Rational Numbers

Concepts Related To Rational Numbers

ou have studied fractional numbers in your earlier classes. Some examples of fractional numbers are $\frac{1}{2}, \frac{-4}{7}, \frac{22}{27}$.

These numbers are also known as **rational numbers.**

What comes first to your mind when you hear the word **rational?**

Yes,you are right. It is something related to the ratios.

The ratio 4:5 can be written as $\frac{4}{5}$ **,** which is a rational number. In ratios, the numerator and denominator both are positive numbers while in rational numbers, they can be negative also.

Thus, rational numbers can be defined as follows.

"Any number which can be expressed in the form $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$, is called a rational number."

For example, $\frac{15}{19}$ is a rational number in which the numerator is 15 and the denominator is 19.

Now, is −34 a rational number?

Yes, it is a rational number. −34 can be written as $\frac{-34}{1}$. It is in the form of $\frac{p}{q}$ and $q \neq 0$.

Thus, we can say that **every integer is a rational number.**

Now, consider the following decimal numbers.

1.6, 3.49, and 2.5

These decimal numbers are also rational numbers as these can be written as $\frac{16}{10}, \frac{349}{100}$, and $\frac{25}{10}$

If in a rational number, either the numerator or the denominator is a negative integer, then the rational number is negative.

For example, $\frac{-5}{12}$ and $\frac{6}{-7}$ are **negative rational numbers.**

If the numerator and the denominator both are either positive integers or negative integers, then the rational number is positive.

For example, $\frac{-9}{-4}$ and $\frac{56}{5}$ **are positive rational numbers.**

Conventions used for writing a rational number:

We know that in a rational number, the numerator and denominator both can be positive or negative.

Conventionally, rational numbers are written with positive denominators.

For example, −9 can be represented in the form of a rational number as $\frac{-9}{1}$ or $-\frac{9}{1}$ or $\frac{9}{-1}$, but generally we do not write the denominator negative and thus, $\frac{9}{-1}$ is eliminated. So, according to the convention, −9 can be represented in the form of a rational number as $\frac{-9}{1}$ or $-\frac{9}{1}$.

Equality relation for rational numbers:

For any four non-zero integers *p*, *q*, *r* and *s*, we have

$\frac{p}{q} = \frac{r}{s}$ if $ps = qr$

Order relation for rational numbers:

If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers such that $q > 0$ and $s > 0$ then it can be said that $\frac{p}{q} > \frac{r}{s}$ if $ps > qr$.

Absolute Value of a Rational Number:

The absolute value of a rational number is its numerical value regardless of its sign. The absolute value of a rational number $\frac{p}{q}$ is denoted as $\left|\frac{p}{q}\right|$.

Therefore, $\left|-\frac{3}{2}\right| = \frac{3}{2}$, $\left|\frac{12}{-7}\right| = \frac{12}{7}$ etc.

Note: The absolute value of any rational number is always non-negative.

Now, let us go through the given example.

Example:

Write each of the following rational numbers according to the convention.

i) $\frac{8}{-15}$

ii) $\frac{-1131}{-729}$

Solution:

According to the convention used in rational numbers, the denominator must be a positive number.

Let us now write the given numbers according to the convention.

i)

In the number $\frac{8}{-15}$, denominator is negative.

We have,

$$\frac{8}{-15} = \frac{-8}{15} = -\frac{8}{15}$$

According to convention, the given number should be written as $\frac{-8}{15}$ or $-\frac{8}{15}$.

ii)

In the number $\frac{-1131}{-729}$, denominator is negative.

We have,

$$\frac{-1131}{-729} = \frac{1131}{729}$$

According to convention, the given number should be written as $\frac{1131}{729}$.

Example:

Find the absolute value of the following:

(i) $\left|-\frac{121}{71}\right|$

(ii) $\left|\frac{12}{19}\right|$

Solution:

(i) Absolute value = $\left|-\frac{121}{71}\right| = \frac{121}{71}$

(ii) Absolute value = $\left|\frac{12}{19}\right| = \frac{12}{19}$

Standard Forms and Equivalent Rational Numbers

$\frac{3}{7}$, $\frac{12}{28}$, and $\frac{9}{21}$

Are these rational numbers representing the same rational number?

Any rational number can be written with different numerators and denominators by multiplying both the numerator and denominator with the same non-zero integer.

