2. Algebraic Expressions

Exercise 2.1

1. Question

Separate the constants and variables from the following:

$$12 + z, 15, \frac{-x}{5}, \frac{-3}{7}, x, 3, \frac{2}{3}xy, \frac{5xy}{2}, 7,$$

$$7 - x, 6x + 4y, -7z, \frac{8yz}{3x}, y + 4, \frac{y}{4} \text{ and } \frac{2x}{8yz}.$$

Answer

In the given question

The symbols which has a fixed value are constant such as 15, 3, –3/7 and 7

The symbols which do not have any fixed value, but may be assigned the value (values) according to the requirement are called the variables such as

$$12 + z, -\frac{x}{5}, x, \frac{2}{3}xy, \frac{5xy}{2}, 7 - x, 6x + 4y, -7z, \frac{8yz}{3x}, y + 4, \frac{y}{4} \text{ and } \frac{2x}{4yz}$$

Where z, x and y are variables which may acquire different values according to the situation.

And a combination of a constant and a variable is a variable.

2. Question

Separate the monomials, binomials and trinomials from the following:

$7xyz, 9 - 4y, 4y^2 - xz, x - 2y + 3z, 7x + z^2, 8xy, \frac{8}{5}x^2y^2, 4 + 5y - 6z.$

Answer

A polynomial which contains only one term is monomials

Such as $7xyz, 8xy \text{ and } \left(\frac{8}{5}\right)x^2y^2$

A Polynomial which contains two terms is called a Polynomial.

Such as $9–4y, 4y^2 - xz, 7x + z^2$

A polynomial which contains three terms is called as trinomial

Such as x –2y +3z, 4+5y –6z

Exercise 2.2

1. Question

Classify into like terms:

$4x^2$, $\frac{1}{3}$ x, $-8x^3$, xy, $6x^3$, 4y, $-74x^3$, 8xy, 7xyz, $3x^2$.

Answer

The terms having same variable with same exponents are called like terms

Here like terms are

($4x^2$,$3x^2$) with same variable x and exponential power as 2

($\frac{1}{3}$x has no like term with x as variable and exponential power as 1)

($-8x^3$, $6x^3$, $-74x^3$ with variable as x and exponential power as 3)

7xyz has no like term as no other term has three variables x, y, z

(xy, 8xy has same variables x, y and exponential power of both x and y as 1)

2. Question

Simplify:

(i) 7x – 9y + 3 – 3x – 5y + 8;

(ii) $3x^2 + 5xy – 4y^2 + x^2 – 8xy – 5y^2$.

Answer

While adding or subtracting, the like term`s numerical coefficient are added or subtracted

(i) 7x – 9y + 3 – 3x – 5y + 8

⇒ (7– 3)x +(–9 –5)y +3+8

⇒ 4x – 14y +11

(ii) $3x^2 + 5xy – 4y^2 + x^2 – 8xy – 5y^2$

⇒ (3+1) x^2 +(–4 –5) y^2 +(5–8) x y

⇒ $4x^2 – 9y^2 – 3xy$

3. Question

Add:

(i) 5a + 3b, a – 2b and 3a + 5b;

(ii) $x^3 - x^2y + 5xy^2 + y$, $-x^3 - 9xy^2 + y$, and $3x^2y + 9xy^2$.

Answer

(i) While adding or subtracting, the like term`s numerical coefficient are added or subtracted

⇒ (5 + 1+ 3)a + (3 – 2 +5) b

⇒ 9 a + 6b

(ii) While adding or subtracting, the like term`s numerical coefficient are added or subtracted

⇒ (1–1) x^3 + (–1+3) x^2y + (5–9 +9) xy^2 + (1+1) y

⇒ $0x^3$ + 2 x^2y +$5xy^2$ +2y

⇒ 2 x^2y +$5xy^2$ +2y

4. Question

Subtract:

(i) $-2xy + 3xy^2$ from 8xy;

(ii) a – b – 2c from 4a + 6b – 2c.

Answer

Given problem asks to subtract

(i) $-2xy + 3xy^2$ from 8xy

⇒ 8xy – ($-2xy + 3xy^2$)

⇒ 8xy +2xy $-3xy^2$

⇒ 10xy $-3xy^2$

(ii) a – b – 2c from 4a + 6b – 2c

⇒ 4a +6b –2c – (a– b – 2c)

⇒ 4a +6b –2c –a + 2c +b

While adding or subtracting, the like term`s numerical coefficient are added or subtracted

⇒ (4–1) a + (6 +1) b +(–2 +2) c

⇒ 3a +7b

Exercise 2.3

1. Question

Complete the following table of products of two monomials:

First → Second ↓	3x	−6y	$4x^2$	−8xy	$9x^2y$	$-11x^3y^2$
3x						
−6y						
$4x^2$						
−8xy						
$9x^2y$						
$-11x^3y^2$						

Answer

To find out product of monomials

We multiple the numerical coefficient together and variables together

The given table hence can be completed as :

