Chapter 1 Similarity

Practice set 1.1

1. Base of a triangle is 9 and height is 5. Base of another triangle is 10 and height is 6. Find the ratio of areas of these triangles.

Solution:
Let base of the first triangle is b_1 and height is h_1. Let base of second triangle is b_2 and height is h_2. Ratio of areas of two triangles is equal to the ratio of the products of their bases and corresponding heights. Here $b_1 = 9$ $h_1 = 5$ $b_2 = 10$ $h_1 = 6$

Then ratio of their areas $= b_1 \times h_1 / b_2 \times h_2$

$= 9 \times 5 / 10 \times 6$

$= 3/4$

Hence, the ratio of the areas of these triangles is 3:4

2. In figure 1.13 BC⊥ AB, AD⊥ AB, BC = 4, AD = 8, then find A(ΔABC) /A(Δ ADB) .

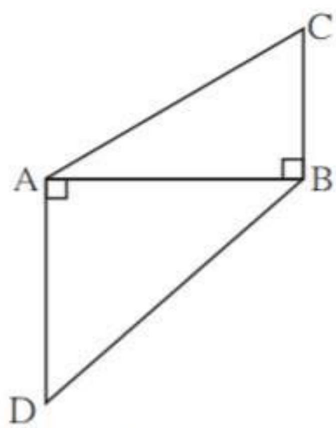


Fig. 1.13

Solution:
Here ΔABC and ΔADB have the same base AB.
Areas of triangles with equal bases are proportional to their corresponding heights.
Since bases are equal, areas are proportional to heights.
Given BC = 4 and AD = 8
So, A(ΔABC) /A(Δ ADB) = BC/AD

$= 4/8$

$= 1/2$

Hence, ratio of areas of ΔABC and ΔADB is 1:2.

3. In adjoining, figure 1.14 seg PS ⊥ seg RQ, seg QT ⊥ seg PR. If RQ = 6, PS = 6 and PR = 12, then find QT.

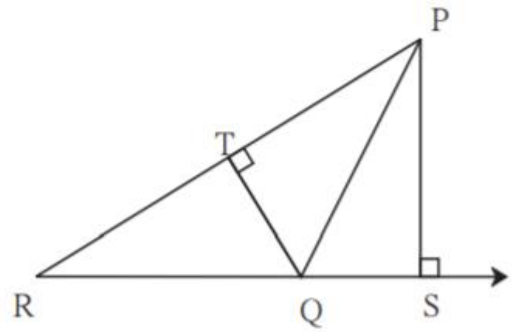


Fig. 1.14

Solution:

Given PS ⊥ RQ and QT ⊥ PR.

RQ = 6

PS = 6

PR = 12

Area of ΔPQR with base PR and height QT = (1/2)×PR×QT

Area of ΔPQR with base QR and height PS = (1/2)×QR×PS

A(ΔPQR)/ A(ΔPQR) = (1/2)×PR×QT /(1/2)×QR×PS

1 = PR×QT/ QR×PS

1 = 12×QT/6×6

6×6 = QT×12

QT = 36/12

QT = 3

Hence, measure of side QT is 3 units.

4. In adjoining figure, AP ⊥ BC, AD || BC, then find A(Δ ABC):A(Δ BCD).

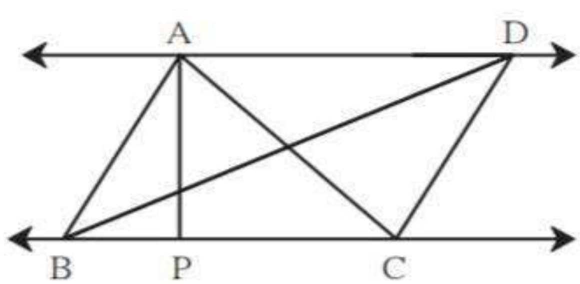


Fig. 1.15

Solution:
Given, AP ⊥ BC, and AD || BC. Δ ABC and Δ BCD has same base BC.
Areas of triangles with equal bases are proportional to their corresponding heights.
Since AP is the perpendicular distance between parallel lines AD and BC, height of ΔABC and height of ΔBCD are same.
∴ A(Δ ABC) /A(Δ BCD) = AP/AP = 1
Hence, A(Δ ABC): A(Δ BCD) = 1:1

5. In adjoining figure PQ ⊥ BC, AD ⊥ BC then find following ratios.
(i) A(Δ PQB) /A(Δ PBC)
(ii) A(ΔPBC) / A(Δ ABC)
(iii) A(Δ ABC) /A(Δ ADC)
(iv) A(Δ ADC) /A(Δ PQC)

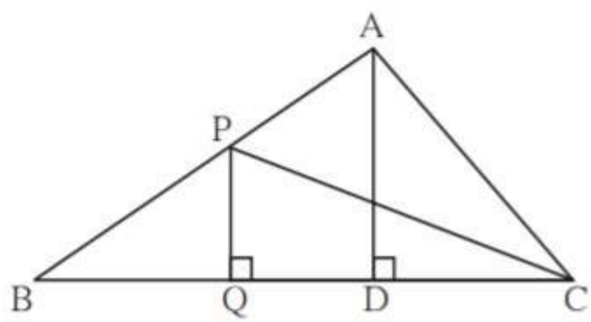


Fig. 1.16

Solution:
(i) ΔPQB and ΔPBC have same height PQ.
Ratio of areas of triangles with equal heights are proportional to their corresponding bases.
∴ A(ΔPQB)/A(ΔPBC) = BQ/BC

(ii) ΔPBC and ΔABC have same base BC.
Ratio of areas of triangles with equal bases are proportional to their corresponding heights.
∴ A(ΔPBC) / A(Δ ABC) = PQ/AD

(iii) Δ ABC) and Δ ADC have equal heights AD.
Ratio of areas of triangles with equal heights are proportional to their corresponding bases.
∴ A(Δ ABC) /A(Δ ADC) = BC/DC

(iv) Ratio of areas of two triangles is equal to the ratio of the products of their bases and corresponding heights.
∴ A(Δ ADC) /A(Δ PQC) = DC×AD/QC×PQ

Practice set 1.2

1. Given below are some triangles and lengths of line segments. Identify in which figures, ray PM is the bisector of ∠QPR.

