SAMPLE QUESTION PAPER - 10

Time: 3 Hrs 15 Min **Subject : Mathematics (35)** **Max Marks: 100**

Instructions: (1) The question paper has five Parts namely A, B, C, D and E. Answer all the parts.
(2) Use the graph sheet for the question on linear programming in PART-E.

PART-A

Answer any Ten of the following. (One Mark each) $\mathbf{10 \times 1 = 10}$

1. Let $*$ be a binary operation on the set of natural numbers N given by $a * b = H.C.F\ of\ a\ and\ b$. Find $22 * 4$.
2. A relation R on $A = \{1,2,3\}$ defined by $R = \{(1,1),(2,1),(3,3)\}$ is not a symmetric. why?
3. Write the set of all principal values of $cosec^{-1}x$
4. Find the value of x or Solve : $tan^{-1}x + 2cot^{-1}x = \frac{2\pi}{3}$
5. If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew symmetric matrix, find the values of a, b and c
6. If a matrix A is of order 3×3 and $|A| = 9$ then find the value of $|A \cdot adjA|$.
7. If $y = e^{6log_e(x-1)}$, then prove that $\frac{dy}{dx} = 6(x-1)^5$.
8. Find the derivative of $sin(\tan^{-1}e^{-x})$ with respect to x
9. Find $\int (3x^2 + cosx)\, dx$
10. Evaluate$\int \frac{x^3-x^2+x-1}{x-1} dx$.
11. Define coplanar vector.
12. Find the unit vector in the direction of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$
13. Find the co-ordinates of the point where the line through the point $A(5,1,6)$ and $B(3,4,1)$ crosses the YZ-plane
14. In an LPP, if the objective function $Z = ax + by$ has the maximum value is 75 on two corner point of the feasible reason, then what is the value of every point on the line segment joining these two points
15. If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$ then find $P(A/B)$

PART-B

Answer any Ten of the following.(Two Marks each) $\mathbf{10 \times 2 = 20}$

16. Let $*$ be a binary operation on set of $R - \{-1\}$ defined by $a * b = \frac{a}{b+1}$ is i) commutative or not or ii) associative or not.
17. Evaluate $sin\left[\frac{\pi}{2} - sin^{-1}\left(-\frac{1}{2}\right)\right]$
18. Prove that $sin^{-1}\frac{1}{x} = cosec^{-1}x\ , x \geq 1\ or\ x \leq -1$
19. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ find the value of x and y
20. Find the area of Triangle whose vertices are $(1,3),(2,5)\ and\ (7,5)$ using determinants
21. If $y = \frac{x^2+1}{x^2-1}$ then prove that $\frac{dy}{dx} = \frac{-4x}{(x^2-1)^2}$.
22. Find $\frac{d^2y}{dx^2}$ If $y = tan^{-1}x$.
23. Find $\frac{dy}{dx}$ if $sin^2x + \cos^2y = 1$.
24. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is strictly increasing in R.
25. Evaluate $\int e^x\left(tan^{-1}x + \frac{1}{1+x^2}\right) dx$

26. Evaluate $\int cos^2x\,dx$.
27. Evaluate $\int xsec^2x\,dx$
28. Find the order and degree of differential equation $\left(\frac{d^3x}{dy^3}\right) + x^2\left(\frac{d^2x}{dy^2}\right)^3 = 0$.
29. Find the unit vector in the direction of sum of the vectors $2\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ and $2\hat{\imath} + \hat{\jmath} + 3\hat{k}$.
30. Find the area of parallelogram whose adjacent sides are given by the vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$.
31. Show that line through the points $(1,-1,2)$, $(3,4,-2)$ is perpendicular to the line through the points $(0,3,2)$, $(3,5,6)$.
32. Find the Cartesian equation of the plane $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} - \hat{k}) = 2$
33. In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII. What is the probability that a student chosen randomly studies in Class XII given that the chosen student is a girl?

