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Vector & 3-D

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VECTORS

1. VECTORS & THEIR REPRESENTATION

Vector quantities are specified by definite magnitude and definite directions. A vector is generally represented by a directed line segment, say $\overrightarrow{AB}$. A is called the **initial point** and B is called the **terminal point**. The magnitude of vector $\overrightarrow{AB}$ is expressed by $|\overrightarrow{AB}|$.

1.1 Zero Vector

A vector of zero magnitude is a zero vector i.e. which has the same initial & terminal point, is called a **Zero Vector**. It is denoted by $\vec{O}$. The direction of zero vector is indeterminate.

1.2 Unit Vector

A vector of unit magnitude in direction of a vector $\vec{a}$ is called unit vector along $\vec{a}$ and is denoted by $\hat{a}$ symbolically $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$.

1.3 Equal Vector

Two vectors are said to be equal if they have the same magnitude, direction & represent the same physical quantity.

1.4 Collinear Vector

Two vectors are said to be collinear if their directed line segments are parallel disregards to their direction. Collinear vectors are also called **Parallel Vectors** . If they have the same direction they are named as like vectors otherwise unlike vectors.

Symbolically, two non – zero vectors $\vec{a}$ and $\vec{b}$ are collinear if and only, if $\vec{a} = K\vec{b}$, where $K \in R - \{0\}$.

1.5 Coplanar Vector

A given number of vectors are called coplanar if their line segments are all parallel to the same plane. Note that **"Two Vectors Are Always Coplanar".**

1.6 Position Vector of A Point

Let O be a fixed origin, then the position vector of a point P is the vector $\overrightarrow{OP}$. If $\vec{a}$ and $\vec{b}$ are positive vectors of two points A and B, then, $\overrightarrow{AB} = \vec{b} - \vec{a}$ = pv of B – pv of A.

If $\vec{a}$ and $\vec{b}$ are the position vectors to two points A and B then the p.v. of a point which divides AB in the ratio m : n is given by :

$\vec{r} = \frac{n\vec{a} + m\vec{b}}{m+n}$. **Note p.v. of mid point of AB =** $\frac{\vec{a}+\vec{b}}{2}$

2. ALGEBRA OF VECTORS

2.1 Addition of vectors

If two vectors $\vec{a}$ & $\vec{b}$ are represented by $\overrightarrow{OA}$ & $\overrightarrow{OB}$, then their sum $\vec{a}+\vec{b}$ is a vector represented by $\overrightarrow{OC}$, where OC is the diagonal of the parallelogram OACB.

- $\vec{a}+\vec{b} = \vec{b}+\vec{a}$ (commutative)
- $(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$ (associativity)
- $\vec{a}+\vec{0} = \vec{a} = \vec{0}+\vec{a}$
- $\vec{a}+(-\vec{a}) = \vec{0} = (-\vec{a})+\vec{a}$

2.2 Multiplication of a Vector by a scalar

If $\vec{a}$ is a vector & m is a scalar, then m $\vec{a}$ is vector parallel to $\vec{a}$ whose modulus is |m| times that of $\vec{a}$. This is multiplication is called **Scalar Multiplication**. If $\vec{a}$ & $\vec{b}$ are vectors & m, n are scalars, then :

$$m(\vec{a}) = (\vec{a})m = m\vec{a}$$

$$m(n\vec{a}) = n(m\vec{a}) = (mn)\vec{a}$$

$$(m+n)\vec{a} = m\vec{a} + n\vec{a}$$

$$m(\vec{a}+\vec{b}) = m\vec{a} + m\vec{b}$$

3. TEST OF COLLINEARITY

Three points A, B, C with position vectors $\vec{a}, \vec{b}, \vec{c}$ respectively are collinear, if & only if there exist scalar x, y, z not all zero simultaneously such that; $x\vec{a} + y\vec{b} + z\vec{c} = 0$, where $x + y + z = 0$

4. TEST OF COPLANARITY

Four points A, B, C, D with position vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ respectively are coplanar if and only if there exist scalars x, y, z w not all zero simultaneously such that $x\vec{a} + y\vec{b} + z\vec{c} + w\vec{d} = 0$ where, $x + y + z + w = 0$

5. PRODUCT OF VECTORS

5.1 Scalar product of two vectors

- $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos\theta \; (0 \le \theta \le \pi)$

  note that if θ is acute then $\vec{a} . \vec{b} > 0$ & if θ is obtuse then $\vec{a} . \vec{b} < 0$

- $\vec{a} . \vec{a} = |\vec{a}|^2 = \vec{a}^2$, $\vec{a} . \vec{b} = \vec{b} . \vec{a}$ (commutative)

  $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$ (distributive)

- $\vec{a} . \vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$

  $(\vec{a} \ne 0 \; \vec{b} \ne 0)$

- $(m\vec{a}) . \vec{b} = \vec{a} . (m\vec{b}) = m(\vec{a} . \vec{b})$ (associative), where m is scalar.

- $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$;

  $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

- projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a} . \vec{b}}{|\vec{b}|}$

- the angle ϕ between $\vec{a}$ & $\vec{b}$ is given by $\cos\phi = \dfrac{\vec{a} . \vec{b}}{|\vec{a}| |\vec{b}|}$

  $0 \le \phi \le \pi$

- if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then

  $\vec{a} . \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

- $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

- $|\vec{b}| = \sqrt{b_1^2 + b_2^2 + b_3^2}$

Note...

(i) Maximum value of $\vec{a} . \vec{b} = |\vec{a}| |\vec{b}|$

(ii) Minimum values of $\vec{a} . \vec{b} = -|\vec{a}| |\vec{b}|$

(iii) Any vector $\vec{a}$ can be written as, $\vec{a} = (\vec{a} \cdot \hat{i})\hat{i} + (\vec{a} \cdot \hat{j})\hat{j} + (\vec{a} \cdot \hat{k})\hat{k}$.

(iv) A vector in the direction of the bisector of the angle between two vectors $\vec{a}$ & $\vec{b}$ is $\dfrac{\vec{a}}{|\vec{a}|} + \dfrac{\vec{b}}{|\vec{b}|}$.

Hence bisector of the angle between the two vectors $\vec{a}$ & $\vec{b}$ is $\lambda(\hat{a} + \hat{b})$, where $\lambda \in R^+$.

Bisector of the exterior angle between $\vec{a}$ & $\vec{b}$ is $\lambda(\hat{a} - \hat{b})$ $\lambda \in R - \{0\}$.

5.2 Vector product of two vectors

- If $\vec{a}$ & $\vec{b}$ are two vectors & θ is the angle between them then $\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin\theta \; \hat{n}$, where $\hat{n}$ is the unit vector perpendicular to both $\vec{a}$ & $\vec{b}$ such that $\vec{a}$, $\vec{b}$ & $\hat{n}$ forms a right handed screw system.

- Geometrically $|\vec{a} \times \vec{b}|$ = area of the parallelogram whose two adjacent sides are represented by $\vec{a}$ & $\vec{b}$

- $\vec{a} \times \vec{b} = \vec{0} \Leftrightarrow \vec{a}$ and $\vec{b}$ are parallel (collinear) (provided $\vec{a} \ne 0, \vec{b} \ne 0$) i.e. $\vec{a} = K\vec{b}$, where K is scalar.

- $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$ (not commutative)
- $(m\vec{a}) \times \vec{b} = \vec{a} \times (m\vec{b}) = m(\vec{a} \times \vec{b})$ where m is scalar
- $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$ (distributive)
- $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$
- $\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$
- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

- Unit vector perpendicular to the plane of $\vec{a}$ & $\vec{b}$ is

$$\hat{n} = \pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

- A vector of magnitude 'r' & perpendicular to the plane of $\vec{a}$ and $\vec{b}$ is $\pm \frac{r(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}$
- If θ is the angle between $\vec{a}$ & $\vec{b}$ then $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$
- If $\vec{a}, \vec{b}$ & $\vec{c}$ are the pv's of 3 points A, B and C then the vector area of triangle ABC = $\frac{1}{2}[\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}]$.

  The point A, B & C are collinear if $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$
- **Area of any quadrilateral** whose diagonal vectors are $\vec{d}_1$ & $\vec{d}_2$ is given by $\frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$
- **Lagranges Identity** : for any two vector $\vec{a}$ & $\vec{b}$;

$$(\vec{a} \times \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} \end{vmatrix}$$

5.3 Scalar triple product

- The scalar triple product of three vectors $\vec{a}, \vec{b}$ & $\vec{c}$ is defined as :

  $\vec{a} \times \vec{b} \cdot \vec{c} = |\vec{a}||\vec{b}||\vec{c}| \sin\theta \cos\phi$ where θ is the angle between $\vec{a}$ & $\vec{b}$ & φ is angle between $\vec{a} \times \vec{b}$ & $\vec{c}$

  It is also defined as $[\vec{a}\ \vec{b}\ \vec{c}]$, spelled as **box product.**
- Scalar triple product geometrically represents the volume of the parallelopied whose three coterminous edges are represented by $\vec{a}, \vec{b}$ & $\vec{c}$ i.e. $V = [\vec{a}\ \vec{b}\ \vec{c}]$
- In a scalar triple product the position of dot & cross can be interchanged i.e.

  $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$ Also $[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$

  $\vec{a} \cdot (\vec{b} \times \vec{c}) = -\vec{a} \cdot (\vec{c} \times \vec{b})$ i.e. $[\vec{a}\ \vec{b}\ \vec{c}] = -[\vec{a}\ \vec{c}\ \vec{b}]$
- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$; $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$.

  In general, if $\vec{a} = a_1\vec{\ell} + a_2\vec{m} + a_3\vec{n}$; $\vec{b} = b_1\vec{\ell} + b_2\vec{m} + b_3\vec{n}$ and

  $\vec{c} = c_1\vec{\ell} + c_2\vec{m} + c_3\vec{n}$ then $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} [\vec{\ell}\ \vec{m}\ \vec{n}]$;

  where $\vec{\ell}, \vec{m}$ & $\vec{n}$ are non coplanar vectors.
- $\vec{a}, \vec{b}, \vec{c}$ are coplanar ⇔ $[\vec{a}\ \vec{b}\ \vec{c}] = 0$.
- Scalar product of three vectors, two of which are equal or parallel is 0.

Note..

If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar then $[\vec{a}\,\vec{b}\,\vec{c}] > 0$ for right handed system & $[\vec{a}\,\vec{b}\,\vec{c}] < 0$ for left handed system.

- $[\hat{i}\ \ \hat{j}\ \ \hat{k}] = 1$
- $[K\,\vec{a}\,\vec{b}\,\vec{c}] = K\,[\vec{a}\,\vec{b}\,\vec{c}]$
- $[(\vec{a}+\vec{b})\,\vec{c}\,\vec{d}] = [\vec{a}\,\vec{c}\,\vec{d}] + [\vec{b}\,\vec{c}\,\vec{d}]$
- **The volume of the tetrahedron OABC** with O as origin & the pv's of A, B and C being $\vec{a}$, $\vec{b}$ & $\vec{c}$ respectively is given by $V = \frac{1}{6}[\vec{a}\,\vec{b}\,\vec{c}]$
- The position vector of the centroid of a tetrahedron if the pv's of its angular vertices are $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are given by $\frac{1}{4}[\vec{a}+\vec{b}+\vec{c}+\vec{d}]$.

Note that this is also the point of concurrrency of the lines joining the vertices to the centroids of the opposite faces and is also called the centre of the tetrahedron. In case the tetrahedron is regular it is equidistant from the vertices and the four faces of the tetrahedron.

- **Remember that :** $[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}] = 0$ &

  $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}] = 2\,[\vec{a}\,\vec{b}\,\vec{c}]$

5.4 Vector triple product

Let $\vec{a}, \vec{b}, \vec{c}$ be any three vectors, then the expression $\vec{a}\times(\vec{b}\times\vec{c})$ is vector & is called vector triple product.

GEOMETRICAL INTERPRETATION OF $\vec{a}\times(\vec{b}\times\vec{c})$

Consider the expression $\vec{a}\times(\vec{b}\times\vec{c})$ which itself is a vector, since it is a cross product of two vectors $\vec{a}$ and $(\vec{b}\times\vec{c})$. Now $\vec{a}\times(\vec{b}\times\vec{c})$ is a vector perpendicular to the plane containing $\vec{a}$ and $(\vec{b}\times\vec{c})$ but $\vec{b}\times\vec{c}$ is a vector perpendicular to the plane containing $\vec{b}$ & $\vec{c}$, therefore $\vec{a}\times(\vec{b}\times\vec{c})$ is a vector lying in the plane of $\vec{b}$ & $\vec{c}$ and perpendicular to $\vec{a}$. Hence we can express $\vec{a}\times(\vec{b}\times\vec{c})$ in terms of $\vec{b}$ & $\vec{c}$ i.e. $\vec{a}\times(\vec{b}\times\vec{c}) = x\vec{b} + y\vec{c}$ where x and y are scalars.

- $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\,.\,\vec{c})\,\vec{b} - (\vec{a}\,.\,\vec{b})\,\vec{c}$

  $(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}\cdot\vec{c})\,\vec{b} - (\vec{b}\cdot\vec{c})\,\vec{a}$
- $(\vec{a}\times\vec{b})\times\vec{c} \neq \vec{a}\times(\vec{b}\times\vec{c})$

6. LINEAR COMBINATIONS

Given a finite set of vector $\vec{a}, \vec{b}, \vec{c}, \ldots..$ then the vector $\vec{r} = x\vec{a} + y\,\vec{b} + z\vec{c} + \ldots.$ is called a linear combination of $\vec{a}, \vec{b}, \vec{c}, \ldots..$ for any x, y, z..... $\in$R. We have the following results :

(a) If $\vec{a}, \vec{b}$ are non zero, non-collinear vectors then $x\vec{a} + y\vec{b} = x'\,\vec{a} + y'\vec{b} \Rightarrow x = x'\,;\ y = y'$

(b) **Fundamental Theorem :** Let $\vec{a}, \vec{b}$ be non zero, non collinear vectors. **Then any vector** $\vec{r}$ **coplanar with** $\vec{a}, \vec{b}$ can be expressed uniquely as a linear combination of $\vec{a}, \vec{b}$ i.e. There exist some unique x, y $\in$ R such that $x\vec{a} + y\vec{b} = \vec{r}$

(c) If $\vec{a}, \vec{b}, \vec{c}$ are non-zero, non-coplanar vectors then :

$x\vec{a} + y\vec{b} + z\vec{c} = x'\vec{a} + y'\vec{b} + z'\vec{c} \Rightarrow x = x',\ y = y',\ z = z'$

(d) **Fundamental Theorem in Space :** Let $\vec{a}, \vec{b}, \vec{c}$ be non zero, non collinear vectors in space. **Then any vector** $\vec{r}$ can be uniquely expressed as a linear combination of $\vec{a}, \vec{b}, \vec{c}$ i.e. There exist some unique x, y, z $\in$ R such that $x\vec{a} + y\vec{b} + z\vec{c} = \vec{r}$.

(e) If $\vec{x}_1, \vec{x}_2$ .... $\vec{x}_n$ are n non zero vectors & k_1, k_2,............k_n are n scalars& if the linear combination $k_1\,\vec{x}_1 + k_2\vec{x}_2 + \ldots\ldots k_n\,\vec{x}_n = 0 \Rightarrow k_1 = 0,\ k_2 = 0, \ldots\ldots. k_n = 0$

then we say that vectors $\vec{x}_1, \vec{x}_2$ .... $\vec{x}_n$ are **Linearly Independent Vectors.**

(f) If $\vec{x}_1, \vec{x}_2$ .... $\vec{x}_n$ are not Linearly Independent then they are said to be Linearly Dependent vectors i.e. if $k_1\vec{x}_1 + k_2\vec{x}_2 + ...k_n\vec{x}_n = 0$ & if there exists at least one $k_r \neq 0$ then $\vec{x}_1, \vec{x}_2$ .... $\vec{x}_n$ are said to be **Linearly Dependent.**

Note...

- If $\vec{a} = 3\hat{i} + 2\hat{j} + 5\hat{k}$ then $\vec{a}$ is expressed as a Linear Combination of vectors $\hat{i}, \hat{j}, \hat{k}$. Also $\vec{a}$ $\hat{i}, \hat{j}, \hat{k}$ form a linearly dependent set of vectors. In general, **every set of four vectors is a linearly dependent system.**
- $\hat{i}, \hat{j}, \hat{k}$ are Linearly Independent set of vectors. For

  $K_1\hat{i} + K_2\hat{j} + K_3\hat{k} = 0 \Rightarrow K_1 = 0 = K_2 = K_3$
- Two vectors $\vec{a}$ & $\vec{b}$ are linearly dependent $\Rightarrow \vec{a}$ is parallel to $\vec{b}$ i.e. $\vec{a} \times \vec{b} = 0 \Rightarrow$ linear dependence of $\vec{a}$ & $\vec{b}$. Conversely if $\vec{a} \times \vec{b} \neq 0$ then $\vec{a}$ & $\vec{b}$ are linearly independent.
- **If three vectors $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent, then they are coplanar i.e.** $[\vec{a}, \vec{b}, \vec{c}] = 0$ conversely, if $[\vec{a}, \vec{b}, \vec{c}] \neq 0$, then the vectors are linearly independent.

7. RECIPROCAL SYSTEM OF VECTORS

If $\vec{a}, \vec{b}, \vec{c}$ & $\vec{a}', \vec{b}', \vec{c}'$ are two sets of non-coplanar vectors such that $\vec{a} \cdot \vec{a}' = \vec{b} \cdot \vec{b}' = \vec{c} \cdot \vec{c}' = 1$ then the two systems are called Reciprocal System of vectors.

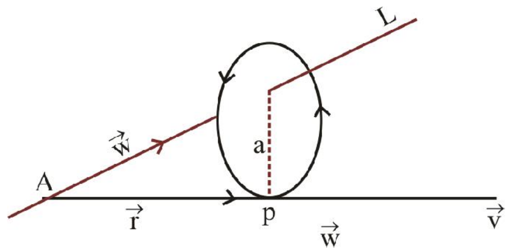
Note...

$$a' = \frac{\vec{b} \times \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]};\ b' = \frac{\vec{c} \times \vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]};\ c' = \frac{\vec{a} \times \vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

Note...

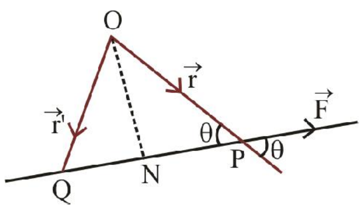
(a) Work done against a constant force $\vec{F}$ over a displacement $\vec{s}$ is defined as $\vec{W} = \vec{F} \cdot \vec{s}$

(b) The tangential velocity $\vec{V}$ of a body moving in a circle is given by $\vec{V} = \vec{w} \times \vec{r}$ where $\vec{r}$ is the pv of the point P.



(c) The moment of $\vec{F}$ about 'O' is defined as $\vec{M} = \vec{r} \times \vec{F}$ where $\vec{r}$ is the pv of P wrt 'O'. The direction of $\vec{M}$ is along the normal to the plane OPN such that $\vec{r}, \vec{F}$ & $\vec{M}$ form a right handed system.

(d) Moment of the couple $(\vec{r}_1 - \vec{r}_2) \times \vec{F}$ where $\vec{r}_1$ & $\vec{r}_2$ are pv's of the point of the application of the force $\vec{F}$ & $-\vec{F}$.



Three Dimensional Geometry

1. CENTRAL IDEA OF 3D

There are infinite number of points in space. We want to identify each and every point of space with the help of three mutually perpendicular coordinate axes OX, OY and OZ.

2. AXES

Three mutually perpendicular lines OX, OY, OZ are considered as three axes.

3. COORDINATE PLANES

Planes formed with the help of x and y axes is known as x–y plane similarly y and z axes y – z plane and with z and x axis z – x plane.

4. COORDINATE OF A POINT

Consider any point P on the space drop a perpendicular form that point to x – y plane then the algebraic length of this perpendicular is considered as z–coordinate and from foot of the perpendicular drop perpendiculars to x and y axes these algebric length of perpendiculars are considered as y and x coordinates respectively.

5. VECTOR REPRESENTATION OF A POINT IN SPAC

If coordinate of a point P in space is (x, y, z) then the position vector of the point P with respect to the same origin is $x\hat{i} + y\hat{j} + z\hat{k}$.

6. DISTANCE FORMULA

Distance between any two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given as $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

Vector method

We know that if position vector of two points A and B are given as $\overrightarrow{OA}$ and $\overrightarrow{OB}$ then

$$|AB| = |\overrightarrow{OB} - \overrightarrow{OA}|$$

$$\Rightarrow \quad |AB| = (x_2 i + y_2 j + z_2 k) - (x_1 i + y_1 j + z_1 k)|$$

$$\Rightarrow \quad |AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

7. DISTANCE OF A POINT P FROM COORDINATE AXES

Let PA, PB and PC are distances of the point P(x, y, z) from the coordinate axes OX, OY and OZ respectively then

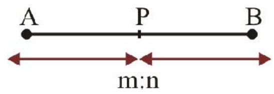
$$PA = \sqrt{y^2 + z^2},\ PB = \sqrt{z^2 + x^2},\ PC = \sqrt{x^2 + y^2}$$

8. SECTION FORMULA

(i) Internal Division :

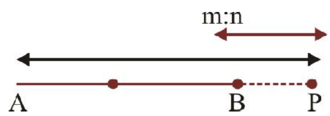
If point P divides the distance between the points A (x_1, y_1, z_1) and B (x_2, y_2, z_2) in the ratio of m : n (internally). The coordinate of P is given as

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$



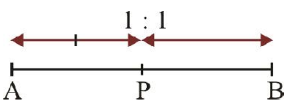
(ii) External division

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$



(iii) Mid point

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

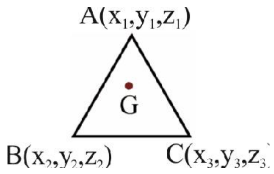


Note...

All these formulae are very much similar to two dimension coordinate geometry.

9. CENTROID OF A TRIANGLE

$$G \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$$



10. IN CENTRE OF TRIANGLE ABC

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}, \frac{az_1 + bz_2 + cz_3}{a+b+c} \right)$$

Where $|AB| = a, |BC| = b, |CA| = c$

11. CENTROID OF A TETRAHEDRON

A (x_1, y_1, z_1) B (x_2, y_2, z_2) C (x_3, y_3, z_3) and D (x_4, y_4, z_4) are the vertices of a tetrahedron then coordinate of its centroid (G) is given as

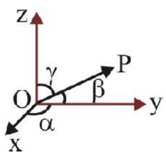
$$\left(\frac{\sum xi}{4}, \frac{\sum yi}{4}, \frac{\sum zi}{4} \right)$$

12. RELATION BETWEEN TWO LINES

Two lines in the space may be coplanar and may be none coplanar. Non coplanar lines are called skew lines if they never intersect each other. Two parallel lines are also non intersecting lines but they are coplanar. Two lines whether intersecting or non intersecting, the angle between them can be obtained.

13. DIRECTION COSINES AND DIRECTION RATIOS

(i) Direction cosines : Let α, β, γ be the angles which a directed line makes with the positive directions of the axes of x, y and z respectively, the cos α, cosβ, cosγ are called the direction cosines of the line. The direction cosines are usually denoted by (l, m, n).



Thus $l = \cos \alpha$, $m = \cos \beta$, $N = \cos \gamma$.

(ii) If l, m, n, be the direction cosines of a lines, then

$l^2 + m^2 + n^2 = 1$

(iii) Direction ratios : Let a, b, c be proportional to the direction cosines, l, m, n, then a, b, c are called the direction ratios.

If a, b, c are the direction ratio of any line L then $a\hat{i} + b\hat{j} + c\hat{k}$ will be a vector parallel to the line L.

If l, m, n are direction cosine of line L then $\ell\hat{i} + m\hat{j} + n\hat{k}$ is a unit vector parallel to the line L.

(iv) If l, m, n be the direction cosines and a, b, c be the direction ratios of a vector, then

$$\left(\ell = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}, n \frac{c}{\sqrt{a^2 + b^2 + c^2}} \right)$$

or $\ell = \dfrac{-a}{\sqrt{a^2 + b^2 + c^2}}, m = \dfrac{-b}{\sqrt{a^2 + b^2 + c^2}}, n = \dfrac{-c}{\sqrt{a^2 + b^2 + c^2}}$

(v) If OP = r, when O is the origin and the direction cosines of OP are l, m, n then the coordinates of P are (lr, mr, nr).

If direction cosine of the line AB are l, m, n, $|AB| = r$, and the coordinate of A is (x_1, y_1, z_1) then the coordinate of B is given as $(x_1 + rl, y_1 + rm, z_1 + rn)$

(vi) If the coordinates P and Q are (x_1, y_1, z_1) and (x_2, y_2, z_2) then the direction ratios of line PQ are, $a = x_2 - x_1$, $b = y_2 - y_1$ and $c = z_2 - z_1$ and the direction cosines of line PQ are $l = \dfrac{x_2 - x_1}{|PQ|}$, $m = \dfrac{y_2 - y_1}{|PQ|}$ and $n = \dfrac{z_2 - z_1}{|PQ|}$

(vii) Direction cosines of axes : Since the positive x–axis makes angles 0°, 90°, 90° with axes of x, y and z respectively. Therefore

Direction cosines of x–axis are (1, 0, 0)

Directio cosines of y–axis are (0, 1, 0)

Direction cosines of z–axis are (0, 0, 1)

14. ANGLE BETWEEN TWO LINE SEGMENTS

If two lines having direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 respectively then we can consider two vector parallel to the lines as $a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$ and $a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$ and angle between them can be given as.

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}.$$

(i) The line will be perpendicular if $a_1a_2+b_1b_2+c_1c_2=0$

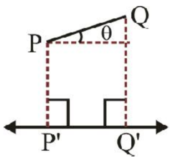
(ii) The lines will be parallel if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

(iii) Two parallel lines have same direction cosines i.e. $l_1=l_2$, $m_1=m_2, n_1=n_2$

15. PROJECTION OF A LINE SEGMENT ON A LINE

(i) If the coordinates P and Q are (x_1, y_1, z_1) and (x_2, y_2, z_2) then the projection of the line segments PQ on a line having direction cosines l, m, n is

$|l(x_2-x_1)+m(y_2-y_1)+n(z_2-z_1)|$



(ii) Vector form : projection of a vector $\vec{a}$ on another vector $\vec{b}$ is $\vec{a}\cdot\hat{b}=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$ In the above case we can consider $\overrightarrow{PQ}$ as $(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}$ in place of $\vec{a}$ and $l\hat{i}+m\hat{j}+n\hat{k}$ in place of $\vec{b}$.

(iii) $l|\vec{r}|, m|\vec{r}|$, and $n|\vec{r}|$ are the projection of $\vec{r}$ in OX, OY and OZ axes.

(iv) $\vec{r}=|\vec{r}|(l\hat{i}+m\hat{j}+n\hat{k})$

A PLANE

If line joining any two points on a surface lies completely on it then the surface is a plane.

OR

If line joining any two points on a surface is perpendicular to some fixed straight line. Then this surface is called a plane. This fixed line is called the normal to the plane.

16. EQUATION OF A PLANE

(i) Normal form of the equation of a plane is $lx+my+nz=p$, where, l, m n are the direction cosines of the normal to the plane and p is the distance of the plane from the origin.

(ii) General form : $ax+by+cz+d=0$ is the equation of a plane, where a, b, c are the direction ratios of the normal to the plane.

(iii) The equation of a plane passing through the point (x_1, y_1, z_1) is given by $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$ where a, b, c are the direction ratios of the normal to the plane.

(iv) Plane through three points : The equation of the plane through three non–collinear points (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) is $\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix}=0$

(v) Intercept Form : The equation of a plane cutting intercept a, b, c on the axes is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

(vi) Vector form : The equation of a plane passing through a point having position vector $\vec{a}$ and normal to vector $\vec{n}$ is $(\vec{r}-\vec{a})\cdot\vec{n}=0$ or $\vec{r}\cdot\vec{n}=\vec{a}\cdot\vec{n}$

Note..

(a) Vector equation of a plane normal to unitvector $\hat{n}$ and at a distance d from the origin is $\vec{r}\cdot\hat{n}=d$

(b) **Planes parallel to the coordinate planes**

(i) Equation of yz–plane is $x=0$

(ii) Equation of xz–plane is $y=0$

(iii) Equation of xy–plane is $z=0$

(c) **Planes parallel to the axes :**

If $a=0$, the plane is parallel to x–axis i.e. equation of the plane parallel to the x–axis is $by+cz+d=0$.

Similarly, equation of planes parallel to y–axis and parallel to z–axis are $ax+cz+d=0$ and $ax+by+d=0$ respectively.

(d) **Plane through origin :** Equation of plane passing through origin is $ax+by+cz=0$.

(e) Transformation of the equation of a plane to the normal form : To reduce any equation $ax + by + cz - d = 0$ to the normal form, first write the constant term on the right hand side and make it positive, then divided each term by $\sqrt{a^2 + b^2 + c^2}$, where a, b, c are coefficients of x, y and z respectively e.g.

$$\frac{ax}{\pm\sqrt{a^2+b^2+c^2}} + \frac{by}{\pm\sqrt{a^2+b^2+c^2}} + \frac{cz}{\pm\sqrt{a^2+b^2+c^2}}$$

$$= \frac{d}{\pm\sqrt{a^2+b^2+c^2}}$$

Where (+) sign is to be taken if $d > 0$ an (–) sign is to be taken if $d > 0$.