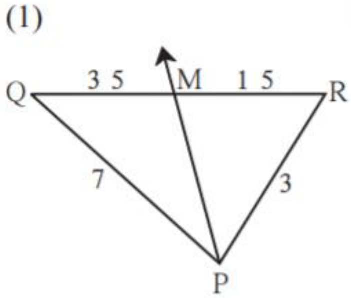


Fig. 1.33

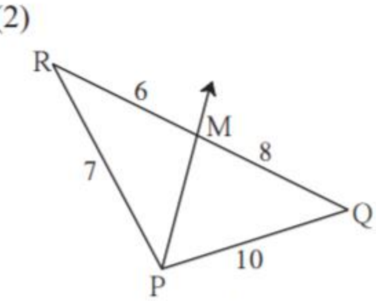


Fig. 1.34

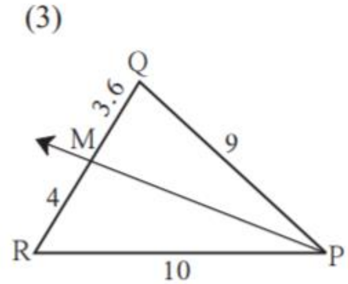


Fig. 1.35

Solution:

(i) In ΔPQR
QM/RM = 35/15 = 7/3 …..(i)
PQ/PR = 7/3……………….(ii)
From (i) and (ii) QM/RM = PQ/PR
∴By Converse of angle bisector theorem, Ray PM is the bisector of ∠QPR.

(ii) In ΔPQR
PR/PQ = 7/10……..(i)
RM/QM = 6/8……..(ii)
From (i) and (ii) PR/PQ ≠ RM/QM
∴Ray PM is not the bisector of ∠QPR

(iii) In ΔPQR PR/PQ = 10/9………(i)
RM/QM = 4/3.6 = 40/36 = 10/9 ……..(ii)
From (i) and (ii) PR/PQ = RM/QM
∴By Converse of angle bisector theorem, Ray PM is the bisector of ∠QPR.

2. In Δ PQR, PM = 15, PQ = 25 PR = 20, NR = 8. State whether line NM is parallel to side RQ. Give reason.

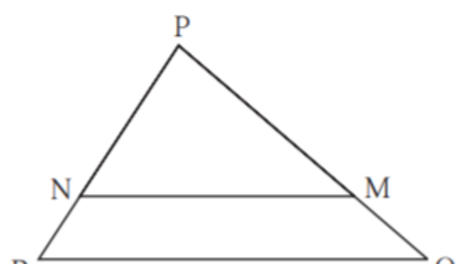


Fig. 1.36

Solution:
Given PM = 15, PQ = 25, PR = 20, NR = 8
PQ = PM+MQ
25 = 15+MQ
MQ = 25-15
MQ = 10
PR = PN+NR
20 = PN+8
PN = 20-8
PN = 12
PM/MQ = 15/10 = 3/2
PN/NR = 12/8 = 3/2
In ΔPQR , PM/MQ = PN/NR.
By Converse of basic proportionality theorem, line NM ǁ side RQ.

3. In Δ MNP, NQ is a bisector of ∠N. If MN = 5, PN = 7 MQ = 2.5 then find QP .

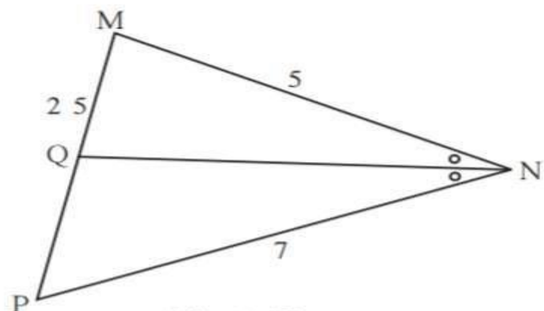


Fig. 1.37

Solution:
Given MN = 5, PN = 7, MQ = 2.5
Since NQ is a bisector of ∠N, PN/MN = QP/MQ [Angle bisector theorem]
7/5 = QP/2.5
QP = 7×2.5/5
QP = 3.5
Hence, measure of QP is 3.5 units.

4. Measures of some angles in the figure are given. Prove that AP/PB = AQ/QC

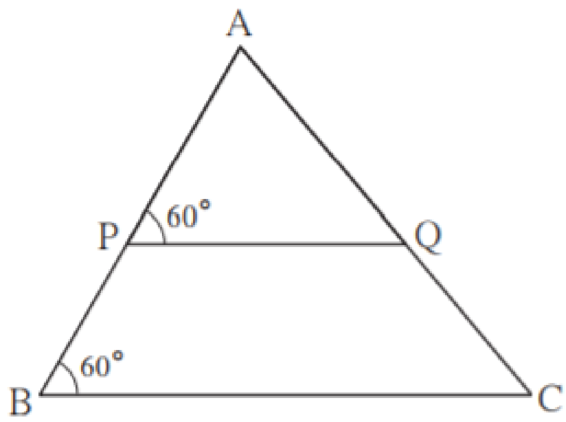


Fig. 1.38

Solution:

∠ABC = 60˚ [Given]

∠APQ = 60˚ [Given]

Since the corresponding angles are equal, line PQ ll BC.

