QUADRATIC EQUATIONS

Quadratic Equations: A quadratic equation in the variable x is an equation of the form $ax^2 + bx + c = 0$ where a, b, c are real numbers and $a \neq 0$

(Or)

Any equation of the form P(x) = 0 Where P(x) is polynomial of degree 2 is a quadratic equation.

$ax^2 + bx + c = 0$ is called the standard form of the quadratic equation

$y = ax^2 + bx + c$ is called quadratic function

There are various uses of quadratic functions. Some of them are:

i. When the rocket is fired upword then the height of the rocket is defined by a **'Quadratic Function '**.

ii. Shapes of the satellite, reflecting mirror in a telescope lens of the eye glasses and orbit of the celestial objects are defined by the quadratic equation

iii. The part of the projectile is defined by quadratic function.

iv. When the breaks are applied to a vehicle, then the stopping point is calculated by using quadratic equation

A real number α is called a root of a quadratic equation $ax^2 + bx + c = 0$ if $a\alpha^2 + b\alpha + c = 0$ we also say that $x = \alpha$ is a solution of the quadratic equation. or α satisfies the quadratic equation.

Ex: 2, 3 are roots of the quadratic equation $x^2 - 5x + 6 = 0$

The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.

Any quadratic equation can have at most two roots.

Solution of a Quadratic Equation By Factorisation

If the quadratic equation $ax^2 + bx + c = 0$ can be written in the form $(px+q)(rx+s)=0;\ p \neq 0, r \neq 0$ then $-\frac{q}{p}$ and $-\frac{s}{r}$ will be the root of quadratic equation. Which are respectively the values of x obtained from $px+q=0$ and $rx+s=0$

Ex: $6x^2 - x - 2 = 0$

$$\Rightarrow (3x-2)(2x+1) = 0$$

The roots of $6x^2 - x - 2 = 0$ are the values of "x" for which

$$\Rightarrow (3x-2)(2x+1) = 0$$

$$\Rightarrow 3x - 2 = 0 \text{ or } 2x + 1 = 0$$

i.e $x = \frac{2}{3}$ or $x = -\frac{1}{2}$

$\therefore$ The roots of $6x^2 - x - 2 = 0$ are $\frac{2}{3}$ and $-\frac{1}{2}$

To factorize a quadratic equation $ax^2 + bx + c = 0$ we find $p, q \in R$ such that $p + q = b$ and $pq = ac$.

This process is called Factorising a quadratic equation by splitting its middle term.

Solution of a Quadratic Equation By Completing The Square

Let the quadratic equation be $ax^2 + bx + c = 0,\ a \neq 0$

Dividing throughout by a, we get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Adding and Subtracting $\left(\frac{b}{2a}\right)^2$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 - \frac{(b^2 - 4ac)}{4a^2} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{(b^2 - 4ac)}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Thus if $b^2 - 4ac \geq 0$ then the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Let $ax^2 + bx + c = 0, a \neq 0$ be a quadratic equation then $b^2 - 4ac$ is called the Discriminate of the quadratic equation.

If $b^2 - 4ac > 0$ then the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

This is called quadratic formula to find the roots.

A quadratic equation $ax^2 + bx + c = 0$ has

i. Two distinct real roots, if $b^2 - 4ac > 0$

ii. Two equal real roots, if $b^2 - 4ac = 0$

iii. No real roots, if $b^2 - 4ac < 0$

Roots of a quadratic equation are those points where the curve cuts the X-axis.

Case - 1: If $b^2 - 4ac > 0$

We get two distinct real roots $\frac{-b - \sqrt{b^2 - 4ac}}{2a}, \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

In such case we get the following figures.

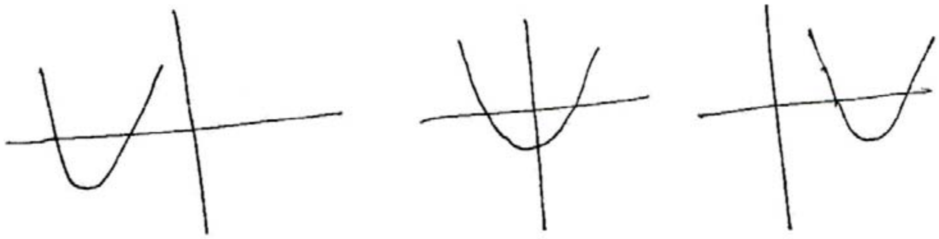


Figure shows that the curve of the quadratic equation cuts the X-axis at two distinct points.

Case – 2 : If $b^2 - 4ac = 0$

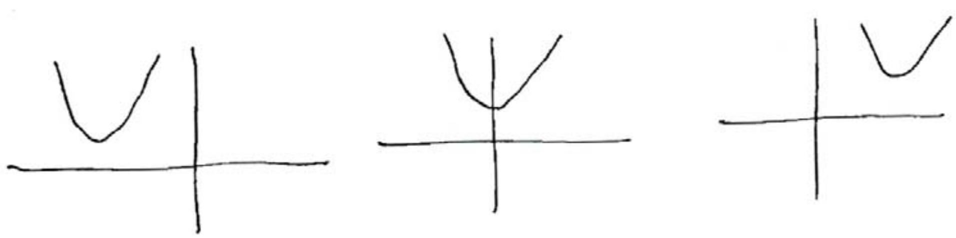
$$x = \frac{-b \pm 0}{2a} \; ; \; x = \frac{-b}{2a}, \frac{-b}{2a}$$

In such case we get the following figures.

Figure shows that the curve of the quadratic equation touching X-axis at one point.

Case – 3 : If $b^2 - 4ac < 0$

There are no real roots. Roots are imaginary. In such case we get the following figures.



In this case graph neither intersects nor touches the X-axis at all. So there are no real roots.

Let $ax^2 + bx + c = 0$ be a given quadratic equation and ∞ , β are the roots of given quadratic equation, then

Sum of the roots $= \alpha + \beta = \frac{-b}{a} = \frac{-x \text{ Coefficient}}{x^2 \text{ Coefficient}}$

Product of the roots $= \alpha \beta = \frac{c}{a} = \frac{\text{Cons tan t term}}{x^2 \text{ Coefficient}}$

Quadratic equation whose roots are α, β is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

ESSAY EQUATIONS

1) Find the roots of the following quadratic equations by factorization

(i) $x - \frac{1}{3x} = \frac{1}{6}$

(ii) $3(x-4)^2 - 5(x-4) = 12$

Sol:

(i) $x - \frac{1}{3x} = \frac{1}{6}$

$\Rightarrow x - \frac{1}{3x} - \frac{1}{6} = 0 \Rightarrow \frac{18x^2 - 6 - 3x}{18x} = 0$

$18x^2 - 3x - 6 = 0$

$\Rightarrow 3\left(6x^2 - x - 1\right)^2 = 0 \Rightarrow 6x^2 - x - 1 = 0$

$\Rightarrow 2x(3x-2) + 1(3x-2) = 0$

$\Rightarrow (3x-2)(2x+1) = 0$

$\Rightarrow 3x - 2 = 0 \text{ (or) } 2x + 1 = 0$

$\Rightarrow x = \frac{2}{3} \text{ (or) } x = -\frac{1}{2}$

$\therefore$ The roots of given quadratic equation are $\frac{2}{3}, -\frac{1}{2}$

(ii) $3(x-4)^2 - 5(x-4) = 12$

$\Rightarrow$ Let $x - 4 = a$

$\Rightarrow 3a^2 - 9a + 4a - 12 = 0 \; (\because 3 \times -12 = -36)$

$\Rightarrow 3a(a-3) + 4(a-3) = 0$

$$\Rightarrow (a-3)(3a+4)=0$$

$$\Rightarrow a=3 \text{ (or) } a=-\frac{4}{3}$$

$$\Rightarrow \text{But } a = x-4$$

i.e. $x-4=3 \Rightarrow x=7$

$$x-4=-\frac{4}{3} \Rightarrow x=-\frac{4}{3}+4$$

$$\Rightarrow x=\frac{-4+12}{3}=\frac{8}{3}$$

The roots of given quadratic equation are $7, \frac{8}{3}$

2) Find two consecutive positive integers, sum of whose squares is 613.

Sol: Let the two consecutive positive integers be x, x+1

Given that sum of the squares of two consecutive integers is 613.

i.e. $x^2+(x+1)^2=613$

$$\Rightarrow x^2+x^2+2x+1=613$$

$$\Rightarrow 2x^2+2x+1-613=0$$

$$\Rightarrow 2x^2+2x-612=0$$

$$\Rightarrow 2\left(x^2+x-306\right)=0 \Rightarrow x^2+x-306=0$$

$$(\because 1\times -306 = -306\ (18\times -17))$$

$$\Rightarrow x^2+18x-17x-306=0$$

$$\Rightarrow x(x+18)-17(x+18)=0$$

$\Rightarrow (x+18)(x-17)=0$

$\Rightarrow (x+18)=0 \text{ (or) } x-17=0$

$\Rightarrow x=-18 \text{ (or) } x=17$

If x is positive, then x = 17.