(f) Any plane parallel to the given plane $ax + by + cz + d = 0$ is $ax + by + cz + \lambda = 0$ distance between two parallel planes $ax + by + cz + d_1 = 0$ and $ax + dy + xz + d_2 = 0$ is given as

$$\frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}}$$

(g) Equation of a plane passing through a given point and parallel to the given vectors : The equation of a plane passing through a point having position vector $\vec{a}$ and parallel to $\vec{b}$ and $\vec{c}$ is $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ parametric form (where λ and μ are scalars).

or $\vec{r}\cdot(\vec{b}\times\vec{c}) = \vec{a}\cdot(\vec{b}\times\vec{c})$ (non parametric form)

(h) A plane $ax + by + cz + d = 0$ divides the line segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2). in the ratio

$$\left(-\frac{ax_1 + by_1 + cz_1 + d}{ax_2 + by_2 + cz_2 + d}\right)$$

(i) The xy–plane divides the line segment joining the point (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $-\frac{z_1}{z_2}$. Similarly

yz – plane in $-\frac{x_1}{x_2}$ and zx–plane in $-\frac{y_1}{y_2}$

17. ANGLE BETWEEN TWO PLANES

(i) Consider two planes $ax + by + cz + d = 0$ and $a'x + b'y + c'z + d' = 0$. Angle between these planes is the angle between their normals. Since direction ratios of their normals are (a, b, c) and (a′, b′, c′) respectively, hence θ the angle between them is given by

$$\cos\theta = \frac{aa'+bb'+cc'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}$$

Planes are perpendicular if $aa' + bb' + cc' = 0$ and planes are parallel if $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$

(ii) The angle θ between the plane $\vec{r}\cdot\vec{n} = d_1$, $\vec{r}\cdot\vec{n}_2 = d_2$ to given by, $\cos\theta = \frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$ Planes are perpendicular if $\vec{n}_1\cdot\vec{n}_2 = 0$ and Planes are parallel if $\vec{n}_1 = \lambda\vec{n}_2$.

18. A PLANE AND A POINT

(i) Distance of the point (x′, y′, z′) from the plane $ax + by + az + d = 0$ is given by $\frac{ax'+by'+cz'+d}{\sqrt{a^2+b^2+c^2}}$

(ii) The length of the perpendicular from a point having position vector $\vec{a}$ to plane $\vec{r}\cdot\vec{n} = d$ to given by

$$p = \frac{|\vec{a}\cdot\vec{n} - d|}{|\vec{n}|}.$$

19. ANGLE BISECTORS

(i) The equations of the planes bisecting the angle between two given planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = \pm\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}$$

(ii) Equation of bisector of the angle containing origin : First make both the constant terms positive. Then the positive sign in $\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = \pm\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}$ gives the bisector of the angle wich contains the origin.

(iii) Bisector of acute/obtuse angle : First make both the constant terms positive. Then

$a_1a_2 + b_1b_2 + c_1c_2 > 0$

$\Rightarrow$ origin lies on obtuse angle

$a_1a_2 + b_1b_2 + c_1c_2 < 0$

$\Rightarrow$ origin lies in acute angle

20. FAMILY OF PLANES

(i) Any plane passing through the line of intersection of non–parallel planes or equation of the plane through the given line in non symmetrical form.

$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is

$a_1x + b_1y + c_1z + d_1 + \lambda\,(a_2x + b_2y + c_2z + d_2) = 0$

(ii) The equation of plane passing through the intersection of the planes $\vec{r}\cdot\vec{n}_1 = d_1$ and $\vec{r}\cdot\vec{n}_2 = d_2$ is $\vec{r}\cdot(\vec{n}_1 + \lambda\vec{n}_2) = d_1 + \lambda d_2$ where λ is arbitrary scalar

(iii) **Plane through a given line :** Equation of any plane through the line in symmetrical form.

$\dfrac{x-x_1}{\ell} = \dfrac{y-y_1}{m} = \dfrac{z-z_1}{n}$ is $A\,(x - x_1) + B\,(y - y_1) + C\,(z - z_1) = 0$ where $Al + Bm + Cn = 0$

21. AREA OF A TRIANGLE

Let A (x_1, y_1, z_1), B (x_2, y_2, z_2), C (x_3, y_3, z_3) be the vertices of a triangle, then $\Delta = \sqrt{(\Delta_x^2 + \Delta_y^2 + \Delta_z^2)}$

where $\Delta_x = \dfrac{1}{2}\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix}$, $\Delta_y = \dfrac{1}{2}\begin{vmatrix} z_1 & x_1 & 1 \\ z_2 & x_2 & 1 \\ z_3 & x_3 & 1 \end{vmatrix}$ and

$$\Delta_z = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Vector Method – From two vector $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Then area is given by

$$\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix}$$

22. VOLUME OF A TETRAHEDRON

Volume of a tetrahedron with vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$ is given by

$$V = \frac{1}{6}\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}$$

23. EQUATION OF A LINE

(i) A straight line in space is characterised by the intersection of two planes which are not parallel and therefore, the equation of a straight line is a solution of the system constituted by the equations of the two planes, $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$. This form is also known as non–symmetrical form.

(ii) The equation of a line passing through the point (x_1, y_1, z_1) and having direction ratios a, b, c is

$\dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c} = r$. This form is called symmetric form. A general point on the line is given by $(x_1 + ar, y_1 + br, z_1 + cr)$.

(iii) Vector equation : Vector equation of a straight line passing through a fixed point with position vector $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\,\vec{b}$ where λ is a scalar.

(iv) The equation of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

(v) Vector equation of a straight line passing through two points with position vectors $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$.

(vi) Reduction of cartesion form of equation of a line to vector form and vice versa

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

$$\Leftrightarrow \vec{r} = (x_1\hat{i} + y\hat{j} + z_1\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k}).$$

Note...

Straight lines parallel to co–ordinate axes :

	Straight lines	Equation
(i)	Through origin	$y = mx, z = nx$
(ii)	x–axis	$y = 0, z = 0$
(iii)	y–axis	$x = 0, z = 0$
(iv)	z–axis	$x = 0, y = 0$
(v)	Parallel to x–axis	$y = p, z = q$
(vi)	Parallel to y–axis	$x = h, z = q$
(vii)	Parallel to z–axis	$x = h, y = p$

24. ANGLE BETWEEN A PLANE AND A LIN

(i) If θ is the angle between line $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and the plane $ax + by + cz + d = 0$, then

$$\sin\theta = \left[\frac{a\ell + bm + cn}{\sqrt{(a^2+b^2+c^2)}\sqrt{\ell^2+m^2+n^2}}\right]$$

(ii) Vector form : If θ is the angle between a line $\vec{r} = (\vec{a} + \lambda\vec{b})$ and $\vec{r}\cdot\vec{n} = d$ then $\sin\theta = \left[\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}\right]$.

(iii) Condition for perpendicularity $\frac{\ell}{a} = \frac{m}{b} = \frac{n}{c}$, $\vec{b}\times\vec{n} = 0$

(iv) Condition for parallel $a\ell + bm + cn = 0$, $\vec{b}\cdot\vec{n} = 0$

25. CONDITION FOR A LINE TO LIE IN A PLANI

(i) Cartesian form : Line $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ would lie in a plane

$ax + by + cz + d = 0$, if $ax_1 + by_1 + cz_1 + d = 0$ and

$a\ell + bm + cn = 0$.

(ii) Vector form : Line $\vec{r} = \vec{a} + \lambda\vec{b}$ would lie in the plane $\vec{r}\cdot\vec{n} = d$ if $\vec{b}\cdot\vec{n} = 0$ and $\vec{a}\cdot\vec{n} = d$

26. COPLANER LINES

(i) If the given lines are $\frac{x-\alpha}{\ell} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and $\frac{x-\alpha'}{\ell'} = \frac{y-\beta'}{m'} = \frac{z-\gamma'}{n'}$, then condition for intersection/ coplanarity is $\begin{vmatrix} \alpha-\alpha' & \beta-\beta' & \gamma-\gamma' \\ \ell & m & n \\ \ell' & m' & n' \end{vmatrix} = 0$ and plane containing the above two lines is $\begin{vmatrix} x-\alpha & y-\beta & z-\gamma \\ \ell & m & n \\ \ell' & m' & n' \end{vmatrix} = 0$

(ii) Condition of coplanarity if both the lines are in general assymetric form :-

$ax + by + cz + d = 0 = a'x + b'y + c'z + d'$ and

$\alpha x + \beta y + \gamma z + \delta = 0 = \alpha' x + \beta' y + \gamma' z + \delta'$

They are coplanar if $\begin{vmatrix} a & b & c & d \\ a' & b' & c' & d' \\ \alpha & \beta & \gamma & \delta \\ \alpha' & \beta' & \gamma' & \delta' \end{vmatrix} = 0$

27. SKEW LINES

(i) The straight lines which are not parallel and non–coplanar i.e. non–intersecting are called skew lines.

If $\Delta = \begin{vmatrix} \alpha'-\alpha & \beta'-\beta & \gamma'-\gamma \\ \ell & m & n \\ \ell' & m' & n' \end{vmatrix} \neq 0$, then lines are skew.

(ii) Vector Form : For lines $\vec{a}_1 + \lambda\vec{b}_1$ and $\vec{a}_2 + \lambda\vec{b}_2$ to be skew

$(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1) \neq 0$ or $[\vec{b}_1\ \vec{b}_2\ (\vec{a}_2 - \vec{a}_1)] \neq 0$.

(iii) Shortest distance between the two parallel lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}$ is $d = \left|\frac{(\vec{a}_2 - \vec{a}_1)\times\vec{b}}{|\vec{b}|}\right|$.

28. COPLANARITY OF FOUR POINTS

The points $A(x_1\ y_1\ z_1)$, $B(x_2\ y_2\ z_2)$ $C(x_3\ y_3\ z_3)$ and $D(x_4\ y_4\ z_4)$ are coplaner then

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \\ x_4-x_1 & y_4-y_1 & z_4-z_1 \end{vmatrix}=0$$

very similar in vector method the points $A\ (\vec{r}_1), B(\vec{r}_2), C(\vec{r}_3)$ and $D(\vec{r}_4)$ are coplanar if $= \quad 4\ \hat{i} \quad + \quad 2\ \hat{k} \quad = 0$

29. SIDES OF A PLANE

A plane divides the three dimensional space in two equal parts. Two points $A\ (x_1\ y_1\ z_1)$ and $B\ (x_2\ y_2\ z_2)$ are on the same side of the plane $ax + by + cz + d = 0$ if $ax_1 + by_1 + cz_1 + d$ and $ax_2 + by_2 + cz_2 + d$ and both positive or both negative and are opposite side of plane if both of these values are in opposite sign.

30. LINE PASSING THROUGH THE GIVEN POINT $(x_1\ y_1\ z_1)$ AND INTERSECTING BOTH THE LINES $(P_1 = 0, P_2 = 0)$ AND $(P_3 = 0, P_4 = 0)$

Get a plane through (x_1, y_1, z_1) and containing the line $(P_1 = 0, P_2 = 0)$ as $P_5 = 0$

Also get a plane through (x_1, y_1, z_1) and containing the line $P_3 = 0, P_4 = 0$ as $P_6 = 0$

equation of the required line is $(P_5 = 0, P_6 = 0)$

31. TO FIND IMAGE OF A POINT W.R.T. A LINE

Let $L \equiv \dfrac{x-x_2}{a} = \dfrac{y-y_2}{b} = \dfrac{z-z_2}{c}$ is a given line

Let (x', y', z') is the image of the point $P\ (x_1, y_1, z_1)$ with respect to the line L.

Then

(i) $a\,(x_1 - x') + b\,(y_1 - y') + c\,(z_1 - z') = 0$

(ii) $$\frac{\dfrac{x_1+x'}{2}-x_2}{a} = \frac{\dfrac{y_1+y'}{2}-y_2}{b} = \frac{\dfrac{z_1+z'}{2}-z_2}{c} = \lambda$$

from (ii) get the value of x', y', z' in terms of λ as

$x' = 2a\lambda + 2x_2 - x_1,\ y' = 2b\alpha - 2y_2 - y_1,$

$z' = 2c\lambda + 2z_2 - z_1$

now put the values of x', y', z' in (i) get λ and resubstitute the value of λ to get $(x'\ y'\ z')$.

32. TO FIND IMAGE OF A POINT W.R.T. A PLANE

Let $P\ (x_1, y_1, z_1)$ is a given point and $ax + by + cz + d = 0$ is given plane Let (x', y', z') is the image point.

then

(i) $x' - x_1 = \lambda a,\ y' - y_1 = \lambda b,\ z' - z_1 = \lambda c$

$\Rightarrow \quad x' = \lambda a + x\ , y' = \lambda b + y_1, z' = \lambda c + z_1$

(ii) $$a\left(\frac{x'+x_1}{2}\right) + b\left(\frac{y'+y_1}{2}\right) + c\left(\frac{z'+z_1}{2}\right) + d = 0$$

from (i) put the values of x', y', z' in (ii) and get the values of λ and resubstitute in (i) to get $(x'\ y'\ z')$.

SOLVED EXAMPLES

Example – 1

Show that the point $P(\vec{a}+2\vec{b}+\vec{c})$, $Q(\vec{a}-\vec{b}-\vec{c})$, $R(3\vec{a}+\vec{b}+2\vec{c})$ and $S(5\vec{a}+3\vec{b}+5\vec{c})$ are coplanar given that $\vec{a},\vec{b},\vec{c}$ are non–coplanar.

Sol. To show that P, Q, R, S are coplanar, we will show that $\overrightarrow{PQ}, \overrightarrow{PR}, \overrightarrow{PS}$ are coplanar.

$\overrightarrow{PQ} = -3\ \vec{b} - 2\vec{c}$

$\overrightarrow{PR} = 2\ \vec{a} - \vec{b} + \vec{c}$

$\overrightarrow{PS} = 4\ \vec{a} + \vec{b} + 4\vec{c}$

Let $\overrightarrow{PQ} = \lambda\ \overrightarrow{PR} + \mu\ \overrightarrow{PS}$

$\Rightarrow$ $-3\vec{b} - 2\vec{c} = \lambda\ (2\vec{a} - \vec{b} + \vec{c}) + \mu\ (4\vec{a} + \vec{b} + 4\vec{c})$

$\Rightarrow$ $-3\vec{b} - 2\vec{c} = (2\lambda + 4\mu)\ \vec{a} + (-\lambda + u)\ \vec{b} + (\lambda + 4\mu)\ \vec{c}$

As the vectors, $\vec{a}, \vec{b}, \vec{c}$ are non–coplanar, we can equate their coefficients.

$\Rightarrow$ $0 = 2\lambda + 4\mu$

$\Rightarrow$ $-3 = -\lambda + \mu$

$\Rightarrow$ $-2 = \lambda + 4\mu$

$\lambda = 2, \mu = -1$ is the unique solution for the above system of equations.

$\Rightarrow$ $\overrightarrow{PQ} = 2\ \overrightarrow{PR} - \overrightarrow{PS}$

$\overrightarrow{PQ}, \overrightarrow{PR}, \overrightarrow{PS}$ are coplanar because $\overrightarrow{PQ}$ is a linear combination of $\overrightarrow{PR}$ and $\overrightarrow{PS}$.

$\Rightarrow$ the points P, Q, R, S are also coplanar.

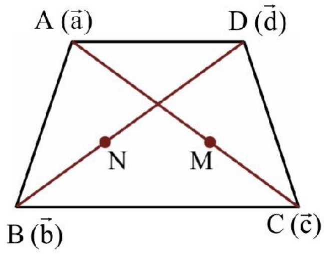
Example – 2

Prove that the segment joining mid–points of the diagonals of a trapezium is parallel to the parallel sides of the trapezium and is equal to half the difference of their lengths.

Sol. Let ABCD be the given trapezium and M, N be the mid points of the diagonals AC and BD.

Let $\vec{a},\vec{b},\vec{c},\vec{d}$ be the position vectors of A, B, C, D respectively.

using section formula, mid points of AC and BD are :



$$\vec{m} = \frac{\vec{a}+\vec{c}}{2}, \quad \vec{n} = \frac{\vec{b}+\vec{d}}{2}$$

$\Rightarrow$ $\overrightarrow{NM} = \vec{m} - \vec{n} = \dfrac{(\vec{a}+\vec{c}) - (\vec{b}+\vec{d})}{2}$

$\Rightarrow$ $\overrightarrow{NM} = \left(\dfrac{\vec{c}-\vec{b}}{2}\right) - \left(\dfrac{\vec{d}-\vec{a}}{2}\right)$

$\Rightarrow$ $\overrightarrow{NM} = 1/2\left(\overrightarrow{BC} - \overrightarrow{AD}\right)$

Let $\overrightarrow{BC} = k\ (\overrightarrow{AD})$

$\Rightarrow$ $\overrightarrow{NM} = 1/2\ (k-1)\ \overrightarrow{AD}$

NM || AD and NM = 1/2 (k – 1) AD

$\Rightarrow$ $NM = \dfrac{k(AD) - AD}{2} = \dfrac{BC - AD}{2}$

$\Rightarrow$ NM is parallel to AD (and BC) and is half the difference of BC and AD.

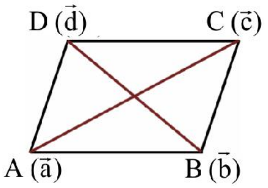
Example – 3

Show that the diagonals of a parallelogram bisect each other.

Sol. Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ be the position vectors of a vertices of a parallelogram ABCD.

AB = DC and AB || DC

(because ABCD is a parallelogram)



$\Rightarrow \quad \overline{AB} = \overline{DC}$

$\Rightarrow \quad \vec{b} - \vec{a} = \vec{c} - \vec{d}$

$\Rightarrow \quad (\vec{b} + \vec{d})/2 = (\vec{a} + \vec{c})/2$

$\Rightarrow$ pv of mid point of BD = pv of mid point of AC

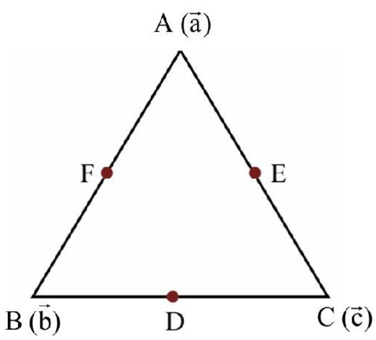
$\Rightarrow$ mid points of BD and AC coincide. Hence AC and BD bisect each other.

Example – 4

Show that the medians of the triangle are concurrent and the point of concurrence divides each median in the ratio 2 : 1.

Sol. Let $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of the vertices of a triangle ABC.

Let D, E, F be the mid–points of sides as shown.



$\Rightarrow \quad \vec{d} = (\vec{b} + \vec{c})/2 \qquad \Rightarrow \quad 2\vec{d} = \vec{b} + \vec{c}$

$\Rightarrow \quad \vec{e} = (\vec{c} + \vec{a})/2 \qquad \Rightarrow \quad 2\vec{e} = \vec{c} + \vec{a}$

$\Rightarrow \quad \vec{f} = (\vec{a} + \vec{b})/2 \qquad \Rightarrow \quad 2\vec{f} = \vec{a} + \vec{b}$

Now try to make the RHS of each equation equal.

$\Rightarrow \quad 2\vec{d} + \vec{a} = \vec{a} + \vec{b} + \vec{c}$

$\Rightarrow \quad 2\vec{e} + \vec{b} = \vec{b} + \vec{c} + \vec{a}$

$\Rightarrow \quad 2\vec{f} + \vec{c} = \vec{c} + \vec{a} + \vec{b}$

$\Rightarrow \quad 2\vec{d} + \vec{a} = 2\vec{e} + \vec{b} = 2\vec{f} + \vec{c} = \vec{a} + \vec{b} + \vec{c}$

Note that the sum of scalar coefficients of vectors is equal to 3 in each expression.We divide each term by 3.

$$\Rightarrow \quad \frac{2\vec{d} + \vec{a}}{3} = \frac{2\vec{e} + \vec{b}}{3} = \frac{2\vec{f} + \vec{c}}{3} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$$

$$\Rightarrow \quad \frac{2\vec{d} + \vec{a}}{2+1} = \frac{2\vec{e} + \vec{b}}{2+1} = \frac{2\vec{f} + \vec{c}}{2+1} = \frac{\vec{a} + \vec{b} + \vec{c}}{2+1}$$

$\Rightarrow$ the point G $[(\vec{a} + \vec{b} + \vec{c})/3]$ divides AD, BE and CF each internally in ratio 2 : 1. Hence G is the common point of intersection of all medians.

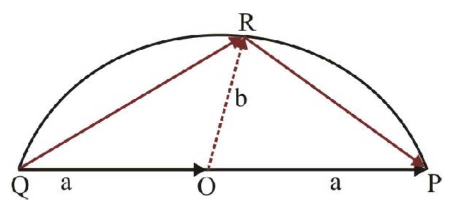
$\Rightarrow$ medians are concurrent and centroid G divides each median in 2 : 1.

$$\text{Centroid } G \equiv \left(\frac{\vec{a} + \vec{b} + \vec{c}}{3}\right)$$

Example – 5

Show that the angle in semi–circle is a right angle.

Sol. Let O be the centre and r be the radius of the semi–circle.



Let $\overline{OP} = \overline{QO} = \vec{a}$ and $\overline{OR} = \vec{b}$

$\Rightarrow \quad \overline{QR} = \vec{a} + \vec{b}$ and $\overline{RP} = \vec{a} - \vec{b}$

Now $\overline{QR} \cdot \overline{RP} = (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$

$= a^2 - b^2$

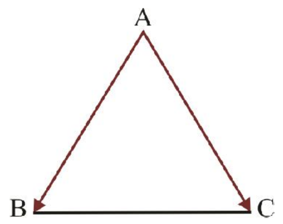
$= a^2 - b^2 = 0$

because a = b = radius of the semi–circle.

Example – 6

The vertices of a triangle are A (2, 3, 0), B (–3, 2, 1) and C (4, –1, 0). Find the area of the triangle ABC and unit vector normal to the plane of this triangle.

Sol. Area of $\Delta ABC = 1/2\ |\overrightarrow{AB} \times \overrightarrow{AC}|$



$\overrightarrow{AB} = (-3 - 2)\,\hat{i} + (2 - 3)\,\hat{j} + (1 - 0)\,\hat{k}$

$\Rightarrow \quad \overrightarrow{AB} = -5\,\hat{i} - \hat{j} + \hat{k}$

and $\quad \overrightarrow{AC} = 2\hat{i} - 4\hat{j} + 0\hat{k}$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & -1 & 1 \\ 2 & -4 & 0 \end{vmatrix} = 4\hat{i} + 2\hat{j} + 22\hat{k}$$

$\Rightarrow \quad$ area of $\Delta ABC = \frac{1}{2}\sqrt{16 + 4 + 484} = \sqrt{126}$ sq. units

and unit vector normal to the plane of this triangle

$$= \frac{\overrightarrow{AB} \times \overrightarrow{AC}}{|\overrightarrow{AB} \times \overrightarrow{AC}|} = \frac{4\hat{i} + 2\hat{j} + 22\hat{k}}{2\sqrt{126}}$$

$$= \frac{2\hat{i} + \hat{j} + 11\hat{k}}{\sqrt{126}}$$

Example – 7

A line makes angles α, β, γ and δ with the diagonals of a cube. Prove that :

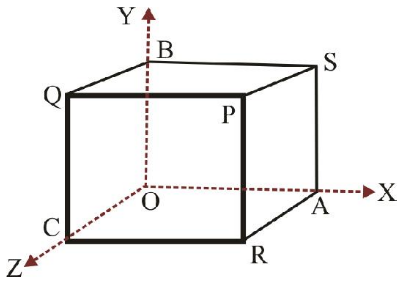
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$$

Sol. Let the origin O be one of the vertices of the cube and OA, OB, OC be the edges thorough O along the axes so that :

$\overrightarrow{OA} = a\,\hat{i}, \quad \overrightarrow{OB} = a\,\hat{j}, \quad \overrightarrow{OC} = a\,\hat{k}$

where a is the length of the edge of the cube

Let P, Q, R, S be the other vertices of the cube opposite to O, A, B, C respectively.



Hence the diagonals of the cube are OP, AQ, BR, & CS.

$\overrightarrow{OP} = a\,\hat{i} + a\,\hat{j} + a\,\hat{k}$

$\overrightarrow{AQ} = a\,\hat{i} + a\,\hat{j} + a\,\hat{k}$

$\overrightarrow{BR} = a\,\hat{i} - a\,\hat{j} + a\,\hat{k}$

$\overrightarrow{CS} = a\,\hat{i} + a\,\hat{j} - a\,\hat{k}$

If $\hat{n} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ is the unit vector along the line which makes the angles α, β, γ & δ with diagonals,

$$\cos\alpha = \frac{\hat{n}\,.\,\overrightarrow{OP}}{|\overrightarrow{OP}|\,|\hat{n}|} = \frac{ax + ay + az}{a\sqrt{3}} = \frac{x + y + z}{\sqrt{3}}$$

$$\cos\beta = \frac{-x + y + z}{\sqrt{3}}\ ; \quad \cos\gamma = \frac{x - y + z}{\sqrt{3}}$$

$$\cos\delta = \frac{x + y - z}{\sqrt{3}}$$

$\Rightarrow \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$

$\frac{1}{3}\,[(x+y+z)^2 + (-x+y+z)^2 + (x-y+z)^2 + (x+y-z)^2]$

$\frac{1}{3}\,4\,(x^2 + y^2 + z^2) = 4/3 \qquad [\because x^2 + y^2 + z^2 = 1]$

Example – 8

Show that : $\hat{i}\times(\vec{a}\times\hat{i})+\hat{j}\times(\vec{a}\times\hat{j})+\hat{k}\times(\vec{a}\times\hat{k})=2\vec{a}.$

Sol. L.H.S. = $[(\hat{i}\cdot\hat{i})\vec{a}-(\hat{i}\cdot\vec{a})\hat{i}]+[(\hat{j}\cdot\hat{j})\vec{a}-(\hat{j}\cdot\vec{a})\hat{j}]+[(\hat{k}\cdot\hat{k})\vec{a}-(\hat{k}\cdot\vec{a})\hat{k}]$

$=\vec{a}+\vec{a}+\vec{a}-[(\hat{i}\cdot\vec{a})\hat{i}+(\hat{j}\cdot\vec{a})\hat{j}+(\hat{k}\cdot\vec{a})\hat{k}]$

$=3\vec{a}-[a_x\hat{i}+a_y\hat{j}+a_z\hat{k}]=3\vec{a}-\vec{a}=2\vec{a}$ = R.H.S.

Note : It is useful to remember that x–component of $\vec{a}=\vec{a}\cdot\hat{i}$ etc ......

Example – 9

Find a vector of magnitude 5 units coplanar with vectors $3\hat{i}-\hat{j}-\hat{k}$ and $\hat{i}+\hat{j}-2\hat{k}$ and perpendicular to the vector $2\hat{i}+2\hat{j}+\hat{k}$.

Sol. Let $\vec{a}=3\hat{i}-\hat{j}-\hat{k}$

$\vec{b}=\hat{i}+\hat{j}-2\hat{k}$

and $\vec{c}=2\hat{i}+2\hat{j}+\hat{k}$

A vector coplanar with $\vec{a}$ and $\vec{b}$ and perpendicualr to $\vec{c}$ can be taken as

$\vec{r}=\ell\vec{c}\times(\vec{a}\times\vec{b})$ where l is a scalar

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & -1\\ 1 & 1 & -2\end{vmatrix}=3\hat{i}+5\hat{j}+4\hat{k}$$

$$\vec{c}\times(\vec{a}\times\vec{b})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 2 & 1\\ 3 & 5 & 4\end{vmatrix}=3\hat{i}-5\hat{j}+4\hat{k}$$

$\Rightarrow$ $\vec{r}=\ell\left(3\hat{i}-5\hat{j}+4\hat{k}\right)$

$|\vec{r}|=|\ell|\sqrt{9+25+15}=5$

$l=\pm\dfrac{5}{5\sqrt{2}}=\pm\dfrac{1}{\sqrt{2}}$

the required vector is $\vec{r}=\pm\dfrac{1}{\sqrt{2}}(3\hat{i}-5\hat{j}+4\hat{k})$

Example – 10

Show that the lines $\vec{r}=3\hat{i}-\hat{j}+\hat{k}+\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$ and $\vec{r}=2\hat{i}+2\hat{j}-2\hat{k}+\mu\left(\hat{i}-\hat{j}+2\hat{k}\right)$ are intersecting and hence find their point of intersection.

Sol. Let $\vec{p}$ be the position vector of their point of intersection.

$\Rightarrow$ $\vec{p}=3\hat{i}-\hat{j}+\hat{k}+\lambda(\hat{i}+\hat{j}+\hat{k})=2\hat{i}+2\hat{j}-2\hat{k}+\mu(\hat{i}-\hat{j}+2\hat{k})$

$\Rightarrow$ $(3+\lambda)\hat{i}+(\lambda-1)\hat{j}+(\lambda+1)\hat{k}=(\mu+2)\hat{i}+(2-\mu)\hat{j}+(2\mu-2)\hat{k}$

$\Rightarrow$ $3+\lambda=\mu+2$ ...(i)

$\Rightarrow$ $\lambda-1=2-\mu$ ...(ii)

$\Rightarrow$ $\lambda+1=2\mu-2$ ...(iii)

The lines are intersecting if these equations are consistent.

from (i) and (ii), we get

$\lambda=1,\quad \mu=2$

Substituting these values in (iii), we get

$1+1=2(2)-2$

$\Rightarrow$ $2=2$

$\Rightarrow$ $\lambda=1,\mu=2$

satisfied (iii) also

Hence lines are intersecting and the point of intersection is :

$\vec{p}=3\hat{i}-\hat{j}+\hat{k}+\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$

$=3\hat{i}-\hat{j}+\hat{k}+\left(\hat{i}+\hat{j}+\hat{k}\right)$

$=4\hat{i}+2\hat{k}$

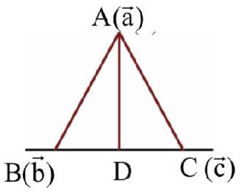
$\Rightarrow$ the coordinates of this point are (4, 0, 2).

