8. STRAIGHT LINE

1. INTRODUCTION

Co-ordinate geometry is the branch of mathematics which includes the study of different curves and figures by ordered pairs of real numbers called Cartesian co-ordinates, representing lines & curves by algebraic equation. This mathematical model is used in solving real world problems.

2. CO-ORDINATE SYSTEM

Co-ordinate system is nothing but a reference system designed to locate position of any point or geometric element in a plane of space.

2.1 Cartesian Co-ordinates

Let us consider two perpendicular straight lines XOX′ and YOY′ passing through the origin O in the plane. Then,

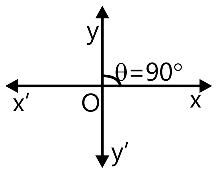


Figure 8.1

Axis of x: The horizontal line xox′ is called axis of x.

Axis of y: The vertical line yoy′ is called axis of y.

Co-ordinate axis: x-axis and y-axis together are called axis of co-ordinates or axis of reference.

Origin: The point ‘O’ is called the origin of co-ordinates or just the origin.

Oblique axis: When xox′ and yoy′ are not at right angle, i.e. if the both axes are not perpendicular, to each other, then axis of co-ordinates are called oblique axis.

2.2 Co-ordinate of a Point

The ordered pair of perpendicular distances of a point from X- and Y-axes are called co-ordinates of that point.

If the perpendicular algebraic distance of a point p from y-axis is x and from x-axis is y, then co-ordinates of the point P is (x, y). Here,

(a) x is called x-co-ordinate or abscissa.

(b) y is called y-co-ordinate or ordinate.

(c) x-co-ordinate of every point lying upon y-axis is zero.

(d) y-co-ordinate of every point lying upon x-axis is zero.

(e) Co-ordinates of origin are (0, 0).

Note: A point whose abscissa and ordinate are both integers is known as lattice point.

2.3 Polar Co-ordinates

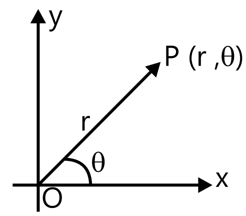


Figure 8.2

Let OX be any fixed line, known as initial line, and O be the origin. If the distance of any point P from the origin O is 'r' and ∠XOP = θ, then (r, θ) are known as polar co-ordinates of point P. If (x, y) are the Cartesian co-ordinates of a point P, then x = rcos θ; y = rsin θ and

$|r| = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ $\theta \in (-\pi, \pi)$

Illustration 1: If the Cartesian co-ordinates of any point are $(\sqrt{3}, 1)$, find the polar co-ordinates. **(JEE MAIN)**

Sol: Polar co-ordinates of any point are (r, θ), where $r = \sqrt{x^2+y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.

$x = \sqrt{3}$; y = 1

Let their polar co-ordinates be (r, θ) ⇒ x = r cos θ; y = r sinθ

So $r \Rightarrow \sqrt{x^2+y^2}$ $\qquad r = \sqrt{3+1}$

$\theta \Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = 2$ $\qquad \theta \Rightarrow \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

$\therefore (r, \theta) = \left(2, \frac{\pi}{6}\right)$.

3. DISTANCE FORMULA

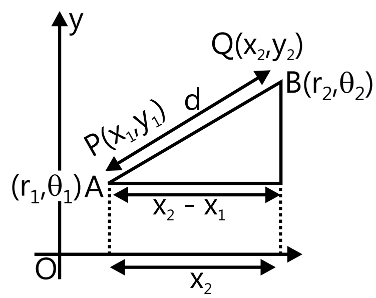


Figure 8.3

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$PQ = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Distance of a point $P(x_1, y_1)$ from the origin O(0, 0) is

$$OP = \sqrt{X_1^2 + y_1^2}$$

Distance between two polar co-ordinates $A(r_1, \theta_1)$ and $B(r_2, \theta_2)$ is given by

$$AB = \sqrt{r_1^2+r_2^2-2r_1r_2\cos(\theta_1-\theta_2)}$$

Proof: $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $x_1 = r_1\cos\theta_1$, $x_2 = r_2\cos\theta_2$, $y_1 = r_1\sin\theta_1$, $y_2 = r_2\sin\theta_2$

$$AB = \sqrt{(r_2\cos\theta_2 - r_1\cos\theta_1)^2 + (r_2\sin\theta_2 - r_1\sin\theta_1)^2}$$

$$AB = \sqrt{(r_2\cos\theta_2)^2 - 2r_1r_2\cos\theta_1\cos\theta_2 + (r_1\cos\theta_1)^2 + (r_2\sin\theta_2)^2 - 2r_1r_2\sin\theta_1\sin\theta_2 + (r_1\sin\theta_1)^2}$$

$$AB = \sqrt{r_1^2+r_2^2-2r_1r_2\cos(\theta_1-\theta_2)}$$

PLANCESS CONCEPTS

Distance between two polar co-ordinates $A(r_1, q_1)$ and $B(r_2, q_2)$ is given by

$$AB = \sqrt{r_1^2+r_2^2-2r_1r_2\cos(\theta_1-\theta_2)}$$

Vaibhav Krishnan (JEE 2009, AIR 22)

Illustration 2: Find the distance between $P\left(2,-\frac{\pi}{6}\right)$ and $Q\left(3,\frac{\pi}{6}\right)$. **(JEE MAIN)**

Sol: The distance between two points = $\sqrt{r_1^2+r_2^2-2r_1r_2\cos(\theta_1-\theta_2)}$. Therefore,

$$PQ=\sqrt{r_1^2+r_2^2-2r_1r_2\cos(\theta_1-\theta_2)}=\sqrt{4+9-2.2.3\cos\left(-\frac{\pi}{6}-\frac{\pi}{6}\right)}==\sqrt{4+9-12\cos\left(\frac{\pi}{3}\right)}=\sqrt{13-12.\frac{1}{2}}=\sqrt{7}$$

Illustration 3: The point whose abscissa is equal to its ordinate and which is equidistant from the points A(1, 0), B(0, 3) is **(JEE MAIN)**

Sol: Given, abscissa = ordinate. Therefore distance can be found by considering the co-ordinates of required point be P(k, k).

Now given PA = PB $\Rightarrow \sqrt{(k-1)^2+k^2}=\sqrt{k^2+(k-3)^2}$

$2k^2-2k+1=2k^2-6k+9\Rightarrow 4k=8\Rightarrow k=2$

4. SECTION FORMULA

Let R divide the two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio m:n.

Let (x, y) be the co-ordinates of R.

Draw PM, QN, RK perpendicular to the x-axis.

Also, draw PE and RF perpendicular to RK and QN.

Here, $\frac{PR}{RQ}=\frac{m}{n}$.

Triangles PRE and RFQ are similar.

$\therefore \frac{PR}{RQ}=\frac{PE}{RF}\Rightarrow\frac{PE}{RF}=\frac{m}{n}$

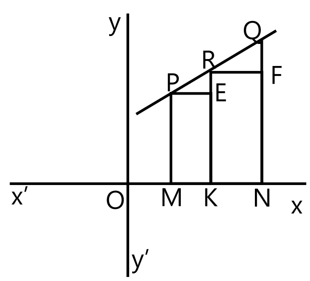


Figure 8.4

But $PE = x - x_1$ and $RF = x_2 - x$.

$\therefore \frac{x-x_1}{x_2-x}=\frac{m}{n}\Rightarrow x=\frac{mx_2+nx_1}{m+n}$

In the same way, $\frac{ER}{FQ}=\frac{m}{n}$

i.e., $\frac{y-y_1}{y_2-y}=\frac{m}{n}\Rightarrow y=\frac{my_2+ny_1}{m+n}$ The co-ordinates of R are $\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$

If R′ divides PQ externally, so that $\frac{PR'}{QR'}=\frac{m}{n}$, triangles PER′ and QR′F are similar.

$\therefore \frac{PR'}{R'Q}=\frac{PE}{R'F}$

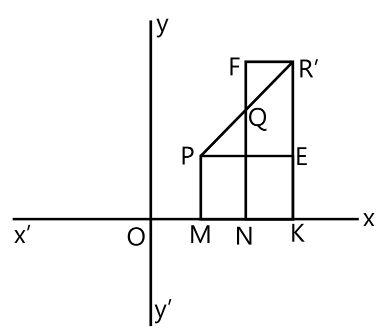


Figure 8.5

But $PE = x - x_1$ and $R'F = x - x_2$.

$\therefore \frac{x-x_1}{x-x_2}=\frac{m}{n}$ i.e., $x=\frac{mx_2-nx_1}{m-n}$

Similarly, $y=\frac{my_2-ny_1}{m-n}$.

The co-ordinates of R′ are $\left(\frac{mx_2-nx_1}{m-n},\frac{my_2-ny_1}{m-n}\right)$

Alternate Method: $\frac{PR'}{R'Q} = -\frac{m}{n} = \frac{m}{-n}$ By changing n into –n in the co-ordinates of R, we can obtain the co-ordinates of R′:

$$\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}$$

Cor. The mid-point joining the two points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Cor. From the above cor., the co-ordinates of a point dividing PQ in the ratio λ:1 are $\left(\frac{x_1 + \lambda x_2}{1+\lambda}, \frac{y_1 + \lambda y_2}{1+\lambda}\right)$. Considering λ as a variable parameter, i.e. of all values positive or negative, the co-ordinates of any point on the line joining the points (x_1, y_1) and (x_2, y_2) can be expressed in the above forms.

5. SPECIAL POINTS OF A TRIANGLE

5.1 Centroid

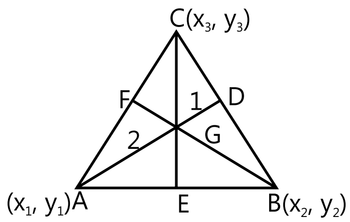


Figure 8.6

Let the vertices of the triangle ABC be (x_1, y_1), (x_2, y_2) and (x_3, y_3), respectively.

The mid-point D of BC is $\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$ G, the centroid, divides AD internally in the ratio 2:1.

Let G be (x, y),

then $x = \frac{2.\left((x_2 + x_3)/2\right) + 1.x_1}{2+1} = \frac{x_1 + x_2 + x_3}{3}$ and

$y = \frac{2.\left((y_2 + y_3)/2\right) + 1.y_1}{2+1} = \frac{y_1 + y_2 + y_3}{3}$ $\therefore$ G is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

5.2 Incentre

Let A (x_1, y_1), B (x_2, y_2), C (x_3, y_3) be the vertices of the triangle.

Let AD bisect angle BAC and cut BC at D.

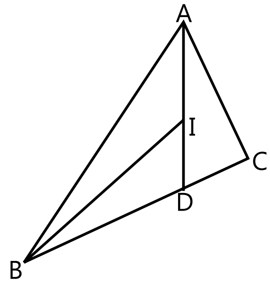


Figure 8.7

We know that $\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$

Hence the co-ordinates of D are $\frac{cx_3 + bx_2}{c+b}, \frac{cy_3 + by_2}{c+b}$

Let (x, y) be the incentre of the triangle

$\frac{CD}{BD} = \frac{b}{c}$ $\therefore$ $\frac{BC}{DB} = \frac{b+c}{c}$ $\therefore$ BD = $\frac{ca}{b+c}$ $\frac{AI}{ID} = \frac{AB}{BD} = \frac{c}{\left(ca/(b+c)\right)} = \frac{b+c}{a}$

$$\therefore\ x = \frac{(b+c)\left((cx_3 + bx_2)/(c+b)\right) + ax_1}{b+c+a} = \frac{ax_1 + bx_2 + cx_3}{a+b+c},\ y = \frac{(b+c)\left((cy_3 + by_2)/(c+b)\right) + ay_1}{b+c+a} = \frac{ay_1 + by_2 + cy_3}{a+b+c}$$

5.3 Ex-centres

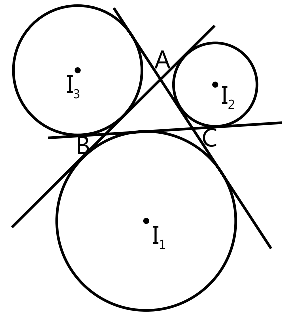


Figure 8.8

The centre of the circle which touches the side BC and the extended portions of sides AB and AC is called the ex-centre of ΔABC with respect to the vertex A. It is denoted by I_1 and its co-ordinates are as follows:

$$I_1 = \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c}\right)$$

Similarly ex-centres of ΔABC with respect to vertices B and C are denoted by I_2 and I_3, respectively, and

$$I_2 = \left(\frac{ax_1 - bx_2 + cx_3}{a-b+c}, \frac{ay_1 - by_2 + cy_3}{a-b+c}\right),$$

$$I_3 = \left(\frac{ax_1 + bx_2 - cx_3}{a+b-c}, \frac{ay_1 + by_2 - cy_3}{a+b-c}\right).$$

5.4 Circumcentre

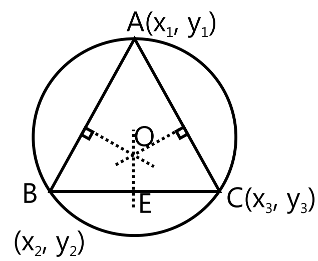


Figure 8.9

It is the point of intersection of perpendicular bisectors of the sides of the triangle. It is also the centre of a circle passing through the vertices of the triangle. If O is the circumcentre of any ΔABC, then, OA = OB = OC.

$$\text{Circumcentre:}\left(\frac{x_1 \sin 2A + x_2 \sin 2B + x_3 \sin 2C}{\Sigma \sin 2A}, \frac{y_1 \sin 2A + y_2 \sin 2B + y_3 \sin 2C}{\Sigma \sin 2A}\right)$$

Note: For a right-angled triangle, its circumcentre is the mid-point of hypotenuse.

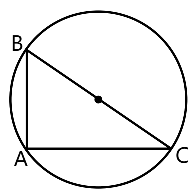


Figure 8.10

5.5 Orthocentre

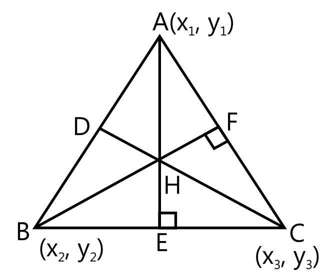


Figure 8.11

The point of intersection of altitudes of a triangle that can be obtained by solving the equation of any two altitudes is called Orthocentre. It is denoted by H

$$\text{Orthocentre: }\left(\frac{x_1 \tan A + x_2 \tan B + x_3 \tan C}{\Sigma \tan A}, \frac{y_1 \tan A + y_2 \tan B + y_3 \tan C}{\Sigma \tan A}\right)$$

Note: In a right angle triangle, orthocentre is the point where right angle is formed.

Remarks:

(a) In an equilateral triangle, centroid, incentre, orthocentre, circumcentre coincide.

(b) Orthocentre, centroid, and circumcentre are always collinear. Centroid divides the Orthocentre and circumcentre joining line in a 2: 1 ratio.

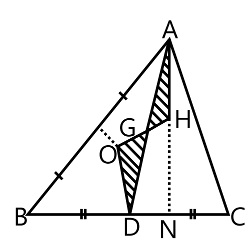


Figure 8.12

Proof: H, G and O are collinear and Δ's OGD & AGH are similar.

But OD (distance of c.c. from BC) = R cos A

HA = distance of orthocentre from vertex A = 2R cos A

$\therefore \frac{AH}{OD} = 2 = \frac{AG}{GD} = \frac{HG}{GO} \Rightarrow$ G divides line joining H and O in 2:1.

(c) In an isosceles triangle centroid, orthocentre, incentre, circumcentre lie on the same line.

5.6 Nine-Point Circle

Nine-point circle can be constructed for any given triangle, and is so named because it touches nine significant concyclic points throughout the triangle.

These nine points are as follows:

- Mid-point of each side of the triangle
- Foot of each altitude
- Mid-point of the line segment from each vertex of the triangle to the orthocentre.

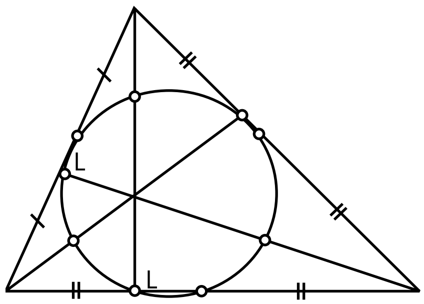


Figure 8.13

PLANCESS CONCEPTS

- The centroid, incentre, orthocentre and circumcentre coincide in an equilateral triangle.
- In an isosceles triangle, centroid, orthocentre, incentre and circumcentre lie on the same line.
- Orthocentre, centroid and circumcentre are always collinear, and centroid divides the line joining orthocentre and circumcentre in the ratio 2:1.

Saurabh Gupta (JEE 2010, AIR 443)

Illustration 4: If G be the centroid of the triangle ABC, prove that $AB^2 + BC^2 + CA^2 = 3(GA^2 + GB^2 + GC^2)$.

(JEE MAIN)

Sol: Distance formula of two points can be used to prove $AB^2 + BC^2 + CA^2 = 3(GA^2 + GB^2 + GC^2)$.

In triangle ABC, let B be the origin and BC the x-axis. Let A be (h, k) and

C be (a, 0). Then centroid G is $\left(\frac{a+h}{3}, \frac{k}{3}\right)$.

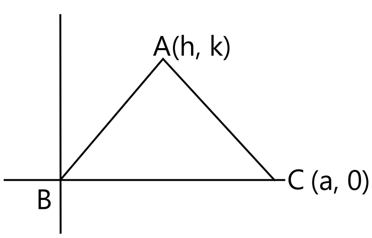


Figure 8.14

LHS

$= AB^2 + BC^2 + CA^2 = (h-0)^2 + (k-0)^2 + a^2 + (h-a)^2 + (k-0)^2$

$= 2h^2 + 2k^2 + 2a^2 - 2ah$

∴ **RHS**

$$= 3\left[\left(\frac{a+h}{3}-h\right)^2+\left(\frac{k}{3}-k\right)^2+\left(\frac{a+h}{3}-0\right)^2+\left(\frac{k}{3}-0\right)^2+\left(\frac{a+h}{3}-a\right)^2+\left(\frac{k}{3}-0\right)\right]$$

$= 1/3\ [(a - 2h)^2 + 4k^2 + (a + h)^2 + k^2 + (h - 2a)^2 + k^2] = 2h^2 + 2k^2 + 2a^2 - 2ah$

Hence, it is equal on both sides.

5.7 Area of a Triangle

Let A, B, C be the vertices of the triangle having (x_1, y_1), (x_2, y_2) and (x_3, y_3) as their respective co-ordinates. Draw AL, BM, CN perpendicular to the x-axis.

Then ΔABC = trapezium ALNC + trapezium CNMB – trapezium ALMB

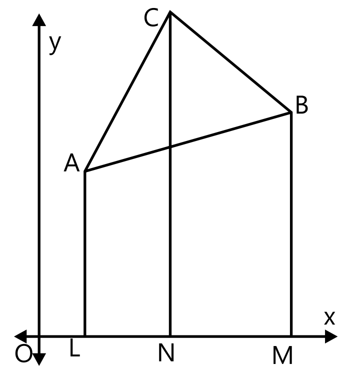


Figure 8.15

$$= \frac{1}{2}(LA + NC)\,LN + \frac{1}{2}(NC + MB)\,NM - \frac{1}{2}(LA + MB)\,LM$$

$$= \frac{1}{2}(y_1 + y_3)(x_3 - x_1) + \frac{1}{2}(y_3 + y_2)(x_2 - x_3) - \frac{1}{2}(y_1 + y_2)(x_2 - x_1)$$

$$= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

If the area of the triangle formed by the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is zero, the points lie on a straight line. Using this, we can determine whether three point are in a straight line. i.e. the condition for (x_1, y_1), (x_2, y_2) and (x_3, y_3) to be collinear is that $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$.

For example, the area of the triangle formed by the points (1, 4), (3, –2) and (–3, 16) is $\frac{1}{2}\{1(-2 - 16) + 3\}\{(16 - 4) - 3(4 + 2)\} = 0$. The three points lie on a straight line.

Illustration 5: The vertices of a triangle ABC are $A(p^2, -p)$, $B(q^2, q)$ and $C(r^2, -r)$. Find the area of the triangle.

(JEE MAIN)

Sol: Area of the triangle formed by the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$. Substituting the given co-ordinates, we can obtain area of given triangle.

$$D = \frac{1}{2}\begin{vmatrix} p^2 & -p & 1 \\ q^2 & q & 1 \\ r^2 & -r & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} p^2 - q^2 & -(p+q) & 0 \\ q^2 - r^2 & q+r & 0 \\ r^2 & -r & 1 \end{vmatrix} = \frac{1}{2}(p + q)(q + r)\begin{vmatrix} p-q & -1 & 0 \\ q-r & 1 & 0 \\ r^2 & -r & 1 \end{vmatrix}$$

$$= \frac{1}{2}(p + q)(q + r)[(p - q) + (q - r)] = \frac{1}{2}(p + q)(q + r)(p - r)$$

Note:

(a) If area of the triangle is zero, then the three points are collinear.

(b) The area of a polygon with vertices $A_1(x_i, y_i)$, i = 1, ..., n (vertices taken in anti-clockwise order)

$$\frac{1}{2}[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + ... + (x_ny_1 - x_1y_n)]$$

6. LOCUS

Locus is a set of points which satisfies a given geometrical data. Thus, for example, locus of a point moving at a constant distance from a given point is a circle. Locus of a point which is equidistance from two fixed points is a perpendicular bisector of the line joining the two points.

All the points in a locus can be represented by an equation. For example,

(a) If the distance of the point (x, y) from (2, 3) is 4, then

$$(x - 2)^2 + (y - 3)^2 = 4^2.$$

i.e. $x^2 + y^2 - 4x - 6y - 3 = 0.$

This equation will represent a circle with its centre at (2, 3) and radius 4.

(b) If (x, y) be the point equidistant from the points (3, 4) and (2, 1), then

$(x - 3)^2 + (y - 4)^2 = (x - 2)^2 + (y - 1)^2$

i.e. $x + 3y = 10.$

From the geometrical constraint, which governs the motion, we can find a relation (locus) between the co-ordinates of the moving point in any of its positions. Equation of locus is therefore merely on equation relating the x and y co-ordinates of every point on the locus.

Steps to find locus

(i) Assume the co-ordinates of point for which locus is to be determined as (h, k).

(ii) Apply the given geometrical conditions.

(iii) Transform the geometrical conditions into algebraic equation and simplify.

(iv) Eliminate variables (if any).

(v) Replace $h \to x$ and $k \to y$ to get the equation of locus.

Note:

- Locus should not contain any other variables except x and y.
- The algebraic relation between x and y satisfied by the co-ordinates at every point on the curve and not off the curve is called the equation of curve.

Illustration 6: Find the equation of locus of a point which moves so that its distance from the point (0, 1) is twice the distance from x-axis. **(JEE MAIN)**

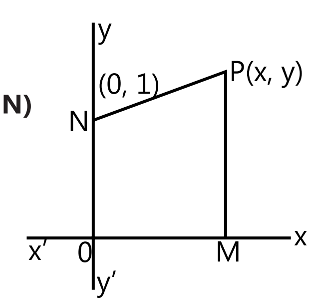


Figure 8.16

Sol: Here we can obtain the equation of locus of given point by using given condition and distance formula of two points.

Let the co-ordinates of such a point be (x, y). Draw PM ⊥ to x-axis.

Hence, PM = y

PN = 2PM (given)

i.e. $(x - 0)^2 + (y - 1)^2 = 4y^2$

i.e. $x^2 - 3y^2 - 2y + 1 = 0$.

Illustration 7: Locus of the centroid of the triangle whose vertices are (a cos t, a sin t), (b sin t, – b cos t) and (1, 0), where t is a parameter is **(JEE MAIN)**

(A) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$ (B) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$

(C) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$ (D) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$

Sol: The centroid (h, k) of a triangle formed by points (x_1, y_1), (x_2, y_2) and (x_3, y_3) will be

$$h = \frac{x_1 + x_2 + x_3}{3} \text{ and } k = \frac{y_1 + y_2 + y_3}{3}.$$

(A) If (h, k) is the centroid, then

$$h = \frac{a\cos t + b\sin t + 1}{3},\ k = \frac{a\sin t - b\cos t + 0}{3} \Rightarrow (3h - 1)^2 + (3k)^2 = (a\cos t + b\sin t)^2 + (a\sin t - b\cos t)^2 = a^2 + b^2$$

$\therefore$ Locus of (h, k) is $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

7. STRAIGHT LINE

Definition: It is defined as the locus of a point such that any two points of this locus have a constant inclination (gradient).

Inclination: If a straight line intersects the x-axis, the inclination of the line is defined as the measure of the smallest non-negative angle which the line makes with the positive direction of the x-axis

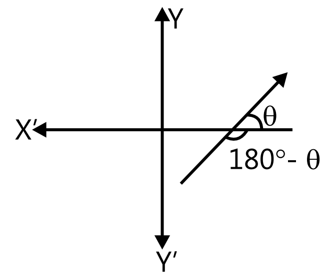


Figure 8.17

Slope (or gradient): If the inclination of a line (i.e. non-vertical line) is θ and $\left(\theta \neq \frac{\pi}{2}\right)$, then the slope of a line is defined to be tan θ and is denoted by m.

$\therefore$ m = tanθ

(a) Slope of x-axis is zero.

(b) Slope of y-axis is not defined.

7.1 Slope

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points on a line, then slope will be

$$m = \tan\theta = \frac{MQ}{MP}$$

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

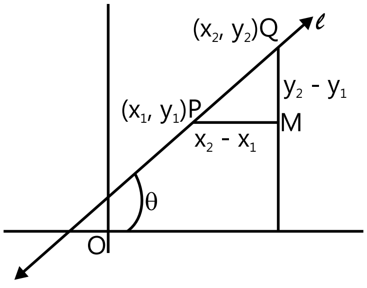


Figure 8.18

(a) Line passing through two points (x_1, y_1) and (x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

Note: Above-mentioned matrix form is a condition for three points to be collinear.

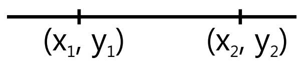


Figure 8.19

(b) Equation of the median through $A(x_1, y_1)$ is

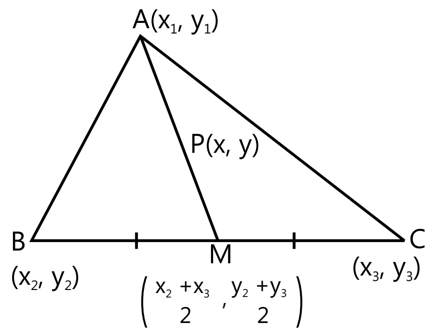


Figure 8.20

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ \frac{x_2+x_3}{2} & \frac{y_2+y_3}{2} & 1 \end{vmatrix} = 0 \text{ or } \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

(c) Equation of internal and external angle bisectors of A are

$$b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} \pm c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

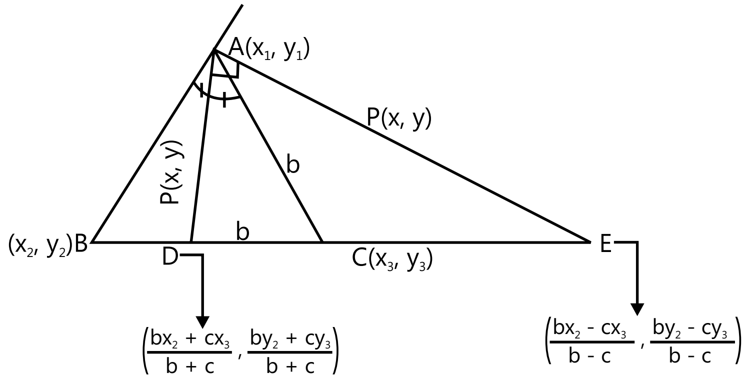


Figure 8.21

7.2 Angle between Two Lines

Two lines intersecting each other make two angles between them, one acute and the other obtuse. Figure 8.22 shows lines L_1 and L_2 intersecting each other, acute angle θ and obtuse angle ϕ.

Let line L_1 makes angle θ_1 with x-axis and L_2 makes θ_2.

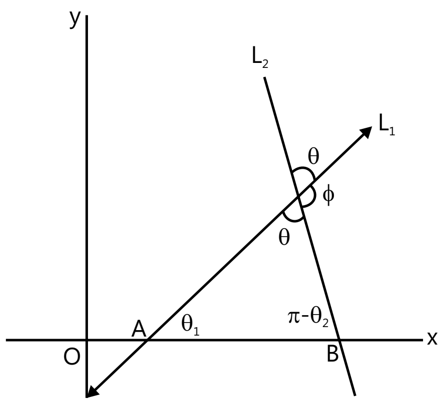


Figure 8.22

Therefore slope of L_1 is $m_1 = \tan\theta_1$

Slope of L_2 is $m_2 = \tan\theta_2$

Now in ΔABC,

$\theta_1 + \pi - \theta_2 + \theta = \pi$

$\theta = \theta_2 - \theta_1$

$\tan\theta = \tan(\theta_2 - \theta_1) \Rightarrow \tan\theta = \dfrac{\tan\theta_2 - \tan\theta_1}{1+\tan\theta_2.\tan\theta_1} \Rightarrow \tan\theta = \left|\dfrac{m_2 - m_1}{1+m_1m_2}\right|$, this gives the acute angle between lines.

Note:

(i) If $m_1 = m_2$, then $\theta = 0^\circ$, i.e. lines are parallel or coincident.

(ii) If $m_1m_2 = -1$, then $\theta = 90^\circ$, i.e. lines are perpendicular to each other.

Illustration 8: If the angle between two lines is $\dfrac{\pi}{4}$ and slope of one of the lines is $\dfrac{1}{2}$. Find the slope of the other. **(JEE MAIN)**

Sol: We know that, $\tan\theta = \left|\dfrac{m_2 - m_1}{1+m_1m_2}\right|$, where m_1 and m_2 are the slope of lines and θ is the angle between them.

Let $m_1 = \dfrac{1}{2}$, $m_2 = m$ and $\theta = \dfrac{\pi}{4}$ So, $\tan\dfrac{\pi}{4} = \left|\dfrac{m-(1/2)}{1+(1/2)m}\right| \Rightarrow 1 = \pm\dfrac{m-(1/2)}{1+(1/2)m} \Rightarrow m = 3$ or $-(1/3)$

Illustration 9: Line through the point (–2, 6) and (4, 8) is perpendicular to the line through the point (8, 12) and (x, 24). Find the value of x. **(JEE MAIN)**

Sol: Given two lines are perpendicular to each other. Therefore, product of their slope will be -1.

Slope of the line through the points (–2, 6) and (4, 8) is $m_1 = \dfrac{8-6}{4-(-2)} = \dfrac{2}{6} = \dfrac{1}{3}$

Slope of the line through the points (8, 12) and (x, 24) is $m_2 = \dfrac{24-12}{x-8} = \dfrac{12}{x-8}$

Since two lines are perpendicular $m_1m_2 = -1$

$\Rightarrow \dfrac{1}{3}\times\dfrac{12}{x-8} = -1 \qquad \Rightarrow x = 4$

7.3 Collinearity

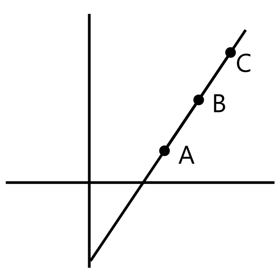


Figure 8.23

If three points A, B, C are collinear, then

Slope of AB = Slope of BC = slope of AC

PLANCESS CONCEPTS

Collinearity of three given points:

Three given points A, B, C are collinear if any one of the following conditions is satisfied.

- Area of triangle ABC is zero.
- Slope of AB = Slope of BC = Slope of AC.
- AC = AB + BC.
- Find the equation of the line passing through two given points, if the third point satisfies the equation of the line, then three points are collinear.

If any one line is parallel to y-axis, then the angle between two straight lines is given by $\tan\theta = \pm\frac{1}{m}$, where m is the slope of other straight line.

A line of gradient m is equally inclined with the two lines of gradient m_1 and m_2.

Then $\frac{m_1 - m}{1 + m_1 m} = -\frac{m_2 - m}{1 + m_2 m}$.

Aman Gour (JEE 2012, AIR 230)

7.4 Equation of a line

(a) Point slope form: Suppose $P_0(x_0, y_0)$ is a fixed point on a non-vertical line L whose slope is m. Let P(x, y) be an arbitrary point on L. Then by definition, the slope of L is given by $m = \frac{y - y_0}{x - x_0} \Rightarrow y - y_0 = m(x - x_0)$

This is called point slope form of a line.

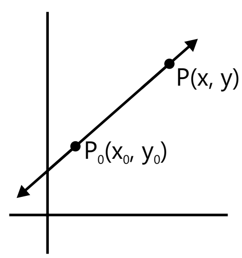


Figure 8.24

(b) Two point form: Let line L passes through two given points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. Let P(x, y) be a point on the line. So slope P_1P = slope P_1P_2

$$\Rightarrow \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

This is called two-point form of the line.

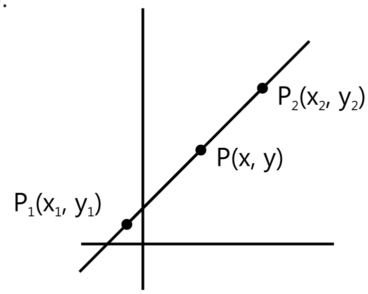


Figure 8.25

(c) Slope intercept from: Case-I: If slope of line is m and makes y-intercept c, then equation is

$(y - c) = m(x - 0) \quad \Rightarrow \quad y = mx + c$

Case-II: If slope of line is m and makes x-intercept d, then equation is

$y = m(x - d)$

These equations are called slope intercept form.

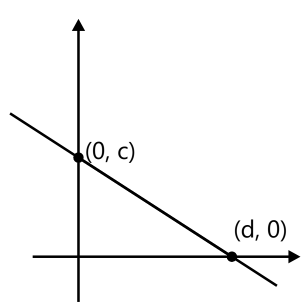


Figure 8.26

(d) Intercept form: Suppose a line L makes intercept a on x-axis and intercept b on y-axis, i.e. the line meets x-axis at (a, 0) and y-axis (0, b).

So, $y - 0 = \dfrac{b-0}{0-a}(x-a)$

i.e. $\dfrac{x}{a} + \dfrac{y}{b} = 1$. This is called intercept form of the line.

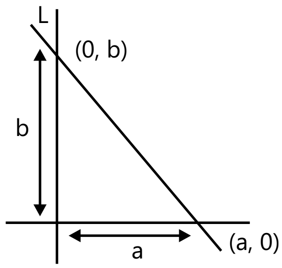


Figure 8.27

(e) Normal form: If P is perpendicular distance from origin to the line AB and makes angle α with x-axis, then equation of the line is $x\cos\alpha + y\sin\alpha = P$

Proof: $\cos\alpha = \dfrac{OM}{OL}$ OM = OL cos α = x cos α

In ΔPNL,

$\sin\alpha = \dfrac{PN}{PL}$

PN = PL sin α = y sin α

MQ = PN = y sin α

Now P = OQ = OM + MQ = x cos α + y sin α

So $x\cos\alpha + y\sin\alpha = P$

This is called normal form of the line.

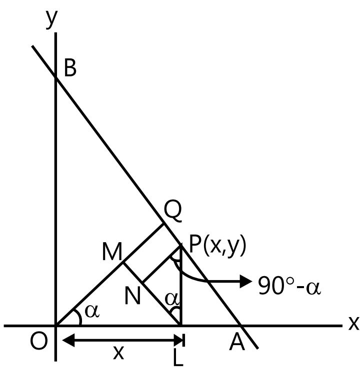


Figure 8.28

(f) Parametric form or distance form: The equation of the line passing through (x_1, y_1) and making an angle θ with the positive x-axis is

$\dfrac{x-x_1}{\cos\theta} = \dfrac{y-y_1}{\sin\theta} = r$, where 'r' is the signed value.

Hence, the co-ordinate of any point at a distance r on this line is

$x = x_1 + r\cos\theta$ $y = y_1 + r\sin\theta$

PLANCESS CONCEPTS

Point of intersection of two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is given by

$$(x', y') = \left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_1 - a_2b_1}\right) = \left(\frac{\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}}, \frac{\begin{vmatrix} c_1 & c_2 \\ a_1 & a_2 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}}\right)$$

Saurabh Gupta (JEE 2010, AIR 443)

Illustration 10: A straight line is drawn through the point P(2, 3) and is inclined at an angle of 30° with positive x-axis. Find the co-ordinate of two points on it at a distance 4 from P on either side of P. **(JEE MAIN)**

Sol: By using formula $\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}=r$, we can obtain co-ordinates of point.

The equation of line

$$\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}=\pm r \Rightarrow \frac{x-2}{\cos 30°}=\frac{y-3}{\sin 30°}=\pm 4 \Rightarrow x=2\pm 2\sqrt{3}, y=3\pm 2$$

So, co-ordinate of two points are $\left(2\pm 2\sqrt{3}, 3\pm 2\right)$

Illustration 11: If two vertices of a triangle are (–2, 3) and (5, –1). Orthocentre of the triangle lies at the origin and centroid on the line x + y = 7, then the third vertex lies at **(JEE MAIN)**

(A) (7, 4) (B) (8, 14)

(C) (12, 21) (D) None of these

Sol: (D) The line passing through the third vertex and orthocentre must be perpendicular to line through (–2, 3) and (5, –1). Therefore, product of their slope will be -1.

Given the two vertices B(–2, 3) and C(5, – 1); let H(0, 0) be the orthocentre; A(h, k) the third vertex.

