AREAS RELATED TO CIRCLES

AREA AND PERIMETER OF CIRCLE, QUADRANT, SEMICIRCLE

Area of Circle = πr^2, Perimeter of Circle = Circumference = $2\pi r$

Area of Semicircle = $\frac{1}{2}\pi r^2$, Perimeter of Semicircle = $\pi r + 2r$

Area of Quadrant = $\frac{1}{4}\pi r^2$, Perimeter of Quadrant = $\frac{1}{2}\pi r + 2r$

IMPORTANT QUESTONS

Find the diameter of the circle whose area is equal to the sum of the areas of the two circles of diameters 20 cm and 48 cm.

Solution: Here, radius r_1 of first circle = 20/2 cm = 10 cm
and radius r_2 of the second circle = 48/2 cm = 24 cm
Therefore, sum of their areas = $\pi r_1^2 + \pi r_2^2 = \pi (10)^2 + \pi (24)^2 = \pi \times 676$
Let the radius of the new circle be r cm. Its area = πr^2
Therefore, $\pi r^2 = \pi \times 676 \quad \Rightarrow r^2 = 676 \quad \Rightarrow r = 26$
Thus, radius of the new circle = 26 cm
Hence, diameter of the new circle = 2×26 cm = 52 cm

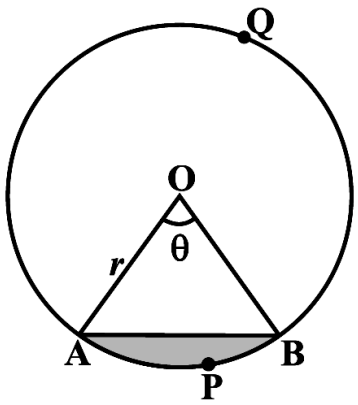
Questions for Practice

1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.
2. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.
3. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?
4. Find the area of a quadrant of a circle whose circumference is 22 cm.

AREAS OF SECTOR AND SEGMENT OF A CIRCLE

Area of the sector of angle $\theta = \frac{\theta}{360^0} \times \pi r^2$, where r is the radius of the circle and θ the angle of the sector in degrees

length of an arc of a sector of angle $\theta = \frac{\theta}{360^0} \times 2\pi r$, where r is the radius of the circle and θ the angle of the sector in degrees



Area of the segment APB = Area of the sector OAPB – Area of Δ OAB

$$= \frac{\theta}{360^0} \times \pi r^2 - \text{area of } \Delta \text{ OAB}$$

- Area of the major sector OAQB = πr^2 – Area of the minor sector OAPB
- Area of major segment AQB = πr^2 – Area of the minor segment APB
- Area of segment of a circle = Area of the corresponding sector – Area of the corresponding triangle

IMPORTANT QUESTIONS

Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector (Use π = 3.14).

Solution : Here, radius, r = 4 cm, $\theta = 30^0$,

We know that Area of sector = $\frac{\theta}{360^0} \times \pi r^2 = \frac{30^0}{360^0} \times 3.14 \times 4 \times 4 = \frac{1}{12} \times 3.14 \times 4 \times 4$

$= \frac{12.56}{3} = 4.19 cm^2$ (approx.)

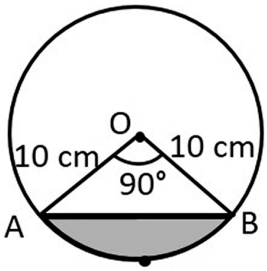
Area of the corresponding major sector
= πr^2 – area of sector OAPB
= (3.14 × 16 – 4.19) cm^2
= 46.05 cm2 = 46.1 cm^2(approx.)

A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding : (i) minor segment (ii) major sector. (Use π = 3.14)

Solutions: Here, radius, r = 10 cm, $\theta = 90^0$,

We know that Area of minor sector = $\frac{\theta}{360^0} \times \pi r^2 = \frac{90^0}{360^0} \times 3.14 \times 10 \times 10 = \frac{1}{4} \times 314 = 78.5 cm^2$

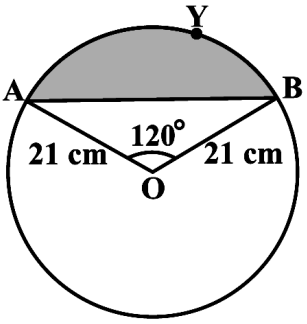
and Area of triangle AOB = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 10 \times 10 = 50 cm^2$



Area of minor segment = Area of minor sector –
Area of triangle AOB = 78.5 – 50 = 28.5 cm^2.
Area of circle = $\pi r^2 = 3.14 \times 10 \times 10 = 314 cm^2$
Area of major sector = Area of circle – Area of minor sector
= 314 – 78.5 = 235.5 cm^2

Questions for Practice

1. Find the area of the segment AYB shown in below figure, if radius of the circle is 21 cm and ∠AOB = 120°.



2. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

4. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find (i) the area of that part of the field in which the horse can graze. (ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)

5. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors. Find : (i) the total length of the silver wire required. (ii) the area of each sector of the brooch.

6. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find: (i) the length of the arc (ii) area of the sector formed by the arc (iii) area of the segment formed by the corresponding chord

7. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

8. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

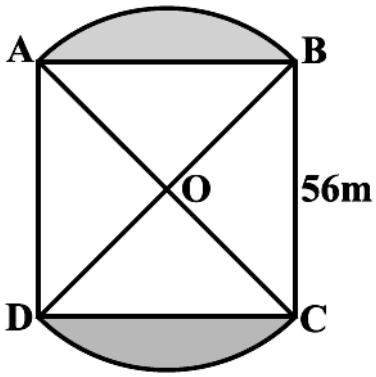
9. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

10. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$)

AREA OF SHADED REGION BASED QUESTIONS

IMPORTANT QUESTIONS

In the adjoining figure, two circular flower beds have been shown on two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and the flower beds.



Solution: Here, side of square ABCD, a = 56 m

diagonal of square = $a\sqrt{2} = 56\sqrt{2}$

radius, r = OA = OB = OC = OD = $\frac{56\sqrt{2}}{2} = 28\sqrt{2}cm$

Now, Area of sector OAB = Area of sector ODC

$= \frac{\theta}{360^0} \times \pi r^2 = \frac{90^0}{360^0} \times \frac{22}{7} \times r^2 = \frac{1}{4} \times \frac{22}{7} \times r^2$

and Area of Δ OAD = Area of Δ OBC = $\frac{1}{2} \times r \times r = \frac{1}{2} \times r^2$

Total area = Area of sector OAB + Area of sector ODC
+ Area of Δ OAD + Area of Δ OBC

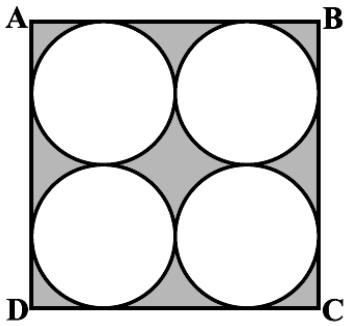
$= \frac{1}{4} \times \frac{22}{7} \times r^2 + \frac{1}{4} \times \frac{22}{7} \times r^2 + \frac{1}{2} \times r^2 + \frac{1}{2} \times r^2$

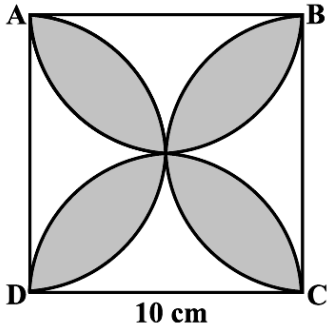
$= 2 \times \frac{1}{4} \times \frac{22}{7} \times r^2 + 2 \times \frac{1}{2} \times r^2 = \frac{11}{7} \times r^2 + r^2 = \left(\frac{11}{7} + 1\right) r^2$

$= \frac{18}{7} \times 28 \times 28 \times 2 = 4032cm^2$

Questions for Practice

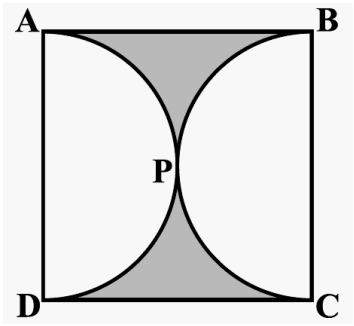
1. Find the area of the shaded region in below left figure, where ABCD is a square of side 14 cm.

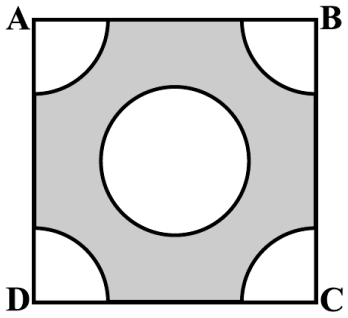




2. Find the area of the shaded design in above right figure, where ABCD is a square of side 10 cm and semicircles are drawn with each side of the square as diameter. (Use $\pi = 3.14$)

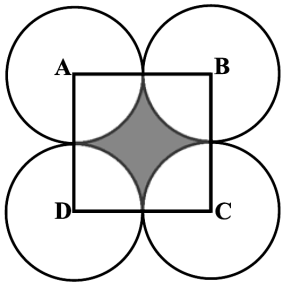
3. Find the area of the shaded region in below left figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.