For example, let us consider the number $\frac{3}{4}$ which can be written as follows:

$$\frac{3}{4} = \frac{3\times 2}{4\times 2} = \frac{6}{8}$$

$$\frac{3}{4} = \frac{3\times(-5)}{4\times(-5)} = \frac{-15}{-20} = \frac{(-1)\times 15}{(-1)\times 20} = \frac{15}{20}$$

Here, the rational numbers $\frac{6}{8}$ and $\frac{15}{20}$ arise from the same rational number $\frac{3}{4}$.

Therefore, $\frac{6}{8}$ and $\frac{15}{20}$ are said to be equivalent to $\frac{3}{4}$.

Here, $\frac{3}{4}$ is already in the standard form.

Thus, we can conclude that

"***Equivalent rational numbers have the same standard form.***"

Now, like multiplication, **does division by the same non-zero integer lead to form an equivalent rational number?**

Let us consider the rational number $\frac{25}{30}$.

On dividing the numerator and denominator by 5, we obtain

$$\frac{25}{30} = \frac{25 \div 5}{30 \div 5} = \frac{5}{6}$$

Now, if we multiply both the numerator and denominator of $\frac{5}{6}$ by 5, then we will obtain the same rational number $\frac{25}{30}$.

Therefore, $\frac{5}{6}$ and $\frac{25}{30}$ are equivalent rational numbers.

Thus, we can say that

"An equivalent rational number of a rational number can be obtained by multiplying or dividing the numerator and the denominator by the same non-zero integer."

Let us solve some more examples to understand the concept better.

Example 1:

Write the next two numbers in the following patterns.

(i) $\frac{-3}{4}, \frac{-6}{8}, \frac{-9}{12}, \ldots$

(ii) $\frac{1}{5}, \frac{3}{15}, \frac{5}{25}, \ldots$

Solution:

(i) $\frac{-3}{4}, \frac{-6}{8}, \frac{-9}{12}, \ldots$

Here, we can observe that in the given pattern, the multiplication of $\frac{-3}{4}$ is carried out with 1, 2, and 3 respectively.

$$\frac{-3}{4} = \frac{(-3)\times 1}{4\times 1} = \frac{-3}{4}$$
$$\frac{-3}{4} = \frac{(-3)\times 2}{4\times 2} = \frac{-6}{8}$$
$$\frac{-3}{4} = \frac{(-3)\times 3}{4\times 3} = \frac{-9}{12}$$

Therefore, the next two numbers can be obtained by multiplication with 4 and 5.

The next two numbers are

$$\frac{-3}{4} = \frac{(-3)\times 4}{4\times 4} = \frac{-12}{16}$$
$$\frac{-3}{4} = \frac{(-3)\times 5}{4\times 5} = \frac{-15}{20}$$

(ii) $\frac{1}{5}, \frac{3}{15}, \frac{5}{25}, \ldots$

Here, we can observe that in the given pattern, the multiplication of $\frac{1}{5}$ is carried out with 1, 3, and 5 respectively.

$$\frac{1}{5}=\frac{1\times 1}{5\times 1}=\frac{1}{5}$$
$$\frac{1}{5}=\frac{1\times 3}{5\times 3}=\frac{3}{15}$$
$$\frac{1}{5}=\frac{1\times 5}{5\times 5}=\frac{5}{25}$$

Therefore, the next two numbers can be obtained by multiplication with 7 and 9.

The next two numbers are

$$\frac{1}{5}=\frac{1\times 7}{5\times 7}=\frac{7}{35}$$
$$\frac{1}{5}=\frac{1\times 9}{5\times 9}=\frac{9}{45}$$

Example 2:

Reduce $\frac{9}{-45}$ to its standard form.

Solution:

Here, the denominator is a negative integer. Therefore, before finding out the H.C.F. of 9 and (−45), we have to make the denominator positive.

On multiplying both the numerator and the denominator by (−1), we obtain

$$\frac{9}{-45}=\frac{9\times(-1)}{(-45)\times(-1)}=\frac{-9}{45}$$

Now,

9 = 3 × 3

45 = 3 × 3 × 5

H.C.F. of 9 and 45 = 3 × 3 = 9

Now, dividing both the numerator and the denominator by the H.C.F., we obtain

$\frac{-9}{45}=\frac{(-9)\div 9}{45\div 9}=\frac{-1}{5}$

Thus, $\frac{-1}{5}$ is the standard form of $\frac{9}{-45}$

Example 3:

In which of the following pairs are the numbers equivalent to each other?

