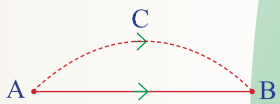
CHAPTER 4

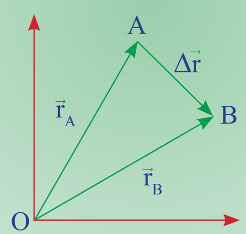
Kinematics

DISTANCE versus DISPLACEMENT

Total length of path (ACB) covered by the particle is called distance. Displacement vector or displacement is the minimum distance (AB) and directed from initial position to final position.



Displacement is Change of Position Vector



From ΔOAB $\quad \Delta\vec{r} = \vec{r}_B - \vec{r}_A$

$$\vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

and
$$\vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

Average velocity $= \dfrac{\text{Displacement}}{\text{Time interval}} \Rightarrow \vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t}$

Average speed $= \dfrac{\text{Distance travelled}}{\text{Time interval}}$

For uniform motion

Average speed = | average velocity | = | instantaneous velocity |

Velocity $\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k}) = \dfrac{dx}{dt}\hat{i} + \dfrac{dy}{dt}\hat{j} + \dfrac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

Average Acceleration $= \dfrac{\text{Total change in velocity}}{\text{Total time taken}} = \vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

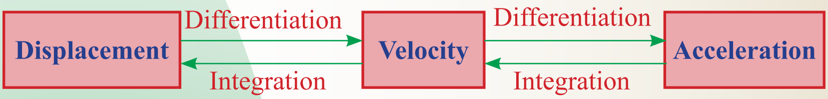
Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Important Points About 1D Motion

- Distance ≥ | displacement | and Average speed ≥ | average velocity |
- If distance > | displacement | this implies

  (a) atleast at one point in path, velocity is zero.



Motion with Constant Acceleration: Equations of Motion

- *In vector form:*

$$\vec{v} = \vec{u} + \vec{a}t \text{ and } \Delta\vec{r} = \vec{r}_2 - \vec{r}_1, \vec{s} = \left(\frac{\vec{u}+\vec{v}}{2}\right)t = \vec{u}t + \frac{1}{2}\vec{a}t^2 = \vec{v}t - \frac{1}{2}\vec{a}t^2$$

$$v^2 = u^2 + 2\vec{a}.\vec{s} \text{ and } \vec{s}_{n^{th}} = \vec{u} + \frac{\vec{a}}{2}(2n-1)$$

($S_{n^{th}} \rightarrow$ displacement in n^{th} second)

- *In scalar form (for one dimensional motion):*

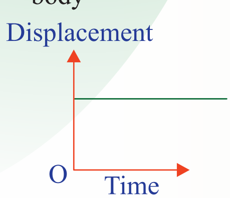
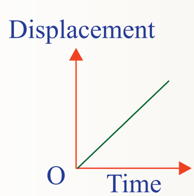
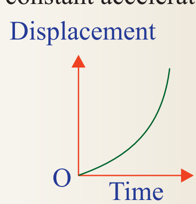
$$v = u + at \qquad s = \left(\frac{u+v}{2}\right)t = ut + \frac{1}{2}at^2 = vt - \frac{1}{2}at^2$$

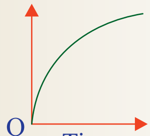
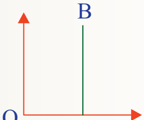
$$v^2 = u^2 + 2as \qquad s_n = u + \frac{a}{2}(2n-1)$$

UNIFORM MOTION

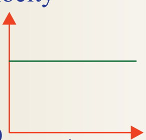
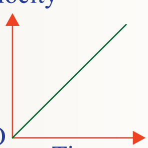
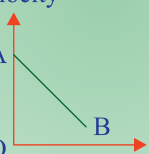
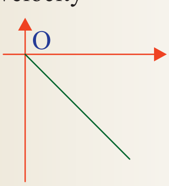
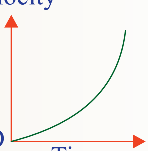
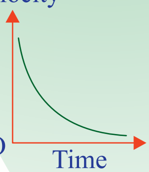
If an object is moving along the straight line covers equal distance in equal interval of time, it is said to be in uniform motion along a straight line.

DIFFERENT GRAPHS OF MOTION DISPLACEMENT-TIME GRAPH

S. No.	Condition	Graph
(a) For a stationary body 	(b) Body moving with a constant velocity 	(c) Body moving with a constant acceleration 

S. No.	Condition	Graph
(d) Body moving with a constant retardation 	(e) Body moving with infinite velocity. But such motion of body is never possible 	

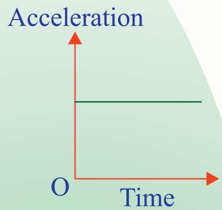
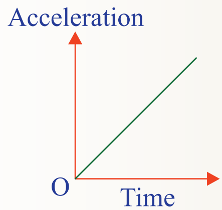
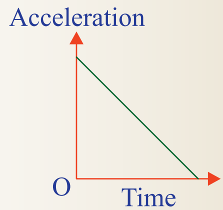
VELOCITY-TIME GRAPH

S.No.	Condition	Graph
(a) Moving with a constant velocity 	(b) Moving with a constant acceleration having zero initial velocity 	(c) Body moving with a constant retardation and its initial retar-dation and its initial velocity is zero 
(d) Moving with a constant retardation with zero initial velocity 	(e) Moving with increasing acceleration 	(f) Moving with decreasing acceleration 

NOTES

Slope of velocity-time graph gives acceleration.

