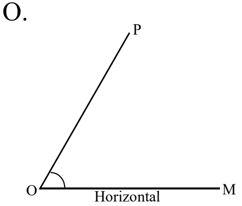
Heights and Distances

INTRODUCTION

This chapter deals with the applications of trigonometry in practical situations concerning measurement of heights and distances, which are otherwise not directly measurable. We need to first define certain terms and state some properties before applying the principles of trigonometry.

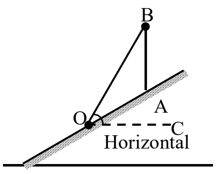
ANGLE OF ELEVATION

Consider a point P being observed from a point O (usually called observer) at a lower horizontal level. Draw a horizontal line OM through O in the direction of P. Then, OP is called the line of sight or observation and ∠POM is called the angle of elevation of point P as seen from O.



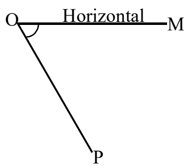
Unless otherwise stated, the height of the observer is neglected because mostly the heights and distances to be measured are very large compared to the height of the observer.

Suppose, there is a flagstaff AB mounted on a sloping level and its top B is observed from O. Then the angle ∠AOB is not the angle of elevation of B because it is not measured w.r.t the horizontal at O. The correct angle of elevation is ∠COB.



ANGLE OF DEPRESSION

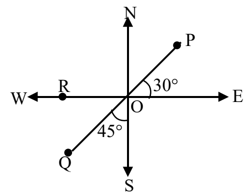
Consider a point P being observed from a point O at a higher horizontal level ∠POM here is called the angle of depression of the point P as seen from O. This is also measured w.r.t the horizontal at O.



DIRECTIONS OF A POINT

When the observer (O) and the object (P) are at the same horizontal level, to specify the direction or location of P w.r.t O, we take the help of cardinal directions North, East, West and South. Bearing of a point is defined as the angle made by the line of sight with one of the principal directions North, East, West or South.

e.g., Bearing of point P can be stated as 30° north of east or 60° east of north, which can be symbolically written as E30°N or N60°E. Bearing of Q is W45°S or S45°W or simply WS or SW and that of R is W.

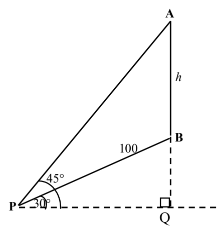


SOLVED EXAMPLE

Example-1

The angle of elevation of the top of a temple from a point on a hill of slope $\frac{1}{\sqrt{3}}$ is 45°. If the point is at a distance of 100m from the temple along the hill, find the height of the temple.

Sol.



Given $\tan\angle BPQ = \frac{1}{\sqrt{3}} \Rightarrow \angle BPQ = 30°$

$\angle APQ = 45°$ and $PB = 100$

Now, $BQ = 100 \sin 30° = 50m$

and $PQ = 100 \cos 30° = 50\sqrt{3}$ m

So, $\Delta APQ \Rightarrow \tan 45° = \frac{AQ}{PQ}$

$\Rightarrow h + 50 = 50\sqrt{3} \Rightarrow h = 50(\sqrt{3}-1) = 36.6m.$

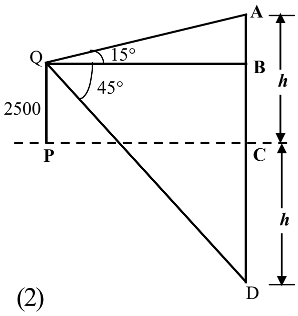
Example-2

The angle of elevation of a stationary cloud from a point 2500 metres above a lake is 15° and the angle of depression of its reflection in the lake is 45°. What is the height of the cloud above the lake level?