(i) $\frac{25}{36},\frac{6}{9}$

(ii) $\frac{-5}{6},\frac{15}{-18}$

Solution:

(i) $\frac{25}{36},\frac{6}{9}$

$\frac{25}{36}$ cannot be obtained by multiplying any integer with $\frac{6}{9}$.

Therefore, $\frac{25}{36}$ and $\frac{6}{9}$ are not equivalent to each other.

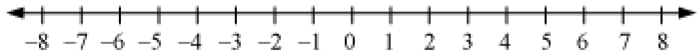
(ii) $\frac{-5}{6},\frac{15}{-18}$

$\frac{15}{-18}$ can be written as $\frac{15}{-18}=\frac{(-5)\times(-3)}{6\times(-3)}=\frac{-5}{6}$

Therefore, $\frac{-5}{6}$ and $\frac{15}{-18}$ are equivalent to each other.

Rational Numbers on Number Line

A number line has numbers marked at equal distances as shown in the figure.



Every point on the number line represents a number. We know how to locate integers and positive fractions in which the numerator is less than the denominator on the number line.

Now, how can we represent the rational numbers in which numerator is greater than the denominator?

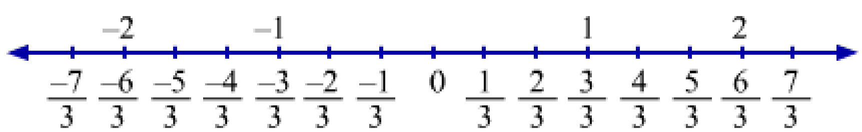
To represent such rational numbers on the number line, we write them as mixed fractions. Let us go through the following video to understand the method of representing $\frac{5}{2}$ and $\frac{-13}{3}$ on the number line.

In this way, we can represent rational numbers on the number line.

The ordinal relationship between rational numbers:

Defining ordinal relationship between two rational numbers is to find out that which number is greater and which one is smaller.

Look at the following number line.



On observing the number line, following points are obtained about the ordinal relationship between rational numbers:

(1) Out of any two numbers on the number line, the number on the left is smaller whereas the number on the right is greater.

(2) On the number line, all negative numbers lie on the left of zero. Thus, all negative numbers are smaller than zero.

(3) All negative numbers along with zero lie on the left of positive numbers. Thus, each positive number is greater than all negative numbers and zero.

(4) Out of the numbers having same denominator, the number having greater numerator is greater.

Let us solve some more examples to understand the concept better.

Example 1:

Represent the rational numbers $\frac{1}{3}, \frac{-5}{3}$, and 2 on a number line.

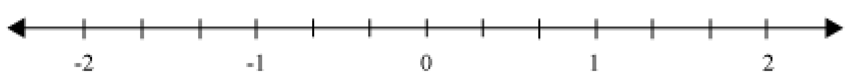
Solution:

Here, $\frac{1}{3}$ and $\frac{-5}{3}$ have a common denominator 3.

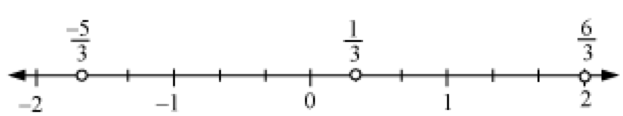
And we can convert 2 into a rational number with denominator 3 by multiplying the numerator and denominator by 3. Therefore, 2 can be written as $\frac{6}{3}$.

By converting all of them into rational numbers having a common denominator, it will become easier to represent them on the number line.

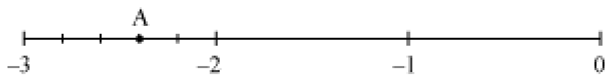
First, each part of the number line between two integers is divided into three equal parts as shown below.



Then, $\frac{1}{3}$ can be marked between 0 and 1. To mark $\frac{-5}{3}$, we move 5 units to the left of 0 and to mark $2\left(=\frac{6}{3}\right)$, we move 6 units to the right of 0.



Example 2:
A is a point on the following number line.



What is the rational number represented by the point A?

Solution:

In the given number line, we may note that the number line between −3 and −2 is divided into five equal parts. The point A is 2 units left to −2. Therefore, the rational number represented by the point A is $-2-2\times\frac{1}{5}=\frac{-12}{5}=-2\frac{2}{5}$.

Finding Rational Numbers between Given Rational Numbers

Let's summarize.

We know that each point on the number line represents a number. Thus, between any two rational numbers, there are infinitely many numbers on the number line.

Let us try to find some rational numbers between $\frac{1}{6}$ and $\frac{7}{8}$.

