms in compound (Q).

2. Number of correct statements from the following are :

- I. Colloidal AgI is prepared by adding KI in slight excess of AgNO₃ solution, the sol particles migrate toward cathode under electric field.
- II. The extent of adsorption of CO₂ is much more higher than of H₂.
- III. Micelles are formed by surfactant molecules above the critical micelle concentration (CMC).
- IV. The conductivity of a solution having surfactant molecules decrease sharply at the CMC
- V. Surface tension of water is reduced by the addition of stearate.
- VI. Brownian movement counters the force of gravity act on colloidal particles.
- VII. Thermal decomposition of KClO₃ (s) in presence of MnO₂ (s) is an example of homogeneous catalysis.
- VIII. The micelle formed by sodium stearate in water has -COO⁻ groups at the surface.
- IX. Coagulation power of Al³⁺ is more than Na⁺
- X. Higher the zeta potential lesser is the stability of colloid.

SPACE FOR ROUGH WORK

3. X : Number of hybrid orbitals in carbon atoms which have 25% s character in $\text{C}(\text{CN})_4$.
 Y : Maximum number of equal S – O bonds in SO_4^{2-}
 Find the value of $X + Y$.
4. X : Temperature in Kelvin at which the most probable speed of SO_2 molecules have the same value of root mean square speed of O_2 molecules at 300 K.
 Y : Compressibility factor (Z) for 0.02 mole of a van der Waal's gas at pressure of 0.1 atm.
 (Assume the size of gas molecules is negligible.)
 (Given : $RT = 20 \text{ L atm mol}^{-1}$ and $a = 1000 \text{ atm L}^2 \text{ mol}^{-2}$)
 Find the value of $X \times Y$.
5. X : Number of minutes taken to plate out 5.2 g of Cr from a $\text{Cr}_2(\text{SO}_4)_3$ solution using a current of 9.65 A.
 (Atomic weight : Cr = 52)
 Y : Total charge in Faraday required for the oxidation of two moles of Mn_3O_4 into MnO_4^{2-} in the presence of alkaline medium.
 Find the value of $X + Y$.



X → Molar mass of (B)

Y → Number of Cl atoms present in 1 molecule of (C)

Find the value of $2X + Y$.

[Given: Atomic mass: H = 1 u, C = 12 u, N = 14 u, O = 16 u, Cl = 35.5 u, Cu = 63.5 u]

SPACE FOR ROUGH WORK

SECTION-1 | Type A

This section consists of 5 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

1. The distance covered by a particle moving in a straight line from a fixed point is 's'.

Where $s^2 = at^2 + 2bt + c$, if t is time and a , b and c are constants then the acceleration is :

- (A) Proportional to square of distance
- (B) Inversely proportional to cube of speed
- (C) Inversely proportional to cube of distance
- (D) Proportional to square of speed

2. If $f(x) = 8x^3 + 3x$ then $\lim_{x \rightarrow \infty} \left(\frac{f^{-1}(8x) - f^{-1}(x)}{x^{1/3}} \right)$ is:

- (A) $\frac{1}{2}$ (B) 2 (C) 8 (D) $\frac{1}{4}$

SPACE FOR ROUGH WORK

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3. The value of $\lim_{n \rightarrow \infty} \frac{2 \sum_{r=1}^n r^3 \cdot \sum_{r=1}^n r^5 - \sum_{r=1}^n r^2 \sum_{r=1}^n r^6}{\left(\sum_{r=1}^n r^4 \right)^2}$ is:
- (A) $\frac{4}{7}$ (B) $\frac{5}{7}$ (C) $\frac{25}{28}$ (D) $\frac{-25}{168}$
4. The range of the functions $f(\theta) = \frac{\sin \theta}{\theta} + \frac{\theta}{\tan \theta}$ $\theta \in \left(0, \frac{\pi}{2}\right)$ is equal to:
- (A) $\left(\frac{2}{\pi}, 2\right)$ (B) $(2, \pi)$ (C) $\left(\frac{1}{\pi}, \pi\right)$ (D) $(0, 2)$
5. A polynomial function $P(x)$ of degree 5 with leading coefficient one, increases in the interval $(-\infty, 1)$ and $(3, \infty)$ and decreases in the interval $(1, 3)$. Given that $P'(2) = 0$ then the value of is $P'(5)$:
- (A) 240 (B) 360 (C) 120 (D) 1200
-

SPACE FOR ROUGH WORK

SECTION-1 | Type B

This section consists of 5 Multiple Correct Answers Type Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE THAN ONE CHOICE** is correct.