If x = 17 then the consecutive positive integers are 17, 18.

3) The altitude of a right triangle is 7cm less than its base. If the hypotenuse is 13 cm; find the other two sides?

Sol: Let the base of a right angle triangle be ‘x’.

Given that the altitude of a right triangle is 7 cm less than its base.

Altitude (or) height = h = x – 7

Given that hypotenuse = 13 cm

By pythogorus theorem

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2 \quad \Rightarrow x^2 - 12x + 5x - 60 = 0$

$\Rightarrow (13)^2 = x^2 + (x-7)^2$

$\Rightarrow x^2 + x^2 - 14x + 49 = 169$

$\Rightarrow 2x^2 - 14x - 120 = 0$

$\Rightarrow 2(x^2 - 7x - 60) = 0, \Rightarrow x^2 - 7x - 60 = 0$

$\Rightarrow x^2 - 12x + 5x - 60 = 0$

$\Rightarrow x(x-12) + 5(x-12) = 0$

$\Rightarrow (x-12)(x+5) = 0$

$\Rightarrow x = 12, -5$

Length of base is always positive.

Length of base = 12 cm

Height (or) Altitude = x – 7 = 12 – 7 = 5 cm

4) **A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs. 90, find the number of articles produced and the cost of each article.**

Sol: Let the number of pottery articles produced by a cottage industry be 'x'.

Given that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day.

$$\therefore \text{Price of each article} = 2x + 3$$

$$\text{Total cost of articles} = \text{Rs. } 90$$

$$\text{i.e. } x(2x + 3) = 90 \Rightarrow 2x^2 + 3x - 90 = 0$$

$$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$$

$$\Rightarrow x(2x + 15) - 6(2x + 15) = 0$$

$$\Rightarrow (x - 6)(2x + 15) = 0$$

$$\Rightarrow 2x + 15 = 0 \text{ (or) } x - 6 = 0$$

$$\Rightarrow x = -\frac{15}{2} \text{ (or) } x = 6$$

No of articles never be negative, So $x \neq -\frac{15}{2}$

Price of each article = 2x + 3 = 2 (6) + 3 = Rs. 15

5) Find the dimensions of a rectangle whose perimeter is 28 meters and whose area is 40 square meters.

Sol: Let the length of a rectangle be 'x' meters i.e $l = x$

$$\text{Perimeter of a rectangle} = 2(l + b) = 28\text{m}(\text{Given})$$

$$\Rightarrow 2(x + b) = 28 \Rightarrow x + b = 14 \Rightarrow b = 14 - x$$

$$\text{Given that area of a rectangle} = lb$$

$$\Rightarrow x(14 - x) = 40 \text{ sq.m}$$

$$\Rightarrow 14x - x^2 = 40$$

$$\Rightarrow 14x - x^2 - 40 = 0$$

$$\Rightarrow x^2 - 14x + 40 = 0$$

$$\Rightarrow x^2 - 10x - 4x + 40 = 0$$

$$\Rightarrow x(x - 10) - 4(x - 10) = 0$$

$$\Rightarrow (x - 10)(x - 4) = 0$$

$$\Rightarrow x = 10 \ (\text{or}) \ x = 4$$

If the length x = 10m, then the width b = 14 – x = 14 – 10 = 4 m

If the length x = 4 cm, then width b = 14 – x = 14 – 4 = 10 m

The dimensions of rectangle are 10m, 4m.

6) The base of a triangle is 4 cm, longer then its altitude. If the area of the triangle is 48 sq. cm then find its base and altitude.

Sol: Let the height of the triangle = x cm

Given that the base of a triangle is 4 cm, longer than its altitude (height)

i.e Base = (x + 4) cm

$$\text{Area of the triangle} = \frac{1}{2}bh = 48 \text{ sq.cm}$$

$$\Rightarrow \frac{1}{2} \times (x + 4)\, x = 48$$

$$\Rightarrow x^2 + 4x = 96$$

$$\Rightarrow x^2 + 4x - 96 = 0$$

$$\Rightarrow x^2 + 12x - 8x - 96 = 0$$

$$\Rightarrow x(x+12) - 8(x+12) = 0$$

$$\Rightarrow (x+12)(x-8) = 0$$

$$x + 12 = 0 \;(\text{or})\; x - 8 = 0$$

$$x = -12 \;(\text{or})\; x = 8$$

Height of the triangle never be negative.

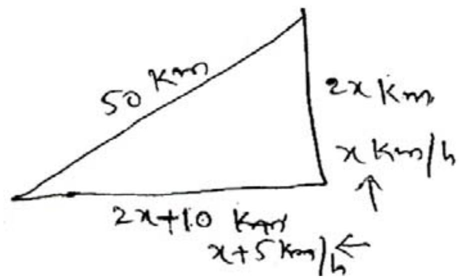
Height x = 8 cm

Base = x + 4 = 8 + 4 = 12 cm

7) Two trains leave a railway station at the same time. The first train travels towards West and the second train towards north. The first train travels 5km/hr faster than the second train. If after two hours they are 50 km apart. Find average speed of each train?

Sol: Let the speed of second train = x km/hour

Speed of first train = x + 5 km/hour



After two hours distance travelled by first train = 2 (x + 5) = 2x + 10 km

Distance travelled by second train = 2x km.

Distance between two trains after two hours = 50 km

By pythagoras theorem

$$(2x+10)^2 + (2x)^2 = 50^2$$

$$\Rightarrow 4x^2 + 40x + 100 + 4x^2 = 2500$$

$$8x^2 + 40x - 2400 = 0 \Rightarrow 8(x^2 + 5x - 300) = 0$$

$$\Rightarrow x^2 + 20x - 15x - 300 = 0$$

$$\Rightarrow x(x+20) - 15(x+20) = 0$$

$$\Rightarrow (x+20)(x-15) = 0$$

$$\Rightarrow x = -20,\ x = 15$$

Speed of the train never be negative

Speed of second train =15 km/hour

Speed of first train = x + 5 = 15 + 5 = 20 km/hour

8) In a class of 60 students, each boy contributed rupees equal to the number of girls and each girl contributes rupees equal to the number of boys. If the total money then collected was Rs. 1600. How many boys are there in the class?

Sol: Total number of students in a class = 60

Let the number of boys = x

Then the number of girls = 60 – x

Each boy contributed rupees equal to the number of girls and each girl contributed rupees equal to the number of boys and the total money collected was Rs. 1600.

i.e $x(60 - x) + x(60 - x) = 1600$

$\Rightarrow 2x(60 - x) = 1600$

$\Rightarrow x(60 - x) = 800$

$\Rightarrow 60x - x^2 - 800 = 0$

$\Rightarrow x^2 - 60x + 800 = 0$

$\Rightarrow x^2 - 20x - 40x + 800 = 0$

$\Rightarrow x(x - 20) - 40(x - 20) = 0$

$\Rightarrow (x - 20)(x - 40) = 0$

$\Rightarrow x = 20 \text{ (or) } x = 40$

Number of boys in the class room = 20 or 40

9) A motor boat heads upstream a distance of 24km on a river whose current is running at 3km per hour. The trip up and back takes 6 hours. Assuming that the motor boat maintained a constant speed, what is its speed?

Sol: Let the speed of a motor boat = x km/hr

Speed of stream = 3 km/hour

The distance of the river = 24 km

The speed of the boat in upstream = (x – 3) km/h

The speed of the boat in downstream = x + 3 km/h

Given that total time taken = 6 hours

i.e. $\frac{24}{x+3}+\frac{24}{x-3}=6$

$\Rightarrow 24(x-3)+24(x+3)=6(x+3)(x-3)$

$6[4(x-3)+4(x+3)]= \ 6(x+3)(x-3)$

$4x-12+4x+12=x^2-9$

$x^2-8x-9=0$

$\Rightarrow x^2-9x+x-9=0$

$\Rightarrow x(x-9)+1(x-9)=0$

$\Rightarrow (x-9)(x+1)=0$

$\Rightarrow x=9, x=-1$

Speed of the boat never be negative.

Speed of boat in still water = 9 km/hour.

10) Solve the equations by completing the square.

i. $5x^2-7x-6=0$

ii. $4x^2+4\sqrt{3}x+3=0$

Sol: (i) Given quadratic equation $5x^2 - 7x - 6 = 0$, dividing with 5 on both sides.