To find the rational numbers between $\frac{1}{6}$ and $\frac{7}{8}$, firstly we have to make their denominators same.

2	6,	8
2	3,	4
2	3,	2
3	3,	1
	1,	1

The L.C.M. of 6 and 8 is 2 × 2 × 2 × 3 = 24

Now, we can write

$$\frac{1}{6}=\frac{1\times4}{6\times4}=\frac{4}{24}$$
$$\frac{7}{8}=\frac{7\times3}{8\times3}=\frac{21}{24}$$

Therefore, between $\frac{4}{24}\left(\frac{1}{6}\right)$ and $\frac{21}{24}\left(\frac{7}{8}\right)$, we can find many rational numbers.

Some of them are

$$\frac{5}{24},\frac{6}{24},\frac{7}{24},\frac{8}{24},\frac{9}{24},\frac{10}{24},\frac{11}{24},\frac{12}{24},\frac{13}{24},\frac{14}{24},\frac{15}{24},\frac{16}{24},\frac{17}{24},\frac{18}{24},\frac{19}{24},\frac{20}{24}$$

Let us solve some more examples to understand the concept better.

Example 1:

Find three rational numbers between $\frac{-1}{15}$ and $\frac{1}{9}$.

Solution:

The first step is to find the L.C.M. of 15 and 9.

3	15,	9
3	5,	3
5	5,	1
	1,	1

The L.C.M. of 15 and 9 is 3 × 3 × 5 = 45

Now, we can write

$$\frac{-1}{15} = \frac{(-1)\times 3}{15\times 3} = \frac{-3}{45}$$
$$\frac{1}{9} = \frac{1\times 5}{9\times 5} = \frac{5}{45}$$

Therefore, three rational numbers between $\frac{-1}{15}$ and $\frac{1}{9}$ are $\frac{-2}{45}, \frac{0}{45}(=0)$, and $\frac{1}{45}$.

Example 2:

Find 10 rational numbers between $\frac{2}{5}$ and $\frac{5}{7}$.

Solution:

The first step is to find the L.C.M. of 5 and 7.

5	5,	7
7	1,	7
	1,	1

The L.C.M. of 5 and 7 is 5 × 7 = 35

Now, we can write

$$\frac{2}{5} = \frac{2\times 7}{5\times 7} = \frac{14}{35}$$
$$\frac{5}{7} = \frac{5\times 5}{7\times 5} = \frac{25}{35}$$

Therefore, 10 rational numbers between

$\frac{2}{5}$ and $\frac{5}{7}$ are $\frac{15}{35}\left(\frac{3}{7}\right), \frac{16}{35}, \frac{17}{35}, \frac{18}{35}, \frac{19}{35}, \frac{20}{35}\left(\frac{4}{7}\right), \frac{21}{35}\left(\frac{3}{5}\right), \frac{22}{35}, \frac{23}{35}$ and $\frac{24}{35}$.

Comparing and Ordering Rational Numbers

Can we find the greater number between $\frac{1}{5}$ and $\frac{7}{6}$?

We know the comparison of fractions. Therefore, we can easily compare $\frac{1}{5}$ and $\frac{7}{6}$.

We compare two or more fractions by making their denominators same.

The common denominator of two or more fractions is the L.C.M. of their denominators.

2	5	6
3	5	3
5	5	1
	1	1

The L.C.M. of 5 and 6 is 2 × 3 × 5 = 30

Now, we can write

$$\frac{1}{5}=\frac{1\times 6}{5\times 6}=\frac{6}{30}$$
$$\frac{7}{6}=\frac{7\times 5}{6\times 5}=\frac{35}{30}$$

Here, the denominators of two rational numbers are same. The numerator of first rational number (i.e., 6) is less than the numerator of the second fraction (i.e., 35).

Thus, we can say that $\frac{6}{30}<\frac{35}{30} \Rightarrow \frac{1}{5}<\frac{7}{6}$

We compare two rational numbers in this way. Using the same rule, we can compare negative rational numbers.

In order to compare two negative rational numbers, we compare the numbers neglecting the negative sign and then we reverse the order of numbers and put the negative sign again.

For example, let us compare $\left(\frac{-3}{4}\right)$ and $\left(\frac{-5}{6}\right)$.

Neglecting the negative sign, we have the numbers $\left(\frac{3}{4}\right)$ and $\left(\frac{5}{6}\right)$.

The L.C.M. of 4 and 6 is 12.