6. A function $f : R \rightarrow R$ is defined by $f(x+y) - kxy = f(x) + 2y^2 \forall x, y \in R$, given that $f(1) = 2, f(2) = 8$ then $f(x+y) \cdot f\left(\frac{1}{x+y}\right)$ is equal to :
- (A) 1 (B) 4 (C) k (D) $f(1)$

7. Let $f(x)$ be a real valued function such that $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t f(t)) dt$. If M and m are maximum and minimum values of the function then :

- (A) $\frac{M}{m} = 3$ (B) $M - m = 2(\pi + 1)$
- (C) $M + m = 2\pi(\pi + 1)$ (D) $M \cdot m = 2(\pi^2 + 1)$

SPACE FOR ROUGH WORK

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8. Let $f(x) = \frac{x^2}{(1+x^2)^2}$, then which of the following statement(s) is (are) correct?
- (A) $f(x)$ is strictly increasing $(-\infty, -1)$ (B) $f(x)$ is strictly decreasing in $(1, \infty)$
 (C) $f(x)$ has two points of local extremum (D) $f(x)$ has a local minimum at some $(-1, 0)$
9. If $f(x)$ and $g(x)$ are thrice differentiable function and $F(x) = f(x).g(x)$ and $f'(x).g'(x) = c$, constant then:
- (A) $F'(x) = c \left(\frac{f(x)}{f'(x)} + \frac{g(x)}{g'(x)} \right)$ (B) $\frac{F''(x)}{F(x)} = \frac{f''(x)}{f(x)} + \frac{g''(x)}{g(x)} + \frac{2c}{f(x).g(x)}$
 (C) $\frac{F'''(x)}{F(x)} = \frac{f'''(x)}{f(x)} + \frac{g'''(x)}{g(x)}$ (D) $\frac{F'''(x)}{F(x)} = \frac{f'''(x)}{f'(x)} + \frac{g'''(x)}{g'(x)}$
10. Let $f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = x^2 - xf'(-3) + f''(-2)$ then which of the following are correct?
- (A) $f'(1) = 4$ (B) $f'(2) = 8$ (C) $g'(-2) = 2$ (D) $g'(-3) = -4$
-

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SECTION-1 | Type C

This section consists of **ONE (01) paragraph**. Based on each paragraph, there are **TWO (02)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

PARAGRAPH FOR Q-11 & 12

Let $x - y + 1 = 0$ be axis of parabola, $x + y - 4 = 0$ is tangent of same parabola at its vertex and $2x - y + 3 = 0$ is one of its tangents.

11. If the equation of directrix of parabola is $lx + my = 29$, then $l + m$ is equal to :
(A) 9 (B) 13 (C) 18 (D) 23
12. If the length of latus rectum of parabola is $\frac{p\sqrt{2}}{q}$ where p and q are relatively prime natural numbers, then $(p + q)$ is equal to :
(A) 7 (B) 11 (C) 19 (D) 23

SPACE FOR ROUGH WORK

SECTION-2

This section consists of 6 Numerical Value Type Questions. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places. If the answer is an Integer value, then do not add zero in the decimal places. *In the OMR, do not bubble the $\oplus$ sign for positive values. However, for negative values, $\ominus$ sign should be bubbled.* (Example: 6, 81, 1.50, 3.25, 0.08)

1. If P is the length of perpendicular from a focus upon a tangent at any point of the ellipse $\frac{x^2}{9a^2} + \frac{y^2}{4a^2} = 1$ and r is the distance of that point from the same focus then the value of $\frac{6a}{r} - \frac{4a^2}{p^2}$ is _____.
2. The four common tangents of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and a circle $x^2 + y^2 = 2r^2$ makes a rectangle with each other. If r_1 and r_2 are radii of director circle of ellipse and circle. Then the value of $\frac{2r_1^2}{r_2^2}$ is _____.
3. If $A(x_1, y_1)$ is a points on the parabola $y^2 = x - 4$ and $B(x_2, y_2)$ is a points on the parabola $x^2 = y - 4$ if d is the shortest distance between the points A and B then the value of $16\sqrt{2}d$ is _____.
4. The parabola $y = ax^2 + bx + c$ has a vertex at $(1, 2)$. If $a \in [2, 4]$ then the difference between the extreme values of the product abc is _____.
5. Let $f(x) = [\tan x [\cot x]]$, $x \in \left[\frac{\pi}{12}, \frac{\pi}{4}\right]$ where $[.]$ denotes the greatest integer less than or equal to x . then the number of points where $f(x)$ is discontinuous is _____.
6. If the tangent drawn at (a, b) to $x^3 + y^3 = r^3$ meets the curve again at (c, d) then the value of $\left(\frac{2c}{a} + \frac{2d}{b} + 5\right)$ is _____.

SPACE FOR ROUGH WORK