First → Second ↓	3x	−6y	$4x^2$	−8xy	$9x^2y$	$-11x^3y^2$
3x	$3\times3\times x = 9x^2$	$-6y\times 3x = -18xy$	$3x\times 4x^2 = 12x^3$	$3x\times-8xy = -24x^2y$	$3x\times 9x^2y = 27x^3y$	$3x\times -11x^3y^2 = -33x^4y^2$
−6y	$-6\times3xy = -18xy$	$-6y\times-6y = 36y^2$	$-6y\times 4x^2 = -24x^2y$	$-6y\times-8xy = 48xy^2$	$-6y\times 9x^2y = -54x^2y^2$	$-6y\times -11x^3y^2 = 66x^3y^3$
$4x^2$	$4\times 3x^2\times = 12x^3$	$4x^2\times-6y = -24x^2y$	$4x^2\times 4x^2 = 16x^4$	$4x^2\times-8xy = -32x^3y$	$4x^2\times 9x^2y = 36x^4y$	$4x^2\times -11x^3y^2 = -44x^5y^2$
−8xy	$-8\times3xy\times = -24x^2y$	$-8xy\times -6y = 48xy^2$	$-8xy\times 4x^2 = -32x^3y$	$-8xy\times-8xy = 64x^2y^2$	$-8xy\times 9x^2y = -72x^3y^2$	$-8xy\times -11x^3y^2 = 88x^4y^3$
$9x^2y$	$9\times3x^2xy = 27x^3y$	$9x^2y\times -6y = -54x^2y^2$	$9x^2y\times 4x^2 = 36x^4y$	$9x^2y\times-8xy = -72x^3y^2$	$9x^2y\times 9x^2y = 81x^4y^2$	$9x^2y\times -11x^3y^2 = -99x^5y^3$
$-11x^3y^2$	$-11\times3x^3y^2x = -33x^4y^2$	$-11x^3y^2\times-6y = 66x^3y^3$	$-11x^3y^2\times 4x^2 = -44x^5y^2$	$-11x^3y^2\times-8xy = 88x^4y^3$	$-11x^3y^2\times 9x^2y = -99x^5y^3$	$-11x^3y^2\times-11x^3y^2 = 121x^6y^4$

2. Question

Find the products:

(i) $(5x + 8)3x$

(ii) $(-3pq)\ (-15p^3q^2 - q^3)$

(iii) $\frac{2x}{5}(3a^3 - 3b^3)$

(iv) $-x^2(x - 15)$.

Answer

(i) By using the distributive law

$(5x + 8)\ 3x$

$= 5x \times 3x + 8 \times 3x$

$= 15x^2 + 24x$

(ii) By using the distributive law

$= (-3pq)(-15\ p^3q^2) - (-3pq)(q^3)$

$= 45p^4q^3 + 3pq^4$

(iii) By using the distributive law

$$= \frac{2x}{5}\ 3a^3 - \frac{2x}{5}\ 3b^3$$

$$= \frac{6a^3x}{5} - \frac{6b^3x}{5}$$

(iv) By using the distributive law

$= -x^2\ (x) - (-x^2)\ (15)$

$= -x^3 + 15x^2$

3. Question

Simplify the following:

(i) $(2xy - xy)(3xy - 5)$

(ii) $(3xy^2 + 1)(4xy - 6xy^2)$

(iii) $(3x^2 + 2x)(2x^2 + 3)$

(iv) $(2m^3 + 3m)(5m - 1)$.

Answer

By using the distributive law

(i) $(2xy - xy)(3xy - 5)$

$= 2xy\ (3xy - 5) - xy\ (3xy - 5)$

$= 6x^2y^2 - 10xy - 3x^2y^2 + 5xy$

$= 3x^2y^2 - 5xy$ (after adding and subtracting the like terns)

(ii) $(3xy^2 + 1)(4xy - 6xy^2)$

By using the distributive law

$3xy^2\ ((4xy - 6xy^2) + 1((4xy - 6xy^2)$

$= 3xy^2 \times 4xy - 3xy^2 \times 6xy^2 + 4xy - 6xy^2$

$= 12x^2y^3 - 18\ x^2y^4 + 4xy - 6xy^2$

(iii) $(3x^2 + 2x)(2x^2 + 3)$

By using the distributive law

$3x^2\ (2x^2 + 3) + 2x\ (2x^2 + 3)$

$= 3x^2 \times 2x^2 + 3x^2 \times 3 + 2x \times 2x^2 + 2x \times 3$

$= 6x^4 + 9x^2 + 4x^3 + 6x$

(iv) $(2m^3 + 3m)(5m - 1)$.

By using the distributive law

$2m^3\ (5m - 1) + 3m\ (5m - 1)$

$= 2m^3 \times 5m - 1 \times 2m^3 + 3m \times 5m - 1 \times 3m$

$= 10m^4 - 2m^3 + 15m^2 - 3m$

Exercise 2.4

1. Question

Find the product:

(i) $(a + 3)(a + 5)$

(ii) $(3t + 1)(3t + 4)$

(iii) $(a - 8)(a + 2)$

(iv) $(a - 6)(a - 2)$

Answer

(i) By using the distributive law

$= a(a + 5) +3((a+5)$

$= a^2 +5a +3a +15$

$= a^2 +8a +15$

(ii) By using the distributive law

$= 3t (3t + 4) + 1(3t + 4)$

$= 9t^2 + 12t +3t + 4$

$= 9t^2 + 15t +4$

(iii) By using the distributive law

$= a(a+2) –8(a+2)$

$= a^2 + 2a – 8a – 16$

$= a^2 – 6a –16$

(iv) By using the distributive law

$= a (a–2) – 6 (a – 2)$

$= a^2 –2a – 6a +12$

$= a^2 – 8a +12$

2. Question

Evaluate using suitable identities:

(i) 53 × 55 (ii) 102 × 106

(iii) 34 × 36 (iv) 103 × 96

Answer

(i) 53 × 55

We can re–write 53 and 55 as

(50+3)× (50+5)