Therefore, we can write

$$\frac{3}{4}=\frac{3\times3}{4\times3}=\frac{9}{12}$$
$$\frac{5}{6}=\frac{5\times2}{6\times2}=\frac{10}{12}$$

Here, the denominators of two rational numbers are the same. The numerator of first rational number (i.e., 9) is less than the numerator of the second rational number (i.e., 10) or 9 < 10.

Therefore, $\frac{9}{12}<\frac{10}{12} \Rightarrow \frac{3}{4}<\frac{5}{6}$

Now, reversing the order and restoring the negative signs, we obtain $\frac{-3}{4}>\frac{-5}{6}$

In comparison of a positive rational number with a negative rational number, the positive rational number is always greater than the negative rational number.

For example, $\frac{6}{7}$ is greater than $\frac{-1}{5}$.

Greatest and smallest rational numbers:

On a number line, there are infinite rational numbers on both sides of 0.

If we move to right of 0 then we observe that the numbers become bigger. **So, it is not possible to determine the greatest rational number.**

Similarly, if we move to left of 0 then we observe that the numbers become smaller. **So, it is not possible to determine the smallest rational number.**

Rules to compare the rational numbers:

Let us take any two rational numbers such as $\frac{p}{q}$ and $\frac{r}{s}$. We can compare these rational numbers using the following rules:

(1) If $p \times s < r \times q$ then $\frac{p}{q} < \frac{r}{s}$
(2) If $p \times s = r \times q$ then $\frac{p}{q} = \frac{r}{s}$
(3) If $p \times s > r \times q$ then $\frac{p}{q} > \frac{r}{s}$

To learn the application of these rules, let us compare few rational numbers.

For $\frac{2}{3}$ and $\frac{15}{18}$, we have $p = 2$, $q = 3$, $r = 15$ and $s = 18$.

Now,

$p \times s = 2 \times 18 = 36$

$r \times q = 15 \times 3 = 45$

Clearly, $p \times s < r \times q$ and thus, $\frac{p}{q} < \frac{r}{s}$.

Thus, $\frac{2}{3} < \frac{15}{18}$.

For $\frac{3}{4}$ and $\frac{21}{28}$, we have $p = 3$, $q = 4$, $r = 21$ and $s = 28$.

Now,

$p \times s = 3 \times 28 = 84$

$r \times q = 21 \times 4 = 84$

Clearly, $p \times s = r \times q$ and thus, $\frac{p}{q} = \frac{r}{s}$.

Thus, $\frac{3}{4} = \frac{21}{28}$.

For $\frac{7}{12}$ and $\frac{11}{19}$, we have $p = 7$, $q = 12$, $r = 11$ and $s = 19$.

Now,

$p \times s = 7 \times 19 = 133$

$r \times q = 11 \times 12 = 132$

Clearly, $p \times s > r \times q$ and thus, $\frac{p}{q} > \frac{r}{s}$.

Thus, $\frac{7}{12} > \frac{11}{19}$.

Let us solve more examples to understand the concept better.

Example 1:

Arrange the rational numbers $\frac{-3}{7}, \frac{6}{-9}, \frac{3}{5}$, and $\frac{-1}{-2}$ in decreasing order.

Solution:

The L.C.M. of 5, 2, 7, and 9 is $5 \times 2 \times 7 \times 9 = 630$

Now, we can write

$$\frac{-3}{7} = \frac{(-3)\times 90}{7\times 90} = \frac{-270}{630}$$
$$\frac{6}{-9} = \frac{-6}{9} = \frac{(-6)\times 70}{9\times 70} = \frac{-420}{630}$$
$$\frac{3}{5} = \frac{3\times 126}{5\times 126} = \frac{378}{630}$$
$$\frac{-1}{-2} = \frac{1}{2} = \frac{1\times 315}{2\times 315} = \frac{315}{630}$$

Denominators of all the rational numbers are same. Therefore, the rational number with the greater numerator will be greater than the others.

Since $378 > 315 > -270 > -420$,

Therefore, $\frac{378}{630} > \frac{315}{630} > \frac{-270}{630} > \frac{-420}{630}$

$\Rightarrow \frac{3}{5} > \frac{-1}{-2} > \frac{-3}{7} > \frac{6}{-9}$

Thus, the numbers in decreasing order are $\frac{3}{5} > \frac{-1}{-2} > \frac{-3}{7} > \frac{6}{-9}$.

Example 2:

Compare the rational numbers given in each pair.