Example – 11

The vertices of a triangle ABC are A (1, 0, 2), B (–2, 1, 3) and C (2, –1, 1). Find the equation of the line BC, the foot of the perpendicular from A to BC and the length of the perpendicular.

Sol. A vector parallel to BC is

$BC = \vec{c} - \vec{b} = 4\hat{i} - 2\hat{j} - 2\hat{k}$



$\Rightarrow$ the equation of BC is : $\vec{r} = \vec{b} + t(\vec{c} - \vec{b})$

$\Rightarrow$ $\vec{r} = -2\hat{i} + \hat{j} + 3\hat{k} + t\left(4\hat{i} - 2\hat{j} - 2\hat{k}\right)$

Let position vector of D be

$\vec{d} = -2\hat{i} + \hat{j} + 3\hat{k} + t\left(4\hat{i} - 2\hat{j} - 2\hat{k}\right)$

because D lies on line BC.

Now $\overrightarrow{AD} \perp \overrightarrow{BC}$

$\Rightarrow$ $\overrightarrow{AD} \cdot \overrightarrow{BC} = 0$

$\Rightarrow$ $(\vec{d} - \vec{a}) \cdot (\vec{c} - \vec{b}) = 0$

$\Rightarrow$ $\left[-3\hat{i} + \hat{j} + \hat{k} + t\left(4\hat{i} - 2\hat{j} - 2\hat{k}\right)\right] \cdot \left(4\hat{i} - 2\hat{j} - 2\hat{k}\right) = 0$

$\Rightarrow$ $(4t - 3)\,4 + (1 - 2t)\,(-2) + (1 - 2t)\,(-2) = 0$

$\Rightarrow$ $24t - 16 = 0$

$\Rightarrow$ $t = 2/3$

$\Rightarrow$ $\vec{d} = -2\hat{i} + \hat{j} + 3\hat{k} + (2/3)\,(4\hat{i} - 2\hat{j} - 2\hat{k})$

$\vec{d} = \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{5}{3}\hat{k}$

$\Rightarrow$ $D \equiv \left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{3}\right)$

$\overrightarrow{AD} = \vec{d} - \vec{a} = \left(\frac{2}{3} - 1\right)\hat{i} - \frac{1}{3}\hat{j} + \left(\frac{5}{3} - 2\right)\hat{k}$

$= -\frac{1}{3}\hat{i} - \frac{1}{3}\hat{j} - \frac{1}{3}\hat{k}$ $\quad AD = |\overrightarrow{AD}| = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$ units

Example – 12

Find the equation of the plane passing through the points A (2, 1, 3), B (–1, 2, 4) and C (0, 2, 1). Hence find the coordinate of the point of intersection of the plane ABC and the line $\vec{r} = \hat{i} - \hat{j} + \hat{k} + \lambda\,(2\hat{i} + \hat{k})$.

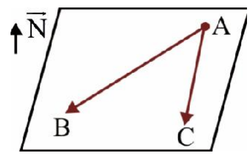
Sol. Let $\vec{N}$ be a vector perpendicular to the plane of ΔABC.

$\Rightarrow$ $\vec{N} = \overrightarrow{AB} \times \overrightarrow{AC}$

$= \left(-3\hat{i} + \hat{j} + \hat{k}\right) \times \left(-2\hat{i} + \hat{j} - 2\hat{k}\right)$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 1 \\ -2 & 1 & -2 \end{vmatrix}$

$= -3\hat{i} - 8\hat{j} - \hat{k}$



$\Rightarrow$ the equation of the plane is

$(\vec{r} - \vec{a}) \cdot \vec{N} = 0$ where $\vec{a} = = 2\hat{i} + \hat{j} + 3\hat{k}$

$\Rightarrow$ $\vec{r} \cdot \vec{N} = \vec{a} \cdot \vec{N}$

$\Rightarrow$ $\vec{r} \cdot \left(-3\hat{i} - 8\hat{j} - \hat{k}\right) = -6 - 8 - 3$

$\Rightarrow$ $\vec{r} \cdot \left(-3\hat{i} - 8\hat{j} - \hat{k}\right) = -17$

The given line is $\vec{r} = \hat{i} - \hat{j} + \hat{k} + \lambda\,(2\hat{i} + \hat{k})$

To find the point of intersection, we solve these equations simultaneously.

$\Rightarrow$ $\left[\hat{i} - \hat{j} + \hat{k} + \lambda\left(2\hat{i} + \hat{k}\right)\right] \cdot \left(-3\hat{i} - 8\hat{j} - \hat{k}\right) = -17$

$\Rightarrow$ $(2\lambda + 1)(-3) + 8 + (\lambda + 1)(-1) = -17$

$\Rightarrow$ $\lambda = 3$

$\Rightarrow$ the point of intersection is

$\vec{r} = \hat{i} - \hat{j} + \hat{k} + 3\,(2\hat{i} + \hat{k})$

$\Rightarrow$ $\vec{r} = 7\hat{i} - \hat{j} + 4\hat{k}$

$\Rightarrow$ coordinates are (7, –1, 4)

Example – 13

From the point A (1, 2, 0), perpendicular is drawn to the plane $\vec{r}\cdot\left(3\hat{i}-\hat{j}+\hat{k}\right)=2$ meeting it at the point P. Find the coordinates of point P and the distance AP.

Sol. Let us first find the equation of line AP. As AP is normal to the plane, the vector $\vec{N}=3\hat{i}-\hat{j}+\hat{k}$ is parallel to AP.

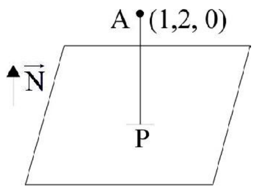
$\Rightarrow$ equation of AP is $\vec{r}=\hat{i}+2\hat{j}+t\,(3\hat{i}-\hat{j}+\hat{k})$

Now we solve equations of AP and plane to get point P.

$\Rightarrow$ $\left[\hat{i}+2\hat{j}+t\left(3\hat{i}-\hat{j}+\hat{k}\right)\right].\left(3\hat{i}-\hat{j}+\hat{k}\right)=2$

$\Rightarrow$ $(3t+1)\,3+(2-t)\,(-1)+t=2$

$\Rightarrow$ $t=1/11$



$\Rightarrow$ point P is $\vec{r}=\hat{i}+2\hat{j}+1/11\,(3\hat{i}-\hat{j}+\hat{k})$

$$\vec{r}=\frac{14}{11}\hat{i}+\frac{21}{11}\hat{j}+\frac{1}{11}\hat{k}$$

$\Rightarrow$ $P\equiv\left(\frac{14}{11},\frac{21}{11},\frac{1}{11}\right)$

$$AP=\sqrt{\left(\frac{14}{11}-1\right)^2+\left(\frac{21}{11}-2\right)^2+\left(\frac{1}{11}-0\right)^2}$$

$$=\frac{1}{\sqrt{11}}$$

Example – 14

The position vectors of the points P and Q are $5\hat{i}+7\hat{j}-2\hat{k}$ and $-3\hat{i}+3\hat{j}+6\hat{k}$ respectively. The vector $\vec{A}=3\hat{i}-\hat{j}+\hat{k}$ passes through the point P and the vector $\vec{B}=-3\hat{i}+2\hat{j}+4\hat{k}$ passes through the point Q. A third vector $2\hat{i}+7\hat{j}-5\hat{k}$ intersects vectors $\vec{A}$ and $\vec{B}$. Find the position vectors of the points of intersection.

Sol. Equation of line AP $\equiv$ $\vec{r}=5\hat{i}+7\hat{j}-2\hat{k}+\lambda_1\left(3\hat{i}-\hat{j}+\hat{k}\right)$

Equation of line BQ $\equiv$

$$\vec{r}=-3\hat{i}+3\hat{j}+6\hat{k}+\lambda_3\left(-3\hat{i}+2\hat{j}+4\hat{k}\right)$$

Since Point D lies on AP, its position vector can be taken as : $\vec{d}=5\hat{i}+7\hat{j}-2\hat{k}+\lambda_1\left(3\hat{i}-\hat{j}+\hat{k}\right)$

A vector parallel to line CD is $2\hat{i}+7\hat{j}-5\hat{k}$

Equation of line CD $\equiv$

$$\vec{r}=5\hat{i}+7\hat{j}-2\hat{k}+\lambda_1\left(3\hat{i}-\hat{j}+\hat{k}\right)+\lambda_2\left(2\hat{i}+7\hat{j}-5\hat{k}\right)$$

solve equation of line BQ with equation of line CD to get point of intersection C.

Solve BQ and CD to get :

$$5\hat{i}+7\hat{j}-2\hat{k}+\lambda_1\left(3\hat{i}-\hat{j}+\hat{k}\right)+\lambda_2\left(2\hat{i}+7\hat{j}-5\hat{k}\right)$$

$$=-3\hat{i}+3\hat{j}+6\hat{k}+\lambda_3\left(-3\hat{i}+2\hat{j}+4\hat{k}\right)$$

Equating the coefficients of i, j and k, we get

$5+3\lambda_1+2\lambda_2=-3\,(1+\lambda_3)$ ....(1)

$7-\lambda_1+7\lambda_2=3+2\lambda_3$ ....(2)

$-2+\lambda_1-5\lambda_2=6+4\lambda_3$ ....(3)

Solve equations (1), (2) and (3) to get :

$\lambda_2=-1,\lambda_3=-1$ and $\lambda_1=-1$

$\Rightarrow$ $D\equiv(2,8,-3)$ and $C\equiv(0,1,2)$

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Algebra of Vectors :

1. In a regular hexagon ABCDEF, $\overrightarrow{AB} = a, \overrightarrow{BC} = \vec{b}$ and $\overrightarrow{CD} = \vec{c}$. Then, $\overrightarrow{AE} =$

(a) $\vec{a} + \vec{b} + \vec{c}$ (b) $2\vec{a} + \vec{b} + \vec{c}$

(c) $\vec{b} + \vec{c}$ (d) $\vec{a} + 2\vec{b} + 2\vec{c}$

2. If ABCDE is a pentagon, then $\overrightarrow{AB} + \overrightarrow{AE} + \overrightarrow{BC} + \overrightarrow{DC} + \overrightarrow{ED} + \overrightarrow{AC}$ equals

(a) $3\overrightarrow{AD}$ (b) $3\overrightarrow{AC}$

(c) $3\overrightarrow{BE}$ (d) $3\overrightarrow{CE}$

3. If G is the intersection of diagonals of a parallelogram ABCD and O is any point, then $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} =$

(a) $2\overrightarrow{OG}$ (b) $4\overrightarrow{OG}$

(c) $5\overrightarrow{OG}$ (d) $3\overrightarrow{OG}$

4. Which of the following is unit vectors

(a) $\hat{i} + \hat{j}$ (b) $\frac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{2}}$

(c) $\hat{i} + \hat{j} + \hat{k}$ (d) $\frac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}}$

5. If $\vec{a} = 2\hat{i} + 5\hat{j}$ and $\vec{b} = 2\hat{i} - \hat{j}$, then unit vector in the direction of $\vec{a} + \vec{b}$ is

(a) $\hat{i} + \hat{j}$ (b) $\sqrt{2}(\hat{i} + \hat{j})$

(c) $(\hat{i} + \hat{j})/\sqrt{2}$ (d) $(\hat{i} - \hat{j})/\sqrt{2}$

6. For any two vector $\vec{a}$ and $\vec{b}$, correct statement is

(a) $|\vec{a} - \vec{b}| = |\vec{a}| - |\vec{b}|$

(b) $|\vec{a} + \vec{b}| \geq |\vec{a}| - |\vec{b}|$

(c) $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$

(d) $|\vec{a} - \vec{b}| \leq |\vec{a}| - |\vec{b}|$

7. If $\hat{i}, \hat{j}, \hat{k}$ are position vectors of A, B, C and $\overrightarrow{AB} = \overrightarrow{CX}$, then position vector of X is

(a) $-\hat{i} + \hat{j} + \hat{k}$ (b) $\hat{i} - \hat{j} + \hat{k}$

(c) $\hat{i} + \hat{j} - \hat{k}$ (d) $\hat{i} + \hat{j} + \hat{k}$

8. If the position vector of points A and B with respect to point P are respectively $\vec{a}$ and $\vec{b}$ then the position vector of middle point of AB is

(a) $\frac{\vec{b} - \vec{a}}{2}$ (b) $\frac{\vec{a} + \vec{b}}{2}$

(c) $\frac{\vec{a} - \vec{b}}{2}$ (d) none of these

9. The points with position vectors

$3\hat{i} - 4\hat{j} - 4\hat{k}, 2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} - 3\hat{j} - 5\hat{k}$ form

(a) an equilateral triangle

(b) an isosceles triangle

(c) a right angle triangle

(d) none of these

10. If vector $2\hat{i} + 3\hat{j} - 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ represents the adjacent sides of any parallelogram then the length of diagonals of parallelogram are

(a) $\sqrt{35}, \sqrt{35}$ (b) $\sqrt{35}, \sqrt{11}$

(c) $\sqrt{25}, \sqrt{11}$ (d) none of these

11. The position vector of two points P and Q are respectively $\vec{p}$ and $\vec{q}$ then the position vector of the point dividing PQ in 2 : 5 is

(a) $\frac{\vec{p} + \vec{q}}{2 + 5}$ (b) $\frac{5\vec{p} + 2\vec{q}}{2 + 5}$

(c) $\frac{2\vec{p} + 5\vec{q}}{2 + 5}$ (d) $\frac{\vec{p} - \vec{q}}{2 + 5}$

12. The position vector of the vertices of triangle ABC are $\hat{i}, \hat{j}$ and $\hat{k}$ then the position vector of its orthocentre is

(a) $\hat{i} + \hat{j} + \hat{k}$ (b) $2(\hat{i} + \hat{j} + \hat{k})$

(c) $\frac{1}{3}(\hat{i} + \hat{j} + \hat{k})$ (d) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

13. If, D, E, F are mid points of sides BC, CA and AB respectively of a triangle ABC, and $\hat{i}+\hat{j}, \hat{j}+\hat{k}, \hat{k}+\hat{i}$ are p.v. of points A,B and C respectively, then p.v. of centroid of ΔDEF is

(a) $\frac{\hat{i}+\hat{j}+\hat{k}}{3}$ (b) $\hat{i}+\hat{j}+\hat{k}$

(c) $2\left(\hat{i}+\hat{j}+\hat{k}\right)$ (d) $\frac{2\left(\hat{i}+\hat{j}+\hat{k}\right)}{3}$

14. The 2 vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represents the two sides AB and AC, respectively of a ΔABC. The length of the median through A is

(a) $\frac{\sqrt{34}}{2}$ (b) $\frac{\sqrt{48}}{2}$

(c) $\sqrt{18}$ (d) None of these

15. P is a point on the line segment joining the points (3, 2, –1) and (6, 2, –2). If x co-ordinate of P is 5, then its y co-ordinate is

(a) 2 (b) 1

(c) –1 (d) –2

16. The plane XOZ divides the join of (1, –1, 5) & (2, 3, 4) in the ratio λ : 1, then λ is :

(a) – 3 (b) 3

(c) –1/3 (d) 1/3

Collinearity & Coplanarity

17. If vectors $(x-2)\ \hat{i}+\hat{j}$ and $(x+1)\ \hat{i}+2\hat{j}$ are collinear, then the value of x is

(a) 3 (b) 4

(c) 5 (d) 0

18. If points $\hat{i}+2\hat{k}, \hat{j}+\hat{k}$ and $\lambda\hat{i}+\mu\hat{j}$ are collinear, then

(a) $\lambda=2, \mu=1$ (b) $\lambda=2, \mu=-1$

(c) $\lambda=-1, \mu=2$ (d) $\lambda=-1, \mu=-2$

19. If position vectors of A, B, C, D are respectively $2\hat{i}+3\hat{j}+5\hat{k}, \hat{i}+2\hat{j}+3\hat{k}, -5\hat{i}+4\hat{j}-2\hat{k}$ and $\hat{i}+10\hat{j}+10\hat{k}$, then

(a) AB || CD (b) DC || AD

(c) A, B, C are collinear (d) B, C, D are collinear

20. If $\vec{a}=3\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=-\hat{i}+\hat{j}+\hat{k}$, then the unit vector parallel to $\vec{a}+\vec{b}$, is

(a) $\frac{1}{3}\left(2\hat{i}-\hat{j}+2\hat{k}\right)$ (b) $\frac{1}{5}\left(2\hat{i}-\hat{j}+2\hat{k}\right)$

(c) $\frac{1}{\sqrt{3}}\left(2\hat{i}-\hat{j}+2\hat{k}\right)$ (d) none of these

21. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors, then the points with p.v. $\vec{a}-2\vec{b}+3\vec{c}, 2\vec{a}+\lambda\vec{b}-4\vec{c}, -7\vec{b}+10\vec{c}$ will be collinear if the value of λ is

(a) 3 (b) 2

(c) 0 (d) none of these

22. If the vector $\vec{b}$ is collinear with the vector $\vec{a}=\left(2\sqrt{2},-1,4\right)$ & $|\vec{b}|=10$, then

(a) $\vec{a}\pm\vec{b}=0$ (b) $\vec{a}\pm2\vec{b}=0$

(c) $2\vec{a}\pm\vec{b}=0$ (d) none

23. Let $\vec{p}$ is the p.v. of the orthocentre & $\vec{g}$ is the p.v. of the centroid of the triangle ABC where circumcentre is the origin. If $\vec{p}=K\vec{g}$ then K =

(a) 3 (b) 2

(c) 1/3 (d) 2/3

24. If $\vec{a}=4\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=-8\hat{i}+4\hat{j}-6\hat{k}$ are two vectors then $\vec{a}$, $\vec{b}$ are

(a) like parallel (b) unlike parallel

(c) non-collinear (d) perpendicular

Product of Vectors :

25. If the moduli of vectors $\vec{a}$ and $\vec{b}$ are 1 and 2 respectively and $\vec{a}.\vec{b}=1$, then the angle θ between them is :

(a) $\theta=\pi/6$ (b) $\theta=\pi/3$

(c) $\theta=\pi/2$ (d) $\theta=2\pi/3$

26. If the angle between $\vec{a}$ and $\vec{b}$ is θ then for $\vec{a}.\vec{b}\geq0$

(a) $0\leq\theta\leq\pi$ (b) $0<\theta$ or $\theta>\pi/2$

(c) $\pi/2\leq\theta\leq\pi$ (d) $0\leq\theta\leq\pi/2$

27. If $\vec{a}$ and $\vec{b}$ are unit vectors and 60° is the angle between them, then $(2\vec{a}-3\vec{b}).(4\vec{a}+\vec{b})$ equals

(a) 5 (b) 0

(c) 11 (d) none of these

28. If vectors $3\hat{i}+2\hat{j}+8\hat{k}$ and $2\hat{i}+x\hat{j}+\hat{k}$ are perpendicular then x is equal to

(a) 7 (b) –7

(c) 5 (d) –4

29. If vector $\vec{a}+\vec{b}$ is perpendicular to $\vec{b}$ and $2\vec{b}+\vec{a}$ is perpendicular to $\vec{a}$, then

(a) $|\vec{a}|=\sqrt{2}|\vec{b}|$ (b) $|\vec{a}|=2|\vec{b}|$

(c) $|\vec{b}|=\sqrt{2}|\vec{a}|$ (d) $|\vec{a}|=|\vec{b}|$

30. $(\vec{A}+\vec{B})^2+(\vec{A}-\vec{B})^2$ equals

(a) $2(\vec{A}^2+\vec{B}^2)$ (b) $4\vec{A}.\vec{B}$

(c) $\vec{A}^2+\vec{B}^2$ (d) none of these

31. If $|\vec{a}|=|\vec{b}|$, then $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})$ is

(a) positive (b) negative

(c) zero (d) none of these

32. If $\vec{a}=2\hat{i}+\hat{j}+\hat{k}, \vec{b}=3\hat{i}-4\hat{j}+2\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+2\hat{k}$ then the projection of $\vec{a}+\vec{b}$ on $\vec{c}$ is

(a) 17/3 (b) 5/3

(c) 4/3 (d) none of these

33. If $\vec{a}=\hat{i}+3\hat{j}-2\hat{k}$ and $\vec{b}=4\hat{i}-2\hat{j}+4\hat{k}$,

then $(2\vec{a}+\vec{b}).(\vec{a}-2\vec{b})$ equals

(a) 14 (b) –14

(c) 0 (d) none of these

34. Angle between the vectors $2\hat{i}+6\hat{j}+3\hat{k}$ and $12\hat{i}-4\hat{j}+3\hat{k}$ is

(a) $\cos^{-1}\left(\frac{1}{10}\right)$ (b) $\cos^{-1}\left(\frac{9}{11}\right)$

(c) $\cos^{-1}\left(\frac{9}{91}\right)$ (d) $\cos^{-1}\left(\frac{1}{9}\right)$

35. If the angle between two vectors $\hat{i}+\hat{k}$ and $\hat{i}-\hat{j}+a\hat{k}$ is π/3, then the value of a is

(a) 2 (b) 4

(c) –2 (d) 0

36. If $\hat{i}+\hat{j}+\hat{k}, 2\hat{i}+5\hat{j}, 3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ be p.v. of four points A,B,C and D respectively, then the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$ is

(a) π/4 (b) π/2

(c) π (d) none of these

37. Projection vector of $\vec{a}$ on $\vec{b}$ is

(a) $\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\vec{b}$ (b) $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

(c) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$ (d) $\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\hat{b}$

38. The number of vectors of unit length perpendicular to the vectors $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$ is

(a) one (b) two

(c) three (d) infinite

39. The angle between the vectors $\vec{a}+\vec{b}$ & $\vec{a}-\vec{b}$, given $|\vec{a}|=2, |\vec{b}|=1$ and angle between $\vec{a}$ & $\vec{b}$ is π/3, is

(a) $\tan^{-1}\frac{2}{\sqrt{3}}$ (b) $\tan^{-1}\sqrt{\frac{2}{3}}$

(c) $\tan^{-1}\sqrt{\frac{3}{7}}$ (d) none

40. Given the vectors $\vec{a}$ & $\vec{b}$ the angle between which equals 120°. If $|\vec{a}|=3$ & $|\vec{b}|=4$ then the length of the vector $2\vec{a}-\frac{3}{2}\vec{b}$ is

(a) $6\sqrt{3}$ (b) $7\sqrt{2}$

(c) $4\sqrt{5}$ (d) none

41. $\left[\dfrac{\vec{a}}{|\vec{a}|^2}-\dfrac{\vec{b}}{|\vec{b}|^2}\right]^2=$

(a) $|\vec{a}|^2-|\vec{b}|^2$ (b) $\left[\dfrac{\vec{a}-\vec{b}}{|\vec{a}||\vec{b}|}\right]^2$

(c) $\left[\dfrac{\vec{a}|\vec{a}|-\vec{b}|\vec{b}|}{|\vec{a}||\vec{b}|}\right]^2$ (d) none

42. The vector $\vec{c}$, directed along the internal bisector of the angle between the vectors, $\vec{a}=7\hat{i}-4\hat{j}-4\hat{k}$ and $\vec{b}=-2\hat{i}-\hat{j}+2\hat{k}$ with $|\vec{c}|=5\sqrt{6}$ is :

(a) $\frac{5}{3}(\hat{i}-7\hat{j}+2\hat{k})$ (b) $\frac{5}{3}(5\hat{i}+5\hat{j}+2\hat{k})$

(c) $\frac{5}{3}(\hat{i}+7\hat{j}+2\hat{k})$ (d) $\frac{5}{3}(-5\hat{i}+5\hat{j}+2\hat{k})$

43. Let $\vec{A}, \vec{B}, \vec{C}$ be vectors of length 3, 4 and 5 respectively. Let $\vec{A}$ be perpendicular to $\vec{B}+\vec{C}$, $\vec{B}$ to $\vec{C}+\vec{A}$ and $\vec{C}$ to $\vec{A}+\vec{B}$. Then length of the vector, $\vec{A}+\vec{B}+\vec{C}$ is :

(a) $-5\sqrt{2}$ (b) $\sqrt{2}$

(c) $5\sqrt{2}$ (d) none of these

44. If the coordinates of the points A, B, C be (–1, 3, 2), (2, 3, 5) and (3, 5, –2) respectively, then $\angle A=$

(a) 0° (b) 45°

(c) 60° (d) 90°

45. The coordinates of the points A, B, C, D are (4, α, 2), (5, –3, 2), (β, 1, 1) & (3, 3, –1). Line AB would be perpendicular to line CD when

(a) $\alpha=-1, \beta=-1$ (b) $\alpha=1, \beta=2$

(c) $\alpha=2, \beta=1$ (d) $\alpha=2, \beta=2$

46. If $\vec{a}$ and $\vec{b}$ are vectors of equal magnitude 2 and α be the angle between them, then magnitude of $\vec{a}+\vec{b}$ will be 2 if

(a) $\alpha=\pi/3$ (b) $\alpha=\pi/4$

(c) $\alpha=\pi/2$ (d) $\alpha=2\pi/3$

47. Two non zero vectors $\vec{a}$ and $\vec{b}$ will be parallel, if

(a) $\vec{a}.\vec{b}=0$ (b) $\vec{a}\times\vec{b}=\vec{0}$

(c) $\vec{a}=\vec{b}$ (d) none of these

48. If $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}, \vec{b}=\hat{i}+3\hat{j}+3\hat{k}$, then $|\vec{a}\times\vec{b}|$ is

(a) $\sqrt{6}$ (b) $2\sqrt{6}$

(c) $\sqrt{70}$ (d) $4\sqrt{6}$

49. If $\vec{a}$ and $\vec{b}$ are two vectors, then -

(a) $|\vec{a}\times\vec{b}|\geq|\vec{a}||\vec{b}|$ (b) $|\vec{a}\times\vec{b}|\leq|\vec{a}||\vec{b}|$

(c) $|\vec{a}\times\vec{b}|>|\vec{a}||\vec{b}|$ (d) $|\vec{a}\times\vec{b}|<|\vec{a}||\vec{b}|$

50. If θ be the angle between vectors $\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}+2\hat{j}+\hat{k}$, then the value of sin θ is

(a) $\sqrt{6/7}$ (b) $\frac{2\sqrt{6}}{7}$

(c) 1/7 (d) none of these

51. If $|\vec{a}\times\vec{b}|=\vec{a}.\vec{b}$ then angle between a and b is

(a) 0° (b) 90°

(c) 60° (d) 45°

52. The unit vector perpendicular to vectors $\hat{i}+\hat{j}$ and $\hat{j}+\hat{k}$ is

(a) $\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}+\hat{k})$ (b) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$

(c) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$ (d) none of these

53. If $|\vec{a}\,.\,\vec{b}| = 3$ and $|\vec{a}\times\vec{b}| = 4$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\cos^{-1} 3/4$ or $\pi - \cos^{-1}\frac{3}{4}$

(b) $\cos^{-1} 3/5$ or $\pi - \cos^{-1}\frac{3}{5}$

(c) $\sin^{-1}\frac{3}{5}$ or $\pi - \sin^{-1}\frac{3}{5}$

(d) $\pi/4$ or $\frac{3\pi}{4}$

54. If $(\vec{a}\times\vec{b})^2 + (\vec{a}.\vec{b})^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is equal to

(a) 3 (b) 8

(c) 12 (d) 16

55. $(\hat{i}+\hat{j}).\left[(\hat{j}+\hat{k})\times(\hat{k}+\hat{i})\right]$ equals

(a) 0 (b) 1

(c) –1 (d) 2

56. If $\vec{a}, \vec{b}, \vec{c}$ are any vectors then which one of the following is a **wrong statement.**

(a) $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (b) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

(c) $\vec{a}\times\vec{b} = \vec{b}\times\vec{a}$ (d) $|\vec{a}\times\vec{b}| = |\vec{b}\times\vec{a}|$

57. If for vectors $\vec{a}$ and $\vec{b}$, $\vec{a}\times\vec{b} = 0$ and $\vec{a}\,.\,\vec{b} = 0$, then

(a) $\vec{a}\,\|\,\vec{b}$ (b) $\vec{a}\perp\vec{b}$

(c) $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$ (d) none of these

58. If $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$ then correct statement is

(a) $\vec{a} = 0$

(b) $\vec{b} = 0 = \vec{c}$

(c) $\vec{b} = \vec{c}$

(d) above three are not necessary

59. For any vectors $\vec{a}, \vec{b}$; $|\vec{a}\times\vec{b}|^2 + (\vec{a}\,.\,\vec{b})^2$ is equal to

(a) $\vec{a}^{\,2}\,\vec{b}^{\,2}$ (b) $\vec{a}^{\,2} + \vec{b}^{\,2}$

(c) $\vec{a}^{\,2} - \vec{b}^{\,2}$ (d) 0

60. If the diagonals of a parallelogram are respectively $\vec{a} = \hat{i}+\hat{j}-2\hat{k}, \vec{b} = \hat{i}-3\hat{j}+4\hat{k}$, then the area of parallelogram is

(a) $\sqrt{14}$ (b) $2\sqrt{14}$

(c) $2\sqrt{6}$ (d) $\sqrt{38}$

61. If $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$ and $\vec{a}\neq\vec{d}, \vec{b}\neq\vec{c}$, Then

(a) $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$.

(b) $\vec{a}-\vec{d}$ is perpendicular to $\vec{b}-\vec{c}$.

(c) $\vec{a}-\vec{d}$ is equal to $\vec{b}-\vec{c}$.