Then, the slope of the line through A and H is k/h, while the line through B and C has the slope (–1 –3)/(5 + 2) = – 4/7. By the property of the orthocentre, these two lines must be perpendicular,

So we have $\left(\frac{k}{h}\right)\left(-\frac{4}{7}\right)=-1 \Rightarrow \frac{k}{h}=\frac{7}{4}$ ... (i)

Also $\frac{5-2+h}{3}+\frac{-1+3+k}{3}=7 \Rightarrow h+k=16$ ... (ii)

Which is not satisfied by the points given in (A), (B) or (C).

Illustration 12: In what direction should a line be drawn passing through point (1, 2) so that its intersection point with line x + y = 4 is at a distance of $\frac{\sqrt{6}}{3}$ units. **(JEE ADVANCED)**

Sol: By using $x=x_1+r\cos\theta$ and $y=y_1+r\sin\theta$, we can obtain the required angle.

For co-ordinates of B

Substitute $r=\frac{\sqrt{6}}{3}$ $\therefore\ x=1+\frac{\sqrt{6}}{3}\cos\theta\ \&\ y=2+\frac{\sqrt{6}}{3}\sin\theta$

Substituting in x + y =4

$\Rightarrow 1+\frac{\sqrt{6}}{3}\cos\theta+2+\frac{\sqrt{6}}{3}\sin\theta=4$ $\therefore (\cos\theta+\sin\theta)=\frac{3}{\sqrt{6}}$

$\therefore\ \frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta=\frac{\sqrt{3}}{2}$ (Multiple by $\frac{1}{\sqrt{2}}$)

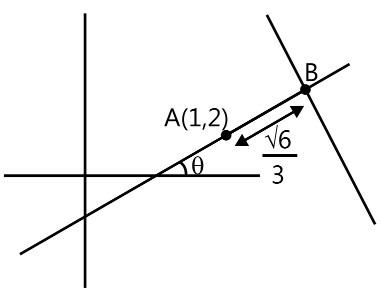


Figure 8.29

$\sin(45°+\theta)=\sin 60°$

$\therefore \theta=15°$

or, $\sin(45°+\theta)=\sin 120°\ \therefore \theta=75°$

Illustration 13: If sum of the distances of the points from two perpendicular lines in a plane is 1, then find its locus. **(JEE ADVANCED)**

Sol: If (h, k) be any point on the locus, then $|h| + |k| = 1$

Let the two perpendicular lines be taken as the co-ordinate axes.

$\Rightarrow$ locus of (h, k) is $|x| + |y| = 1$

This consists of four line segments which enclose a square as shown in figure.

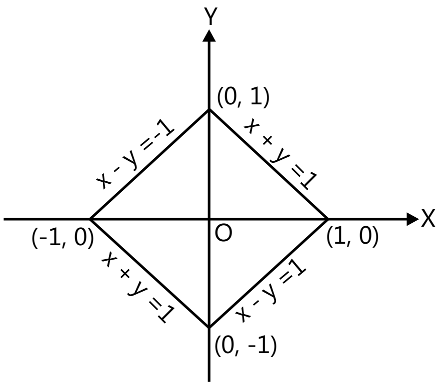


Figure 8.30

Illustration 14: If the circumcentre of a triangle lies at the origin and the centroid is the mid-point of the line joining the points $(a^2 + 1, a^2 + 1)$ and $(2a, -2a)$, then the orthocentre lies on the line. **(JEE ADVANCED)**

(A) $y = (a^2 + 1)x$ (B) $y = 2ax$ (C) $x + y = 0$ (D) $(a - 1)^2 x - (a + 1)^2 y = 0$

Sol: (D) We know from geometry that the circumcentre, centroid and orthocentre of a triangle lie on a line. So the orthocentre of the triangle lies on the line joining the circumcentre (0, 0) and the centroid $\left(\frac{(a+1)^2}{2}, \frac{(a-1)^2}{2}\right)$

$\frac{(a+1)^2}{2}y = \frac{(a-1)^2}{2}x$ or $(a - 1)^2 x - (a + 1)^2 y = 0$.

PLANCESS CONCEPTS

Equation of parallel and perpendicular lines:

- Equation of a line which is parallel to $ax + by + c = 0$ is $ax + by + k = 0$.
- Equation of a line which is perpendicular to $ax + by + c = 0$ is $bx - ay + k = 0$.
- If $y = m_1x + c_1$, $y = m_1x + c_2$, $y = m_2x + d_1$ and $y = m_2x + d_2$ are sides of a parallelogram then its area is $\left|\frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2}\right|$.
- The equation of a line whose mid-point is (x_1, y_1) in between the axes is $\frac{x}{x_1} + \frac{y}{y_1} = 2$.
- Area of the triangle made by the line $ax + by + c = 0$ with the co-ordinate axes is $\frac{c^2}{2|ab|}$.
- A line passing through (x_1, y_1) and if the intercept between the axes is divided in the ratio m:n at this point then the equation is $\frac{nx}{x_1} + \frac{my}{y_1} = m + n$.
- The equation of a straight line which makes a triangle with the co-ordinates axes whose centroid is (x_1, y_1) is $\frac{x}{3x_1} + \frac{y}{3y_1} = 1$.

B Rajiv Reddy (JEE 2012, AIR 11)

7.5 Foot of the Perpendicular

The foot of the perpendicular (h, k) from (x_1, y_1) to the line ax + by + c = 0 is given by

$$\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-(ax_1+by_1+c)}{a^2+b^2}.$$

Hence, the co-ordinates of the foot of perpendicular is

$$\left(\frac{b^2x_1-aby_1-ac}{a^2+b^2},\frac{a^2y_1-abx_1-bc}{a^2+b^2}\right).$$

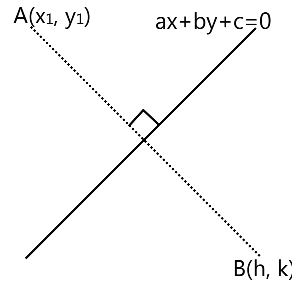


Figure 8.31

The image of a point with respect to the line mirror: The image of A(x_1, y_1) with respect to the line mirror ax + by + c = 0, B (h, k) is given by $\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-2(ax_1+bx_1+c)}{a^2+b^2}$.

Special Cases

(a) Image of the point P(x_1, y_1) with respect to x-axis is (x_1, – y_1).

(b) Image of the point P(x_1, y_1) with respect to y-axis is (– x_1, y_1).

(c) Image of the point P(x_1, y_1) with respect to the line mirror y = x is Q(y_1, x_1).

(d) Image of the point P(x, y) with respect to the origin is the point (-x, -y).

Illustration 15: Find the equation of the line which is at a distance 3 from the origin and the perpendicular from the line makes an angle of 30° with the positive direction of the x-axis. **(JEE MAIN)**

Sol: By using xcosα + ysinα = P, we can solve this problem. Here α = 30° and P = 3.

So equation is x cos 30° + y sin 30° = 3 $x\frac{\sqrt{3}}{2}+\frac{y}{2}=3 \quad \Rightarrow \sqrt{3}x+y=6$

Position of a point w.r.t. a line

Let the equation of the given line be ax + by + c = 0 and let the co-ordinates of the two given points be P(x_1, y_1) and Q(x_2, y_2). Let R_1 be a point on the line.

The co-ordinates of R_1 which divides the line joining P and Q in the ratio m:n are $\frac{m}{n}=\frac{-ax_1-by_1-c}{ax_2+by_2+c}$.

Thus, the two points (x_1, y_1) and (x_2, y_2) are on the same (or opposite) sides of the straight line ax + by + c = 0 whether Point R_1 divides internally or externally or sign of $\frac{m}{n}$.

Note:

⇒ A point (x_1, y_1) will lie on the side of the origin relative to a line ax + by + c = 0, if ax_1 +by_1 + c and c have the same sign.

⇒ A point (x_1, y_1) will lie on the opposite side of the origin relative to the line ax + by + c = 0, if

ax_1 + by_1 + c and c have the opposite sign.

Illustration 16: For what values of the parameter α does the point M (α, α + 1) lies within the triangle ABC of vertices A(0, 3), B(– 2, 0) and C(6, 1). **(JEE ADVANCED)**

Sol: Here, the point M will be inside the triangle if and only if |Area ΔMBC| + |Area ΔMCA| + |Area ΔMAB| = |Area ΔABC|. And each individual area must be non-zero.

Area MBC = $\frac{1}{2}\begin{vmatrix} \alpha & \alpha+1 & 1 \\ -2 & 0 & 1 \\ 6 & 1 & 1 \end{vmatrix} = \frac{1}{2}|7\alpha+6|$

Area MCA = $\frac{1}{2}\begin{vmatrix} \alpha & \alpha+1 & 1 \\ 6 & 1 & 1 \\ 0 & 3 & 1 \end{vmatrix} = \frac{1}{2}|-8\alpha+12|$

Area MAB = $\frac{1}{2}\begin{vmatrix} \alpha & \alpha+1 & 1 \\ 0 & 3 & 1 \\ -2 & 0 & 1 \end{vmatrix} = \frac{1}{2}|\alpha+4|$

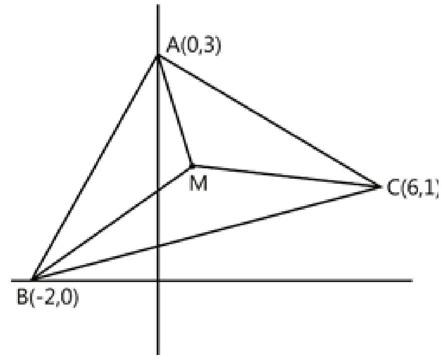


Figure 8.32

Area ABC = $\frac{1}{2}\begin{vmatrix} 0 & 3 & 1 \\ -2 & 0 & 1 \\ 6 & 1 & 1 \end{vmatrix} = \frac{1}{2}.22$

The above equation has critical points – 4, $-\frac{6}{7}$ and $\frac{3}{2}$.

For $\alpha \le -4$, the equation is

$-7\alpha - 6 - 8\alpha + 12 - \alpha - 4 = 22$

$\Rightarrow \alpha = -\frac{5}{4}$ which is not a solution, since $-\frac{5}{4} > -4$

For $\alpha \in \left(-4, -\frac{6}{7}\right)$, then equation is $-7\alpha - 6 - 8\alpha + 12 + \alpha + 4 = 22 \Rightarrow \alpha = -\frac{6}{7}$

which is solution of equation but area MBC = 0 $\Rightarrow$ M lies on BC $\Rightarrow \alpha = -\frac{6}{7}$ is not the desired value.

For $\alpha \in \left(-\frac{6}{7}, \frac{3}{2}\right)$, the equation is $7\alpha + 68\alpha + 12 + \alpha + 4 = 22$.

$\Rightarrow$ All α in the interval $\left(-\frac{6}{7}, \frac{3}{2}\right)$ satisfy the equation.

Finally over $\left(\frac{3}{2}, \infty\right)$, we get $\alpha = \frac{3}{2}$ implies area MCA become zero.

$\Rightarrow$ The desired values of α lie in the interval $\left(-\frac{6}{7}, \frac{3}{2}\right)$.

7.6 Length of the Perpendicular

The perpendicular distance 'p' of a point $P(x_1, y_1)$ from the line

$ax + by + c = 0$ is $\quad p = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

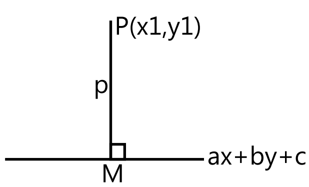


Figure 8.33

(a) Distance between parallel lines: The distance between the parallel lines a

x+ by + c_1 = 0 and ax + by + c_2 = 0 is $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$

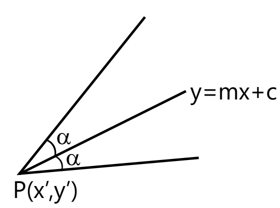


Figure 8.34

(b) Lines making angle α with given line: The equations of the two straight lines passing through P(x′, y′) and making an angle α with the line y = mx + c (where m = tan θ) are

$y - y' = \tan(\theta + \alpha)(x - x')$

Note: If $\theta + \alpha$ or $\theta - \alpha$ is an odd multiple of $\frac{\pi}{2}$, the corresponding line has equation $x = x'$.

(c) Concurrency of lines: Lines $a_i x + b_i y + c_1 = 0$, where i = 1, 2, 3 are concurrent if they meet at a point. The condition for concurrency is $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_2 & c_3 \end{vmatrix} = 0$

Illustration 17: The equation of the two tangents to the circle are $3x - 4y + 10 = 0$ and $6x - 8y + 30 = 0$. Find diameter of the circle. **(JEE MAIN)**

Sol: By using formula of distance between two parallel line, i.e. $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$, we can find the diameter of given circle.

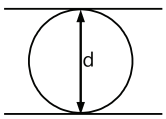


Figure 8.35

These are two parallel lines

$3x - 4y + 10 = 0$ .....(i)

$6x - 8y + 30 = 0$ .....(ii)

Dividing second equation by 2 gives $3x - 4y + 15 = 0$; $\therefore$ $d = \left|\frac{15 - 10}{\sqrt{3^2 + 4^2}}\right| = 1$

PLANCESS CONCEPTS

(a) A triangle is isosceles if any two of its median are equal.

(b) Triangle having integral co-ordinates can never be equilateral.

(c) If $a_r x + b_r y + c_r = 0$ (r = 1, 2, 3) are the sides of a triangle then the area of the triangle is given by $\frac{1}{2C_1C_2C_3}\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2$ where C_1, C_2 and C_3 are the cofactors of c_1, c_2 and c_3 in the determinant.

(d) Area of parallelogram:

(i) Whose sides are a and b and angle between them is θ is given by ab sin θ. Area of ABCD = ab.sinθ

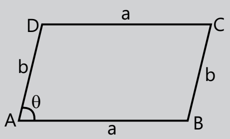


Figure 8.36

(ii) Whose length of perpendicular from one vertices to the opposite sides are p_1 and p_2 and angle between sides is θ is given by Area = $\frac{P_1P_2}{\text{Sin}\theta}$

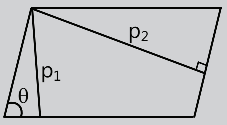


Figure 8.37

Krishan Mittal (JEE 2012, AIR 199)

8. FAMILY OF LINES

Consider two intersecting lines L_1: $a_1x + b_1y + c_1 = 0$ and L_2: $a_2x + b_2y + c_2 = 0$, then

Type-1: The equation of the family of lines passing through the intersection of the lines

$L_1 + \lambda L_2 = 0$

$\Rightarrow$ $(a_1x + b_1y + c_1) + \lambda(a_2x + b_2y + c_2) = 0$ where λ is a parameter.

Type-2: Converse, $L_1 + \lambda L_2 = 0$ is a line which passes through a fixed point, where $L_1 = 0$ and $L_2 = 0$ are fixed lines and the fixed point is the intersection of L_1 and L_2.

Type-3: Equation of $AC \equiv u_2u_3 - u_1u_4 = 0$ and $BD \equiv u_3u_4 - u_1u_2 = 0$

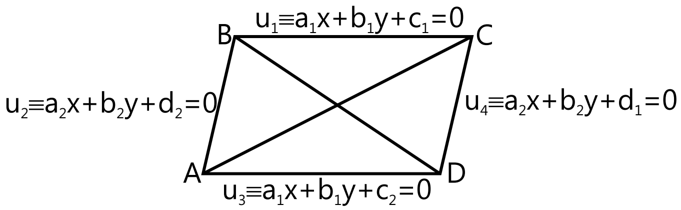


Figure 8.38

Note that second degree terms cancel and the equation $u_2u_3 - u_1u_4 = 0$ is satisfied by the co-ordinate points B and D.

Illustration 18: If a, b, c are in A.P., then prove that the variable line $ax + by + c = 0$ passes through a fixed point. **(JEE MAIN)**

Sol: By using given condition we can reduce $ax + by + c = 0$ to as $L_1 + \lambda L_2 = 0$. Hence we can obtain co-ordinate of fixed point by taking $L_1 = 0$ and $L_2 = 0$.

$2b = a + c \quad \Rightarrow c = 2b - a \quad \Rightarrow ax + by + 2b - a = 0$

$\therefore$ $a(x - 1) + b(y + 2) = 0$ This is of the form $L_1 + \lambda L_2 = 0$, where $b/a = l$

$\therefore$ Co-ordinates of fixed point is $(1, -2)$.

9. ANGULAR BISECTOR

9.1 Bisectors of the Angle Between Two Lines

(a) Equations of the bisectors of angle between the lines $ax + by + c = 0$ and $a_1x + b_1y + c_1 = 0$ are

$$\frac{ax+by+c}{\sqrt{a^2+b^2}} = \pm\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} \quad (ab_1 \neq a_1b)$$

(b) To discriminate between the bisectors of the angle containing the origin and that of angle not containing the origin, rewrite the equations, $ax + by + c = 0$ and $a'x + b'y + c' = 0$ such that the terms c, c′ are positive,

then $\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = +\dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$ gives the equation of the bisector of the angle containing origin and $\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = -\dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$ gives the equation of the bisector of the angle not containing origin.

(c) Acute angle bisector and obtuse angle bisector can be differentiated from the following methods: Let two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ intersect such that constant terms are positive.

If $a_1a_2 + b_1b_2 < 0$, then the angle between the lines that contain the origin is acute and the equation for the acute angle bisector is $\frac{a_1x+b_1y+c}{\sqrt{a^2+b^2}} = +\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$. Therefore $\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}} = -\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$ is the equation of other bisector. If, however, $a_1a_2 + b_1b_2 > 0$, then the angle between the lines containing the origin is obtuse and the equation of the bisector of the obtuse angle is $\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = +\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$; therefore $\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = -\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$ is acute angle bisector.

(d) Few more methods of identifying an acute and obtuse angle bisectors are as follows:

Let $L_1 = 0$ and $L_2 = 0$ are the given lines and $u_1 = 0$ and $u_2 = 0$ are the bisectors between $L_1 = 0$ and $L_2 = 0$. Take a point P on any one of the lines $L_1 = 0$ or $L_2 = 0$ and draw a perpendicular on $u_1 = 0$ and $u_2 = 0$ as shown. If

$|p| < |q| \Rightarrow u_1$ is the acute angle bisector.

$|p| > |q| \Rightarrow u_1$ is the obtuse angle bisector.

$|p| = |q| \Rightarrow$ the lines L_1 and L_2 are perpendicular.

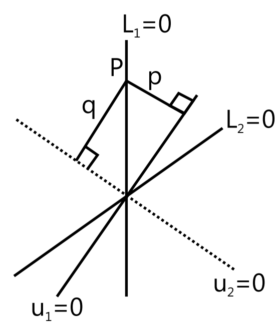


Figure 8.39

Note: The straight lines passing through $P(x_1, y_1)$ and equally inclined with the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are those which are parallel to the bisectors between lines and passing through the point P.

PLANCESS CONCEPTS

(a) Algorithm to find the bisector of the angle containing the origin: Let the equations of the two lines be $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$. The following methods are used to find the bisector of the angle containing the origin:

Step I: In the equations of two lines, check if the constant terms c_1 and c_2 are positive. If the terms are negative, then make them positive by multiply both the sides of the equation by –1.

Step II: Obtain the bisector corresponding to the positive sign, i.e. $\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$

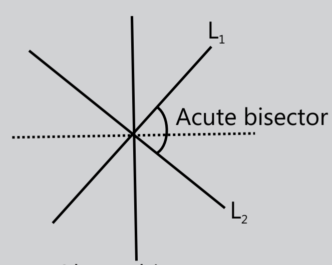


Figure: 8.40

This is the required bisector of the angle containing the origin, i.e. the bisectors of the angle between the lines which contain the origin within it.

(b) Method to find acute angle bisector and obtuse angle bisector

(i) Make the constant term positive by multiplying the equation by –1.

(ii) Now determine the sign of the expression $a_1a_2 + b_1b_2$.

PLANCESS CONCEPTS

(iii) If $a_1a_2 + b_1b_2 > 0$, then the bisector corresponding to '+ve' and '–ve' signs give the obtuse and acute angle bisectors, respectively, between the lines.

(iv) If $a_1a_2 + b_1b_2 < 0$, then the bisector corresponding to '+ve' and '–ve' signs give the acute and obtuse angle bisectors, respectively.

Both the bisectors are perpendicular to each other. If $a_1a_2 + b_1b_2 > 0$, then the origin lies in the obtuse angle and if $a_1a_2 + b_1b_2 < 0$, then the origin lies in the acute angle.

T P Varun (JEE 2012, AIR 64)

PLANCESS CONCEPTS

Incentre divides the angle bisectors in the ratios (b + c):a, (c + a):b and (a + b):c . Angle bisector divides the opposite sides in the ratio of remaining sides.

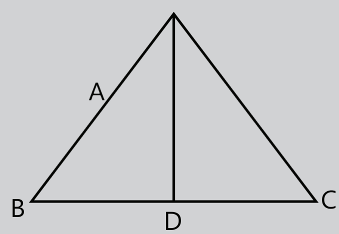


Figure: 8.41

$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$$

Aishwarya Karnawat (JEE 2012, AIR 839)

9.2 Bisectors in Case of Triangle

Two possible models are as follows:

Case-I: When vertices of a triangle are known, compute the sides of the triangle and the incentre. All the internal bisectors can be known, using the co-ordinates of incentre and vertices of triangle.

Note: If the triangle is isosceles/equilateral, then one can easily get the incentre.

Case-II: When the equations of the sides are given,compute tan A, tan B, tan C by arranging the lines in descending order of their slope. Compute the acute/obtuse angle bisectors as the case may be. Plot the lines approximately and bisectors containing or not containing the origin.

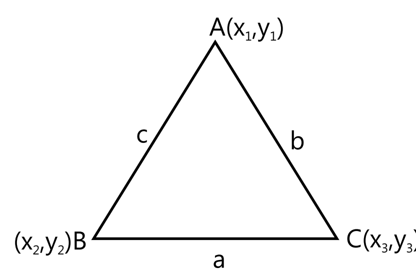


Figure 8.42

Illustration 19: The line x + y =a meets the x- and y-axes at A and B, respectively. A triangle AMN is inscribed in the triangle OAB, O being the origin, with right angle at N. M and N lie respectively on OB and AB. If the area of the triangle AMN is 3/8 of the area of the triangle OAB, then AN/BN is equal to. **(JEE ADVANCED)**

(A) 3 (B) 1/3 (C) 2 (D) 1/2

Sol: (A) Here simply by using the formula of area of triangle,

i.e. $\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$, we can solve the problem.

Let $\frac{AN}{BN} = \lambda$. Then the co-ordinates of N are $\left(\frac{a}{1+\lambda}, \frac{\lambda a}{1+\lambda}\right)$,

where (a, 0) and (0, a) are the co-ordinates of A and B, respectively.

Now equation of MN perpendicular to AB is

$y - \frac{\lambda a}{1+\lambda} = x - \frac{a}{1+\lambda}$ or $x - y = \frac{1-\lambda}{1+\lambda}a$ So the co-ordinates of M are $\left(0, \frac{\lambda - 1}{\lambda + 1}a\right)$

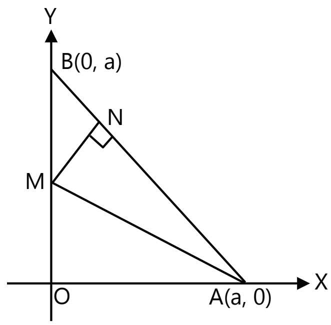


Figure 8.43

Therefore, area of the triangle AMN is

$$= \frac{1}{2}\left|\left[a\left(\frac{-a}{\lambda+1}\right) + \frac{1-\lambda}{(1+\lambda)^2}a^2\right]\right| = \frac{\lambda a^2}{(1+\lambda)^2}$$

Also area of the triangle OAB = $a^2/2$.

So that according to the given condition: $\frac{\lambda a^2}{(1+\lambda)^2} = \frac{3}{8}.\frac{1}{2}a^2 \Rightarrow 3l^2 - 10\lambda + 3 = 0$; $\Rightarrow$ $\lambda = 3$ or $\lambda = 1/3$.

For $\lambda = 1/3$, M lies outside the segment OB and hence the required value of λ is 3.

10. PAIR OF STRAIGHT LINES

10.1 Pair of Straight Lines Through Origin

(a) A homogeneous equation of degree two of the type $ax^2 + 2hxy + by^2 = 0$ always represents a pair of straight lines passing through the origin and if

(i) $h^2 > ab \Rightarrow$ lines are real and distinct.

(ii) $h^2 = ab \Rightarrow$ lines are coincident.

(iii) $h^2 < ab \Rightarrow$ lines are imaginary with real point of intersection, i.e. (0, 0)

(b) If $y = m_1x$ and $y = m_2x$ be the two equations represented by $ax^2 + 2hxy + by^2 = 0$, then

$m_1 + m_2 = -\frac{2h}{b}$ and $m_1m_2 = \frac{a}{b}$

Angle between two straight lines:

(c) If θ is the acute angle between the pair of straight lines represents by $ax^2 + 2hxy + b$, then

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$$

The condition that these lines are:

(i) At right angles to each other if a + b = 0, i.e. sum of coefficients of x^2 and y^2 is zero.

(ii) Coincident if $h^2 = ab$ and $(ax^2 + 2hxy + by^2)$ is a perfect square of $(\sqrt{a}x+\sqrt{b}y)^2$.

(iii) Equally inclined to the axis of x if h = 0, i.e. coefficient of xy =0.

Combined equation of angle bisectors passing through origin: The combined equation of the bisectors of the angles between the lines $ax^2+2hxy+by^2=0$ (a pair of straight lines passing through origin) is given by $\frac{x^2-y^2}{a-b}=\frac{xy}{h}$.

10.2 General Equation for Pair of Straight Lines

(a) $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of straight lines if:

$$abc+2fgh-af^2-bg^2-ch^2=0,\ \text{if}\ \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0$$

(b) The slope of the two lines represented by a general equation is the same as that between the two lines represented by only its homogeneous part.

10.3 Homogenisation

The equation of the two lines joining the origin to the points of intersection of the line lx + my +n = 0 and the curve $ax^2+2hxy+by^2+2gx+2fy+c=0$ is obtained by homogenising the equation of the curve using the equation of the line.

The combined equation of pair of straight lines joining origin to the points of intersection of the line given by $lx + my + n = 0$ .... (i)

The second degree curve:

$$ax^2+2hxy+by^2+2gx+2fy+c=0 \quad \text{.... (ii)}$$

Using equation (i) and (ii)

$$ax^2+2hxy+by^2+2gx\left(\frac{lx+my}{-n}\right)+2fy\left(\frac{lx+my}{-n}\right)+c\left(\frac{lx+my}{-n}\right)^2=0 \quad \text{.... (iii)}$$

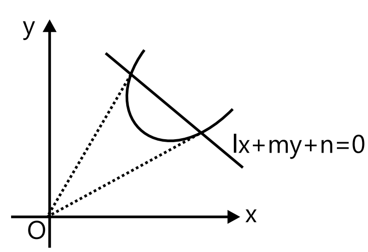


Figure 8.44

Obtained by homogenizing (ii) with the help of (i), by writing (i) in the form: $\left(\frac{lx+my}{-n}\right)=1$.

PLANCESS CONCEPTS

Through a point A on the x-axis, a straight line is drawn parallel to y-axis so as to meet the pair of straight lines.

$ax^2+2hxy+by^2=0$ in B and C. If AB = BC, then $8h^2 = 9ab$.

Krishan Mittal (Jee 2012, Air 199)

Illustration 20: The orthocentre of the triangle formed by the lines xy = 0 and x + y = 1 is **(JEE MAIN)**

(A) (1/2, 1/2) (B) (1/3, 1/3) (C) (0, 0) (D) (1/4, 1/4)

Sol: (C) Here the three lines are x = 0, y = 0 and x + y = 1.

Since the triangle formed by the line x = 0, y = 0 and x + y = 1 is right angled, the orthocentre lies at the vertex (0, 0), the point of intersection of the perpendicular lines x = 0 and y = 0.

Illustration 21: If θ is an angle between the lines given by the equation $6x^2+5xy-4y^2+7x+13y-3=0$ then equation of the line passing through the point of intersection of these lines and making an angle θ with the positive x-axis is **(JEE ADVANCED)**

(A) 2x + 11y + 13 = 0 (B) 11x – 2y + 13 = 0

(C) 2x – 11y + 2 = 0 (D) 11x + 2y – 11 = 0

Sol: (B) By taking the term y constant and using the formula of roots of quadratic equation, we can get the equation of two lines represented by the given equation and then by using $\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$, we will get the required result.

Writing the given equation as a quadratic in x, we have

$$6x^2+(5y+7)x-(4y^2-13y+3)=0 \Rightarrow x=\frac{-(5y+7)\pm\sqrt{(5y+7)^2+24(4y^2-13y+3)}}{12}$$

$$=\frac{-(5y+7)\pm\sqrt{121y^2-242y+121}}{12}=\frac{-(5y+7)\pm 11(y-1)}{12}=\frac{6y-18}{12}\text{ or }\frac{-16y+4}{12}$$

$\Rightarrow$ 2x – y + 3 = 0 and 3x + 4y – 1 = 0 are the two lines represented by the given equation and the point of intersection is (– 1, 1), obtained by solving these equations.

Also $\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$, where a = 6, b = – 4, h = 5/2 = $\frac{2\sqrt{(5/2)^2-6(-4)}}{6-4}=\sqrt{\frac{121}{4}}=\frac{11}{2}$

So the equation of the required line is $y-1=\frac{11}{2}(x+1) \Rightarrow$ 11x – 2y + 13 = 0

Illustration 22: If the equation of the pair of straight lines passing through the point (1, 1), and making an angle θ with the positive direction of x-axis and the other making the same angle with the positive direction of y-axis is $x^2-(a+2)xy+y^2+a(x+y-1)=0$, $a \neq -2$, then the value of $\sin 2\theta$ is **(JEE ADVANCED)**

(A) a – 2 (B) a + 2 (C) $\frac{2}{(a+2)}$ (D) $\frac{2}{a}$

Sol: (C) As both line passes through (1, 1) and one line makes angle θ with x-axis and other line with y–axis, slopes of line are $\tan\theta$ and $\cot\theta$

Equations of the given lines are $y-1=\tan\theta\,(x-1)$ and $y-1=\cot\theta\,(x-1)$

So, their combined equation is $[(y-1)-\tan\theta\,(x-1)]\,[(y-1)-\cot\theta\,(x-1)]=0$

$\Rightarrow (y-1)^2-(\tan\theta+\cot\theta)(x-1)(y-1)+(x-1)^2=0$

$\Rightarrow x^2-(\tan\theta+\cot\theta)\,xy+y^2+(\tan\theta+\cot\theta-2)(x+y-1)=0$

Comparing with the given equation we get $\tan\theta+\cot\theta=a+2$

$$\Rightarrow \frac{1}{\sin\theta\cos\theta}=a+2 \Rightarrow \sin 2\theta=\frac{2}{a+2}$$

Illustration 23: If two of the lines represented by $x^4 + x^3y + cx^2y^2 - xy^3 + y^4 = 0$ bisect the angle between the other two, then the value c is **(JEE ADVANCED)**

(A) 0 (B) –1 (C) 1 (D) – 6

Sol: (D) As the product of the slopes of the four lines represented by the given equation is 1 and a pair of line represents the bisectors of the angles between the other two, the product of the slopes of each pair is –1.

So let the equation of one pair be $ax^2 + 2hxy - ay^2 = 0$.

The equation of its bisectors is $\frac{x^2 - y^2}{2a} = \frac{xy}{h}$.

By hypothesis $x^4 + x^3y + cx^2y^2 - xy^3 + y^4$

$\equiv (ax^2 + 2hxy - ay^2)(hx^2 - 2axy - hy^2) = ah(x^4 + y^4) + 2(h^2 - a^2)(x^3y - xy^3) - 6ahx^2y^2$

Comparing the respective coefficients, we get $ah = 1$ and $c = -6ah = -6$

11. TRANSLATION AND ROTATION OF AXES

11.1 Translation of Axes

Let OX and OY be the original axes, and let the new axes, parallel to original axes, be O′X′ and O′Y′. Let the co-ordinates of the new origin O′ referred to the original axes be (h, k). If the point P has co-ordinates (x, y) and (x′, y′) with respect to original and new axes, respectively, then $x = x' + h$; $y = y' + k$

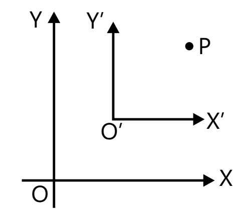


Figure 8.45

11.2 Rotation of Axes

Let OX and OY be the original system of axes and let OX′ and OY′ be the new system of axes and angle XOX′ = θ (the angle through which the axes are turned). If the point P has co-ordinates (x, y) and (x′, y′) with respect to original and new axes, respectively, then

$x = x' \cos\theta - y' \sin\theta$ and $y = x' \sin\theta + y' \cos\theta$

in matrix form it is as follows:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

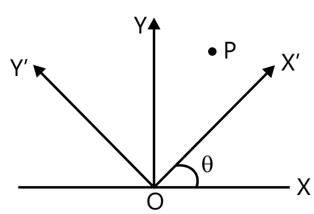


Figure 8.46

PLANCESS CONCEPTS

If origin is shifted to point (α, β) , then new equation of curve can be obtained by putting x + α in place of x and y + β in placed of y.

Vaibhav Krishnan (JEE 2009, AIR 22)

Illustration 24: The line L has intercepts a and b on the co-ordinate axes. The co-ordinate axes are rotated through a fixed angle, keeping the origin fixed. If p and q are the intercepts of the line L on the new axes, then

$\frac{1}{a^2} - \frac{1}{p^2} + \frac{1}{b^2} - \frac{1}{q^2}$ is equal to **(JEE MAIN)**

(A) –1 (B) 0 (C) 1 (D) None of these

Sol: (B) By using intercept form of equation of line, we will get equation of line before and after rotation. As their perpendicular length from the origin does not change, by using distance formula the result can be obtained.

Equation of the line L in the two co-ordinate system is $\frac{x}{a}+\frac{y}{b}=1, \frac{X}{p}+\frac{Y}{q}=1$ Where (x, y) are the new co-ordinates of a point (x, y) when the axes are rotated through a fixed angle, keeping the origin fixed. As the length of the perpendicular from the origin has not changed:

$$\frac{1}{\sqrt{(1/a^2)+(1/b^2)}}=\frac{1}{\sqrt{(1/p^2)+(1/q^2)}} \Rightarrow \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2} \Rightarrow \frac{1}{a^2}-\frac{1}{p^2}+\frac{1}{b^2}-\frac{1}{q^2}=0$$

Illustration 25: Let $0 < \alpha < \pi/2$ be a fixed angle. If P = $(\cos\theta, \sin\theta)$ and Q = $(\cos(\alpha-\theta), (\sin(\alpha-\theta))$. Then Q is obtained from P by **(JEE ADVANCED)**

(A) Clockwise rotation around origin through an angle α.

(B) Anti-clockwise rotation around origin through an angle α.

(C) Reflection in the line through the origin with slope tan α.

(D) Reflection in the line through the origin with slope tan $\alpha/2$.

Sol: As we know angle decreases during clockwise rotation and increases during anticlockwise rotation.

D Clockwise rotation of P through an angle α takes it to the point $(\cos(\theta-\alpha), \sin(\theta-\alpha))$ and anticlockwise takes it to $(\cos(\alpha+\theta), \sin(\alpha+\theta))$

Now slope of PQ $=\frac{\sin\theta-\sin(\alpha-\theta)}{\cos\theta-\cos(\alpha-\theta)}=\frac{2\cos(\alpha/2)\sin(\theta-\alpha/2)}{-2\sin(\alpha/2)-\sin(\theta-\alpha/2)}=-\cot(\alpha/2)$

$\Rightarrow$ PQ is perpendicular to the line with slope $\tan(\alpha/2)$. Hence, Q is the reflection of P in the line through the origin with slope $\tan\left(\frac{\alpha}{2}\right)$.

PLANCESS CONCEPTS

RELATION BETWEEN THE COEFFICIENT

Conditions for two lines to be coincident, parallel, perpendicular and intersecting: Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

- Coincident, if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$
- Parallel, if $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$
- Intersecting, if $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$
- Perpendicular, if $a_1a_2 + b_1b_2 = 0$

The three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are concurrent, if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$$

Aman Gour (JEE 2012, AIR 230)

PROBLEM-SOLVING TACTICS

(a) In most of the questions involving figures like triangle or any parallelogram, taking origin as (0,0) helps a lot in arriving at desired solution. One must ensure that conditions given are not violated.

(b) One must remember that in an isosceles triangle, centroid, orthocentre, incentre and circumcentre lie on the same line.

(c) The centroid, incentre, orthocentre and circumcentre coincide in an equilateral triangle.

(d) If area of the triangle is zero, then the three points are collinear.

(e) Find the equation of the line passing through two given points, if the third point satisfies the equation of the line, then three points are collinear

(f) Whenever origin is shifted to a new point (α, β), then new equation can be obtained by putting x + α in place of x and y + β in placed of y.