$x^2-\frac{7}{5}x-\frac{6}{5}=0$

$$\Rightarrow x^2 - \frac{7}{5}x = \frac{6}{5}$$

$$\Rightarrow x^2 - 2(x)\left(\frac{7}{10}\right) = \frac{6}{5}\left(\because 2\left(\frac{7}{10}\right) = \frac{7}{5}\right)$$

Adding $\left(\frac{7}{10}\right)^2$, onbothsides

$$\Rightarrow x^2 - 2(x)\left(\frac{7}{10}\right) + \left(\frac{7}{10}\right)^2 = \frac{6}{5} + \left(\frac{7}{10}\right)^2$$

$$\Rightarrow \left(x - \frac{7}{10}\right)^2 = \frac{6}{5} + \frac{49}{100}\left(\because x^2 - 2xy + y^2 = (x-y)^2\right)$$

$$\Rightarrow \left(x - \frac{7}{10}\right)^2 = \frac{120+49}{100} = \frac{169}{100}$$

$$\Rightarrow \left(x - \frac{7}{10}\right)^2 = \left(\frac{13}{10}\right)^2$$

$$\Rightarrow x - \frac{7}{10} = \pm\frac{13}{10}\left(\because x^2 = a^2 \Rightarrow x = \pm a\right)$$

$$\Rightarrow x = \pm\frac{13}{10} + \frac{7}{10}$$

$$= \frac{13}{10} + \frac{7}{10}\ (\text{or}) - \frac{13}{10} + \frac{7}{10}$$

$$= \frac{20}{10}(\text{or})\frac{-6}{10} = 2\ (\text{or}) - \frac{3}{5}$$

(ii) Given quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$

Dividing on both sides by 4, we get $x^2 + \sqrt{3}x + \frac{3}{4} = 0$

$$\Rightarrow x^2 + \sqrt{3}x = -\frac{3}{4}$$

$$\Rightarrow x^2 + 2(x)\left(\frac{\sqrt{3}}{2}\right) = -\frac{3}{4}\left(\because \sqrt{3} = 2\left(\frac{\sqrt{3}}{2}\right)\right)$$

Adding $\left(\frac{\sqrt{3}}{2}\right)^2$ on both sides, we getAdding $\left(\frac{\sqrt{3}}{2}\right)^2$ on both sides, we get

$$\Rightarrow x^2 + 2(x)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)^2 = -\frac{3}{4} + \left(\frac{\sqrt{3}}{2}\right)^2 = -\frac{3}{4} + \frac{3}{4}$$

$$\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 = 0$$

$$\Rightarrow x + \frac{\sqrt{3}}{2} = 0 \Rightarrow x = -\frac{\sqrt{3}}{2}$$

11) Find the roots of the following quadratic equations, if they exist, using the quadratic formula.

i. $2x^2 - 2\sqrt{2}x + 1 = 0$

ii. $x + \frac{1}{x} = 3 \ (x \neq 0)$

Sol:

(i) Given quadratic equation $2x^2 - 2\sqrt{2}x + 1 = 0$

Compare $ax^2+bx+c=0$; $a=2$, $b=-2\sqrt{2}$, $c=1$

So, $b^2-4ac=\left(-2\sqrt{2}\right)^2-4(2)(1)=8-8=0$

Since $b^2-4ac\geq 0$, the roots are exist.

$$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-\left(-2\sqrt{2}\right)\pm\sqrt{0}}{2(2)}=\frac{2\sqrt{2}}{4}=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$$

So the roots are $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

(ii)

Given that $x+\frac{1}{x}=3$

$$\Rightarrow \frac{x^2+1}{x}=3\Rightarrow x^2+1=3x\Rightarrow x^2-3x+1=0$$

Compare with $ax^2+bx+c=0$, we get $a=1$, $b=-3$, $c=1$

$b^2-4ac=(-3)^2-4(1)(1)=9-4=5$

$$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(1)}}{2(1)}$$

$$x=\frac{3\pm\sqrt{9-4}}{2}=\frac{3\pm\sqrt{5}}{2}$$

So the roots are $\frac{3+\sqrt{5}}{2}$ and $\frac{3-\sqrt{5}}{2}$

12) Find the roots of the following quadratic equations?

i. $\frac{1}{x} - \frac{1}{x-2} = 3, \left(x \neq 0, 2\right)$

ii. $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, \left(x \neq -4, 7\right)$

Sol:

$\left(i\right)$ Given $\frac{1}{x} - \frac{1}{x-2} = 3$

Multiplying the equation by $x\left(x-2\right)$ we get

$\left(x - 2\right) - x = 3x\left(x - 2\right)$

$\Rightarrow 3x\left(x-2\right) = -2$

$3x^2 - 6x = -2 \Rightarrow 3x^2 - 6x + 2 = 0$

Which is a quadratic equation compare with $ax^2 + bx = c = 0$

$a = 3,\ b = -6,\ c = 2$

So, $b^2 - 4ac = \left(-6\right)^2 - 4\left(3\right)\left(2\right) = 36 - 24 = 12 > 0$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-\left(-6\right) \pm \sqrt{12}}{2\left(3\right)}$$

$$x = \frac{6 \pm 2\sqrt{3}}{6} = \frac{2\left(3 \pm \sqrt{3}\right)}{6} = \frac{\left(3 \pm \sqrt{3}\right)}{3}$$

So the roots are $\frac{3+\sqrt{3}}{3}$ and $\frac{3-\sqrt{3}}{3}$

(ii) Given $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}, (x \neq -4, 7)$

Multiplying the equation by $30(x+4)(x-7)$, we get

$30(x-7)-30(x+4)=11(x+4)(x-7)$

$\Rightarrow 30x-210-30x-120=11(x^2-7x+4x-28)$

$\Rightarrow -330=11(x^2-3x-28)$

$\Rightarrow x^2-3x-28+30=0$

$\Rightarrow x^2-3x+2=0$

$\Rightarrow x^2-2x-x+2=0$

$\Rightarrow x(x-2)-1(x-2)=0$

$\Rightarrow (x-2)(x-1)=0$

$\Rightarrow x-2=0 \text{ (or) } x-1=0$

$\Rightarrow x=2 \text{ (or) } 1$

The roots of the given equation are 1 (or) 2

13) The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is 1/3. Find his present age.

Sol: Let the present age of Rehman = x years

3 years age ago of Rehman = x - 3 years

After 5 years, the age of Rehman = x + 5 years

Given that the sum of the reciprocals of Rehman's ages, 3 years ago and 5 years from now is 1/3.

$$i.e.\ \frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$

Multiplying with 3 (x – 3) (x + 5), we get

$\Rightarrow 3(x + 5) + 3(x – 3) = (x – 3)(x + 5)$

$\Rightarrow 3x + 15 + 3x – 9 = x^2 + 5x – 3x – 15$

$\Rightarrow 6x + 6 = x^2 + 2x – 15$

$\Rightarrow x^2 + 2x – 15 – 6x – 6 = 0$

$\Rightarrow x^2 – 4x – 21 = 0$

$\Rightarrow x^2 – 7x + 3x – 21 = 0$

$\Rightarrow x(x – 7) + 3(x – 7) = 0$

$\Rightarrow (x – 7)(x + 3) = 0$

$\Rightarrow (x – 7) = 0$ (or) $(x + 3) = 0$

$\Rightarrow x = 7$ (or) $x = –3$

But age is not negative.

$\therefore$ Present age of Rehman = 7 years.

14) In a class test, the sum of Moulika's marks in mathematics and English is 30. If she got 22 marks more in mathematics and 3 marks less in English, the product of her marks would have been 210. Find her marks in the two subjects?

Sol: Sum of Moulika's marks in Mathematics and English is 30.

Let the marks in Maths = x

Then the marks in English = 30 – x

If she got 2 marks more in Maths, and 3 marks less in English, the product of marks = 210.

i.e. $(x + 2)(30 – x – 3) = 210$

$\Rightarrow (x + 2)(27 - x) = 210$

$\Rightarrow 27x - x^2 + 54 - 2x - 210 = 0$

$\Rightarrow -x^2 + 25x - 156 = 0$

$\Rightarrow x^2 - 25x + 156 = 0$

$\Rightarrow x^2 - 13x - 12x + 156 = 0$ $(\therefore\ 1 \times 156 = 156 \rightarrow -13 \times -12)$

$\Rightarrow x(x - 13) - 12(x - 13) = 0$

$\Rightarrow (x - 13)(x - 12) = 0$

$\Rightarrow (x - 13) = 0$ (or) $(x - 12) = 0$

$\Rightarrow x = 13$ (or) $x = 12$

If marks in Maths $x = 13$, then marks in English $= 30 - x = 30 - 13 = 17$

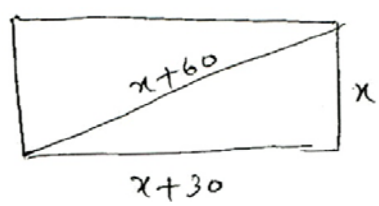
If marks in Maths $x = 12$, then marks in English $= 30 - 12 = 18$

15) The diagonal of a rectangular field is 60 meters more than the shorter side. If the longer side is 30 meters more than the shorter side, find the sides of the field?

Sol: Let the length of the shorter side (bread th) = x meters.

Then the length of longer side = x + 30 meters (Given)

The length of diagonal = x + 60 (Given)



By pythogorus theorem

$(x+60)^2 = (x+30)^2 + x^2$

$\Rightarrow x^2 + 120x + 3600 = x^2 + 60x + 900 + x^2$

$\Rightarrow x^2 + 60x + 900 - 120x - 3600 = 0$

$\Rightarrow x^2 - 60x - 2700 = 0$

$\Rightarrow x^2 - 90x + 30x - 2700 = 0$ $(\therefore\ 1 \times -270 = -270)$

$\Rightarrow x(x-90)+30(x-90)=0$

$\Rightarrow (x-90)(x+30)=0$

$\Rightarrow x=90$ (or) $x=-30$

Length of the shorter side (x) never be negative.