(d) none of these

62. The area of the parallelogram constructed on the vectors $\vec{a} = \vec{p}+2\vec{q}$ & $\vec{b} = 2\vec{p}+\vec{q}$ where $\vec{p}$ & $\vec{q}$ are unit vectors forming an acute angle of 30° is

(a) 3/2 (b) 5/2

(c) 7/2 (d) none

63. Vectors $\vec{a}$ & $\vec{b}$ make an angle $\theta = \frac{2\pi}{3}$. If $|\vec{a}| = 1, |\vec{b}| = 2$ then $\{(\vec{a}+3\vec{b})\times(3\vec{a}-\vec{b})\}^2 =$

(a) 225 (b) 250

(c) 275 (d) 300

64. Unit vector perpendicular to the plane of the triangle ABC with pv's $\vec{a}, \vec{b}, \vec{c}$ of the vertices A, B, C is

(a) $\frac{(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})}{\Delta}$ (b) $\frac{(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})}{2\,\Delta}$

(c) $\frac{(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})}{4\,\Delta}$ (d) none

65. The value of $[(\vec{a}+2\vec{b}-\vec{c})\ \ (\vec{a}-\vec{b})\ \ (\vec{a}-\vec{b}-\vec{c})]$ is equal to the box product

(a) $[\vec{a}\ \vec{b}\ \vec{c}]$ (b) $2[\vec{a}\ \vec{b}\ \vec{c}]$

(c) $3[\vec{a}\ \vec{b}\ \vec{c}]$ (d) $4[\vec{a}\ \vec{b}\ \vec{c}]$

66. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors & $\vec{p}, \vec{q}, \vec{r}$ are vectors defined by the relations

$\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}, \vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$. Then the value of the expression ; $(\vec{a}+\vec{b}).\vec{p} + (\vec{b}+\vec{c}).\vec{q} + (\vec{c}+\vec{a}).\vec{r} =$

(a) 0 (b) 1
(c) 2 (d) 3

67. For non-zero vectors $\vec{a}, \vec{b}, \vec{c}, |\vec{a} \times \vec{b}.\vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$ holds if and only if

(a) $\vec{a}.\vec{b} = 0. \vec{b}.\vec{c} = 0$ (b) $\vec{c}.\vec{a} = 0, \vec{a}.\vec{b} = 0$
(c) $\vec{a}.\vec{c} = 0. \vec{b}.\vec{c} = 0$ (d) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

68. The volume of the parallelopiped whose sides are given by $\overrightarrow{OA} = 2\hat{i} - 3\hat{j}$. $\overrightarrow{OB} = \hat{i} + \hat{j} - \hat{k}$ and $\overrightarrow{OC} = 3\hat{i} - \hat{k}$ is :

(a) 4/13 (b) 4
(c) 2/7 (d) none

69. Volume of the tetrahedron whose vertices are represented by the position vectors, A (0, 1, 2) ; B (3, 0, 1); C (4, 3, 6) and D (2, 3, 2) is :

(a) 3 (b) 6
(c) 36 (d) none

70. The number of distinct real values of λ for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}, \hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar is

(a) 0 (b) 1
(c) 2 (d) 3

DC's & DR's :

71. In a line makes angles 90°, 60° and 30° with positive direction of x, y and z-axis respectively, then its direction-cosines are :

(a) < 0, 0, 0 > (b) < 1, 1, 1 >
(c) $< 0, \frac{1}{2}, \frac{\sqrt{3}}{2} >$ (d) $< \frac{\sqrt{3}}{2}, \frac{1}{2}, 0 >$

72. If a line makes angles α, β, γ with the co-ordinate axes, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is :

(a) 3 (b) –2
(c) 2 (d) –1

73. A parallelopiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7), parallel to the co-ordinate planes. The length of a diagonal of the parallelopiped is:

(a) 7 (b) $\sqrt{38}$
(c) $\sqrt{155}$ (d) None of these

74. A line makes equal angles with co-ordinate axes. Direction cosines of this line are

(a) $\pm 1, \pm 1, \pm 1$ (b) $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$
(c) $\pm\frac{1}{3}, \pm\frac{1}{3}, \pm\frac{1}{3}$ (d) $\pm\sqrt{3}, \pm\sqrt{3}, \pm\sqrt{3}$

75. The area of the quadrilateral ABCD, where A(0, 4, 1), B(2, 3, –1), C(4, 5, 0) and D(2, 6, 2), is equal to

(a) 9 sq. units (b) 18 sq. units
(c) 27 sq. units (d) 81 sq. units

76. If a line passes through the points (–2, 4, –5) and (1, 2, 3) then its direction-cosines will be :

(a) $< -\frac{2}{\sqrt{77}}, \frac{8}{\sqrt{77}}, \frac{3}{\sqrt{77}} >$ (b) $< -3, 2, +8 >$
(c) $< +\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}} >$ (d) $< 3, -2, 8 >$

77. A line makes acute angles of α, β and γ with the co-ordinate axes such that $\cos\alpha \cos\beta = \cos\beta \cos\gamma = \frac{2}{9}$ and $\cos\gamma \cos\alpha = \frac{4}{9}$, then $\cos\alpha + \cos\beta + \cos\gamma$ is equal to :

(a) 25/9 (b) 5/9
(c) 5/3 (d) 2/3

78. If a line has direction ratios $<2, -1, -2>$, then its direction-cosines will be :

(a) $<\frac{2}{3}, \frac{1}{3}, \frac{2}{3}>$ (b) $<\frac{1}{3}, \frac{1}{3}, \frac{1}{3}>$

(c) $<\frac{-2}{3}, -\frac{1}{3}, \frac{-2}{3}>$ (d) $<\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}>$

79. Two lines, whose direction ratios are :
$<a_1, b_1, c_1>$ and $<a_2, b_2, c_2>$ respectively are perpendicular if

(a) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{c_1}{c_2}$ (b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(c) $a_1a_2 + b_1b_2 + c_1c_2 = 0$ (d) $a_1a_2 + b_1b_2 + c_1c_2 = 1$

Straight Line

80. A line passes through a point A with p.v. $3\hat{i} + \hat{j} - \hat{k}$ and is parallel to the vector $2\hat{i} - \hat{j} + 2\hat{k}$. If P is a point on this line such that AP = 15 units, then the p.v. of the point P may be:

(a) $13\hat{i} + 4\hat{j} - 9\hat{k}$ (b) $13\hat{i} - 4\hat{j} + 9\hat{k}$

(c) $7\hat{i} - 6\hat{j} + 11\hat{k}$ (d) $+7\hat{i} + 6\hat{j} + 11\hat{k}$

81. Image of the point P with position vector $7\hat{i} - \hat{j} + 2\hat{k}$ in the line whose vector equation is, $\vec{r} = 9\hat{i} + 5\hat{j} + 5\hat{k} + \lambda\ (\hat{i} + 3\hat{j} + 5\hat{k})$ has the position vector :

(a) (–9, 5, 2) (b) (9, 5, –2)

(c) (9, –5, –2) (d) none

82. Find the angle between the two straight lines, $\vec{r} = 3\hat{i} - 2\hat{j} + 4\hat{k} + \lambda\ (-2\hat{i} + \hat{j} + 2\hat{k})$ and

$\vec{r} = \hat{i} + 3\hat{j} - 2\hat{k} + \mu\ (3\hat{i} - 2\hat{j} + 6\hat{k})$:

(a) $\cos^{-1}(4/21)$ (b) $\sin^{-1}(4/21)$

(c) $\sin^{-1}(17/21)$ (d) $\cos^{-1}(17/21)$

83. The lines,

$\vec{r}_1 = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\ (2\hat{i} + 3\hat{j} + 4\hat{k})$

and $\vec{r}_2 = 2\hat{i} + 3\hat{j} + 4\hat{k} + \mu\ (3\hat{i} + 4\hat{j} + 5\hat{k})$ are :

(a) coplanar

(b) skew

(c) such that shortest distance between them is 1

(d) none

84. The equations of x-axis in space are

(a) $x = 0, y = 0$ (b) $x = 0, z = 0$

(c) $x = 0$ (d) $y = 0, z = 0$

85. The direction cosines of the line, $x = y = z$ are :

(a) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ (b) $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$

(c) $\sqrt{5}, \sqrt{13}, \sqrt{10}$ (d) $\frac{1}{\sqrt{13}}, \frac{1}{\sqrt{10}}, \frac{1}{\sqrt{5}}$

86. A line makes angles α, β, γ with the coordinate axes. If $\alpha + \beta = 90^\circ$, then $\gamma =$

(a) 0 (b) 90°

(c) 180° (d) None of these

87. Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ would be coplanar if :

(a) $[\vec{a}_1\ \vec{b}_1\ \vec{b}_2] = [\vec{a}_2\ \vec{b}_1\ \vec{b}_2]$

(b) $(\vec{a}_1 . \vec{b}_1)\vec{b}_2 = (\vec{a}_2 . \vec{b}_1)\vec{b}_2$

(c) $\vec{a}_1 (\vec{b}_1 . \vec{b}_2) = \vec{a}_2 (\vec{b}_1 . \vec{b}_2)$

(d) $\vec{a}_1 . \vec{b}_1 - \vec{a}_1 . \vec{b}_2 = \vec{a}_2 . \vec{b}_1 - \vec{a}_2 . \vec{b}_2$

88. The point of intersection of lines,

$\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ & $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is :

(a) (–1, –1, –1) (b) (–1, –1, 1)

(c) 1, –1, –1) (d) (–1, 1, –1)

89. The straight lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-1}{2}=\frac{y-2}{2}=\frac{z-3}{-2}$ are

(a) parallel lines (b) Intersecting at 60º

(c) Skew lines (d) Intersecting at right angle

90. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The co-ordinates of each of the points of intersection are given by

(a) (3a, 3a, 3a), (a, a, a) (b) (3a, 2a, 3a), (a, a, a)

(c) (3a, 2a, 3a), (a, a, 2a) (d) (2a, 3a, 3a), (2a, a, a)

91. If the straight lines $x = 1 + s, y = -3 - \lambda s, z = 1 + \lambda s$ and $x = t/2, y = 1 + t, z = 2 - t$, with parameter s and t respectively, are coplanar, then λ equals to

(a) –2 (b) –1

(c) – 1/2 (d) 0

92. The shortest distance between the skew lines $\ell_1 : \vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\ell_2 : \vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is:

(a) $\frac{\left|(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$ (b) $\frac{\left|(\vec{a}_2-\vec{a}_1).(\vec{a}_2\times\vec{b}_2)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$

(c) $\frac{\left|(\vec{a}_2-\vec{b}_2).(\vec{a}_1\times\vec{b}_1)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$ (d) $\frac{\left|(\vec{a}_1-\vec{b}_2).(\vec{b}_1\times\vec{a}_2)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$

Planes :

93. The reflection of the point (α,β,γ) in the xy – plane is

(a) $(\alpha,\beta,0)$ (b) $(0,0,\gamma)$

(c) $(-\alpha,-\beta,\gamma)$ (d) $(\alpha,\beta,-\gamma)$

94. The plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(\alpha)$ with x-axis. The value of α is equal to

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{2}}{3}$

(c) $\frac{2}{7}$ (d) $\frac{3}{7}$

95. If a plane passes through the point (1, 1, 1) and is perpendicular to the line $\frac{x-1}{3}=\frac{y-1}{0}=\frac{z-1}{4}$ then its perpendicular distance from the origin is

(a) 3/4 (b) 4/3

(c) 7/5 (d) 1

96. The distance of the point, (–1, –5, –10) from the point of intersection of the line, $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane, $x - y + z = 5$, is :

(a) 10 (b) 11

(c) 12 (d) 13

97. The angle between the planes, $2x - y + z = 6$ and $x + y + 2z = 7$ is

(a) 30º (b) 45º

(c) 0º (d) 60º

98. The length of the perpendicular from origin on plane $\vec{r}.(3\hat{i}-4\hat{j}+12\hat{k}) = 5$ is

(a) $\frac{5}{69}$ (b) $\frac{25}{69}$

(c) $\frac{5}{13}$ (d) $\sqrt{\frac{5}{13}}$

99. The line, $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is parallel to the plane :

(a) $3x + 4y + 5z = 7$ (b) $2x + y - 2z = 0$

(c) $x + y - z = 2$ (d) $2x + 3y + 4z = 0$

100. The angle between the line, $\frac{x+1}{3}=\frac{y-1}{2}=\frac{z-2}{4}$ and the plane, $2x + y - 3z + 4 = 0$, is :

(a) $\sin^{-1}\left(\frac{4}{\sqrt{406}}\right)$ (b) $\sin^{-1}\left(\frac{14}{\sqrt{406}}\right)$

(c) $\sin^{-1}\left(\frac{4}{14\sqrt{29}}\right)$ (d) None of these

101. If the given planes, $ax + by + cz + d = 0$ and $a'x + b'y + c'z + d' = 0$ be mutually perpendicular, then:

(a) $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$ (b) $\frac{a}{a'} + \frac{b}{b'} + \frac{c}{c'} = 0$

(c) $aa' + bb' + cc' + dd' = 0$

(d) $aa' + bb' + cc' = 0$

102. The angle between the line $\frac{x-2}{a} = \frac{y-2}{b} = \frac{z-2}{c}$ and the plane $ax + by + cz + 6 = 0$ is

(a) $\sin^{-1}\frac{1}{\sqrt{a^2+b^2+c^2}}$ (b) 45°

(c) 60° (d) 90°

103. The point at which the line joining the points (2, –3, 1) & (3, –4, –5) intersects the plane, $2x + y + z = 7$ is :

(a) (1, 2, 7) (b) (1, –2, 7)

(c) (–1, 2, 7) (d) (1, –2, –7)

104. The point of intersection of the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+2}{3}$ and the plane, $2x + 3y + z = 0$, is :

(a) (0, 1, –2) (b) (1, 2, 3)

(c) (–1, 9, –25) (d) $\left(\frac{-1}{11}, \frac{9}{11}, \frac{-25}{11}\right)$

105. The equation of the plane passing through the points (3, 2, 2) and (1, 0 –1) and parallel to the line $\frac{x-1}{2} = \frac{y-1}{-2} = \frac{z-2}{3}$, is

(a) $4x - y - 2z + 6 = 0$ (b) $4x - y + 2z + 6 = 0$

(c) $4x - y - 2z - 6 = 0$ (d) $3x - 2z - 5 = 0$

106. The equation of a plane which passes through (2, –3, 1) and is normal to the line joining the points (3, 4, –1) and (2, –1, 5) is given by :

(a) $x + 5y - 6z + 19 = 0$ (b) $x - 5y + 6z - 19 = 0$

(c) $x + 5y + 6z + 19 = 0$ (d) $x - 5y - 6z - 19 = 0$

107. A plane meets the coordinate axes in A, B, C and (α, β, γ) is the centroid of the triangle ABC. Then the equation of the plane is

(a) $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$ (b) $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$

(c) $\frac{3x}{\alpha} + \frac{3y}{\beta} + \frac{3z}{\gamma} = 1$ (d) $\alpha x + \beta y + \gamma z = 1$

108. The direction ratio of normal to the plane through (1, 0, 0), (0, 1, 0), which makes an angle $\frac{\pi}{4}$ with plane $x + y = 3$, are :

(a) $1, \sqrt{2}, 1$ (b) $1, 1, \sqrt{2}$

(c) 1, 1, 2 (d) $\sqrt{2}, 1, 1$

109. The plane $ax + by + cz = 1$ meets the co-ordinate axes in A, B and C. The centroid of the triangle is :

(a) (3a, 3b, 3c) (b) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

(c) $\left(\frac{3}{a}, \frac{3}{b}, \frac{3}{c}\right)$ (d) $\left(\frac{1}{3a}, \frac{1}{3b}, \frac{1}{3c}\right)$

110. If line $\vec{r} = (\hat{i} - 2\hat{j} - \hat{k}) + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$ is parallel to the plane $\vec{r} \cdot (3\hat{i} - 2\hat{j} - m\hat{k}) = 14$ then the value of m is

(a) +2

(b) –2

(c) 0

(d) can not be predicted with this much informations

111. The equation of the plane which is right bisector of the line joining (2, 3, 4) and (6, 7, 8), is :

(a) $x+y+z-15=0$ (b) $x-y+z-15=0$

(c) $x-y-z-15=0$ (d) $x+y+z+15=0$

112. The equation of the plane containing the line $\frac{x-x_1}{\ell}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ is

$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$, where :

(a) $ax_1+by_1+cz_1=0$ (b) $a\ell+bm+cn=0$

(c) $\frac{a}{\ell}=\frac{b}{m}=\frac{c}{n}$ (d) $\ell x_1+my_1+nz_1=0$

113. The image of the point P (1, 3, 4) in the plane $2x-y+z+3=0$ is

(a) (3, 5, –2) (b) (–3, 5, 2)

(c) (3, –5, 2) (d) (–1, 4, 2)

114. The vector equation of the plane passing through the origin and the line of intersection of the plane $\vec{r}.\,\vec{a}=\lambda$ and $\vec{r}.\,\vec{b}=\mu$ is :

(a) $\vec{r}.\left(\lambda\vec{a}-\mu\vec{b}\right)=0$ (b) $\vec{r}.\left(\lambda\vec{b}-\mu\vec{a}\right)=0$

(c) $\vec{r}.\left(\lambda\vec{a}+\mu\vec{b}\right)=0$ (d) $\vec{r}.\left(\lambda\vec{b}+\mu\vec{a}\right)=0$

115. Equation of the plane passing through the line of intersection of the planes

$P\equiv ax+by+cz+d=0$, $P'\equiv a'x+b'y+c'z+d'=0$, and parallel to x-axis is :

(a) $Pa-P'a'=0$ (b) $P/a=P'/a'=0$

(c) $Pa+P'a'=0$ (d) $P/a=P'/a'$

116. The locus represented by $xy+yz=0$ is

(a) A pair of perpendicular lines

(b) A pair of parallel lines

(c) A pair of parallel planes

(d) A pair of perpendicular planes

117. The Plane $2x-(1+\lambda)y+3\lambda z=0$ passes through the intersection of the planes

(a) $2x-y=0$ and $y-3z=0$

(b) $2x+3z=0$ and $y=0$

(c) $2x-y+3z=0$ and $y-3z=0$

(d) none of these

118. The equation of the plane through the intersection of the planes $x+2y+3z=4$ and $2x+y-z=-5$ and perpendicular to the plane $5x+3y+6z+8=0$ is

(a) $7x-2y+3z+81=0$ (b) $23x+14y-9z+48=0$

(c) $51x-15y-50z+173=0$ (d) none of these

119. The equation of the plane containing the two lines

$\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-0}{3}$ and $\frac{x}{-2}=\frac{y-2}{11}=\frac{z+1}{-1}$ is

(a) $8x+y-5z-7=0$ (b) $8x+y+5z-7=0$

(c) $8x-y-5z-7=0$ (d) none of these

120. The equation of the plane through the intersection of the planes $ax+by+cz+d=0$ and $lx+my+nz+p=0$ and parallel to the line $y=0, z=0$

(a) $(bl-am)y+(cl-an)z+dl-ap=0$

(b) $(am-bl)x+(mc-bn)z+md-bp=0$

(c) $(na-cl)x+(bn-cm)y+nd-cp=0$

(d) none of these

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. Given, two vectors are $\hat{i}-\hat{j}$ and $\hat{i}+2\hat{j}$ the unit vector coplanar with the two vectors and perpendicular to first is **(2002)**

(a) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ (b) $\frac{1}{\sqrt{5}}(2\hat{i}+\hat{j})$

(c) $\pm\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ (d) None of these

2. The vector $\hat{i}+x\hat{j}+3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i}+(4x-2)\hat{j}+2\hat{k}$. The value of x are **(2002)**

(a) $\left\{-\frac{2}{3},2\right\}$ (b) $\left(\frac{1}{3},2\right)$

(c) $\left\{\frac{2}{3},0\right\}$ (d) {2, 7}

3. If the vectors $\vec{a},\vec{b}$ and $\vec{c}$ from the sides BC, CA and AB respectively of a triangle ABC, then **(2002)**

(a) $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{b}=0$ (b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

(c) $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$ (d) $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=0$

4. If the vectors $\vec{c},\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$ and $\vec{b}=\hat{j}$ are such that $\vec{a},\vec{c}$ and $\vec{b}$ form a right handed system, then $\vec{c}$ is **(2002)**

(a) $z\hat{i}-x\hat{k}$ (b) $\vec{0}$

(c) $y\hat{j}$ (d) $-z\hat{i}+x\hat{k}$

5. A tetrahedron has vertices at O(0, 0, 0), A(1, 2, 1), B(2, 1, 3) and C(–1, 1, 2). Then, the angle between the faces OAB and ABC will be **(2003)**

(a) $\cos^{-1}\left(\frac{19}{35}\right)$ (b) $\cos^{-1}\left(\frac{17}{31}\right)$

(c) 30° (d) 90°

6. $\vec{a},\vec{b},\vec{c}$ are three vectors, such that $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=1$, $|\vec{b}|=2$, $|\vec{c}|=3$, then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is equal to **(2003)**

(a) 0 (b) –7

(c) 7 (d) 1

7. If $\vec{u},\vec{v}$ and $\vec{w}$ are three non-coplanar vector, then $(\vec{u}+\vec{v}-\vec{w})\cdot[(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})]$ equals **(2003)**

(a) 0 (b) $\vec{u}\cdot\vec{v}\times\vec{w}$

(c) $\vec{u}\cdot\vec{w}\times\vec{v}$ (d) $3\vec{u}.\vec{v}$

8. Consider points A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}$, $\hat{i}-6\hat{j}+10\hat{k}$, $-\hat{i}-3\hat{j}+4\hat{k}$, and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then, ABCD is a **(2003)**

(a) square (b) rhombus

(c) rectangle (d) none of these

9. The vectors $\overline{AB}=3\hat{i}+4\hat{k}$, and $\overline{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is **(2003)**

(a) $\sqrt{18}$ (b) $\sqrt{72}$

(c) $\sqrt{33}$ (d) $\sqrt{288}$

10. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$ and vectors $(1,a,a^2)$, $(1,b,b^2)$ and $(1,c,c^2)$ are non–coplanar, then the product abc equals **(2003)**

(a) 2 (b) –1

(c) 1 (d) 0

11. Let $\vec{u}=\hat{i}+\hat{j}, \vec{v}=\hat{i}-\hat{j}$ and $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}\cdot\hat{n}=0$ and $\vec{v}\cdot\hat{n}=0$, then $|\vec{w}\cdot\hat{n}|$ is equal to **(2003)**

(a) 0 (b) 1

(c) 2 (d) 3

12. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar) then $\vec{a}+2\vec{b}+6\vec{c}$ equals **(2004)**

(a) $\lambda\vec{a}$ (b) $\lambda\vec{b}$

(c) $\lambda\vec{c}$ (d) $\vec{0}$

13. A particle is acted upon by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ which displace it from a point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The work done in standard units by the forces is given by **(2004)**

(a) 40 unit (b) 30 unit

(c) 25 unit (d) 15 unit

14. If $\vec{a},\vec{b},\vec{c}$ are non-coplanar vectors and λ be a real number, then the vectors $\vec{a}+2\vec{b}+3\vec{c}$, $\lambda\vec{b}+4\vec{c}$ and $(2\lambda-1)\vec{c}$ are non-coplanar for **(2004)**

(a) all values of λ

(b) all except one value of λ

(c) all except two values of λ

(d) no value of λ

15. Let $\vec{u},\vec{v},\vec{w}$ be such that $|\vec{u}|=1, |\vec{v}|=2, |\vec{w}|=3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}$, $\vec{w}$ are perpendicular to each other, then $|\vec{u}-\vec{v}+\vec{w}|$ are equals **(2004)**

(a) 2 (b) $\sqrt{7}$

(c) $\sqrt{14}$ (d) 14

16. Let $\vec{a},\vec{b}$ and $\vec{c}$ be non–zero vectors such that $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between the vector $\vec{b}$ and $\vec{c}$, then $\sin\theta$ equals **(2004)**

(a) $\frac{1}{3}$ (b) $\frac{\sqrt{2}}{3}$

(c) $\frac{2}{3}$ (d) $\frac{2\sqrt{2}}{3}$

17. If C is the mid point of AB and P is any point outside AB, then **(2005)**

(a) $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=0$ (b) $\overrightarrow{PA}+\overrightarrow{PB}+2\overrightarrow{PC}=\vec{0}$

(c) $\overrightarrow{PA}+\overrightarrow{PB}=\overrightarrow{PC}$ (d) $\overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$

18. The distance between the line $\vec{r}=2\hat{i}-2\hat{j}+3\hat{k}+\lambda(\hat{i}-\hat{j}+4\hat{k})$ and the plane $\vec{r}.(\hat{i}+5\hat{j}+\hat{k})=5$ is **(2005)**

(a) $\frac{10}{3}$ (b) $\frac{3}{10}$

(c) $\frac{10}{3\sqrt{3}}$ (d) $\frac{10}{9}$

19. For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{i})^2+(\vec{a}\times\hat{j})^2+(\vec{a}\times\hat{k})^2$ is equal to **(2005)**

(a) $4\vec{a}^2$ (b) $2\vec{a}^2$

(c) $\vec{a}^2$ (d) $3\vec{a}^2$

20. Let $\vec{a}=\hat{i}-\hat{k}$, $\vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$ and $\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$. Then $[\vec{a}\ \vec{b}\ \vec{c}]$ depends on **(2005)**

(a) neither x nor y (b) both x and y

(c) only x (c) only y

21. Let a, b and c be distinct non–negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is **(2005)**

(a) the harmonic mean of a and b

(b) equal to zero

(c) the arithmetic mean of a and b

(d) the geometric mean of a and b

22. If $\vec{a},\vec{b},\vec{c}$ are non–coplanar vector and λ is a real number then $[\lambda(\vec{a}+\vec{b})\ \ \lambda^2\vec{b}\ \ \lambda\vec{c}] = [\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{b}]$ for **(2005)**

(a) exactly two values of λ

(b) exactly three values of λ

(c) non value of λ

(d) exactly one value of λ

23. If $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$, where $\vec{a},\vec{b},\vec{c}$ are any three vectors such that $\vec{a}\cdot\vec{b}\neq 0$, $\vec{b}\cdot\vec{c}\neq 0$, then $\vec{a}$ and $\vec{c}$ are **(2006)**

(a) inclined at an angle of $\frac{\pi}{6}$ between them

(b) perpendicular

(c) parallel

(d) inclined at an angle of $\frac{\pi}{3}$ between them

24. The values of a, for which the points A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $a\hat{i}-3\hat{j}+\hat{k}$ respectively are the vertices of a right–angled triangle with $C=\frac{\pi}{2}$ are **(2006)**

(a) –2 and –1 (b) –2 and 1

(c) 2 and –1 (d) 2 and 1

25. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u}\times 3\hat{v}$ is a unit vector for **(2007)**

(a) exactly two values of θ

(b) more than two values of θ

(c) no value of θ

(d) exactly one value of θ

26. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$. If the vectors $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals **(2007)**

(a) 0 (b) 1

(c) –4 (d) –2

27. The vector $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$ lies in the plane of the vectors $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{j}+\hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β ? **(2008)**

(a) $\alpha=1,\beta=1$ (b) $\alpha=2,\beta=2$

(c) $\alpha=1,\beta=2$ (d) $\alpha=2,\beta=1$

28. The non–zero vectors $\vec{a},\vec{b}$ and $\vec{c}$ are related by $\vec{a}=8\vec{b}$ and $\vec{c}=-7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is **(2008)**

(a) π (b) 0

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

29. If $\vec{u},\vec{v},\vec{w}$ are non-coplanar vectors and p, q, are real numbers, then the equality

$[3\vec{u}\ p\vec{v}\ p\vec{w}]-[p\vec{v}\ \vec{w}\ q\vec{u}]-[2\vec{w}\ q\vec{v}\ q\vec{u}]=0$ holds for **(2009)**

(a) exactly two values of (p, q)

(b) more than two but not all values of (p, q)

(c) all values of (p, q)

(d) exactly one value of (p, q)

30. Let $\vec{a}=\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a}\times\vec{b}+\vec{c}=0$ and $\vec{a}\cdot\vec{b}=3$, is **(2010)**

(a) $-\hat{i}+\hat{j}-2\hat{k}$ (b) $2\hat{i}-\hat{j}+2\hat{k}$

(c) $\hat{i}-\hat{j}-2\hat{k}$ (d) $\hat{i}+\hat{j}-2\hat{k}$

31. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then (λ, μ) is equal to **(2010)**

(a) (–3, 2) (b) (2, –3)
(c) (–2, 3) (d) (3, –2)

32. If $\vec{a} = \frac{1}{\sqrt{10}}(3\hat{i} + \hat{k})$ and $\vec{b} = \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})$, then the value of $(2\vec{a} - \vec{b}).[(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})]$ is **(2011)**

(a) –3 (b) 5
(c) 3 (d) –5

33. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a}.\vec{d} = 0$. Then the vectors $\vec{d}$ is equal to **(2011)**

(a) $\vec{c} + \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (b) $\vec{b} + \left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

(c) $\vec{c} - \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (d) $\vec{b} - \left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

34. If the vectors $p\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + q\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + r\hat{k}$ $(p \neq q \neq r \neq 1)$ are coplanar, then the value of $pqr - (p + q + r)$ is **(2011)**

(a) –2 (b) 2
(c) 0 (d) –1

35. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors which are pair wise non-collinear. If $\vec{a} + 3\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 2\vec{c}$ is collinear with $\vec{a}$, then $\vec{a} + 3\vec{b} + 6\vec{c}$ is **(2011)**

(a) $\vec{a} + \vec{c}$ (b) $\vec{a}$

(c) $\vec{c}$ (d) $\vec{0}$

36. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is **(2012)**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

37. Let ABCD be a parallelogram such that $\overrightarrow{AB} = \vec{q}$, $\overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by **(2012)**

(a) $\vec{r} = 3\vec{q} - \frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$ (b) $\vec{r} = -\vec{q} + \left(\frac{\vec{p}.\vec{q}}{\vec{p}.\vec{p}}\right)\vec{p}$

(c) $\vec{r} = \vec{q} - \left(\frac{\vec{p}.\vec{q}}{\vec{p}.\vec{p}}\right)\vec{p}$ (d) $\vec{r} = -3\vec{q} + \frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

38. If the vector $AB = 3\hat{i} + 4\hat{k}$ and $AC = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a ΔABC, then the length of the median through A is **(2013)**

(a) $\sqrt{18}$ (b) $\sqrt{72}$

(c) $\sqrt{33}$ (d) $\sqrt{45}$

39. If $\left[\vec{a} \times \vec{b}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}\right] = \lambda\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$ then λ is equal to : **(2014)**

(a) 1 (b) 2
(c) 3 (d) 0

40. If $|\vec{a}| + 2, |\vec{b}| = 3$ and $|2\vec{a} - \vec{b}| = 5$, then $|2\vec{a} + \vec{b}|$ equals: **(2014/Online Set–1)**

(a) 17 (b) 7
(c) 5 (d) 1

41. If $|\vec{c}|^2 = 60$ and $\vec{c}\times(i+2j+5k)=0$, then a value of $\vec{c}.(-7i+2j+3k)$ is: **(2014/Online Set–2)**

(a) $4\sqrt{2}$ (b) 12

(c) 24 (d) $12\sqrt{2}$

42. If $\hat{x}, \hat{y}$ and $\hat{z}$ are three unit vectors in three dimensional space, thent he minimum value of $|\hat{x}+\hat{y}|^2 + |\hat{y}+\hat{z}|^2 + |\hat{z}+\hat{x}|^2$ is **(2014/Online Set–3)**

(a) $\frac{3}{2}$ (b) 3

(c) $3\sqrt{3}$ (d) 6

43. If $x = 3\hat{i}-6\hat{j}-\hat{k}$, $y = \hat{i}+4\hat{j}-3\hat{k}$ and $z = 3\hat{i}-4\hat{j}-12\hat{k}$, then the magnitude of the projection of $x \times y$ on z is: **(2014/Online Set–4)**

(a) 12 (b) 15

(c) 14 (d) 13

44. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and

$(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vector $\vec{b}$ and $\vec{c}$, then a value of sin θ is: **(2015)**

(a) $\frac{2}{3}$ (b) $\frac{-2\sqrt{3}}{3}$

(c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{-\sqrt{2}}{3}$

45. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is : **(2015)**

(a) $\frac{\pi}{2}$ (b) $\frac{2\pi}{3}$

(c) $\frac{5\pi}{6}$ (d) $\frac{3\pi}{4}$

46. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $|\vec{a}+\vec{b}| = \sqrt{3}$.