FORMULAE SHEET

(a) **Distance Formula:** The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

And between two polar co-ordinate $A(r_1, \theta_1)$ and $B(r_2, \theta_2)$ is $AB = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}$

(b) **Section Formula:** If $P(x_1, y_1)$, $Q(x_2, y_2)$ and the point R(x, y) divide the line PQ internally in the ratio m:n then the co-ordinates of R will be

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 - ny_1}{m-n}, \text{ i.e. } R\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

And if R is a mid-point of line PQ, then the co-ordinates of R will be $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

(c) **Centroid of Triangle:** If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of the triangle ABC and G is Centroid, then co-ordinate of G will be $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

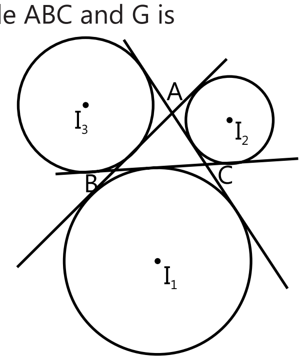


Figure 8.47

(d) **Co-ordinates of Incentre:** $x = \frac{ax_1 + bx_2 + cx_3}{a+b+c}$, $y = \frac{ay_1 + by_2 + cy_3}{a+b+c}$

(e) **Co-ordinates of Ex-centre:** As shown in figure, ex-centres of ΔABC with respect to vertices A, B and C are denoted by I_1, I_2 and I_3, respectively,

$$I_1 = \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c}\right); \quad I_2 = \left(\frac{ax_1 - bx_2 + cx_3}{a-b+c}, \frac{ay_1 - by_2 + cy_3}{a-b+c}\right),$$

$$I_3 = \left(\frac{ax_1 + bx_2 - cx_3}{a+b-c}, \frac{ay_1 + by_2 - cy_3}{a+b-c}\right)$$

(f) Co-ordinates of Circumcentre: If O is the circumcentre of any ΔABC, then its co-ordinates will be

$$O = \left(\frac{x_1 \sin 2A + x_2 \sin 2B + x_3 \sin 2C}{\Sigma \sin 2A}, \frac{y_1 \sin 2A + y_2 \sin 2B + y_3 \sin 2C}{\Sigma \sin 2A}\right)$$

(g) Co-ordinates of Orthocentre: If H is the orthocentre of any ΔABC, then its co-ordinates will be

$$H = \left(\frac{x_1 \tan A + x_2 \tan B + x_3 \tan C}{\Sigma \tan A}, \frac{y_1 \tan A + y_2 \tan B + y_3 \tan C}{\Sigma \tan A}\right)$$

(h) Slope of Line: Slope of line made by joining of points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by $m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$

(i) Angle between two Lines: $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

(j) Equation of a Line:

(i) Slope point form: $y - y_1 = m(x - x_1)$; **(ii) Two point form:** $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

(iii) Slope intercept form: $y = mx + c$; **(iv) Intercept form:** $\frac{x}{a} + \frac{y}{b} = 1$

(v) Normal form: $x\cos\alpha + y\sin\alpha = P$

(vi) Parametric form or distance form: $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$; and $x = x_1 + r\cos\theta$ and $y = y_1 + r\sin\theta$

(k) Length of Perpendicular: The perpendicular distance 'p' of a point $P(x_1, y_1)$ from the line $ax + by + c = 0$ is

$$p = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

(i) Distance between parallel lines: $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$

(ii) Lines making angle α with given line: $y - y' = \tan(\theta + \alpha)(x - x')$ and $y - y' = \tan(\theta - \alpha)(x - x')$

(iii) Concurrency of lines: The lines are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_2 & c_3 \end{vmatrix} = 0$

(l) Equation of bisector of the angle between two lines: $\frac{ax + by + c}{\sqrt{a^2 + b^2}} = \pm\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}}$ $(ab_1 \neq a_1b)$

(m) Pair of straight line:

(i) $ax^2 + 2hxy + by^2 = 0$ always represents a pair of straight lines passing through the origin and if

- $h^2 > ab \Rightarrow$ lines are real and distinct.
- $h^2 = ab \Rightarrow$ lines are coincident.
- $h^2 < ab \Rightarrow$ lines are imaginary with real point of intersection, i.e. (0, 0)

(ii) $m_1 + m_2 = -\frac{2h}{b}$ and $m_1 m_2 = \frac{a}{b}$

(iii) $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$

(n) General equation for pair of straight lines: $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines, if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

Solved Examples

JEE Main/Boards

Example 1: Find the ratio in which y – x + 2 = 0 divides the line joining A (3, – 1) and B (8, 9).

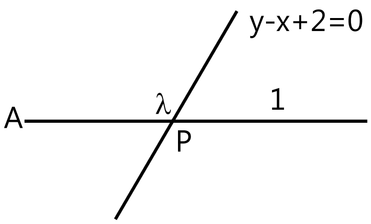
Sol: By considering the required ratio be λ:1, and using section formula, we can solve above problem.

The point of division P is internal as A and B lie on opposite sides of given line.

Let required ratio be λ:1.

Since, P $\left(\frac{8\lambda+3}{\lambda+1},\frac{9\lambda-1}{\lambda+1}\right)$ lies on y – x + 2 = 0,

$$\therefore\ \frac{9\lambda-1}{\lambda+1}-\frac{8\lambda+3}{\lambda+1}+2=0 \text{ or } \lambda=\frac{2}{3}$$



Hence, required ratio is 2/3:1 or 2:3

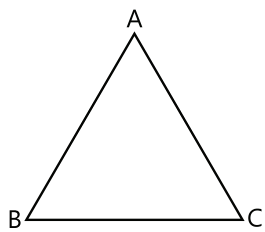
Example 2: Find the incentre I of ΔABC, if A is (4, – 2) B is (– 2, 4) and C is (5, 5).

Sol: Using $x=\frac{ax_1+bx_2+cx_3}{a+b+c}$, $y=\frac{ay_1+by_2+cy_3}{a+b+c}$, we can obtain the incentre.

$$a = BC = \sqrt{(5+2)^2+(5-4)^2}=5\sqrt{2}$$

$$b = CA = \sqrt{(5-4)^2+(5+2)^2}=5\sqrt{2}$$

$$c = AB = \sqrt{(-2-4)^2+(4+2)^2}=6\sqrt{2}$$



If incentre I is $(\overline{x},\overline{y})$, then,

$$\overline{x}=\frac{ax_1+bx_2+cx_3}{a+b+c}$$

$$=\frac{20\sqrt{2}-10\sqrt{2}+30\sqrt{2}}{5\sqrt{2}+5\sqrt{2}+6\sqrt{2}}=\frac{5}{2}$$

$$\overline{y}=\frac{ay_1+by_2+cy_3}{a+b+c}$$

$$=\frac{-10\sqrt{2}+20\sqrt{2}+30\sqrt{2}}{5\sqrt{2}+5\sqrt{2}+6\sqrt{2}}=\frac{5}{2}$$

Example 3: A rectangle PQRS has its side PQ parallel to the line y = mx and vertices P, Q, S lie on lines y = a, x = b and x = –b, respectively. Find the locus of the vertex R.

Sol: Here sides PQ and QR must be perpendicular to each other. Therefore product of their slopes will be –1.

Let R(h, k) be any point on the locus and let S and Q have co-ordinates (–b, β) and (b, α), respectively, as T is mid-point of SQ and PR.

Thus P has co-ordinates (–h, a) $\frac{\alpha-a}{b+h}=m$

⇒ α = a + m (b + h)

$-\frac{1}{m}$ = slope of QR = $\frac{\alpha-k}{b-h}$

⇒ α = k – $\frac{1}{m}(b-h)$

a + m(b + h) = k – $\frac{1}{m}(b-h)$

∴ locus of R is

x(m^2 – 1) 2 my + b + am + bm^2 = 0.

Example 4 Two equal sides AB and AC of an isosceles triangle ABC have equation 7x – y + 3 = 0 and x + y – 3 = 0, respectively. The third side BC of the triangle passes through point P(1, – 10). Find the equation of BC.

Sol: For isosceles triangle ABC, AD is perpendicular bisector of side BC and it also bisects angle BAC. Hence by using equation of bisector formula, i.e. $\frac{ax+by+c}{\sqrt{a^2+b^2}}=\pm\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}$,

we can obtain slope of AD.

Equations of AB and AC are 7x –y + 3 = 0 and – x – y + 3 = 0, respectively.

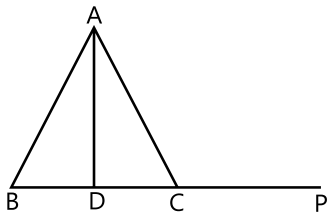
a_1 = 7; b_1 = – 1, c_1 = 3;

$a_2 = b_2 = -1, c_2 = 3$.

As $c_1 > 0, c_2 > 0$ and $a_1a_2 + b_1b_2 = -6 < 0$

Equation of the bisector of the acute angle BAD is

$$\frac{7x-y+3}{\sqrt{49+1}} = \frac{-x-y+3}{\sqrt{2}}$$, i.e. $3x + y = 3$



As slope of AD is –3, slope of BC is $\frac{1}{3}$

Equation of BC through P(1, –10) is

$y + 10 = \frac{1}{3}(x-1)$ or $x - 3y = 31$.

Example 5: Find the equation of the line passing through the intersection of lines $x - 3y + 1 = 0$, $2x + 5y - 9 = 0$ and whose distance from the origin is $\sqrt{5}$.

Sol: Equation of any line passing through the intersection of two other lines will be $L_1 + \lambda L_2 = 0$. Therefore, by using perpendicular distance formula of point to line, i.e

$$p = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$

we can obtain required equation of line.

Any line through the point of intersection of given lines is

$$x - 3y + 1 + \lambda(2x + 5y - 9) = 0$$

or $(1 + 2\lambda)x + (-3 + 5\lambda)y + 1 - 9\lambda = 0$

$$\sqrt{5} = \frac{|0+0+1-9\lambda|}{\sqrt{(1+2\lambda)^2+(-3+5\lambda)^2}}$$

Squaring and simplifying, we get $\lambda = \frac{7}{8}$.

Hence, required line has the equation $2x + y - 5 = 0$.

Example 6: Show that $bx^2 - 2hxy + ay^2 = 0$ represent a pair of straight lines which are at right angles to the pair of lines given by $ax^2 + 2hxy + by^2 = 0$.

Sol: Here if the product of slopes of a pair of straight lines represented by the given equations is –1, then they are right angle to each other.

Let $ax^2 + 2hxy + by^2$

$= (y - m_1x)(y - m_2x)$

Comparing the coefficients on both sides, we have

$m_1 + m_2 = -\frac{2h}{b}$; $m_1m_2 = \frac{a}{b}$ .....(i)

Now, $bx^2 - 2hxy + ay^2 = 0$

$\Rightarrow x^2 - \frac{2h}{b}xy + \frac{a}{b}y^2 = 0$

$\Rightarrow x^2 + (m_1 + m_2)xy + m_1m_2y^2 = 0$ (by (i))

$\Rightarrow (x + m_1y)(x + m_2y) = 0$

$\Rightarrow y = \frac{-1}{m_1}x$; $y = \frac{-1}{m_2}x$

Example 7: Find the angle ϕ between the straight lines $(x^2 + y^2)\sin^2\alpha = (x\cos\theta - y\sin\theta)^2$, where $0 < 2\alpha < \frac{\pi}{2}$.

Sol: We know $\tan\phi = \left|\frac{2\sqrt{h^2-ab}}{(a+b)}\right|$. Solving it, angle ϕ can be obtained.

$$x^2(\cos^2\theta - \sin^2\alpha) - 2xy\cos\theta\sin\theta + y^2(\sin^2\theta - \sin^2\alpha) = 0$$

$a = \cos^2\theta - \sin^2\alpha$, $2h = -2\cos\theta\sin\theta$,

$b = \sin^2\theta - \sin^2\alpha$

$$= \tan\phi = \left|\frac{2\sqrt{h^2-ab}}{(a+b)}\right|$$

$$= \frac{2\sqrt{\cos^2\theta\sin^2\theta - (\cos^2\theta - \sin^2\alpha)(\sin^2\theta - \sin^2\alpha)}}{|(\cos^2\theta - \sin^2\alpha) + \sin^2\theta - \sin^2\alpha|}$$

$$= \left|\frac{2\sin\alpha\cos\alpha}{\cos2\alpha}\right| = \left|\frac{\sin2\alpha}{\cos2\alpha}\right| = \tan2\alpha \therefore \phi = 2a$$

Example 8: The point A divides the line joining $P \equiv (-5, 1)$ and $Q = (3, 5)$ in the ratio k:1. Find the two values of k for which the area of ΔABC where $B \equiv (1, 5)$, $C \equiv (7, 2)$ is equal to two square units.

Sol: By using section formula, we can obtain the co-ordinates of point A and then values of k by using the triangle formula.

Co-ordinates of A, dividing the line joining points $P \equiv (-5, 1)$ and $Q \equiv (3, 5)$ in the ratio k:1, are given by $(3k-5/k+1, 5k+1/k+1)$. Also, area of the ΔABC

is given by $\Delta = \left|\frac{1}{2}\sum x_1(y_2 - y_3)\right|$

$$= \frac{1}{2}|[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]|$$

$$\left|\frac{1}{2}\left\{\frac{3k-5}{k+1}(7) + \left(-2 - \frac{5k+1}{k+1}\right) + 7\left(\frac{5k+1}{k+1} - 5\right)\right\}\right| = 2$$

$$\frac{3k-5}{k+1}(7)+\left(-2-\frac{5k+1}{k+1}\right)+7\left(\frac{5k+1}{k+1}-5\right)=\pm 4$$

$\Rightarrow$ $14k - 66 = 4k + 4, 10k = 70, k = 7$

or $14k - 66 = -4k - 4$, $18k = 62$,

$k = (31/9)$.

Therefore, value of the k = 7, 31/9

Example 9: Prove that the sum of the reciprocals of the intercepts made on the co-ordinate axes by any line not passing through the origin and through the point of intersection of the lines 2x + 3y = 6 and 3x + 2y = 6 is constant.

Sol: Equation of any line through the points of intersection of the given lines is $L_1 + \lambda L_2 = 0$.

$$2x+3y-6+k(3x+2y-6)=0$$

$$(2+3k)x+(3+2k)y-6(k+1)=0$$

$$\Rightarrow \frac{x}{\left((6(k+1))/(2+3k)\right)}+\frac{y}{\left((6(k+1))/(3+2k)\right)}=1$$

Where $k \neq -1$

and in this case, sum of the reciprocals of the intercepts made by this line on the co-ordinate axis is equal to

$$\frac{2+3k+3+2k}{6(k+1)}=\frac{5(k+1)}{6(k+1)}=\frac{5}{6}.$$

However, for k = – 1, the line become

x = y, which passes through the origin.

Example 10: Find the straight lines represented by $6x^2 + 13xy + 6y^2 + 8x + 7y + 2 = 0$ and also find their point of intersection.

Sol: Taking term y as a constant and using quadratic roots formula, i.e. $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, we can obtain equations of required straight lines and after that by solving them we will get their point of intersection.

Rewrite the given equation as

$$x=\frac{-(13y+8)\pm\sqrt{(13y+8)^2-24(6y^2+7y+2)}}{12}$$

$$=\frac{-(13y+8)\pm(5y+4)}{12}=\frac{-(2y+1)}{3},\frac{-(3y+2)}{2}$$

Hence, 3x + 2y + 1 = 0 and 2x + 3y + 2 = 0

are the required lines and they intersect at $\left(\frac{1}{5},-\frac{4}{5}\right)$.

JEE Advanced/Boards

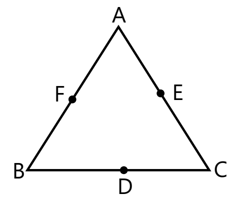
Example 1: If A(–1, 5), B(3,1) and C(5, 7) are vertices of a ΔABC and D, E, F are the mid-points of BC, CA and AB, respectively, then show that area ΔABC = 4 times area (ΔDEF).

Sol: Co-ordinates of D, E and F are first obtained by using mid-point formula, and prove the given equation by using formula of area of triangle.

Co-ordinates of D, E, F are (4, 4), (2, 6) and (1, 3), respectively.

$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} -1 & 5 & 1 \\ 3 & 1 & 1 \\ 5 & 7 & 1 \end{vmatrix} = 16$$

$$\text{Area of } \Delta DEF = \frac{1}{2}\begin{vmatrix} 4 & 4 & 1 \\ 2 & 6 & 1 \\ 1 & 3 & 1 \end{vmatrix} = 4$$



Hence, area of ΔABC = 4 area (ΔDEF)

Example 2: Point $P(a^2, a+1)$ is a point of the angle (which contains the origin) between the lines 3x – y + 1 = 0, x + 2y – 5 = 0. Find interval for values of 'a'.

Sol: Given origin and P lie on same side of each line. Substituting P in the given equation of line, we can obtain the required interval.

$a^2 + 2a + 2 - 5 < 0$ and $3a^2 - (a + 1) + 1 > 0$

i.e. $(a + 3)(a - 1) < 0$ and $a(3a - 1) > 0$

$\therefore$ $a \in (-3, 1)$ and $a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$

$\therefore$ $a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$

Example 3: Find the equations of the lines passing through P(2, 3) and making an intercept AB of length 2 units between the lines y + 2x = 3 and y + 2x = 5.

Sol: Using equation of line in parametric form, i.e. $x - x_1 = r\cos\theta$ and $y - y_1 = r\sin\theta$, we can obtain the required equation of line.

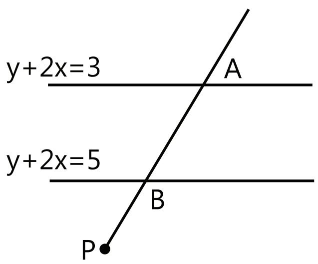
Let equation of the line, in parametric form, be $x - 2 = r\cos\theta$; $y - 3 = r\sin\theta$.

Then, A($2 + r_1\cos\theta$, $3 + r_1\sin\theta$) and B($2 + r_2\cos\theta$, $3 + r_2\sin\theta$) lie on y+2x=3 and y+2x=5, respectively.

$\therefore (3 + r_1\sin\theta) + 2(2 + r_1\cos\theta) = 3$ .... (i)

and $(3 + r_2\sin\theta) + 2(2 + r_2\cos\theta) = 5$ .... (ii)

$\therefore (r_2 - r_1)(\sin\theta + 2\cos\theta) = 2$



$\therefore \sin\theta + 2\cos\theta = \pm 1$ (as $|r_2 - r_1| = 2$)

$3\cos^2\theta + 4\sin\theta\cos\theta = 0$

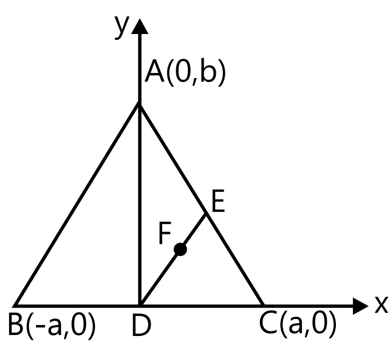
$\theta = \frac{\pi}{2}$ or $\tan\theta = -\frac{3}{4}$ $\therefore$ Required lines are x = 2 and 4y + 3x = 18

Example 4: In a triangle ABC, AB = AC. If D is mid-point of BC, E is the foot of perpendicular from D on AC, and F is the mid-point of DE, show that AF is perpendicular to BE.

Sol: As the geometrical fact to be established does not depend on position of ABC, we may assume that ''D is the origin; BC and AD are along x and y axes respectively (as shown)''. Therefore by using intercept form of equation of line, we can obtain required result.

Let BD = DC = a, and A and E have co-ordinates (0, b) and (h, k), respectively.

Line AC has the equation $\frac{x}{a} + \frac{y}{b} = 1$



$\therefore \frac{h}{a} + \frac{k}{b} = 1$ ....(i)

Also, $(k/h)(-b/a) = -1$

($\because$ AC $\perp$ DE) ....(ii)

By (i) and (ii) $h = \frac{ab^2}{a^2 + b^2}$, $k = \frac{a^2 b}{a^2 + b^2}$ F is $\left(\frac{h}{2}, \frac{k}{2}\right)$

Slope of BE = $\frac{ab}{a^2 + 2b^2}$.

Slope of AF = $\frac{a^2 + 2b^2}{-ab}$

Product of slopes of BE and AF is equal to (–1). Hence AF $\perp$ BE.

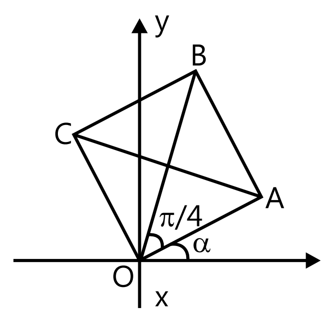
Example 5: A square lying above the x-axis and has one vertex at the origin. A side passing through the origin makes an angle $\alpha\left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of x-axis. Prove that the equations of its diagonals are $y(\cos\alpha - \sin\alpha) = x(\sin\alpha + \cos\alpha)$ and $y(\sin\alpha + \cos\alpha) + x(\cos\alpha - \sin\alpha) = 0$, where a is the length of a side of the square.

Sol: Using slope point form of equation of line, i.e. $y - y_1 = m(x - x_1)$, we can obtain the result. Here m = tan θ and x_1, y_1 is 0.

Equation of diagonal OB is $y = \tan\left(\alpha + \frac{\pi}{4}\right)x$

or $y\cos\left(\alpha + \frac{\pi}{4}\right) = x\sin\left(\alpha + \frac{\pi}{4}\right)$

or $y(\cos\alpha - \sin\alpha) = x(\cos\alpha + \sin\alpha)$ ....(i)



From the figure, point A is ($a\cos\alpha$, $a\sin\alpha$).

As diagonal AC is perpendicular to diagonal OB, equation of AC is

$Y - a\sin\alpha = \frac{\cos\alpha - \sin\alpha}{\cos\alpha + \sin\alpha}(x - a\cos\alpha)$

or $x(\cos\alpha - \sin\alpha) + y(\cos\alpha + \sin\alpha) = a$

Example 6: Two sides of a rhombus lying in the first quadrant are given by $3x - 4y = 0$ and $12x - 5y = 0$. If the length of the longer diagonal is 12 units, find the equations of the other two sides of the rhombus.

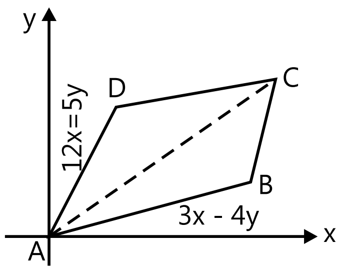
Sol: Using formula of equation of bisector of the angle, we can obtain equation of AC. Given the length AC, we can obtain co-ordinates of C.

Let ABCD be the rhombus with AC as the longer diagonal, where A has co-ordinates (0, 0). AC is bisector of angle ΔBAD. The equations of the two angle bisectors (of angles formed by given lines) are:

$\frac{3x-4y}{5} = \pm\frac{12x-5y}{13}$ or $21x + 27y = 0$ and $99x - 77y = 0$.

Since diagonal AC has positive slope, its equation is

$\frac{x}{7} = \frac{y}{9}$



In parametric form, we get

$\frac{x}{7/\sqrt{130}} = \frac{y}{9/\sqrt{130}}$; Since AC = 12 units,

C has co-ordinates $\left(\frac{7.12}{\sqrt{130}}, \frac{9.12}{\sqrt{130}}\right)$.

Let sides DC and BC have equations $3x - 4y = a$ and $12x - 5y = b$, respectively. Substituting co-ordinates of C in these equations yields

$$a = \frac{3.84}{\sqrt{130}} - \frac{4.108}{\sqrt{130}} = \frac{-180}{\sqrt{130}};$$

$$b = \frac{12.84}{\sqrt{130}} - \frac{5.108}{\sqrt{130}} = \frac{468}{\sqrt{130}}$$

Hence, equation of sides DC and BC are $3x - 4y + \frac{180}{\sqrt{130}} = 0$ and $12x - 5y = \frac{468}{\sqrt{130}}$, respectively.

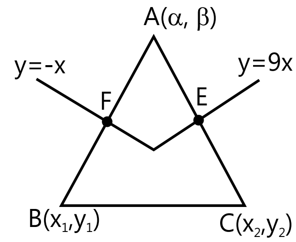
Example 7: The base of a triangle ABC passes through a fixed point (f, g) and its other two sides are bisected at right angles by the lines $y^2 - 8xy - 9x^2 = 0$. Find the locus of the vertex

Sol: Let vertices be $A(\alpha, \beta)$, $B(x_1, y_1)$ and $C(x_2, y_2)$ and let (f, g) lie on BC. The mid-point of side AB and AC must be lies on two perpendicular lines represented by $y^2 - 8xy - 9x^2 = 0$, respectively. Hence by solving them we will get locus of the vertex.

Let $y^2 - 8xy - 9x^2 = 0$

$\Rightarrow \quad (y - 9x)(y + x) = 0$

$\Rightarrow \quad y - 9x = 0$ or $y + x = 0$



The mid-point E of AC lies on

$y - 9x = 0$

$\beta + y_2 - 9(\alpha + x_2) = 0$ ....(i)

Since AC is perpendicular to $y - 9x = 0$,

We have $\frac{y_2-\beta}{x_2-\alpha}(9) = -1$ Rewrite (i) and (ii) as $9x_2 - y_2 = \beta - 9a$

$x_2 + 9y_2 = \alpha + 9\beta$

$\therefore x2 = \frac{9\beta - 40\alpha}{41}$ $y_2 = \frac{9\alpha + 40\beta}{41}$

Similarly, $F\left(\frac{\alpha + x_1}{2}, \frac{\beta + y_1}{2}\right)$, the mid-point of AB lies on $y + x = 0$.

$\therefore \alpha + x_1 + \beta + y_1 = 0$

or $x_1 + y_1 = -(\alpha + \beta)$ .....(iii)

Since, AB is perpendicular to $y + x = 0$,

We have,

$\frac{\beta - y_1}{\alpha - x_1}(-1) = -1$ or $\beta - y_1 = \alpha - x_1$

or $x_1 - y_1 = \alpha - \beta$ ....(iv)

Solving (iii) and (iv), we have

$x_1 = -\beta$ and $y_1 = -a$

As $B(x_1, y_1)$, $C(x_2, y_2)$ and D(f, g) are collinear we have,

$$\begin{vmatrix} f & g & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

or $\quad f(y_1 - y_2) - g(x_1 - x_2) + (x_1y_2 - x_2y_1) = 0$

or $\quad f\left[-\alpha - \frac{9\alpha + 40\beta}{41}\right] - g\left[-\beta - \frac{9\beta - 40\alpha}{41}\right]$

$$-\beta\frac{9\alpha + 40\beta}{41} + \alpha\frac{9\beta - 40\alpha}{41} = 0$$

or $f(50\alpha + 40\beta) + g(40\alpha - 50\beta) + 40(\alpha^2 + \beta^2) = 0$ Locus of (α, β) is $4(x^2 + y^2) + (4g + 5f)x + (4f - 5g)y = 0$

Example 8: If the vertices of a triangle have integral co-ordinates, prove that the triangle cannot be equilateral.

Sol: Obtaining the area of triangle using Δ = (1/2) bc sin A and using the co-ordinate form, we can conclude that the triangle cannot be equilateral if vertices have integral co-ordinates.

Consider a triangle ABC with vertices $A \equiv (x_1, y_1)$, $B \equiv (x_2, y_2)$, $C \equiv (x_3, y_3)$. Let $x_1, x_2, x_3, y_1, y_2, y_3$ be the integers.

$BC^2 = (x_2 - x_3)^2 + (y_2 - y_3)^2$ a positive integer.

If the triangle is equilateral, then AB = BC = CA = a and $\angle A = \angle B = \angle C = 60°$.

Area of the triangle = (1/2) bc sin A = (1/2) a^2 sin 60°

$= (a^2/2).(\sqrt{3}/2) = (\sqrt{3}/4)a^2$ which is irrational.

$\because$ a^2 is a positive integer.

Now, the area of the triangle in terms of the co-ordinates

$= (1/2)\,[(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

which is rational number.

This contradicts that the area is an irrational number, if the triangle is equilateral.

Example 9: A line L intersects the three sides BC, CA and AB of a triangle ABC at P, Q and R, respectively.

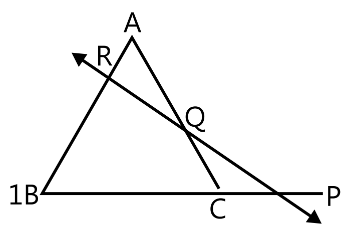
Show that $\left(\frac{BP}{PC}\right)\left(\frac{CQ}{QA}\right)\left(\frac{AR}{RB}\right) = -1$.

Sol: Using equation of line lx + my + n = 0 and section formula, we can prove the given equation.

Consider a triangle ABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, and let lx + my + n = 0 be equation of the line L. If P divides BC in the ratio λ:1, then the co-ordinates of P are $\left(\frac{\lambda x_3 + x_2}{\lambda + 1}, \frac{\lambda y_3 + y_2}{\lambda + 1}\right)$

Also, as P lies on L, we have

$$l\left(\frac{\lambda x_3 + x_2}{\lambda + 1}\right) + m\left(\frac{\lambda y_3 + y_2}{\lambda + 1}\right) + n = 0$$



$$\Rightarrow \quad -\frac{lx_2 + my_2 + n}{lx_3 + my_3 + n} = \lambda = \frac{BP}{PC} \qquad \text{.... (i)}$$

Similarly, we obtain

$$\frac{CQ}{QA} = -\frac{lx_3 + my_3 + n}{lx_1 + my_1 + n} \qquad \text{.... (ii)}$$

and $$\frac{AR}{RB} = -\frac{lx_1 + my_1 + n}{lx_2 + my_2 + n} \qquad \text{.... (iii)}$$

Multiplying (i), (ii) and (iii), we get the required result.

Example 10: The circumcentre of a triangle having vertices A = (a, a tan α), B = (b, b tan β), C = (c, c tan γ) is the origin, where $\alpha + \beta + \gamma = \pi$. Show that the orthocentre lies on the line.

$$\left(4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}\right)x - 4\left(\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}\right)y - y = 0$$

Sol: Consider the circumcentre 'O' to be the origin and the equation of the circumcircle be $x^2 + y^2 = r^2$. As vertices of triangle lies on this circle, we can obtain the co-ordinates of centroid by using the respective formula.

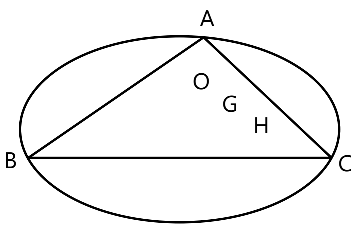
Since vertex A(a, a tan α) is r distance from the circumcenter.

Therefore $a^2(1 + \tan^2\alpha) = r^2 \Rightarrow a = r\cos\alpha$

A = (r cos α, r sin α)

Similarly B = (r cos β, r sin β)

C = (r cos γ, r sin γ)



Centroid G

$$\left(\frac{r(\cos\alpha + \cos\beta + \cos\gamma)}{3}, \frac{r(\sin\alpha + \sin\beta + \sin\gamma)}{3}\right)$$

Circumcentre O' (0, 0) and let orthocentre H (h, k). We know that O, G, H are collinear. Therefore slope of OG = slope of OH

i.e. $\frac{\sin\alpha + \sin\beta + \sin\gamma}{\cos\alpha + \cos\beta + \cos\gamma} = \frac{k}{h}$,

Point (h, k) will be on

$x(\sin\alpha + \sin\beta + \sin\gamma) - y(\cos\alpha + \cos\beta + \cos\gamma) = 0$

$$\Rightarrow x\left(4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}\right) - y\left(1 + 4\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}\right) = 0$$

And hence the result is $\alpha + \beta + \gamma = \pi$.

Example 11: ABC is a variable triangle with the fixed vertex C(1, 2) and vertices A and B with co-ordinates (cos t, sin t) and (sin t, – cos t),

Respectively, where t is a parameter. Find the

locus of the centroid of the ΔABC.

Sol: We can obtain co-ordinates of centroid $G(\alpha, \beta)$ using the formula $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$ and we will get required equation of locus of centroid by solving them simultaneously.

Let $G(\alpha, \beta)$ be the centroid in any position .Then $G(\alpha, \beta)$

$$= \left(\frac{1 + \cos t + \sin t}{3} - \frac{2 + \sin t - \cos t}{3}\right) \text{ or}$$

$$\therefore\ \alpha = \frac{1 + \cos t + \sin t}{3}, \beta = \frac{2 - \sin t - \cos t}{3}$$

$\therefore$ or $3\alpha - 1 = \cos t + \sin t$ .... (i)

$3\beta = 2 - \sin t - \cos t$ .... (ii)

Squaring and adding equations (i) and (ii), we get

$(3\alpha - 1)^2 + (3\beta - 2)^2$

$= (\cos t + \sin t)^2 + (\sin t - \cos t)^2$

$= 2(\cos^2 t + \sin^2 t) = 2$

$\therefore$ the equation of the locus of the centroid is $(3x - 1)^2 + (3y - 2)^2 = 2$

$9(x^2 + y^2) - 6x - 12y + 3 = 0$

$\therefore\quad 3(x^2 + y^2) - 2x - 4y + 1 = 0$

Example 12: Find equations of the sides of the triangle having (3, –1) as a vertex, $x - 4y + 10 = 0$ and $6x + 10y - 59 = 0$ being the equations of an angle bisector and a median, respectively, drawn from different vertices.

Sol: Consider the vertices of the triangle to be A(3, – 1), $B(x_1, y_1)$ and $C(x_2, y_2)$. Here the mid-point of AC lies on the median through B.

Equation of the median through B be $6x + 10y - 59 = 0$ and the equation of the angle bisector from C be

$x - 4y + 10 = 0$; $x_2 - 4y_2 + 10 = 0$ ....(i)

Also $D\left(\frac{x_2 + 3}{2}, \frac{y_2 - 1}{2}\right)$, the mid-point of AC lies on the median through B,

i.e. $6x + 10y - 59 = 0$

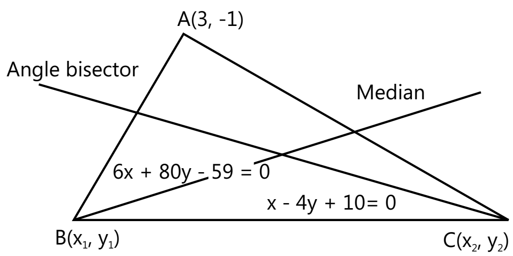
$$\Rightarrow 6\left(\frac{x_2 + 3}{2}\right) + 10\left(\frac{y_2 - 1}{2}\right) - 59 = 0$$

$\Rightarrow 3x_2 + 5y_2 - 55 = 0$ ....(ii)

Solving (i) and (ii), we get $x_2 = 10$, $y_2 = 5$,

i.e. the co-ordinates of C are (10, 5) and thus

the equation of AC is $6x - 7y = 25$



Let the slope of BC be m_1. Since BC and AC are equally inclined to the angle bisector

$x - 4y + 10 = 0$,

$$\frac{(1/4) - m}{1 + (1/4)m} = \frac{(6/7) - (1/4)}{1 + (6/7) \times (1/4)} \Rightarrow \frac{1 - 4m}{4 + m} = \frac{17}{34}$$

$\Rightarrow m = -\frac{2}{9}$ Equation of BC is

$y - 5 = -\frac{2}{9}(x - 10)$ and $6x_1 + 10y_1 = 59$

Solving these equations, we get $X_1 = -7/2$, $y_1 = 8$

Hence, equation of AB is $y + 1 = \frac{8 + 1}{-7/2 - 3}(x - 3)$

Example 13: A triangle has the lines $y = m_1x$ and $y = m_2x$ for two of its sides, with m_1 and m_2 being roots of the equation $bx^2 + 2hx + a = 0$. If H(a, b) is the orthocentre of the triangle, show that the equation of the third side is $(a + b)(ax + by) = ab(a + b - 2h)$.

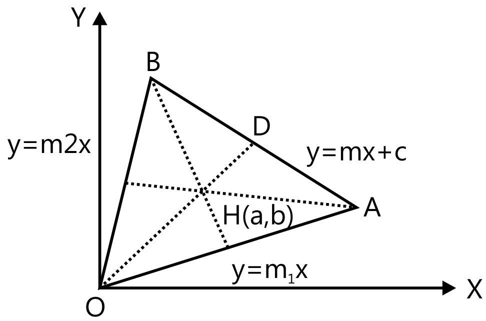
Sol: Line OD passes from orthocentre. Therefore it must be perpendicular to the side AB. By considering equation of AB as $y = mx + c$, we will get co-ordinates of A and B. Using slope point form of equation of line, we can solve the problem.

The given lines $y = m_1x$ and $y = m_2x$ intersect at the origin O (0, 0). Thus one vertex of the triangle is at the origin O. Therefore, let OAB be the triangle and OA and

OB be the lines

$y = m_1x$ ....(i)

and $y = m_2x$ ....(ii)



Let the equation of the third side AB be

$y = mx + c$ .....(iii)

Given that H(a, b) is the orthocentre of the OAB,

$\therefore$ OH $\perp$ AB

$\Rightarrow (b/a) \times m = -1 \Rightarrow m = -a/b$ ....(iv)

Solving (iii) with (i) and (ii), the co-ordinates of

$A = \left(\frac{c}{m_1 - m}, \frac{cm_1}{m_1 - m}\right)$ and

$B = \left(\frac{c}{m_2 - m}, \frac{cm_2}{m_2 - m}\right)$

Now equation of line through A

perpendicular to OB is

$y - \frac{cm_1}{m_1 - m} = -\frac{1}{m_2}\left(x - \frac{c}{m_1 - m}\right)$ or

$y = -\frac{x}{m_2} + \frac{c(m_1m_2 + 1)}{m_2(m_1 - m)}$ ....(v)

Similarly, equation of line through B

perpendicular to OA is

$y = -\frac{x}{m_1} + \frac{c(m_1 m_2 + 1)}{m_1(m_2 - m)}$ ....(vi)

The point of intersection of (v) and (vi) is the orthocentre H (a, b).