Length of the shorter side = 90m

Length of longer side = 90 + 30 = 120m

Length of diagonal = 90 + 60 = 150 m

16) The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers?

Sol: Let the larger number = x

Square of the larger number = x^2

Square of the smaller number = 8 times the larger number (Given)

= 8x

Given that difference of squares of two numbers is 180

i.e. $x^2-8x=180$

$\Rightarrow x^2-8x-180=0$

$\Rightarrow x^2-18x+10x-180=0$

$\Rightarrow x(x-8)\ 10(x-18)=0$

$\Rightarrow (x-18)(x+10)=0$

$\Rightarrow (x-18)=0$ (or) $(x+10)=0$

$\Rightarrow x=18$ (or) -10

$\therefore$ The larger number $x=18$ $(x\neq 10)$

The square of smaller number = $8\times x=8\times 18=144$

The smaller number = $\sqrt{144}=\pm 12$

The two numbers are 18 and 12 (or) 18 and –12

17) A train travels 360 km at a uniform speed. If the speed had been 5km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train?

Sol: The distance travelled by a train = 360 km

Let the speed of the train = x km/h

If the speed of the train increased 5 km/h, then the speed of the train = (x + 5) km/h

The time taken by the train, to cover 360 km distance with the speed x km/h is $\frac{360}{x}$.

The time taken by the train, to cover 360 km distance with the speed x + 5 km/h is $\frac{360}{x+5}$.

Difference between the two timings = 1 hour

i.e $\frac{360}{x} - \frac{360}{x+5} = 1$

Multiplying with x (x + 5), we get

$\Rightarrow 360(x+5) - 360x = x(x+5)$

$\Rightarrow 360x + 1800 - 360x = x^2 + 5x$

$\Rightarrow x^2 + 5x - 1800 = 0$

$\Rightarrow x^2 + 45x - 40x - 1800 = 0$

$\Rightarrow x(x+45) - 40(x+45) = 0$

$\Rightarrow (x+45)(x-40) = 0$

$\Rightarrow (x-40) = 0$ (or) $(x+45) = 0$

$\Rightarrow x = 40$ (or) $x = -45$

$\therefore$ Speed of the train x = 40 km/h (x = – 45 net negative)

18) Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Sol: Total time takes by two water taps together can fill a tank

$$= 9\frac{3}{8} = \frac{75}{8} Hours$$

The part of the tank filled by the two taps together in 1 hour is

$$= \frac{1}{\frac{75}{8}} = \frac{8}{75}$$

Time taken by the smaller diameter tap to fill the tank = x hours.

The tap of larger diameter takes 10 hours less than the smaller one to fill the tank.

i.e The time taken by the larger diameter tap to fill the tank = x – 10 hours

Part of the tank filled by the smaller diameter tap in 1 hour = $\frac{1}{x}$

Part of the tank filled by the larger diameter tap in 1 hour = $\frac{1}{x-10}$.

$$But\ \frac{1}{x} + \frac{1}{x-10} = \frac{8}{75} (From\ \ the\ \ problem)$$

Multiplying with 75x (x – 10) on both sides

$\Rightarrow 75(x-10) + 75x = 8(x)(x-10)$

$\Rightarrow 75x - 750 + 75x = 8x^2 - 80x$

$\Rightarrow 8x^2 - 80x - 150x + 750 = 0$

$\Rightarrow 8x^2 - 230x + 750 = 0$

$\Rightarrow 8x^2 - 200x - 30x + 750 = 0$

$\Rightarrow 8x(x-25) - 30(x-25) = 0$

$\Rightarrow (x-25)(8x-30) = 0$

$\Rightarrow$ (8x – 30) = 0 (or) (x – 25) = 0

$\Rightarrow$ x = 30/8 = 15/4 (or) x = 25

$x \neq \frac{15}{4}$ (The big tap takes 10 hours less than the small tap)

$\therefore$ x = 25 hours

Time taken by the smaller diameter tap to fill the tank separately = 25 hours

Time taken by the larger diameter tap to fill the tank separately

= x – 10

= 25 – 10

= 15 hours.

19) An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (Without taking into consideration the time they stop at intermediate stations). If the Average Speed of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Sol: Let the speed of passenger train = x km/h.

Then the speed of express train = x + 11 km/h

The distance travelled by two trains = 132 km.

Difference time between the two trains = 1 hour

$i.e.\ \frac{132}{x} - \frac{132}{x+11} = 1$

Multiplying with x (x + 11) on both sides we get,

$\Rightarrow$ 132 (x + 11) – 132 x = x (x + 11)

$\Rightarrow$ 132x + 1452 – 132x = x^2 + 11x

$\Rightarrow$ x^2 + 11x – 1452 = 0

$\Rightarrow$ x^2 + 44x – 33x – 1452 = 0

$\Rightarrow$ x(x + 44) – 33 (x + 44) = 0

$\Rightarrow (x + 44)(x - 33) = 0$

$\Rightarrow (x + 44) = 0$ (or) $(x - 33) = 0$

$\Rightarrow x = -44$ (or) $x = 33$

Speed of the passenger train = 33 km/h ($x \neq -44$ not negative)

Speed of the express train = $x + 11 = 33 + 11 = 44$ km/h

20) Sum of the areas of two squares is $468m^2$. If the difference of their perimeters is 24m, find the sides of the two squares.

Sol: Let the side of the big square = x m.

Then the perimeter of the big square = 4x m.

Difference of the two squares perimeters = 24m.

The perimeter of the small square = $4x - 24$

Side of the small square = $\frac{4x-24}{4} \quad x \quad 6$

Area of the big square = x^2

Area of the small square = $(x - 6)^2$

Given that sum of the areas of two squares is 468 m^2

$\Rightarrow x^2 + (x - 6)^2 = 468$

$\Rightarrow x^2 + x^2 - 12x + 36 = 468$

$\Rightarrow 2x^2 - 12x + 36 - 468 = 0$

$\Rightarrow 2x^2 - 12x - 432 = 0$

$\Rightarrow x^2 - 6x - 216 = 0$

$\Rightarrow x^2 - 18x + 12x - 216 = 0$

$\Rightarrow x(x - 18) + 12(x - 18) = 0$

$\Rightarrow (x - 18)(x + 12) = 0$

$\Rightarrow (x - 18) = 0$ (or) $(x + 12) = 0$

$\Rightarrow$ X = 18 (or) x = –12

Side of the big square x = 18 m(x $\neq$ –12 Not Negative)

Side of the small square = x – 6 = 18 – 6 = 12 m.

21) A ball is thrown vertically upward from the top of the building 96 feet tall with an initial velocity 80 m/second. The distance 's' of the ball from the ground after 't' seconds is $s = 96 + 80t - 16t^2$. After how many seconds does the ball strike the ground?

Sol: Let the ball strike the ground at 't sec.

Distance between the ball and the ground after 't' secs is '0'.

Given that the distance 's' of the ball from the ground after 't' seconds is

$s = 96 + 80t - 16t^2$

i.e $\Rightarrow 96 + 80t - 16t^2 = 0$

$\Rightarrow -16(t^2 - 5t - 6) = 0$

$\Rightarrow (t^2 - 5t - 6) = 0$

$\Rightarrow t^2 - 6t + t - 6 = 0$

$\Rightarrow t(t - 6) + 1(t - 6) = 0$

$\Rightarrow (t + 1) = 0$ (or) $(t - 6) = 0$

$\Rightarrow t = -1$ (or) $t = 6$

Time, 't' never be negative, i.e $t \neq -1$

$\therefore t = 6$

After 6 seconds the ball strike the ground.

22) **If a polygon of 'n' sides has $\frac{1}{2}n(n-3)$ diagonal. How many sides will a polygon having 65 diagonals? Is there a problem with a 50 diagonals?**

Sol: Given that No. of diagonals of a polygon of 'n' sides $\frac{1}{2}n(n-3)$

Number of diagonals of a given polygon = 65

$\frac{1}{2}n(n-3) = 65$

$\Rightarrow$ n(n – 3) = 65 × 2 = 130

$\Rightarrow n^2 - 3n - 130 = 0$

$\Rightarrow n^2 - 13n + 10n - 130 = 0$

$\Rightarrow$ n(n – 13) + 10(n – 13) = 0

$\Rightarrow$(n – 13) (n + 10)=0

$\Rightarrow$ (n – 13) = 0(or) (n + 10) = 0

$\Rightarrow$ n = 13 (or) n = –10

No. of sides are not negative

$\therefore$ Number of sides of a given polynomial = 13

To check there is a polygon with 50 diagonals

i.e. $\frac{1}{2}n(n-3) = 50$

$\Rightarrow$ n(n – 3) = 100

$\Rightarrow n^2 - 3n - 100 = 0$

Compare with $ax^2 + bx + c = 0$, a = 1, b = – 3, c = – 100

$b^2 - 4ac = (-3)^2 - 4(1)(-100) = 9 + 400 = 409$

$n = \quad c$

$= \frac{(\ \ 3)\quad 409}{2(1)}$

$n = \frac{3 \pm \sqrt{409}}{2}$

Since n is not a natural number (countable number)

$\therefore$ We can't find the sides of the polynomial.