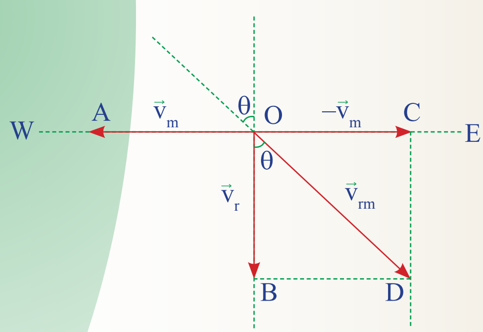
ACCELERATION-TIME GRAPH

S.No.	Condition	Graph
(a) When object is moving with constant acceleration 	(b) When object is moving with constant increasing acceleration 	(c) When object is moving with constant decreasing acceleration 

RELATIVE MOTION

Relative Velocity of Rain w.r.t. the Moving Man

A man walking west with velocity $\vec{v}_m$, represented by $\overrightarrow{OA}$. Let the rain be falling vertically downwards with velocity $\vec{v}_r$, represented by $\overrightarrow{OB}$ as shown in figure.



The relative velocity of rain w.r.t. man $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$ will be represented by diagonal $\overrightarrow{OD}$ of rectangle OBDC.

$$\therefore \qquad v_{rm} = \sqrt{v_r^2 + v_m^2 + 2v_r v_m \cos 90^\circ} = \sqrt{v_r^2 + v_m^2}$$

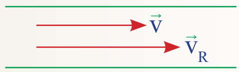
If θ is the angle which $\vec{v}_{rm}$ makes with the vertical direction then

$$\tan\theta = \frac{BD}{OB} = \frac{v_m}{v_r} \Rightarrow \theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$$

Swimming into the River

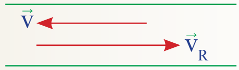
A man can swim with velocity $\vec{v}$, i.e., it is the velocity of man w.r.t. still water. If water is also flowing with velocity $\vec{v}_R$ then velocity of man relative to ground $\vec{v}_m = \vec{v} + \vec{v}_R$.

If the swimming is in the direction of flow of water or along the downstream then



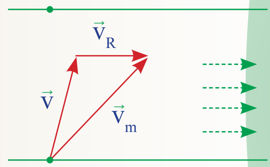
$$v_m = v + v_R$$

If the swimming is in the direction opposite to the flow of water or along the upstream then

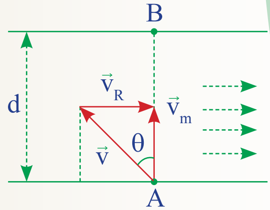


$$v_m = v - v_R$$

If man is crossing the river as shown in the figure i.e. $\vec{v}$ and $\vec{v}_R$ not collinear then use the vector algebra $\vec{v}_m = \vec{v} + \vec{v}_R$ **(assuming $v > v_R$)**



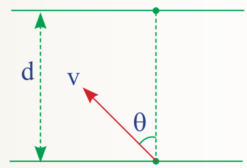
For shortest path



For minimum displacement

To reach at B, $v \sin \theta = v_R \Rightarrow \sin \theta = \frac{v_R}{v}$

Time of crossing

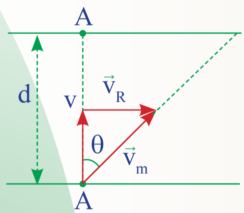


$t = (d/v \cos \theta)$

NOTES

If $v_R > v$ then for minimum drifting $\sin\theta = \frac{v}{v_R}$.

For minimum time



then $t_{min} = \frac{d}{v}$

(For minimum time)

MOTION UNDER GRAVITY (No Air Resistance)

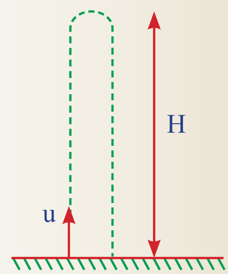
If an object is falling freely (u = 0) under gravity, then equations of motion becomes

(*i*) $v = u + gt$ (*ii*) $h = ut + \frac{1}{2}gt^2$ (*iii*) $v^2 = u^2 + 2gh$

NOTES

If an object is thrown upward then g is replaced by –g in above three equations.