(a) $\frac{13}{15}$ and $\frac{104}{120}$

(b) $\frac{9}{17}$ and $\frac{12}{23}$

(c) $\frac{-15}{7}$ and $\frac{-27}{13}$

Solution:

(a)

For $\frac{13}{15}$ and $\frac{104}{120}$, we have $p = 13$, $q = 15$, $r = 104$ and $s = 120$.

Now,

$p \times s = 13 \times 120 = 1560$

$r \times q = 104 \times 15 = 1560$

Clearly, $p \times s = r \times q$ and thus, $\frac{p}{q} = \frac{r}{s}$.

Thus, $\frac{13}{15} = \frac{104}{120}$.

(b)

For $\frac{9}{17}$ and $\frac{12}{23}$, we have $p = 9$, $q = 17$, $r = 12$ and $s = 23$.

Now,

$p \times s = 9 \times 23 = 207$

$r \times q = 12 \times 17 = 204$

Clearly, $p \times s > r \times q$ and thus, $\frac{p}{q} > \frac{r}{s}$.

Thus, $\frac{9}{17} > \frac{12}{23}$.

(c)

For $\frac{-15}{7}$ and $\frac{-27}{13}$, we have $p = -15$, $q = 7$, $r = -27$ and $s = 13$.

Now,

$p \times s = -15 \times 13 = -195$

$r \times q = -27 \times 7 = -189$

Clearly, $p \times s < r \times q$ and thus, $\frac{p}{q} < \frac{r}{s}$.

Thus, $\frac{-15}{7} < \frac{-27}{13}$.

Addition And Subtraction Of Rational Numbers

Many times instead of adding whole numbers, we have to add rational numbers in which we have to follow a specific method. This method is explained in the following example.

Vikram went to a market and bought $\frac{3}{4}$ kg apples and $\frac{2}{4}$ kg mangoes.

Hence, we can make the following conclusions.

1) Addition of rational numbers having the same denominator

When two rational numbers having the same denominator are added, only their numerators are added keeping the denominator same as that of the given numbers and the resultant number is reduced to the simplest form, if possible.

2) Addition of rational numbers with different denominators

To add the rational numbers having different denominators, first the rational numbers are converted to their equivalent rational numbers having common denominator. Then, the numerators are added keeping the common denominator same and the resultant number is reduced to the simplest form, if possible.

Now, let us add $\frac{-3}{5}$ and $\frac{3}{5}$.

Now, let us learn about the **subtraction** of rational numbers.

Rahul gave $\frac{2}{3}$ of a chocolate to Sonu and Sonu gave $\frac{1}{4}$ of it to his younger brother.

How much chocolate is left with Sonu?

Hence, we can conclude that the subtraction of a rational number from another rational number is same as the addition of the additive inverse of the rational number that is being subtracted to the other rational number.

Let us find the value of $\left(23\frac{1}{3}-5\frac{3}{4}-6\frac{1}{6}\right)$.

$$\begin{aligned}23\frac{1}{3}-5\frac{3}{4}-6\frac{1}{6} &= \frac{70}{3}-\frac{23}{4}-\frac{37}{6}\\ &= \frac{70}{3}+\frac{(-23)}{4}+\frac{(-37)}{6}\end{aligned}$$

Now, we have to find the L.C.M. of 3, 4, and 6.

2	3 4 6
2	3 2 3
3	3 1 3
	1 1 1

The L.C.M. of 3, 4, and 6 is 2 × 2 × 3 = 12

Now, $\frac{70}{3}+\frac{(-23)}{4}+\frac{(-37)}{6}$

$=\frac{70\times 4}{3\times 4}+\frac{(-23)\times 3}{4\times 3}-\frac{(-37)\times 2}{6\times 2}$

$=\frac{280}{12}+\frac{(-69)}{12}+\frac{(-74)}{12}$

$=\frac{280-69-74}{12}$

$=\frac{137}{12}$

$=\frac{(12\times 11)+5}{12}$

$=11\frac{5}{12}$

Opposite Rational Numbers: If the sum of two rational numbers is 0 then the numbers are called opposite numbers. Also, each number is said to be the additive inverse of the other.

For example, −11 + 11 = 0. Thus, −11 and 11 are opposite numbers as well as these are additive inverse of each other.

Similarly, $-\frac{2}{3}+\frac{2}{3}=0$. Thus, $-\frac{2}{3}$ and $\frac{2}{3}$ are opposite numbers. Also, these are additive inverse of each other.

Let us solve a few more examples to understand the concept better.

Example 1:

Find the value of $\left(\frac{13}{4}+\frac{29}{3}+\frac{11}{2}\right)$.