In ΔABC, PQ ll BC.

∴ By basic proportionality theorem, AP/PB = AQ/QC
Hence proved.

5. In trapezium ABCD, side AB ||side PQ ||side DC, AP = 15, PD = 12, QC = 14, find BQ.

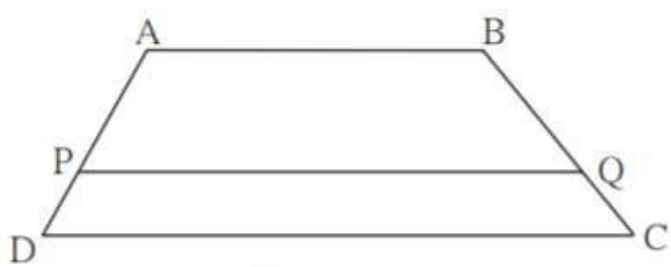


Fig. 1.39

Solution:

Given AB ll PQ ll DC.

AP = 15

PD = 12

QC = 14

AP/PD = BQ/QC [Property of three parallel lines and their transversals]

15/12 = BQ /14

BQ = 15×14/12

BQ = 17.5 units.

Hence, measure of BQ is 17.5 units.

6. Find QP using given information in the figure.

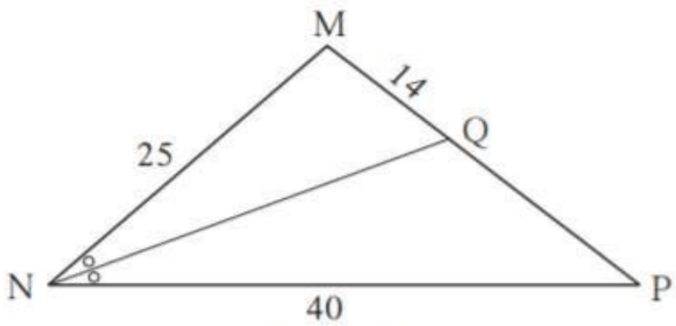


Fig. 1.40

Solution:
From figure MN = 25, NP = 40, MQ = 14
Given, NQ bisects ∠MNP.

∴ MN/NP = MQ/QP [Angle bisector theorem]
25/40 = 14/QP
QP = 40×14/25
QP = 22.5
Hence, measure of QP is 22.5 units.

7. In figure 1.41, if AB || CD || FE then find x and AE.

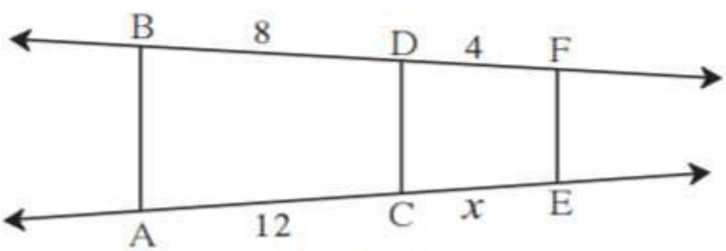


Fig. 1.41

Solution:
From figure BD = 8, DF = 4, AC = 12 and CE = x
Given AB ll CD ll FE
∴ BD/DF = AC/CE [Property of three parallel lines and their transversals]
8/4 = 12/x
x = 12×4/8
x = 6
∴ CE = 6
AE = AC+CE
∴ AE = 12+6
∴ AE = 18 Hence, measure of x is 6 units and AE is 18 units.

8. In D LMN, ray MT bisects ∠LMN. If LM = 6, MN = 10, TN = 8, then find LT.

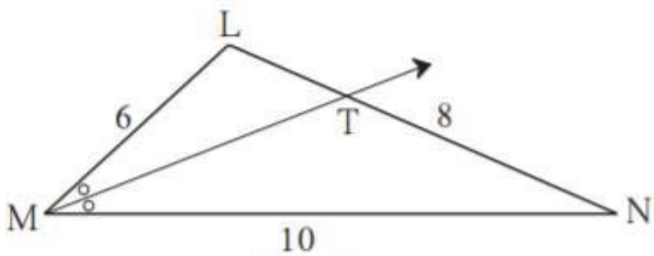


Fig. 1.42

Solution:

Given, ray MT bisects ∠LMN.

LM = 6
MN = 10
TN = 8
Since ray MT bisects ∠LMN , LM/MN = LT/TN [Angle bisector theorem] 6/10 = LT/8
LT = 6×8/10
LT = 4.8
Hence, measure of LT is 4.8 units.