$\therefore$ Subtracting (vi) from (v), we get

$x = a = \frac{-cm(m_1m_2 + 1)}{(m_1 - m)(m_2 - m)}$

or $c = \frac{-[m_1m_2 - m(m_1 + m_2) + m^2]a}{m(m_1m_2 + 1)}$ .....(vii)

since m_1 and m_2 are the roots of the equation

$bx^2 + 2hx + a = 0$

$m_1 + m_2 = -2h/b$ and $m_1m_2 = a/b$

From (vii), we have

$c = \frac{-[a/b + 2hm/b + m^2]a}{m(a/b + 1)} = \frac{-[a + 2hm + bm^2]a}{m(a + b)}$

From (iii), the equation of third side AB is

$y = mx - \frac{(a + 2hm + bm^2)a}{m_1(a + b)}$

or $y = -\frac{a}{b}x - \frac{(a - 2ha/b + ba^2/b^2)a}{(-a/b)(a + b)}$

or $(ax + by)(a + b) = ab(a + b - 2h)$

Example 14: Find the co-ordinates of the centroid, circumcentre and orthocentres of the triangle formed by the lines $3x - 2y = 6$, $3x + 4y + 12 = 0$ and $3x - 8y + 12 = 0$.

Sol: Solving the given equations, we can obtain the co-ordinates of vertices of triangle. Using appropriate formula for finding the co-ordinates of centroid, circumcentre and orthocentre, the problem can be solved.

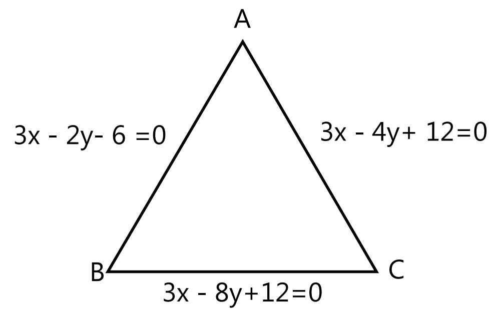
Let sides AB, BC and CA have the

equations $3x - 2y - 6 = 0$ ....(i)

$3x - 8y + 12 = 0$ ....(ii)

$3x + 4y + 12 = 0$ ....(iii)

Solving (ii), (iii) we get $y = 0$, $x = -4$,



C = (– 4, 0)

Solving (i), (ii) we get y = 3, x = 4

B = (4, 3)

Solving (i,) (iii) we get y = – 3, x = 0;

A = (0 – 3)

Centroid $G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

where vertices are (x, y), etc.

$$G = \left(\frac{0+4-4}{3}, \frac{-3+3+0}{3}\right) = (0,0)$$

To find the circumcentre:

Let M(α, β) be the circumcentre.

MA = MB = MC

$(\alpha - 0)^2 + (\beta + 3)^2 = (\alpha - 4)^2 + (\beta - 3)^2 = (\alpha + 4)^2 + (\beta - 0)^2$

$\alpha^2 + \beta^2 + 6\beta + 9$

$= \alpha^2 + \beta^2 - 8\alpha - 6\beta + 25 = \alpha^2 + \beta^2 - 8\alpha + 16$

$= 6\beta + 9 = -8\alpha - 6\beta + 25 = 8\alpha + 16$

$6\beta + 9 = -8\alpha - 6\beta + 25$ and $6\beta + 9 = 8\alpha + 16$

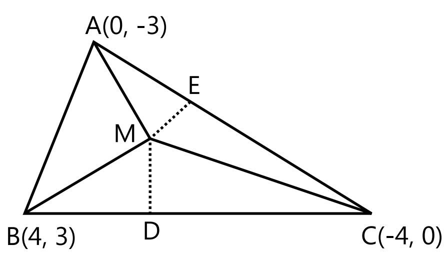
$8\alpha + 12\beta - 16 = 0$

$2\alpha + 3\beta - 4 = 0$ .... (i)

$8\alpha - 6\beta + 7 = 0$ .... (ii)

Solving (i) and (ii), we get $\alpha = \frac{1}{12}, \beta = \frac{23}{18}$

Circumcentre $= \left(\frac{1}{12}, \frac{23}{18}\right)$



Use $= \frac{MG}{GH} = \frac{1}{2}$

Let H(α, β) be the orthocentre

$$0 = \frac{\alpha + 2\frac{1}{12}}{3} \Rightarrow \alpha = -\frac{1}{6} \quad .... \text{(i)}$$

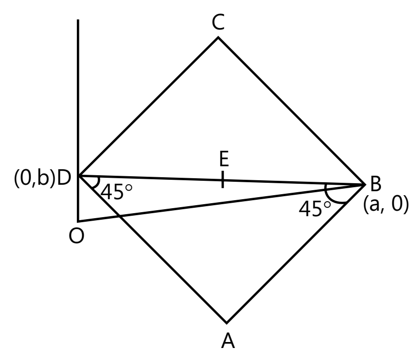
$$0 = \frac{\beta + 2\frac{23}{18}}{3} \Rightarrow \beta = -\frac{23}{9} \text{ Then H } \left(\frac{-1}{6}, \frac{-23}{9}\right).$$

Example 15: One diagonal of a square is the position of the line $\tan(\pm 45°) = \frac{m + \frac{b}{a}}{1 - m\frac{b}{a}} = \pm 1$ which is intercepted between the axes. Find the co-ordinates of other two vertices of the square. Also prove that if two opposite vertices of a square move on two perpendicular lines, the other two vertices also move on two perpendicular lines.

Sol: Using $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$, we can obtain the slopes of AB and AD. As the slope of the given lines is –b/a, the two vertices are clearly on the diagonal BD of the square ABCD.

If m be the slope of the line inclined at an angle of 45° to BD,

$$\tan(\pm 45°) = \frac{m + (b/a)}{1 - m(b/a)} = \pm 1$$



$m = \frac{a-b}{a+b}$ or $-\frac{(a+b)}{a-b}$ AB is $y - 0 = \frac{a-b}{a+b}(x - a)$

AD is $y - b = -\frac{a+b}{a-b}(x - 0)$

By solving these equation we get

The point A is $\left(\frac{a-b}{2}, \frac{b-a}{2}\right)$. C is obtained by using the fact that mid-point of AC and BD is same.

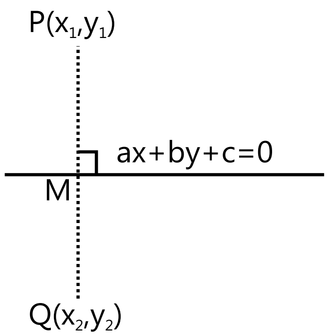
$$C = \left(\frac{a+b}{2}, \frac{a+b}{2}\right)$$

The opposite vertices B, D move on two perpendicular lines x-axis and y-axis. Now the point $A\left(\frac{a-b}{2}, \frac{b-a}{2}\right)$ lies on y = –x and point $C\left(\frac{a+b}{2}, \frac{a+b}{2}\right)$ lies on y = x.

Example 16: If the image of the point (x_1, y_1) with respect to the mirror ax + by + c = 0 be (x_2, y_2), show that $\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$

Sol: As the line PQ joining the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is perpendicular to the line ax + by +c = 0. Also the mid-point M of PQ is on the lines ax + by +c= 0. Hence product of their slopes will be –1 and co-ordinates of M lies on ax + by +c= 0.

$$\frac{y_1 - y_2}{x_1 - x_2}\left(-\frac{a}{b}\right) = -1 \quad .....(i)$$



and $$a.\frac{x_1 + x_2}{2} + b.\frac{y_1 + y_2}{2} + c = 0 \quad ....(ii)$$

From (ii), $a(x_1 + x_2) + b(y_1 + y_2) + 2c = 0$

or $(ax_1 + by_1 + c) + (ax_1 + by_1 + c) = 0$ ....(iii)

From (i), $\frac{x_1 - x_2}{a} = \frac{y_1 - y_2}{b} = \frac{a(x_1 - x_2) + b(y_1 - y_2)}{a^2 + b^2}$

By ratio and proportion

$$= \frac{(ax_1 + by_1 + c) - (ax_2 + by_2 + c)}{a^2 + b^2}$$

$$= \frac{2(ax_1 + by_1 + c)}{a^2 + b^2}, \text{ using (iii)}$$

$$\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$$

JEE Main/Boards

Exercise 1

Q.1 Find the slope of the line joining (4, – 6) and (– 2, – 5).

Q.2 Show that the line joining (2, – 3) and (– 5, 1) is (i) Parallel to the line joining (7, – 1) and (0, 3), (ii) Perpendicular to the line joining (4, 5) and (0, – 2).

Q.3 A quadrilateral has the vertices at the points (– 4, 2), (2, 6), (8, 5) and (9, – 7). Show that the mid-points of the sides of this quadrilateral are vertices of a parallelogram.

Q.4 Find the values of x and y for which A(2, 0), B(0, 2), C(0, 7) and D(x, y) are the vertices of an isosceles trapezium in which AB ||DC.

Q.5 Find the equations of the diagonals of the rectangle, whose sides are x = a, x = a′, y = b and y= b′.

Q.6 In what ratio is the line joining the points (2, 3) and (4, –5) divided by the line joining the points (6, 8) and (– 3, – 2) ?

Q.7 Find the coordinates of the vertices of a square inscribed in the triangle with vertices A(0, 0), B(2, 1) and C(3, 0); Given the two vertices are on the side AC.

Q.8 Find the equation of the straight line which passes through the origin and trisects the intercept of line 3x + 4y = 12 between the axes.

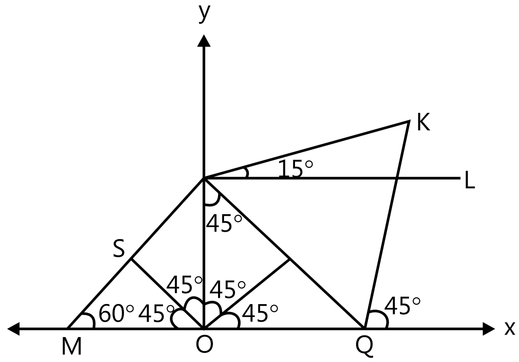
Q.9 A straight line passes through the point (3, – 2). Find the locus of the middle point of the portion of the line intercepted between the axes.

Q.10 Find the equation of the straight line which passes through the point (3, 2) and whose gradient is 3/4. Find the coordinates of the point on the line that are 5 units away from the point (3, 2).

Q.11 Find the distance of the point (2, 5) the lines 3x +y + 4 = 0 measured parallel to line having slope 3/4.

Q.12 The extremities of a diagonal of a square are (1, 1), (– 2, – 1). Obtain the other two vertices and the equation of the other diagonal.

Q.13 In the given figure, PQR is an equilateral triangle and OSPT is a square. If OT $=2\sqrt{2}$ units, find the equation of the lines OT, OS, SP, OR, PR and PQ.



Q.14 Find the equation of the medians of a triangle formed by the lines $x + y - 6 = 0$, $x - 3y - 2 = 0$ and $5x - 3y + 2 = 0$.

Q.15 Find the coordinates of the orthocentre of the triangle whose vertices are (0, 0), 2, – 1) and (–1, 3).

Q.16 Two vertices of a triangle are (3, – 1) and (–2, 3) and its orthocentre is origin. Find the coordinates of the third vertix.

Q.17 If the lines $3x + y - 2 = 0$, $px + 2y - 3 = 0$ and $2x - y - 3 = 0$ are concurrent, find the value of p.

Q.18 Find the angle between the lines $y - 3x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$.

Q.19 Prove that the points (2, – 1), (0, 2), (3, 3) and (5, 0) are the vertices of a parallelogram. Also find the angle between its diagonals.

Q.20 A and B are the points (–2, 0) and (0, 5). Find the Coordinates of two points C and D such that ABCD is a square.

Q.21 Find the equations of the lines through the point (3, 2), which make an angle of 45° with the line $x - 2y = 3$.

Q.22 A line $4x + y = 1$ through the point A(2, – 7) meets the line BC whose equation is $3x - 4y + 1 = 0$ at the point B. Find the equation of the line AC, so that AB = AC.

Q.23 A ray of light is sent along the line $x - 2y - 3 = 0$. Upon reaching the line $3x - 2y - 5 = 0$, the ray is reflected from it. Find the equation of the line containing the reflected ray.

Q.24 Find the image of the point (– 8, 12) with respect to the line mirror $4x + 7y + 13 = 0$.

Q.25 The equations of two sides of a triangle are $3x - 2y + 6 = 0$ and $4x + 5y = 20$ and the orthocentre is (1, 1). Find the equation of the third side.

Q.26 Find the equations of the straight lines passing through the point of intersection of the lines $x + 3y + 4 = 0$ and $3x + y + 4 = 0$ and equally inclined to the axis.

Q.27 Show that the straight lines $x(a + 2b) + y(a + 3b) = a + b$, for different values of a and b pass through a fixed point.

Q.28 The equation of the base of an equilateral triangle is $x + y = 2$ and the vertex is (2, – 1). Find the length of the side of the triangle.

Exercise 2

Single Correct Choice Type

Q.1 The pair of points which lie on the same side of the straight line, $3x - 8y - 7 = 0$ is

(A) (0, – 1), (0, 0) (B) (0, 1), (3, 0)

(C) (– 1, – 1), (3, 7) (D) (– 4, –3), (1, 1)

Q.2 Equation of the bisector of the acute angle between the lines, $3x - 4y + 7 = 0$ and $12x + 5y - 2 = 0$ is

(A) $11x - 3y + 9 = 0$ (B) $11x + 3y - 9 = 0$

(C) $3x - 11y + 9 = 0$ (D) None

Q.3 A ray of light passing through the point A (1, 2) is reflected at a point B on the x-axis and then passes through (5, 3). Then the equation of AB is

(A) $5x + 4y = 13$ (B) $5x - 4y = -3$

(C) $4x + 5y = 14$ (D) $4x - 5y = -6$

Q.4 The line $x + 3y - 2 = 0$ bisects the angle between a pair of straight lines of which one has equation $x - 7y + 5 = 0$. The equation of the other line is

(A) $3x + 3y - 1 = 0$ (B) $x - 3y + 2 = 0$

(C) $5x + 5y - 3 = 0$ (D) None

Q.5 A is point (3, – 5) with respect to a given system of axes. If the origin is moved to (4, – 3) by a translation of axes, then the new co-ordinates of the point A are given by

(A) (1, – 2) (B) (– 1, 2)

(C) (– 1, – 2) (D) None of these

Q.6 The set of lines given by $\frac{x}{a}+\frac{y}{b}=1$, where $\frac{3}{a}+\frac{4}{b}=5$ are concurrent at a fixed point, then point is -

(A) $\left(\frac{3}{5}, \frac{4}{5}\right)$ (B) (0, b)

(C) (a, 0) (D) None

Q.7 If P = (1, 0); Q = (– 1, 0) & R= (2, 0) are three given points, then the locus of the points S satisfying the relation, $SQ^2 + SR^2 = 2\,SP^2$ is

(A) A straight line parallel to x-axis.

(B) A circle passing through the origin.

(C) A circle with the centre at the origin.

(D) A straight line parallel to y-axis.

Q.8 Area of the rhombus bonded by the four lines, $ax \pm by \pm c = 0$ is:

(A) $\frac{c^2}{2ab}$ (B) $\frac{2c^2}{ab}$

(C) $\frac{4c^2}{ab}$ (D) $\frac{ab}{4c^2}$

Q.9 If the sum of the distances of a point from two perpendicular lines in a plane is 1, then its locus is

(A) A square (B) Circle

(C) A straight line (D) Two intersecting lines

Q.10 If the straight line $x + 2y = 9$, $3x - 5y = 5$ & $ax + by = 1$ are concurrent, then the straight line $5x + 2y = 1$ passes through the point

(A) (a, – b) (B) (– a, b)

(C) (a. b) (D) (–a, –b)

Q.11 The straight line, $ax + by = 1$, makes with the curve $px^2 + 2axy + qy^2 = r$ a chord which subtends a right angle at the origin. Then:

(A) $r(a^2 + b^2) = p + q$ (B) $r(a^2 + p^2) = q + b$

(C) $r(b^2 + q^2) = p + a$ (D) None

Q.12 The lines $y - y_1 = m(x - x_1) \pm a\sqrt{1+m^2}$ are tangents to the same circle. The radius of the circle is:

(A) a/2 (B) a (C) 2a (D) None

Q.13 The equation of the pair of bisectors of the angles between two straight lines is, $12x^2 - 7xy - 12y^2 = 0$. If the equation of one line is $2y - x = 0$, then the equation of the other line is

(A) $41x - 38y = 0$ (B) $38x - 41y = 0$

(C) $38x + 41y = 0$ (D) $41x + 38y = 0$

Q.14 If the point B is symmetric to the point A(4, – 1) with respect to the bisector of the first quadrant, then the length AB is

(A) $3\sqrt{2}$ (B) $4\sqrt{2}$ (C) $5\sqrt{2}$ (D) None

Q.15 The co-ordinates of the points A, B, C are (– 4, 0), (0, 2) & (– 3, 2) respectively. The point of intersection of the line which bisects the angle CAB internally and the line joining C to the middle point of AB is

(A) $\left(-\frac{7}{3}, \frac{4}{3}\right)$ (B) $\left(-\frac{5}{2}, \frac{13}{2}\right)$

(C) $\left(-\frac{7}{3}, \frac{10}{3}\right)$ (D) $\left(-\frac{5}{2}, \frac{3}{2}\right)$

Q.16 The sides of ΔABC are $2x - y + 5 = 0$, $x + y - 5 = 0$ and $x - 2y - 5 = 0$. Sum of the tangents of its interior angle is

(A) 6 (B) 27/4 (C) 9 (D) None

Q.17 Equation of a straight line passing through the origin and making with x-axis an angle twice the size of the angle made by the line $y = 0.2x$ with the x-axis, is:

(A) $y = 0.4x$ (B) $y = (5/12)x$

(C) $6y - 5x = 0$ (D) None of these

Q.18 The shortest distance from the point M(– 7, 2) to the circle $x^2 + y^2 - 10x - 14y - 151 = 0$ is

(A) 1 (B) 2 (C) 3 (D) None

Q.19 The image of the pair of lines represented by $ax^2+2hxy + by^2 = 0$ by the line mirror y=0 is

(A) $ax^2 - 2hxy - by^2 = 0$ (B) $bx^2 - 2hxy + ay^2 = 0$

(C) $bx^2 + 2hxy + ay^2 = 0$ (D) $ax^2 - 2hxy + by^2 = 0$

Q.20 The pair of straight lines $x^2 - 4xy + y^2 = 0$ together with the line $x + y + 4\sqrt{6} = 0$ form a triangle which is

(A) Right angle but not isosceles

(B) Right isosceles

(C) Scalene

(D) Equilateral

Q.21 Points, A & B are in the first quadrant; point 'O' is the origin. If the slope of OA is 1, slope of OB is 7 and OA = OB, then the slope of AB is

(A) – 1/5 (B) – 1/4 (C) –1/3 (D) – 1/2

Previous Years' Questions

Q.1 The points (–a, – b), (0, 0), (a, b) and are ***(1979)***

(A) Collinear

(B) Vertices of a rectangle

(C) Vertices of a parallelogram

(D) None of the above

Q.2 Given the four lines with the equations, $x + 2y - 3 = 0$, $3x + 4y - 7 = 0$, $2x + 3y - 4 = 0$, $4x + 5y - 6 = 0$, then ***(1980)***

(A) They are all concurrent

(B) They are the sides of a quadrilateral

(C) Only three lines are concurrent

(D) None of these

Q.3 The point (4, 1) undergoes the following three transformations successively

(I) Reflection about the line y = x.

(II) Translation through a distance 2 unit along the positive direction of x-axis.

(III) Rotation through an angle $\frac{\pi}{4}$ about the origin in the counter clockwise direction. Then, the final position of the point is given by the coordinates ***(1980)***

(A) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (B) $\left(-\sqrt{2}, 7\sqrt{2}\right)$

(C) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (D) $\left(\sqrt{2}, 7\sqrt{2}\right)$

Q.4 The straight lines $x + y = 0$, $3x + y - 4 = 0$, $x + 3y - 4 = 0$ form a triangle which is ***(1983)***

(A) Isosceles (B) Equilateral

(C) Right angled (D) None of these

Q.5 If the sum of the distance of a point from two perpendicular lines in a plane is 1, then its locus is ***(1992)***

(A) Square (B) Circle

(C) Straight line (D) Two intersecting lines

Q.6 The orthocentre of the triangle formed by the lines xy = 0 and x + y = 1, is ***(1995)***

(A) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (B) $\left(\frac{1}{3}, \frac{1}{3}\right)$

(C) (0, 0) (D) $\left(\frac{1}{4}, \frac{1}{4}\right)$

Q.7 The graph of the function $\cos x \cos(x + 2) - \cos^2(x + 1)$ is ***(1997)***

(A) A straight line passing through $(0, -\sin^2 1)$ with slope 2.

(B) A straight line passing through (0, 0).

(C) A parabola with vertex $(1, -\sin^2 1)$.

(D) A straight line passing through the point $\left(\frac{\pi}{2}, -\sin^2 1\right)$ and parallel to the x-axis.

Q.8 The diagonals of a parallelogram PQRS are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then PQRS must be a ***(1998)***

(A) Rectangle (B) Square

(C) Cyclic quadrilateral (D) Rhombus

Q.9 Let PS be the median of the triangle with vertices P (2, 2), Q(6, – 1) and R(7, 3). The equation of the line passing through (1, – 1) and parallel to PS is ***(2000)***

(A) $2x - 9y - 7 = 0$ (B) $2x - 9y - 11 = 0$

(C) $2x + 9y - 11 = 0$ (D) $2x + 9y + 7 = 0$

Q.10 The incentre of the triangle with vertices $(1, \sqrt{3}), (0, 0)$ and $(2, 0)$ is ***(2000)***

(A) $\left(1, \frac{\sqrt{3}}{2}\right)$ (B) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$

(C) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (D) $\left(1, \frac{1}{\sqrt{3}}\right)$

Q.11 The number of integer values of m, for which the x-coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is ***(2001)***

(A) 2 (B) 0 (C) 4 (D) 1

Q.12 Area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$ and $y = nx + 1$ equals ***(2001)***

(A) $\frac{|m+n|}{(m-n)^2}$ (B) $\frac{2}{|m+n|}$

(C) $\frac{1}{|m+n|}$ (D) $\frac{1}{|m-n|}$

Q.13 Let P = (– 1, 0), Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three points. Then, the equations of the bisector of the angle PQR is ***(2002)***

(A) $\frac{\sqrt{3}}{2}x + y = 0$ (B) $x + \sqrt{3}y = 0$

(C) $\sqrt{3}x + y = 0$ (D) $x + \frac{\sqrt{3}}{2}y = 0$

Q.14 If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals ***(2012)***

(A) $\frac{29}{5}$ (B) 5 (C) 6 (D) $\frac{11}{5}$

Q.15 The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is ***(2013)***

(A) $2-\sqrt{2}$ (B) $1+\sqrt{2}$

(C) $1-\sqrt{2}$ (D) $2+\sqrt{2}$

Q.16 Let PS be the median of the triangle with vertices P(2, 2), Q(6, -1) and R(7, 3). The equation of the line passing through (1, -1) and parallel to PS is ***(2014)***

(A) $4x - 7y - 11 = 0$ (B) $2x + 9y + 7 = 0$

(C) $4x + 7y + 3 = 0$ (D) $2x - 9y - 11 = 0$

Q.17 Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then ***(2014)***

(A) $2bc - 3ad = 0$ (B) $2bc + 3ad = 0$

(C) $3bc - 2ad = 0$ (D) $3bc + 2ad = 0$

Q.18 Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in R$, is a: ***(2015)***

(A) Straight line parallel to y-axis

(B) Circle of radius $\sqrt{2}$

(C) Circle of radius $\sqrt{3}$

(D) Straight line parallel to x-axis.

Q.19 The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0), is: ***(2015)***

(A) 861 (B) 820 (C) 780 (D) 901

Q.20 Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at (–1, –2), then which one of the following is a vertex of this rhombus? ***(2016)***

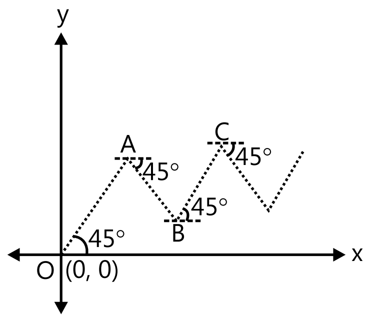
(A) (-3, -8) (B) $\left(\frac{1}{3}, -\frac{8}{3}\right)$

(C) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ (D) (-3, -9)

JEE Advanced/Boards

Exercise 1

Q.1 Points O, A, B, C.........are shown in figure where OA = 2AB = 4BC =.........so on. Let A is the centroid of a triangle whose orthocentre and circumcentre are (2, 4) and $\left(\frac{7}{2},\frac{5}{2}\right)$ respectively. If an insect starts moving from the point O(0, 0) along the straight line is zig-zag fashions and terminates ultimately at point P(α, β), then find the value of ($\alpha + \beta$)



Q.2 Let ABC be a triangle such that the coordinates of A are (– 3, 1). Equation of the median through B is 2x + y – 3 = 0 and equation of the angular bisector of C is 7x – 4y – 1 = 0. Then match the entries of column-1 with their corresponding correct entries of column-II.

Column I	Column II
(A) Equation of the line AB is	(p) 2x + y – 3 = 0
(B) Equation of the line BC is	(q) 2x – 3y + 9 = 0
(C) Equation of CA is	(r) 4x + 7y + 5 = 0
	(s) 18x – y – 49 = 0

Q.3 The equations of the perpendicular of sides AB and AC of triangle ABC are x – y – 4 = 0 and 2x – y – 5 = 0 respectively. If the vertex A is (–2, 3) and point of intersection of perpendicular bisector is $\left(\frac{3}{2},\frac{5}{2}\right)$, find the equation of medians to the sides AB and AC respectively.

Q.4 The interior angle bisector of angle A for the triangle ABC whose coordinates of the vertices are A(– 8, 5); B(– 15, – 19) and C(1, – 7) has the equation ax + 2y + c = 0. Find 'a' and 'c'.

Q.5 Find the equation of the straight lines passing through (– 2, – 7) & having an intercept of length 3 between the straight lines 4x + 3y = 12, 4x + 3y = 3.

Q.6 Two sides of a rhombus ABCD are parallel to the lines y = x + 2 & y = 7x + 3. If the diagonals of the rhombus intersect at the point (1, 2) & the vertex A is on the y-axis, find the possible coordinates of A.

Q.7 Let O(0, 0), A(6, 0) and B(3, $\sqrt{3}$) be the vertices of ΔOAB. Let R be the region consisting of all those points P inside OAB which satisfy d(P, OA) = minimum {d(P, OB)), d(P, AB)}, where d(P, OA), d(P, OB) and d(P, AB) represent the distance of P from the sides OA, OB and AB respectively. If the area of region R is $9(a-\sqrt{b})$, where a and b are coprime. Then, find the value of (a + b).

Q.8 Find the equations of the sides of a triangle having (4, – 1) as a vertex. If the lines x – 1 = 0 and x – y – 1 = 0 are the equations of two internal bisectors of its angles.

Q.9 P is the point (– 1, 2), a variable line through P cuts the x & y axes at A & B respectively. Q is the point on AB such that PA, PQ, PB are in HP. Find the locus of Q.

Q.10 The equations of the altitudes AD, BE, CF of a triangle ABC are x + y = 0, x + 4y = 0 and 2x– y = 0 respectively. The coordinates of A are (t, –t). Find coordinates of B & C. Prove that if t varies the locus of the centroid of the triangle ABC is x + 5y = 0.

Q.11 The distance of a point (x_1, y_1) from each of two straight lines which passes through the origin of co-ordinates is δ; find the combined equation of these straight lines.

Q.12 Consider a, ΔABC whose sides AB, BC and CA are represented by the straight lines 2x + y = 0. x + py = q and x – y = 3 respectively. The point P is (2, 3)

(i) If P is the centroid, then find the value of (p + q)

(ii) If P is the orthocentre, then find the value of (p + q)

(iii) If P is the circumcentre, then find the values of (p + q)

Q.13 The sides of a triangle have the combined equation $x^2 - 3y^2 - 2xy + 8y - 4 = 0$. The third side, which is variable always passes through the point (– 5, – 1). If the range of values of the slope of the third line so that the origin is an interior point of the triangle, lies in the interval (a, b), then find $\left(a + \frac{1}{b^2}\right)$.

Q.14 Consider a line pair $2x^2 + 3xy - 2y^2 - 10x + 15y - 28 = 0$ and another line L passing through origin with gradient 3. The line pair and line L form a triangle whose vertices are A, B and C.

(i) Find the sum of the cotangents of the inter ior angles of the triangle ABC.

(ii) Find the area of triangle ABC.

(iii) Find the radius of the circle touching all the 3 sides of the triangle.

Q.15 Show that all the chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ which subtend a right angle at the origin are concurrent. Does this result also hold for the curve, $3x^2 + 3y^2 + 2x + 4y = 0$? If yes, what is the point of concurrency & if not, give reasons.

Q.16 A straight line is drawn from the point (1, 0) to the curve $x^2 + y^2 + 6x - 10y + 1 = 0$, such that the intercept made on it by the curve subtends a right angle at the origin. Find the equations of the line.

Q.17 The two line pairs $y^2 - 4y + 3 = 0$ and$x^2 + 4xy + 4y^2 - 5x - 10y + 4 = 0$ enclose a 4 sided convex polygon find.

(i) Area of the polygon.

(ii) Length of the diagonals.

Q.18 Find the equations of the two straight lines which together with those given by the equation $6x^2 - xy - y^2 + x + 12y - 35 = 0$ will make a parallelogram whose diagonals intersect in the origin.

Q.19 A straight line passing through O(0, 0) cuts the lines $x = \alpha$, $y = \beta$ and $x + y = 8$ at A, B and C respectively such that OA . OB . OC = 482 and $f(\alpha, \beta) = 0$ where

$$f(x,y) = \left|\frac{y}{x} - \frac{3}{2}\right| + (3\pi - 2y)^6 + \sqrt{ex + 2y - 2e - 6}$$

(i) Find the point of intersection of lines $x = \alpha$ and $y = \beta$.

(ii) Find the value of (OA + OB + OC)

(iii) Find the equation of line OA.

Q.20 The triangle ABC, right angled at C, has median AD, BE and CF. AD lies along the line $y = x + 3$, BE lies along the line $y = 2x + 4$. If the length of the hypotenuse is 60, find the area of the triangle ABC.

Q.21 A triangle has side lengths 18, 24 and 30. Find the area of the triangle whose vertices are the incentre, circumcentre and centroid of the triangle.

Q.22 The points (1, 3) & (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on the lines $y = 2x + c$. Find c & the remaining vertices.

Q.23 A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L & the coordinate axes is 5. Find the equation of the line.

Q.24 Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ & its third side passes through the point (1, – 10). Determine the equation of the third side.

Q.25 The equations of the perpendicular bisectors of the sides AB & AC of a triangle ABC are $x - y + 5 = 0$ & $x + 2y = 0$, respectively. If the point A is (1, –2). Find the equation of the line BC.

Q.26 Let P be the point (3, 2). Let Q be the reflection of P about the x-axis. Let R be the reflection of Q about the lines $y = -x$ and Let S be the reflection of R through the origin. PQRS in a convex quadrilateral. Find the area of PQRS.

Q.27 Two parallel lines ℓ_1 and ℓ_2 having non-zero slope, are passing through the points (0, 1) and (– 1,0) respectively. Two other lines ℓ_1 and ℓ_2 are drawn through (0, 0) and (1, 0) which are perpendicular to ℓ_1 and ℓ_2 respectively. The two sets of lines intersect in four points which are vertices of a square. If the area of this square can be expressed is the form $\frac{p}{q}$ where $p \in N$, then the least value of (p + q)?

Q.28 In an acute triangle ABC, the base BC has the equation $4x - 3y + 3 = 0$. If the coordinates of the orthocentre (H) and circumcentre (P) of the triangle are (1, 2) and (2, 3) respectively, then the radius of the circle circumscribing the triangle is $\sqrt{\frac{m}{a}}$, where m and n are relatively prime. Find the value of (m+ n).

(You may use the fact that the distance between orthocentre and circumcentre of the triangle is given $R\sqrt{1-8\cos A\cos B\cos C}$)

Q.29 The points (– 6, 1), (6, 10), (9, 6) and (– 3, – 3) are the vertices of a rectangle. If the area of the portion of this rectangle that lies above the x axis is a/b, find the value of (a +b), given a and b are coprime.

Q.30 Consider the triangle ABC with sides AB and AC having the equation $L_1 = 0$ and $L_2 = 0$. Let the centroid. Orthocentre and circumcentre of the ΔABC and G, H and S respectively. L = 0 denotes the equation of sides BC.

(i) If L_1: 2x – y = 0 and L_2: x + y = 3 and G(2, 3) then find the slope of the line L= 0.

(ii) If L_1: 2x + y = 0 and L_2: x – y + 2 = 0 and H(2, 3) then find the y-intercept of L = 0.

(iii) If L_1: x + y – 1 = 0 and L_2: 2x – y + 4 = 0 and S(2, 1) then find the x-intercept of the line L= 0.

Exercise 2

Single Correct Choice Type

Q.1 Given the family of lines, a (3x + 4y + 6) + b (x + y + 2) = 0. The line of the family situated at the greatest distance from the point P(2, 3) has equation:

(A) 4x + 3y + 8 = 0 (B) 5x + 3y + 10 = 0

(C) 15x + 8y + 30 = 0 (D) None

Q.2 On the portion of the straight line, x + 2y = 4 intercepted between the axes, a square is constructed on the side of the line away from the origin. Then the point of intersection of its diagonals has co-ordinates:

(A) (2, 3) (B) (3, 2) (C) (3, 3) (D) None

Q.3 The base BC of a triangle ABC is bisected at the point (p, q) and the equation to the side AB & AC are px + qy = 1 and qx + py = 1. The equation of the median through A is:

(A) (p – 2q) x + (q – 2p)y + 1 = 0

(B) (p + q) (x + y) – 2 = 0

(C) $(2pq - 1)(px+qy-1) = (p^2 + q^2 - 1)(qx+py - 1)$

(D) None

Q.4 The lines 3x + 4y = 9 & 4x – 3y + 12 = 0 intersect at P. The first line intersects x-axis at A and the second line intersects y-axis at B. Then the circum radius of the triangle PAB is

(A) 3/2 (B) 5/2 (C) 10 (D) None

Q.5 If the lines ax + y + 1 = 0, x + by + 1 = 0 & x + y + c = 0, where a, b & c are distinct real numbers different from 1 are concurrent, then the value of

$$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=$$

(A) 4 (B) 3 (C) 2 (D) 1

Q.6 The points A(a, 0), B(0, b), C(c, 0) & D(0, d) are such that ac = bd & a, b, c, d are all non zero. The points thus:

(A) Form a parallelogram (B) Do not lie on a circle

(C) Form a trapezium (D) Are concyclic

Q.7 The angles between the straight lines joining the origin to the points common to $7x^2 + 8y^2 - 4xy + 2x - 4y - 8 = 0$ and 3x – y = 2 is

(A) $\tan^{-1}\sqrt{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

Q.8 Distance between two lines represented by the line pair, $x^2 - 4xy + 4y^2 + x - 2y - 6 = 0$ is

(A) $\frac{1}{\sqrt{5}}$ (B) $\sqrt{5}$ (C) $2\sqrt{5}$ (D) None

Q.9 If the straight lines joining the origin and the points of intersection of the curve

$5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$.