Solution:

Since the denominators of rational numbers are different, we have to take the L.C.M. of the denominators.

The L.C.M. of 4, 3, and 2 is 12.

$$
\begin{aligned}
&=\frac{13}{4}+\frac{29}{3}+\frac{11}{2} \\
&=\frac{13\times 3}{4\times 3}+\frac{29\times 4}{3\times 4}+\frac{11\times 6}{2\times 6} \\
&=\frac{39}{12}+\frac{116}{12}+\frac{66}{12} \\
&=\frac{39+116+66}{12}=\frac{221}{12} \\
&=\frac{(12\times 18)+5}{12}=18+\frac{5}{12} \\
&=18\frac{5}{12}
\end{aligned}
$$

Thus, the value of the given expression $\left(\frac{13}{4}+\frac{29}{3}+\frac{11}{2}\right)$ is $18\frac{5}{12}$.

Example 2:

Write the additive inverse of each of the following numbers.

(a) 625 **(b)** $-\frac{5}{12}$ **(c)** $\frac{11}{25}$

(d) –15 (e) 21 (f) $\frac{46}{45}$

Solution:

(a)

We have,

625 + (–625) = 625 – 625 = 0

Thus, –625 is the additive inverse of 625.

(b)

We have,

$$-\frac{5}{12}+\frac{5}{12}=0$$

Thus, $\frac{5}{12}$ is the additive inverse of $-\frac{5}{12}$.

(c)

We have,

$$\frac{11}{25}+\left(-\frac{11}{25}\right)=\frac{11}{25}-\frac{11}{25}=0$$

Thus, $-\frac{11}{25}$ is the additive inverse of $\frac{11}{25}$.

(d)

We have,

–15 + 15 = 0

Thus, 15 is the additive inverse of –15.

(e)

We have,

21 + (−21) = 21 − 21 = 0

Thus, 21 is the additive inverse of −21.

(f)

We have,

$$\frac{46}{45}+\left(-\frac{46}{45}\right)=\frac{46}{45}-\frac{46}{45}=0$$

Thus, $-\frac{46}{45}$ is the additive inverse of $\frac{46}{45}$.

Example 3:

The distance between two stations A and B is $3\frac{5}{6}$ km, that of B and C is $6\frac{3}{5}$ km, and that of C and D is $5\frac{1}{2}$ km. The stations are in a straight line as shown in the given figure. Find the distance between A and D. If Ritu went from A to D and returned on the same path through a distance of $2\frac{1}{3}$ km, then how much distance is she away from A?

Solution:

Distance between A and D = Distance between A and B + Distance between B and C +

Distance between C and D

$$=\left(3\frac{5}{6}+6\frac{3}{5}+5\frac{1}{2}\right)\text{ km}$$

$$=\left(\frac{23}{6}+\frac{33}{5}+\frac{11}{2}\right)\text{ km}$$

Now, in order to add the above rational numbers, we have to find the L.C.M. of 6, 5, and 2 to make their denominators same.

The L.C.M. of 6, 5, and 2 is 30.

Now,

$$\frac{23}{6}+\frac{33}{5}+\frac{11}{2} \quad =\frac{23\times 5}{6\times 5}+\frac{33\times 6}{5\times 6}+\frac{11\times 15}{2\times 15}$$
$$=\frac{115}{30}+\frac{198}{30}+\frac{165}{30}$$
$$=\frac{115+198+165}{30}$$
$$=\frac{478}{30}$$

$$=\frac{(15\times 30)+28}{30}$$
$$=15+\frac{28}{30}$$
$$=15+\frac{14}{15}$$
$$=15\frac{14}{15}$$

Thus, the distance between A and D is $15\frac{14}{15}$ km.

After reaching D, Ritu returned towards A (opposite direction). Therefore, the distance between A and her current position will be obtained by subtracting $2\frac{1}{3}$ km from the distance AD.

Let this distance be *x* km.

Therefore, $x=15\frac{14}{15}\text{ km}+\left(-2\frac{1}{3}\right)\text{ km}=\frac{239}{15}+\frac{(-7)}{3}\text{ km}$

The L.C.M. of 15 and 3 is 15.

$$\frac{239}{15}+\frac{(-7)}{3} = \frac{239\times1}{15\times1}+\frac{(-7)\times5}{3\times5}=\frac{239}{15}+\frac{(-35)}{15}=\frac{239-35}{15}=\frac{204}{15}$$

$$=\frac{(13\times15)+9}{15}$$

$$=13+\frac{9}{15}$$

$$=13+\frac{3}{5}$$

$$=13\frac{3}{5}$$

Thus, the distance x is $13\frac{3}{5}$ km.