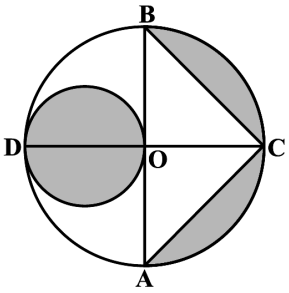




4. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in above right sided figure. Find the area of the remaining portion of the square.

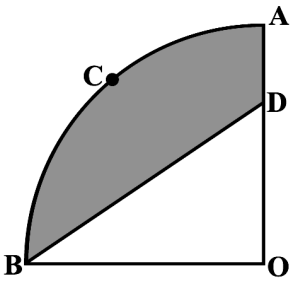
5. In the below left figure, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.

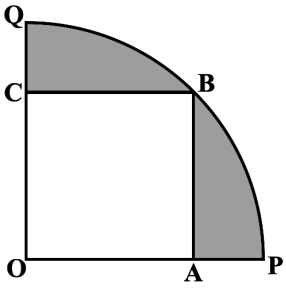




6. In the above right sided figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

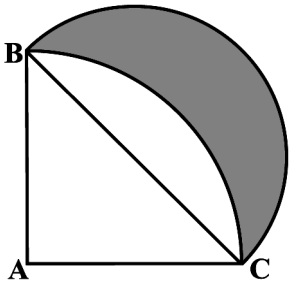
7. In the below left figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.

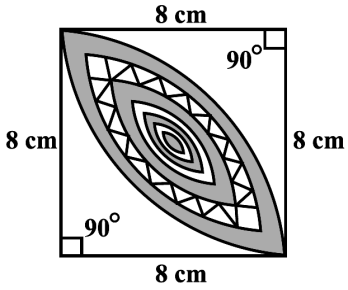




8. In the above right sided figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the (i) quadrant OACB, (ii) shaded region.

9. In the below figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use $\pi = 3.14$)





10. Calculate the area of the designed region in above right sided figure, common between the two quadrants of circles of radius 8 cm each.

MCQ QUESTIONS (1 mark)

1. If the area of a circle is 154 cm^2, then its perimeter is
(a) 11 cm (b) 22 cm (c) 44 cm (d) 55 cm

2. If θ is the angle (in degrees) of a sector of a circle of radius r, then area of the sector is
(a) $\frac{\pi r^2\theta}{360^0}$ (b) $\frac{\pi r^2\theta}{180^0}$ (c) $\frac{2\pi r\theta}{360^0}$ (d) $\frac{2\pi r\theta}{180^0}$

3. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then
(a) $R_1 + R_2 = R$ (b) $R_1^2 + R_2^2 = R^2$ (c) $R_1 + R_2 < R$ (d) $R_1^2 + R_2^2 < R^2$

4. Area of the largest triangle that can be inscribed in a semi-circle of radius r units is
(a) r^2 sq. units (b) $\frac{1}{2}$ r^2 sq. units (c) 2 r^2 sq. units (d) $\sqrt{2}$ r^2 sq. units

5. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is
(a) 22 : 7 (b) 14 : 11 (c) 7 : 22 (d) 11: 14

6. The area of the circle that can be inscribed in a square of side 6 cm is
(a) 36 π cm^2 (b) 18 π cm^2 (c) 12 π cm^2 (d) 9 π cm^2

7. The area of the square that can be inscribed in a circle of radius 8 cm is
(a) 256 cm^2 (b) 128 cm^2 (c) $64\sqrt{2}$ cm^2 (d) 64 cm^2

8. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36cm and 20 cm is
(a) 56 cm (b) 42 cm (c) 28 cm (d) 16 cm

9. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is
(a) 31 cm (b) 25 cm (c) 62 cm (d) 50 cm

10. A wire is looped in the form of a circle of radius 28 cm. It is rebent into a square form. Determine the length of the side of the square.
(a) 42 cm (b) 44 cm (c) 46 cm (d) 48 cm

11. A circular part, 42 m in diameter has a path 3.5 m wide running round it on the outside. Find the cost of gravelling the path at Rs. 4 per m^2.
(a) Rs. 2800 (b) Rs. 2020 (c) Rs. 2002 (d) none of these

12. The diameter of the wheels of a bus is 140 cm. How many revolutions per minute must a wheel make in order to move a t a speed of 66km/hr?
(a) 240 (b) 250 (c) 260 (d) 270