9. In ΔABC, seg BD bisects ∠ABC. If AB = x, BC = x+5, AD = x-2, DC = x+2, then find the value of x.

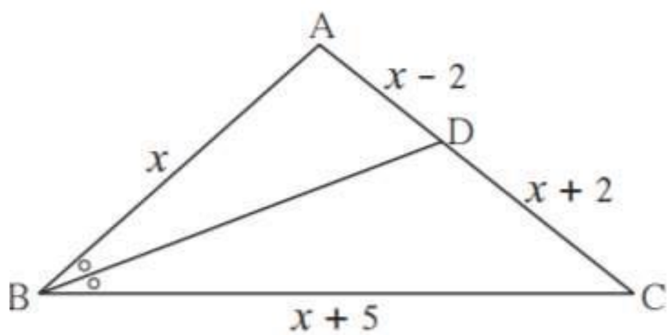


Fig. 1.43

Solution:

Given, BD bisects ∠ABC.
Also AB = x, BC = x+5
AD = x-2, DC = x+2
Since BD bisects ∠ABC, AB/BC = AD/DC [Angle bisector theorem]
x/(x+5) = (x-2)/(x+2)
Cross-multiplying, we get
x(x+2) = (x+5)(x-2)

$x^2+2x = x^2+5x-2x-10$
$x^2+2x = x^2+3x-10$
$x = 10$
∴ The value of x is 10.

10. In the figure 1.44, X is any point in the interior of triangle. Point X is joined to vertices of triangle. Seg PQ || seg DE, seg QR || seg EF. Fill in the blanks to prove that, seg PR || seg DF.

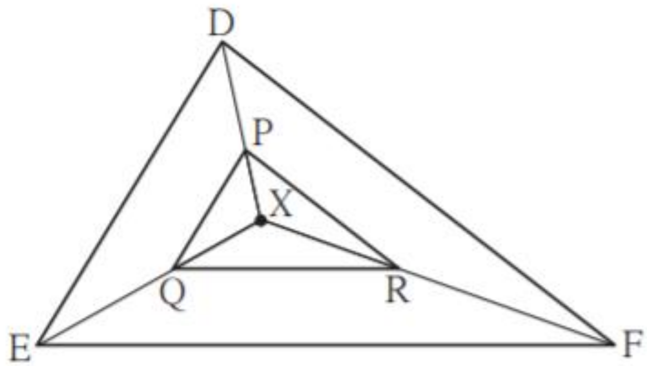


Fig. 1.44

Proof : In Δ XDE, PQ || DE ...._______

∴ XP/________ = ______/ QE .... (I) (Basic proportionality theorem)

In Δ XEF, QR || EF .... ________

∴ _______ /_______= _______ /_______ ....(II) _______

∴ _______ /_______= _______ /_______ .... from (I) and (II)

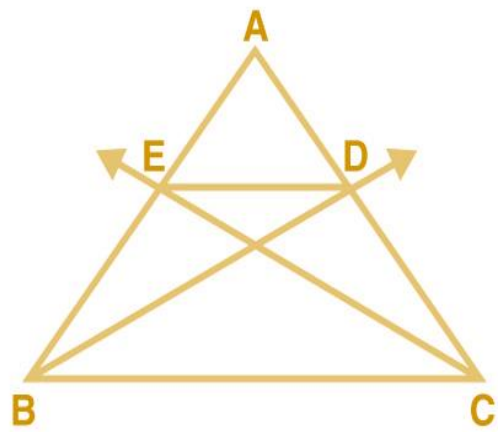
seg PR || seg DE .... (Converse of basic proportionality theorem)

Solution:

In Δ XDE, PQ || DE……. Given
∴ XP/PD = XQ/QE…….. (I) (Basic proportionality theorem)
In Δ XEF, QR || EF……. Given
∴ XR/RF = XQ/QE……. .(II) (Basic proportionality theorem)
∴ XP/PD = XR/RF .... from (I) and (II)
∴ seg PR || seg DE .... (Converse of basic proportionality theorem)

11*. In Δ ABC, ray BD bisects ∠ABC and ray CE bisects ∠ACB. If seg AB ≅ seg AC then prove that ED || BC.

Solution:



Given, In ΔABC ray BD bisects ∠ABC.

∴ AB/BC = AD/CD ………..(i) [Angle bisector theorem]

Since ray CE bisects ∠ACB

AC/BC = AE/BE …………….(ii) [Angle bisector theorem] Given seg AB = seg AC.

Substitute AB in (ii)

AB/BC = AE/BE…..(iii)

From (i) AD/CD = AE/BE [in (i) AB/BC = AD/CD]

∴ED || BC [converse of basic proportionality theorem]

Hence proved.

Practice set 1.3

1. In figure1.55, ∠ABC = 75°, ∠EDC = 75° state which two triangles are similar and by which test? Also, write the similarity of these two triangles by a proper one to one correspondence.

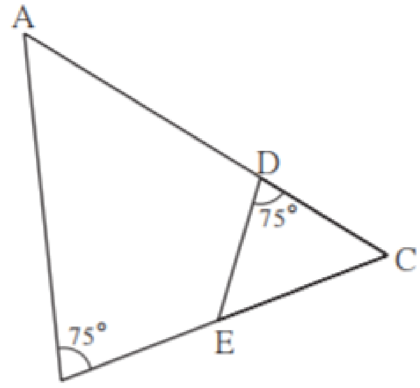


Fig. 1.55

Solution:

Given ∠ABC = 75°, ∠EDC = 75°

Consider ΔABC and ΔEDC

∠ABC = ∠EDC [Given ∠ABC = 75°, ∠EDC = 75°]

∠ACB = ∠DCE [Common angle]

∴ ΔABC ~ ΔEDC [AA test of similarity]

One to one correspondence is ABC↔ EDC

2. Are the triangles in figure 1.56 similar? If yes, by which test?

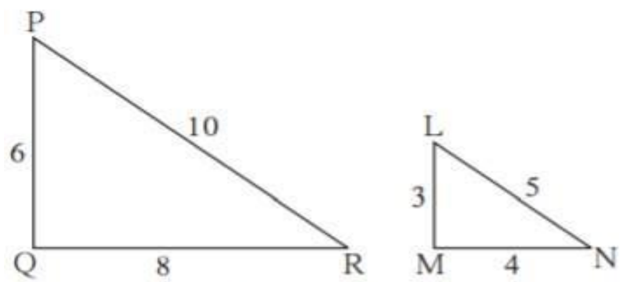


Fig. 1.56

Solution:

Consider ΔPQR and ΔLMN ,

PQ/LM = 6/3 = 2/1 …………(i)

QR/MN = 8/4 = 2/1 ……….(ii)

PR/LN = 10/5 = 2/1 ……….(iii)

PQ/LM = QR/MN = PR/LN

ΔPQR ~ ΔLMN [SSS test of similarity]

3. As shown in figure 1.57, two poles of height 8 m and 4 m are perpendicular to the ground. If the length of shadow of smaller pole due to sunlight is 6 m then how long will be the shadow of the bigger pole at the same time?

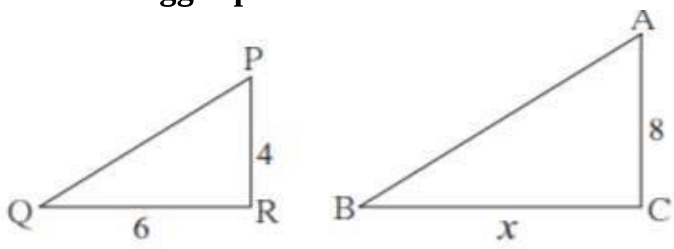


Fig. 1.57

Solution:

Here PR and AC represents the smaller and bigger poles, and QR and BC represents their shadows respectively.

Given PR = 4m, QR = 6m, AC = 8m, BC = x

ΔPRQ ~ ΔACB [∵Vertical poles and their shadows form similar figures]

∴ PR/AC = QR/BC [Corresponding sides of similar triangles]

4/8 = 6/x

x = 6×8/4

x = 12

Hence, the length of shadow of the bigger pole is 12 m.

4. In Δ ABC, AP ⊥ BC, BQ ⊥ AC B- P-C, A-Q - C then prove that, Δ CPA ~ Δ CQB. If AP = 7, BQ = 8, BC = 12 then find AC.

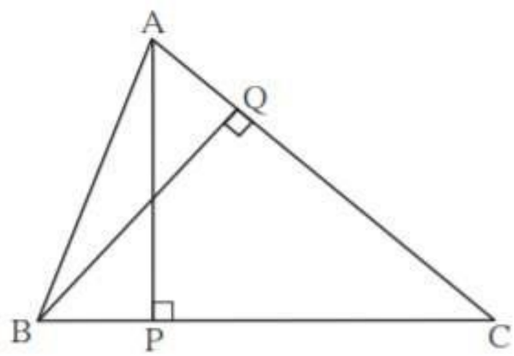


Fig. 1.58

Solution:

Consider ΔCPA and ΔCQB,

∠CPA ≅ ∠CQB [From figure, angle is equal to 90˚]

∠PCA ≅ ∠QCB [Common angle]

∴ ΔCPA ~ ΔCQB, [AA test of similarity]

Hence proved.

AC/BC = AP/BQ [corresponding sides of similar triangles]
Given AP = 7, BQ = 8, BC = 12
AC/12 = 7/8
AC = 12×7/8
AC = 10.5
Hence, measure of AC is 10.5 units.

5. Given: In trapezium PQRS, side PQ || side SR, AR = 5AP, AS = 5AQ then prove that, SR = 5PQ

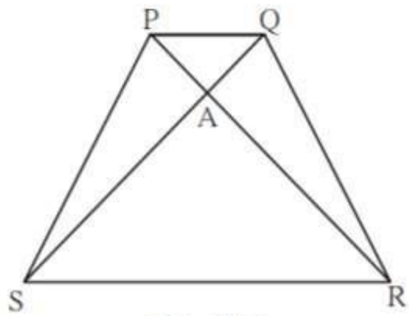


Fig. 1.59

Solution:
Given side PQ || side SR.
Also AR = 5AP, AS = 5AQ
SQ is the transversal of parallel sides PQ and SR.
∠QSR = ∠PQS [Alternate interior angles]
∠ASR = ∠AQP….(i) [Alternate interior angles]
Consider ΔASR and ΔAQP
∠ASR = ∠AQP From (i)
∠SAR = ∠QAP [vertical opposite angles]
Δ ASR ~ ΔAQP [AA test of similarity]
AS/AQ = SR/PQ [Corresponding sides of similar triangles]
AS = 5AQ [Given]
AS/AQ = 5/1
SR/PQ = 5/1
∴ SR = 5PQ
Hence proved.

Practice set 1.4

1. The ratio of corresponding sides of similar triangles is 3:5; then find the ratio of their areas

Solution:

When two triangles are similar, the ratio of areas of those triangles is equal to the ratio of the squares of their corresponding sides.

Given, the ratio of corresponding sides of the triangle is 3:5.