Using the identity

$(x+a) (x+b) = x^2 + (a+b) x + ab$

(50+3)× (50+5) where x = 50, a = 3 and b = 5

$= 50^2 + (3+5)\,50 + 3\times 5$

$=2500 + 400 + 15$

$= 2915$

(ii) 102×106

$= (100 + 2)\,(100 + 6)$

Using the identity

$(x+a)\,(x+b) = x^2 + (a+b)\,x + ab$

Here $x= 100$, $a = 2$ and $b = 6$

$\Rightarrow (100 + 2)\,(100 + 6)$

$= 100^2 + (2+6)\,100 + 2\times 6$

$= 10000 + 800 +12$

$= 10812$

(iii) 34×36

$= (30+4) + (30+6)$

Using the identity

$(x+a)\,(x+b) = x^2 + (a+b)\,x + ab$

Here $x = 30$, $a= 4$ and $b = 6$

So, $30^2 +(4+6)\,30 +(4\times 6)$

$= 900 + 300 +24$

$= 1224$

(iv) 103×96

$= (90 + 13)\,(90 +6)$

Using the identity

$(x+a)\,(x+b) = x^2 + (a+b)\,x + ab$

Here $x = 90$, $a = 13$ and $b = 6$

So, $(90 + 13)\,(90 +6)$

$= 90^2 + (13+6)\,90 +(13\times 6)$

$= 8100 + 1710 +78$

= 9888

3. Question

Find the expression for the product (x + a)(x + b)(x + c) using the identity (x + a)(x + b) = x^2 + (a + b)x + ab

Answer

first we will expand (x + a)(x + b)

Using the identity

(x+a) (x+b) = x^2 + (a+b)x + ab

= x^2 + (a+b)x + ab

Now multiplying the expansion with (x+c)

(x^2 + (a+b)x + ab) (x+c)

By using the distributive law

x(x^2 + (a+b) x + ab) + c(x^2 + (a+b) x + ab)

= x^3 + (a + b)x^2 + abx + cx^2 + (a + b)cx + abc

Arranging the like terms

= x^3 + (a + b)x^2 + cx^2 + abx + (a + b)cx + abc

= x^3 + (a + b + c)x^2 + abx + acx + bcx + abc

= x^3 + (a + b + c) x^2 + x(ab+ ac+ bc) + abc

4. Question

Using the identity $(a+b)^2 = a^2 + 2ab + b^2$, simplify the following:

(i) $(a + 6)^2$ (ii) $(3x + 2y)^2$

(iii) $(2p + 3q)^2$ (iv) $(x^2 + 5)^2$

Answer

Given the identity

$(a+b)^2 = a^2 + 2ab + b^2$

(i) $(a + 6)^2$

Using the given identity

Here a = a, b = 6

$= a^2 + 2 \times a \times 6 + 6^2$

$= a^2 + 12a + 36$

(ii) $(3x + 2y)^2$

Using the given identity

Here a = 3x, b = 2y

$= (3x)^2 + 2 \times 3x \times 2y + (2y)^2$

$= 9x^2 + 12xy + 4y^2$

(iii) $(2p + 3q)^2$

Using the given identity

Here a = 2p, b = 3q

$= (2p)^2 + 2 \times 2p \times 3q + (3q)^2$

$= 4p^2 + 12pq + 9q^2$

(iv) $(x^2 + 5)^2$

Using the given identity

Here $a = x^2$, b = 5

$= (x^2)^2 + 2 \times x^2 \times 5 + 5^2$

$= x^4 + 10x^2 + 25$

5. Question

Evaluate using the identity $(a + b)^2 = a^2 + 2ab + b^2$

(i) $(34)^2$ (ii) $(10.2)^2$

(iii) $(53)^2$ (iv) $(41)^2$

Answer

Given identity $(a + b)^2 = a^2 + 2ab + b^2$

(i) $(34)^2$

$= (30+4)^2$

Here a = 30 and b = 4

$= 30^2 + 2 \times 30 \times 4 + 4^2$

= 900 + 240 +16

= 1156

(ii)$(10.2)^2$

= $(10 +0.2)^2$

Using the given identity

Here a = 10, b = 0.2

= 10^2 + 2× 10 × 0.2 +$(0.2)^2$

= 100 + 4 + 0.04

= 104.04

(iii) $(53)^2$

= $(50+3)^2$

Using the given identity

Here a = 50, b = 3

= $(50)^2$ +2× 50 × 3 + 3^2

= 2500 + 300 +9

= 2809

(iv) $(41)^2$

= $(40+1)^2$

Using the given identity

Here a = 40, b = 1

= 40^2 +2 × 40 × 1 + 1^2

= 1600 + 80 +1

= 1681

6. Question

Use the identity $(a – b)^2 = a^2 – 2ab + b^2$ to compute:

(i) $(x – 6)^2$ (ii) $(3x – 5y)^2$

(iii) $(5a – 4b)^2$ (iv) $(p^2 + q^2)^2$

Answer

Given identity

$(a - b)^2 = a^2 - 2ab + b^2$

(i) $(x - 6)^2$

Using the given identity

Here a = x, b = 6

$= x^2 - 2\times x \times 6 + 6^2$

$= x^2 - 12x + 36$

(ii)$(3x - 5y)^2$

Using the given identity

Here a = 3x, b = 5y

$= (3x)^2 - 2\times 3x \times 5y + (5y)^2$

$= 9x^2 - 30xy + 25y^2$

(iii)$(5a - 4b)^2$

Using the given identity

Here a = 5a, b = 4b

$= (5a)^2 - 2\times 5a \times 4b + (4b)^2$

$= 25a^2 - 40\ ab + 16b^2$

(iv) $(p^2 - q^2)^2$

Using the given identity

Here $a = p^2$, $b = q^2$

$= (p^2)^2 - 2 \times p^2 \times q^2 + (q^2)^2$

$= p^4 - 2\ p^2\ q^2 + q^4$

7. Question

Evaluate using the identity $(a - b)^2 = a^2 - 2ab + b^2$

(i) (49)2 (ii) (9.8)2

(iii) (59)2 (iv) (198)2

Answer

The given identity is $(a - b)^2 = a^2 - 2ab + b^2$

(i) $(49)^2$

$= (50 - 1)^2$

Using the given identity

Here a = 50, b = 1

$= 50^2 - 2\times 50 \times 1 + 1^2$

$= 2500 - 100 + 1$

$= 2401$

(ii) $(9.8)^2$

$= (10 - 0.2)^2$

Using the given identity

Here a = 10, b = 0.2

$= 10^2 - 2\times 10 \times 0.2 + (0.2)^2$

$= 100 - 4 + 0.04$

$= 96.04$

(iii) (59)2

$= (60 - 1)^2$

Using the given identity

Here a = 60, b = 1

$= 60^2 - 2\times 60\times 1 + 1^2$

$= 3600 - 120 + 1$

$= 3481$

(iv) (198)2

$= (200 - 2)^2$

Using the given identity

Here a = 200, b = 2

$= 200^2 - 2\times 200 \times 2 + 2^2$

$= 40000 - 800 + 4$

$= 39204$

8. Question

Use the identity $(a + b)(a - b) = a^2 - b^2$ to find the products:

(i) $(x - 6)(x + 6)$

(ii) $(3x + 5)(3x + 5)$

(iii) $(2a + 4b)(2a - 4b)$

(iv) $\left(\frac{2x}{3}+1\right)\left(\frac{2x}{3}-1\right)$

Answer

The given identity is $(a+b)(a-b) = a^2 - b^2$

(i) $(x - 6)(x + 6)$

Using the given identity

Here $a = x$ and $b = 6$

$= x^2 - 6^2$

$= x^2 - 36$

(ii) $(3x + 5)(3x + 5)$

Using the given identity

Here $a = 3x$ and $b = 5$

$= (3x)^2 - 5^2$

$= 9x^2 - 25$

(iii) $(2a + 4b)(2a - 4b)$

Using the given identity

Here $a = 2a$ and $b = 4b$

$= (2a)^2 - (4b)^2$

$= 4a^2 - 16b^2$

(iv) $\left(\frac{2x}{3}+1\right)\left(\frac{2x}{3}-1\right)$

Using the given identity

Here $a = \frac{2x}{3}$ and $b = 1$

$= \left(\frac{2x}{3}\right)^2 - 1^2$

$= \frac{4x^2}{9} - 1$

9. Question

Evaluate these using identity:

(i) 55 × 45 (ii) 33 × 27

(iii) 8.5 × 9.5 (iv) 102 × 98

Answer

(i) 55 × 45

We can split 55 as (50+5)

And 45 as (50–5)

Now 55 × 45

= (50+5) (50–5)

Using the identity (a +b) (a–b) = $a^2 - b^2$

Here a = 50 and b = 5

= $50^2 - 5^2$

= 2500 –25

= 2475

(ii) 33 × 27

= (30+3) (30–3)

Using the identity (a +b) (a–b) = $a^2 - b^2$

Here a = 30 and b = 3

= $(30)^2 - 3^2$

= 900 – 9

= 891

(iii) 8.5 × 9.5

= (9 – 0.5) (9 + 0.5)

Using the identity $(a+b)(a-b) = a^2 - b^2$

Here a = 9 and b = 0.5

$= 9^2 - (0.5)^2$

= 81 – 0.25

= 80.75

(iv) 102 × 98

= (100 + 2) (100 – 2)

Using the identity $(a+b)(a-b) = a^2 - b^2$

Here a = 100 and b = 2

$= (100)^2 - 2^2$

= 10000 – 4

= 9996

10 A. Question

Find the product:

$(x - 3)(x + 3)(x^2 + 9)$

Answer

First solving (x – 3)(x + 3)

Using the identity $(a+b)(a-b) = a^2 - b^2$

Here a = x and b = 3

$= x^2 - 3^2$

$= x^2 - 9$

Now $(x^2 - 9)(x^2 + 9)$

Again, using the identity $(a+b)(a-b) = a^2 - b^2$

Here $a = x^2$ and b = 9

$= (x^2)^2 - 9^2$

$= x^4 - 81$ ($\because (a^x)^m = a^{xm}$)

10 B. Question

Find the product:

$(2a + 3)(2a - 3)(4a^2 + 9)$

Answer

First solving $(2a + 3)(2a - 3)$

Using the identity $(a + b)(a - b) = a^2 - b^2$

Here $a = 2a$ and $b = 3$

$= (2a)^2 - 3^2$

$= 4a^2 - 9$

Now $(4a^2 - 9)(4a^2 + 9)$

Using the identity $(a + b)(a - b) = a^2 - b^2$

Here $a = 4a$ and $b = 9$

$= (4a)^2 - 9^2$

$= 16a^4 - 81$

10 C. Question

Find the product:

$(p + 2)(p - 2)(p^2 + 4)$

Answer

First solving $(p+2)(p-2)$

Using the identity $(a + b)(a - b) = a^2 - b^2$

Here $a = p$ and $b = 2$

$= p^2 - 2^2$

$= p^2 - 4$

Now solving $(p^2 + 4)(p^2 - 4)$

Again Using the identity $(a + b)(a - b) = a^2 - b^2$

Here a = p^2 and b = 4

= $(p^2)^2 - 4^2$

= $p^4 - 16$ ($\because (a^x)^m = a^{xm}$)