$\therefore$ There can't be a polygon with 50 diagonals.

23) Find the discriminant of the following quadratic equations and hence find the nature of its roots. Find them, if they are real?

i. $3x^2 - 2x + 1/3 = 0$ ii. $2x^2 - 3x + 5 = 0$

Sol:

i. Given quadratic equation $3x^2 - 2x + 1/3 = 0$.

Compare with $ax^2 + bx + c = 0$, $a = 3$, $b = -2$, $c = 1/3$

Discriminant $= b^2 - 4ac = (-2)^2 - 4(3)(1/3) = 4 - 4 = 0$

The roots are

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-(-2) \pm \sqrt{0}}{2(3)}$$

$$n = \frac{2}{6} = \frac{1}{3}$$

The two equal roots are $\frac{1}{3}, \frac{1}{3}$

ii. Given quadratic equation $2x^2 - 3x + 5 = 0$.

Compare with $ax^2 + bx + c = 0$, $a = 2$, $b = -3$, $c = 5$

$\therefore$ Discriminant $= b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40 = -31 < 0$

So, the given equation has no real roots.

24) It is possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth?

Sol: Let the breadth of the rectangular mango grove = x m.

$\therefore$ Given that length is twice its breadth.

Length = 2x m.

Area = l × b = 2x × x = 800

$\Rightarrow 2x^2 = 800$

$\Rightarrow x^2 = \frac{800}{2} = 400$

$\Rightarrow x^2 - 400 = 0$ -----(1)

Compare with $ax^2 + bx + c = 0$, a = 1, b = 0, c = – 400

$\therefore$ Discriminant $= b^2 - 4ac = (0)^2 - 4(1)(-400) = 1600 > 0$

(Discriminant > 0)

i.e. $\therefore$ It is possible to find breadth (x), and length (2x).

$\therefore$ It is possible to design a rectangular mango grove.

From equation (1)

$x = - \pm \quad - \quad c$

a

0 1600

2(1)

$\Rightarrow \; = \; 40 \; = 20$

$\therefore$ Breadth (x) = 20m, Length (2x) = 2 × 20 = 40 m.

25) The sum of the ages of two friends is 25 years. Four years ago, the product of their ages in years was 48. Is the situation possible? If so, determine their present ages?

Sol: Let the ages of two friends be x, 20 – x

(Given that sum of the ages of two friends is 20 years)

4 years ago their ages are x – 4 ; 20 – x – 4

Given that $(x-4)(20-x-4) = 48$

$\Rightarrow (x-4)(16-x) = 48$

$\Rightarrow 16x - x^2 - 64 + 4x = 48$

$\Rightarrow -x^2 + 20x - 64 - 48 = 0$

$\Rightarrow x^2 - 20x + 112 = 0$ -------(1)

Compare with $ax^2 + bx + c = 0$, $a = 1$, $b = -20$, $c = 112$

$\therefore$ Discriminant $= b^2 - 4ac = (-20)^2 - 4(1)(112) = 400 - 448 = -48 < 0$

(Discriminant < 0)

It is not possible to find real root for equation (1)

$\therefore$ The situation is not possible.

26) It is possible to design a rectangular park of perimeter 80 m. and area 400m^2 ? If so, find its length and breadth?

Sol: Let the length and breadth be l and b.

Given that perimeter 80m, and area 400m^2.

i.e. $2(l + b) = 80$

$\Rightarrow l + b = 40$ ---- (1)

$\Rightarrow lb = 400$ ----- (2)

Let the breadth of the rectangle be (b) = x m.

Then from equation (1) we get $l + x = 40 \Rightarrow l = 40 - x$

From equation (2) lb = 400

$\Rightarrow$ (40 – x) (x) = 400

$\Rightarrow$ 40x – x^2 = 400

$\Rightarrow$ 40x – x^2 – 400 = 0

$\Rightarrow$ x^2 – 40x + 400 = 0 -----(3)

Compare with $ax^2 + bx + c = 0$, a = 1, b = –40, c = 400

$\therefore$ Discriminant = $b^2 - 4ac = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$

Since discriminant = 0, we can find real roots of equation (3)

i.e. $\therefore$ It is possible to design a rectangular park.

From equation (3)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-40) \pm \sqrt{0}}{2(1)}$$

$$\Rightarrow x = \frac{40}{2} = 20$$

$\therefore$ Breadth (x) = 20m

Length = 40 – x = 40 – 20 = 20 m.

27) Solve the following quadratic equations by factorization method.

i. $4x^2 - 2(a^2 + b^2)x + a^2 b^2 = 0$

ii. $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

Sol:

i. Given quadratic equation $4x^2 - 2(a^2 + b^2)x + a^2 b^2 = 0$

$(4 \times a^2 b^2 = 4a^2 b^2)(4a^2 b^2 = -2a^2 \times -2b^2)$

$\therefore$ $4x^2 - 2(a^2 + b^2)x + a^2 b^2 = 0$

$\Rightarrow 4x^2 - 2a^2 x - 2b^2 x + a^2 b^2 = 0$

$\Rightarrow 2x (2x - a^2) - b^2 (2x - a^2) = 0$

$\Rightarrow (2x - a^2) (2x - b^2) = 0$

$\Rightarrow (2x - a^2) = 0$ (or) $(2x - b^2) = 0$

$$\Rightarrow x = \frac{a^2}{2} \text{ or } \frac{b^2}{2}$$

ii. Given quadratic equation $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

Co-efficient of x^2 × constant term

$= 9 \times (2a^2 + 5ab + 2b^2)$

$= 9 \times (2a^2 + 4ab + ab + 2b^2)$

$= 9 (2a(a + 2b) + b (a + 2b))$

$= 9 (a + 2b) (2a + b)$

$-3 ((a+2b), (2a +b))$

$\therefore 9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

$\Rightarrow 9x^2 - 3(a + 2 b + 2a + b) x + (2a^2 + 5ab + 2b^2) = 0$

$\Rightarrow 9x^2 - 3(a + 2 b)x - 3(2a + b)x +(a + 2 b) (2a + b) = 0$

$\Rightarrow 3x[3x - (a+2b)] - 3 (2a+b)[3x - (a+2b)] = 0$

$\Rightarrow [3x - (a+2b)] [3x - (2a+b)] =0$

$$\Rightarrow \quad = \frac{a+2b}{3} \ (\ r) \ \frac{(2a+b)}{3}$$

Model Problem

Solve the following quadratic equations by factorization method.

(i) $x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1=0$

(ii) $\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$

(iii) $\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}\left(x\neq 2, 4\right)$

(iv) $x^2-2(a^2+b^2)x+(a^2-b^2)^2=0$

Sol: (i) $x=-\frac{a}{a+b}(or)-\frac{(a+b)}{a}$

(ii) x = – a (or) x = – b

(iii) $x=5\ (or)\ \frac{5}{2}$

(iv) x = $(a^2+b^2)\pm 2ab$

28) Solve the following equations by the method of completing the square?

(i) $4x^2+4bx-(a^2-b^2)=0$

(ii) $a^2x^2-3abx+2b^2=0$

Sol:

(i) Given quadratic equation $4x^2+4bx-(a^2-b^2)=0$

Dividing on both sides by 4, we get

$$\Rightarrow x^2+bx-\left(\frac{a^2-b^2}{4}\right)=0$$

$$\Rightarrow x^2+bx=\left(\frac{a^2-b^2}{4}\right)$$

$$\Rightarrow x^2 + 2(x)\left(\frac{b}{2}\right) = \left(\frac{a^2 - b^2}{4}\right)$$