Sol. $\tan 15° = \dfrac{AB}{BQ}$

$\Rightarrow \tan(45°–30°) = \dfrac{h-2500}{BQ}$

$\Rightarrow \dfrac{1-\dfrac{1}{\sqrt{3}}}{1+\dfrac{1}{\sqrt{3}}} = \dfrac{h-2500}{BQ}$



$\tan 45° = \dfrac{BD}{BQ}$

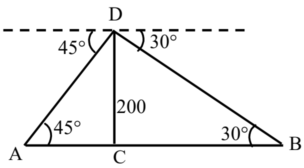
$\Rightarrow 1 = \dfrac{h+2500}{BQ}$ (2)

$(1) \div (2) \Rightarrow \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = \dfrac{h-2500}{h+2500} \Rightarrow h = 2500\sqrt{3}$ m

Example-3

An observer on the top of a cliff 200 m above the sea level observes the angle of depression of two objects on the opposite sides of the cliff. to be 45° and 30°. Find the distance between the objects.

Sol.



$AC = 200 \cot 45°$

$= 200\ m$

$BC = 200 \cot 30°$

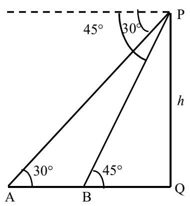
$= 200\sqrt{3}\ m$

$\Rightarrow AB = 200(\sqrt{3}+1) = 546.4\ m.$

Example-4

The shadow of a tower standing on a level plane is found to be 60 m longer when the sun's elevation is 30° then when it is 45°. Find the height of tower.

Sol.



$AB = 60\ m$

$= AQ - BQ$

$= h \cot 30° - h \cot 45°$

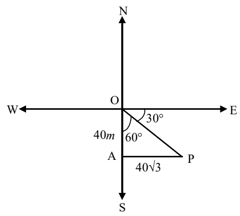
$\Rightarrow h = \dfrac{60}{\sqrt{3}-1}$ m

$= \dfrac{60\left(\sqrt{3}+1\right)}{3-1} = 81.96$ m

Example-5

Find the distance and bearing of a point situated 40 m south and 40√3 m east of the reference point.

Sol.



$\tan\angle POA = \dfrac{40\sqrt{3}}{40} = \sqrt{3}$

$\Rightarrow \angle POA = 60°$

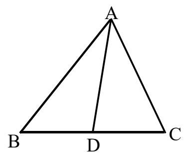
∴ Bearing of P can be written as E30°S or S60°E or 30° south of east 'or' 60^0 east of south.

Distance OP $= \sqrt{(40) + \left(40\sqrt{3}\right)^2} = 80$m

APOLLONIUS THEOREM

If AD is the median of a ΔABC, then

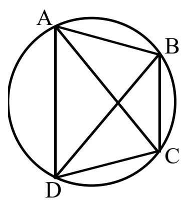
$AB^2 + AC^2 = 2(AD^2 + BD^2)$ or $2(AD^2 + DC^2)$



PTOLEMY'S THEOREM

In a cyclic quadrilateral ABCD

AC.BD = AB.CD + AD.BC

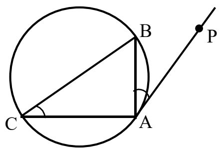


PROPERTIES OF CIRCLES

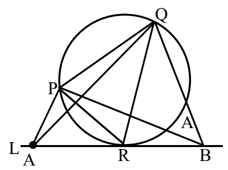
(i) If AB subtends equal angles at two points P and Q, the points A, B, P and Q are concyclic. (∵ angles on the same segment of a circle are equal)

(ii) Angle subtended by a chord at the center is twice the angle subtended at any point on the circumference.

(iii) Let AP be the tangent at a point A on the circumference of a circle passing through A, B and C. Then $\angle BAP = \angle ACB$.



(iv) A line segment PQ subtends various angles at different points on another line L. Obviously, at some point (say R) PQ will subtend the greatest angle also. Then, the circle passing through P, Q, R touches the line L at R.

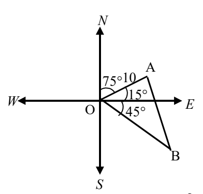


Three dimensional problems in heights and distances are generally solved by decomposing the picture into two 2-dimensional views [top (or plane) view and front view] and then principles of plane trigonometry are used.

Example-6

Two cars leave a place at the same time. One travels 10 km in the direction N75°E and the other 20 km in the south-east direction. What is the final distance between the cars?