If a body is thrown vertically up with a velocity u in the uniform gravitational field then



(*i*) Maximum height attained $H = \frac{u^2}{2g}$

(*ii*) Time of ascent = time of descent = $\frac{u}{g}$

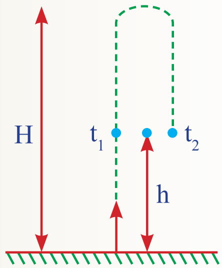
(*iii*) Total time of flight = $\frac{2u}{g}$

(*iv*) Velocity of fall at the point of projection = u (downwards)

(v) **Gallileo's law of odd numbers:** For a freely falling body ratio of successive distance covered in equal time interval 't'

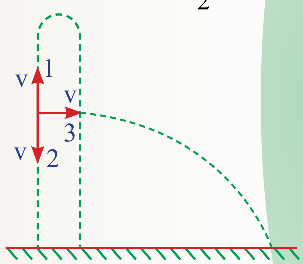
$S_1 : S_2 : S_3 : \ldots, S_n = 1 : 3 : 5 : \ldots, 2n - 1$

At any point on its path the body will have same speed for upward journey and downward journey.



If a body thrown upwards crosses a point in time t_1 and t_2 respectively then height of point $h = \frac{1}{2} g t_1 t_2$. Maximum height $H = \frac{1}{8} g(t_1 + t_2)^2$.

A body is thrown upward, downward and horizontally with same speed takes time t_1, t_2 and t_3 respectively to reach the ground then $t_3 = \sqrt{t_1 t_2}$ and height from where the particle was throw is $H = \frac{1}{2} g t_1 t_2$.



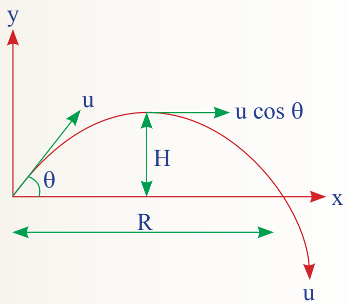
PROJECTION MOTION

Horizontal Motion of Projectile

$u \cos \theta = u_x$

$a_x = 0$

$x = u_x t = (u \cos \theta)t$



Vertical Motion of Projectile

$v_y = u_y - gt$ where $u_y = u \sin\theta$; $y = u_y t - \frac{1}{2}gt^2 = u \sin\theta t - \frac{1}{2}gt^2$

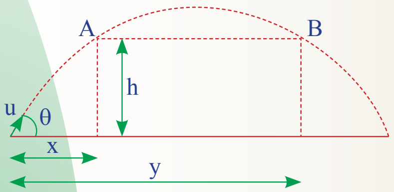
Net acceleration = $\vec{a} = a_x\hat{i} + a_y\hat{j} = -g\hat{j}$

At any Instant

$v_x = u \cos\theta, \quad v_y = u \sin\theta - gt$

For Projectile Motion

A body crosses two points at same height in time t_1 and t_2 the points are at distance x and y from starting point then



(a) $x + y = R$

(b) $t_1 + t_2 = T$

(c) $h = \frac{1}{2} g t_1 t_2$

(d) Average velocity from A to B is $u \cos\theta$

If a person can throw a ball to a maximum distance 'x' then the maximum height to which he can throw the ball will be (x/2).

Velocity of Particle at Time t

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = u_x\hat{i} + (u_y - gt)\hat{j} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$$

If angle of velocity $\vec{v}$ from horizontal is α, then

$$\tan\alpha = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x} = \frac{u\sin\theta - gt}{u\cos\theta} = \tan\theta - \frac{gt}{u\cos\theta}$$

At highest point: $v_y = 0,\ v_x = u\cos\theta$

Time of flight: $T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$

Horizontal range: $R = (u\cos\theta)T = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g} = \frac{2u_x u_y}{g}$

It is same for q and (90° – θ) and maximum for θ = 45°.

Maximum height: $H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g} = \frac{1}{8}gT^2$

$$\frac{H}{R} = \frac{1}{4}\tan\theta$$

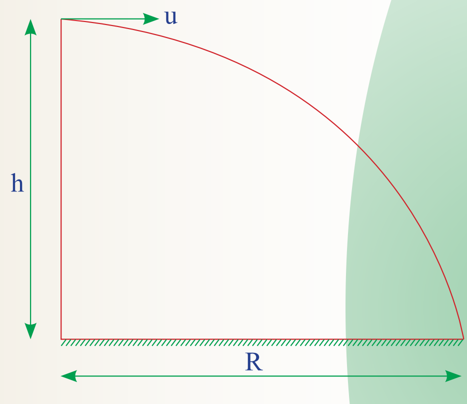
Equation of trajectory $y = x \tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta} = x\tan\theta\left(1 - \dfrac{x}{R}\right)$

Horizontal Projection from a Height h

Time of flight $T = \sqrt{\dfrac{2h}{g}}$

Horizontal range $R = uT = u\sqrt{\dfrac{2h}{g}}$

Angle of velocity at any instant with horizontal $\theta = \tan^{-1}\left(\dfrac{gt}{u}\right)$



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