Adding $\left(\frac{b}{2}\right)^2$ on both sides, we get

$$\Rightarrow x^2 + 2(x)\left(\frac{b}{2}\right) + \left(\frac{b}{2}\right)^2 = \left(\frac{a^2 - b^2}{4}\right) + \left(\frac{b}{2}\right)^2$$

$$\Rightarrow \left(x + \frac{b}{2}\right)^2 = \frac{a^2 - b^2 + b^2}{4} = \frac{a^2}{4} = \left(\frac{a}{2}\right)^2$$

$$\Rightarrow x + \frac{b}{2} = \sqrt{\left(\frac{a}{2}\right)^2}$$

$$\Rightarrow x + \frac{b}{2} = \pm\frac{a}{2}$$

$$\Rightarrow x = -\frac{b}{2} \pm \frac{a}{2}$$

$$\Rightarrow x = -\frac{b}{2} + \frac{a}{2} \,(or)\, = -\frac{b}{2} \; -$$

$$\Rightarrow x = \frac{a - b}{2} \,(or)\, x = -\frac{(a + b)}{}$$

(ii) Given quadratic equation $a^2x^2 - 3abx + 2b^2 = 0$

Dividing both sides by a^2, we get

$$\Rightarrow x^2 - \frac{3ab}{a^2}x + 2\frac{b^2}{a^2} = 0$$

$$\Rightarrow x^2 - 3\left(\frac{b}{a}\right)x + 2\frac{b^2}{a^2} = 0$$

$$\Rightarrow x^2 - 3\left(\frac{b}{a}\right)x = -2\frac{b^2}{a^2}$$

$$\Rightarrow x^2 - 2x\left(\frac{3b}{2a}\right)x = -2\frac{b^2}{a^2} \ldots$$

Adding $\left(\frac{3b}{2a}\right)^2$ on both sides

$$\Rightarrow x^2 - 2x\left(\frac{3b}{2a}\right)x + \left(\frac{3b}{2a}\right)^2 = -2\frac{b^2}{a^2} + \left(\frac{3b}{2a}\right)^2$$

$$\Rightarrow \left(x - \frac{3b}{2a}\right)^2 = -2\frac{b^2}{a^2} + \frac{9b^2}{4a^2} = \frac{-8b^2 + 9b^2}{4a^2} = \frac{b^2}{4a^2} = \left(\frac{b}{2a}\right)^2$$

$$\Rightarrow x - \frac{3b}{2a} = \sqrt{\left(\frac{b}{2a}\right)^2} = \pm\frac{b}{2a}$$

$$\Rightarrow x = \pm\frac{b}{2a} + \frac{3b}{2a}$$

$$\Rightarrow x = \frac{\pm b + 3b}{2a}$$

$$\Rightarrow x = \frac{-b + 3b}{2a} (or) x = \frac{+b + 3b}{2a}$$

$$\Rightarrow x = \frac{2b}{2a} (or) x = \frac{4b}{2a}$$

$$\Rightarrow x = \frac{b}{a} (or) x = \frac{2b}{a}$$

Model Problem:

Solve the following quadratic equations by the method of completing the square.

(i) $ax^2 + bx + c = 0$

(ii) $x^2 - 4ax + 4a^2 - b^2 = 0$

(iii) $x^2 - \left(\sqrt{3} + 1\right)x + \sqrt{3} = 0$

(iv) $\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

Answers:

(i) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(ii) $x = 2a - b$ (or) $2a + b$

(iii) $x = \sqrt{3}, 1$

(iv) $-\frac{1}{\sqrt{2}}, 2\sqrt{2}$

29) Solve the following problems, using quadratic formula.

(i) $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

(ii) $a^2 b^2 x^2 - (4b^4 - 3a^4) x - 12a^2b^2 = 0$

Sol:

(i) Given quadratic equation $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

Compare with $Ax^2 + Bx + C = 0$, we get

$A = 12ab$; $B = -(9a^2 - 8b^2)$; $C = -6ab$

Using the quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-\left[-\left(9a^2 - 8b^2\right)\right] \pm \sqrt{\left[-\left(9a^2 - 8b^2\right)\right]^2 - 4(12ab)(-6ab)}}{2(12ab)}$$

$$x = \frac{9a^2 - 8b^2 \pm \sqrt{81a^4 + 64b^4 - 144a^2b^2 + 288a^2b^2}}{24ab}$$

$$x = \frac{9a^2 - 8b^2 \pm \sqrt{81a^4 + 64b^4 + 144a^2b^2}}{24ab}$$

$$x = \frac{9a^2 - 8b^2 \pm \sqrt{\left(9a^2 + 8b^2\right)^2}}{24ab}$$

$$x = \frac{9a^2 - 8b^2 \pm 9a^2 + 8b^2}{24ab}$$

$$x = \frac{9a^2 - 8b^2 + 9a^2 + 8b^2}{24ab} (or)\, x = \frac{9a^2 - 8b^2 - 9a^2 - 8b^2}{24ab}$$

$$x = \frac{18a^2}{24ab} (or)\, x = \frac{-16b^2}{24ab}$$

$$x = \frac{3a}{4b} (or)\, x = \frac{-2b}{3a}$$

(ii) Given quadratic equation $a^2 b^2 x^2 - (4b^4 - 3a^4) x - 12a^2b^2 = 0$

Compare with $Ax^2 + Bx + C = 0$, we get

$A = a^2 b^2$; $B = -(4b^4 - 3a^4)$; $C = -12$

Using the quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-\left[-\left(4b^4 - 3a^4\right)\right] \pm \sqrt{\left(-\left(4b^4 - 3a^4\right)\right)^2 - 4\left(a^2b^2\right)\left(-12a^2b^2\right)}}{2a^2b^2}$$

$$x = \frac{4b^4 - 3a^4 \pm \sqrt{16b^8 + 9a^8 - 24b^4a^4 + 48a^4b^4}}{2a^2b^2}$$

$$x = \frac{4b^4 - 3a^4 \pm \sqrt{16b^8 + 9a^8 + 24b^4a^4}}{2a^2b^2}$$

$$x = \frac{4b^4 - 3a^4 \pm \sqrt{\left(4b^4 + 3a^4\right)^2}}{2a^2b^2}$$

$$x = \frac{4b^4 - 3a^4 \pm 4b^4 + 3a^4}{2a^2b^2}$$

$$x = \frac{4b^4 - 3a^4 + 4b^4 + 3a^4}{2a^2b^2} (or)\, x = \frac{4b^4 - 3a^4 - 4b^4 - 3a^4}{2a^2b^2}$$

$$x = \frac{8b^4}{2a^2b^2} (or)\, x = \frac{-6a^4}{2a^2b^2}$$

$$x = \frac{4b^2}{a^2} (or)\, x = \frac{-3a^2}{2b^2}$$

<u>Model Problem:</u>

Solve the following problem, using quadratic formula.

(i) $(a + b)^2 x^2 + 8 (a^2 - b^2) + 16 (a - b)^2 = 0, a + b \neq 0, a \neq b$

(ii) $3x^2a^2 + 8abx + 4b^2 = 0, a \neq 0$

Answers:

(i) $-4\left(\frac{a-b}{a+b}\right), -4\left(\frac{a-b}{a+b}\right)$

(ii) $-\frac{2b}{a}, -\frac{2b}{3a}$

30) Find the values of k for which the following equation has equal roots.

$(k - 12) x^2 + 2 (k - 12) x + 2 = 0$

Sol: Given quadratic equation $(k - 12)^2 x^2 + 2 (k - 12) x + 2 = 0$ ----(1)

Compare with $ax^2 + bx + c = 0$, we get

$a = k - 12$; $b = 2(k - 12)$; $c = 2$

Given that the roots of equation (1) are equal

i.e. discriminant D = 0

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (2(k - 12))^2 - 4 (k - 12) (2) = 0$

$\Rightarrow 4(k - 12)^2 - 8(k - 12) = 0$

$\Rightarrow 4 [(k - 12)^2 -2(k - 12)] = 0$

$\Rightarrow (k - 12)^2 -2(k - 12)= 0$

$\Rightarrow (k - 12) [(k - 12 -2]= 0$

$\Rightarrow (k - 14) (k - 12) = 0$

$\Rightarrow (k - 14)= 0$ (or) $(k - 12) = 0$

$\Rightarrow k = 14$ (or) $k = 12$

31) Prove that the equation $x^2 (a^2 + b^2) + 2x (ac + bd) + (c^2 + d^2) = 0$ has no real roots, if $ad \neq bc$.

Sol: Given quadratic equation

$x^2 (a^2 + b^2) + 2x (ac + bd) + (c^2 + d^2) = 0$

Compare with $ax^2 + bx + c = 0$, we get

$a = a^2 + b^2$; $b = 2 (ac + bd)$; $c = c^2 + d^2$

Discriminant (d) = $b^2 - 4ac$

$= [2(ac + bd)]^2 - 4 (a^2 + b^2)(c^2 + d^2)$

$= 4(a^2 c^2 + a^2 c^2 + 2abcd) - 4 (a^2 c^2 + a^2 d^2 + b^2 c^2 + b^2 d^2)$

$= 4 [a^2 c^2 + a^2 c^2 + 2abcd - a^2 c^2 - a^2 d^2 - b^2 c^2 - b^2 d^2]$

$= 4[- a^2 d^2 - b^2 c^2 + 2abcd]$

$= -4[a^2 d^2 + b^2 c^2 - 2abcd]$

$= -4[(ad - bc)^2]$

Given that $ad \neq bc$

$\Rightarrow ad - bc \neq 0$

$\Rightarrow (ad - bc)^2 > 0$

$D = -4(ad - bc)^2 < 0$

$(\therefore (ad - bc)^2 < 0 \quad -(ad - bc)^2 > 0)$

Since $D < 0$, the given equation has no real roots.

32) If the roots of the equation $x^2 + 2cx + ab = 0$ are real unequal, prove that the equation $x^2 - 2 (a + b)x + a^2 + b^2 + 2c^2 = 0$ has no real roots.