If $\vec{c} = \vec{a}+2\vec{b}+3(\vec{a}\times\vec{b})$, then $2|\vec{c}|$ is equal to: **(2015/Online Set–1)**

(a) $\sqrt{55}$ (b) $\sqrt{51}$

(c) $\sqrt{43}$ (d) $\sqrt{37}$

47. In a triangle ABC, right angled at the vertex A, if the position vectors of A, B and C are respectively $3\hat{i}+\hat{j}-\hat{k}$, $-\hat{i}+3\hat{j}+p\hat{k}$ and $5\hat{i}+q\hat{j}-4\hat{k}$, then the point (p, q) lies on a line : **(2016/Online Set–1)**

(a) parallel to x-axis.

(b) parallel to y-axis.

(c) making an acute angle with the positive direction of x-axis.

(d) making an obtuse angle with the positive direction of x-axis.

48. Let ABC be a triangle whose circumcentre is at P. If the position vectors of A, B, C and P are $\vec{a}, \vec{b}, \vec{c}$ and $\frac{\vec{a}+\vec{b}+\vec{c}}{4}$ respectively, then the position vector of the orthocenter of this triangle, is : **(2016/Online Set–2)**

(a) $\vec{a}+\vec{b}+\vec{c}$ (b) $-\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

(c) $\vec{0}$ (d) $\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

49. The area (in sq. units) of the parallelogram whose diagonals are along the vectors, $8\hat{i}-6\hat{j}$ and $3\hat{i}+4\hat{j}-12\hat{k}$ is : **(2017)**

(a) 26 (b) 65

(c) 20 (d) 52

50. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c}-\vec{a}|=3, |(\vec{a}\times\vec{b})\times\vec{c}|=3$ and the angle between $\vec{c}$ and $\vec{a}\times\vec{b}\vec{c}$ be 30°. Then $\vec{a}.\vec{c}$ is equal to **(2017)**

(a) $\frac{25}{8}$ (b) 2

(c) 5 (d) $\frac{1}{8}$

51. If the vector is written $\vec{b}=3\hat{j}+4\hat{k}$ as the sum of a vector $\hat{b}_1$, parallel to $\vec{a}=\hat{i}+\hat{j}$ and a vector $\vec{b}_2$, perpendicular to $\vec{a}$, then $\vec{b}_1\times\vec{b}_2$ is equal to : **(2017/Online Set–2)**

(a) $-3\hat{i}+3\hat{j}-9\hat{k}$ (b) $6\hat{i}-6\hat{j}+\frac{9}{2}\hat{k}$

(c) $-6\hat{i}+6\hat{j}-\frac{9}{2}\hat{k}$ (d) $3\hat{i}-3\hat{j}+9\hat{k}$

52. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$. . If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}.\vec{b}=24$, then $|\vec{u}|^2$ is equal to : **(2018)**

(a) 84 (b) 336

(c) 315 (d) 256

53. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+2\vec{b}+2\vec{c}=\vec{0}$, then $|\vec{a}\times\vec{c}|$ is equal to : **(2018/Online Set–1)**

(a) $\frac{\sqrt{15}}{4}$ (b) $\frac{1}{4}$

(c) $\frac{15}{16}$ (d) $\frac{\sqrt{15}}{16}$

54. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{c}=\hat{j}-\hat{k}$ and a vector $\vec{b}$ be such that $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{a}.\vec{b}=3$. Then $|\vec{b}|$ equals : **(2018/Online Set–3)**

(a) $\frac{11}{3}$ (b) $\frac{11}{\sqrt{3}}$

(c) $\sqrt{\frac{11}{3}}$ (d) $\frac{\sqrt{11}}{3}$

3D

55. A parallelopiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7), parallel to the coordinate planes. The length of a diagonal of the parallelopiped is **(2002)**

(a) 7 unit (b) $\sqrt{38}$ unit

(c) $\sqrt{155}$ unit (d) None of these

56. The equation of the plane containing the line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$, where **(2002)**

(a) $ax_1+by_1+cz_1=0$ (b) $al+bm+cn=0$

(c) $\frac{a}{l}=\frac{b}{m}=\frac{c}{n}$ (d) $lx_1+my_1+nz_1=0$

57. The centre of the circle given by $\vec{r}.(\hat{i}+2\hat{j}+2\hat{k})=15$ and $|\vec{r}-(\hat{j}+2\hat{k})|=4$ is **(2002)**

(a) (0, 1, 2) (b) (1, 3, 4)

(c) (–1, 3, 4) (d) None of these

58. The radius of the circle in which the sphere $x^2+y^2+z^2+2x-2y-4z-19=0$ is cut by the plane $x+2y+2z+7=0$ is **(2003)**

(a) 1 (b) 2

(c) 3 (d) 4

59. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, if **(2003)**

(a) $k=0$ or -1 (b) $k=1$ or -1

(c) $k=0$ or -3 (d) $k=3$ or -3

60. The shortest distance from the plane $12x+4y+3z=327$ to the sphere $x^2+y^2+z^2+4x-2y-6z=155$ is **(2003)**

(a) 26 (b) $11\frac{4}{13}$

(c) 13 (d) 39

61. Two systems of rectangular axes have the same origin. If a plane cuts them at distance a, b, c and a′, b′, c′ from the origin, then **(2003)**

(a) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(b) $\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(c) $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(d) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

62. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y–axis is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals **(2004)**

(a) 2/3 (b) 1/5

(c) 3/5 (d) 2/5

63. Distance between two parallel planes $2x+y+2z=8$ and $4x+2y+4z+5=0$ is **(2004)**

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$

(c) $\frac{7}{2}$ (d) $\frac{9}{2}$

64. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x=y+a=z$ and $x+a=2y=2z$. The co-ordinates of each of the points of intersection are given by **(2004)**

(a) (3a, 3a, 3a) (a, a, a)

(b) (3a, 2a, 3a), (a, a, a)

(c) (3a, 2a, 3a), (a, a, 2a)

(d) (2a, 3a, 3a), (2a, a, a)

65. If the straight lines $x=1+s$, $y=-3-\lambda s$, $z=1+\lambda s$ and $x=\frac{t}{2}$, $y=1+t$, $z=2-t$, with parameters s and t respectively, are coplanar, then λ equals **(2004)**

(a) –2 (b) –1

(c) $-\frac{1}{2}$ (d) 0

66. The intersection of the spheres
$x^2+y^2+z^2+7x-2y-z=13$ and
$x^2+y^2+z^2-3x+3y+4z=8$ is the same as the intersection of one of the sphere and the plane **(2004)**

(a) $x-y-z=1$ (b) $x-2y-z=1$

(c) $x-y-2z=1$ (d) $2x-y-z=1$

67. If the plane $2ax-3ay+4az+6=0$ passes through the mid-point of the line joining the centres of the spheres $x^2+y^2+z^2+6x-8y-2z=13$ and $x^2+y^2+z^2-10x+4y-2z=8$, then a equals **(2005)**

(a) 2 (b) –2

(c) 1 (d) –1

68. If the angle θ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}\,z+4=0$ is such that $\sin\theta=\frac{1}{3}$. The value of λ is **(2005)**

(a) $-\frac{4}{3}$ (b) $\frac{3}{4}$

(c) $-\frac{3}{5}$ (d) $\frac{5}{3}$

69. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is **(2005)**

(a) 30° (b) 45°

(c) 90° (d) 0°

70. The plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius **(2005)**

(a) $\sqrt{2}$ (b) 2

(c) 1 (d) 3

71. The two lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular to each other if **(2006)**

(a) $aa' + cc' = 1$ (b) $\frac{a}{a'} + \frac{c}{c'} = -1$

(c) $\frac{a}{a'} + \frac{c}{c'} = 1$ (d) $aa' + cc' = -1$

72. The image of the points (–1, 3, 4) in the plane $x - 2y = 0$ is **(2006)**

(a) (15, 11, 4) (b) $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$

(c) (8, 4, 4) (d) $\left(\frac{9}{5}, -\frac{13}{5}, 4\right)$

73. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x–axis, then cos α equals **(2007)**

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$

(c) 1 (d) $\frac{1}{\sqrt{2}}$

74. If a line makes an angle of $\pi/4$ with the positive directions of each of x–axis and y–axis, then the angle that the line makes with the positive direction of the z–axis is **(2007)**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

75. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the coordinates of the other end of the diameter are **(2007)**

(a) (4, 9, –3) (b) (4, –3, 3)

(c) (4, 3, 5) (d) (4, 3, –3)

76. The line passing through the point (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then **(2008)**

(a) a = 8, b = 2 (b) a = 2, b = 8

(c) a = 4, b = 6 (d) a = 6, b = 4

77. If the straight lines

$$\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3} \text{ and } \frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$$

intersect at a point, then the integer k is equal to **(2008)**

(a) – 2 (b) – 5

(c) 5 (d) 2

78. Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$. Then (α, β) equals. **(2009)**

(a) (6, –17) (b) (–6, 7)

(c) (5, –15) (d) (–5, 15)

79. The projections of a vector on the three coordinate axes are 6, –3, 2 respectively. The direction cosines of the vector are **(2009)**

(a) 6, –3, 2 (b) $\frac{6}{5}, -\frac{3}{5}, \frac{2}{5}$

(c) $\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$ (d) $-\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$

80. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals. **(2010)**

(a) 30° (b) 45°

(c) 60° (d) 75°

81. **Statement–I :** The point A (3, 1, 6) is the mirror image of the point B (1, 3, 4) in the plane $x - y + z = 5$

Statement–II : The plane $x - y + z = 5$ bisects the line segment joining A(3, 1, 6) and B(1, 3, 4). **(2010)**

(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.

(b) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

82. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals to **(2011)**

(a) $\frac{3}{2}$ (b) $\frac{2}{5}$

(c) $\frac{5}{3}$ (d) $\frac{2}{3}$

83. **Statement–I :** The point A (1, 0, 7) is the mirror image of the point B (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

Statement–II : The line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segement joining A(1, 0, 7) and B(1, 6, 3). **(2011)**

(a) Statement I is true, Statement II is ture; Statement II is not a correct explanation for Statement I.

(b) Statement I is true, Statement II is false.

(c) Statement I is false, Statement II is true.

(d) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.

84. The length of the perpendicular drawn from the point (3, –1, 11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is **(2011)**

(a) $\sqrt{66}$ (b) $\sqrt{29}$

(c) $\sqrt{33}$ (d) $\sqrt{53}$

85. The distance of the point (1, –5, 9) from the plane $x - y + z = 5$ measured along a straight line $x = y = z$ is **(2011)**

(a) $3\sqrt{5}$ (b) $10\sqrt{3}$

(c) $5\sqrt{3}$ (d) $3\sqrt{10}$

86. An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is **(2012)**

(a) $x - 2y + 2z - 3 = 0$ (b) $x - 2y + 2z + 1 = 0$

(c) $x - 2y + 2z - 1 = 0$ (d) $x - 2y + 2z + 5 = 0$

87. If the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to **(2012)**

(a) –1 (b) $\frac{2}{9}$

(c) $\frac{9}{2}$ (d) 0

88. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is **(2013)**

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$

(c) $\frac{7}{2}$ (d) $\frac{9}{2}$

89. If the lines

$$\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k} \text{ and } \frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$$

are coplanar, then k can have **(2013)**

(a) any value (b) exactly one value

(c) exactly two values (d) exactly three value

90. The image of the line

$\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane

$2x-y+z+3=0$ is the line : **(2014)**

(a) $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$

(b) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

(c) $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$

(d) $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$

91. The angle between the lines whose direction cosines satisfy the equations $l+m+n=0$ and $l^2=m^2+n^2$ is : **(2014)**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

92. Equation of the plane which passes through the point of intersection of lines $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and has the largest distance from the origin is: **(2014/Online Set–1)**

(a) $7x+2y+4z=54$ (b) $3x+4y+5z=49$

(c) $4x+3y+5z=50$ (d) $5x+4y+3z=57$

93. A line in the 3-dimensinal space makes and angle $\theta\left(0<\theta\le\frac{\pi}{2}\right)$ with both the x and y axis. Then the set of all values of θ is the interval: **(2014/Online Set–1)**

(a) $\left(0,\frac{\pi}{4}\right]$ (b) $\left[\frac{\pi}{6},\frac{\pi}{3}\right]$

(c) $\left[\frac{\pi}{4},\frac{\pi}{2}\right]$ (d) $\left(\frac{\pi}{3},\frac{\pi}{2}\right]$

94. The plane contaning the line $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and parallel to the line $\frac{x}{1}=\frac{y}{1}=\frac{z}{4}$ passes through the point: **(2014/Online Set–2)**

(a) (1, -2, 5) (b) (1, 0, 5)

(c) (0, 3, -5) (d) (-1, -3, 0)

95. A symmetrical form of the line of intersection of the planes $x=ay+b$ and $z=cy+d$ is: **(2014/Online Set–3)**

(a) $\frac{x-b}{a}=\frac{y-1}{1}=\frac{z-d}{c}$

(b) $\frac{x-b-a}{a}=\frac{y-1}{1}=\frac{z-d-c}{c}$

(c) $\frac{x-a}{b}=\frac{y-0}{1}=\frac{z-c}{d}$

(d) $\frac{x-b-a}{b}=\frac{y-1}{0}=\frac{z-d-c}{d}$

96. If the distance between planes, $4x-2y-4z+1=0$ and $4x-2y-4z+d=0$ is 7, then d is: **(2014/Online Set–3)**

(a) 41 or - 42 (b) 42 or - 43

(c) -41 or 43 (d) -42 or 44

97. Equation of the line of the shortest distance between the lines $\frac{x}{1}=\frac{y}{-1}=\frac{z}{1}$ and $\frac{x-1}{0}=\frac{y+1}{-2}=\frac{z}{1}$ is **(2014/Online Set–4)**

(a) $\frac{x}{1}=\frac{y}{-1}=\frac{z}{-2}$ (b) $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z}{-2}$

(c) $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z}{1}$ (d) $\frac{x}{-2}=\frac{y}{1}=\frac{z}{2}$

98. If the angle between the line 2(x+1) = y = z + 4 and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is $\frac{\pi}{6}$, then the value of λ is **(2014/Online Set–4)**

(a) $\frac{135}{7}$ (b) $\frac{45}{11}$

(c) $\frac{45}{7}$ (d) $\frac{135}{11}$

99. The equation of the plane containing the line $2x - 5y + z = 3$; $x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is: **(2015)**

(a) $x + 3y + 6z = 7$ (b) $2x + 6y + 12z = -13$

(c) $2x + 6y + 12z = 13$ (d) $x + 3y + 6z = -7$

100. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$, is: **(2015)**

(a) $3\sqrt{21}$ (b) 13

(c) $2\sqrt{14}$ (d) 8

101. If the points $(1, 1, \lambda)$ and $(-3, 0, 1)$ are equidistant from the plane, $3x + 4y - 12z + 13 = 0$, then λ satisfies the equation : **(2015/Online Set–1)**

(a) $3x^2 - 10x + 21 = 0$

(b) $3x^2 + 10x - 13 = 0$

(c) $3x^2 - 10x + 7 = 0$

(d) $3x^2 + 10x + 7 = 0$

102. If the shortest distance between the lines $\frac{x-1}{\alpha} = \frac{y+1}{-1} = \frac{z}{1}$, $(\alpha \neq -1)$ and $x + y + z + 1 = 0 = 2x - y + z + 3$ is $\frac{1}{\sqrt{3}}$, then a value of α is:

(2015/Online Set–1)

(a) $\frac{32}{19}$ (b) $\frac{19}{32}$

(c) $-\frac{16}{19}$ (d) $\frac{19}{16}$

103. The shortest distance between the z-axis and the line $x + y + 2z - 3 = 0 = 2x + 3y + 4z - 4$, is **(2015/Online Set–2)**

(a) 1 (b) 2

(c) 4 (d) 3

104. A plane containing the point (3, 2, 0) and the line $\frac{x-1}{1} = \frac{y-2}{5} = \frac{z-3}{4}$ also contains the point : **(2015/Online Set–2)**

(a) (0, 3, 1) (b) (0, 7, -10)

(c) (0, -3, 1) (d) (0, 7, 10)

105. If the line, $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the plane, $lx + my - z = 9$, then $l^2 + m^2$ is equal to : **(2016)**

(a) 18 (b) 5

(c) 2 (d) 26

106. The distance of the point (1, –5, 9) from the plane $x - y + z = 5$ measured along the line $x = y = z$ is **(2016)**

(a) $10\sqrt{3}$ (b) $\frac{10}{\sqrt{3}}$

(c) $\frac{20}{3}$ (d) $3\sqrt{10}$

107. The shortest distance between the lines lies $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$ in the interval : **(2016/Online Set–1)**

(a) [0, 1) (b) [1, 2)

(c) (2, 3] (d) (3, 4]

108. The distance of the point (1, -2, 4) from the plane passing through the point (1, 2, 2) and perpendicular to the planes x"y+2z=3 and 2x"2y+z+12=0, is :

(2016/Online Set–1)

(a) $2\sqrt{2}$ (b) 2

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

109. The number of distinct real values of λ for which the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{\lambda^2}$ c and $\frac{x-3}{1}=\frac{y-2}{\lambda^2}=\frac{z-1}{2}$ are coplanar is :

(2016/Online Set–2)

(a) 4 (b) 1

(c) 2 (d) 3

110. If the image of the point P(1, –2, 3) in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to the line, $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is Q then PQ is equal to : **(2017)**

(a) $3\sqrt{5}$ (b) $2\sqrt{42}$

(c) $\sqrt{42}$ (d) $6\sqrt{5}$

111. The coordinates of the foot of the perpendicular from the point (1, –2, 1) on the plane containing the lines,

$\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$ is :

(2017/Online Set–1)

(a) (2, –4, 2) (b) (-1, 2, –1)

(c) (0, 0, 0) (d) (1, 1, 1)

112. The line of intersection of the planes =1 $\vec{r}.\left(3\hat{i}-\hat{j}+\hat{k}\right)$ and $\vec{r}.\left(\hat{i}+4\hat{j}-2\hat{k}\right)=2,$ is: **(2017/Online Set–1)**

(a) $\frac{x-\frac{4}{7}}{-2}=\frac{y}{7}=\frac{z-\frac{5}{7}}{13}$

(b) $\frac{x-\frac{4}{7}}{2}=\frac{y}{-7}=\frac{z+\frac{5}{7}}{13}$

(c) $\frac{x-\frac{6}{13}}{2}=\frac{y+\frac{5}{13}}{-7}=\frac{z}{-13}$

(d) $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{-7}=\frac{z}{-13}$

113. If the line, $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+\lambda}{-2}$ lies in the plane, $2x - 4y + 3z = 2$, then the shortest distance between this line and the line, $\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}$ is :

(2017/Online Set–2)

(a) 2 (b) 1

(c) 0 (d) 3

114. If a variable plane, at a distance of 3 units from the origin, intersects the coordinate axes at A, B and C, then the locus of the centroid of ΔABC is :

(2017/Online Set–2)

(a) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$ (b) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=3$

(c) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{9}$ (d) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=9$

115. The length of the projection of the line segment joining the points (5, -1, 4) and (4, -1, 3) on the plane, $x + y + z = 7$ is : **(2018)**

(a) $\sqrt{\frac{2}{3}}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

116. If L_1 is the line of intersection of the planes $2x-2y+3z-2=0,\ x-y+z+1=0$ and L_2 is the line of intersection of the planes $x+2y-z-3=0,\ 3x-y+2z-1=0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is : **(2018)**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{4\sqrt{2}}$

(c) $\frac{1}{3\sqrt{2}}$ (d) $\frac{1}{2\sqrt{2}}$

117. A variable plane passes through a fixed point (3, 2, 1) and meets x, y and z axes at A, B and C respectively. A plane is drawn parallel to yz-plane through A, a second plane is drawn parallel zx-plane through B and a third plane is drawn parallel to xy-plane through C. Then the locus of the point of intersection of these three planes, is: **(2018/Online Set–1)**

(a) $\frac{x}{3}+\frac{y}{2}+\frac{z}{1}=1$ (b) $x+y+z=6$

(c) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{11}{6}$ (d) $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$

118. An angle between the plane, x + y + z = 5 and the line of intersection of the planes, 3x + 4y + z "1 = 0 and $5x+8y+2z+14=0$, is: **(2018/Online Set–1)**

(a) $\sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$ (b) $\cos^{-1}\left(\sqrt{\frac{3}{17}}\right)$

(c) $\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)$ (d) $\sin^{-1}\left(\frac{3}{\sqrt{17}}\right)$

119. If the position vectors of the vertices A, B and C of a ΔABC are respectively

$4\hat{i}+7\hat{j}+8\hat{k},\ 2\hat{i}+3\hat{j}+4\hat{k}$ and $2\hat{i}+5\hat{j}+7\hat{k}$, then the position vector of the point, where the bisector of $\angle A$ meets BC is : **(2018/Online Set–2)**

(a) $\frac{1}{2}\left(4\hat{i}+8\hat{j}+11\hat{k}\right)$ (b) $\frac{1}{3}\left(6\hat{i}+11\hat{j}+15\hat{k}\right)$

(c) $\frac{1}{3}\left(6\hat{i}+13\hat{j}+18\hat{k}\right)$ (d) $\frac{1}{4}\left(8\hat{i}+14\hat{j}+19\hat{k}\right)$

120. An angle between the lines whose direction cosines are given by the equations,

$l+3m+5n=0$ and $5\,lm-2mn+6nl=0$, is :

(2018/Online Set–2)

(a) $\cos^{-1}\left(\frac{1}{3}\right)$ (b) $\cos^{-1}\left(\frac{1}{4}\right)$

(c) $\cos^{-1}\left(\frac{1}{6}\right)$ (d) $\cos^{-1}\left(\frac{1}{8}\right)$

121. A plane bisects the line segment joining the points (1, 2, 3) and (-3, 4, 5) at right angles. Then this plane also passes through the point : **(2018/Online Set–2)**

(a) (-3, 2, 1) (b) (3, 2, 1)

(c) (-1, 2, 3) (d) (1, 2, -3)

122. The sum of the intercepts on the coordinate axes of the plane passing through the point (-2, -2, 2) and containing the line joining the points (1, -1, 2) and (1, 1, 1), is :

(2018/Online Set–3)

(a) 4 (b) -4

(c) -8 (d) 12

123. If the angle between the lines, $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{5-x}{-2}=\frac{7y-14}{p}=\frac{z-3}{4}$ is $\cos^{-1}\left(\frac{2}{3}\right)$, then p is equal to :

(2018/Online Set–3)

(a) $\frac{7}{2}$ (b) $\frac{2}{7}$

(c) $-\frac{7}{4}$ (d) $-\frac{4}{7}$

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. If $\vec{a}\,\&\,\vec{b}$ lie on a plane normal to the plane containing $\vec{c}\,\&\,\vec{d}$ then, $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})$ is equal to

(a) -1 (b) 1

(c) 0 (d) none

2. Let $\vec{a}=\hat{i}-\hat{k},\ \vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$ and $\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$. Then $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ depends on

(a) only x (b) only y

(c) neither x nor y (d) both x and y

3. The value of a so that the volume of the parallelopiped formed by the vectors $\hat{i}+a\hat{j}-\hat{k},\ \hat{j}+a\hat{k},\ \hat{i}+\hat{j}+\hat{k}$ becomes minimum is

(a) $\sqrt{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{7}{4}$

4. The vectors $\vec{a},\vec{b},\vec{c}$ are of the same length & pairwise form equal angles. If $\vec{a}=\hat{i}+\hat{j}$ & $\vec{b}=\hat{j}+\hat{k}$ then $\vec{c}$ can be

(a) (1, 0, 1) (b) $\left(-\frac{4}{3},\frac{1}{3},-\frac{4}{3}\right)$

(c) $\left(\frac{1}{3},-\frac{4}{3},\frac{1}{3}\right)$ (d) $\left(-\frac{1}{3},\frac{4}{3},-\frac{1}{3}\right)$

5. The vector $\vec{a}\times(\vec{b}\times\vec{a})$ is

(a) perpendicular to $\vec{a}$ (b) perpendicular to $\vec{b}$

(c) coplanar with $\vec{a}$ & $\vec{b}$ (d) perpendicular to $\vec{a}\times\vec{b}$

6. $(\vec{u}\times\vec{v}).(\vec{u}\times\vec{v})$ is equal to

(a) $\begin{vmatrix}\vec{u}.\vec{u} & \vec{u}.\vec{v}\\ \vec{u}.\vec{v} & \vec{v}.\vec{v}\end{vmatrix}$ (b) $(\vec{u}.\vec{v})^2-\vec{u}^2.\vec{v}^2$

(c) $|\vec{u}|^2\,|\vec{v}|^2-(\vec{u}.\vec{v})^2$ (d) none

7. For any vector $\vec{A},\ \hat{i}\times(\hat{i}\times\vec{A})+\hat{j}\times(\hat{j}\times\vec{A})+\hat{k}\times(\hat{k}\times\vec{A})$ simplifies to

(a) $3\vec{A}$ (b) $\vec{A}$

(c) $-\vec{A}$ (d) $-2\vec{A}$

8. For a non zero vector $\vec{A}$ if the equations $\vec{A}.\vec{B}=\vec{A}.\vec{C}$ and $\vec{A}\times\vec{B}=\vec{A}\times\vec{C}$ hold simultaneously, then

(a) $\vec{A}$ is perpendicular to $\vec{B}-\vec{C}$

(b) $\vec{A}=\vec{B}$

(c) $\vec{B}=\vec{C}$ (d) $\vec{C}=\vec{A}$

9. If $\vec{a},\vec{b},\vec{c}$ be the unit vectors such that $\vec{b}$ is not parallel to $\vec{c}$ and $\vec{a}\times(2\vec{b}\times\vec{c})=\vec{b}$ then the angle that $\vec{a}$ makes with $\vec{b}\,\&\,\vec{c}$ are respectively

(a) $\frac{\pi}{3}\,\&\,\frac{\pi}{4}$ (b) $\frac{\pi}{3}\,\&\,\frac{2\pi}{3}$

(c) $\frac{\pi}{2}\,\&\,\frac{2\pi}{3}$ (d) $\frac{\pi}{2}\,\&\,\frac{\pi}{3}$

10. The vectors $\vec{p}\,\&\,\vec{q}$ satisfy the system of equations $2\vec{p}+\vec{q}=\vec{a},\ \vec{p}+2\vec{q}=\vec{b}$ and the angle between $\vec{p}\,\&\,\vec{q}$ is θ. If it is known that in the rectangular system of co-ordinates the vectors $\vec{a}\,\&\,\vec{b}$ have the forms $\vec{a}=(1,1)$ & $\vec{b}=(1,-1)$ then $\cos\theta=$

(a) $\frac{4}{5}$ (b) $-\frac{4}{5}$

(c) $-\frac{3}{5}$ (d) none

11. If $\vec{z}_1 = a\,\hat{i} + b\,\hat{j}$ & $\vec{z}_2 = c\,\hat{i} + d\,\hat{j}$ are two vectors in $\hat{i}$ & $\hat{j}$ system where $|\vec{z}_1| = |\vec{z}_2| = r$ & $\vec{z}_1 . \vec{z}_2 = 0$ then $\vec{w}_1 = a\,\hat{i} + c\,\hat{j}$ and $\vec{w}_2 = b\,\hat{i} + d\,\hat{j}$ satisfy

(a) $|\vec{w}_1| = r$ (b) $|\vec{w}_2| = r$

(c) $\vec{w}_1 . \vec{w}_2 = 0$ (d) none of these

12. 'P' is a point inside the triangle ABC such that $BC\,(\overrightarrow{PA}) + CA\,(\overrightarrow{PB}) + AB\,(\overrightarrow{PC}) = 0$ then for the triangle ABC the point P is its

(a) incentre (b) circumcentre

(c) centroid (d) orthocentre

13. Let $\vec{p}$ is the p.v. of the orthocentre & $\vec{g}$ is the p.v. of the centroid of the triangle ABC where circumcentre is the origin. If $\vec{p} = K\vec{g}$ then $K =$

(a) 3 (b) 2

(c) 1/3 (d) 2/3

14. $[(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c})\ (\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\ (\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})] =$

(a) $[\vec{a}\,\vec{b}\,\vec{c}]^2$ (b) $[\vec{a}\,\vec{b}\,\vec{c}]^3$

(c) $[\vec{a}\,\vec{b}\,\vec{c}]^4$ (d) none

15. Consider a tetrahedron with faces f_1, f_2, f_3, f_4. Let $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ be the vectors whose magnitudes are respectively equal to the areas of f_1, f_2, f_3, f_4 & whose directions are perpendicular to these faces in the outward direction. Then

(a) $\vec{a}_1 + \vec{a}_2 + \vec{a}_3 + \vec{a}_4 = 0$ (b) $\vec{a}_1 + \vec{a}_3 = \vec{a}_2 + \vec{a}_4$

(c) $\vec{a}_1 + \vec{a}_2 = \vec{a}_3 + \vec{a}_4$ (d) none

16. If $A_1, A_2, A_3, \ldots\ldots, A_n$ are the vertices of a regular plane polygon with n sides & O is its centre then

$$\sum_{i=1}^{n-1} (\overrightarrow{OA}_i \times \overrightarrow{OA}_{i+1}) =$$

(a) $(1-n)\,\overrightarrow{OA}_2 \times \overrightarrow{OA}_1$ (b) $(n-1)\,\overrightarrow{OA}_2 \times \overrightarrow{OA}_1$

(c) $n\,\overrightarrow{OA}_2 \times \overrightarrow{OA}_1$ (d) none

17. The triple product $(\vec{d}+\vec{a}) . [\vec{a}\times(\vec{b}\times(\vec{c}\times\vec{d}))]$ simplifies to

(a) $(\vec{b}\,.\,\vec{d})\,[\vec{a}\,\vec{c}\,\vec{d}]$ (b) $(\vec{b}\cdot\vec{c})\,[\vec{a}\,\vec{b}\,\vec{d}]$

(c) $(\vec{b}\,.\,\vec{a})\,[\vec{a}\,\vec{b}\,\vec{d}]$ (d) none

18. If the unit vectors $\vec{e}_1$ and $\vec{e}_2$ are inclined at an angle 2θ and $|\vec{e}_1 - \vec{e}_2| < 1$, then for $\theta \in [0, \pi]$, θ may lie in the interval :

(a) $\left[0, \frac{\pi}{6}\right)$ (b) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

(c) $\left(\frac{5\pi}{6}, \pi\right]$ (d) $\left[\frac{\pi}{2}, \frac{5\pi}{6}\right]$

19. If the vectors $a\,\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ $(a \neq b \neq c \neq 1)$ are coplanar then the value of

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} =$$

(a) 1 (b) –1

(c) 0 (d) none

20. Two given points P and Q in the rectangular cartesian co–ordinates lie on $y = 2^{x+2}$ such that $\overrightarrow{OP}.\hat{i} = -1$ and $\overrightarrow{OQ}.\hat{i} = +2$ where $\hat{i}$ is a unit vector along the x–axis. The magnitude of $\overrightarrow{OQ} - 4\,\overrightarrow{OP}$ will be :

(a) 10 (b) 20

(c) 30 (d) none

21. A, B, C and D are four points in a plane with pv's $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ respectively such that

$(\vec{a}-\vec{d}) . (\vec{b}-\vec{c}) = (\vec{b}-\vec{d}) . (\vec{c}-\vec{a}) = 0$.