And x + ky – 1= 0 are equally inclined to the co-ordinate axes then the value of k:

(A) Is equal to 1

(B) Is equal to – 1

(C) Is equal to 2

(D) Does not exist in the set of real numbers

Q.10 If the vertices P and Q of a triangle PQR are given by (2,5) and (4,–11) respectively, and the point R moves along the line N: 9x + 7y + 4 = 0, then the locus of the centroid of the triangle PQR is a straight line parallel to

(A) PQ (B) QR (C) RP (D) N

Q.11 Let the co-ordinates of the two points A & B be (1, 2) and (7, 5) respectively. The line AB is rotated through 45° in anticlockwise direction about the point of trisection of AB which is nearer to B. The equation of the line in new position is

(A) $2x - y - 6 = 0$ (B) $x - y - 1 = 0$

(C) $3x - y - 11 = 0$ (D) None of these

Q.12 If the line $y = mx$ bisects the angle between the lines $ax^2 + 2hxy + by^2 = 0$, then m is a root of the quadratic equation:

(A) $hx^2 + (a - b)x - h = 0$

(B) $x^2 + h(a - b)x - 1 = 0$

(C) $(a - b)x^2 + hx - (a - b) = 0$

(D) $(a - b)x^2 - hx - (a - b) = 0$

Q.13 A Triangle is formed by the lines $2x - 3y - 6 = 0$; $3x - y + 3 = 0$ and $3x + 4y - 12 = 0$. If the points $P(\alpha, 0)$ and $Q(0, \beta)$ always lie on or inside the ΔABC, then

(A) $\alpha \in [-1, 2]$ & $\beta \in [-2, 3]$

(B) $\alpha \in [-1, 3]$ & $\beta \in [-2, 4]$

(C) $\alpha \in [-2, 4]$ & $\beta \in [-3, 4]$

(D) $\alpha \in [-1, 3]$ & $\beta \in [-2, 3]$

Q.14 In a triangle ABC, side AB has the equation $2x + 3y = 29$ and the side AC has the equation, $x + 2y = 16$. If the mid-point of BC is (5, 6), then the equation of BC is

(A) $x - y = -1$ (B) $5x - 2y = 13$

(C) $x + y = 11$ (D) $3x - 4y = -9$

Q.15 The vertex of a right angle o triangle lies on the straight line $2x + y - 10 = 0$ and the two other vertices, at point (2, – 3) and (4, 1) then the area of triangle in sq. units is -

(A) $\sqrt{10}$ (B) 3 (C) $\frac{33}{5}$ (D) 11

Multiple Correct Choice Type

Q.16 The area of triangle ABC is 20 cm². The co-ordinates of vertex A are (– 5, 0) and B are (3, 0). The vertex C lies upon the line, $x - y = 2$. The co-ordinates of C are

(A) (5, 3) (B) (– 3, – 5) (C) (– 5, – 7) (D) (7, 5)

Q.17 A is a point on either of two lines $y + \sqrt{3}|x| = 2$ at a distance of $\frac{4}{\sqrt{3}}$ units from their point of intersection. The co-ordinates of the foot of perpendicular from A on the bisector of the angle between them are

(A) $\left(-\frac{2}{\sqrt{3}}, 2\right)$ (B) (0, 0) (C) $\left(\frac{2}{\sqrt{3}}, 2\right)$ (D) (0, 4)

Q.18 All the points lying inside the triangle formed by the points (1, 3), (5, 6) & (– 1, 2) satisfy

(A) $3x + 2y \geq 0$ (B) $2x + y + 1 \geq 0$

(C) $2x + 3y - 12 \geq 0$ (D) $-2x + 11 \geq 0$

Q.19 Line $\frac{x}{a} + \frac{y}{b} = 1$ cuts the co-ordinate axes at A(a, 0) & B(0, b) & the line $\frac{x}{a'} + \frac{y}{b'} = -1$ at A′ (–a′, 0) & B′(0, – b′). If the points A, B, A′, B′ are concyclic then the orthocentre of the triangle ABA′ is

(A) (0, 0) (B) (0, b′) (C) $\left(0, \frac{aa'}{b}\right)$ (D) $\left(0, \frac{bb'}{a}\right)$

Q.20 If one vertex of an equilateral triangle of side 'a' lies at the origin and the other lies on the line $x - \sqrt{3}y = 0$ then the co-ordinates of the third vertex are:

(A) $(0, a)$ (B) $\left(\frac{\sqrt{3}a}{2}, -\frac{a}{2}\right)$ (C) (0, – a) (D) $\left(-\frac{\sqrt{3}a}{2}, \frac{a}{2}\right)$

Q.21 Three vertices of a triangle are A(4, 3); B(1, –1) and C(7, k). Value(s) of k for which centroid, orthocentre, incentre and circumcentre of the ABC lie on the same straight line is/are:

(A) 7 (B) – 1 (C) $-\frac{19}{8}$ (D) None

Q.22 Equation of a line through (7, 4) and touching the circle $x^2 + y^2 - 6x + 4y - 3 = 0$ is

(A) $5x - 12y + 13 = 0$ (B) $12x - 5y - 64 = 0$

(C) $x - 7 = 0$ (D) $y = 4$

Q.23 The circumcentre of the triangle formed by the lines, $xy + 2x + 2y + 4 = 0$ and $x + y + 2 = 0$ is

(A) (– 2, – 2) (B) (–1 , – 1)

(C) (0, 0) (D) (– 1, – 2)

Q.24 The sides of a triangle are x + y = 1, 7y = x and – 3y + x = 0. Then the following is an interior point of the triangle.

(A) Circumcentre (B) Centroid

(C) Incentre (D) Orthocentre

Q.25 Equation of a straight line passing through the point (2, 3) and inclined at an angle of $\tan^{-1}\left(\frac{1}{2}\right)$ with the line y + 2x = 5 is

(A) y =3 (B) x = 2

(C) 3x + 4y – 18 = 0 (D) 4x + 3y – 17 = 0

Q.26 A ray of light travelling along the line x + y = 1 is incident on the x-axis and after refraction it enters the other side of the x-axis by turning π/6 away from the x-axis. The equation of the line along which the refracted ray travels is

(A) $x+(2-\sqrt{3})y=1$ (B) $(2-\sqrt{3})x+y=1$

(C) $y+(2+\sqrt{3})x=2+\sqrt{3}$ (D) None of these

Q.27 Consider the equation $y - y_1 = m(x - x_1)$. If m & x_1 are fixed and different lines are drawn for different values of y_1, then

(A) The lines will pass through a fixed point

(B) There will be a set of parallel lines

(C) All the lines interest the line $x = x_1$

(D) All the lines will be parallel to the line

Previous Years' Questions

The codes (A), (B), (C) and (D) deformed as follows.

(A) Statement-I is true, statement-II is also true; statement-II is the correct explanation of statement-I.

(B) Statement-I is true, statement-II is also true; statement-II is not the correct explanation of statement-I.

(C) Statement-I is true; statement-II is false.

(D) Statement-I is false; statement-II is true.

Q.1 Lines L_1: y – x = 0 and L_2: 2x + y =0 intersect the line L_3: y + 2 = 0 at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. ***(2007)***

Statement-I: The ratio PR: RQ equals $2\sqrt{2}:\sqrt{5}$. Because

Statement-II: In any triangle, bisector of an angle divides the triangle into two similar triangles.

Match the conditions expressions in column I with statement in column II

Q.2 Consider the lines given by ***(2008)***

L_1: x + 3y – 5 = 0 ; L_2: 3x – ky – 1 = 0

L_3: 5x + 2y – 12 = 0

Column I	Column II
(A) L_1, L_2, L_3 are concurrent, if	(p) k = – 9
(B) One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(q) $k=-\frac{6}{5}$
(C) L_1, L_2, L_3 form a triangle, if	(r) $k=\frac{5}{6}$
(D) L_1, L_2, L_3 do not form a triangle, if	(s) k = 5

Q.3 Three lines px + qy + r = 0, qx + ry + p = 0 and rx + py + q = 0 are concurrent, if ***(1985)***

(A) p + q + r = 0 (B) $p^2 + q^2 + r^2 = pr + rq$

(C) $p^3 + q^3 + r^3 = 3pqr$ (D) None of these

Q.4 All points lying inside the triangle formed by the points (1, 3), (5, 0) and (– 1, 2) satisfy ***(1986)***

(A) 3x + 2y ≥ 0 (B) 2x +y – 13 ≥ 0

(C) 2x – 3y – 12 ≤ 0 (D) – 2x + y ≥ 0

Q.5 Let L_1 be a straight line passing through the origin and L_2 be the straight line x+y=1. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equation can represent L_3? ***(1999)***

(A) x + y = 0 (B) x – y = 0

(C) x + 7y = 0 (D) x – 7y = 0

Q.6 If the distance between the plane Ax – 2y + z = d and the plane containing the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is $\sqrt{6}$, then |d| is........... ***(2010)***

Q.7 The straight line 2x – 3y = 1 divides the circular region $x^2 + y^2 \le 6$ into two parts. If

$$S=\left\{\left(2,\frac{3}{4}\right),\left(\frac{5}{2},\frac{3}{4}\right),\left(\frac{1}{4},-\frac{1}{4}\right),\left(\frac{1}{8},\frac{1}{4}\right)\right\},$$ ***(2011)***

Then the number of point(s) lying inside the smaller part is..............

Q.8 For points P = (x_1, y_1) and Q = (x_2, y_2) of the coordinate plane, a new distance d(P,Q) is defined by $d(P, Q) = |x_1 - x_2| + |y_1 - y_2|$. Let Q = (0, 0) and A = (3, 2). Prove that the set of points in the first quadrant which are equidistant (with respect to the new distance) from O and A consists of the union of a line segment of finite length and an infinite ray. Sketch this set in a labelled diagram. ***(2000)***

Q.9 A straight line L through the origin meets the line x + y = 1 and x + y = 3 at P and Q respectively. Through P and Q two straight lines L_1 and L_2 are drawn, parallel to 2x – y = 5 and 3x + y = 5 respectively. Lines L_1 and L_2 intersect at R, show that the locus of R as L varies, is a straight line. ***(2002)***

Q.10 Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side of S. If a, b, c and d denote the length of the sides of the quadrilateral, prove that $2 \le a^2 + b^2 + c^2 + d^2 \le 4$. ***(1997)***

Q.11 Using coordinate geometry, prove that the three altitudes of any triangle are concurrent. ***(1998)***

Q.12 A Straight line L through the point (3, -2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is ***(2011)***

Q.13 For a > b > c > 0, the distance between (1, 1) and the point of intersection of the lines ax + by + c = 0 and bx + ay + c = 0 is less than $2\sqrt{2}$. Then ***(2013)***

(A) a + b – c > 0 (B) a – b + c < 0

(C) a – b + c > 0 (D) a + b – c < 0

Q.14 For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distance of the point P from the lines x – y = 0 and x + y = 0 respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$ ***(2014)***

PlancEssential Questions

JEE Main/Boards

Exercise 1

Q.10 Q.13 Q.23
Q.24 Q.26 Q.27

Exercise 2

Q.11 Q.12 Q.16
Q.18

Previous Years' Questions

Q.2 Q.7 Q.10
Q.11 Q.13

JEE Advanced/Boards

Exercise 1

Q.2 Q.6 Q.10
Q.15 Q.19 Q.24
Q.27

Exercise 2

Q.2 Q.6 Q.8
Q.12 Q.18 Q.21
Q.23 Q.26

Previous Years' Questions

Q.2 Q.5 Q.7
Q.9 Q.10

Answer Key

JEE Main/Boards

Exercise 1

Q.1 $-\frac{1}{6}$

Q.4 (7, 0) or (2, 5)

Q.5 $(b-b')x+(a'-a)y+ab'-a'b = 0, (b-b')x+(a-a')y+a'b'-ab=0$

Q.6 5 : 97 externally

Q.7 $E\left[\frac{3}{2},0\right]$, $F\left[\frac{9}{4},0\right]$, $G\left[\frac{9}{4},\frac{3}{4}\right]$, $H\left[\frac{3}{2},\frac{3}{4}\right]$

Q.8 3x – 8y = 0, 3x – 2y = 0

Q.9 3y – 2x = 2xy

Q.10 (7, 5) and (– 1, – 1)

Q.11 5

Q.12 $C = \left(-\frac{3}{2},\frac{3}{2}\right)$, $A = \left(\frac{1}{2},-\frac{3}{2}\right)$

Q.13 OT → y = x, OS → y = –x, SP → y = x + 4, PQ → y = –x + 4, PR → $y = (2-\sqrt{3})x + 4$

Q.14 x = 2, x + 9y – 14 = 0, 7x – 9y – 2 = 0

Q.15 (– 4, – 3)

Q.16 $-\frac{36}{7},-\frac{45}{7}$

Q.17 5

Q.18 30° or 150°

Q.19 $\theta = \tan^{-1}(-\frac{22}{3})$

Q.20 D = (– 7, 2), C = (– 5, 7)

Q.21 3x – y – 7 = 0 and x + 3y – 9 = 0

Q.22 52x + 89y + 519 = 0 or 4x + y = 1

Q.23 29x – 2y – 31 = 0

Q.24 (– 16, – 2)

Q.25 26x – 122y – 1675 = 0

Q.26 x + y + 2 = 0

Q.27 P (2, – 1)

Q.28 $\sqrt{\frac{2}{3}}$

Exercise 2

Single Correct Choice Type

Q.1 C	**Q.2** A	**Q.3** A	**Q.4** C	**Q.5** C	**Q.6** A
Q.7 D	**Q.8** B	**Q.9** A	**Q.10** C	**Q.11** A	**Q.12** B
Q.13 A	**Q.14** C	**Q.15** D	**Q.16**B	**Q.17** B	**Q.18** B
Q.19 D	**Q.20** D	**Q.21** D			

Previous Years Questions

Q.1 A	**Q.2** C	**Q.3** C	**Q.4** A	**Q.5** A	**Q.6** C
Q.7 D	**Q.8** D	**Q.9** D	**Q.10** D	**Q.11** A	**Q.12** D
Q.13C	**Q.14** C	**Q.15** A	**Q.16** B	**Q.17** C	**Q.18** B
Q.19 C	**Q.20** B				

JEE Advanced/Boards

Exercise 1

Q.1 8

Q.2 (A) R; (B) S; (C) Q

Q.3 x + 4y = 4 ; 5x + 2y = 8

Q.4 a = 11, c = 78

Q.5 7x + 24y - 182 = 0 or x = – 2

Q.6 (0, 0) or $\left(0, \frac{5}{2}\right)$

Q.7 5

Q.8 2x – y +3 =0, 2x + y – 7 =0, x – 2y – 6 =0

Q.9 y = 2x

Q.10 $B\left(-\frac{2t}{3}.\frac{-t}{6}\right).C\left(\frac{t}{2}.t\right)$

Q.11 $(y_1^2-\delta^2)x^2-2x_1y_1xy+(x_1^2-\delta^2)y^2=0$

Q.12 (a) 74 ; (b) 50 ; (c) 47

Q.13 24

Q.14 (a) $\frac{50}{7}$; (b) $\frac{63}{10}$; (c) $\frac{3}{10}(8\sqrt{5}-5\sqrt{10})$

Q.15 (1,– 2), yes$(-1,-2)$

Q.16 x +y = 1; x + 9y = 1

Q.17 (i) area – 6 sq. units, (ii) diagonals ar $\sqrt{5}$ & $\sqrt{53}$

Q.18 $6x^2-xy-y^2-x-12y-35=0$

Q.19 (i) α =2 and β =3; (ii) $9\sqrt{2}$; (iii) x – y = 0

Q.20 400 sq. units

Q.21 3 units

Q.22 c = – 4; B (2, 0) ; D (4, 4)

Q.23 x + 5y + $5\sqrt{2}$ = 0 or x + 5y – $5\sqrt{2}$ = 0

Q.24 x – 3y – 31 = 0 or 3x+ y + 7 = 0

Q.25 14x + 23y = 40

Q.26 15

Q.27 6

Q.28 63

Q.29 533

Q.30 (a) 5 ; (b) 2 ; (c) $\frac{3}{2}$

Exercise 2

Single Correct Choice Type

Q.1 A	**Q.2** C	**Q.3** C	**Q.4** B	**Q.5** D	**Q.6** D
Q.7 D	**Q.8** B	**Q.9** B	**Q.10** D	**Q.11** C	**Q.12** A
Q.13 D	**Q.14** C	**Q.15** B			

Multiple Correct Choice Type

Q.16 B, D	**Q.17** B, C	**Q.18** A, B, D	**Q.19** B, C	**Q.20** A, B, C, D	**Q.21** B, C
Q.22 A, C	**Q.23** B, C	**Q.24** B, C	**Q.25** B, C	**Q.26** A, C	**Q.27** B, C

Previous Years Questions

Q.1 C

Q.2 A $\rightarrow$ s; B $\rightarrow$ p, q; C $\rightarrow$ r; D $\rightarrow$ p, q, s

Q.3 A , C

Q.4 A, C

Q.5 B , C

Q.6 6

Q.7 2

Q.13 A

Q.14 6

Solutions

JEE Main/Boards

Exercise 1

Sol 1: Slope of line joining [4, –6] & [–2, –5]

$= \frac{-5+6}{-2-4} = \frac{1}{-6}$

Sol 2: Line ℓ_1 joining (2, –3) and (–5, 1)

$\Rightarrow$ Slope $= -\frac{4}{7}$

$\ell_2 \rightarrow$ line joining (7, –1) and (0, 3)

$\Rightarrow$ Slope $= \frac{-4}{7}$

$\ell_3 \rightarrow$ line joining (4, 5) and (0, –2)

$\Rightarrow$ Slope $= \frac{7}{4}$

$\ell_1 \parallel \ell_2$ and $\because m_2 \cdot m_3 = -1$

i.e. ℓ_1 and ℓ_2 are perpendicular to ℓ_3

Sol 3: A(–4, 2) ; B(2, 6) ; C(8, 5); D(9, –7) mid points of

$AB = E\left(\frac{-4+2}{2}, \frac{2+6}{2}\right) = (-1, 4)$

$BC = F\left[\frac{2+8}{2}, \frac{6+5}{2}\right] = \left[5, \frac{11}{2}\right]$

$CD = G\left[\frac{8+9}{2}, \frac{5-7}{2}\right] = \left[\frac{17}{2}, -1\right]$

$AD = H\left[\frac{9-4}{2}, \frac{-7+2}{2}\right] = \left[\frac{5}{2}, \frac{-5}{2}\right]$

Slope of EF $= \left[\frac{\frac{11}{2}-4}{5+1}\right] = \frac{3}{12} = \frac{1}{4}$

Slope of FG $= \frac{\frac{11}{2}+1}{5-\frac{17}{2}} = \frac{13}{-7} = \frac{-13}{7}$

Slope of GH $= \frac{-1+\frac{5}{2}}{\frac{17}{2}-\frac{5}{2}} = \frac{3}{12} = \frac{1}{4}$

Slope of EH $= \frac{4+\frac{5}{2}}{-1-\frac{5}{2}} = \frac{-13}{7}$

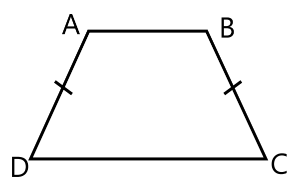
EF || GH and FG || EH, hence midpoints E, F, G, H form a parallelogram.

Sol 4: AB || DC

Slope of AB $= \frac{2-0}{0-2} = -1$

Slope of DC $= \frac{7-y}{0-x} = \frac{y-7}{x}$

$y - 7 = -x \Rightarrow x + y = 7 - x$ ...(i)



Trapezium is isosceles i. e. AD = BC

$(x-2)^2 + y^2 = 25$

$x^2 + y^2 - 4x = 21$ ...(ii)

$x^2 + y^2 + 2xy = 49$

$4x + 2xy = 28 \Rightarrow 2x(2 + y) = 28$ from equation (i)

y= 7-x

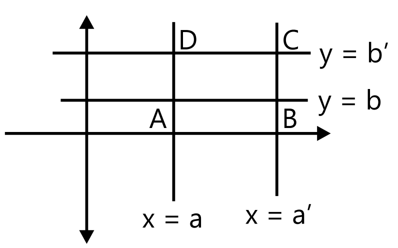
$\Rightarrow x(9-x) = 14 \Rightarrow x^2 - 9x + 14 = 0$

$\Rightarrow x = 7, 2$

If x = 7, y = 0 and if x = 2, y = 5

Ans is (7, 0) and (2, 5)

Sol 5:



Coordinates are A(a, b), B(a′, b), C(a′, b′) and D(a, b′)

Slope of line AC = $\frac{b'-b}{a'-a}$

Slope of line BD = $\frac{b'-b}{a-a'}$

Equation of line AC

$\Rightarrow y - b = \left(\frac{b'-b}{a'-a}\right)(x-a)$

$\Rightarrow (b' - b)x + (a - a')y - ab' + a \Rightarrow b = 0$

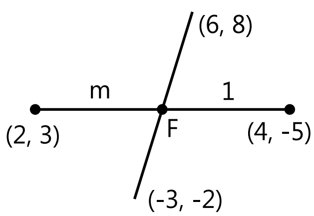
Equation of line BD

$\Rightarrow y - b = \frac{b'-b}{a-a'}(x-a')$

$\Rightarrow (b' - b)x - (a - a')y - a'b' + ab = 0$

Sol 6: ℓ_1 (line joining (2, 3) and (4, –5))

ℓ_2 (line joining (6, 8) and (–3, –2))



Lets assume it divides ℓ_1 in ratio m : 1

Coordinates of F are $\left[\frac{4m+2}{m+1}, \frac{3-5m}{1+m}\right]$

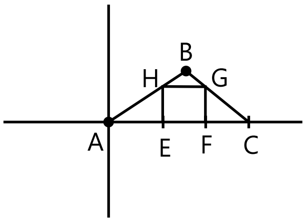
Equation of ℓ_2 is $\frac{y-8}{x-6} = \frac{10}{9}$

$\Rightarrow \frac{\frac{3-5m}{1+m}-8}{\frac{4m+2}{m+1}-6} = \frac{10}{9} \Rightarrow \frac{-5-13m}{-2m-4} = \frac{10}{9}$

$\Rightarrow 45 + 117m = 20m + 40 \Rightarrow m = \frac{-5}{97}$

i. e. in ratio 5 : 97 (externally).

Sol 7:



E = (a, 0), F = (b, 0), G = (b, c)

H = (a, c)

It is a square hence b – a = c

H lies of x = 2y $\Rightarrow$ a = 2c $\Rightarrow$ b = 3c

G lies on x + y = 3 $\Rightarrow$ b + c = 3

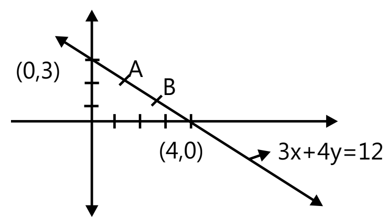
4c = 3

$\Rightarrow c = \frac{3}{4}; b = \frac{9}{4};$ $\qquad a = \frac{6}{4}$

Coordinates of square are

$E\left[\frac{3}{2}, 0\right]$, $F\left[\frac{9}{4}, 0\right]$, $G\left[\frac{9}{4}, \frac{3}{4}\right]$, $H\left[\frac{3}{2}, \frac{3}{4}\right]$

Sol 8:



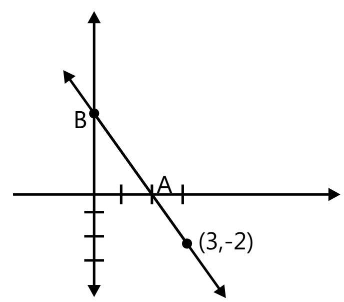
$A = \left[\frac{1(4,0)+2(0,3)}{3}\right] \Rightarrow \left[\frac{4}{3}, \frac{6}{3}\right]$

$B = \left[\frac{2(4,0)+1(0,3)}{3}\right] \Rightarrow \left[\frac{8}{3}, \frac{3}{3}\right]$

Line OA $\rightarrow y = \frac{3x}{2} \Rightarrow 3x - 2y = 0$

Line OB $\rightarrow y = \frac{3x}{8} \Rightarrow 3x - 8y = 0$

Sol 9:



$\frac{y+2}{x-3} = m$

y + 2 = m(x – 3)

$A \Rightarrow \left(3 + \frac{2}{m}, 0\right)$ [x intercept]

B $\Rightarrow$ (0, –2 – 3m) [y intercept]

Mid point of AB $\left[\frac{3+\frac{2}{m}}{2}, \frac{-2-3m}{2}\right]$

$y = \frac{-3m-2}{2}$, $x = \frac{3m+2}{2m}$

$m = -\left(\frac{2y+2}{3}\right)$

$\Rightarrow 2x\left(\frac{-2y-2}{3}\right) = 3\left(\frac{-2y-2}{3}\right) + 2$

$\Rightarrow -4xy - 4x = -6y - 6 + 6$

$\Rightarrow 2xy + 2x - 3y = 0$

Sol 10: $\frac{y-2}{x-3} = \frac{3}{4}$ and $y = 3x - 1$

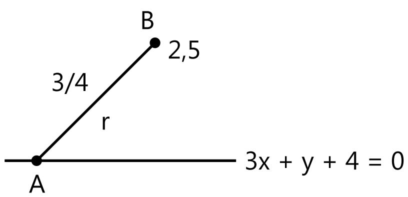
Points which are 5 units away from (3, 2) are

$(3 \pm 5\ \cos\theta, 2 \pm 5\sin\theta)$

$\tan\theta = \frac{3}{4} \Rightarrow (3 \pm 4, 2 \pm 3) \Rightarrow [7, 5]\ [-1, -1]$

Sol 11: Coordinates of A at r units from B

$A(2 + r\cos\theta, 5 + r\sin\theta)$



$\tan\theta = \frac{3}{4}$

$A\left[2+\frac{4r}{5}, 5+\frac{3r}{5}\right]$

A lies on given line

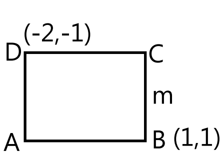
$\Rightarrow 3\left(2+\frac{4r}{5}\right) + 5 + \frac{3r}{5} + 4 = 0$

$15 + 3r = 0 \Rightarrow r = -5$

$A[2 - 4, 5 - 3] = [-2, 2]$

r = 5 units

Sol 12:



Slope of BD = $\frac{2}{3}$

Slope of AC = $\frac{-3}{2}$

BD = $\sqrt{13}$

Length of side = $\sqrt{\frac{13}{2}}$

Let slope of AB be m

$\tan 45^o = \left|\frac{m-\frac{2}{3}}{1+\frac{2m}{3}}\right| = 1$

$\frac{m}{3} = \frac{5}{3} \Rightarrow m = 5$ or $\frac{5m}{3} = \frac{-1}{3} \Rightarrow m = \frac{-1}{5}$

$A = \left[1+\sqrt{\frac{13}{2}}\cos\theta, 1+\sqrt{\frac{13}{2}}\sin\theta\right]$

Now $\cos\theta = \frac{-1}{\sqrt{26}}$, $\sin\theta = \frac{-5}{\sqrt{26}}$

$\Rightarrow A\left[\frac{1}{2}, \frac{-3}{2}\right]$

$m = \frac{-1}{5}$, $\sin\theta = \frac{1}{\sqrt{26}}$, $\cos\theta = \frac{-5}{\sqrt{26}}$

$C = \left[1+\sqrt{\frac{13}{2}}\cos\theta, 1+\sqrt{\frac{13}{2}}\sin\theta\right]$

$\Rightarrow C = \left[-\frac{3}{2}, \frac{3}{2}\right]$

Sol 13: OT $\rightarrow y = x$ [O(0, 0), T (2, 2)]

OS $\rightarrow y = -x$ [O(0, 0), S (–2, 2)]

SP $\rightarrow y = x + 4$ [as OP = 4]

PQ $\rightarrow y = -x + 4$ [as OQ = 4]

PR $\rightarrow y = mx + 4$

$m = \tan 15^0 \rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3}$

$\Rightarrow y = (2 - \sqrt{3})x + 4$

OR $\rightarrow y = mx$

$R[4 + 4\sqrt{2}\cos 75, 4\sqrt{2}\sin 75]$

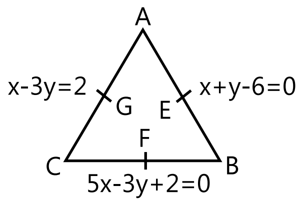
$R\left[4+4\sqrt{2}\frac{(\sqrt{3}-1)}{2\sqrt{2}}, 4\sqrt{2}\frac{(\sqrt{3}+1)}{2\sqrt{2}}\right]$

$R[4 + 2(\sqrt{3}-1), 2(\sqrt{3}+1)]$

$R[2(\sqrt{3}+1), 2(\sqrt{3}+1)]$

$\Rightarrow y = x$

Sol 14 :



Solving

$x + y - 6 = 0$, $x - 3y - 2 = 0$ gives A[5,1]

$x + y - 6 = 0$, $5x - 3y + 2 = 0$ gives B[2,4]

$x - 3y - 2 = 0$, $5x - 3y + 2 = 0$ gives C[–1,–1]

Midpoints are $E\left[\frac{7}{2},\frac{5}{2}\right]$, $F\left[\frac{1}{2},\frac{3}{2}\right]$, G[2, 0]

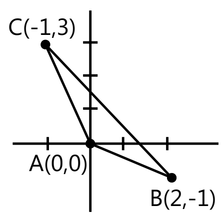
Equation of median

AF $\Rightarrow \frac{y-1}{x-5} = \frac{\frac{1}{2}}{-\frac{9}{2}} \Rightarrow x + 9y = 14$

BG $\Rightarrow \frac{y-4}{x-2} = \frac{1}{0} \Rightarrow x = 2$

CE $\Rightarrow \frac{y+1}{x+1} = \frac{7}{9} \Rightarrow 7x - 9y = 2$

Sol 15:



Slope of line BC $= \frac{-4}{3}$

Slope of line perpendicular to BC $= \frac{3}{4}$

$\perp$ bisector through A

$y = \frac{3x}{4}$ .... (i)

Slope of line AC = –3

Slope of line perpendicular to AC $= \frac{1}{3}$

$\perp$ bisector through B

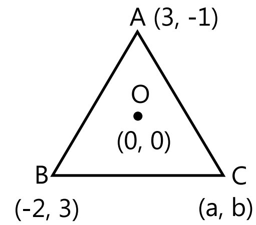
$3y = x - 5$ .... (ii)

Solving (i) & (ii), we get

$x = -4$; $y = -3$

Ans is (–4, –3)

Sol 16:



Orthocentre is (0, 0)

$(\text{slope})_{OA} = -\frac{1}{3}$

$(\text{slope})_{BC} = 3 = \frac{b-3}{a+2}$ .... (i)

$(\text{slope})_{OB} = \frac{-3}{2}$

$(\text{slope})_{AC} = \frac{2}{3} = \frac{b+1}{a-3}$ .... (ii)

Solving (i) and (ii), we get

$b = 3a + 9$ and $3b = 2a - 9$

$9a + 27 = 2a - 9$

$a = -\frac{36}{7}$; $b = \frac{-108+63}{7} = -\frac{45}{7}$

$\Rightarrow C\left(-\frac{36}{7}, -\frac{45}{7}\right)$

Sol 17: Lines are concurrent intersection of ℓ_1 and $\vdots\ \ell_3$ gives x = 1 y = –1

It lies on $\ell_2 \Rightarrow p - 2 - 3 = 0 \Rightarrow p = 5$

Sol 18: $\ell_1 \Rightarrow y = \sqrt{3x} + 5$ $\quad \ell_2 \Rightarrow y = \frac{x-6}{\sqrt{3}}$

$m_1 = \sqrt{3}$ $\quad m_2 = \frac{1}{\sqrt{3}}$

$$\tan\theta = \left|\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \frac{\sqrt{3}}{\sqrt{3}}}\right| = \left|\frac{1}{\sqrt{3}}\right|$$

$\Rightarrow \theta = 30^o, 150^o$

Sol 19:

A(2, –1) $\quad m_{AB} = -\frac{3}{2}$

B(0, 2) $\quad m_{BC} = \frac{1}{3}$

C(3, 3) $\quad m_{CD} = \frac{-3}{2}$

D(5, 0) $\qquad m_{AD} = \frac{1}{3}$

BC || AD and AB || CD

$\Rightarrow$ ABCD is a parallelogram

$\Rightarrow$ Diagonals are AC & BD

Slope of AC $\rightarrow \frac{-1-3}{2-3} = 4$

Slope of BD $\rightarrow \frac{2-0}{0-5} = \frac{-2}{5}$

$m_1 = 4; \qquad m_2 = \frac{-2}{5}$

$$\tan\theta = \frac{4+\frac{2}{5}}{1-\frac{8}{5}} = \frac{22}{-3} = \frac{-22}{3}$$

$$\phi = \tan^{-1}\left(\frac{-22}{3}\right)$$

Sol 20: A(–2, 0) ; B(0, 5)

$m_{AB} = \frac{5}{2} \quad r_{AB} = \sqrt{29}$

$C = \left[0+\sqrt{29}\cos\theta, 5+\sqrt{29}\sin\theta\right]$

$D = \left[-2+\sqrt{29}\cos\theta, 0+\sqrt{29}\sin\theta\right]$

$\tan\theta = \frac{-2}{5}$; $\sin\theta = \frac{2}{\sqrt{29}}$; $\cos\theta = \frac{-5}{\sqrt{29}}$

C = [–5, 7] $\qquad$ D = [–7, 2]

Sol 21: $\frac{y-2}{x-3} = m$

$$\tan 45 = \left|\frac{m-\frac{1}{2}}{1+\frac{m}{2}}\right|$$

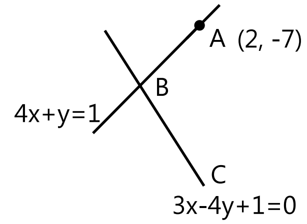
$\frac{2m-1}{m+2} = \pm 1$

$m = +3, m = -\frac{1}{3}$

$\Rightarrow y - 2 = 3x - 9 \Rightarrow y = 3x - 7$

& $y - 2 = -\frac{1}{3}(x-3) \Rightarrow x + 3y = 9$

Sol 22:



Solving equations we get B

$\frac{3x+1}{4} = 1 - 4x \Rightarrow 19x = 3$

$\Rightarrow x = \frac{3}{19}, y = \frac{7}{19}$

$\therefore B\left[\frac{3}{19}, \frac{7}{19}\right]$

$$AB = \sqrt{\left(2-\frac{3}{19}\right)^2 + \left(\frac{7}{19}+7\right)^2}$$

$$= \sqrt{\frac{(35)^2+(140)^2}{19}} = \frac{35}{19}\sqrt{17}$$

C = (2 + r cosθ, –7 + r sinθ)

3(2 + r cosθ) – 4(–7 + r sinθ) + 1 = 0

35 + 3r cosθ – 4r sinθ = 0

$1 + \frac{3\sqrt{17}}{19}\cos\theta - \frac{4\sqrt{17}}{19}\sin\theta = 0$

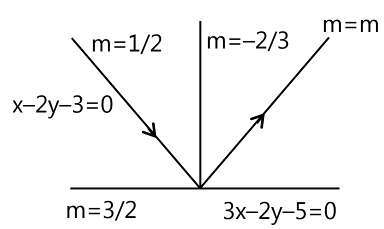
$-3\cos\theta + 4\sin\theta = \frac{19}{\sqrt{17}}$

$\tan\theta = -4$ or $\frac{-52}{89}$

Equation of AC $\Rightarrow \frac{y+7}{x-2} = -4$ or $= \frac{-52}{89}$

52x + 89y + 519 = 0 or 4x + y = 1

Sol 23:



Points of intersection is (1, –1) and both the lines x – 2y – 3 = 0 and reflected Line are equally inclined to normal on 3x – 2y 5 = 0

$$\frac{m+\frac{2}{3}}{1-\frac{2m}{3}} = \frac{\frac{-2}{3}-\frac{1}{2}}{1-\frac{1}{3}}$$

$\Rightarrow \dfrac{3m+2}{3-2m} = \dfrac{-7}{4}$

$\Rightarrow 2(6m + 4) = -21 + 14m$

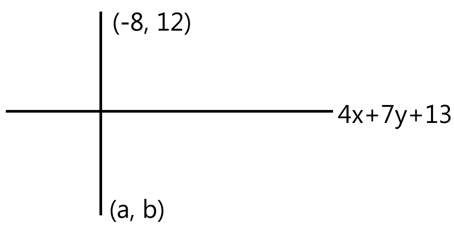
$\Rightarrow 2m = +29$

$\Rightarrow m = +\dfrac{29}{2}$

$\Rightarrow (y + 1) = \dfrac{29}{2}(x - 1)$

$\Rightarrow 2y + 31 = 29x$

Sol 24:



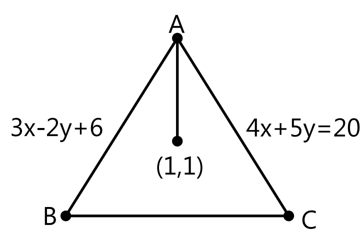
$\dfrac{a+8}{4} = \dfrac{b-12}{7} = -2 \cdot \dfrac{65}{65}$

$\Rightarrow \dfrac{a+8}{4} = \dfrac{b-12}{7} = -2$

$\Rightarrow a = -16 \quad \& \quad b = -2$

Image $\equiv (-16, -2)$

Sol 25:



$A = \left[\dfrac{10}{23}, \dfrac{84}{23}\right]$

$B = (a, b) \Rightarrow 3a - 2b + 6 = 0$

$\dfrac{b-1}{a-1} = \dfrac{5}{4} \Rightarrow 4b = 5a - 1$

Solving these, we get

$6a + 12 = 5a - 1$

$a = -13 \qquad b = \dfrac{-33}{2}$

AB slope $= \dfrac{\frac{84}{23}-1}{\frac{10}{23}-1} = \dfrac{-61}{13}$

$\Rightarrow$ Slope of BC $= \dfrac{13}{61}$

$\Rightarrow \dfrac{y-b}{x-a} = \dfrac{13}{61} \Rightarrow \dfrac{y+\frac{33}{2}}{x+13} = \dfrac{13}{61}$

$\Rightarrow 61y = 13x + 13\times13 - \dfrac{61\times33}{2}$

$\Rightarrow 26x - 122y - 1675 = 0$

Sol 26: The equation of line passes through point of intersection of x+3y+y=0 and 3x+y+4=0 is

$3x+y+4 + \lambda (x+3y+4) = 0$

$(\lambda+3)x + (1+3\lambda)y + 4 + 4\lambda = 0$

The obtained line is equally inclined to axes, then

Slope of line = ± 1

$-\dfrac{\lambda+3}{1+3\lambda} = 0$ and $-\dfrac{\lambda+3}{1+3\lambda} = -1$

$\Rightarrow \lambda + 3 = -1 - 3\lambda$ or $\lambda + 3 = 1 + 3\lambda$

$\Rightarrow \lambda = -1$ or $\Rightarrow \lambda = 1$

Eqn of line is $(-1 + 3)x + (1 - 3)y + 4 - 4 = 0$

$\Rightarrow 2x - 2y = 0 \quad \Rightarrow x = y$ and

$(1+3)x + (1 + 3)y + 4 + 4 = 0$

$\Rightarrow \quad 4x + 4y + 8 = 0$

$\Rightarrow \quad x + y + 2 = 0$

Sol 27: $x(a + 2b) + y(a + 3b) - a - b = 0$

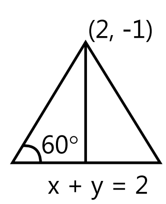
$a(x + y - 1) + b(2x + 3y - 1) = 0$

Fixed point is the intersection of given 2 lines

$x + y = 1$ & $2x + 3y = 1$

$\Rightarrow y = -1; x = 2;$ P (2, – 1)

Sol 28:



Distance of vertex from line is $\left|\dfrac{-1}{\sqrt{2}}\right| = \dfrac{1}{\sqrt{2}}$

Length of base = 2 a

$= 2\sqrt{\dfrac{2}{3}}$

$\sin 60^\circ = \frac{1}{\sqrt{2}a} \Rightarrow a = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$

Exercise 2

Single Correct Choice Type

Sol 1: (C) $3x - 8y - 7 = 0$

(A) $\left.\begin{matrix} 3(0)-8(-1)-7=1 \\ 3(0)-8(0)-7=-7 \end{matrix}\right\}$ different sides

(B) $\left.\begin{matrix} 3(0)-8(1)-7=-15 \\ 3(3)-8(0)-7=2 \end{matrix}\right\}$ different sides

(C) $\left.\begin{matrix} 3(-1)-8(1)-7=-2 \\ 3(3)-8(7)-7=-54 \end{matrix}\right\}$ same sides

Sol 2: (A) $3x - 4y + 7 = 0$; $12x + 5y - 2 = 0$

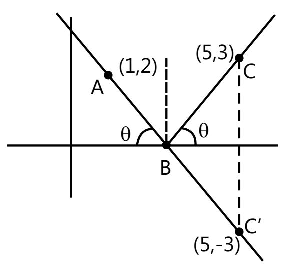
$p_1p_2 + q_1q_2 = 36 - 20 = 16 > 0$

$\frac{3x-4y+7}{5} = \frac{-(12x+5y-2)}{13}$

$39x - 52y + 91 = -60x - 25y + 10$

$99x - 27y + 81 = 0 \Rightarrow 11x - 3y + 9 = 0$

Sol 3: (A)



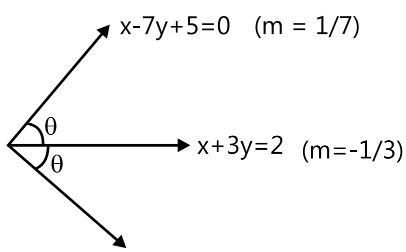
C′ is the reflection of C w.r.t. x-axis.