Sol: Given quadratic equations,

$x^2 + 2cx + ab = 0$ ---- (i)

$x^2 - 2 (a + b)x + a^2 + b^2 + 2c^2 = 0$ ----(ii)

Since the roots of equation (i) are real and unequal.

$\therefore$ Discriminant $d = b^2 - 4ac > 0$

$\Rightarrow (2c)^2 - 4(1)(ab) > 0$

$\Rightarrow 4(c^2 - ab) > 0$

$\Rightarrow c^2 - ab > 0$ ($\therefore$ $4 > 0$)

From the equation (2)

Discriminant $d = b^2 - 4ac$

$\Rightarrow (-2(a + b))^2 - 4(a)(a^2 + b^2 + 2c^2)$

$\Rightarrow 4[a^2 + b^2 + 2ab - a^2 - b^2 - 2c^2]$

$\Rightarrow 4[2ab - 2c^2]$

$\Rightarrow 8(ab - c^2)$

$\Rightarrow -8(c^2 - ab) < 0$ ($\therefore$ $c^2 - ab > 0$)

Since $d < 0$, roots of equation (2) are not real.

SHORT ANSWER QUESTIONS

1) Check whether the following are quadratic equations?

(i) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

(ii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

Sol:

(i) Given $x^3 - 4x^2 - x + 1 = (x - 2)^3$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 + 3x(2)^2 - 3x^2(2)$

$((a - b)^3 = a^3 - b^3 - 3a^2 b + 3ab^2)$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 + 12x - 6x^2$

$\Rightarrow 6x^2 - 4x^2 - x - 12x + 1 + 8 = 0$

$\Rightarrow 2x^2 - 13x + 9 = 0$

It is in the form of $ax^2 + bx + c = 0, a \neq 0$.

Hence the given equation is a quadratic equation.

(ii) Given $(x - 2)(x + 1) = (x - 1)(x + 3)$

$\Rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3$

$\Rightarrow - x - 2 - 2x + 3 = 0$

$\Rightarrow - 3x + 1 = 0$

$\Rightarrow 3x - 1 = 0$

It is not in the form of $ax^2 + bx + c = 0, a \neq 0$.

Hence the given equation is not a quadratic equation.

2) Report the following situation in the form of quadratic equation?

(i) Rohan's mother is 26 years older than him. The product of their ages after 3 years will be 360 years, we need to find Rohan's Present age.

(ii) A train travels a distance of 480km at a uniform speed. If the speed had been 8km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of train?

Sol:

(i) Let Rohan's present age = x years.

Then, the present age of his mother = (x + 26) years (Given)

3 years from now (After 3 years)

Age of Rohan = (x + 3) years

Age of his mother = x + 26 + 3 = x + 29

Given that product of their ages will be 360

i.e. $(x + 3)(x + 29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 = 360$

$\Rightarrow x^2 + 32x - 87 = 0$

$\therefore$ Rohan's present age satisfies the quadratic equation

$x^2 + 32x - 87 = 0$

(ii) Let the uniform speed of the train = x km/hour

The distance travelled by the train = 480 km.

Time taken by the train $= \frac{distance}{speed} = \frac{480}{x}$

If the speed had been 8 km/h less, then the speed of the train = (x – 8) km/h

Time taken by the train when the speed increase $= \frac{480}{x-8}$

Difference between the two timings s = 3 hours

i.e. $\frac{480}{x-8} - \frac{480}{x} = 3$

Uniform speed of train satisfies the quadratic equation.

$\frac{480}{x-8} - \frac{480}{x} = 3$

3) Find two numbers whose sum is 27 and product is 182.

Sol: Let the numbers are x,(27 – x)

($\therefore$ Given sum is 27)

Given that product of that two numbers = 182

$\Rightarrow$ x (27 – x) = 182

$\Rightarrow 27x - x^2 = 182$

$\Rightarrow 27x - x^2 - 182 = 0$

$\Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow x^2 - 13x - 14x + 182 = 0$

$\Rightarrow$ x (x – 13) – 14 (x – 13) = 0

$\Rightarrow$ (x – 13) (x – 14) = 0

$\Rightarrow$ (x – 13) = 0 (or) (x – 14) = 0

$\Rightarrow$ x = 13 (or) x = 14

27 – x = 27 – 13 (or) 27 – 14 = 14 (or) 13

So the required two numbers are 13, 14.

4) Solve the quadratic equation $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$ by factorization method?

Sol:

Given quadratic equation $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$

$$\Rightarrow \frac{2x^2 - 5x - 3}{5} = 0$$

$\Rightarrow 2x^2 - 5x - 3 = 0$

$\Rightarrow 2x^2 - 6x + x - 3 = 0$

$\Rightarrow$ 2x (x – 3) + 1 (x – 3) = 0

$\Rightarrow (x - 3)(2x + 1) = 0$

$\Rightarrow (x - 3) = 0$ (or) $(2x + 1) = 0$

$\Rightarrow x = 3$ (or) $x = -1/2$

5) Solve $9x^2 - 6ax + (a^2 - b^2) = 0$ by factorization method?

Sol: Given $9x^2 - 6ax + (a^2 - b^2) = 0$

$\Rightarrow 9x^2 - [3(a + b) + 3(a - b)]x + (a^2 - b^2) = 0$

$\Rightarrow 9x^2 - 3(a + b)x - 3(a - b)x + (a + b)(a - b) = 0$

$\Rightarrow 3x[3x - (a + b)] - (a - b)[3x - (a + b)] = 0$

$\Rightarrow [3x - (a + b)][3x - (a - b)] = 0$

$\Rightarrow 3x - (a + b) = 0$ (or) $3x - (a - b) = 0$

$x = \frac{a+b}{3} (or) x = \frac{a-b}{3}$

6) The sum of a number and its reciprocal is $2\frac{1}{42}$. Find the number?

Sol: Let the number $= x$

Reciprocal of that number $= 1/x$

Given that, $x + \frac{1}{x} = 2\frac{1}{42}$

$\Rightarrow x + \frac{1}{x} = \frac{85}{42}$

Multiplying both sides with "42x"

$42x^2 + 42 = 85x$

$\Rightarrow 42x^2 + 42 - 85x = 0$

$\Rightarrow 42x^2 - 49x - 36x + 42 = 0$

$\Rightarrow 7x(6x - 7) - 6(6x - 7) = 0$

$\Rightarrow (6x - 7)(7x - 6) = 0$

$\Rightarrow$ (6x – 7) = 0 (or) (7x – 6) = 0

$\Rightarrow$ x = 7/6 (or) x = 6/7

7) Find the roots of the quadratic equation $2x^2 - 7x + 3 = 0$, by using method of completing the square?

Sol: Given $2x^2 - 7x + 3 = 0$

Dividing both sides with "2"

$$\Rightarrow x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

$$\Rightarrow x^2 - \frac{7}{2}x = -\frac{3}{2}$$

$$\Rightarrow x^2 - 2.(x).\frac{7}{4} = -\frac{3}{2}$$

Adding both sides by $\left(\frac{7}{4}\right)^2$

$$\Rightarrow x^2 - 2.(x).\frac{7}{4} + \left(\frac{7}{4}\right)^2 = -\frac{3}{2} + \left(\frac{7}{4}\right)^2$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 = -\frac{3}{2} + \frac{49}{16} = \frac{-24+49}{16} = \frac{25}{16} = \left(\frac{5}{4}\right)^2$$

$$\Rightarrow x - \frac{7}{4} = \sqrt{\left(\frac{5}{4}\right)^2} = \pm\frac{5}{4}$$

$$\Rightarrow x = \pm\frac{5}{4} + \frac{7}{4}$$

$$\Rightarrow x = \frac{5}{4} + \frac{7}{4} (or) x = -\frac{5}{4} + \frac{7}{4} = \frac{12}{4} (or) \frac{2}{4}$$

$$\Rightarrow x = 3 (or) \frac{1}{2}$$

8) Find the roots of the equation $x+\frac{1}{x}=3$ by using quadratic formula?

Sol: Given $x+\frac{1}{x}=3$

Multiplying with ‘x’ on both sides

$\Rightarrow x^2 + 1 = 3x$

$\Rightarrow x^2 - 3x + 1 = 0$

Comparing with $ax^2 + bx + c = 0$, we get a =1, b = -3 c = 1

Discriminant (d) = $b^2 - 4ac = (-3)^2 - 4(1)(1) = 9 - 4 = 5 > 0$

Since d > 0, we can find the real root of given equation.

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2(1)}$$

$$\Rightarrow x = \frac{-(-3) \pm \sqrt{5}}{2(1)}$$

$$\Rightarrow \frac{3 \pm \sqrt{5}}{2}$$

$$\Rightarrow \frac{3+\sqrt{5}}{2} (or) \Rightarrow \frac{3-\sqrt{5}}{2}$$

9) Find the values of k for the quadratic equation kx(x – 2) + 6 = 0. So that they have two real equal roots?