Ratio of their areas = $3^2/5^2$ [Theorem of areas of similar triangles]

= 9/25

Hence ratio of their areas = 9:25

2. If ΔABC ~ ΔPQR and AB: PQ = 2:3, then fill in the blanks.
A(ΔABC)/ A(ΔPQR) = AB^2/____ = $2^2/3^2$ = ____/____

Solution:

A(ΔABC)/ A(ΔPQR) = AB^2/PQ^2

= $2^2/3^2$ = 4/9 [Theorem of areas of similar triangles]

3. If Δ ABC ~ Δ PQR, A(Δ ABC) = 80, A(Δ PQR) = 125, then fill in the blanks. A(Δ ABC) /A(Δ…) = 80/125 ∴ AB/PQ = ____/____

Solution:

Given A(Δ ABC) = 80, A(Δ PQR) = 125

(Δ ABC) / A(Δ PQR) = 80/125 = 16/25

(Δ ABC) / A(Δ PQR) = AB^2/PQ^2 [Theorem of areas of similar triangles]

AB^2/PQ^2 = 16/25

Taking square root on both sides

AB/PQ = 4/5

Hence AB/PQ = 4/5

4. Δ LMN ~ Δ PQR, 9 ×A(ΔPQR) = 16 ×A(ΔLMN). If QR = 20 then find MN.

Solution:

Given 9 ×A(ΔPQR) = 16 ×A(ΔLMN)

∴ A(ΔPQR)/ A(ΔLMN) = 16/9…………(i)

Δ LMN ~ Δ PQR

∴ A(ΔPQR)/ A(ΔLMN) = QR^2/MN^2 …..(ii)

From (i) and (ii)

$QR^{2/}MN^2 = 16/9$

Given QR = 20

∴ $20^2/MN^2 = 16/9$

Taking square root on both sides

$20/MN = 4/3$

$MN = 20 \times 3/4$

$MN = 15$

Hence, the measure of MN is 15 units.

Problem Set 1

1. Select the appropriate alternative.

(1) In Δ ABC and Δ PQR, in a one to one correspondence AB/QR = BC/ PR = CA/ PQ then

(A) Δ PQR ~ Δ ABC

(B) Δ PQR ~ Δ CAB

(C) Δ CBA ~ Δ PQR

(D) Δ BCA ~ Δ PQR

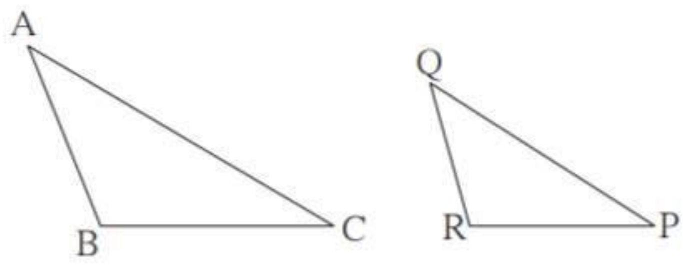


Fig. 1.67

Solution:

Given AB/QR = BC/ PR = CA/ PQ

By SSS test of similarity, Δ PQR ~ Δ CAB**.**

Correct option is (B).

(2) If in ΔDEF and Δ PQR, ∠D ≅ ∠Q, ∠R ≅ ∠E then which of the following statements is false?

(A) EF/PR = DF/ PQ

(B) DE/ PQ = EF/ RP

(C) DE/ QR = DF /PQ

(D) EF/ RP = DE/ QR

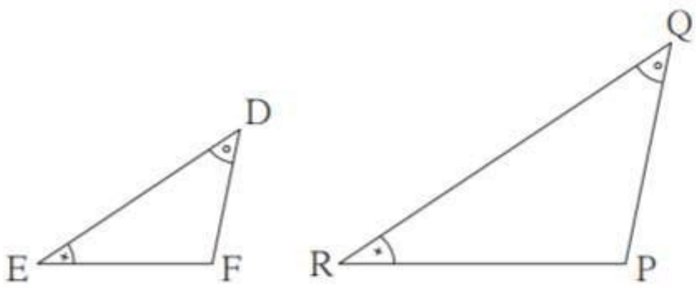


Fig. 1.68

Solution:

Given ∠D ≅ ∠Q, ∠R ≅ ∠E

ΔDEF ~ΔQRP…. [AA test of similarity]

DE/QR = EF/RP = DF/QP [Corresponding sides of similar triangles]

DE/ PQ ≠ EF/ RP

Hence, option (B) is false.

(3) In Δ ABC and Δ DEF ∠B = ∠E, ∠F = ∠C and AB = 3DE then which of the statements regarding the two triangles is true?

(A)The triangles are not congruent and not similar

(B) The triangles are similar but not congruent.

(C)The triangles are congruent and similar.

(D) None of the statements above is true.

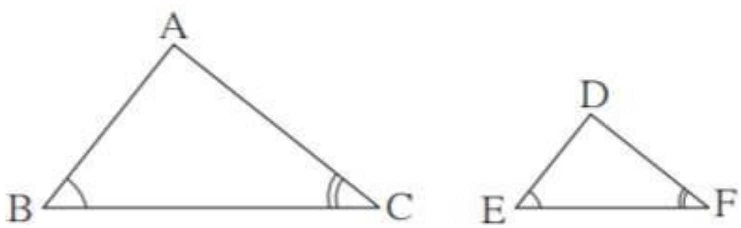


Fig. 1.69

Solution:

Given ∠B = ∠E

∠F = ∠C

∴ ΔABC ~ΔDEF [AA test of similarity]

Hence, option B is the true statement.

(4) Δ ABC and Δ DEF are equilateral triangles, A (ΔABC):A(ΔDEF)=1:2 If AB = 4 then what is length of DE?