10 D. Question

Find the product:

$$\left(\frac{1}{2}m - \frac{1}{3}\right)\left(\frac{1}{2}m + \frac{1}{3}\right)\left(\frac{1}{2}m^2 + \frac{1}{9}\right)$$

Answer

First solving $\left(\frac{1}{2}m - \frac{1}{3}\right)\left(\frac{1}{2}m + \frac{1}{3}\right)$

Using the identity (a +b) (a–b) = $a^2 - b^2$

Here $a = \frac{1}{2}m$ and $b = \frac{1}{3}$

$$= \left(\frac{1}{2}m\right)^2 - \left(\frac{1}{3}\right)^2$$

$$= \frac{1}{4}m^2 - \frac{1}{9}$$

Now solving $\left(\frac{1}{4}m^2 - \frac{1}{9}\right)\left(\frac{1}{4}m^2 + \frac{1}{9}\right)$

Again, Using the identity (a +b) (a–b) = $a^2 - b^2$

Here $a = \frac{1}{4}m^2$ and $b = \frac{1}{9}$

$$= \left(\frac{1}{4}m^2\right)^2 - \left(\frac{1}{9}\right)^2$$

$$= \frac{1}{16}m^4 - \frac{1}{81}$$

10 E. Question

Find the product:

$(2x - y)(2x + y)(4x^2 + y^2)$

Answer

First solving (2x – y) (2x+y)

Using the identity (a +b) (a–b) = $a^2 - b^2$

Here a = 2x and b = y

$= (2x)^2 - y^2$

$= 4x^2 - y^2$

Now solving $(4x^2 - y^2)(4x^2 + y^2)$

Again using the identity $(a+b)(a-b) = a^2 - b^2$

Here $a = 4x^2$ and $b = y^2$

$= (4x^2)^2 - (y^2)^2$

$= 16\,x^4 - y^4$

10 F. Question

Find the product:

$(2x - 3y)(2x + 3y)(4x^2 + 9y^2)$

Answer

First solving for $(2x - 3y)(2x + 3y)$

Using the identity $(a+b)(a-b) = a^2 - b^2$

Here a = 2x and b = 3y

$= (2x)^2 - (3y)^2$

$= 4x^2 - 9y^2$

Now solving for $(4x^2 - 9y^2)(4x^2 + 9y^2)$

Again Using the identity $(a+b)(a-b) = a^2 - b^2$

Here $a = 4x^2$ and $b = 9y^2$

$= (4x^2)^2 - (9y^2)^2$

$= 16\,x^4 - 81\,y^4$

Additional Problems 2

1 A. Question

Terms having the same literal factors with same exponents are called.

A. exponents

B. like terms

C. factors

D. unlike terms

Answer

Terms having the same literal factors with same exponents are called like terms

So B is the correct option

1 B. Question

The coefficient of ab in 2ab is:

A. ab

B. 2

C. 2a

D. 2b

Answer

The coefficient is the term excluding ab in 2ab.

So B is the correct option

1 C. Question

The exponential form of a × a × a is:

A. 3a

B. 3 + a

C. a^3

D. 3 – a

Answer

The exponential form of a × a × a is a^3.

So C is the correct option.

1 D. Question

Sum of two negative integers is:

A. negative

B. positive

C. zero

D. infinite

Answer

Sum of two negative integers is always negative

So A is the correct option.

1 E. Question

What should be added to $a^2 + 2ab$ to make it a complete square?

A. b^2

B. 2ab

C. ab

D. 2a

Answer

$(a + b)^2 = a^2 + 2ab + b^2$

In the above expression we have to add b^2 to make it $(a + b)^2$

So A is the correct option.

1 F. Question

What is the product of (x + 2)(x–3)?

A. 2x – 6

B. 3x – 2

C. $x^2 – x – 6$

D. $x^2 – 6x$

Answer

(x + 2)(x-3)

$\Rightarrow (x + 2)(x-3) = x(x-3) + 2(x-3)$

$\Rightarrow (x + 2)(x-3) = x^2-3x + 2x-6$

$\Rightarrow (x + 2)(x-3) = x^2 – x – 6$

So C is the correct option.

1 G. Question

The value of $(7.2)^2$ is (use an identity to expand)

A. 49.4

B. 14.4

C. 51.84

D. 49.04

Answer

$(7.2)^2 = (7 + 0.2)^2$

$\Rightarrow (7.2)^2 = 7^2 + 0.2^2 + 2\times 7\times 0.2$

$\Rightarrow (7.2)^2 = 49 + 0.04 + 2.8$

$\Rightarrow (7.2)^2 = 51.84$

So C is the correct option.

1 H. Question

The expansion of $(2x - 3y)^2$ is:

A. $2x^2 + 3y^2 + 6xy$

B. $4x^2 + 9y^2 - 12xy$

C. $2x^2 + 3y^2 - 6xy$

D. $4x^2 + 9y^2 + 12xy$

Answer

$(a + b)^2 = a^2 + 2ab + b^2$

$\Rightarrow (2x - 3y)^2 = 4x^2 + 9y^2 - 12xy$

So B is the correct option.

1 I. Question

The product 58 × 62 is

A. 4596

B. 2596

C. 3596

D. 6596

Answer

58× 62 = (60-2)× (60 + 2)

We use the form (a + b)(a-b) = a^2 - b^2

⇒ 58× 62 = 60^2-2^2

⇒ 58× 62 = 3600-4 = 3596

So C is the correct option.