Sol: Given quadratic equation kx(x – 2) + 6 = 0

$\Rightarrow kx^2 - 2kx + 6 = 0$

Comparing with $ax^2 + bx + c = 0$, we get a =k, b = –2k , c = 6

Since the given quadratic has two equal real roots discriminant (d) = 0

$\Rightarrow$ i.e. $b^2 - 4ac = 0$

$\Rightarrow (-2k)^2 - 4(k)(6) = 0$

$\Rightarrow 4k^2 - 24k = 0$

$\Rightarrow 4k(k-6) = 0$

$\Rightarrow k = 0$ (or) $k - 6 = 0$

$\Rightarrow k = 0$ (or) $k = 6$

If $k = 0$ then the equation $0.x(x-2) + 6 = 0 \Rightarrow 6 = 0$. This is not a quadratic equation. So $k \neq 0$.

$\therefore K = 6$

10)If –4 is a root of the quadratic equation $x^2 + px - 4 = 0$ and the quadratic equation $x^2 + px + k = 0$ has equal roots, find the value of k.

Sol: Given equations

$x^2 + px - 4 = 0$ ----(1)

$x^2 + px + k = 0$ ----(2)

–4 is a root of equation(1).

i.e. $(-4)^2 + p(-4) - 4 = 0$

$\Rightarrow 16 - 4p - 4 = 0$

$\Rightarrow 12 - 4p = 0$

$\Rightarrow p = 3$

Substitute $p = 3$ in equation (2) we get,

$x^2 + 3x + k = 0$ ----(3)

Equation (3) has equal roots.

Discriminant $b^2 - 4ac = 0$

$\Rightarrow (3)^2 - 4(1)(k) = 0$

$\Rightarrow 4k = 9$

$\Rightarrow k = 9/4$

MULTIPLE CHOICE QUESTIONS

1) Which of the following is not a quadratic equation. [**B**]

A) $(x-2)^2+1=2x-3$ B) $x(x+1)+8=(x+2)(x-2)$

C) $x(2x+3)=x^2+1$ D) $(x+2)^3=x^3-4$

2) Which of the following is a quadratic equation? [**A**]

A) $(x+1)^2=2(x-3)$ B) $(x-2)(x+1)=(x-1)(x+3)$

C) $x^2+3x+1=(x-2)^2$ D) $x^4-1=0$

3) The sum of a number and its reciprocal is 50/7, then the number is ….[**A**]

A) 1/7 B) 5 C) 2/7 D) 3/7

4) The roots of the equation $3x^2-2\sqrt{6}x+2=0$ are: [**C**]

A) $\frac{2}{\sqrt{3}},-\frac{2}{\sqrt{3}}$ B) $\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}}$ C) $\sqrt{\frac{2}{3}},\sqrt{\frac{2}{3}}$ D) $\frac{1}{\sqrt{3}},\frac{5}{\sqrt{3}}$

5) Which of the following equations has 1/5 as a root? [**A**]

A) $35x^2-2x-1=0$ B) $2x^2-7x-6=0$

C) $10x^2-3x-1=0$ D) $3x^2-2x-1=0$

6) If $x^2-2x+1=0$, then x +1/x = ….then k = …. [**B**]

A) 0 B) 2 C) 1 D) None

7) If 3 is a solution of $3x^2+(k-1)x+9=0$, then k = …. [**B**]

A) 11 B) –11 C) 13 D) –13

8) The roots of $x^2-2x-(r^2-1)=0$ are …. [**B**]

A) 1 – r , –r – 1 B) 1 – r, r + 1 C) 1, r D) 1 – r , r

9) The sum of the roots of the equation $3x^2-7x+11=0$ is …. [**C**]

A) 11/3 B) –7/3 C) 7/3 D) 3/7

10) The roots of the equation $\frac{x^2-8}{x^2+20}=\frac{1}{2}$ are …. [**C**]

A) ± 3 B) ± 2 C) ± 6 D) ± 4

11) The roots of the quadratic equation $\frac{9}{x^2-27}=\frac{25}{x^2-11}$ are …. [**C**]

A) ± 3 B) ± 4 C) ± 6 D) ± 5

12) The roots of the equation $\sqrt{2x^2+9}=9$ are …. [**B**]

A) x = 6 B) x = ±6 C) x = –6 D) 0

13) Which of the following equations has the product of its roots as 4? [**A**]

A) $x^2 + 4x + 4 = 0$ B) $x^2 + 4x – 4 = 0$

C) $– x^2 + 4x + 4 = 0$ D) $x^2 + 4x – 24 = 0$

14) The two roots of a quadratic equation are 2 and –1. The equation is ….[**D**]

A) $x^2 + 2x – 2 = 0$ B) $x^2 + x + 2 = 0$

C) $x^2 + x + 2 = 0$ D) $x^2 – x – 2 = 0$

15) If the sum of a quadratic equation are $3x^2 + (2k + 1)x – (k+5) = 0$, is equal to the product of the roots, then the value of k is….. [**C**]

A) 2 B) 3 C) 4 D) 5

16) The value of k for which 3 is a root of the equation $kx^2 – 7x + 3 = 0$ is ….

[**B**]

A) – 2 B) 2 C) 3 D) – 3

17) If the difference of the roots of the quadratic equation $x^2 – ax + b$ is 1, then[**C**]

A) $a^2 – 4b = 0$ B) $a^2 – 4b = –1$

C) $a^2 – 4b = 1$ D) $a^2 – 4b = 4$

18) The quadratic equation whose one root $2-\sqrt{3}$ is …. [**A**]

A) $x^2 – 4x + 1 = 0$ B) $x^2 + 4x – 1 = 0$

C) $x^2 – 4x –1 = 0$ D) $x^2 – 2x –3 = 0$

19) What is the condition that one root of the quadratic equation $ax^2 + bx + c$ is reciprocal of the other? [**A**]

A) a = c B) a = b C) b = c D) a + b + c = 0

20) The roots of a quadratic equation $\frac{x}{p} = \frac{p}{x}$ are [A]

A) $\pm p$ B) p, 2p C) $-p, 2p$ D) $-p, -2p$

21) If the roots of the equation $12x^2 + mx + 5 = 0$ are real and equal then **m** is equal to[C]

A) $8\sqrt{15}$ B) $2\sqrt{15}$ C) $4\sqrt{15}$ D) $10\sqrt{15}$

22) Which of the following equations has the equal roots? [B]

A) $x^2 + 6x + 5 = 0$ B)) $x^2 - 8x + 16 = 0$

C) $6x^2 - x - 2 = 0$ D) $10x - \frac{1}{x} = 3$

23) If the equation $x^2 - 4x + 9$ has no real roots, then …. [D]

A) $a < 4$ B) $a \leq 4$ C) $a < 2$ D) $a > 4$

24) The discrimination of the quadratic equation $7\sqrt{3}x^2 + 10x - \sqrt{3} = 0$ is …. [C]

A) 142 B) $\frac{-10}{7\sqrt{3}}$ C)184 D) 26

25) The value of $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+......}}}}$ is …. [B]

A) 4 B) 3 C) -2 D) 3.5

FILL IN THE BLANKS

1) Standard form of a quadratic equation is …. ($ax^2 + bx + c = 0, a \neq 0$)

2) The sum of a number and its reciprocal is 5/2. This is represent as …. ($x + \frac{1}{x} = \frac{5}{2}$)

3) "The sum of the squares of two consecutive natural numbers is 25", is represent as …. ($x^2 + (x - 1)^2 = 25$)

4) If one root of a quadratic equation is $7 - \sqrt{3}$ then the other root is …. ($7 + \sqrt{3}$)

5) The discriminant of $5x^2 - 3x - 2 = 0$ is …. (49)

6) The roots of the quadratic equation $x^2 - 5x + 6 = 0$ are …. (2, 3)

7) If x = 1 is a common root of the equations $ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$ then the value of ab is …. (3)

8) If the discriminant of the quadratic equation $ax^2 + bx + c = 0$ is zero, then the roots of the equation are …. (Real and equal)

9) The product of the roots of the quadratic equation $\sqrt{2}x^2 - 3x + 5\sqrt{2} = 0$ is …. (5)

10) The nature of the roots of a quadratic equation $4x^2 - 12x + 9 = 0$ is …. (real and equal)

11) If the equation $x^2 - bx + 1 = 0$ does not possess real roots, then ….

($b^2 - 4 < 0$ (or) $b^2 < 4$ (or) $-2 < b < 2$)

12) If the sum of the roots of the equation $x^2 - (k + 6)x + 2(2k - 1) = 0$ is equal to half of their product, then k = …. (7)

13) If one root of the equation $4x^2 - 2x + (\lambda - 4) = 0$ be the reciprocal of the other, then λ = …. (8)

14) If $\sin\alpha$ and $\cos\alpha$ are the roots of the equation $ax^2 + bx + c = 0$, then b^2 = ….($a^2 + 2ac$)

15) If the roots of the equation $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$ are equal, then b^2 = …. (ac)

16) The quadratic equation whose roots are –3, –4 is …. ($x^2 + 7x + 12 = 0$)

17) If $b^2 - 4ac < 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ are ….

(Not real or imaginary)