(A) 2√2

(B) 4

(C) 8

(D) 4√2

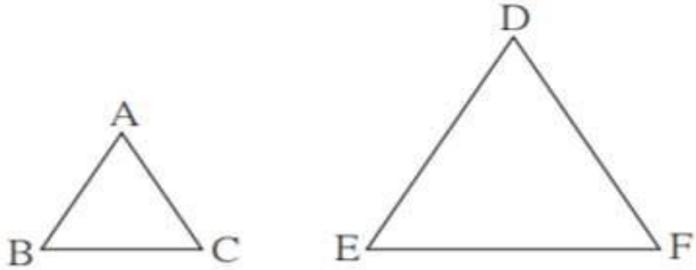


Fig. 1.70

Solution:

Given A (ΔABC):A(ΔDEF) = 1:2

ΔABC and ΔDEF are equilateral triangles.

∠A = ∠D [Angle equals 60˚]

∠B = ∠E [Angle equals 60˚]

ΔABC ~ ΔDEF [AA test of similarity]

A (ΔABC):A(ΔDEF) = AB^2/DE^2 [Theorem of areas of similar triangles]

$1/2 = 4^2/DE^2$

Taking square root on both sides

$1/\sqrt{2} = 4/DE$

∴ $DE = 4\sqrt{2}$

Hence, option (D) is the correct answer.

(5) In figure 1.71, seg XY || seg BC, then which of the following statements is true?

(A) AB / AC = AX / AY

(B) AX / XB = AY / AC

(C) AX / YC = AY / XB

(D) AB / YC = AC / XB

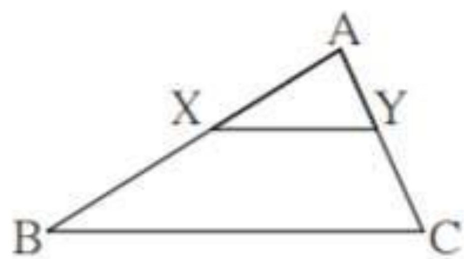


Fig. 1.71

Solution:

Given seg XY || seg BC

AX/BX = AY/YC [Basic proportionality theorem]

(BX/AX) +1 = (YC/AY) +1

(BX+AX)/AX = (YC+AY)/AY

AB/AX = AC/AY

AB/AC = AX/AY

Hence, correct option is (A).

2. In Δ ABC, B - D – C and BD = 7, BC = 20 then find following ratios.

(1) A(Δ ABD) /A(Δ ADC)

(2) A(Δ ABD) /A(Δ ABC)

(3) A(Δ ADC) /A(Δ ABC)

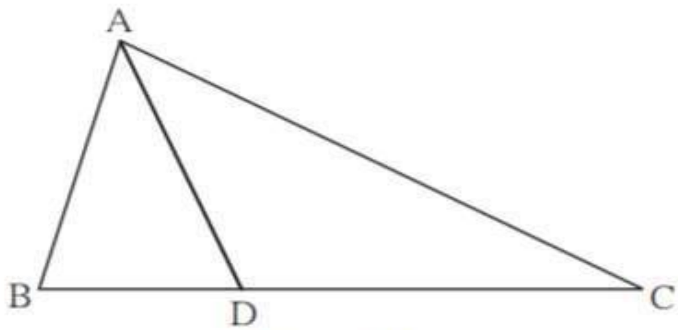
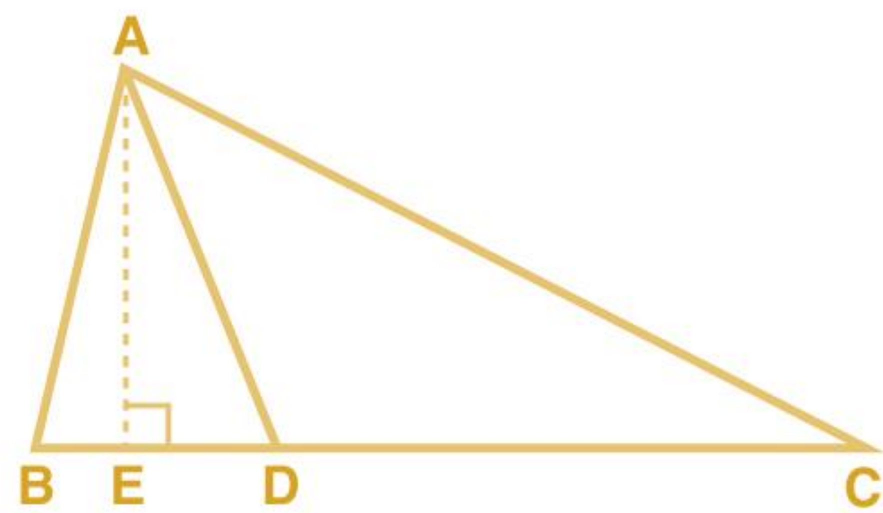


Fig. 1.72

Solution:



Given BD = 7, BC = 20
Construction:
Draw a perpendicular from A to BC meeting at E.
BC = BD+DC
20 = 7+DC
DC = 13

(1)A(Δ ABD) /A(Δ ADC) = BD/DC [Triangles having same height]
A(Δ ABD) /A(Δ ADC) = 7/13

(2) A(Δ ABD) /A(Δ ABC) = BD/BC [Triangles having same height]
A(Δ ABD) /A(Δ ABC) = 7/20

(3) A(Δ ADC) /A(Δ ABC) = DC/BC [Triangles having same height]
A(Δ ADC) /A(Δ ABC) = 13/20

3. Ratio of areas of two triangles with equal heights is 2:3. If the base of the smaller triangle is 6cm then what is the corresponding base of the bigger triangle?

Solution:
Given ratio of two triangles with equal height is 2:3
Let b_1 be base of smaller triangle and b_2 be base of bigger triangle. $b_1 = 6$ cm
Let a1 and a_2 be areas of the triangles.
Since triangles have equal height, $a_1/a_2 = b_1/b_2$
$2/3 = 6/b_2$
$b_2 = 3 \times 6/2$
$b_2 = 9$
Hence, base of bigger triangle is 9 cm.