Example 4:

Two months ago, weight of Raj was $65\frac{1}{2}$ kg. He reduced $3\frac{3}{4}$ kg weight in two months. How much does he weigh now?

Solution:

Weight of Raj two months ago = $65\frac{1}{2}$ kg

Weight reduced = $3\frac{3}{4}$ kg

∴ Present weight = Weight two months ago − Weight reduced

$$=\left(65\frac{1}{2}-3\frac{3}{4}\right)\text{ kg}=\left(\frac{131}{2}-\frac{15}{4}\right)\text{ kg}=\left[\frac{131}{2}+\frac{(-15)}{4}\right]\text{ kg}$$

The L.C.M. of 2 and 4 is 4.

∴ Present weight $=\frac{131}{2}+\frac{(-15)}{4}$

$$= \frac{131 \times 2}{2 \times 2} + \frac{(-15) \times 1}{4 \times 1}$$
$$= \frac{262}{4} + \frac{(-15)}{4}$$
$$= \frac{262 - 15}{4}$$
$$= \frac{247}{4}$$
$$= \frac{(61 \times 4) + 3}{4}$$
$$= 61 + \frac{3}{4}$$
$$= 61\frac{3}{4}$$

Thus, his present weight is $61\frac{3}{4}$ kg.

Multiplication And Division Of Rational Numbers

Multiplication and division of rational numbers are required in many real life situations to make the calculations easier. Following examples explain the method used to multiply and divide rational numbers.

There are fifty sticks, each of length $\frac{1}{3}$ m. Bimal joins them end to end. What is the total length of the arrangement?

Hence, we can conclude the following.

To multiply a rational number by another rational number, the denominator of one rational number is multiplied with the denominator of the other and the numerator of one rational number is multiplied with the numerator of the other. Then, the resultant is simplified, if possible.

The division of a rational number by another rational number is same as the multiplication of the dividend by the reciprocal of the divisor.

Now, what will be $\left(-3\frac{1}{2}\right)\div\frac{3}{7}$?

$$\left(-3\frac{1}{2}\right)\div\frac{3}{7}=\left(-\frac{7}{2}\right)\div\frac{3}{7}=\left(-\frac{7}{2}\right)\times\frac{7}{3}=-\frac{49}{6}$$

Note: Division of a non-zero rational number by itself gives 1.

For example,

$$2\div 2=\frac{2}{2}=1,$$
$$(-10)\div(-10)=\frac{-10}{-10}=1,$$
$$\frac{4}{5}\div\frac{4}{5}=\frac{4}{5}\times\frac{5}{4}=1 \text{ and}$$
$$\left(-\frac{7}{11}\right)\div\left(-\frac{7}{11}\right)=\left(-\frac{7}{11}\right)\times\left(-\frac{11}{7}\right)=1$$

Let us solve some examples to understand the concept better.

Example 1:

There are six baskets full of fruits each weighing $1\frac{4}{5}$ kg. What is the total weight of the six baskets?

Solution:

Number of baskets = 6

Weight of each basket = $1\frac{4}{5}$ kg

Therefore,

Total weight of the six baskets $=6\times 1\frac{4}{5}\text{kg}$

$= \left(6 \times \frac{9}{5}\right) \text{kg}$

$= \left(\frac{6 \times 9}{5}\right) \text{kg}$

$= \frac{54}{5} \text{kg}$

Now, converting into standard form, we obtain

$= \left[\frac{(10 \times 5) + 4}{5}\right] \text{kg}$

$= \left(10 + \frac{4}{5}\right) \text{kg}$

$= 10\frac{4}{5} \text{kg}$

Thus, the total weight of the six baskets is $10\frac{4}{5} \text{kg}$.

Example 2:

Amit bought a pipe of length $4\frac{1}{5}$ m. He cut the pipe into smaller pieces each of length $\frac{3}{5}$ m. How many pieces was the pipe cut into?

Solution:

It is given that length of each piece = $\frac{3}{5}$ m

Total length of pipe = $4\frac{1}{5}$ m = $\frac{21}{5}$ m

∴ Number of pieces $= \frac{21}{5} \div \frac{3}{5} = \frac{21}{5} \times \frac{5}{3} = \frac{21}{3} = 7$

Thus, the pipe was cut into 7 pieces.