Then for the triangle ABC, D is its :

(a) incentre (b) circumcentre

(c) othocentre (d) centroid

22. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the pv's of the point A, B, C and D respectively in three dimensional space and satisfy the relation $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$, then :

(a) A, B, C and D are coplanar

(b) the line joining the points B and D divides the line joining the point A and C in the ratio 2 : 1.

(c) the line joining the points A and C divides the line joining the points B and D in the ratio 1 : 1.

(d) the four vectors $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ are linearly dependents.

23. Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{k}$. The point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is :

(a) $-\hat{i} + \hat{j} + \hat{k}$ (b) $3\hat{i} - \hat{j} + \hat{k}$

(c) $3\hat{i} + \hat{j} - \hat{k}$ (d) $\hat{i} - \hat{j} - \hat{k}$

24. If a vector $\vec{a}$ is expressed as the sum of two vectors $\vec{a}'$ and $\vec{a}''$ along and perpendicular to a given vector $\vec{b}$ then $\vec{a}''$ is

(a) $\frac{(\vec{a} \times \vec{b}) \times \vec{b}}{\vec{b}}$ (b) $\frac{\vec{b} \times (\vec{a} \times \vec{b})}{\vec{b}^2}$

(c) $\frac{\vec{b} \times (\vec{a} \times \vec{b})}{\vec{b}}$ (d) $\frac{\vec{a} \times \vec{b}}{\vec{b}^2}\vec{b}$

25. $\vec{a}$ and $\vec{b}$ are two non collinear unit vectors. Then $\vec{a}, \vec{b}, x\vec{a} - y\vec{b}$ form a triangle, if:

(a) $x = -1; y = 1$ and $\left|\vec{a} + \vec{b}\right| = 2\cos\left(\frac{\vec{a} \wedge \vec{b}}{2}\right)$

(b) $x = -1; y = 1$ and $\cos(\vec{a} \wedge \vec{b}) +$

$|\vec{a} + \vec{b}| \cos[\vec{a} \wedge -(\vec{a} + \vec{b})] = -1$

(c) $\left|\vec{a} + \vec{b}\right| = -2\cot\left(\frac{\vec{a} \wedge \vec{b}}{2}\right)\cos\left(\frac{\vec{a} \wedge \vec{b}}{2}\right)$ and

$x = -1, y = 1$

(d) none of these

26. If $\vec{a} = \hat{i} + \hat{j} + \hat{k} . \vec{b} = \hat{i} - \hat{j} + \hat{k} . \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$, then the value of $\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} =$

(a) 2 (b) 4

(c) 16 (d) 64

27. If $\overrightarrow{OA} = \vec{a}; \overrightarrow{OB} = \vec{b}; \overrightarrow{OC} = 2\vec{a} + 3\vec{b}; \overrightarrow{OD} = \vec{a} - 2\vec{b}$, the length of $\overrightarrow{OA}$ is three times the length of $\overrightarrow{OB}$ and $\overrightarrow{OA}$ is prependicular to $\overrightarrow{DB}$ then $(\overrightarrow{BD} \times \overrightarrow{AC}) . (\overrightarrow{OD} \times \overrightarrow{OC})$ is :

(a) $7|\vec{a} \times \vec{b}|^2$ (b) $42|\vec{a} \times \vec{b}|^2$

(c) 0 (d) none of these

28. If the vector product of a constant vector $\overrightarrow{OA}$ with a variable vector $\overrightarrow{OB}$ in a fixed plane OAB be a constant vector, then locus of B is :

(a) a straight line perpendicular to $\overrightarrow{OA}$

(b) a circle with centre O radius equal to $\left|\overrightarrow{OA}\right|$

(c) a straight line parallel to $\overrightarrow{OA}$

(d) none of these

29. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non–zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to :

(a) 0

(b) 1

(c) $\frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$

(d) none of these

30. Let $\vec{b} = 4\hat{i} + 3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in xy–plane. All vectors in the same plane having projections 1 and 2 along $\vec{b}$ and $\vec{c}$ respectively, are given by :

(a) $2\hat{i} - \hat{j} \& \frac{2}{5}\hat{i} + \frac{11}{5}\hat{j}$ (b) $2\hat{i} - \hat{j} \& -\frac{2}{5}\hat{i} + \frac{11}{5}\hat{j}$

(c) $-2\hat{i} - \hat{j} \& -\frac{2}{5}\hat{i} - \frac{11}{5}\hat{j}$ (d) $2\hat{i} - \hat{j} \& -\frac{2}{5}\hat{i} - \frac{11}{5}\hat{j}$

31. If $\vec{b} \& \vec{c}$ are any two perpendicular unit vectors and $\vec{a}$ is any vector, then, $(\vec{a}.\vec{b})\,\vec{b} + (\vec{a}.\vec{c})\,\vec{c} + \frac{\vec{a}.(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|^2}(\vec{b}\times\vec{c})$ is equal to :

(a) $\vec{a}$ (b) $\vec{b}$

(c) $\vec{c}$ (d) none of these

32. If $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors, then

$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\times\vec{c}$ is true if :

(a) $\vec{b} \& \vec{c}$ are collinear (b) $\vec{a} \& \vec{c}$ are collinear

(c) $\vec{a} \& \vec{b}$ are collinear (d) none of these

33. $\hat{a} \& \hat{b}$ are two given unit vectors at right angle. The unit vector equally inclined with $\hat{a} \& \hat{b}$ and $\hat{a}\times\hat{b}$ will be :

(a) $-\frac{1}{\sqrt{3}}(\hat{a}+\hat{b}+\hat{a}\times\hat{b})$ (b) $\frac{1}{\sqrt{3}}(\hat{a}+\hat{b}+\hat{a}\times\hat{b})$

(c) $\frac{1}{\sqrt{3}}(\hat{a}+\hat{b}-\hat{a}\times\hat{b})$ (d) $-\frac{1}{\sqrt{3}}(\hat{a}+\hat{b}-\hat{a}\times\hat{b})$

34. Let $\vec{A}, \vec{B}, \vec{C}$ be vectors of length 3, 4 and 5 respectively. Let $\vec{A}$ be perpendicular to $\vec{B}+\vec{C}, \vec{B}$ to $\vec{C}+\vec{A}$ and $\vec{C}$ to $\vec{A}+\vec{B}$. Then length of the vector, $\vec{A}+\vec{B}+\vec{C}$ is :

(a) $-5\sqrt{2}$ (b) $\sqrt{2}$

(c) $5\sqrt{2}$ (d) none of these

35. $(\vec{r}\,.\,\hat{i})\,(\hat{i}\times\vec{r}) + (\vec{r}\,.\,\hat{j})\,(\hat{j}\times\vec{r}) + (\vec{r}\,.\,\hat{k})\,(\hat{k}\times\vec{r}) =$

(a) 0 (b) $\vec{r}$

(c) $2\,\vec{r}$ (d) $3\,\vec{r}$

36. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non–coplanar unit vectors equally inclined to one another at an acute angle θ. Then $|[\vec{a}\ \vec{b}\ \vec{c}]|$ in terms of θ is equal to :

(a) $(1+\cos\theta)\sqrt{\cos 2\theta}$ (b) $(1+\cos\theta)\sqrt{1-2\cos 2\theta}$

(c) $(1-\cos\theta)\sqrt{1+2\cos\theta}$ (d) none of these

37. If line makes angle α,β,γ,δ with the four diagonals of a cube, then the value of $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta =$

(a) 1 (b) 4/3

(c) 2/3 (d) Variable

38. If the sum of the squares of the distance of a point from the three coordinate axes be 36, then its distance from the origin is

(a) 6 (b) $3\sqrt{2}$

(c) $2\sqrt{3}$ (d) $6\sqrt{2}$

39. The direction ratio's of the line $x-y+z-5=0=x-3y-6$ are

(a) 3, 1, –2 (b) 2, –4, 1

(c) $\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}$ (d) $\frac{2}{\sqrt{21}}, \frac{-4}{\sqrt{21}}, \frac{1}{\sqrt{21}}$

40. A variable plane passes through a fixed point (1, 2, 3). The locus of the foot of the perpendicular drawn from origin to this plane is :

(a) $x^2+y^2+z^2-x-2y-3z=0$

(b) $x^2+2y^2+3z^2-x-2y-3z=0$

(c) $x^2+4y^2+9z^2+x+2y+3=0$

(d) $x^2+y^2+z^2+x+2y+3z=0$

41. The straight lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-1}{2}=\frac{y-2}{2}=\frac{z-3}{-2}$ are

(a) parallel lines (b) Intersecting at 60º

(c) Skew lines (d) Intersecting at right angle

42. The equation of the plane which bisects the angle between the planes $3x-6y+2z+5=0$ and $4x-12y+3z-3=0$ which contains the origin is

(a) $33x-13y+32z+45=0$

(b) $x-3y+z-5=0$

(c) $33x+13y+32z+45=0$

(d) None of these

43. If a plane cuts off intercepts OA = a, OB = b, OC = c from the coordinate axes, then the area of the triangle ABC =

(a) $\frac{1}{2}\sqrt{b^2c^2+c^2a^2+a^2b^2}$ (b) $\frac{1}{2}(bc+ca+ab)$

(c) $\frac{1}{2}abc$

(d) $\frac{1}{2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}$

44. P is fixed point (a, a, a) on a line through the origin equally inclined to the axes, then any plane through P perpendicular to OP, makes intercepts on the axes, the sum of whose reciprocals is equal to

(a) a (b) a/2

(c) 3a/2 (d) 1/a

45. If $\vec{a}.\vec{b}$ are unit vectors such that $(\vec{a}+\vec{b})\cdot(2\vec{a}+3\vec{b})\times(3\vec{a}-2\vec{b})=0$, then angle between $\vec{a}$ and $\vec{b}$

(a) 0 (b) $\pi/2$

(c) π (d) indeterminate

46. Let $\vec{r},\vec{a},\vec{b}$ and $\vec{c}$ be four non–zero vectors such that $\vec{r}\cdot\vec{a}=0$, $|\vec{r}\times\vec{b}|=|\vec{r}||\vec{b}|, |\vec{r}\times\vec{c}|=|\vec{r}||\vec{c}|$, then [a b c] =

(a) |a| |b| |c| (b) –|a| |b| |c|

(c) 0 (d) none of these

47. If $\vec{a}_1,\vec{a}_2$ and $\vec{a}_3$ are non–coplanar vectors and $(x+y-3)\,\vec{a}_1+(2x-y+2)\,\vec{a}_2+(2x+y+\lambda)\,\vec{a}_3=\vec{o}$ holds for some 'x' and 'y' then 'λ' is

(a) $\frac{7}{3}$ (b) 2

(c) $-\frac{10}{3}$ (d) $\frac{5}{3}$

48. A unit vector $\vec{a}$ in the plane of $\vec{b}=2\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}-\hat{j}+\hat{k}$ is such that $\vec{a}\wedge\vec{b}=\vec{a}\wedge\vec{d}$ where $\vec{d}=\hat{j}+2\hat{k}$ is

(a) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ (b) $\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

(c) $\frac{2\hat{i}+\hat{j}}{\sqrt{5}}$ (d) $\frac{2\hat{i}+\hat{j}}{\sqrt{5}}$

49. If $\vec{a},\vec{b},\vec{c}$ are such that $[\vec{a}\ \vec{b}\ \vec{c}]=1, \vec{c}=\lambda\,\vec{a}\times\vec{b},\ \vec{a}\wedge\vec{b}<\frac{2\pi}{3}$ and $|\vec{a}|=\sqrt{2},|\vec{b}|=\sqrt{3},\ |\vec{c}|=\frac{1}{\sqrt{3}}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

50. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{o}$, then $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is equal to

(a) $6(\vec{b}\times\vec{c})$ (b) $6(\vec{c}\times\vec{a})$

(c) $6(\vec{a}\times\vec{b})$ (d) none of these

51. If $((\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}))\cdot(\vec{a}\times\vec{d})=0$, then which of the following is always true

(a) $\vec{a},\vec{b},\vec{c},\vec{d}$ are necessarily coplanar

(b) either $\vec{a}$ or $\vec{d}$ must lie in the plane of $\vec{b}$ and $\vec{c}$

(c) either $\vec{b}$ or $\vec{c}$ must lie in plane of $\vec{a}$ and $\vec{d}$

(d) either $\vec{a}$ or $\vec{b}$ must lie in plane of $\vec{c}$ and $\vec{d}$

52. If the foot of the perpendicular from the origin to a plane is P (a, b, c), the equation of the plane is

(a) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=3$

(b) $ax+by+cz=3$

(c) $ax+by+cz=a^2+b^2+c^2$

(d) $ax+by+cz=a+b+c$

53. Equation of line in the plane $P \equiv 2x - y + z - 4 = 0$ which is perpendicular to the line l whose equation is $\frac{x-2}{1} = \frac{y-2}{-1} = \frac{z-3}{-2}$ and which passes through the point of intersection of l and P is

(a) $\frac{x-2}{3} = \frac{y-1}{5} = \frac{z-1}{-1}$ (b) $\frac{x-1}{3} = \frac{y-3}{5} = \frac{z-5}{-1}$

(c) $\frac{x+2}{2} = \frac{y+1}{-1} = \frac{z+1}{1}$ (d) $\frac{x-2}{2} = \frac{y-1}{-1} = \frac{z-1}{1}$

54. Equation of plane which passes through the point of intersection of lines $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2}$ and $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and at greatest distance from the point (0, 0, 0) is :

(a) $4x + 3y + 5z = 25$ (b) $4x + 3y + 5z = 50$

(c) $3x + 4y + 5z = 49$ (d) $x + 7y - 5z = 2$

55. Let A $(\vec{a})$ amd B $(\vec{b})$ be points on two skew lines $\vec{r} = \vec{a} + \lambda\vec{p}$ and $\vec{r} = \vec{b} + u\vec{q}$ and the shortest distance between the skew lines is 1, where $\vec{p}$ and $\vec{q}$ are unit vectors forming adjacent sides of a parallelogram enclosing an area of $\frac{1}{2}$ units. If an angle between AB and the line of shortest distance is 60°, then AB =

(a) $\frac{1}{2}$ (b) 2

(c) 1 (d) $\lambda \in R - \{0\}$

56. If $P_1 : \vec{r}\cdot\vec{n}_1 - d_1 = 0, P_2 : \vec{r}\cdot\vec{n}_2 - d_2 = 0$ and $P_3 : \vec{r}\cdot\vec{n}_3 - d_3 = 0$ are three planes and $\vec{n}_1, \vec{n}_2$ and $\vec{n}_3$ are three non–coplanar vectors then, the three lines $P_1=0$, $P_2=0$ and $P_2=0$, $P_3=0$ and $P_3 = 0$, $P_1 = 0$ are

(a) parallel lines (b) coplanar lines

(c) coincident lines (d) concurrent lines

57. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors equally inclined to each other at an angle α. Then the angle between $\vec{a}$ and plane of $\vec{b}$ and $\vec{c}$ is

(a) $\theta = \cos^{-1}\left(\frac{\cos\alpha}{\cos\frac{\alpha}{2}}\right)$ (b) $\theta = \sin^{-1}\left(\frac{\cos\alpha}{\cos\frac{\alpha}{2}}\right)$

(c) $\theta = \cos^{-1}\left(\frac{\sin\frac{\alpha}{2}}{\sin\alpha}\right)$ (d) $\theta = \sin^{-1}\left(\frac{\sin\frac{\alpha}{2}}{\sin\alpha}\right)$

58. Let A(1, 1, 1), B(2, 3, 5), C(–1, 0, 2) be three points, then equation of a plane parallel to the plane ABC which is at a distance 2 is

(a) $2x - 3y + z + 2\sqrt{14} = 0$

(b) $2x - 3y + z - \sqrt{14} = 0$

(c) $2x - 3y + z + 2 = 0$

(d) $2x - 3y + z - 2 = 0$

59. If $\vec{a}$ and $\vec{b}$ unequal unit vectors such that $(\vec{a} - \vec{b}) \times [(\vec{b} + \vec{a}) \times (2\vec{a} + \vec{b})] = \vec{a} + \vec{b}$, then smaller angle θ between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{2}$ (b) 0

(c) π (d) $\frac{\pi}{4}$

60. A vector $(\vec{d})$ is equally inclined to three vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{j} - 2\hat{k}$. Let $\vec{x}, \vec{y}, \vec{z}$ be three vector in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{c}; \vec{c}, \vec{a}$ respectively then

(a) $\vec{x} \cdot \vec{d} = 14$

(b) $\vec{y} \cdot \vec{d} = 3$

(c) $\vec{z} \cdot \vec{d} = 0$

(d) $\vec{r} \cdot \vec{d} = 0$ where $\vec{r} = \lambda\vec{x} + \mu\vec{y} + \delta\vec{z}$

61. Identify the statement(s) which is/are INCORRECT ?

(a) $\vec{a}\times[\vec{a}\times(\vec{a}\times\vec{b})]=(\vec{a}\times\vec{b})(\vec{a}^2)$

(b) If $\vec{a},\vec{b},\vec{c}$ are non–zero, non coplanar vectors, and $\vec{v}.\vec{a}=\vec{v}.\vec{b}=\vec{v}.\vec{c}=0$ then $\vec{v}$ must be a null vector

(c) If $\vec{a}$ and $\vec{b}$ lie in a plane normal to the plane containing the vectors $\vec{a}-\vec{b},\vec{c}-\vec{d}$, where $\vec{a},\vec{b},\vec{c},\vec{d}$ are non–zero vectors, then $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{o}$

(d) If $\vec{a},\vec{b},\vec{c}$ and $\vec{a}',\vec{b}'\vec{c}'$ are reciprocal system of vectors then $\vec{a}\cdot\vec{b}'+\vec{b}\cdot\vec{c}'+\vec{c}\cdot\vec{a}'=3$

62. Consider the planes $3x - 6y + 2z + 5 = 0$ and $4x - 12y + 3z = 3$. The plane $67x - 162y + 47z + 44 = 0$ bisects that angle between the given planes which

(a) contains origin (b) is acute

(c) is obtuse (d) none of theses

63. **Assertion :** If $\vec{a}=3\hat{i}+\hat{k},\vec{b}=-\hat{i}+2\hat{j}+\hat{k},\vec{c}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{d}=2\hat{i}-\hat{j}$, then there exist real numbers α, β, γ such that $\vec{a}=\alpha\vec{b}+\beta\vec{c}+\gamma\vec{d}$

Reason : $\vec{a},\vec{b},\vec{c},\vec{d}$ are four vectors in a 3–dimensional space. If $\vec{b},\vec{c},\vec{d}$ are non–coplanar, then there exist real numbers α, β, γ such that $\vec{a}=\alpha\vec{b}+\beta\vec{c}+\gamma\vec{d}$

(a) A (b) B
(c) C (d) D
(e) E

64. **Assertion :** Let $\vec{a}=2\hat{i}+3\hat{j}-\hat{k},\vec{b}=4\hat{i}+6\hat{j}-2\hat{k}$, then $\vec{a}\times\vec{b}=\vec{o}$

Reason : If $\vec{a}\neq\vec{o}$, $\vec{b}\neq\vec{o}$ and $\vec{a}$ and $\vec{b}$ are non–collinear vectors, then $\vec{a}\times\vec{b}$ = ab sin θ $\hat{n}$, where θ is the smaller angle between the vectors $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is unit vector such that $\vec{a},\vec{b},\hat{n}$ taken in this order form right handed orientation

(a) A (b) B
(c) C (d) D
(e) E

65. **Assertion :** Let $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are position vectors of four points A, B, C and D and $3\vec{a}-2\vec{b}+5\vec{c}-6\vec{d}=\vec{0}$, then points A, B, C and D are coplaner.

Reason : Three non zero, linearly dependent co–initial vectors $(\overrightarrow{PQ},\overrightarrow{PR}$ and $\overrightarrow{PS})$ are coplaner.

(a) A (b) B
(c) C (d) D
(e) E

66. **Assertion :** Let $\vec{a}=3\hat{i}-\hat{j},\ \vec{b}=2\hat{i}+\hat{j}-3\hat{k}$. If $\vec{b}=\vec{b}_1+\vec{b}_2$ such that $\vec{b}_1$ is collinear with $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$ is possible, then $\vec{b}_2=\hat{i}+3\hat{j}-3\hat{k}$.

Reason : If $\vec{a}$ and $\vec{b}$ are non–zero, non–collinear vectors, then $\vec{b}$ can be expressed as $\vec{b}=\vec{b}_1+\vec{b}_2$, where $\vec{b}_1$ is collinear with $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$.

(a) A (b) B
(c) C (d) D
(e) E

67. **Assertion :** A point on the straight line $2x + 3y - 4z = 5$, $3x - 2y + 4z = 7$ can be determined by taking $x = k$ and then solving the two equations for y and z, where k is any real number except 12/5.

Reason : If $c' \neq kc$, then the straight line $ax + by + cz + d = 0$, $kax + kby + c'z + d' = 0$, does not intersect the plane $z = \alpha$, where α is any real number except $\frac{d'-kd}{kc-c'}$.

(a) A (b) B
(c) C (d) D
(e) E

Using the following passage, solve Q.68 to Q.71

PASSAGE –1

Three vectors $\hat{a},\ \hat{b}$ and $\hat{c}$ are such that $\hat{a}\times\hat{b}=\hat{c},\ \hat{b}\times\hat{c}=\hat{a},\ \hat{c}\times\hat{a}=\hat{b}$.

Answer the following questions :

68. If vector $3\hat{a}-2\hat{b}+2\hat{c}$ and $-\hat{a}-2\hat{c}$ are adjacent sides of a parallelogram, then an angle between the diagonals is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

69. Vectors $2\hat{a}-3\hat{b}+4\hat{c}$, $\hat{a}+2\hat{b}-\hat{c}$ and $x\hat{a}-\hat{b}+2\hat{c}$ are coplanar, then x =

(a) $\frac{8}{5}$ (b) $\frac{5}{8}$

(c) 0 (d) 1

70. Let $\vec{x}=\hat{a}+\hat{b}, \vec{y}=2\hat{a}-\hat{b}$, then the point of intersection of straight lines $\vec{r}\times\vec{x}=\vec{y}\times\vec{x}, \vec{r}\times\vec{y}=\vec{x}\times\vec{y}$ is

(a) $2\hat{b}$ (c) $3\hat{b}$

(c) $3\hat{a}$ (d) $2\hat{a}$

71. $\hat{a}\cdot(\hat{b}\times\hat{c})+\hat{b}\cdot(\hat{c}\times\hat{a})+\hat{c}\cdot(\hat{a}\times\hat{b})$ is equal to

(a) 1 (b) 3

(c) 0 (d) – 12

Using the following passage, solve Q. 72 to Q. 74

PASSAGE –2

Let $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ be two planes, where $d_1, d_2 > 0$. Then origin lies in acute angle if $a_1a_2+b_1b_2+c_1c_2<0$ and origin lies in obtuse angle if $a_1a_2+b_1b_2+c_1c_2>0$.

Further point (x_1, y_1, z_1) and origin both lie either in acute angle or in obtuse angle, if $(a_1x_1+b_1y_1+c_1z_1+d_1)(a_2x_1+b_2y_1+c_2z_1+d_2)>0$. One of (x_1, y_1, z_1) and origin lie in acute angle and the other in obtuse angle, if

$(a_1x_1+b_1y_1+c_1z_1+d_1)(a_2x_1+b_2y_1+c_2z_1+d_2)<0$

72. Given the planes $2x+3y-4z+7=0$ and $x-2y+3z-5=0$, if a point P is (1, –2, 3), then

(a) O and P both lie in acute angle between the planes

(b) O and P both lie in obtuse angle

(c) O lies in acute angle, P lies in obtuse angle

(d) O lies in obtuse angle, P lies an acute angle.

73. Given the planes $x+2y-3z+5=0$ and $2x+y+3z+1=0$. If a point P is (2, –1, 2), then

(a) O and P both lie in acute angle between the planes

(b) O and P both lie in obtuse angle

(c) O lies in acute angle, P lies in obtuse angle

(d) O lies in obtuse angle, P lies an acute angle.

74. Given the planes $x+2y-3z+2=0$ and $x-2y+3z+7=0$, if the point P is (1, 2, 2), then

(a) O and P both lie in acute angle between the planes

(b) O and P both lie in obtuse angle

(c) O lies in acute angle, P lies in obtuse angle

(d) O lies in obtuse angle, P lies an acute angle.

75.

Column–I	**Column–II**
(A) If $\vec{a}+\vec{b}=\hat{j}$ and $2\vec{a}-\vec{b}=3\hat{i}+\frac{\hat{j}}{2}$, then cosine of the angle between $\vec{a}$ and $\vec{b}$ is	(P) 1
(B) If $\lvert\vec{a}\rvert=\lvert\vec{b}\rvert=\lvert\vec{c}\rvert$, angle between each pair of vectors is $\frac{\pi}{3}$ and $\lvert\vec{a}+\vec{b}+\vec{c}\rvert=\sqrt{6}$, then $\lvert\vec{a}\rvert=$	(Q) $5\sqrt{3}$
(C) Area of the parallelogram whose diagonals represent the vectors $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-3\hat{j}+4\hat{k}$ is	(R) 7
(D) If $\vec{a}$ is perpendicular to $\vec{b}+\vec{c}$, $\vec{b}$ is perpendicular to $\vec{c}+\vec{a}$, $\vec{c}$ is perpendicular to $\vec{a}+\vec{b}$, $\lvert\vec{a}\rvert=2, \lvert\vec{b}\rvert=3$ and $\lvert\vec{c}\rvert=6$, then $\lvert\vec{a}+\vec{b}+\vec{c}\rvert=$	(S) $-\frac{3}{5}$

76. **Column–I** **Column–II**

	Column–I		Column–II
(A)	Foot of perp. drawn for point (1, 2, 3) to the line $\frac{x-2}{2}=\frac{y-1}{3}=\frac{z-2}{4}$ is	(P)	$\left(\frac{107}{29},\frac{30}{29},\frac{69}{29}\right)$
(B)	Image of line point (1, 2, 3) in the line $\frac{x-2}{2}=\frac{y-1}{3}=\frac{z-2}{4}$ is	(Q)	$\left(\frac{88}{29},\frac{125}{29},\frac{69}{29}\right)$
(C)	Foot of perpendicular from the point (2, 3, 5) to the plane $2x+3y-4z+17=0$ is	(R)	$\left(\frac{68}{29},\frac{44}{29},\frac{78}{29}\right)$
(D)	Image of the point (2, 5, 1) in the plane $3x-2y+4z-5=0$ is	(S)	$\left(\frac{38}{29},\frac{57}{29},\frac{185}{29}\right)$

77. In a regular tetrahedron let θ be the angle between any edge and a face not containing the edge. If $\cos^2\theta = \frac{a}{b}$ where a, b $\in I^+$ also a and b are coprime, then find the value of $10a + b$

78. If equation of the plane through the straight line $\frac{x-1}{2}=\frac{y+2}{-3}=\frac{z}{5}$ and perpendicular to the plane $x-y+z+2=0$ is $ax-by+cz+4=0$, then find the value of 10^3a+10^2b+10c.