PART-C

Answer any Ten of the following. (Three Marks each) $\mathbf{10 \times 3 = 30}$

34. Check whether the relation R in R of real numbers defined by $R = \{(a,b): a \leq b^3\}$ is reflexive, symmetric or transitive.
35. Prove that $tan^{-1}\frac{1}{2} + tan^{-1}\frac{2}{11} = tan^{-1}\frac{3}{4}$
36. For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that
    (a) $A + A'$ is symmetric matrix, and
    (b) $A - A'$ is skew symmetric matrix
37. Verify that the value of the determinant remains unchanged if its row and columns are interchanged by considering third order determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$
38. Differentiate $(sinx - cosx)^{(sinx-cosx)}$, $\frac{\pi}{4} < x < \frac{3\pi}{4}$ with respect to x
39. Find $\frac{dy}{dx}$ If $x = acos\theta\ and y = asin\theta$
40. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8, x \in [-4,2]$
41. If the radius of sphere is measured as 7m with an error 0.02m, then find the approximate error in calculating its volume.
42. Evaluate $\int_0^2 x\sqrt{2-x}\,dx$.
43. Evaluate $\int \frac{x^2}{(x^2+1)(x^2+4)}dx$..
44. Evaluate $\int e^{3logx}(x^4+1)^{-1}\,dx$.
45. Find the area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$.
46. Form the differential equation representing family of curves $y = mx$ where m is arbitrary constants
47. Find the solution of differential equation $(1 + e^{2x})dy + (1 + y^2)e^x dx = 0$.
48. Consider two points P and Q with position vectors $\overrightarrow{OP} = 3\vec{a} - 2\vec{b}$ and $\overrightarrow{OQ} = \vec{a} + \vec{b}$. Find the position vector of a point R which divides the line joining the points P and Q in the ratio $2:1$
    (i) Internally (ii) Externally
49. Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$ for any two non zero vectors $\vec{a}$ and $\vec{b}$.
50. Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.
51. In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B?.

PART-D

Answer any Six of the following. (Five Marks each) $\mathbf{6 \times 5 = 30}$

52. If $f: R \rightarrow R$ defined by $f(x) = x^3$ show that f is one-one and onto
53. Show that the function $f: R_* \rightarrow R_*$ defined by $f(x) = \frac{1}{x}$ is one one and onto, where R_* is the set of all non zero real numbers
54. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & -2 \\ 3 & 0 \end{bmatrix}$
    Calculate $A(BC)$and $(AB)C,$ show that $A(BC) = (AB)C$
55. Solve the following system of linear equation by matrix method
    $x + y + z = 6$
    $x - y - z = -4$
    $x + 2y - 2z = -1$
56. If $y = 3e^{2x} + 2e^{3x}$, show that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0.$
57. The volume of a cube is increasing at a rate of 8 cubic centimetres per second. How fast is the surface area increasing when the length of an edge is 12 centimetres?
58. Find the integral $\sqrt{a^2 - x^2}$ w.r.t x and hence evaluate, $\int \sqrt{5 - x^2 + 2x}\, dx$
59. Using the method of integration find the area enclosed by the circle $x^2 + y^2 = a^2$.
60. Find the general solution of differential equation $(x + y)\frac{dy}{dx} = 1.$
61. Derive the Equation of a plane passing through three non collinear point both in the vector and Cartesian form.
62. If a fair coin is tossed 8 times, find the probability of
    (i) Exactly five heads (ii) At least five heads (iii) At most five heads
63. One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent?
    i) E : 'the card drawn is a spade' F:'the card drawn is an ace'
    ii) E : 'the card drawn is black' F:'the card drawn is a king'
    iii) E : 'the card drawn is a king or queen' F:'the card drawn is a queen or jack'

PART-E

Answer any One of the following. (Ten Marks) $\mathbf{1 \times 10 = 10}$

64. a) Prove that $\int_0^a f(x)dx = \int_0^a f(a - x)dx$ and hence find the value of $\int_0^{\pi/2} \frac{sin^{3/2}x}{sin^{3/2}x + cos^{3/2}x} dx$
    b) If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, satisfies the equation $A^2 - 4A + 7I = O$, then find the inverse of A using this equation, where I is the identity matrix of order 2
65. a) Minimise and Maximise $Z = 3x + 9y$ by graphical method
    Subjected to $x + 3y \leq 60$
    $x + y \geq 10$
    $x \leq y$
    $x \geq 0, y \geq 0$
    b) Find the value of k if $f(x) = \begin{cases} kx + 1 & if\ x \leq \pi \\ cosx & if\ x > \pi \end{cases}$ is continuous at $x = \pi$.
66. a) Prove that the volume of largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere
    b) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(a + b + c)$