4. In figure 1.73, ∠ABC = ∠DCB = 90° AB = 6, DC = 8 then A(Δ ABC) /A(Δ DCB) = ?

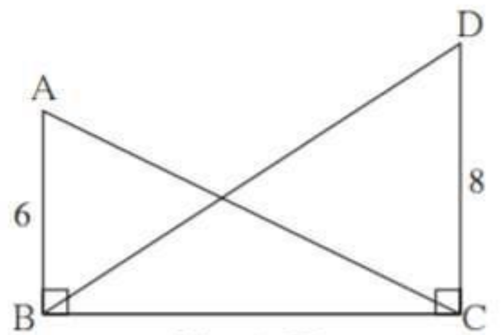


Fig. 1.73

Solution:
Given ∠ABC = ∠DCB = 90° AB = 6, DC = 8
BC is the common base of Δ ABC and Δ DCB
∴ A(Δ ABC) /A(Δ DCB) = AB/DC
= 6/8
= 3/4

5. In figure 1.74, PM = 10 cm A(ΔPQS) = 100 sq.cm A(ΔQRS) = 110 sq.cm then find NR.

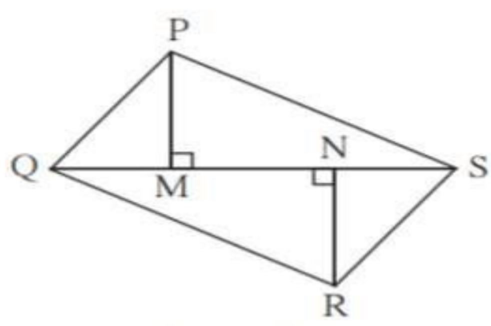


Fig. 1.74

Solution:

Given PM = 10 cm

A(ΔPQS) = 100 sq.cm

A(ΔQRS) = 110 sq.cm

ΔPQS and ΔQRS have common base QS

A(ΔPQS)/ A(ΔQRS) = PM/NR

100/110= 10/NR

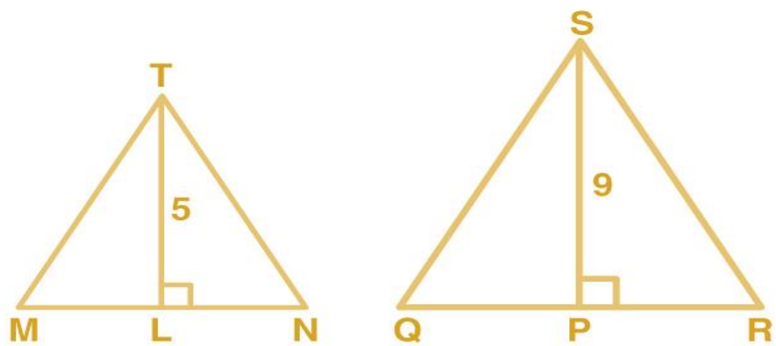
NR = 110×10/100

NR = 11

Hence, NR = 11 cm.

6. Δ MNT ~ Δ QRS. Length of altitude drawn from point T is 5 and length of altitude drawn from point S is 9. Find the ratio A(Δ MNT) /A(Δ QRS) .

Solution:



Given ΔMNT ~ ΔQRS

∠TMN ≅ ∠SQR [corresponding angles of similar triangles]

Construction:

Draw altitude from T to MN meeting at L.

Draw altitude from S to QR meeting at P.

∠TLM = ∠SPQ = 90°

In ΔMLT and ΔQPS

∠TMN ≅ ∠SQR

∠TLM ≅ ∠SPQ

ΔMLT ~ ΔQPS [AA test of similarity]

MT/QS = TL/SP

MT/QS = 5/9

ΔMNT ~ ΔQRS [Given]

A(Δ MNT) /A(Δ QRS) = MT^2/QS^2 [Theorem of areas of similar triangles]

A(Δ MNT) /A(Δ QRS) = $5^2/9^2$

A(Δ MNT) /A(Δ QRS) = 25/81

Hence A(Δ MNT):A(Δ QRS) = 25:81

7. In figure 1.75, A – D – C and B – E – C seg DE || side AB If AD = 5, DC = 3, BC = 6.4 then find BE.

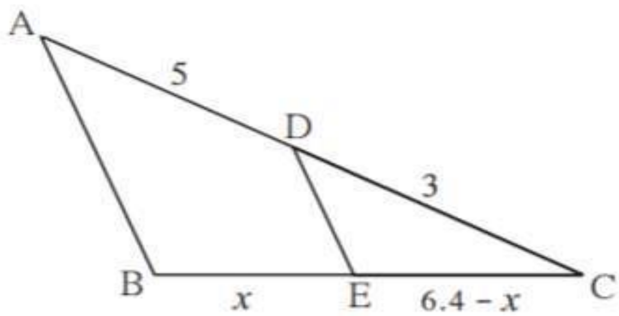


Fig. 1.75

Solution:

Given DE ∥ AB.

∴ AD/DC = BE/EC [Basic proportionality theorem]

AD = 5, DC = 3, BC = 6.4 [Given]

BE = x , EC = 6.4-x [Given]

∴ 5/3 = x/(6.4-x)

Cross-multiplying we get

5×(6.4-x) = 3× x

32-5x = 3x

32 = 8x

x = 32/8 = 4

Hence, BE = 4 units.