2. Question

Take away 8x – 7y – 8p + 10q from 10x + 10y – 7p + 9q.

Answer

We need to subtract the two expressions

10x + 10y – 7p + 9q-8x + 7y + 8p - 10q = 2x-17y + p-q

3. Question

Expand:

(i) $(4x + 3)^2$

(ii) $(x + 2y)^2$

(iii) $\left(x + \frac{1}{x}\right)^2$;

(iv) $\left(x - \frac{1}{x}\right)^2$.

Answer

(i) We use $(a + b)^2 = a^2 + 2ab + b^2$

⇒ $(4x + 3)^2 = (4x)^2 + 2\times 4x \times 3 + 3^2$

⇒ $(4x + 3)^2 = 16x^2 + 24x + 9$

(ii) We use $(a + b)^2 = a^2 + 2ab + b^2$

⇒$(x + 2y)^2 = x^2 + 4xy + 4y^2$

(iii) We use $(a + b)^2 = a^2 + 2ab + b^2$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

(iv) We use $(a - b)^2 = a^2 - 2ab + b^2$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

4. Question

Expand:

(i) (2t + 5)(2t – 5);

(ii) (xy + 8)(xy – 8);

(iii) (2x + 3y)(2x – 3y).

Answer

(i) We use the form $(a + b)(a-b) = a^2 - b^2$

$\Rightarrow (2t + 5)(2t - 5) = 4t^2 - 25$

(ii) We use the form $(a + b)(a-b) = a^2 - b^2$

$\Rightarrow (xy + 8)(xy - 8) = x^2y^2 - 64$

(iii) We use the form $(a + b)(a-b) = a^2 - b^2$

$\Rightarrow (2x + 3y)(2x - 3y) = 4x^2 - 9y^2$

5 A. Question

Expand:

$(n - 1)(n + 1)(n^2 + 1)$

Answer

Applying the formula $(a + b)(a - b) = a^2 - b^2$ on first two terms

$(n - 1)(n + 1)(n^2 + 1) = (n^2 - 1)(n^2 + 1)$

Applying the formula $(a + b)(a - b) = a^2 - b^2$ again

$\Rightarrow (n - 1)(n + 1)(n^2 + 1) = (n^4 - 1)$

5 B. Question

Expand:

$$\left(n - \frac{1}{n}\right)\left(n + \frac{1}{n}\right)\left(n^2 + \frac{1}{n^2}\right)$$

Answer

Applying the formula $(a + b)(a - b) = a^2 - b^2$ on first two terms

$$\Rightarrow \left(n - \frac{1}{n}\right)\left(n + \frac{1}{n}\right)\left(n^2 + \frac{1}{n^2}\right) = \left(n^2 - \frac{1}{n^2}\right)\left(n^2 + \frac{1}{n^2}\right)$$

Applying the formula $(a + b)(a - b) = a^2 - b^2$ again

$$\Rightarrow \left(n - \frac{1}{n}\right)\left(n + \frac{1}{n}\right)\left(n^2 + \frac{1}{n^2}\right) = \left(n^4 - \frac{1}{n^4}\right)$$

5 C. Question

Expand:

$(x – 1)(x + 1)(x^2 + 1)(x^4 + 1)$

Answer

Applying the formula $(a + b)(a - b) = a^2 - b^2$ on first two terms

$(x – 1)(x + 1)(x^2 + 1)(x^4 + 1) = (x^2 - 1)(x^2 + 1)(x^4 + 1)$

Applying the formula $(a + b)(a - b) = a^2 - b^2$ again

$\Rightarrow (x – 1)(x + 1)(x^2 + 1)(x^4 + 1) = (x^4 - 1)(x^4 + 1)$

Applying the formula $(a + b)(a - b) = a^2 - b^2$ again

$\Rightarrow (x – 1)(x + 1)(x^2 + 1)(x^4 + 1) = (x^8 - 1)$

5 D. Question

Expand:

$(2x – y)(2x + y)(4x^2 + y^2)$

Answer

Applying the formula $(a + b)(a - b) = a^2 - b^2$ on first two terms

$(2x – y)(2x + y)(4x^2 + y^2) = (4x^2 - y^2)(4x^2 + y^2)$

Applying the formula $(a + b)(a - b) = a^2 - b^2$ again

$\Rightarrow (2x – y)(2x + y)(4x^2 + y^2) = (16x^4 – y^4)$

6. Question

Use appropriate formulae and compute:

$(103)^2$

Answer

$(103)^2 = (100 + 3)^2$

We use $(a + b)^2 = a^2 + 2ab + b^2$

$\Rightarrow (103)^2 = 100^2 + 2\times 100\times 3 + 3^2$

$\Rightarrow (103)^2 = 10000 + 600 + 9 = 10609$

6 B. Question

Use appropriate formulae and compute:

$(96)^2$

Answer

$(96)^2 = (100-4)^2$

We use $(a - b)^2 = a^2 - 2ab + b^2$

$\Rightarrow (96)^2 = 100^2-2\times 100\times 4 + 4^2$

$\Rightarrow (96)^2 = 10000-800 + 16 = 9216$

6 C. Question

Use appropriate formulae and compute:

107×93

Answer

$107 \times 93 = (100 + 7)(100 - 7)$

We use $(a + b)(a - b) = a^2 - b^2$

$\Rightarrow 107 \times 93 = 100^2-7^2$

$\Rightarrow 107 \times 93 = 10000-49$

$\Rightarrow 107 \times 93 = 9951$

6 D. Question

Use appropriate formulae and compute:

1008×992

Answer

$1008 \times 992 = (1000 + 8)(1000 - 8)$

We use $(a + b)(a - b) = a^2 - b^2$

$\Rightarrow 1008 \times 992 = 1000^2-8^2$

$\Rightarrow 1008 \times 992 = 1000000-64$

$\Rightarrow 1008 \times 992 = 999936$

6 E. Question

Use appropriate formulae and compute:

$185^2 – 115^2$

Answer

$185^2 – 115^2= (150 + 35)^2-(150-35)^2$

$(a + b)^2-(a-b)^2 = 4ab$

$\Rightarrow 185^2 – 115^2= 4\times 150\times 35$

$\Rightarrow 185^2 – 115^2 = 21000$

7. Question

If $x + y = 7$ and $xy = 12$, find $x^2 + y$.

Answer

$x + y = 7$

$\Rightarrow y = 7-x$ …Equation (i)

$xy = 12$

Putting the value of y from above Equation (i) we get

$x(7-x) = 12$

$\Rightarrow 7x-x^2 = 12$

$\Rightarrow x^2-7x + 12 = 0$

Solving the above equation by the method of factorization we get

$\Rightarrow x^2-4x-3x + 12 = 0$

$\Rightarrow x(x-4)-3(x-4) = 0$

$\Rightarrow (x-4)(x-3) = 0$

$x = 4,3$

When x = 4, y = 3

$\Rightarrow x^2 + y = 19$

When x = 3, y = 4

$\Rightarrow x^2 + y = 13$

8. Question

If x + y = 12 and xy = 32, find x^2 + y.

Answer

x + y = 12

⇒ y = 12-x ...Equation (i)

xy = 32

Putting the value of y from above Equation (i) we get

x(12-x) = 32

$\Rightarrow 12x-x^2 = 32$

$\Rightarrow x^2-12x + 32 = 0$

Solving the above equation by the method of factorization we get

$\Rightarrow x^2-8x-4x + 12 = 0$

⇒x(x-8)-4(x-8) = 0

⇒(x-4)(x-8) = 0

x = 4,8

When x = 4, y = 8

$\Rightarrow x^2 + y = 24$

When x = 8, y = 4

$\Rightarrow x^2 + y = 68$

9. Question

If $4x^2 + y^2 = 40$ and xy = 6, find 2x + y.

Answer

$4x^2 + y^2 = 40$ ...Equation (i)

$xy = 6$

$\Rightarrow 4xy = 24$ ...Equation (ii)

Adding Equation (i) and (ii)

$4x^2 + y^2 + 2xy = 64$

$\Rightarrow (2x + y)^2 = 8^2$

$\Rightarrow 2x + y = \pm 8$

10. Question

If $x - y = 3$ and $xy = 10$, find $x^2 + y$.

Answer

$x - y = 3$

$\Rightarrow y = x-3$ ...Equation (i)

$xy = 10$

Putting the value of y from above Equation (i) we get

$x(x-3) = 10$

$\Rightarrow x^2-3x = 10$

$\Rightarrow x^2-3x-10 = 0$

Solving the above equation by the method of factorization we get

$\Rightarrow x^2-5x + 2x + 10 = 0$

$\Rightarrow x(x-5) + 2(x-5) = 0$

$\Rightarrow (x + 2)(x-5) = 0$

$x = -2,5$

Neglecting the negative value

When $x = 5$, $y = 2$

$\Rightarrow x^2 + y = 27$

11. Question

If $x + \frac{1}{x} = 3$, find $x^2 + \frac{1}{x^2}$ and $x^3 + \frac{1}{x^3}$.

Answer

$x + \frac{1}{x} = 3$ ...Equation (i)

Squaring both sides of the equation we get

$$x^2 + \frac{1}{x^2} + 2 = 9$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 7$$

Cubing both sides of the equation (i) we get

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 9 = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 18$$

12. Question

If $x + \frac{1}{x} = 6$, find $x^2 + \frac{1}{x^2}$ and $x^4 + \frac{1}{x^4}$.

Answer

$$x + \frac{1}{x} = 6$$

Squaring both sides of the equation we get

$$x^2 + \frac{1}{x^2} + 2 = 36$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 34$$

Again Squaring both sides of the above equation we get

$$x^4 + \frac{1}{x^4} + 2 = 1156$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 1154$$

13. Question

Simplify:

(i) $(x + y)^2 + (x – y)^2$;

(ii) $(x + y)^2 \times (x – y)^2$.

Answer

(i) We use the formula $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

$(x + y)^2 + (x – y)^2 = 2(x^2 + y^2)$

(ii) Applying the formula $(a + b)^2(a - b)^2 = (a^2 - b^2)^2$

$(x + y)^2 \times (x – y)^2 = (x^2 - y^2)^2$

$\Rightarrow (x + y)^2 \times (x – y)^2 = x^4-2x^2y^2 + y^4$

14. Question

Express the following as difference of two squares:

(i) $(x + 2z)(2x + z)$;

(ii) $4(x + 2y)(2x + y)$;

(iii) $(x + 98)(x + 102)$;

(iv) 505×495.