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Objective Question I [Only one correct option]

1. The scalar $\vec{A}.\{(\vec{B}+\vec{C})\times(\vec{A}+\vec{B}+\vec{C})\}$ equals : **(1981)**

(a) 0 (b) $[\vec{A}\,\vec{B}\,\vec{C}]+[\vec{B}\,\vec{C}\,\vec{A}]$

(c) $[\vec{A}\,\vec{B}\,\vec{C}]$ (d) none of these

2. For non-zero vectors $\vec{a},\vec{b},\vec{c}$, $|(\vec{a}\times\vec{b})\cdot\vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$ holds, if and only if : **(1982)**

(a) $\vec{a}\cdot\vec{b}=0, \vec{b}\cdot\vec{c}=0$ (b) $\vec{b}\cdot\vec{c}=0, \vec{c}\cdot\vec{a}=0$

(c) $\vec{c}\cdot\vec{a}=0, \vec{a}\cdot\vec{b}=0$ (d) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$

3. The volume of the parallelopiped whose sides are given by $\overrightarrow{OA}=2\hat{i}-3\hat{j}, \overrightarrow{OB}=\hat{i}+\hat{j}-\hat{k}, \overrightarrow{OC}=3\hat{i}-\hat{k}$, is : **(1983)**

(a) $\frac{4}{13}$ (b) 4

(c) $\frac{2}{7}$ (d) none of these

4. The points with position vectors $60\hat{i}+3\hat{j}, 40\hat{i}-8\hat{j}, a\hat{i}-52\hat{j}$ are collinear, if : **(1983)**

(a) $a=-40$ (b) $a=40$

(c) $a=20$ (d) none of these

5. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then

$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to : **(1986)**

(a) 0 (b) 1

(c) $\frac{1}{4}(a_1^2+a_2^2+a_3^3)(b_1^2+b_2^2+b_3^2)$

(d) $\frac{3}{4}(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)(c_1^2+c_2^2+c_3^2)$

6. A vector $\vec{a}$ has components 2 p and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to the new system, $\vec{a}$ has components p + 1 and 1, then : **(1986)**

(a) $p=0$ (b) $p=1$ or $p=-\frac{1}{3}$

(c) $p=1$ or $p=\frac{1}{3}$ (d) $p=1$ or $p=-1$

7. The number of vectors of unit length perpendicular to vectors $\vec{a}=(1, 1, 0)$ and $\vec{b}=(0, 1, 1)$ is: **(1987)**

(a) one (b) two

(c) three (d) infinite

8. Let a, b, c be distinct non-negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is: **(1993)**

(a) the Arithmetic Mean of a and b

(b) the Geometric Mean of a and b

(c) the Harmonic Mean of a and b

(d) equal to zero

9. Let $\vec{a}=\hat{i}-\hat{j}, \vec{b}=\hat{j}-\hat{k}, \vec{c}=\hat{k}-\hat{i}$. If $\vec{d}$ is a unit vector such that $\vec{a}\cdot\vec{d}=0=[\vec{b}\;\vec{c}\;\vec{d}]$, then $\vec{d}$ equals : **(1995)**

(a) $\pm\frac{\hat{i}+\hat{j}-2\hat{k}}{\sqrt{6}}$ (b) $\pm\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

(c) $\pm\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ (d) $\pm\hat{k}$

10. If $\vec{a},\vec{b},\vec{c}$ are non coplanar unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{(\vec{b}+\vec{c})}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is : **(1995)**

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

11. Let $\vec{u},\vec{v}$ and $\vec{w}$ be vectors such that $\vec{u}+\vec{v}+\vec{w}=0$. If $|\vec{u}|=3, |\vec{v}|=4$ and $|\vec{w}|=5$, then $\vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{w}+\vec{w}\cdot\vec{u}$ is : **(1995)**

(a) 47 (b) –25

(c) 0 (d) 25

12. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non coplanar vectors, then $(\vec{a}+\vec{b}+\vec{c}).[(\vec{a}+\vec{b}) \times (\vec{a}+\vec{c})]$ equals : **(1995)**

(a) 0 (b) $[\vec{a}\ \vec{b}\ \vec{c}]$

(c) $2\cdot[\vec{a}\ \vec{b}\ \vec{c}]$ (d) $-[\vec{a}\ \vec{b}\ \vec{c}]$

13. If $\vec{p}, \vec{q}, \vec{r}$ be three mutually perpendicular vectors of the same magnitude. If a vectors $\vec{x}$ satisfies the equation $\vec{p}\times((\vec{x}-\vec{q})\times\vec{p})+\vec{q}\times((\vec{x}-\vec{r})\times\vec{q})+\vec{r}\times((\vec{x}-\vec{p})\times\vec{r})=0$, then $\vec{x}$ is given by : **(1997)**

(a) $\frac{1}{2}(\vec{p}+\vec{q}-2\vec{r})$ (b) $\frac{1}{2}(\vec{p}+\vec{q}+\vec{r})$

(c) $\frac{1}{3}(\vec{p}+\vec{q}+\vec{r})$ (d) $\frac{1}{3}(2\vec{p}+\vec{q}-\vec{r})$

14. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=4\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$ are linearly dependent vectors and $|\vec{c}|=\sqrt{3}$, then : **(1998)**

(a) $\alpha=1, \beta=-1$ (b) $\alpha=1, \beta=\pm 1$

(c) $\alpha=-1, \beta=\pm 1$ (d) $\alpha=\pm 1, \beta=1$

15. For three vector $\vec{u}, \vec{v}, \vec{w}$ which of the following expressions is not equal to any of the remaining three : **(1998)**

(a) $\vec{u}\cdot(\vec{v}\times\vec{w})$ (b) $(\vec{v}\times\vec{w})\cdot\vec{u}$

(c) $\vec{v}\cdot(\vec{u}\times\vec{w})$ (d) $(\vec{u}\times\vec{v})\cdot\vec{w}$

16. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c}=|\vec{c}|, |\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is 30°, then $|(\vec{a}\times\vec{b})\times\vec{c}|$ is equal to : **(1999)**

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) 2 (d) 3

17. Let $\vec{a}=2\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$, then $\vec{c}$ is equal to : **(1999)**

(a) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ (b) $\frac{1}{\sqrt{3}}(-\hat{i}-\hat{j}-\hat{k})$

(c) $\frac{1}{\sqrt{5}}(\hat{i}-2\hat{j})$ (d) $\frac{1}{\sqrt{5}}(\hat{i}-\hat{j}-\hat{k})$

18. If the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ from the sides BC, CA and AB respectively of a triangle ABC, then : **(2000)**

(a) $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=0$ (b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

(c) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}$ (d) $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=0$

19. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be such that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=0$. Let P_1 and P_2 be planes determined by the pairs of vectors $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ respectively, then the angle between P_1 and P_2 is: **(2000)**

(a) 0 (b) $\pi/4$

(c) $\pi/3$ (d) $\pi/2$

20. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit coplanar vectors, then the scalar triple product $[2\vec{a}-\vec{b}\quad 2\vec{b}-\vec{c}\quad 2\vec{c}-\vec{a}]$ is equal to : **(2000)**

(a) 0 (b) 1

(c) $-\sqrt{3}$ (d) $\sqrt{3}$

21. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors, then

$|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2$ does not exceed : **(2001)**

(a) 4 (b) 9

(c) 8 (d) 6

22. Let $\vec{a}=\hat{i}-\hat{k}, \vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$ and

$\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$ Then $[\vec{a}\ \vec{b}\ \vec{c}]$ depends on : **(2001)**

(a) only x (b) only y

(c) neither x nor y (d) both x and y

23. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+2\vec{b}$ and $5\vec{a}-4\vec{b}$ are perpendicular to each other, then the angle between $\vec{a}$ and $\vec{b}$ is : **(2002)**

(a) 45° (b) 60°

(c) $\cos^{-1}\left(\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\frac{2}{7}\right)$

24. Let $\vec{V}=2\vec{i}+\vec{j}-\vec{k}$ and $\vec{W}=\vec{i}+3\vec{k}$. If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[\vec{U}\ \vec{V}\ \vec{W}]$ is : **(2002)**

(a) -1 (b) $\sqrt{10}+\sqrt{6}$

(c) $\sqrt{59}$ (d) $\sqrt{60}$

25. The positive value of 'a' so that the volume of parallelopiped formed by $\hat{i}+a\hat{j}+\hat{k}, \hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ become minimum is : **(2003)**

(a) 4 (b) 3

(c) $1/\sqrt{3}$ (d) $\sqrt{3}$

26. If $\vec{a}=(\hat{i}+\hat{j}+\hat{k}), \vec{a}\cdot\vec{b}=1$ and $\vec{a}\times\vec{b}=\hat{j}-\hat{k}$, then $\vec{b}$ is : **(2003)**

(a) $\hat{i}-\hat{j}+\hat{k}$ (b) $2\hat{j}-\hat{k}$

(c) $\hat{i}$ (d) $2\hat{i}$

27. The unit vector which is orthogonal to the vector $3\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar with the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is : **(2004)**

(a) $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$ (b) $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$

(c) $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$ (d) $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

28. If $\vec{a},\vec{b},\vec{c}$ are three non-zero, non-coplanar vectors and

$\vec{b}_1=\vec{b}-\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}$, $\quad \vec{b}_2=\vec{b}+\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}$,

$\vec{c}_1=\vec{c}-\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}-\frac{\vec{c}\cdot\vec{b}}{|\vec{b}|^2}\vec{b}$, $\quad \vec{c}_2=\vec{c}-\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}-\frac{\vec{c}\cdot\vec{b}_1}{|\vec{b}_1|^2}\vec{b}_1$,

$\vec{c}_3=\vec{c}-\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}-\frac{\vec{c}\cdot\vec{b}_2}{|\vec{b}|^2}\vec{b}_2$, $\quad \vec{c}_4=\vec{a}-\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}$.

Then which of the following is a set of mutually orthogonal vectors ? **(2005)**

(a) $\{\vec{a},\vec{b}_1,\vec{c}_1\}$ (b) $\{\vec{a},\vec{b}_1,\vec{c}_2\}$

(c) $\{\vec{a},\vec{b}_2,\vec{c}_3\}$ (d) $\{\vec{a},\vec{b}_2,\vec{c}_4\}$

29. Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}, \vec{c}=\hat{i}+\hat{j}-\hat{k}$.

A vector coplanar to $\vec{a}$ and $\vec{b}$ has a projection along $\vec{c}$ of magnitude $\frac{1}{\sqrt{3}}$, then the vector is : **(2006)**

(a) $4\hat{i}-\hat{j}+4\hat{k}$ (b) $4\hat{i}+\hat{j}-4\hat{k}$

(c) $2\hat{i}+\hat{j}+\hat{k}$ (d) none of these

30. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i}+\hat{j}+\hat{k}, \hat{i}-\lambda^2\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\lambda^2\hat{k}$ are coplanar, is : **(2007)**

(a) zero (b) one

(c) two (d) three

31. Let $\vec{a},\vec{b},\vec{c}$ be unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Which one of the following is correct ? **(2007)**

(a) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}=\vec{0}$

(b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq\vec{0}$

(c) $\vec{b}\times\vec{b}=\vec{b}\times\vec{c}=\vec{a}\times\vec{c}=0$

(d) $\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}$ are mutually perpendicular

32. The edges of a parallelopiped are of unit length and are parallel to non–coplanar unit vector $\hat{a},\hat{b},\hat{c}$ such that $\hat{a}\cdot\hat{b}=\hat{b}\cdot\hat{c}=\hat{c}\cdot\hat{a}=\frac{1}{2}$. Then, the volume of the parallelopiped is **(2008)**

(a) $\frac{1}{\sqrt{2}}$ cu unit (b) $\frac{1}{2\sqrt{2}}$ cu unit

(c) $\frac{\sqrt{3}}{2}$ cu unit (d) $\frac{1}{\sqrt{3}}$ cu unit

33. Let two non–collinear unit vectors $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}$ cos t + $\hat{b}$ sin t. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overline{OP}$. Then, **(2008)**

(a) $\hat{u}=\frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M=(1+\hat{a}\cdot\hat{b})^{1/2}$

(b) $\hat{u}=\frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M=(1+\hat{a}\cdot\hat{b})^{1/2}$

(c) $\hat{u}=\frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M=(1+2\hat{a}\cdot\hat{b})^{1/2}$

(d) $\hat{u}=\frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M=(1+2\hat{a}\cdot\hat{b})^{1/2}$

34. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the unit vector such that $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=1$ and $\vec{a}\cdot\vec{c}=\frac{1}{2}$, then **(2009)**

(a) $\vec{a}, \vec{b}, \vec{c}$ are non–coplanar

(b) $\vec{a}, \vec{b}, \vec{d}$ are non–coplanar

(c) $\vec{b}, \vec{d}$ are non–parallel

(d) $\vec{a}, \vec{d}$ are parallel and $\vec{b}, \vec{c}$ are parallel

35. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}, 4\hat{i}, 3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral PQRS must be a **(2010)**

(a) parallelogram, which is neither a rhombus nor a rectangle

(b) square

(c) rectangle, but not a square

(d) rhombus, but not a square

36. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB}=2\hat{i}+10\hat{j}+11\hat{k}$ and $\overrightarrow{AD}=-\hat{i}+2\hat{j}+2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD′. If AD′ makes a right angle with the side AB, then the cosine of the angle α is given by **(2010)**

(a) $\frac{8}{9}$ (b) $\frac{\sqrt{17}}{9}$

(c) $\frac{1}{9}$ (d) $\frac{4\sqrt{5}}{9}$

37. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by **(2011)**

(a) $\hat{i}-3\hat{j}+3\hat{k}$ (b) $-3\hat{i}-3\hat{j}-\hat{k}$

(c) $3\hat{i}-\hat{j}+3\hat{k}$ (d) $\hat{i}+3\hat{j}-3\hat{k}$

38. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}|=\sqrt{29}$ and $\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k})=(2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$, then a possible value of $(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k})$ is **(2012)**

(a) 0 (b) 3

(c) 4 (d) 8

39. Let $\overrightarrow{PR}=3\hat{i}+\hat{j}-2\hat{k}$ and $\overrightarrow{SQ}=\hat{i}-3\hat{j}-4\hat{k}$ determine diagonals of a parallologram PQRS and $\overrightarrow{PT}=\hat{i}+2\hat{j}+3\hat{k}$ be another vector. Then, the volume of the parallelopiped determined by the vectors $\overrightarrow{PT}, \overrightarrow{PQ}$ and $\overrightarrow{PS}$ is **(2013)**

(a) 5 (b) 20

(c) 10 (d) 30

40. The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes $2x+y-2z=5$ and $3x-6y-2z=7$, is **(2017)**

(a) $-14x+2y+15z=3$ (b) $14x-2y+15z=27$

(c) $14x+2y-15z=1$ (d) $14x+2y+15z=31$

41. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$\overrightarrow{OP}.\overrightarrow{OQ}+\overrightarrow{OR}.\overrightarrow{OS}=\overrightarrow{OR}.\overrightarrow{OP}+$

$\overrightarrow{OQ}.\overrightarrow{OS}=\overrightarrow{OQ}.\overrightarrow{OR}+\overrightarrow{OP}.\overrightarrow{OS}$

Then the triangle PQR has S as its **(2017)**

(a) incentre (b) circumcentre

(c) orthocenter (d) centroid

Paragraph

Let O be the origin, and $\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overrightarrow{QR}, \overrightarrow{RP}, \overrightarrow{PQ}$, respectively, of a triangle PQR. **(2017)**

42. $|\overrightarrow{OX}\times\overrightarrow{OY}|=$

(a) $\sin(P+R)$ (b) $\sin 2R$

(c) $\sin(P+Q)$ (d) $\sin(Q+R)$

43. If the triangle PQR varies, then the minimum value of $\cos(P+Q)+\cos(Q+R)+\cos(R+P)$

(a) $-\frac{3}{2}$ (b) $\frac{3}{2}$

(c) $\frac{5}{3}$ (d) $-\frac{5}{3}$

Matrix–Match Type Questions

44. Match List–I with List–II and select the correct answer using the code give below the lists.

List–I	List–II
P. Volume of parallelopiped determined by vectors $\vec{a}, \vec{b}$ & $\vec{c}$ is 2. Then, the volume of the parallelopied determined by vectors $2\left(\vec{a}\times\vec{b}\right), 3\left(\vec{b}\times\vec{c}\right)$ and $\left(\vec{c}\times\vec{a}\right)$ is	1. 100
Q. Volume of parallelopiped determined by vectors $\vec{a}, \vec{b}$ & $\vec{c}$ is 5. Then, the volume of the parallelopiped determined by vectors $3\left(\vec{a}+\vec{b}\right), \left(\vec{b}+\vec{c}\right)$ and $2\left(\vec{c}+\vec{a}\right)$ is	2. 30
R. Area of a triangle with adjacent sides determined by vector $\vec{a}$ and $\vec{b}$ is 20. Then, the area of the triangle with adjacent sides determined by vectors $\left(2\vec{a}+3\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$ is	3. 24
S. Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then, the area of the parallelogram with adjacent sides determined by vectors $\left(\vec{a}+\vec{b}\right)$ and $\vec{a}$ is	4. 60

	P	Q	R	S		P	Q	R	S
(a)	4	2	3	1	(b)	2	3	1	4
(c)	3	4	1	2	(d)	1	4	3	2

(2013)

Assertion and Reason

For the following questions choose the correct answer from the codes (a), (b), (c) and (d) defined as follows.

(a) Statement I is true, Statement II is also true; Statement II is the correct explanation of Statement I.

(b) Statement I is true, Statement II is also true; Statement II is not the correct explanation of Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

45. Let the vectors $\overrightarrow{PQ}$, $\overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.

Assertion : $\overrightarrow{PQ}\times(\overrightarrow{RS}+\overrightarrow{ST})\neq\vec{0}$.

Reason : $\overrightarrow{PQ}\times\overrightarrow{RS}=\vec{0}$ and $\overrightarrow{PQ}\times\overrightarrow{ST}\neq\vec{0}$. **(2007)**

(a) A (b) B

(c) C (d) D

Objective Questions II [One or more than one correct option]

46. Let $\vec{a}=2\hat{i}-\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-2\hat{k}$ be three vactor. A vector in the plane of $\vec{b}$ and $\vec{c}$, whose projection on $\vec{a}$ is of magnitude $\sqrt{2/3}$, is **(1993)**

(a) $2\hat{i}+3\hat{j}-3\hat{k}$ (b) $2\hat{i}+3\hat{j}+3\hat{k}$

(c) $-2\hat{i}-\hat{j}+5\hat{k}$ (d) $2\hat{i}+\hat{j}+5\hat{k}$

47. Which of the following expressions are meaningfull question ? **(1998)**

(a) $\vec{u}\cdot(\vec{v}\times\vec{w})$ (b) $(\vec{u}\cdot\vec{v})\cdot\vec{w}$

(c) $(\vec{u}\cdot\vec{v})\,\vec{w}$ (d) $\vec{u}\times(\vec{v}\cdot\vec{w})$

48. Let $\vec{a}$ and $\vec{b}$ be two non–collinear unit vectors. If $\vec{u}=\vec{a}-(\vec{a}\cdot\vec{b})\,\vec{b}$ and $\vec{v}=\vec{a}\times b$, then $|\vec{v}|$ is **(1999)**

(a) $|\vec{u}|$ (b) $|\vec{u}|+|\vec{u}\cdot\vec{a}|$

(c) $|\vec{u}|+|\vec{u}\cdot\vec{b}|$ (d) $|\vec{u}|+\vec{u}\cdot(\vec{a}+\vec{b})$

49. Let $\vec{A}$ be vector parallel to line of intersection of planes P_1 and P_2 through origin. P_1 is parallel to the vectors $2\hat{j}+3\hat{k}$ and $4\hat{j}-3\hat{k}$ and P_2 is parallel to $\hat{j}-\hat{k}$ and $3\hat{i}+3\hat{j}$, then the angle between vector $\vec{A}$ and $2\hat{i}+\hat{j}-2\hat{k}$ is **(2006)**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{3\pi}{4}$

50. The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, are perpendicular to the vectors $\hat{i}+\hat{j}+\hat{k}$ is/are **(2011)**

(a) $\hat{j}-\hat{k}$ (b) $-\hat{i}+\hat{j}$

(c) $\hat{i}-\hat{j}$ (d) $-\hat{j}+\hat{k}$

51. A line l passing through the origin is perpendicular to the lines

$l_1:(3+t)\hat{i}+(-1+2t)\hat{j}+(4+2t)\hat{k}, -\infty<t<\infty$

$l_2:(3+2s)\hat{i}+(3+2s)\hat{j}+(2+s)\hat{k}, -\infty<s<\infty$

Then, the coordinate(s) of the point(s) on l_2 at a distance of $\sqrt{17}$ from the point of intersection of l and l_1 is are **(2013)**

(a) $\left(\frac{7}{3},\frac{7}{3},\frac{5}{3}\right)$ (b) $(-1,-1,0)$

(c) $(1,1,1)$ (d) $\left(\frac{7}{9},\frac{7}{9},\frac{8}{9}\right)$

52. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a nonzero vector perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$ and $\vec{b}$ is a nonzero vector perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$, then **(2014)**

(a) $\vec{b}=(\vec{b}.\vec{z})(\vec{z}-\vec{x})$ (b) $\vec{a}=(\vec{a}.\vec{y})(\vec{y}-\vec{z})$

(c) $\vec{a}.\vec{b}=-(\vec{a}.\vec{y})(\vec{b}.\vec{z})$ (d) $\vec{a}=(\vec{a}.\vec{y})(\vec{z}-\vec{y})$

53. Let ΔPQR be a triangle. Let $\vec{a}=\overrightarrow{QR}$, $\vec{b}=\overrightarrow{RP}$ and $\vec{c}=\overrightarrow{PQ}$. If $|\vec{a}|=12$, $|\vec{b}|=4\sqrt{3}$, $\vec{b}.\vec{c}=24$, then which of the following is (are) true ? **(2015)**

(a) $\frac{|\vec{c}|}{2}-|\vec{a}|=12$ (b) $\frac{|\vec{c}|}{2}+|\vec{a}|=30$

(c) $|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}|=48\sqrt{3}$ (d) $\vec{a}.\vec{b}=-72$

54. Let $\hat{u}=u_1\hat{i}+u_2\hat{j}+u_3\hat{k}$ be a unit vector in $\mathbb{R}^3$ and $\hat{w}=\frac{1}{\sqrt{6}}(\hat{i}+\hat{j}+2\hat{k})$. Given that there exists a vector $\vec{v}$ in $\mathbb{R}^3$ such that $|\hat{u}\times\vec{v}|=1$ and $\hat{w}.|\hat{u}\times\vec{v}|=1$. Which of the following statement(s) is(are) correct ? **(2016)**

(a) There is exactly one choice for such $\vec{v}$

(b) There are infinitely many choice for such

(c) If $\hat{u}$ lies in the xy-plane then $|u_1|=|u_2|$

(d) If $\hat{u}$ lies in the xz-plane then $2|u_1|=|u_3|$

55. Let $P_1: 2x+y-z=3$ and $P_2: x+2y+z=2$ be two planes. Then, which of the following statements(s) is (are) TRUE? **(2018)**

(a) The line of intersection of P_1 and P_2 has direction ratios $1, 2, -1$

(b) The line $\frac{3x-4}{9}=\frac{1-3y}{9}=\frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2

(c) The acute angle between P_1 and P_2 is $60°$

(d) If P_3 is the plane passing through the point $(4, 2, -2)$ and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point $(2, 1, 1)$ from the plane P_3 is $\frac{2}{\sqrt{3}}$.

Integer Answer Type Questions

56. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a}=\frac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\vec{b}=\frac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$, then the value of $\left(2\vec{a}+\vec{b}\right).\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}-2\vec{b}\right)\right]$ is.... **(2010)**

57. If $\vec{a}=-\hat{i}-\hat{k}$, $\vec{b}=-\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}.\vec{a}=0$, then the value of $\vec{r}.\vec{b}$ is..... **(2011)**

58. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is **(2012)**

59. Consider the set of eight vectors $V=\left[a\hat{i}+b\hat{j}+c\hat{k}:a,b,c\in\{-1,1\}\right]$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then, p is **(2013)**

60. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$, where p,q and r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is **(2014)**

Fill in the Blanks.

61. Let $\vec{A},\vec{B},\vec{C}$ be vectors of length 3, 4, 5 respectively. Let $\vec{A}$ be perpendicular to $\vec{B}+\vec{C}$, $\vec{B}$ to $\vec{C}+\vec{A}$ and $\vec{C}$ to $\vec{A}+\vec{B}$. Then the length of vector $\vec{A}+\vec{B}+\vec{C}$ is... **(1981)**

62. A, B, C and D, are four points in a plane with position vectors $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ respectively such that $(\vec{a}-\vec{d}).(\vec{b}-\vec{c})=(\vec{b}-\vec{d}).(\vec{c}-\vec{a})=0$. The point D, then, is the.... of the triangle ABC. **(1984)**

63. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$ and the vectors

$\vec{A}=(1,a,a^2)$, $\vec{B}=(1,b,b^2)$, $\vec{C}=(1,c,c^2)$

are non-coplanar, then the product abc = ... **(1985)**

64. If $\vec{A}$, $\vec{B}$, $\vec{C}$ are three non-coplanar vectors, then

$$\frac{\vec{A}.(\vec{B}\times\vec{C})}{(\vec{C}\times\vec{A}).\vec{B}}+\frac{\vec{B}.(\vec{A}\times\vec{C})}{\vec{C}.(\vec{A}\times\vec{B})}=....$$

65. If $\vec{A}=(1,1,1)$, $\vec{C}=(0,1,-1)$ are given vectors, then a vector $\vec{B}$ satisfying the equations $\vec{A}\times\vec{B}=\vec{C}$ and $\vec{A}.\vec{B}=3$ is ....

66. If the vectors $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ $(a\neq b\neq c\neq 1)$ are coplanar, then the value of

$$\frac{1}{(1-a)}+\frac{1}{(1-b)}+\frac{1}{(1-c)}=.....$$ **(1987)**

67. Let $\vec{b}=4\hat{i}+3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the xy-plane. All vectors in the same plane having projections 1 and 2 along $\vec{b}$ and $\vec{c}$, respectively, are given by .... **(1987)**

68. The components of a vector $\vec{a}$ along and perpendicular to a non-zero vector $\vec{b}$ are ...... and ...... respectively. **(1988)**

69. A unit vector coplanar with $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ and perpendicular to $\hat{i}+\hat{j}+\hat{k}$ is....... **(1992)**

70. A non-zero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i},\hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}$, $\hat{i}+\hat{k}$. The angle between $\vec{a}$ and the vector $\hat{i}-2\hat{j}+2\hat{k}$ is........ **(1996)**

71. If $\vec{b}$ and $\vec{c}$ are any two perpendicular unit vectors and $\vec{a}$ is any vector, then

$$(\vec{a}.\vec{b})\vec{b}+(\vec{a}.\vec{c})\vec{c}+\frac{\vec{a}.(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|}(\vec{b}\times\vec{c})=.......$$ **(1996)**

72. Let $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=10\vec{a}+2\vec{b}$, and $\overrightarrow{OC}=\vec{b}$, where O, A and C are non-collinear points. Let p denotes, the area of the quadrilateral OABC, and let q denotes, the area of the parallelogram with OA and OC as adjacent sides. If p = kq, then k = ... **(1997)**

73. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1, 1 and 2 respectively. If $\vec{a}\times(\vec{a}\times\vec{c})+\vec{b}=\vec{0}$, then the acute angle between $\vec{a}$ and $\vec{c}$ is .... **(1997)**

74. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a}.\vec{b}=0$. For some $x,y\in\mathbb{R}$, let $\vec{c}=x\vec{a}+y\vec{b}+(\vec{a}\times\vec{b})$. If $|\vec{c}|=2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$ and then the value of $8\cos^2\alpha$ is ____. **(2018)**

75. Let P be a point in the first octant, whose image Q in the plane $x+y=3$ (that is, the line segment PQ is perpendicular to the plane $x+y=3$ and the mid-point of PQ lies in the plane $x+y=3$) lies on the z-axis. Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane, then the length of PR is _____. **(2018)**

76. Consider the cube in the first octant with sides OP, OQ and OR of length 1, along the x-axis, y-axis and z-axis, respectively, where O(0, 0, 0) is the origin. Let $S\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT. If, $\vec{p}=\overrightarrow{SP}$, $\vec{q}=\overrightarrow{SQ}$, $\vec{r}=\overrightarrow{SR}$ and $\vec{t}=\overrightarrow{ST}$, then the value of $|(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})|$ is ____ **(2018)**

True/False.