∴ Eq. of AB = Eq. of AC′

AC′ $\frac{y+3}{x-5} = \frac{2+3}{1-5} = -\frac{5}{4}$

$\Rightarrow 5x + 4y = 13$

Sol 4: (C)



$\frac{m+\frac{1}{3}}{1-\frac{m}{3}} = \frac{-\frac{1}{3}-\frac{1}{7}}{1-\frac{1}{21}}$

$\Rightarrow \frac{3m+1}{3-m} = \frac{-10}{20} \Rightarrow \frac{3m+1}{3-m} = \frac{-1}{2}$

$\Rightarrow 6m + 2 = m - 3 \Rightarrow m = -1$

It passes through $\left[\frac{-1}{10}, \frac{7}{10}\right]$

$\frac{y-\frac{7}{10}}{x+\frac{1}{10}} = -1 \Rightarrow x + y = \frac{7}{10} - \frac{1}{10} \Rightarrow 5x + 5y = 3$

Sol 5:(C) $X = x - h = 3 - 4 = -1$

$Y = y - k = -5 + 3 = -2$

Sol 6:(A) $\frac{x}{a} + \frac{y}{b} = 1$

$\frac{3}{5a} + \frac{4}{5b} = 1$ are concurrent at a fixed point.

Point is $x = \frac{3}{5}$ and $y = \frac{4}{5}$

$\Rightarrow \left[\frac{3}{5}, \frac{4}{5}\right]$

Sol 7: (D) P (1, 0) ; Q (–1, 0) ; R (2, 0)

$2SP^2 = SR^2 + SQ^2$

$(x+1)^2+y^2+(x-2)^2+y^2 = 2(x-1)^2+2y^2$

$\Rightarrow x^2+1+2x+x^2+4-4x=2(x^2+1-2x)$

$\Rightarrow 5 - 2x = 2 - 4x$

$2x = -3 \Rightarrow x = -\frac{3}{2}$

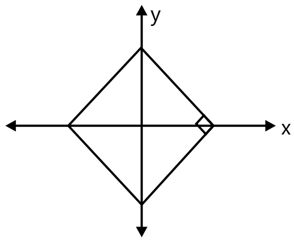
Which is a straight line parallel to y-axis.

Sol 8: (B) $ax \pm by \pm c = 0$

$\Rightarrow \pm\frac{x}{c/a} \pm \frac{y}{c/b} = 1$

$\therefore$ Area $= 4\left(\frac{1}{2}\cdot\frac{c}{a}\cdot\frac{c}{b}\right) = \frac{2c^2}{ab}$

Sol 9: (A) Lets assume 2 lines are x and y axis so the distances from 2 lines are .... .



$|x| + |y| = 1$

It forms a square.

Sol 10: (C) Given 3 lines are concurrent

$$\begin{vmatrix} 1 & 2 & 9 \\ 3 & -5 & 5 \\ a & b & 1 \end{vmatrix} = 0$$

$-5 - 5b - 2(3 - 5a) + 9(3b + 5a) = 0$

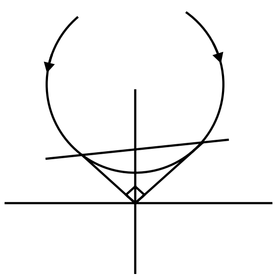
$-5 - 5b - 6 + 10a + 27b + 45a = 0$

$55a + 22b - 11 = 0$

$5a + 2b = 1$

The straight line $5x + 2y = 1$ passes through (a, b)

Sol 11: (A)



$px^2 + 2axy + qy^2 = r$ (homogenising)

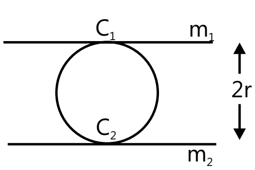
$px^2 + 2axy + qy^2 = r[ax + by]^2$

$x^2(p - ra^2) + y^2(q - rb^2) + 2axy - 2abxy = 0$

Lines are perpendicular i. e. $a + b = 0$

$p + q = r(a^2 + b^2)$

Sol 12: (B) $y - y_1 = m(x - x_1) \pm a\sqrt{1+m^2}$



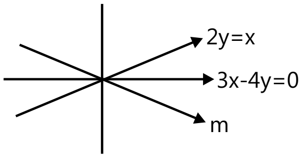
$$2r = \frac{C_1 - C_2}{\sqrt{1+m^2}}$$

$$2r = \frac{2a\sqrt{1+m^2}}{\sqrt{1+m^2}} = 2a \Rightarrow r = a$$

Sol 13: (A) $12x^2 - 16xy + 9xy - 12y^2 = 0$

$4x(3x - 4y) + 3y(3x - 4y)$

$(4x + 3y)(3x - 4y) = 0$



$$\frac{m - \frac{3}{4}}{1 + \frac{3m}{4}} = \frac{\frac{3}{4} - \frac{1}{2}}{1 + \frac{3}{4} \times \frac{1}{2}}$$

$$\frac{4m-3}{4+3m} = \frac{2}{11} \Rightarrow 44m - 33 = 8 + 6m$$

$$38m = 41 \Rightarrow y = \frac{41x}{38}$$

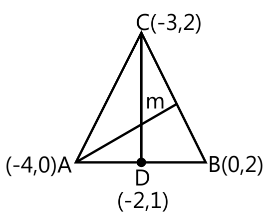
Sol 14: (C) $A = (4, -1)$

B is symmetric to A w. r. t. $y = x$

B is $[-1, 4]$

$AB = \sqrt{25+25} = 5\sqrt{2}$

Sol 15: (D) $A(-4, 0)$; $B(0, 2)$; $C(-3, 2)$



$$\text{Eq. of CD} \Rightarrow \frac{y-1}{x+2} = \frac{-1}{1}$$

$y - 1 = -2 - x$

$x + y = -1$

$m_{AB} = \frac{1}{2}$ $\qquad m_{AC} = +2$

$$\Rightarrow \frac{2-m}{1+2m} = \frac{m - \frac{1}{2}}{1 + \frac{m}{2}} = \frac{2m-1}{2+m}$$

$\Rightarrow 4 - m^2 = 4m^2 - 1 \Rightarrow m = \pm 1$

$$\frac{y}{x+4} = 1$$

Eq. AD $y = x + 4$

Intersection point $-1 - x = x + 4 \Rightarrow 2x = -5; 2y = 3$

Sol 16: (B) $2x - y + 5 = 0$ $m = 2$

$x + y - 5 = 0$ $m = -1$

$x - 2y - 5 = 0$ $m = \frac{1}{2}$

$$\sum \tan\theta = \left|\frac{3}{1-2}\right| + \left|\frac{-1-\frac{1}{2}}{1-\frac{1}{2}}\right| + \left|\frac{\frac{1}{2}-2}{1+1}\right|$$

$$= 3 + 3 + \frac{3}{4} = 6 + \frac{3}{4} = \frac{27}{4}$$

Sol 17: (B) $y = 0.2x$

$m = 0.2$

$\tan\theta = 0.2$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{0.4}{1-0.04} = \frac{0.4}{0.96} = \frac{5}{12}$$

$$\Rightarrow y = \frac{5x}{12}$$

Sol 18: (B) $x^2 + y^2 - 10x - 14y - 151 = 0$

M(–7, 2)

$49 + 4 + 70 - 28 - 151 < 0$

Point is inside

A(–7, +2) and Centre (5, 7)

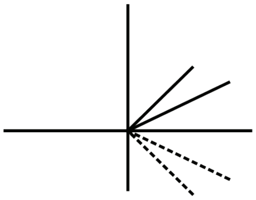
$r = \sqrt{25+49+151} = 15$

$AC = \sqrt{144+25} = 13$

$(r - AC) = 2$ (minimum distance)

Sol 19: (D) $ax^2 + 2hxy + by^2 = 0$

Image by line mirror $y = 0$



Replace y by (–y)

$ax^2 - 2hxy + by^2 = 0$

Sol 20: (D) $x^2 + y^2 - 4xy = 0$

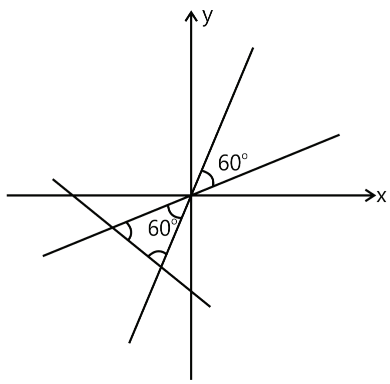
$m_1 . m_2 = 1$

$m_1 + m_2 = \frac{+4}{1} = +4$

$\Rightarrow m_1 = 2 - \sqrt{3}$

$m_2 = 2 + \sqrt{3}$ & $m_3 = -1$

It forms an equilateral triangle.



Sol 21: (D) $y = x$ (A lies on line ℓ_1)

A(x, x)

$y = 7x$ (B lies on this line ℓ_2)

(y, 7y)

$OA = OB \Rightarrow x\sqrt{2} = y\sqrt{50}$

$x = 5y$

Slope of line AB $= \frac{7y-x}{y-x} = \frac{2y}{-4y} = \frac{-1}{2}$

Previous Years' Questions

Sol 1: (A) The point O(0,0) is the mid point of A(-a,-b) and B(a,b).

Therefore, A,O,B are collinear and equation of line AOB is $y = \frac{b}{a}x$

Since, the fourth point D(a^2, ab) satisfies the above equation. Hence, the four points are collinear.

Sol 2: (C) Given lines, $x+2y-3=0$ and $3x+4y-7=0$ intersect at (1,1), which does not satisfy $2x+3y-a=0$ and $4x+5y-6=0$. Also, $3x+4y-7=0$ and $2x+3y-4=0$ intersect at (5,-2) which does not satisfy $x+2y-3=0$

$4x+5y-6=0$. Lastly, intersection point of $x+2y-3=0$ and $2x+3y-4=0$ is (-1,2) which satisfy $4x+5y-6=0$. Hence, only three lines are concurrent.

Sol 3: (C) Let B, C, D be the position of the point A(4, 1) after the three operations I, II and III respectively. Then, B(1, 4), C(1+2, 4) ie, (3, 4). The point D is obtained from C by rotating the coordinate axes through an angle $\pi/4$ in anticlockwise direction.

Therefore, the coordinates of D are given by

$$x = 3\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

and $Y = 3\sin\frac{\pi}{4} + 4\cos\frac{\pi}{4} = \frac{7}{\sqrt{2}}$

$\therefore$ Coordinates of D are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$

Sol 4: (A) The point of intersection of three lines are A(1, 1), B(2, -2), C(-2, 2).

Now, $|AB| = \sqrt{1+9} = \sqrt{10}, |BC| = \sqrt{16+16} = 4\sqrt{2},$

and $|CA| = \sqrt{9+1} = \sqrt{10}$

$\therefore$ Triangle is an isosceles

Sol 5: (A) By the given condition, we can take two perpendicular lines as x and y axes. If (h, k) is any point on the locus, then |h|+|k|=Sol 1 Therefore, the locus is |x|+|y|=1 This consist of a square of side 1.

Hence, the required locus is a square.

Sol 6: (C) Orthocentre of right angled triangle is at the vertex of right angle. Therefore, orthocentre of the triangle is at (0, 0).

Sol 7: (D) Let $y = \cos x \cos(x+2) - \cos^2(x+1)$

$= \cos(x+1-1)\cos(x+1+1) - \cos^2(x+1)$

$= \cos^2(x+1) - \sin^2 1 - \cos^2(x+1)$

$y = -\sin^2 1.$

This is a straight line which is parallel to x-axis It passes through $(\pi/2, -\sin^2 1)$.

Sol 8: (D) Slope of line x+3y=4 is -1/3

And slope of line 6x-2y=7 is 3.

Here, $3 \times \left(\frac{-1}{3}\right) = -1$

Therefore, these two lines are perpendicular which show that both diagonals are perpendicular.

Hence, PQRS must be a rhombus.

Sol 9: (D) Since, S is the mid point of Q and R.

$\therefore\ S \equiv \frac{7+6}{2}, \frac{3-1}{2} = \frac{13}{2}, 1$

Now, slope of PS $= m = \frac{2-1}{2-13/2} = -\frac{2}{9}$

Now, equation of the line passing through (1, -1) and parallel to PS is

$y + 1 = -\frac{2}{9}(x-1)$

$\Rightarrow 2x + 9y + 7 = 0$

Sol 10: (D) Let the vertices of triangle be $A(1, \sqrt{3})$, B(0, 0) and C (2, 0).

Here AB = BC = CA = 2.

Therefore, it is an equilateral triangle. So the incentre coincides with centroid.

$$\therefore\quad I = \left(\frac{0+1+2}{3}, \frac{0+0+\sqrt{3}}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right)$$

Sol 11: (A) On solving equations 3x + 4y = 9 and y = mx + 1, we get

$$x = \frac{5}{3+4m}$$

Now, for x to be an integer

$3 + 4m = \pm 5$ or ± 1

The integral values of m satisfying these conditions are – 2 and – 1

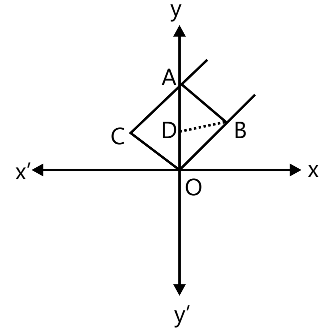
Sol 12: (D) Let lines OB : y = mx

CA : y = mx + 1

BA : y = nx + 1

and OC : y = nx

The point of intersection B of OB and AB has x-coordinate $\frac{1}{m-n}$.



Now, area of parallelogram

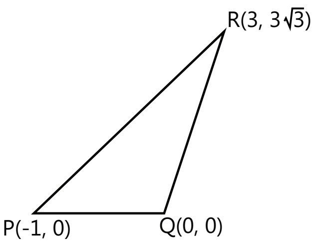
OBAC = 2 × area of ΔOBA

$$= 2\times\frac{1}{2}\times OA\times DB = 2\times\frac{1}{2}\times\frac{1}{m-n}$$

$$= \frac{1}{m-n} = \frac{1}{|m-n|}$$

Sol 13: (C) The equation of the line passing through points P(-1, 0) and Q(0, 0) is;

$Y = 0$ ... (i)



Equation of the line passing through points Q(0, 0) and R(3, $\sqrt{3}$) is;

$$\frac{y-0}{x-0} = \frac{3\sqrt{3}}{3-0} \quad \Rightarrow \frac{y}{x} = \frac{3\sqrt{3}}{3}$$

$\Rightarrow y = \sqrt{3}x$ ... (ii)

Therefore, the equations of the bisector of the angle PQR is

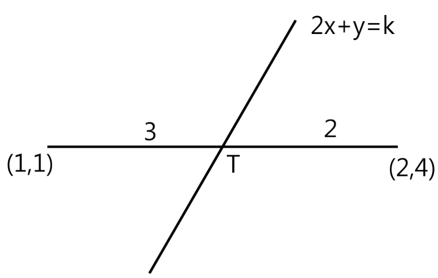
$$\frac{y}{\sqrt{1^2}} = \pm\frac{\sqrt{3}x - y}{\sqrt{3+1}} \quad \Rightarrow y = \pm\frac{\sqrt{3}x - y}{2}$$

$$\Rightarrow y = \frac{\sqrt{3}x - y}{2} \quad \text{or} \quad y = -\frac{\sqrt{3}x + y}{2}$$

$$\Rightarrow \sqrt{3}x - 3y = 0 \quad \text{or} \quad \sqrt{3}x + y = 0.$$

Sol 14: (C) Point T is given by

$$T \equiv \left(\frac{3\times2+2\times1}{3+2}, \frac{3\times4+2\times1}{3+2}\right) \equiv \left(\frac{8}{5}, \frac{14}{5}\right)$$



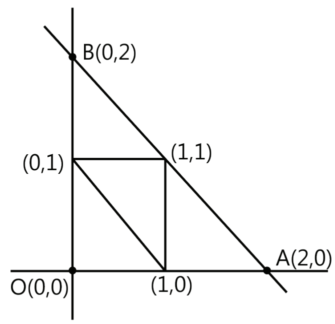
Which lies on the line 2x + y = k

$$2\times\frac{8}{5}+\frac{14}{5} = k \Rightarrow \frac{30}{5} = k$$

$\Rightarrow k = 6$

Sol 15: (A) Sides OA = OB = 2 and AB = $2\sqrt{2}$

X-coordinates of incentre of Δ OAB

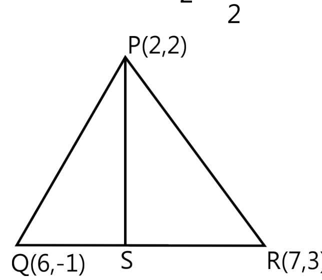


$$\equiv \frac{2\sqrt{2}\times0+2\times0+2\times2}{2\sqrt{2}+2+2}$$

$$= \frac{4}{4+2\sqrt{2}} = \frac{2}{2+2\sqrt{2}} = 2-\sqrt{2}$$

Sol 16: (B) $S \equiv \left(\frac{6+7}{2}, \frac{-1+3}{2}\right); \quad \equiv \left(\frac{13}{2}, 1\right)$

$$\text{Slope of PS} = \frac{2-1}{2-\frac{13}{2}} = -\frac{2}{9}$$



Equation of line passes through having slope $-\frac{2}{9}$

$$y+1 = -\frac{2}{9}(x-1) \Rightarrow 9y+9 = -2x+2$$

$$\Rightarrow 2x+9y-7 = 0$$

Sol 17: (C) The given equation of lines are

4ax + 2ay + c = 0 and 5bx + 2by + d = 0

Since, these lines intersect in fourth quadrant and point is equidistant from axes, then

The point can be of form (k, -k), k > 0

$\Rightarrow 4ak - 2ak + c = 0$ and $5bk - 2bk + d = 0$

$\Rightarrow 2ak + c = 0$ and $3bk + d = 0$

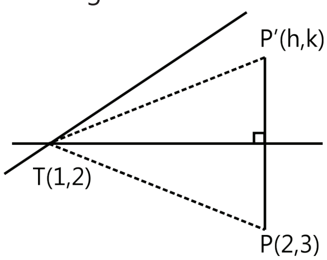
$\Rightarrow -\frac{c}{2a} = -\frac{d}{3b}$

$\Rightarrow 3bc - 2ad = 0$

Sol 18: (B) (2x - 3y + 4) + k (x – 2y + 3) = 0

Is a family of equation passes through T (1, 2)

From figure



PT = P'T

$\Rightarrow PT^2 = (P'T)^2$

$\Rightarrow (h-1)^2 + (k-2)^2 = (2-1)^2 + (3-1)^2$

$\Rightarrow (h-1)^2 + (k-2)^2 = 1 + 1 = 2$

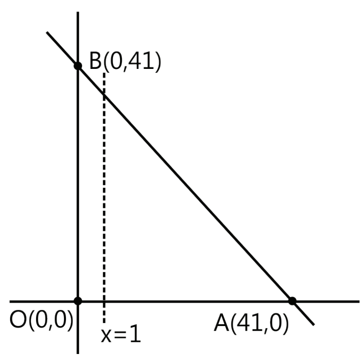
$\Rightarrow (x-1)^2 + (y-2)^2 = 2$

Locus is a circle.

Sol 19: (C) Equation of AB, x + y = 41

On line x = 1, there are 39 points inside Δ OAB

Similarly



On line x = 2, there are 38 points inside Δ DAB

Total points

= 39 + 38 + ... + 2 + 1

$= \frac{39(40)}{2} = 780$

Sol 20: (B) Let two sides AB and BC be x – y + = 0 and 7x – y – 5 = 0 respectively.

On solving, we get

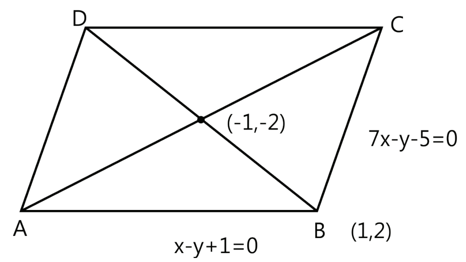
$B \equiv (1, 2)$

We know that diagonals in rhombus bisect each other, so $D \equiv (-3, -6)$

Since BC || AD

$\Rightarrow$ Equation of AD $7x - y + \lambda_1 = 0$, Passes through (-3, -6)

$\Rightarrow 7x - y + 15 = 0$



Similarly, AB || DC

$\Rightarrow$ Equation of DC x – y + λ_2 = 0, passes through (-3, -6)

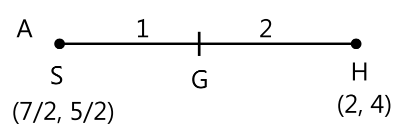
$\Rightarrow x - y - 3 = 0$

$\Rightarrow C\left(\frac{1}{3}, -\frac{8}{3}\right)$ and $A\left(-\frac{7}{3}, -\frac{4}{3}\right)$

JEE Advanced/Boards

Exercise 1

Sol 1: OA = 2AB = 4BC



G = $\frac{2+7}{3}, \frac{4+5}{3}$ = (3, 3)

A = (3, 3) = (a, a)

B = $\left(\frac{3a}{2}, a - \frac{a}{2}\right)$

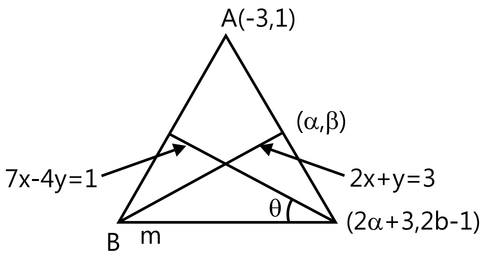
C = $\left(a + \frac{a}{2} + \frac{a}{4}, a - \frac{a}{2} + \frac{a}{4}\right)$

$\Rightarrow \left(a + \frac{a}{2} + \frac{a}{4} + \frac{a}{8} + ..., a - \frac{a}{2} + \frac{a}{4} - \frac{a}{8} ....x\right)$

$$N = \left(\frac{a}{1-\frac{1}{2}}, \frac{a}{1+\frac{1}{2}}\right) = \left[2a, \frac{2a}{3}\right]$$

$$\alpha + \beta = \frac{8a}{3} = 8$$

Sol 2:



$7(2\alpha + 3) - 4(2\beta - 1) = 1$

$14\alpha - 8\beta + 24 = 0 \Rightarrow 7\alpha - 4\beta + 12 = 0$

$2\alpha + \beta - 3 = 0$

$\alpha = 0, \beta = 3 \Rightarrow C = (3, 5)$

Slope of AC $= \frac{2}{3}$

$$\frac{m-\frac{7}{4}}{1+\frac{7m}{4}} = \frac{\frac{7}{4}-\frac{2}{3}}{1+\frac{7}{4}.\frac{2}{3}} \Rightarrow \frac{4m-7}{7m+4} = \frac{13}{26} = \frac{1}{2}$$

$8m - 14 = 7m + 4 \Rightarrow m = 18$

Equation of AC $= \frac{y-5}{x-3} = \frac{2}{3}$

$\Rightarrow 3y - 9 = 2x$

Equation of BC $= \frac{y-5}{x-3} = 18$

$\Rightarrow 18x - y = 49$

$2x + y = 3; \quad 18x - y = 49$

$x = \frac{26}{10}, y = \frac{-11}{5}$

$B = \left(\frac{26}{10}, \frac{-11}{5}\right)$

Equation of AB $= \frac{y-1}{x+3} = \frac{\frac{-11}{5}-1}{\frac{26}{10}+3} = \frac{-16}{28} = \frac{-4}{7}$

$\Rightarrow 7y - 7 = -4x - 12 \Rightarrow 4x + 7y + 5 = 0$

Sol 3: $x - y = 4$ [Line perpendicular to AB]

$2x - y = 5$ [Line perpendicular to AC]

$A = (-2, 3)$

$\Rightarrow$ Eq. of AB is $x + y = 1$ ... (i)

Eq. of AC is $x + 2y = 4$ ... (ii)

$S = \left(\frac{3}{2}, \frac{5}{2}\right)$

$\Rightarrow$ Eq. of perpendicular bisector of AB and AC are

$y = x + 1$ ... (iii)

$y = 2x - \frac{1}{2}$ ... (iv) respectively.

Eq. (i) and (iii) $\Rightarrow E = (0, 1)$ [Midpoint of A, B]

Eq. (ii) and (iv) $\Rightarrow F = (1, 3/2)$ [Midpoint of A, C]

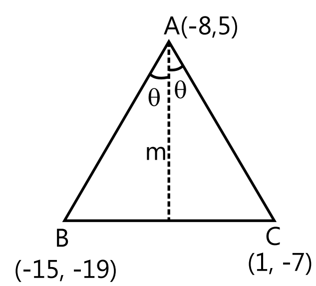
$\Rightarrow B = (2, -1)$ and $C = (4, 0)$

Eq. of median to AB is $\frac{y-0}{x-4} = \frac{1-0}{0-4} \Rightarrow x + 4y = 4$

Eq. of median to AC is

$$\frac{y+1}{x-2} = \frac{\frac{3}{2}+1}{1-2} \Rightarrow 5x + 2y = 8$$

Sol 4:



$m_{AB} = \frac{24}{7}$

$m_{AC} = \frac{12}{-9} = \frac{-4}{3}$

$$\frac{\frac{24}{7}-m}{1+\frac{24m}{7}} = \frac{m+\frac{4}{3}}{1-\frac{4m}{3}}$$

$\Rightarrow \frac{24-7m}{24m+7} = \frac{3m+4}{3-4m}$

$\Rightarrow 72 - 117m + 28m^2 = 72m^2 + 117m + 28$

$\Rightarrow 44m^2 + 234m - 44 = 0$

$\Rightarrow 22m^2 + 117m - 22 = 0$

$\Rightarrow (11m - 2)(2m + 11) = 0$

$\Rightarrow m = 2/11, -11/2$

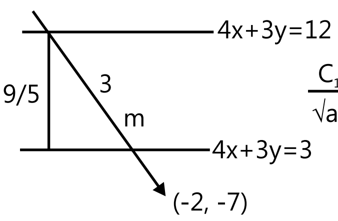
$\frac{y-5}{x+8} = \frac{2}{11}$ or $-\frac{11}{2}$

$\Rightarrow 11y - 55 = 2x + 16$ or $2y - 10 = -11x - 88$

$\Rightarrow 2x - 11y + 71 = 0$ or $2y + 11x + 78 = 0$

$\Rightarrow a = 11;\ c = 78$

Sol 5:



$$\frac{C_1 - C_2}{\sqrt{a^2 + b^2}} = \frac{9}{5}$$

$(-2 + r\cos\theta, -7 + r\sin\theta)$ lies on $4x + 3y = 3$

$-2 + (r+3)\cos\theta, -7 + (r+3)\sin\theta$ lies on $4x + 3y = 12$

$4(r\cos\theta - 2) + 3(r\sin\theta - 7) = 3$ .... (i)

$4((r+3)\cos\theta - 2) + 3((r+3)\sin\theta - 7) = 12$ ....(ii)

On solving (i) and (ii), we get

$-12\cos\theta - 9\sin\theta = -9$

$\Rightarrow 4\cos\theta + 3\sin\theta = 3$

$\Rightarrow 9 + 16\cos^2\theta - 24\cos\theta = 9 - 9\cos^2\theta$

$\Rightarrow 25\cos^2\theta = 24\cos\theta$

$\Rightarrow \cos\theta = 0$ or $\cos\theta = 24/25$

$\Rightarrow \sin\theta = 1$ or $\sin\theta = \frac{-7}{24}$

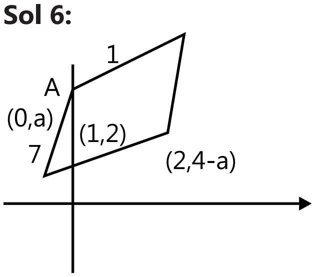
$\Rightarrow m = \infty, \frac{-7}{24}$

$\Rightarrow \frac{y+7}{x+2} = \frac{-7}{24}$ or ∞

$\Rightarrow x = -2$ or $24y + 168 = -7x - 14$

$\Rightarrow 7x + 24y = 182$ or $x = -2$

Sol 6:



Sides of the rhombus are

$y = x + C_1;\ y = 7x + C_2$

Eq. of diagonals

$$\Rightarrow \frac{y - 7x - C_2}{\sqrt{50}} = \pm\left(\frac{y - x - C_1}{\sqrt{2}}\right)$$

$\Rightarrow y - 7x - C_2 = \pm(5y - 5x - 5C_1)$

$\Rightarrow 4y + 2x - 5C_1 + C_2 = 0$ and $6y - 12x - 5C_1 - C_2 = 0$

go through point (1, 2) (given)

$\Rightarrow 10 - 5C_1 + C_2 = 0$ and $5C_1 + C_2 = 0$

$-2C_2 = 10 \Rightarrow C_2 = -5$

$-5C_1 = 5 \Rightarrow C_1 = 1$

$\Rightarrow$ Diagonals are

$4y + 2x - 10 = 0 \Rightarrow 2y + x - 5 = 0$

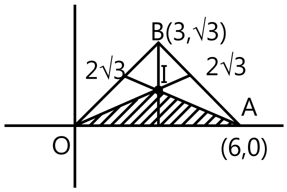
$6y - 12x + 0 = 0 \Rightarrow y = 2x$

If A vertex is at y-axis $\Rightarrow x = 0$

$y = 2(0) = 0 \Rightarrow (0, 0)$

$2y + 0 - 5 = 0 \Rightarrow y = 5/2 \Rightarrow (0, 5/2)$

Sol 7: O(0, 0) ; A(6, 0) ; B(3, $\sqrt{3}$)



d(P, OA) = d(P, OB) if P lies along the angular bisector of AOB. Similarly if d(P, OA) = d(P, AB) then P lies along the angular bisector of OAB

Hence the region where

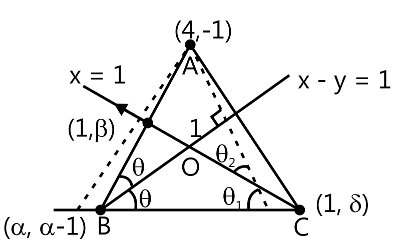
d(P, OA) $\le$ minimum {d(P, OB), d(P, AB)} is OAI.

Area of OAI $= \frac{1}{2}OA\left(\frac{OA}{2}\tan\frac{AOB}{2}\right)$

$= \frac{1}{2}6\left(3\tan 15^0\right) = 9\left(2 - \sqrt{3}\right)$

$\Rightarrow a + b = 2 + 3 = 5$

Sol 8:



Reflection of (4, –1) about both of bisectors lie on the line BC

So reflection about $x - 1 = 0$ is A′(2, 1)

Reflection about $x - y = 1$ is A″(0, 3)

Hence Eq. of BC is $y - 3 = 2x$

B and C are intersections of BC with

$x - y - 1 = 0$ and $x = 1$ respectively

$\Rightarrow B = (4, 5); C = (1, 5)$

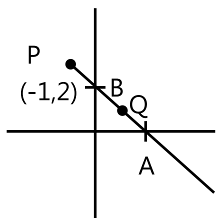
$\Rightarrow$ Eq. of AB is $x - 2y = 6$

Eq. of AC is $2x + y = 7$

Sol 9: The equation of line

$(y - 2) = m(x + 1)$ .... (i)

$\Rightarrow A = \left(\frac{-2}{m} - 1, 0\right); B = (0, 2 + m)$



Assume Q = (h, k)

Q is on the line

$\Rightarrow k = 2 + m(h + 1)$

$PQ = \sqrt{(h+1)^2 + (k-2)^2}$

$= \sqrt{(h+1)^2 + m^2(h+1)^2} = |h+1|\sqrt{m^2+1}$

$PA = \sqrt{\left(\frac{-2}{m} - 1 + 1\right)^2 + 2^2} = \left|\frac{2}{m}\right|\sqrt{1+m^2}$

$PB = \sqrt{1+m^2}$

It's given that PA, PQ, PB are in H. P.