Answer

(i) We use $ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$

$(x + 2z)(2x + z) = \left(\frac{x+2z+2x+z}{2}\right)^2 - \left(\frac{x+2z-2x-z}{2}\right)^2$

$\Rightarrow (x + 2z)(2x + z) = \left(\frac{3x+3z}{2}\right)^2 - \left(\frac{z-x}{2}\right)^2$

(ii) $4(x + 2y)(2x + y) = (2x + 4y)(4x + 2y)$

We use $ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$

$\Rightarrow 4(x + 2y)(2x + y) = \left(\frac{2x+4y+4x+2y}{2}\right)^2 - \left(\frac{2x+4y-4x-2y}{2}\right)^2$

$\Rightarrow 4(x + 2y)(2x + y) = (3x + 3y)^2-(2y – 2x)^2$

(iii) We use $ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$

$(x + 98)(x + 102) = \left(\frac{2x+200}{2}\right)^2 - \left(\frac{2x-200}{2}\right)^2$

$\Rightarrow (x + 98)(x + 102) = (x + 100)^2\text{-}(x\text{-}100)^2$

(iv) $505 \times 495 = (500 + 5)(500\text{-}5)$

We use $(a + b)(a - b) = a^2 - b^2$

$\Rightarrow 505 \times 495 = 500^2\text{-}5^2$

15. Question

If $a = 3x - 5y$, $b = 6x + 3y$ and $c = 2y - 4x$, find

(i) $a + b - c$;

(ii) $2a - 3b + 4c$.

Answer

(i) $a + b - c = 3x - 5y + 6x + 3y\text{-}2y + 4x$

$\Rightarrow a + b - c = 13x - 4y$

(ii) $2a - 3b + 4c = 2(3x - 5y) + 3(6x + 3y)\text{-}4(2y - 4x)$

$\Rightarrow 2a - 3b + 4c = 6x\text{-}10y + 18x + 9y\text{-}8y + 16x$

$\Rightarrow 2a - 3b + 4c = 40x\text{-}9y$

16. Question

The perimeter of a triangle is $15x^2 - 23x + 9$ and two of its sides are $5x^2 + 8x - 1$ and $6x^2 - 9x + 4$. Find the third side.

Answer

Perimeter = $15x^2 - 23x + 9$

First side = $5x^2 + 8x - 1$

Second side = $6x^2 - 9x + 4$

Sum of first two side = $11x^2\text{-}x + 3$

Third side = (Perimeter- Sum of first two side)

$\Rightarrow$ Third side = $4x^2\text{-}22x + 6$

17. Question

The two adjacent sides of a rectangle are $2x^2 - 5xy + 3z^2$ and $4xy - x^2 - z$.

Answer

Two adjacent sides are $2x^2 - 5xy + 3z^2$ and $4xy - x^2 - z$.

Area of rectangle = $(2x^2 - 5xy + 3z^2) \times (4xy - x^2 - z)$

$\Rightarrow$ Area of rectangle = $2x^2(4xy - x^2 - z) - 5xy(4xy - x^2 - z) + 3z^2(4xy - x^2 - z)$

$\Rightarrow$ Area of rectangle = $8x^3y - 2x^4 - 2x^2z - 20x^2y^2 + 5x^3y + 5xyz + 12xyz^2 - 3x^2z^2 - 3z^3$

Perimeter of rectangle = $2 \times (2x^2 - 5xy + 3z^2 + 4xy - x^2 - z)$

$\Rightarrow$ Perimeter of rectangle = $2 \times (x^2 - xy + 3z^2 - z)$

$\Rightarrow$ Perimeter of rectangle = $(2x^2 - 2xy + 6z^2 - 2z)$

18. Question

The base and the altitude of a triangle are $(3x - 4y)$ and $(6x + 5y)$ respectively. Find its area.

Answer

Base of triangle = $(3x - 4y)$

Altitude of triangle = $(6x + 5y)$

Area of triangle $= \frac{1}{2} \times \text{base} \times \text{altitude} = \frac{1}{2} \times (3x - 4y) \times (6x + 5y)$

$\Rightarrow$ Area of triangle $= \frac{1}{2} \times (18x^2 - 24xy + 15xy - 20y^2)$

$\Rightarrow$ Area of triangle $= \frac{(18x^2 - 9xy - 20y^2)}{2}$

$\Rightarrow$ Area of triangle $= 9x^2 - \frac{9}{2}xy - 10y^2$

19. Question

The sides of a rectangle are $2x + 3y$ and $3x + 2y$. From this a square of side length $x + y$ is removed. What is the area of the remaining region?

Answer

Length of rectangle = $2x + 3y$

Breadth of rectangle = $3x + 2y$

Area of rectangle = (Length× breadth) = (2x + 3y)× (3x + 2y)

⇒ Area of rectangle = $6x^2 + 13xy + 6y^2$

Side of square = x + y

Area of Square = (Side× Side) = $(x + y)^2$

⇒ Area of Square = $x^2 + y^2 + 2xy$

Area of remaining region = (Area of rectangle- Area of Square) = $5x^2 + 5y^2 + 11xy$

20. Question

If a, b are rational numbers such that $a^2 + b^2 + c^2 – ab – bc – ca = 0$, prove that a = b = c.

Answer

$a^2 + b^2 + c^2 – ab – bc – ca = 0$

Multiplying both sides by 2 we get

$2 (a^2 + b^2 + c^2 – ab – bc – ca) = 0$

⇒ $(a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (c^2 + a^2 -2ac) = 0$

The individual terms inside the brackets can be expressed as a whole square

⇒ $(a – b)^2 + (b – c)^2 + (c – a)^2 = 0$

Since a, b, c are rational and none of the term is equal to zero so each of the terms inside the bracket must individually be equal to zero

⇒ a – b = 0

⇒ a = b

⇒ b – c = 0

⇒ b = c

⇒ c – a = 0

⇒ c = a

So together we can say that

a = b = c