77. Let $\vec{A}, \vec{B}$ and $\vec{C}$ be unit vectors. Suppose that $\vec{A}.\vec{B} = \vec{A}.\vec{C} = 0$, and the angle between $\vec{B}$ and $\vec{C}$ is $\pi/6$. Then $\vec{A} = \pm 2\left(\vec{B} \times \vec{C}\right)$. **(1981)**

78. If $\vec{X}.\vec{A} = 0, \vec{X}.\vec{B} = 0, \vec{X}.\vec{C} = 0$ for some non-zero vector $\vec{X}$, then $\left[\vec{A}\vec{B}\vec{C}\right] = 0$. **(1983)**

79. The points with position vectors $\vec{a}+\vec{b}, \vec{a}-\vec{b}$ and $\vec{a}+k\vec{b}$ are collinear for all real values of k. **(1984)**

80. For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$,

$$\left(\vec{a}-\vec{b}\right).\left\{\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)\right\} = 2\vec{a}.\left(\vec{b}\times\vec{c}\right)$$ **(1989)**

Analytical and Descriptive Questions.

81. $A_1, A_2, \ldots\ldots, A_n$ are the vertices of a regular plane polygon with n sides and O is its centre. Show that

$$\sum_{i=1}^{n-1}\left(\vec{OA}_i \times \vec{OA}_{i+1}\right) = (1-n)\left(\vec{OA}_2 \times \vec{OA}_1\right)$$ **(1982)**

82. Find all the values of λ such that x, y, z $\neq$ (0, 0, 0) and $\left(\hat{i}+\hat{j}+3\hat{k}\right)x+\left(3\hat{i}-3\hat{j}+\hat{k}\right)y+\left(-4\hat{i}+5\hat{j}\right)z=\lambda\left(\hat{i}x+\hat{j}y+\hat{k}z\right)$ where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the coordinate axes. **(1982)**

83. (a) If c be a given non-zero scalar and $\vec{A}$ and $\vec{B}$ be given non-zero vectors such that $\vec{A} \perp \vec{B}$, find the vector $\vec{X}$ which satisfies the equation $\vec{A}.\vec{X}=c$ and $\vec{A}\times\vec{X}=\vec{B}$. **(1983)**

(b) $\vec{A}$ vector A has components A_1, A_2, A_3 in a right-handed rectangular cartesian coordinate system oxyz. The coordinate system is rotated about the z-axis through an angle $\frac{\pi}{2}$. Find the components of A in the new coordinate system, in terms of A_1, A_2, A_3. **(1983)**

84. The position vectors of the points A, B, C and D are $3\hat{i}-2\hat{j}-\hat{k}$, $2\hat{i}+3\hat{j}-4\hat{k}$, $-\hat{i}+\hat{j}+2\hat{k}$ and $4\hat{i}+5\hat{j}+\lambda\hat{k}$, respectively. If the points A, B, C and D lie on a plane, find the value of λ. **(1986)**

85. If A, B, C, D are any four points in space, prove that

$$\left|\vec{AB}\times\vec{CD}+\vec{BC}\times\vec{AD}+\vec{CA}\times\vec{BD}\right| = 4 \text{ (area of } \Delta ABC).$$ **(1987)**

86. Let OACB be a parallelogram with O at the origin and OC a diagonal. Le D be the mid point of OA. Using vector methods prove that BD and CO intersect in the same ratio. Determine this ratio. **(1988)**

87. If vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, show that

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \end{vmatrix} = \vec{0}$$ **(1989)**

88. In a triangle OAB, E is the mid point of BO and D is a point on AB such that AD : DB = 2 : 1. If OD and AE intersect at P, determine the ratio OP : PD using vector methods. **(1989)**

89. Let $\vec{A}=2\hat{i}+\hat{k}$, $\vec{B}=\hat{i}+\hat{j}+\hat{k}$, and $\vec{C}=4\hat{i}-3\hat{j}+7\hat{k}$. Determine a vector $\vec{R}$ satisfying $\vec{R}\times\vec{B} = \vec{C}\times\vec{B}$ and $\vec{R}.\vec{A} = 0$. **(1990)**

90. Determine the value of 'c' so that for all real x, the vector $cx\hat{i}-6\hat{j}-3\hat{k}$ and $x\hat{i}+2\hat{j}+2cx\hat{k}$ make an obtuse angle with each other. **(1991)**

91. In a triangle ABC, D and E are points on BC and AC respectively, such that BD = 2DC and AE = 3 EC. Let P be the point of intersection of AD and BE. Find BP/PE using vector methods. **(1993)**

92. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non-coplanar unit vectors, equally inclined to one another at an angle θ. If $\vec{a}\times\vec{b} + \vec{b}\times\vec{c} = p\vec{a}+q\vec{b}+r\vec{c}$, find scalars p, q and r in terms of θ. **(1997)**

93. Prove, by vector methods or otherwise, that the point of intersection of the diagonals of a trapezium lies on the line passing through the mid points of the parallel sides. (You may assume that the trapezium is not a parallelogram). **(1998)**

94. If the vectors $\vec{b}, \vec{c}, \vec{d}$, are not coplanar, then prove that the vector $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)+\left(\vec{a}\times\vec{c}\right)\times\left(\vec{d}\times\vec{b}\right)$ $+\left(\vec{a}\times\vec{d}\right)\times\left(\vec{b}\times\vec{c}\right)$ is parallel to $\vec{a}$. **(1994)**

95. The position vectors of the vertices A, B and C of a tetrahedron ABCD are $\hat{i}+\hat{j}+\hat{k}$, $\hat{i}$ and $3\hat{i}$, respectively. The altitude from vertex D to the opposite face ABC meets the median line through A of the triangle ABC at a point E. If the length of the side AD is 4 and the volume of the tetrahedron is $\frac{2\sqrt{2}}{3}$, find the position vector of the point E for all its possible positions. **(1996)**

96. If $\vec{A}, \vec{B}$ and $\vec{C}$ are vectors such that $|\vec{B}| = |\vec{C}|$. Prove that

$$\left[\left(\vec{A}+\vec{B}\right)\times\left(\vec{A}+\vec{C}\right)\right]\times\left(\vec{B}\times\vec{C}\right).\left(\vec{B}+\vec{C}\right)=\vec{0}.$$ **(1997)**

97. For any two vectors $\vec{u}$ and $\vec{v}$ prove that

(a) $|\vec{u}.\vec{v}|^2 + |\vec{u}\times\vec{v}|^2 = |\vec{u}|^2|\vec{v}|^2$ and

(b) $\left(1+|\vec{u}|^2\right)\left(1+|\vec{v}|^2\right)$

$= |1-\vec{u}.\vec{v}|^2 + \left|\vec{u}+\vec{v}+\left(\vec{u}\times\vec{v}\right)\right|^2$ **(1998)**

98. Let $\vec{u}$ and $\vec{v}$ be unit vectors. If $\vec{w}$ is a vector such that $\vec{w}+\left(\vec{w}\times\vec{u}\right)=\vec{v}$, then prove that $\left|\left(\vec{u}\times\vec{v}\right).\vec{w}\right| \le \frac{1}{2}$ and that the equality holds if and only if $\vec{u}$ is perpendicular to $\vec{v}$. **(1999)**

99. Let ABC and PQR be any two triangles in the same plane. Assume that the perpendiculars from the points A, B, C to the sides QR, RP, PQ respectively are concurrent. Using vector methods or otherwise, prove that the perpendiculars from P, Q, R to BC, CA, AB respectively are also concurrent. **(2000)**

100. Show, by vector methods, that the angular bisectors of a triangle are concurrent and find an expression for the position vector of the point of concurrency in terms of the position vectors of the vertices. **(2001)**

101. Find 3–dimensional vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ satisfying

$\vec{v}_1\cdot\vec{v}_1 = 4, \vec{v}_1\cdot\vec{v}_2 = -2, \vec{v}_1\cdot\vec{v}_3 = 6,$

$\vec{v}_2\cdot\vec{v}_2 = 2, \vec{v}_2\cdot\vec{v}_3 = -5, \vec{v}_3\cdot\vec{v}_3 = 29$ **(2001)**

102. Let $\vec{A}(t) = f_1(t)\,\hat{i} + f_2(t)\,\hat{j}$ and

$\vec{B}(t) = g_1(t)\,\hat{i} + g_2(t)\,\hat{j}, t \in [0, 1]$, where

f_1, f_2, g_1, g_2 are continuous functions. If $\vec{A}(t)$ and $\vec{B}(t)$ are non–zero vectors

for all t and $\vec{A}(0) = 2\hat{i} + 3\hat{j}$, $\vec{A}(1) = 6\hat{i} + 2\hat{j}$,

$\vec{B}(0) = 3\hat{i} + 2\hat{j}$ and $\vec{B}(1) = 2\hat{i} + 6\hat{j}$

Then show that $\vec{A}(t)$ and $\vec{B}(t)$ are parallel for some t. **(2001)**

103. Let V be the volume of the parallelopiped formed by the vectors

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$.

If a_r, b_r, c_r, where r = 1, 2, 3 are non–negative real numbers and $\sum_{r=1}^{3}(a_r + b_r + c_r) = 3L$. Show that $V \le L^3$. **(2002)**

104. If $\vec{u}, \vec{v}, \vec{w}$ are three non–coplanar unit vectors and α, β, γ are the angles between $\vec{u}$ and $\vec{v}$, $\vec{v}$ and $\vec{w}$, $\vec{w}$ and $\vec{u}$ respectively and $\vec{x}, \vec{y}, \vec{z}$ are unit vectors along the bisectors of the angles α, β, γ respectively. Prove that

$[\vec{x}\times\vec{y}\ \ \vec{y}\times\vec{z}\ \ \vec{z}\times\vec{x}]$

$= \frac{1}{16}[\vec{u}\ \vec{v}\ \vec{w}]^2 \sec^2\frac{\alpha}{2}\sec^2\frac{\beta}{2}\sec^2\frac{\gamma}{2}$. **(2003)**

105. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are four distinct vectors satisfying the conditions $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$, then prove that $\vec{a}.\vec{b} + \vec{c}.\vec{d} \ne \vec{a}.\vec{c} + \vec{b}.\vec{d}$. **(2004)**

106. Incident ray is along the unit vector $\hat{v}$ and the reflected ray is along the unit vector $\hat{w}$. The normal is along unit vector $\hat{a}$ outwards. Express $\hat{w}$ in terms of $\hat{a}$ and $\hat{v}$. **(2005)**

Objective Question I [Only one correct option]

107. The value of k such that $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane $2x - 4y + z = 7$, is **(2003)**

(a) 7 (b) –7

(c) No real value (d) 4

108. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then the value of k is **(2004)**

(a) $\frac{3}{2}$ (b) $\frac{9}{2}$

(c) $-\frac{2}{9}$ (d) $-\frac{3}{2}$

109. A variable plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ at a unit distance from origin cuts the coordinate axes at A, B and C. Centroid (x, y, z) satisfies the equation $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = K$. Then value of K is **(2005)**

(a) 9 (b) 3

(c) 1/9 (d) 1/3

110. A plane passes through (1, –2, 1) and is perpendicular to two planes 2x – 2y + z = 0 and x – y + 2 z = 4, then the distance of the plane from the point (1, 2, 2) is **(2006)**

(a) 0 (b) 1

(c) $\sqrt{2}$ (d) $2\sqrt{2}$

111. Let P (3, 2, 6) be a point in space and Q be a point on the line $\vec{r}=\left(\hat{i}-\hat{j}+2\hat{k}\right)+\mu\left(-3\hat{i}+\hat{j}+5\hat{k}\right)$. Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x-4y+3z=1$ is **(2009)**

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

112. A line with positive direction cosines passes through the point P(2, –1, 2) and makes equal angles with the coordinate axes. The line meets the plane 2x + y + z = 9 at point Q. The length of the line segment PQ equals **(2009)**

(a) 1 (b) $\sqrt{2}$

(c) $\sqrt{3}$ (d) 2

113. The point P is the intersection of the straight line joining the points Q(2, 3, 5) and R(1, –1, 4) with the plane $5x-4y-z=1$. If S is the foot of the perpendicular drawn from the point T (2, 1, 4) to QR, then the length of the line segment PS is **(2012)**

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) 2 (d) $2\sqrt{2}$

114. The equation of a plane passing through the line of intersection of the planes x + 2y + 3z = 2 & x – y + z = 3 & at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, –1) is **(2012)**

(a) $5x-11y+z=17$ (b) $\sqrt{2}x+y=3\sqrt{2}-1$

(c) $x+y+z=\sqrt{3}$ (d) $x-\sqrt{2}y=1-\sqrt{2}$

115. Perpendicular are drawn from points on the line $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z}{3}$ to the plane x + y + z = 3. The feet of perpendiculars lie on the line. **(2013)**

(a) $\frac{x}{5}=\frac{y-1}{8}=\frac{z-2}{-13}$ (b) $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-5}$

(c) $\frac{x}{4}=\frac{y-1}{3}=\frac{z-2}{-7}$ (d) $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

116. From a point $P(\lambda,\lambda,\lambda)$, perpendicular PQ and PR are drawn respectively on the lines y = x, z = 1 and y = – x, z = – 1. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is(are) **(2014)**

(a) $\sqrt{2}$ (b) 1

(c) –1 (d) $-\sqrt{2}$

117. Let P be the image of the point (3, 1, 7) with respect to the plane $x-y+z=3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1}=\frac{y}{2}=\frac{z}{1}$ is **(2016)**

(a) $x+y-3z=0$ (b) $3x+z=0$

(c) $x-4y+7z=0$ (d) $2x-y=0$

Objective Questions II [One or more than one correct option]

118. If the straight lines $\frac{x-1}{2}=\frac{y+1}{K}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{K}$ are coplanar, then the plane(s) containing these two lines is/are **(2012)**

(a) $y+2z=-1$ (b) $y+z=-1$

(c) $y-z=-1$ (d) $y-2z=-1$

119. Two lines $L_1: x=5, \frac{y}{3-\alpha}=\frac{z}{-2}$ and $L_2: x=\alpha, \frac{y}{-1}=\frac{z}{2-\alpha}$ are coplanar. Then, α can take value(s). **(2013)**

(a) 1 (b) 2

(c) 3 (d) 4

120. In R^3, consider the planes $P_1 : y = 0$ and $P_2 : x + z = 1$. Let P_3 be the plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0, 1, 0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, the which of the following relations is (are) true ? **(2015)**

(a) $2\alpha + \beta + 2\gamma + 2 = 0$ (b) $2\alpha - \beta + 2\gamma + 4 = 0$

(c) $2\alpha + \beta - 2\gamma - 10 = 0$ (d) $2\alpha - \beta + 2\gamma - 8 = 0$

121. In R^3, let L be a straight line passing through the origin. Suppose that all the points on L at a constant distance from the two planes $P_1 : x + 2y - z + 1 = 0$ & $P_2 : 2x - y + z - 1 = 0$. Let M be the locus of the foot of the perpendiculars drawn from the points on L to plane P_1. Which of the following point(s) lie(s) on M. **(2015)**

(a) $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$ (b) $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$

(c) $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$ (d) $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

122. Consider a pyramid OPQRS located in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ with O as origin, and OP and OR along the x-axis and the y-axis, respectively. The base OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid-point T of diagonal OQ such that TS =3. Then **(2016)**

(a) the acute angle between OQ and OS is $\frac{\pi}{3}$

(b) the equation of the plane containing the triangle OQS is $x - y = 0$

(c) the length of the perpendicular fr om P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$

(d) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

Assertion and Reason

For the following questions choose the correct answer from the codes (a), (b), (c) and (d) defined as follows.

(a) Statement I is true, Statement II is also true; Statement II is the correct explanation of Statement I.

(b) Statement I is true, Statement II is also true; Statement II is not the correct explanation of Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

123. Consider the planes

$3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.

Assertion : The parametric equations of the line of intersection of the given planes are $x = 3 + 14t$, $y = 1 - 2t$, $z = 15t$.

Because

Reason : The vectors $14\hat{i} + 2\hat{j} + 15\hat{k}$ is parallel to the line of intersection of the given planes. **(2007)**

(a) A (b) B

(c) C (d) D

124. Consider three planes

$P_1 : x - y + z = 1$

$P_2 : x + y - z = 1$ and $P_3 : x - 3y + 3z = 2$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, P_1 and P_2, respectively.

Assertion : At least two of the lines L_1, L_2 and L_3 are non-parallel.

Reason : The three planes do not have a common point. **(2008)**

(a) A (b) B

(c) C (d) D

Passage Based Problem

Read the following passage and answer the questions.

Consider the lines

$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$, $L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$ **(2008)**

125. The unit vector perpendicular to both L_1 and L_2 is

(a) $\frac{-\hat{i} + 7\hat{j} + 7\hat{k}}{\sqrt{99}}$ (b) $\frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$

(c) $\frac{-\hat{i} + 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$ (d) $\frac{7\hat{i} - 7\hat{j} - \hat{k}}{\sqrt{99}}$

126. The shortest distance between L_1 and L_2 is

(a) 0 unit (b) $17/\sqrt{3}$ unit

(c) $41/5\sqrt{3}$ unit (d) $17/5\sqrt{3}$ unit

127. The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is

(a) $2/\sqrt{75}$ unit (b) $7/\sqrt{75}$ unit

(c) $13/\sqrt{75}$ unit (d) $23/\sqrt{75}$ unit

Matrix–Match Type Questions

Match the conditions/expressions in Column I with statement in Column II.

128. Consider the following linear equations

$ax + by + cz = 0,$

$bx + cy + az = 0,$

$cx + ay + bz = 0$ **(2007)**

Column–I	**Column–II**
(A) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(p) the equations represent planes meeting only at a single point
(B) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(q) the equations represent the lines $x = y = z$
(C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(r) the equations represent identical planes
(D) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(s) the equations represent the whole of the three dimensional space.

129. Consider the lines

$L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$, $L_2 : \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3$, $P_2 : 3x + 5y - 6z = 4$. Let $ax + by + cz = d$ the equation of the plane passing through the point of intersection of lines L_1 and L_2 and perpendicular to planes P_1 and P_2.

Match List–I with List–II and select the correct answer using the code give below the lists.

List–I	**List–II**
P. a =	1. 13
Q. b =	2. –3
R. c =	3. 1
S. d =	4. –2

	P	Q	R	S		P	Q	R	S
(a)	3	2	4	1	(b)	1	3	4	2
(c)	3	2	1	4	(d)	2	4	1	3

(2013)

130. Match the following. **(2015)**

Column–I	**Column–II**
(A) In a triangle ΔXYZ, let a, b, and c be the lengths of the sides opposite to the angles X, Y and Z, respectively. If $2(a^2 - b^2) = c^2$ and $\lambda = \frac{\sin(X-Y)}{\sin Z}$, then possible value of n for which $\cos(n\pi\lambda) = 0$ is	(P) 1
(B) In a triangle ΔXYZ, let a, b (are) and c be the lengths of the sides opposite to the angles X, Y and Z respectively. If $1 + \cos 2X - 2\cos 2Y = 2 \sin X \sin Y$, then possible value(s) of $\frac{a}{b}$ is are	(Q) 2
(C) In R^2, let $\sqrt{3}\hat{i} + \hat{j}$, $\hat{i} + \sqrt{3}\hat{j}$ and $\beta\hat{i} + (1-\beta)\hat{j}$ be the position vectors X, Y and Z with respect to the origin O, respectively. If the distance of Z from the bisector of the acute angle of $\overrightarrow{OX}$ with $\overrightarrow{OY}$ is $\frac{3}{\sqrt{2}}$, the possible value(s) of $\lvert\beta\rvert$ is(are)	(R) 3
(D) Suppose that $F(\alpha)$ denotes the area of the region bounded by $x = 0, x = 2, y^2 = 4x$ and $y = \lvert ax - 1\rvert + \lvert \alpha x - 2\rvert + ax$, where $\alpha \in \{0, 1\}$. Then the value(s) of $F(\alpha) + \frac{8}{3}\sqrt{2}$, when $\alpha = 0$ and $\alpha = 1$, is (are)	(S) 5
	(T) 6

Fill in the Blanks.

131. The area of the triangle whose vertices are A (1, –1, 2), B(2, 1, –1), C(3, –1, 2) is ...... **(1983)**

132. The unit vector perpendicular to the plane determined by P(1, –1, 2), Q(2, 0, –1) & R(0, 2, 1) is ...... **(1983)**

Analytical and Descriptive Questions.

133. (a) Find the equation of the plane passing through the points (2, 1, 0), (5, 0, 1) and (4, 1, 1).

(b) If P is the point (2, 1, 6), then the point Q such that PQ is perpendicular to the plane in (a) and the mid point of PQ lies on it. **(2003)**

134. T is a parallelopiped in which A, B, C and D are vertices of one face and the face just above it has corresponding vertices A′, B′, C′, D′, T is now compressed to S with face ABCD remaining same and A′, B′, C′, D′ shifted to A″, B″, C″, D″ in S. The volume of parallelopiped S is reduced to 90% of T. Prove that locus of A″ is a plane.

(2004)

135. A plane is parallel to two lines whose direction ratios are (1, 0, –1) & (–1, 1, 0) and it contains the point (1, 1, 1,). If it cuts coordinate axes at A, B, C. Then find the volume of the tetrahedron OABC. **(2004)**

136. Find the equation of the plane containing the line $2x - y + z - 3 = 0, 3x + y + z = 5$ and at a distance of $\frac{1}{\sqrt{6}}$ from the point (2, 1, –1). **(2005)**

137. Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is **(2014)**

ANSWER KEY

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (c)	2. (b)	3. (b)	4. (d)	5. (c)	6. (b)	7. (a)	8. (b)	9. (c)	10. (b)
11. (b)	12. (c)	13. (d)	14. (a)	15. (a)	16. (d)	17. (c)	18. (c)	19. (a)	20. (a)
21. (a)	22. (c)	23. (a)	24. (b)	25. (b)	26. (d)	27. (b)	28. (b)	29. (a)	30. (a)
31. (c)	32. (a)	33. (b)	34. (c)	35. (d)	36. (c)	37. (a)	38. (b)	39. (a)	40. (a)
41. (b)	42. (a)	43. (c)	44. (d)	45. (a)	46. (d)	47. (b,c)	48. (c)	49. (b)	50. (b)
51. (d)	52. (a)	53. (b)	54. (a)	55. (d)	56. (c)	57. (c)	58. (d)	59. (a)	60. (a)
61. (a)	62. (a)	63. (d)	64. (b)	65. (c)	66. (d)	67. (d)	68. (b)	69. (b)	70. (c)
71. (c)	72. (d)	73. (a)	74. (b)	75. (a)	76. (c)	77. (c)	78. (d)	79. (c)	80. (b)
81. (b)	82. (a)	83. (a)	84. (d)	85. (a)	86. (b)	87. (a)	88. (a)	89. (d)	90. (b)
91. (a)	92. (a)	93. (d)	94. (c)	95. (c)	96. (d)	97. (d)	98. (c)	99. (b)	100. (a)
101. (d)	102. (d)	103. (b)	104. (d)	105. (d)	106. (a)	107. (a)	108. (b)	109. (d)	110. (a)
111. (a)	112. (b)	113. (b)	114. (b)	115. (d)	116. (d)	117. (a)	118. (d)	119. (a)	120. (a)

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. (c)	2. (a)	3. (b)	4. (a)	5. (a)	6. (b)	7. (b)	8. (d)	9. (c)	10. (b)
11. (d)	12. (d)	13. (a)	14. (c)	15. (c)	16. (d)	17. (d)	18. (c)	19. (b)	20. (a)
21. (d)	22. (c)	23. (c)	24. (d)	25. (d)	26. (d)	27. (a)	28. (a)	29. (d)	30. (a)
31. (a)	32. (d)	33. (c)	34. (a)	35. (d)	36. (c)	37. (b)	38. (c)	39. (a)	40. (c)
41. (c)	42. (b)	43. (c)	44. (c)	45. (c)	46. (a)	47. (c)	48. (d)	49. (b)	50. (b)
51. (b)	52. (b)	53. (a)	54. (c)	55. (a)	56. (b)	57. (b)	58. (c)	59. (c)	60. (c)
61. (d)	62. (c)	63. (c)	64. (b)	65. (a)	66. (d)	67. (b)	68. (d)	69. (c)	70. (c)
71. (d)	72. (d)	73. (a)	74. (d)	75. (a)	76. (d)	77. (b)	78. (b)	79. (c)	80. (c)
81. (b)	82. (d)	83. (a)	84. (d)	85. (b)	86. (a)	87. (c)	88. (c)	89. (c)	90. (b)
91. (b)	92. (c)	93. (c)	94. (b)	95. (b)	96. (c)	97. (b)	98. (c)	99. (a)	100. (b)
101. (c)	102. (a)	103. (b)	104. (d)	105. (c)	106. (a)	107. (c)	108. (a)	109. (d)	110. (b)
111. (c)	112. (d)	113. (c)	114. (a)	115. (a)	116. (c)	117. (d)	118. (a)	119. (c)	120. (c)
121. (a)	122. (b)	123. (a)							

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (c)	2. (c)	3. (b)	4. (a, d)	5. (a, c, d)	6. (a, c)	7. (d)	8. (c)	9. (d)	10. (b)
11. (a, b, c)	12. (a)	13. (a)	14. (c)	15. (a)	16. (a)	17. (a)	18. (a)	19. (a)	20. (a)
21. (c)	22. (a,c,d)	23. (c)	24. (b)	25. (a, b)	26. (c)	27. (a,b,c)	28. (c)	29. (c)	30. (b)
31. (a)	32. (b)	33. (a,b)	34. (c)	35. (a)	36. (c)	37. (b)	38. (b)	39. (a,c)	40. (a)
41. (d)	42. (d)	43. (a)	44. (d)	45. (d)	46. (c)	47. (c)	48. (b)	49. (b)	50. (a)
51. (c)	52. (c)	53. (b)	54. (b)	55. (b)	56. (d)	57. (a)	58. (a)	59. (a, c)	60. (c,d)
61. (a, c, d)	62. (a, b)	63. (b)	64. (b)	65. (a)	66. (d)	67. (d)	68. (a)	69. (a)	70. (c)
71. (b)	72. (b)	73. (c)	74. (a)	75. (A – S; B – P; C – Q; D – R)			76. (A– R; B– P; C– S; D– Q)		
77. 0013	78. 1710								

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (a)	2. (d)	3. (b)	4. (a)	5. (c)	6. (b)	7. (b)	8. (b)	9. (a)	10. (a)
11. (b)	12. (d)	13. (b)	14. (d)	15. (c)	16. (b)	17. (a)	18. (b)	19. (a)	20. (a)
21. (b)	22. (c)	23. (b)	24. (c)	25. (c)	26. (c)	27. (c)	28. (b)	29. (a)	30. (c)
31. (b)	32. (a)	33. (a)	34. (c)	35. (a)	36. (b)	37. (c)	38. (c)	39. (c)	40. (d)
41. (d)	42. (c)	43. (a)	44. (c)	45. (c)	46. (a,c)	47. (a,c)	48. (a,c)	49. (b,d)	50. (a,d)
51. (b,d)	52. (a,b,c)	53. (a, c, d)	54. (b,c)	55. (c,d)	56. 5	57. 9	58. 3	59. 5	60. (4)

61. $5\sqrt{2}$ 62. Orthocentre 63. –1 64. 0 65. $\left(\frac{5}{3},\frac{2}{3},\frac{2}{3}\right)$ 66. 1

67. $\left(2\hat{i}-\hat{j}\right)$, $-\frac{2}{5}\hat{i}+\frac{11}{5}\hat{j}$ 68. $\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$ and $\vec{a}-\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$ 69. $\pm\frac{1}{\sqrt{2}}.\left(-\hat{j}+\hat{k}\right)$ 70. $\frac{\pi}{4}$ or $\frac{3\pi}{4}$

71. $\vec{a}$ 72. 6 73. $\frac{\pi}{6}$ 74. (3) 75. (8) 76. (0.50) 77. True 78. True 79. True 80. False

82. 0, –1 83. (a) $\vec{X}=\left(\frac{c}{|\vec{A}|^2}\right)\vec{A}-\left(\frac{1}{|\vec{A}|^2}\right)\left(\vec{A}\times\vec{B}\right)$ (b) $\left(A_2\hat{i}-A_1\hat{j}+A_3\hat{k}\right)$ 84. $\frac{146}{17}$ 86. 2 : 1 88. 3 : 2

89. $-\hat{i}-8\hat{j}+2\hat{k}$ 90. $\left(-\frac{4}{3},0\right)$ 91. $\frac{8}{3}$ 92. $p=r=\frac{1}{\sqrt{1+2\cos\theta}}$, $q=\frac{-2\cos\theta}{\sqrt{1+2\cos\theta}}$

95. $-\hat{i}+3\hat{j}+3\hat{k}$ and $3\hat{i}-\hat{j}-\hat{k}$ 101. $\vec{v}_1=2\hat{i}$, $\vec{v}_2=-\hat{i}+\hat{j}$ and $\vec{v}_3=3\hat{i}-2\hat{j}\pm4\hat{k}$

106. $\hat{w}=\hat{v}-2(\hat{a}.\hat{v})\hat{a}$ 107. (a) 108. (b) 109. (a) 110. (d) 111. (a) 112. (c) 113. (a) 114. (a)

115. (d) 116. (c) 117. (c) 118. (b, c) 119. (a,d) 120. (b, d) 121. (a, b) 122. (b,c,d) 123. (d) 124. (d)

125. (b) 126. (d) 127. (c) 128. (A–r, B–q; C–p; D–s) 129. (a) 130. (A$\rightarrow$ P, R, S; B$\rightarrow$ P; C$\rightarrow$ P,Q; D$\rightarrow$ S,T)

131. $\sqrt{13}$ sq.unit 132. $\pm\frac{\left(2\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{6}}$ 133. (a) $x+y-2z=3$, (b) Q (6, 5, –2)

135. $\frac{9}{2}$ cu unit 136. $2x-y+z-3=0$ and $62x+29y+19z-105=0$ 137. (5)

Dream on !!