$\Rightarrow \frac{2}{PQ} = \frac{1}{PA} + \frac{1}{PB}$

$\Rightarrow \frac{2}{|h+1|\sqrt{m^2+1}} = \left|\frac{m}{2}\right|\frac{1}{\sqrt{1+m^2}} + \frac{1}{\sqrt{1+m^2}}$

$\Rightarrow \frac{2}{|h+1|} = \frac{1}{2}\left|\frac{k-2}{h+1}\right| + 1$

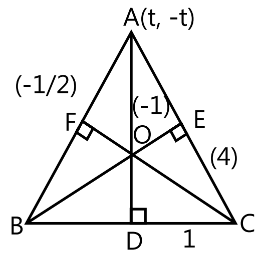
$\Rightarrow 2|x + 1| + |y - 2| = 4$

If $m < 0$ then $y = 2x$

If $m > 0$ then $2x + y = 4$ (If $x > 1$ and $y > 2$)

$= 4$ (If $x < 1$ and $y < 2$)

Sol 10:



$m_{BC} = 1$

$m_{AC} = 4 \Rightarrow y = 4x - 5t$ ... (i)

$m_{AB} = \frac{-1}{2} \Rightarrow 2y + x + t = 0$ ...(ii)

Solving AB and BE we get $x = -4y$

$t = 2y \Rightarrow y = \frac{t}{2} \Rightarrow B = \left(\frac{-4t}{2}, \frac{t}{2}\right)$

CF and AC $\Rightarrow y = 2x$

$2x = 5t \Rightarrow C = \left[\frac{5t}{2}, 5t\right]$

G = (h, k)

$3h = t + \frac{5t}{2} - \frac{4t}{2}; 3k = t + 5t + \frac{t}{2}$

$h = \frac{t}{2}; k = \frac{3t}{2} \Rightarrow k = 3h$

Sol 11: $y = m_1x$; $y = m_2x$

$y^2 - (m_1 + m_2)xy + m_1m_2x^2 = 0$

$\frac{m_1x_1 - y_1}{\sqrt{1+m_1^2}} = \frac{m_2x_1 - y_1}{\sqrt{1+m_2^2}} = \delta$

$(mx_1 - y_1)^2 = \delta^2(1 + m^2)$

$m^2x_1^2 + y_1^2 - 2mx_1y_1 = \delta^2 + \delta^2m^2$

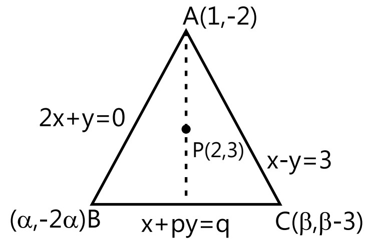
$m^2(x_1^2 - \delta^2) + m(-2x_1y_1) + y^2 - \delta^2 = 0$

$m_1 + m_2 = \frac{2x_1y_1}{x_1^2 - \delta^2}; m_1m_2 = \frac{y^2 - \delta^2}{x_1^2 - \delta^2}$

$y^2 - \frac{2x_1y_1}{x_1^2 - \delta^2}xy + \frac{y_1^2 - \delta^2}{x_1^2 - \delta^2}x^2 = 0$

$(x_1^2 - \delta^2)y^2 + (y_1^2 - \delta^2)x^2 - 2x_1y_1xy = 0$

Sol 12:



If P is centroid

$\Rightarrow \alpha + \beta + 1 = 6$ ...(i)

$\Rightarrow -2 - 2\alpha + \beta - 3 = 9$

$\Rightarrow 2\alpha - \beta = -14$ ...(ii)

$\Rightarrow \alpha + \beta = 5$

$\Rightarrow \alpha = -3; \beta = 8$

B and C lie on line BC

$\Rightarrow \alpha - 2p\alpha = q$

$\Rightarrow \beta + p(\beta - 3) = q$

$\Rightarrow -3 + 6p = q$

$\Rightarrow 8 + 5p = q$

$\Rightarrow p = 11; q = 63 \Rightarrow p + q = 74$

If P is orthocentre

$m_{(AP)} = 5$

$m_{BC} = \frac{-1}{5} = \frac{-1}{p} \Rightarrow p = 5$

m(AC) = 1

$m(BP) = -1 = \frac{3+2\alpha}{2-\alpha}$

$\Rightarrow \alpha = -5$

$\Rightarrow -5 - 10(-5) = q \Rightarrow q = 45$

$\Rightarrow p + q = 50$

If P is circumcentre

Mid point of AB = $\left(\frac{\alpha+1}{2}, \frac{-2-2\alpha}{2}\right) = \left(\frac{\alpha+1}{2}, -(\alpha+1)\right)$

$m_{(PM)} = \frac{1}{2} = \frac{3-(-\alpha-1)}{2-\left(\frac{\alpha+1}{2}\right)}$

$\Rightarrow \frac{1}{2} = \frac{2(4+\alpha)}{(3-\alpha)}$

$\Rightarrow 3 - \alpha = 16 + 4\alpha \Rightarrow \alpha = \frac{-13}{5}$

Mid point of AC = $\left(\frac{1+\beta}{2}, \frac{\beta-5}{2}\right)$

$\Rightarrow -1 = \frac{3-\left(\frac{\beta-5}{2}\right)}{2-\left(\frac{\beta+1}{2}\right)} \Rightarrow -1 = \frac{11-\beta}{3-\beta}$

$\Rightarrow \beta - 3 = 11 - \beta \Rightarrow \beta = \frac{14}{2} = 7$

$\Rightarrow 7 + 4p = q$

$\Rightarrow -13 + 26p = 5q$

$\Rightarrow 35 + 20p = 5q$

$\Rightarrow -48 + 6p = 0$

$\Rightarrow p = 8 ; q = 39$

$\Rightarrow p + q = 47$

Sol 13: $x^2 - 3y^2 - 2xy + 8y - 4 = 0$

Assume eq. are

$(x + ay + c)(x + by + d) = 0$

$\Rightarrow db = -3; dc = -4$

$\Rightarrow a+b = -2; bc+ad = 8$

$\Rightarrow (a-b)^2 = \sqrt{4+12} = 4^2 d + c = 0$

$a - b = 4 \Rightarrow d = -c$

$\Rightarrow a = 1 \Rightarrow -c^2 = -4$

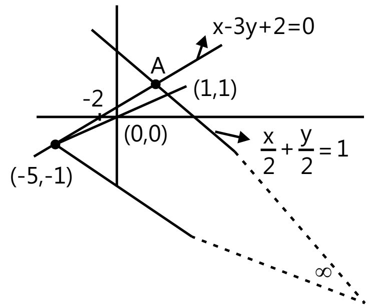
$\Rightarrow d = -3\ c = \pm 2$

$bc - ac = 8$

$(b - a) = \frac{8}{c}$

$\Rightarrow c = -2, \Rightarrow d = 2$

Eq. are $(x + y - 2)(x - 3y + 2) = 0$



For A $\Rightarrow x + y = 2 \Rightarrow x = 2 - y$

$\Rightarrow 2 - y - 3y + 2 = 0$

$4y = 4 \Rightarrow y = 1$

$\Rightarrow x = 2 - 1 = 1$

Slope of L_1 [(–5, –1) to (0, 0)] = $\frac{1}{5}$

Slope of $\frac{x}{2}+\frac{y}{2} = 1 \Rightarrow \frac{-2}{2} = -1$

Range (–1, 1/5)

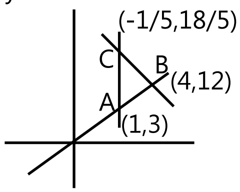
($\therefore$ Third line is go through (0, 0) and for triangle parallel to $\frac{x}{2}+\frac{y}{2} = 1$ meet at infinity)

$\left(-1, \frac{1}{5}\right) = (a, b)$

$\Rightarrow \left(a+\frac{1}{b^2}\right) = -1 + 5^2 = -1 + 15 = 24$

Sol 14: $2x^2 + 3xy - 2y^2 - 10x + 15y - 28 = 0$

y = 3x



$2x^2+9x^2-2(9x^2)-10x+15(3x)-28 = 0$

$-7x^2 + 35x - 28 = 0$

$x^2 - 5x + 4 = 0 \Rightarrow x = 1, 4$

$\therefore$ y = 3, 12

$\frac{\partial p}{dx} = 4x + 3y - 10 = 0$

$\frac{\partial p}{dy} = -4y + 3x + 15 = 0$

$25y = 90 \Rightarrow y = \frac{18}{5}$

$x = \frac{10-\frac{54}{5}}{4} = \frac{-1}{5}$

$\Rightarrow m_1 = 3,\ m_2 = \frac{\frac{3}{5}}{\frac{-6}{5}} = \frac{-1}{2},$

$\Rightarrow m_3 = \frac{12-\frac{18}{5}}{4+\frac{1}{5}} = \frac{42}{21} = 2$

$\Rightarrow \tan\theta_1 = \frac{3-2}{1+6} = \frac{1}{7}$

$\Rightarrow \tan\theta_2 = \frac{3+\frac{1}{2}}{1-\frac{3}{2}} = -7$

$\Rightarrow \cot\theta_1 + \cot\theta_2 + \cot\theta_3 = 17 + \frac{1}{7} + 0 = \frac{50}{7}$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 4 & 12 & 1 \\ -\frac{1}{5} & \frac{18}{5} & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[1\left(12-\frac{18}{5}\right)-3\left(4+\frac{1}{5}\right)+1\left(\frac{18.4}{5}+\frac{1.12}{5}\right)\right]$$

$$= \frac{1}{2}\left[\frac{42}{5}-\frac{3.21}{5}+\frac{72+12}{5}\right]$$

$$= \frac{1}{2}\left[\frac{84+42-63}{5}\right] = \frac{63}{10}$$

$\Rightarrow a_1 = \sqrt{9+81} = 3\sqrt{10}$

$\Rightarrow a_2 = \sqrt{\frac{36}{25}+\frac{9}{25}} = \sqrt{45} = \frac{3}{5}\sqrt{45}$

$\Rightarrow a_3 = \sqrt{\left(\frac{21}{5}\right)^2+\left(12-\frac{18}{5}\right)^2} = \sqrt{\frac{441+(42)^2}{25}} = \frac{21}{5}\sqrt{5}$

Incentre will be

$$\frac{\frac{1\times21\sqrt{5}}{5}+\frac{4\times3\sqrt{5}}{5}-\frac{1}{5}\times3\sqrt{10}}{\frac{15\sqrt{10}}{5}+\frac{3\sqrt{5}}{5}+\frac{21\sqrt{5}}{5}},$$

$$\frac{\frac{3\times21\sqrt{5}}{5}+\frac{12\times3\sqrt{5}}{5}+\frac{18}{8}\sqrt{10}}{\frac{24\sqrt{5}+15\sqrt{10}}{5}}$$

$$= \left[\frac{11\sqrt{5}-\sqrt{10}}{8\sqrt{5}+5\sqrt{10}}, \frac{33\sqrt{5}+18\sqrt{10}}{8\sqrt{5}+5\sqrt{10}}\right]$$

Radius = distance of incentre from any of the sides.

$$\frac{4-3x}{\sqrt{10}} = \frac{18\sqrt{10}+3\sqrt{10}}{\sqrt{10}(8\sqrt{5}+5\sqrt{10})} = \frac{21}{8\sqrt{5}+5\sqrt{10}}$$

$$= \frac{21(8\sqrt{5}-5\sqrt{10})}{70} = \frac{3}{10}(8\sqrt{5}-5\sqrt{10})$$

Sol 15: y = mx + c

$3x^2-y^2- 2x\left(\frac{y-mx}{c}\right)+4y\left(\frac{y-mx}{c}\right)=0$

$3x^2-y^2- \frac{2xy}{c}+\frac{2mx^2}{c}+\frac{4y^2}{c}-\frac{4mxy}{c} = 0$

$$x^2\left(3+\frac{2m}{c}\right)+y^2\left(\frac{4}{c}-1\right)+xy\left(-\frac{4m}{c}-\frac{2}{c}\right)=0$$

Now $\frac{a}{b}=-1$

$\Rightarrow \frac{3c+2m}{4-c}=-1$

$\Rightarrow 3c + 2m = c - 4$

$\Rightarrow 2c + 2m = -4$

$\Rightarrow c + m = -2$

Point is (1, –2)

for $3x^2 + 3y^2 + 2x + 4y = 0$

equation will be

$$x^2\left(3-\frac{3m}{c}\right)+y^2\left(\frac{4}{c}-1\right)+xy\left(\frac{-4m}{c}+\frac{2}{c}\right)=0$$

$\frac{3c-2m}{4-c}=-1 \Rightarrow 3c-2m=c-4$

$c - m = -2 \Rightarrow$ point is (–1, –2)

Sol 16: $y = m(x-1) - \frac{y}{m} + x = 1$

$$\Rightarrow x^2+y^2+6x\left(x-\frac{y}{m}\right)-10\left(x-\frac{y}{m}\right)+1\left(x-\frac{y}{m}\right)^2=0$$

$$\Rightarrow x^2+y^2+6x^2-\frac{6xy}{m}-10xy+\frac{10y^2}{m}+x^2$$

$$+\frac{y^2}{m^2}-\frac{2xy}{m}=0$$

$$\Rightarrow 8x^2+y^2\left[1+\frac{10}{m}+\frac{1}{m^2}\right]+xy\left[-\frac{6}{m}-10-\frac{2}{m}\right]=0$$

$$\Rightarrow \frac{a}{b}=\frac{8}{1+\frac{10}{m}+\frac{1}{m^2}}=-1$$

$\Rightarrow -8=\frac{1}{m^2}+\frac{10}{m}+1$

$\Rightarrow -9m^2 = 1 + 10m$

$\Rightarrow 9m^2 + 10m + 1 = 0$

$\Rightarrow (9m^2 + 9m + m + 1) = 0$

$\Rightarrow 9m(m + 1) + 1(m + 1) = 0$

$\Rightarrow m = -1, \frac{-1}{9}$

$\Rightarrow y = 1 - x, 9y = 1 - x$

Sol 17: $x^2 + 4xy + 4y^2 - 5x - 10y + 4 = 0$

$\Rightarrow y^2 - 4y + 3 = 0$

$\Rightarrow y^2 - 3y - y + 3 = 0 \Rightarrow y = 1, 3$

For y = 1

$\Rightarrow x^2 + 4x + 4 - 5x - 10 + 4 = 0$

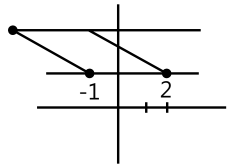
$x^2 - x - 2 = 0 \Rightarrow x = -1, 2$

For y = 3

$\Rightarrow x^2 + 12x + 36 - 5x - 30 + 4 = 0$

$\Rightarrow x^2 + 7x + 10$

$\Rightarrow x = -5, -2$



The points are (–1, 1) (2, 1)

(–5, 3) (–2, 2)

$$\frac{1}{2}\begin{vmatrix}-1 & 1\\ 2 & 1\\ -2 & 3\\ -5 & 3\\ -1 & 1\end{vmatrix}=\frac{1}{2}[-1+6-5-6-(2-2-15-3)]$$

$=\frac{1}{2}[-6-(-18)] = 6$ units

Diagonals are $=\sqrt{49+4}, \sqrt{-1+4}=\sqrt{53}, \sqrt{5}$

Sol 18: $6x^2 - xy - y^2 + x + 12y - 35 = 0$

$(y - m_1x - c_1)(y - m_2x - c_2) = 0$

$y^2 - m_2xy - c_2y - m_1xy + m_1m_2x^2$

$+ m_1xc_2 - c_1y + c_1m_2x + c_1c_2$

$m_1m_2x^2 + y^2 + xy(-m_2 - m_1) + x(m_2c_1 + m_1c_2)$

$+ y(-c_1 - c_2) + c_1c_2 = 0$

$m_1m_2 = -6;$ $m_1 + m_2 = -1$

This gives $m_1 = 2$ and $m_2 = -3, -3, 2$

$\Rightarrow c_1c_2 = 35$

$\Rightarrow c_1 + c_2 = 12$

$\Rightarrow m_2c_1 + m_1c_2 = -1$

$\Rightarrow -3c_2 + 2c_1 = -1$

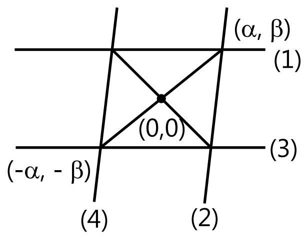
$\Rightarrow c_2 = 5, c_1 = 7$

$y = -3x + c_1$ .... (i)

$y = 2x + c_2$ .... (ii)

$y = -3x + 7$ .... (iii)

$y = 2x + 5$ .... (iv)



$\Rightarrow 3\alpha + \beta + 7 = 0$

$3\alpha + \beta = c_1$

$\Rightarrow c_1 = -7$

$y = 2\alpha + 5$

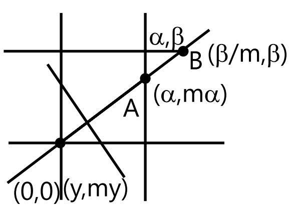
$-y = -2\alpha + c_2$

$c_2 = -5$

Combined equation will be

$6x^2 - xy - y^2 + x(-1) + y(-12) - 35 = 0$

Sol 19:



$y = mx$

$y = \dfrac{8}{1+m}$

OA. OB. OC = 48

$$\sqrt{y^2 + m^2y^2}\ \sqrt{\beta^2 + \frac{\beta^2}{m^2}}\ \sqrt{\alpha^2 + \alpha^2 m^2} = 48$$

$$\frac{8}{(1+m)}\alpha\beta\frac{(1+m^2)^{3/2}}{(m)} = 48$$

$\Rightarrow (1 + m^2)^{3/2} = \sqrt{2}\, m(1 + m)$

$\Rightarrow (1 + m^2)^3 = 2m^2(1+m^2+2m)$

$\Rightarrow m^6 + 1 + 3m^2 + 3m^4 = (m^4 + m^2 + 2m^3)^2$

$m^6 + m^4 + m^2 + 1 = 4m^3$

$\Rightarrow m = 1$

$$f(\alpha, \beta) = \left|\frac{\beta}{\alpha} - \frac{3}{2}\right| + (3x-2y)^2 + \sqrt{e(x-2)+2(y-3)} = 0$$

i. e. $\alpha = 2$, $\beta = 3$

OA + OB + OC

$$= \frac{8\sqrt{1+m^2}}{1+m} + \beta\sqrt{\frac{m^2+1}{m^2}} + \alpha\sqrt{1+m^2}$$

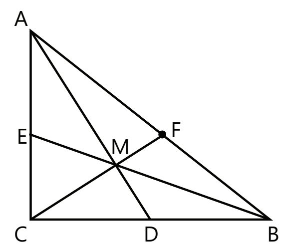
$$= \frac{8\sqrt{1+m^2}}{1+m} + \frac{3}{m}\sqrt{1+m^2} + 2\sqrt{1+m^2}$$

$$= 4\sqrt{2} + 3\sqrt{2} + 2\sqrt{2} = 9\sqrt{2}$$

Equation is $y = mx \Rightarrow y = x$

Sol 20: Area(ABC) = 3 Arc (AMB)

In ΔACB



$AF=BF=CF= \dfrac{AB}{2} = 30$

CM:MF=2:1

$\Rightarrow MF = 10$

By Apollonius theorem

$$AM^2 + BM^2 = 2\left(CF^2 + AF^2\right)$$

$$AM^2 + BM^2 = 2000$$

Let $\angle AMB = \theta$

$$\tan\theta = \frac{M_{BE} - M_{AD}}{1 + M_{BE}M_{AD}} = \frac{2-1}{1+2} = \frac{1}{3}$$

$\Rightarrow \cos\theta\dfrac{3}{\sqrt{10}}$ and $\sin\theta = \dfrac{1}{\sqrt{10}}$

In ΔAMB

$$\cos\theta = \frac{AM^2 + BM^2 - AB^2}{2AM.BM}$$

$$\frac{3}{\sqrt{10}} = \frac{2000 - 3600}{2AM.BM}$$

$$\Rightarrow AM.BM = -\frac{1600}{6}\sqrt{10}$$

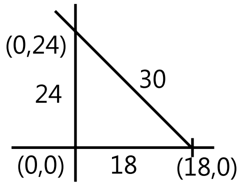
Area of $\Delta AMB = \dfrac{1}{2}(AM)(BM)\sin\theta$

$$= \frac{1}{2} \times \frac{1600}{6}\sqrt{10} \times \frac{1}{\sqrt{10}} = \frac{400}{3}$$

Area of ABC $3 \times$ Area of ΔAMB

$$= 3 \times \frac{400}{3} = 400 \text{ Sq. units}$$

Sol 21:



Centroid = $\frac{18}{3}, \frac{24}{3}$ = (6, 8)

Circumcentre = $\left(\frac{18}{2}, \frac{24}{3}\right)$ = (9, 12)

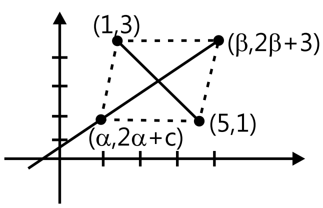
Incentre

$= \left(\frac{24.18}{30+24+18}, \frac{18.24}{30+24+18}\right) = \left(\frac{24.18}{72}, 6\right)$ = (6, 6)

Area = $\frac{1}{2}\begin{vmatrix} 6 & 6 & 1 \\ 6 & 8 & 1 \\ 9 & 12 & 1 \end{vmatrix}$

$= \frac{1}{2}[6(-4)-6(6-9)+1(72-72)] = \frac{1}{2}[-24+18]$ = 3 sq. units

Sol 22:



$\left(\frac{2\beta+c-1}{\beta-5}\right)\left(\frac{2\beta+c+3}{\beta-1}\right) = -1$

$\beta^2 - 6\beta + 5 = -(2\beta + c - 1)(2\beta + c - 3)$

$\frac{\alpha+\beta}{2} = 3; \qquad \frac{2\alpha+2\beta+2c}{2} = 2$

$\alpha + \beta = 6; \qquad \alpha + \beta + c = 2$

$c = -4$

$\beta^2 - 6\beta + 5 = -(2\beta - 5)(2\beta - 7) = -(4\beta^2 - 24\beta + 35)$

$5\beta^2 - 30\beta + 40 = 0$

$\beta = 2, 4$

Coordinates are (2, 0) and (4, 4)

Sol 23: $5x - y = 1$

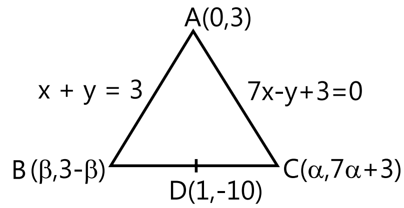
$y = \frac{-1}{5}x + c$

A(0, 0) ; B(0, c) ; C(5c, 0)

$\frac{1}{2} \times 5c^2 = 5$

$c^2 = 2; \qquad c = \pm\sqrt{2}$

$5y + x = \pm 5\sqrt{2}$



$m_{BD} = m_{BC}$

$\frac{13-\beta}{\beta-1} = \frac{7\alpha+13}{\alpha-1}$

$13\alpha - 13 - \alpha\beta + \beta = 7\alpha\beta - 7\alpha + 13\beta - 13$

$8\alpha\beta = 20\alpha - 12\beta$

$\sqrt{\beta^2+\beta^2} = \sqrt{\alpha^2+(7\alpha)^2}$

$2\beta^2 = 50\alpha^2$

$\beta = \pm 5\alpha$

Case I: $40\alpha^2 = 20\alpha - 60\alpha$

$40\alpha^2 = -40\alpha$

$\alpha = -1; \quad \beta = -5$

$\frac{y+10}{x-1} = \frac{13+5}{-6} = -3$

$y + 3x = -7$

Case II: $\beta = -5\alpha$

$-40\alpha^2 = 80\alpha$

$\alpha = -2; \qquad \beta = +10$

$\frac{y+10}{x-1} = \frac{3}{9} = \frac{1}{3}$

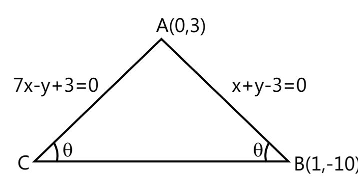
$3y + 30 = x - 1$

$x - 3y = 31$

Sol 24: Let slope of BC is m, then equations is y + 10 = m(x -1)

$\angle ACB = \angle ABC$

$\tan\angle ACB = \tan\angle ABC$



$\frac{7-m}{1+7m} = \frac{m-(1)}{1+m(-1)}$

$\Rightarrow \frac{7-m}{1+7m} = \frac{m+1}{1-m}$

$\Rightarrow 7 - 7m - m + m^2 = 1 + m + 7m^2 + 7m$

$\Rightarrow 6m^2 + 16m - 6 = 0$

$\Rightarrow 3m^2 + 8m - 3 = 0$

$\Rightarrow 3m^2 + 8m - 3 = 0$

$\Rightarrow 3m^2 + 9m - m - 3 = 0$

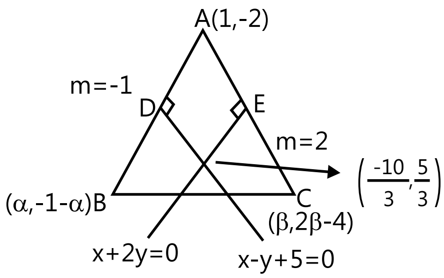
$\Rightarrow (3m-1)(m+3) = 0$

$\Rightarrow m = -3,\ 1/3$

Equations

$3x + y + 7 = 0$ or $x - 3y - 31 = 0$

Sol 25:



Eq. of AB = $\dfrac{y+2}{x-1} = -1$

$y + 2 = 1 - x$

$x + y + 1 = 0$

Eq. of AC = $\dfrac{y+2}{x-1} = 2$

$y + 2 = 2x - 2 \Rightarrow y + 4 = 2x$

$D \rightarrow \left(\dfrac{\alpha+1}{\alpha}, \dfrac{-\alpha-3}{\alpha}\right)$; $\quad E \rightarrow \left(\dfrac{\beta+1}{2}, \dfrac{2\beta-6}{2}\right)$

$\dfrac{\alpha+1}{2} + \left(\dfrac{\alpha+3}{2}\right) + 5 = 0$

$\alpha + 7 = 0 \Rightarrow \alpha = -7$

$\dfrac{\beta+1}{2} = -2\left(\dfrac{2\beta-6}{2}\right)$

$\beta + 1 = -4\beta + 12;\quad \beta = \dfrac{11}{5}$

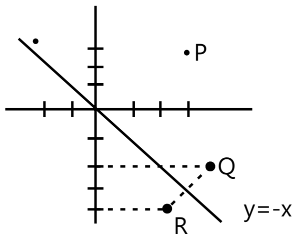
$(-7, 6)\ \left(\dfrac{11}{5}, \dfrac{2}{5}\right)$

$\dfrac{y-6}{x+7} = \dfrac{28}{-46} = \dfrac{-14}{23}$

$23y + 14x = 138 - 98$

$14x + 23y = 40$

Sol 26:

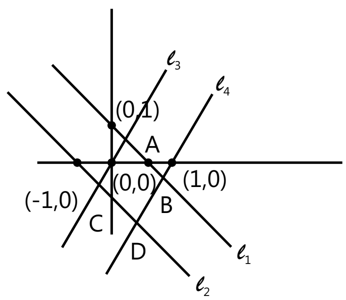


P(3, 2), Q(3, –2), R(2, –3), S = (–2, 3)

Area of PQRS will be, A = $\dfrac{1}{2}\begin{vmatrix} 3 & 2 \\ 3 & -2 \\ 2 & -3 \\ -2 & 3 \end{vmatrix}$

$= \dfrac{1}{2}[-6-9+6-4-(6-4+6+9)] = \dfrac{1}{2}[-13-17] = 15$ units

Sol 27:



$y - 1 = mx \rightarrow \ell_1$

$y = m(x+1) \rightarrow \ell_2$

$y = \dfrac{-1x}{m} \rightarrow \ell_3$

$y = \dfrac{-1(x-1)}{m} \rightarrow \ell_4$

ℓ_1 intersection $\ell_3 \rightarrow mx + 1 = \dfrac{-x}{m}$

$x = \dfrac{-m}{1+m^2},\ y = \dfrac{1}{1+m^2}$

ℓ_1 intersection $\ell_4 \rightarrow mx + 1 = \dfrac{-(x-1)}{m}$

$mx + 1 = \dfrac{-x}{m} + \dfrac{1}{m}$

$\left[\dfrac{m^2+1}{m}\right]x = \dfrac{1-m}{m}$

$x = \dfrac{1-m}{1+m^2}$

$y = \frac{m-m^2}{1+m^2} + 1 = \frac{m+1}{m^2+1}$

$B\left[\frac{1-m}{m^2+1}, \frac{m+1}{m^2+1}\right]$

ℓ_2 intersect $\ell_3 \to m(x + 1) = \frac{-x}{m}$

$mx + \frac{x}{m} = -m$

$x = \frac{-m^2}{1+m^2}$

$y = \frac{m}{1+m^2}$ $C\left[\frac{-m^2}{1+m^2}, \frac{m}{1+m^2}\right]$

ℓ_2 intercept $\ell_4 \Rightarrow m(x + 1) = \frac{-1}{m}(x-1)$

$\Rightarrow mx + \frac{x}{m} = \frac{1}{m} - m$

$x = \frac{(1-m^2)}{1+m^2}$

$y = m\left(\frac{2}{1+m^2}\right)$

$\Rightarrow D\left[\frac{(1-m^2)}{1+m^2}, \frac{2m}{m^2+1}\right]$

Area of square $= \left[\frac{m-1}{\sqrt{1+m^2}}\right]^2 = \frac{(m-1)^2}{m^2+1}$

AB = BD

$m^2 + m^4 = 2m(1 + m^2)$

$m^2 = 2m$

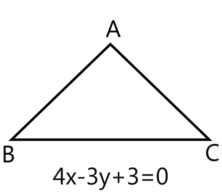
$m = 2$

Area $= \frac{1}{5} = \frac{p}{q}$

$p + q = 6$

Sol 28: H(1,2) —2— C —1— P(2,3)

Centroid (c) $= \left(\frac{5}{3}, \frac{3}{3}\right)$



Let coordinates of vertices of ΔABC be A (x, y), B(x_2, y_2), C (x_3, y_3)

$4x_2 - 3y_2 + 3=0$ ... (i)

$4x_3 - 3y_3 + 3=0$ ... (ii)

and $\frac{x_1+x_2+x_3}{3} = \frac{5}{3}$

$\Rightarrow x_1 + x_2 + x_3 = 5$ ... (iii)

$\Rightarrow y_1 + y_2 + y_3 = 8$ ... (iv)

From (i), (ii), (iii) and (iv), we get

$\Rightarrow 4(x_2 + x_3) - 3(y_2 + y_3) + 6 = 0$

$\Rightarrow 4(5 - x_1) - 3(8 - y_1) + 6 = 0$

$\Rightarrow 20 - 4x_1 - 24 + 3y_1 + 6 = 0$

$\Rightarrow -4x_1 + 3y_1 + 2 = 0$ ... (v)

Now $AH \perp BC$

$\frac{y_1-2}{x_1-1} \times \frac{4}{3} = -1$

$\Rightarrow 4y_1 - 8y = -3x_1 + 3$

$\Rightarrow 3x_1 - 4y_1 = 11$ ... (vi)

From (v) and (vi), we get

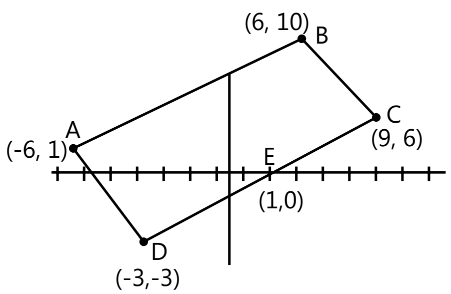
$(x_1, y_1) = \left(\frac{41}{25}, \frac{38}{25}\right)$

Radius = AO= $\sqrt{\left(2-\frac{41}{25}\right)^2 + \left(3-\frac{38}{25}\right)^2}$

$= \sqrt{\frac{58}{(25)}} = \sqrt{\frac{m}{n}}$

$\Rightarrow m = 58,$ $n = 25$ $\Rightarrow m + n = 83$

Sol 29: (–6, 1) (6, 10), (9, 6), (–3, –3)



Area of rectangle – Area of $\Delta = a/b$

Area of rect = $\sqrt{81+144}$ $\sqrt{16+9}$ = 15. 5 = 75

Eq. of CD $\frac{y+3}{x+3} = \frac{3}{4}$

Point E = (1, 0)

Eq. of AD = $\frac{y+3}{x+3} = \frac{-4}{3}$

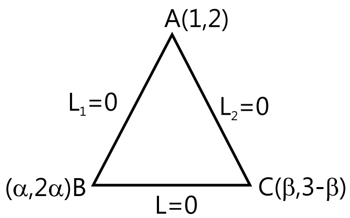
$\frac{-9}{4} - 3 = x;$ $x = \frac{-21}{4}$

$\frac{1}{2}[x_1 - x_2] + 3$ = Area of $\Delta = \frac{1}{2}\left[\frac{25}{4}\right] \times 3 = \frac{75}{8}$

Area $= \frac{75\times 7}{8} = \frac{525}{8} = \frac{a}{b}$

a + b = 533

Sol 30:



(i) $L_1 = 2x - y = 0$

$L_2 = x + y = 3$

G = (2, 3), A(1, 2)

$\Rightarrow \alpha + \beta + 1 = 6$

$\Rightarrow 2 + 2\alpha + 3 - \beta = 9$

$\Rightarrow 2\alpha - \beta = 4 \Rightarrow 3\alpha = 9$

$\alpha = 3, \beta = 2$

B(3, 6)C(2, 1)

Eq. $\Rightarrow \frac{y-1}{x-2} = 5$

y + 5 = 5x

m = 5

(ii) If H = (2, 3)

$L_1 = 2x + y = 0$ / $L_2 = x - y + 2 = 0$

$x = \frac{-2}{3} y = \frac{4}{3} A\left(\frac{-2}{3}, \frac{4}{3}\right)$

B(α, –2α), C (β, β + 2)

Slope of AH $= \frac{3-\frac{4}{3}}{2+\frac{2}{3}} = \frac{5}{8}$

Slope of BC $= \frac{-8}{5} = \frac{B+2\alpha+2}{\beta-\alpha}$

Slope of BH $= \frac{3+2\alpha}{2-\alpha}$

Slope of AC $= \frac{\alpha-2}{3+2\alpha} = \frac{\beta+\frac{2}{3}}{\beta+\frac{2}{3}} = 1$

$\alpha - 2 = 2\alpha + 3 \Rightarrow \alpha = -5$

and $-8\beta + 8\alpha = 5\beta + 10\alpha + 10$

$\alpha = \frac{-13\beta - 10}{2}, \beta = 0$

β(–5, +10)C, (0, 2)

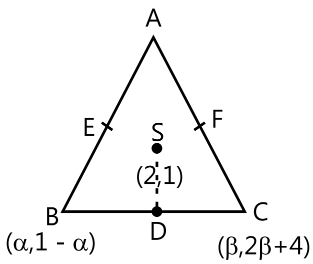
$m(BC) = \frac{-8}{5}$

$\frac{y-2}{x} = \frac{-8}{5}$

y intercept = 2

(iii) $L_1 = x + y - 1 = 0$

$L_2 = 2x - y + 4 = 0$



S(2, 1)

A(–1, 2)

$D\left(\frac{\alpha+\beta}{2}, \frac{2\beta-\alpha+5}{2}\right)$

$E = \left(\frac{\alpha-1}{2}, \frac{3-\alpha}{2}\right)$; $F = \left(\frac{\beta-1}{2}, \beta+3\right)$

Now, m(SE) = 1

$\Rightarrow \frac{\frac{\alpha-1}{2}-2}{\frac{3-\alpha}{2}-1} = 1$

$\Rightarrow \alpha - 5 = 1 - \alpha; \alpha = 3$

$m(SF) = \frac{-1}{2}$

$\Rightarrow \frac{\frac{\beta-1}{2}-2}{\beta+3-1} = \frac{\beta-5}{2(\beta+2)} = -2$

$-4(\beta + 2) = \beta - 5$

$5\beta = -3\beta = \frac{-3}{5}$

$\beta = (3, -2), C\left(\frac{-3}{5}, \frac{14}{5}\right)$

$\frac{y+2}{x-3} = \frac{-24}{18} \Rightarrow \frac{y+2}{x-3} = \frac{-4}{3}$

$\Rightarrow$ x intercept $3 - \frac{3}{2} = \frac{3}{2}$

Exercise 2

Single Correct Choice Type

Sol 1: (A) a(3x + 4y + 6) + b(x + y + 2) = 0

(2, 3) is situated at greater distance

$$D = \frac{a(6+12+6)+b(2+3+2)}{\sqrt{(3a+b)^2+(4a+b)^2}}$$

$$D = \frac{24a+7b}{\sqrt{(3a+b)^2+(4a+b)^2}} = \frac{24T+7}{\sqrt{(3T+1)^2+(4T+1)^2}}$$

Where $\left(T = \frac{a}{b}\right)$

$$\frac{dD}{dT} = 0$$

$$\Rightarrow \sqrt{(3T+1)^2+(4T+1)^2} \times 24$$

$$= \frac{(24T+7)2[(3T+1)3+4(4T+1)]}{2\sqrt{(3T+1)^2+(4T+1)^2}}$$

$\Rightarrow 24[(3T+1)^2 + (4T+1)^2$

$= (24T + 7)(9T + 16T + 7)$

$\Rightarrow 24[25T^2 + 2 + 14T] = [24T + 7][25T + 7]$

$\Rightarrow 48 + 24\times14T = 24T\times7+25\times7T+7\times7$

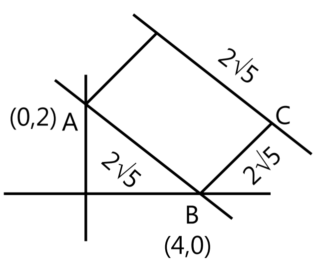
$\Rightarrow 24\times7T = 49 - 48 + 25\times7T$

$\Rightarrow -7T = 1 \Rightarrow T = \frac{-1}{7} = \frac{a}{b}$

$\Rightarrow -(3x + 4y + 6) + 7(x + y + 2) = 0$

4x + 3y + 8 = 0

Sol 2: (C)



$C = 4 + 2\sqrt{5}\cos\theta,\ 2\sqrt{5}\sin\theta$

$\tan\theta = 2$

$\sin\theta = \frac{2}{\sqrt{5}}$, $\cos\theta = \frac{1}{\sqrt{5}}$

C = (6, 4)

$D = 0 + 2\sqrt{5}\cos\theta,\ 2 + 2\sqrt{5}\sin\theta = (2, 6)$

Eq. of AC

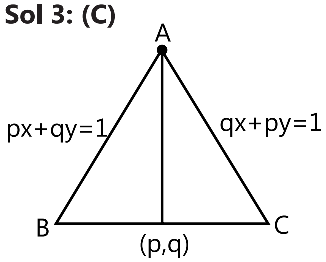
$\frac{y-2}{x-0} = \frac{1}{3} \Rightarrow x = 3y - 6$ ....(i)

Eq. of BD $= \frac{y-0}{x-4} = -3$

$x = 4 - \frac{y}{3}$ ....(ii)

Solving eq. (i) & (ii) we get the required Point is (3, 3)

Sol 3: (C)



px + qy = 1 × q

qx + py = 1 × p

$pqx + q^2y = q$

$pqr + p^2y = p$

$y(q^2 - p^2) = (q - p)$

$$y = \frac{1}{p+q}$$

$$x = \frac{1}{p+q}$$

$$A = \left(\frac{1}{p+q}, \frac{1}{p+q}\right)$$

Median through AB

$$\Rightarrow \frac{y-q}{x-p} = \frac{q - \frac{1}{p+q}}{p - \frac{1}{p+q}}$$

$$\Rightarrow \frac{y-q}{x-p} = \left[\frac{pq+q^2-1}{p^2+pq-1}\right]$$

$(p^2 + pq - 1)y - q(p^2 + pq - 1)$

$= x(q^2 + qp - 1) - p(q^2 + pq - 1)$

$x(pq+q^2-1) + p = q + y(p^2 + pq - 1)$

$= (2pq - 1)(px + qy - 1)$

$= (p^2 + q^2 - 1)(qx + py - 1)$

Sol 4: (B) $3x + 4y = 9 \times 4$

$\Rightarrow 12x + 16y = 36$

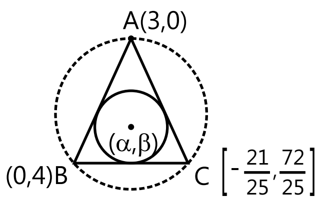
$4x - 3y + 12 = 0 \times 3$

$\Rightarrow 12x - 9y = -36$

$25y = 72$

$y = \frac{72}{25}$

$$\Rightarrow x = \frac{9 - \frac{4 \times 72}{25}}{3} = 3 - \frac{4 \times 24}{25} = -\frac{21}{25}$$



$A(3, 0)\ B(0, 4]C\left[-\frac{21}{25}, \frac{72}{25}\right]$

$$\Delta PAB = \frac{1}{2}\begin{vmatrix} 3 & 0 & 1 \\ 0 & 4 & 1 \\ -\frac{21}{25} & \frac{72}{25} & 0 \end{vmatrix}$$

$$= \frac{1}{2}\left(3\left(4 - \frac{72}{25}\right) + 1\left(0 + \frac{84}{25}\right)\right)$$

$$= \frac{1}{2}\left[\frac{3 \times 28}{25} + \frac{84}{25}\right] = \frac{84}{25}$$

$$m_{AC} = \frac{\frac{72}{25}}{-\frac{21}{25} - 3} = \frac{-72}{96} = \frac{-3}{4}$$

$$m_{BC} = \frac{4 - \frac{72}{25}}{\frac{21}{25}} = \frac{24}{21} = \frac{8}{7}$$

$$m_{AB} = -\frac{4}{3}$$

Mid point of AB $\left[\frac{3}{2}, 2\right] \Rightarrow \frac{\beta - 2}{\alpha - \frac{3}{2}} = \frac{3}{4}$ (Slope of $\perp$ AB)

$\Rightarrow 4\beta - 8 = 3\alpha - \frac{9}{2} \Rightarrow 3\alpha - 4\beta + \frac{7}{2} = 0$

Mid point of AC= $\left(\frac{27}{25}, \frac{36}{25}\right) = \frac{25\beta - 36}{25\alpha - 27} = \frac{4}{3}$

(Slope of $\perp$ to AC)

$\Rightarrow 100\alpha - 108 = 75\beta - 108 \Rightarrow \beta = \frac{4\alpha}{3}$

$\alpha = \frac{3}{2}\ \beta = 2$

Circumradius = $\sqrt{\frac{9}{4} + 4} = \frac{5}{2}$

Sol 5: (D) $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$a(bc - 1) - 1(c - 1) + 1(1 - b) = 0$

$abc - a - b - c + 2 = 0$

$a + b + c = abc + 2$

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$$

$$= \frac{1 + bc - b - c + 1 + ab - a - b + 1 + ac - a - c}{(1-a)(1-b)(1-c)}$$

$$= \frac{3 + ab + bc + ca - 2a - 2b - 2c}{1 - a - b - c + ab + bc + ca - abc}$$

$$= \frac{3 - 2(abc + 2) + ab + bc + ca}{1 - abc - (abc + 2) + ab + bc + ca}$$

$$= \frac{-1 - 2abc + ab + bc + ca}{-1 - 2abc + ab + bc + ca} = 1$$

Sol 6: (D) A(a, 0) ; B(0, b) ; C (c, 0) ; D(0, d)

$\frac{d}{a} = \frac{c}{b}$ lines are not parallel i. e. not a trapezium, not a parallelogram

OA OC = OB OD (For concyclic points)

Assuming origin as centre

ac = bd i. e. ABCD are concyclic.

Sol 7: (D) $7x^2 + 8y^2 - 4xy + 2x - 4y - 8 = 0$

Homogenising

$7x^2 + 8y^2 - 4xy + 2x\left[\frac{3x - y}{2}\right]$

$$-4\left[\frac{3x-y}{2}\right]y-8\left[\frac{3x-y}{2}\right]^2 = 0$$

$7x^2 + 8y^2 - 4xy + 3x^2 - xy - 6xy$

$+ 2y^2 - 2(9x^2 + y^2 - 6xy) = 0$

$\Rightarrow -8x^2 + 8y^2 - 11xy + 12xy = 0$

$\Rightarrow 8x^2 - 8y^2 - xy = 0$

$\Rightarrow \tan\theta = \dfrac{2\sqrt{h^2-ab}}{a+b} = \infty$

$\Rightarrow h = -\dfrac{1}{2}a = 8b = -8$

$\Rightarrow \theta = \dfrac{\pi}{2}$

Sol 8: (B) $x^2 - 4xy + 4y^2 + x - 2y - 6 = 0$

$y = mx + C_1 y = mx + C_2$

$(y - mx - a)(y - mx - C_2) = 0$

$y^2 - mxy - C_2 y - mxy + m^2x^2$

$+ C_2 mx - C_2 y + C_1 mx + C_1C_2 = 0$

$m^2x^2 + y^2 - 2mxy + x (C_1 m + C_2 m)$

$+ y(-C_1 - C_2) + C_1C_2 = 0$

$\dfrac{1}{m^2} = 4 \Rightarrow -\dfrac{2}{m} = -4 \Rightarrow m = +\dfrac{1}{2}$

$(C_1 + C_2) + \dfrac{2}{a} = 1$

$(C_1 - C_2) = \dfrac{1}{2}$

$C_1C_2 = -\dfrac{6}{4} = -\dfrac{3}{2}$

Distance between lines = $\dfrac{C_1 - C_2}{\sqrt{1+m^2}}$

$C_1 + C_2 = \dfrac{1}{2}$

$C_1C_2 = \dfrac{-6}{4}$

$$\frac{C_1-C_2}{\sqrt{1+m^2}} = \frac{\sqrt{\frac{1}{4}+6}}{\sqrt{1+\frac{1}{4}}} = \frac{5}{2\sqrt{\frac{5}{4}}} = \sqrt{5}$$

Sol 9: (B) $5x^2 + 12xy - 6y^2 + 4x(x + ky)$ [Homogenising]

$- \alpha y(x + ky) + 3(x + ky)^2 = 0$

$9x^2 - 6y^2 - 2ky^2 + 10xy + 4kxy + 3(x^2 + k^2y^2 + 2kxy) = 0$

$12x^2 + y^2(3k^2 - 2k - 6) + xy(6k + 4k + 10) = 0$

$m_1 + m_2 = \dfrac{-2h}{b} = \dfrac{-(10k+10)}{3k^2-2k-6} = 0 \Rightarrow k = -1$

Sol 10: (D) R = (h, k)

Centroid is $\left(\dfrac{h+6}{3}, \dfrac{k-6}{3}\right)$

$9\left(\dfrac{h+6}{3}\right) + 7\left(\dfrac{k-6}{3}\right) + 4 = 0$

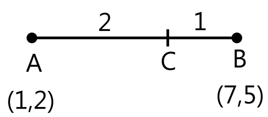
$9h + 7k + 54 - 42 + 12 = 0$

$9h + 7k + 24 = 0$

parallel to N.

Sol 11: (C) A(1, 2) & B(7, 5)

AB line $\Rightarrow m = \dfrac{3}{6} = \dfrac{1}{2}$



$C = \left(\dfrac{14+1}{3}, \dfrac{10+2}{3}\right) \Rightarrow (5,4)$

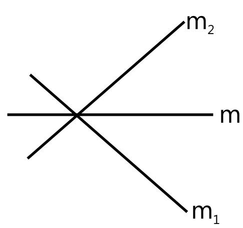
$\dfrac{2m-1}{m+2} = \pm 1$

$2m - 1 = -2 - m$

$m = -\dfrac{1}{3}$ or $m = 3$ ($m = -\dfrac{1}{3}$ is not possible)

$\Rightarrow y = 3x - 11 \Rightarrow 3x - y - 11 = 0$

Sol 12: (A) $ax^2 + 2hxy + by^2 = 0$



$\dfrac{m-m_1}{1+mm_1} = \dfrac{m_2-m}{1+m_2m}$

$\Rightarrow m + m_2m^2 - m_1 - m_1m_2m$

$= m_2 - m + mm_1m_2 - m^2m_1$

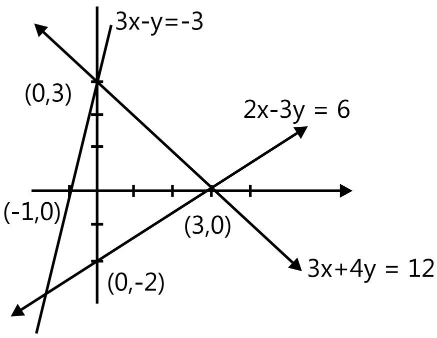
$\Rightarrow 2m - (m_1 + m_2) + (m_1 + m_2)m^2 = 2m_1m_2m$

$$\Rightarrow\ 2m + \frac{2h}{b} - \frac{2h}{b}m^2 = \frac{2a}{b}m$$

$$\Rightarrow\ 2mb - 2am = -2h + 2hm^2$$

$hm^2 + m(a - b) - h = 0$

Sol 13: (D)

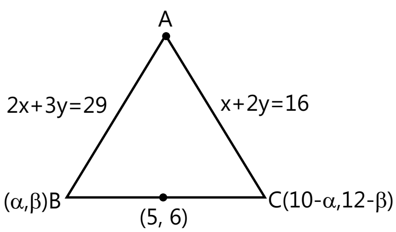


From figure

$\alpha \in [-1, 3]$

$\beta \in [-2, 3]$

Sol 14: (C)



$2\alpha + 3\beta = 29$

$10 - \alpha + 24 - 2\beta = 16$

$\alpha + 2\beta = 18$ ...(i)

$2\alpha + 4\beta = 36$ ...(ii)

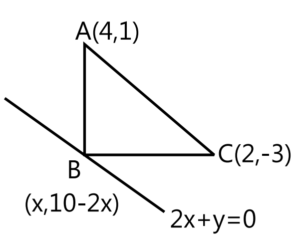
On solving, we get

$\beta = 7, \alpha = 4$

So we have, B(4, 7)

Eq. of BC = $\frac{y-7}{x-4} = -1 \Rightarrow x + y = 11$

Sol 15: (B)



$m_{AB} . m_{BC} = -1$

$$m_{AB} = \frac{9-2x}{x-4}$$

$$m_{BC} = \frac{13-2x}{x-2}$$

$(9 - 2x)(13 - x) = -1(x^2 - 6x + 8)$

$117 - 18x - 26x + 4x^2 = -x^2 + 6x - 8$

$5x^2 - 50x + 125 = 0$

$x^2 - 10x + 25 = 0 \Rightarrow x = 5; y = 0$

$$\text{Area} = \frac{1}{2} \times BC \times AB = \frac{1}{2}\sqrt{9+9}\,\sqrt{1+1} = 3$$

Multiple Correct Choice Type

Sol 16: (B, D) Let the co-ordinate of the vertex C is (h, h – 2) then area of triangle will be

$$\Rightarrow \frac{1}{2}\begin{vmatrix} -5 & 0 & 1 \\ 3 & 0 & 1 \\ h & h-2 & 1 \end{vmatrix} = 20$$

$\Rightarrow (h - 2)8 = \pm 40$

$\Rightarrow h - 2 = \pm 5$

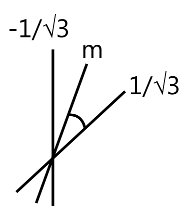
$\Rightarrow h = 7$ or -3

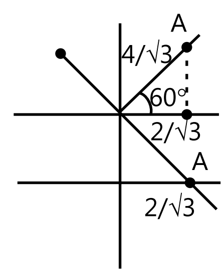
$\therefore$ Co-ordinate are (7, 5) or (–3, –5)

Sol 17: (B, C) $y + \sqrt{3}x = 2$

$y - \sqrt{3}x = 2$

$x = 0, y = 2$





$$\frac{\frac{1}{\sqrt{3}} - m}{1 + \frac{m}{\sqrt{3}}} = \frac{m + \frac{1}{\sqrt{3}}}{1 - \frac{m}{\sqrt{3}}} \Rightarrow \frac{1 - \sqrt{3}m}{m + \sqrt{3}} = \frac{\sqrt{3}m + 1}{\sqrt{3} - m}$$

$$\Rightarrow \sqrt{3} - m - 3m + m^2\sqrt{3} = m^2/\sqrt{3} + m + 3m + \sqrt{3}$$

$\Rightarrow m = 0$

Bisector line is y = 2

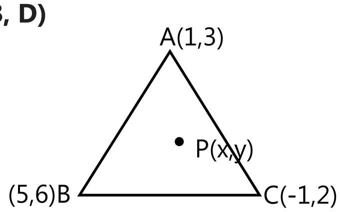
$\Rightarrow \cos 60^\circ = \frac{P}{4/\sqrt{3}}$; $P = \frac{2}{\sqrt{3}}$

$\Rightarrow$ Given point is $\left(\frac{2}{\sqrt{3}}, 2\right)$

Or the other possibility is $m = \infty$

$\Rightarrow$ Foot of perpendicular = (0, 0)

Sol 18: (A, B, D)



AB - $\frac{y-6}{x-5} = \frac{3}{4}$

$4y - 24 = 3x - 15 \Rightarrow 3x - 4y + 9 = 0$

BC - $\frac{y-6}{x-5} = \frac{2}{3}$

$3y - 18 = 2x - 10 \Rightarrow 2x - 3y + 8 = 0$

AC - $\frac{y-3}{x-1} = \frac{1}{2}$

$2y - 6 = x - 1 \Rightarrow x - 2y + 5 = 0$

C and P on same side

$3(-1) - 4(2) + 5 < 0$ $3x - 4y + 9 < 0$

B and P on same side of AC

$5 - 2(6) + 5 < 0$ $x - 2y + 5 < 0$

A and P on same side of BC

$2(1) - 3(3) + 8 > 0$ $2x - 3y + 8 > 0$

So we can see [A,B,D] are correct.

Sol 19: (B, C) $\frac{x}{a} + \frac{y}{b} = 1$ A (a, 0) & B(0, b)

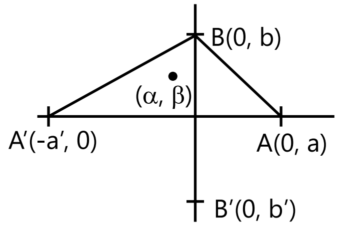
line $\frac{x}{a'} + \frac{y}{b'} = 1$ C(–a′, 0) D(0, –b′)

AB A′B′ are concyclic

i. e. OAOA′ = OBOB′

aa′ = bb′

$b' = \frac{aa'}{b}$



$m_{(AB)} = \frac{-b}{a}$

$m_{(BA')} = \frac{b}{a'}$

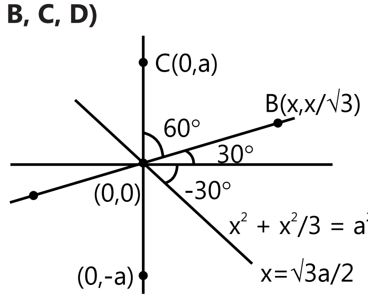
Let orthocentre be at (α, β)

$\frac{b-\beta}{0-\alpha} = \infty \Rightarrow \alpha = 0$

$\frac{\beta - 0}{\alpha + a'} = \frac{a}{b}$

$\beta = \frac{aa'}{b} = b'$ [From (i)]

Sol 20: (A, B, C, D)



C either lies on y axis so C(0,a) or C(0,–a)

or on $y = -\frac{1}{\sqrt{3}x}$

$C\left(x, \frac{-1}{\sqrt{3}x}\right)$

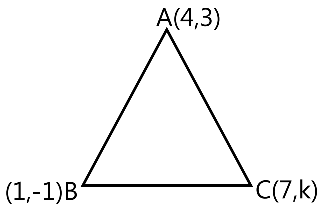
$\Rightarrow x^2 + \frac{x^2}{3} = a^2$

$x = \frac{\pm a\sqrt{3}}{2}$

$y = \mp\frac{a}{2}$

$\Rightarrow (0, a), (0, -a), \left(\frac{\sqrt{3}a}{2}, \frac{-a}{2}\right)\left(\frac{-\sqrt{3}a}{2}, \frac{a}{2}\right)$

Sol 21: (B, C) Vertices are (4, 3)(1, –1) & (7, k)



$C = \left(4, \frac{2+k}{3}\right)$

That occurs only in isosceles Δ

$(1-4)^2 + (3+1)^2 = (k-3)^2 + 9$

AC = AB

$16 = (k-3)^2$

$\Rightarrow k - 3 = \pm 4$

$\Rightarrow k = 7, -1$ or

$25 = (k+1)^2 + 36$

≠ not possible (AB = BC)

or $(k-3)^2 + 9 = (k+1)^2 + 36$

(BC = AC)

$18 - 6k = 37 + 2k$

$\Rightarrow 8k = -19 \Rightarrow k = \frac{-19}{8}$

For k = 7AB = 5

AC = 5, BC = 10

[Δ is not possible]

So $k = -1$ or $\frac{-19}{8}$

Sol 22: (A, C) (7, 4)

$y - 4 = m(x - 7)$

Centre ≡ (3, –2)

$$\Rightarrow \frac{(3-7)m+4+2}{\sqrt{1+m^2}} = 4$$

$\Rightarrow (-4m+6)^2 = 16(1+m^2)$

$\Rightarrow 9 + 4m^2 - 12m = 4 + 4m^2$

$\Rightarrow m = \frac{5}{12}$ or $m = \infty$

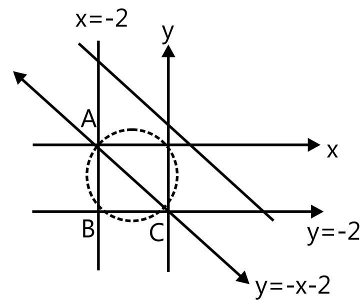
$\Rightarrow y - 4 = \frac{5}{12}(x-7)$

$\Rightarrow 5x - 12y + 13 = 0$ and $x = 7$

Sol 23: (B, C) $(x+2)(y+2) = 0$; $x + y + 2 = 0$

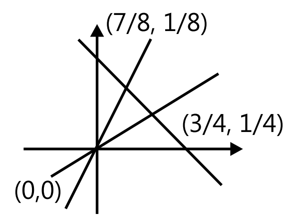
Circumcentre lies on mid-point of hypotenuse i. e. AB

$$\frac{(-2,0)+(0,-2)}{2} = (-1, -1)$$

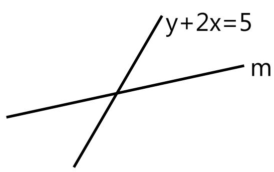


Sol 24: (B, C) $x + y = 1$; $x = 7y$; $x = 3y$

Centroid and In centre always Lie inside of the triangle.



Sol 25: (B, C)



$$\tan\theta = \left|\frac{m-(-2)}{1+m(-2)}\right|$$

$$\frac{m+2}{1-2m} = \pm\frac{1}{2}$$

$2m + 4 = \pm(1 - 2m)$

$\Rightarrow 2m + 4 = 2m - 1 \Rightarrow (m = \infty)$

$\Rightarrow x = 2$ (B)

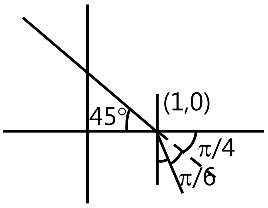
$2m + 4 = 1 - 2m$

$4m = -3 \Rightarrow m = -\frac{3}{4}$

$$\frac{y-3}{x-2} = \frac{-3}{4}$$

$3x + 4y = 18$ (C)

Sol 26: (A, C)



$y - a = m(x - 1)$

$y = m(x - 1)$

$m = -\tan 75^o = \dfrac{-\sin 75^o}{\cos 75^o}$

$= \dfrac{-(\sqrt{3}+1)}{(\sqrt{3}-1)} = -(2+\sqrt{3})$

$y = -(2+\sqrt{3})\,(x - 1)$

$(2+\sqrt{3})\,x + y = (2+\sqrt{3})$ (C)

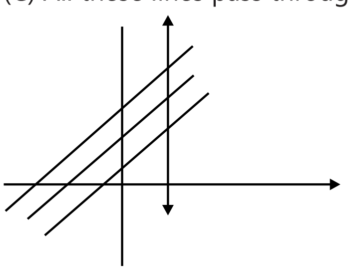
$x + (2-\sqrt{3})\,y = 1$

Sol 27: (B, C) $y - y_1 = m(x - x_1)$

$y = y_1 + m(x - x_1)$

(B) Set of parallel lines

(C) All these lines pass through $x = x_1$



Previous Years' Questions

Sol 1: (C) It is not necessary that the bisector of an angle will divide the triangle into two similar triangle, therefore, statement-II is false.

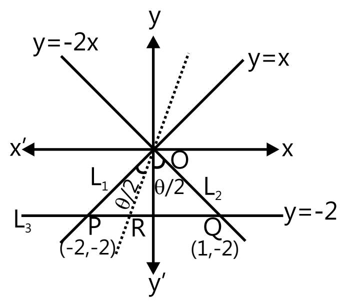
Now we verify statement-I

Δ OPQ, OR is the internal bisector of $\angle POQ$

$\therefore \quad \dfrac{PR}{RQ} = \dfrac{OP}{OQ}$

$\Rightarrow \quad \dfrac{PR}{RQ} = \dfrac{\sqrt{2^2+2^2}}{1^2+2^2}$

$= \dfrac{2\sqrt{2}}{\sqrt{5}}$



Sol 2: (A) Solving equations L_1 and L_2.

$\Rightarrow \dfrac{x}{-36+10} = \dfrac{y}{-25+12} = \dfrac{1}{2-15}$

$\therefore \quad x = 2, y = 1$

L_1, L_2, L_3 are concurrent if point (2, 1) lies on L_2

$\therefore \quad 6 - k - 1 = 0 \Rightarrow k = 5$

(A) $\rightarrow$ (S)

(B) Either L_1 is parallel to L_2 or L_3 is parallel to L_2, then

$\dfrac{1}{3} = \dfrac{3}{-k}$ or $\dfrac{3}{5} = \dfrac{-k}{2}$

$\Rightarrow \quad k = -9 \qquad$ or $k = \dfrac{-6}{5}$

(B) $\rightarrow$ (p, q)

(C) L_1, L_2, L_3 form a triangle, if they are not concurrent, or not parallel.

$\therefore \quad k \neq 5, -9, -\dfrac{6}{5} \Rightarrow k = \dfrac{5}{6}$

(c) $\rightarrow$ (r)

(D) L_1, L_2, L_3 do not form a triangle. If

$k = 5, -9, -\dfrac{6}{5}$

(D) $\rightarrow$ (p, q, s)

Sol 3: (A, C) Given lines px + qy + r = 0, qx + ry + p = 0

and rx + py + q = 0 are concurrent.

$\therefore \quad \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$ and taking common from R_1

$\Rightarrow \quad (p + q + r)\begin{vmatrix} 1 & 1 & 1 \\ q & r & p \\ r & p & q \end{vmatrix}$

$\Rightarrow$ (p + q + r) ($p^2 + q^2 + r^2 - pq - qr - pr$) = 0

$\Rightarrow$ $p^3 + q^3 + r^3 - 3pqr = 0$.

Sol 4: (A, C) Since, $3x + 2y \geq 0$ ....(i)

Where (1, 3) (5, 0) and (– 1, 2) satisfy Equation (i)

$\therefore$ Option (a) is true.

Again $2x + y - 13 \geq 0$

is not satisfied by (1, 3),

$\therefore$ Option (b) is false.

$2x - 3y - 12 \leq 0$,

is satisfied for all points,

$\therefore$ Option (c) is true.

And $-2x + y \geq 0$,

is not satisfied by (5, 0),

$\therefore$ Option (d) is false,

Sol 5: (B, C) Let equation of line L_1 be y =mx. Intercepts made by L_1 and L_2 on the circle will be equal ie, L_1 and L_2 are at the same distance from the centre of the circle.

Centre of the given circle is (1/2, – 3/2). Therefore,

$$\frac{|1/2-3/2-1|}{\sqrt{1+1}} = \left|\frac{\frac{m}{2}+\frac{3}{2}}{\sqrt{m^2+1}}\right|$$

$\Rightarrow$ $\frac{2}{\sqrt{2}} = \frac{|m+3|}{2\sqrt{m^2+1}}$

$\Rightarrow$ $8m^2 + 8 = m^2 + 6m + 9$

$\Rightarrow$ $7m^2 - 6m - 1 = 0$

$\Rightarrow$ (7m + 1) (m – 1) = 0

$\Rightarrow$ $m = \frac{-1}{7}, m = 1$

Thus, two chords are x + 7y = 0 and y – x = 0 Therefore, (b) and (c) are correct answers.

Sol 6: Equation of plane containing the given lines is

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$$

$\Rightarrow$ (x – 1)(– 1) – (y – 2)(– 2) + (z – 3)(– 1) = 0

$\Rightarrow$ – x +1 + 2y – 4 – z + 3 = 0

$\Rightarrow$ x + 2y – z = 0 ......(i)

Given plane is

x – 2y +z = d ....(ii)

Equations (i) and (ii) are parallel.

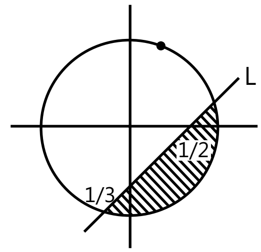
Clearly, A = 1

Now, distance between plane

$$= \left|\frac{d}{\sqrt{1+4+1}}\right| = \sqrt{6}$$

$\Rightarrow$ |d| = 6

Sol 7: $x^2 + y^2 \leq 6$ and 2x – 3y = 1 is shown as



For the point to lie in the shaded part, origin and the point lie on opposite side of straight line L.

$\therefore$ For any point in shaded part L > 0 and for any point inside the circle S < 0.

Now, for $\left(2, \frac{3}{4}\right)$, L : 2x – 3y – 1

$$L: 4 - \frac{9}{4} - 1 = \frac{3}{4} > 0$$

and $S: x^2 + y^2 - 6, \quad S: 4 + \frac{9}{16} - 6 < 0$

$\Rightarrow$ $\left(2, \frac{3}{4}\right)$ lies in shaded part.

For $\left(\frac{5}{2}, \frac{3}{4}\right)$ L : 5 – 9 – 1 < 0 (neglect)

For $\left(\frac{1}{4}, -\frac{1}{4}\right)$ $L: \frac{1}{2} + \frac{3}{4} - 1 > 0$

$\therefore$ $\left(\frac{1}{4} - \frac{1}{4}\right)$ lies in the shaded part.

For $\left(\frac{1}{8}, \frac{1}{4}\right)$ $L: \frac{1}{4} - \frac{3}{4} - 1 < 0$ (neglect)

$\Rightarrow$ Only 2 points lie in the shaded part.

Sol 8: Note: d : (P, Q) = | $x_1 - x_2$ | + | $y_1 - y_2$ |. It is new method representing distance between two points P and Q and in future very important in coordinate geometry.

Now, let P(x, y) be any point in the first quadrant. We have

d(P, 0) = | x – 0 | + | y – 0 | = | x | + | y | = x + y (∵ x, y > given)

d (P, A) = |X – 3| + |Y – 2| (given)

d(P, 0) = D(P, A)

$\Rightarrow$ x + y = |x – 3| + |y – 2|

Case I : When 0 < x < 3, 0 < y < 2

In this case I Eq.(i) becomes

x + y= 3 – x + 2 – y

$\Rightarrow 2x + 2y = 5 \qquad \Rightarrow x + y = \frac{5}{2}$

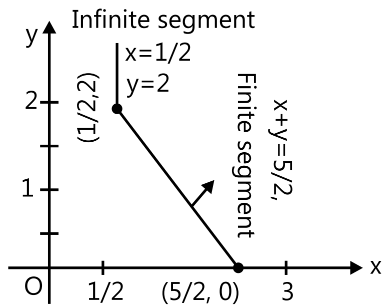
Case II : When 0 < x < 3, y ≥ 2

Now, Eq. (i) becomes

x + y = 3 – x + y – 2

$\Rightarrow 2x = 1 \qquad \Rightarrow x = \frac{1}{2}$

Case III : When x ≥ 3, 0 < y < 2



Now, Eq.(i) becomes x + y = x – 3 + 2 – y

$\Rightarrow 2y = -1 \qquad$ or $\qquad y = -\frac{1}{2}$

Hence, no solution.

Case IV : When x ≥ 3, y ≥ 2

In this case (i) changes to x + y = x – 3 + y – 2

$\Rightarrow 0 = -5$

Which is not possible.

Hence, this solution set is {(x, y)} | x = 12, y ≥ 2} ∪ {(x, y)} |

x + y = 5/2, 0 < x < 3, 0 < y > 2 }

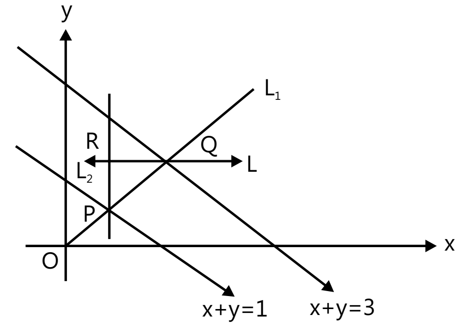
The graph is given in adjoining figure.

Sol 9: Let the equation of straight line L be

y = mx

$$P \equiv \left(\frac{1}{m+1}, \frac{m}{m+1}\right)$$

$$Q \equiv \left(\frac{3}{m+1}, \frac{3m}{m+1}\right)$$



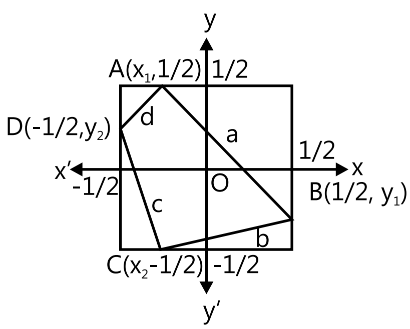
Now, equation of L_1 : $y - 2x = \frac{m-2}{m+1}$ .... (i)

And equation of L_2 : $y + 3x = \frac{3m+9}{m+1}$ ....(ii)

By eliminating 'm' from Equ. (i) and (ii), we get locus of R as x – 3y + 5 = 0, which represents a straight line.

Sol 10: Let the square S is to be bounded by the lines $x = \pm\frac{1}{2}$ and $y = \pm\frac{1}{2}$

We have, $a^2 = \left(x_1 - \frac{1}{2}\right)^2 + \left(\frac{1}{2} - y_1\right)^2$



$$= x_1^2 - y_1^2 - x_1 - y_1 + \frac{1}{2}$$

Similarly, $b^2 = x_2^2 - y_1^2 - x_2 + y_1 + \frac{1}{2}$

$c^2 = x_2^2 - y_2^2 + x_2 + y_2 + \frac{1}{2}$

$d^2 = x_1^2 - y_2^2 + x_1 + y_2 + \frac{1}{2}$

$\therefore \quad a^2 + b^2 + c^2 + d^2 = 2(x_1^2 + y_1^2 + x_2^2 + y_2^2) + 2$

Therefore $0 \le x_1^2, x_2^2, y_1^2, y_2^2 \le \frac{1}{4}$

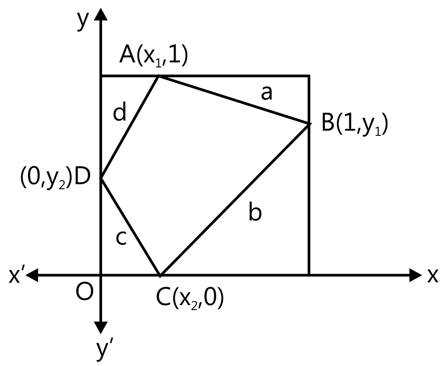
$0 \le x_1^2 + x_2^2 + y_1^2 + y_2^2 \le 1$

$\Rightarrow \quad 0 \le 2(x_1^2 + x_2^2 + y_1^2 + y_2^2) \le 2$

But $\quad 2 \le 2(x_1^2 + x_2^2 + y_1^2 + y_2^2) + 2 \le 4$

Alternate Solution

$c^2 = x_2^2 + y_2^2$ ....(i)



$b^2 = (1 - x_2)^2 + y_1^2$ .... (ii)

$a^2 = (1 - y_1)^2 + (1 - x_1)^2$ .....(iii)

$d^2 = x_1^2 + (1 - y_2)^2$ .....(iv)

On adding Eqs. (i), (ii), (iii) and (iv), we get

$a^2 + b^2 + c^2 + d^2 = \{x_1^2 + (1 - x_1)^2\} + \{y_1^2 + (1 - y_1)^2\}$

$+ \{x_2^2 + (1 - x_2)^2\} + \{y_2^2 + (1 - y_2)^2\}$

Where x_1, y_1, x_2, y_2 all vary in the interval [0, 1].

Now, consider the function $y = x^2 + (1 - x)^2,\ 0 \le x \le 1$

Differentiating $\Rightarrow \Rightarrow \frac{dy}{dx} = 2x - 2(1 - x)$. For maximum or minimum $\frac{dy}{dx} = 0$.

$\Rightarrow \quad 2x - 2(1 - x) = 0 \Rightarrow 2x - 2 + 2x = 0$

$\Rightarrow \quad 4x = 2 \quad \Rightarrow x = \frac{1}{2}$

Again, $\quad \frac{d^2y}{dx^2} = 2 + 2 = 4$

$\Rightarrow$ Which is positive.

Hence, y is minimum at $x = \frac{1}{2}$ and its minimum value is $\frac{1}{4}$.

Clearly, value is maximum when x = 1.

$\therefore$ Minimum value of $a^2 + b^2 + c^2 + d^2 = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$

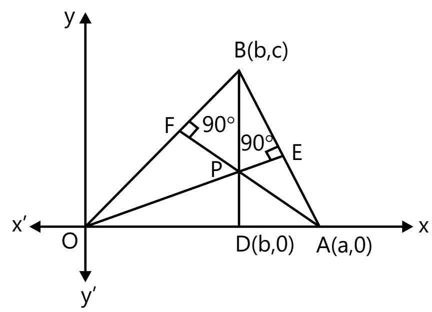
And maximum value is 1 + 1 + 1 + 1 = 4

Sol 11: Let the vertices of a triangle be, O(0, 0) A(a, 0) and B(b, c) equation of altitude BD is x =b.

Slope of OB is $\frac{c}{b}$.

Slope of AF is $-\frac{b}{c}$.

Now, the equation of altitude AF is



$y - 0 = -\frac{b}{c}(x - a)$

Suppose, BD and OE intersect at P.

Coordinates of P are $\left[b, b\left(\frac{a-b}{c}\right)\right]$

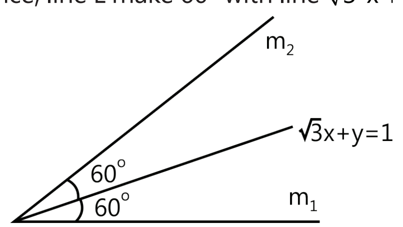
Let m_1 be the slope of OP = $\frac{a-b}{c}$

and m_2 be the slope of AB $= \frac{c}{b-a}$

Now, $m_1 m_2 = \left(\frac{a-b}{c}\right)\left(\frac{c}{b-a}\right) = -1$

We get that the line through O and P is perpendicular to AB.

Sol 12: Since, line L make 60° with line $\sqrt{3}\,x + y = 1$, then



$m_1 = \frac{-\sqrt{3} + \tan 60°}{1 - \left(-\sqrt{3}\right)\left(\tan 60°\right)} = 0$

$m_2 = \frac{-\sqrt{3} - \tan 60°}{1 + \left(-\sqrt{3}\right)\tan 60°} = \frac{-2\sqrt{3}}{1-3} = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$

Equation of line having slope $\sqrt{3}$ and passes through (3, -2)

$y+2=\sqrt{3}(x-3)$

$\Rightarrow y+2=\sqrt{3}x-3\sqrt{3}$

$\Rightarrow y-\sqrt{3}x+2+3\sqrt{3}=0$

Sol 13: (A) The point of intersection of lines ax + by + c = 0 and bx + ay + c = 0 is

$P\left(\frac{-c}{a+b},\frac{-c}{a+b}\right)$

Given that distance of point P from (1, 1) is less than $2\sqrt{2}$

$$\sqrt{\left(1+\frac{c}{a+b}\right)^2+\left(1+\frac{c}{a+b}\right)^2}<2\sqrt{2}$$

$$\Rightarrow \sqrt{2}\left|\frac{a+b+c}{a+b}\right|<2\sqrt{2}$$

$$\Rightarrow \left|\frac{a+b+c}{a+b}\right|<2 \Rightarrow -2<\frac{a+b+c}{a+b}<2$$

$\Rightarrow a+b+c<2a+2b \Rightarrow a+b-c>0$

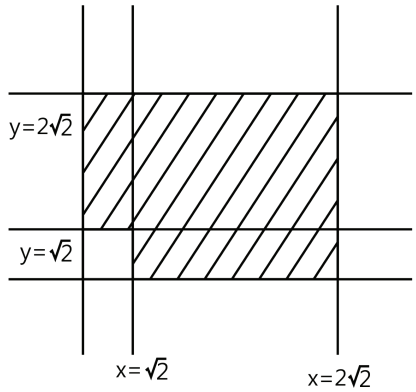
Sol 14: Let P be (α,β), then

$$d_1(P)=\frac{|\alpha-\beta|}{\sqrt{2}}$$

$$d_2(P)=\frac{|\alpha+\beta|}{\sqrt{2}}$$

$\Rightarrow 2\le d_1(P)+d_2(P)\le 4$

$$\Rightarrow 2\le\frac{|\alpha-\beta|}{\sqrt{2}}+\frac{|\alpha+\beta|}{\sqrt{2}}\le 4$$



Since, Point P lies in the first quadrant $\alpha;\beta>0$

Case 1: $\alpha\ge\beta$

$$2\le\frac{\alpha-\beta}{\sqrt{2}}+\frac{\alpha+\beta}{\sqrt{2}}\le 4$$

$\Rightarrow \sqrt{2}\le\alpha\le 2\sqrt{2}$

Case 2: $\alpha<\beta$

$$2\le\frac{-\alpha+\beta}{\sqrt{2}}+\frac{\alpha+\beta}{\sqrt{2}}\le 4 \Rightarrow \sqrt{2}\le\beta\le 2\sqrt{2}$$

Area = $\left(2\sqrt{2}\right)^2-\left(\sqrt{2}\right)^2=8-2$

= 6 sq. unit