Sol.



$$\cos\angle AOB = \frac{OA^2 + OB^2 - AB^2}{2(OA)(OB)}$$

$\Rightarrow AB^2 = (10)^2 + (20)^2 - 2(10)(20)\cos 60°$

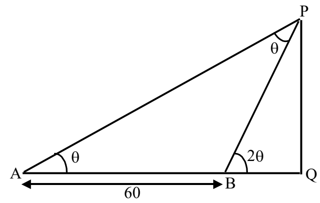
$= 100 + 400 - 200 = 300$

$\Rightarrow AB = 10\sqrt{3}\ km$

Example-7

A man observes the angle of elevation of the top of a tower to be θ°. He moves a distance 60 m towards the tower and finds that the elevation has doubled. Find the value of sin2θ. If height of the tower is 50 m.

Sol.



In ΔPAB, $\angle ABP = \pi - 2\theta$ and $\angle APB = \theta$ (exterior angle is the sum of two opposite interior angles)

Applying sine rule,

$$\frac{\sin\theta}{AB} = \frac{\sin(\pi - 2\theta)}{AP}$$

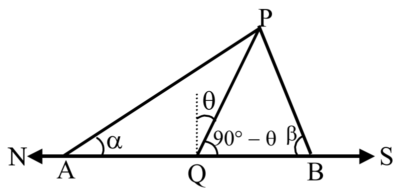
$$\Rightarrow \frac{\sin\theta}{60} = \frac{\sin 2\theta}{PQ/\sin\theta} = \frac{\sin 2\theta.\sin\theta}{50}$$

$$\Rightarrow \sin 2\theta = \frac{5}{6}$$

Example-8

A chimney leans towards the south. At equal distances due north and south of it the angles of elevation of the top of chimney are α and β respectively. Find the inclination of the chimney to the vertical.

Sol.



We require the value of θ.

Applying m – n theorem to ΔPAB, We have

$(1 + 1)\cot(90° - \theta) = 1.\cot\alpha - 1.\cot\beta$

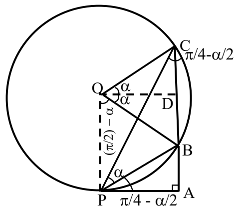
$$\Rightarrow \tan\theta = \frac{\cot\alpha - \cot\beta}{2}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{\cot\alpha - \cot\beta}{2}\right)$$

Example-9

A man is moving towards a building on which a flagstaff is mounted. The flagstaff subtends its maximum angle α at the man's eye when he is at a distance 'c' from the base of building. Obtain the height of building and flagstaff in terms of c and α.

Sol.



Let AB be the building and BC be the flagstaff.

Since angle subtended by BC is maximum at P, a circle drawn through P, B and C touches PA at P. The perpendicular bisector of chord BC and a line perpendicular to PA at P intersect at the center O of the circle. Consider chord BC.

$\because \angle BPC = \alpha \Rightarrow \angle BOC = 2\alpha$

$\Rightarrow \angle COD = \alpha$

$\therefore$ Height of flagstaff $= BC = 2CD$

$= 2OD \tan \angle COD$

$= 2PA \tan \alpha = 2c \tan \alpha$

Now consider chord PB.

$\because \angle POB = \pi/2 - \alpha$

$\Rightarrow \angle PCB = \pi/4 - \alpha/2$

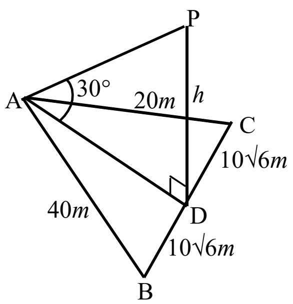
$\Rightarrow \angle BPA = \pi/4 - \alpha/2$ (alternate segment theorem)

$\therefore$ Height of building $= AB = AP \tan \angle BPA$ $= C \tan(\pi/4 - \alpha/2)$

Example-10

A vertical lamp post stands at the mid-point of the edge BC of a park in the shape of a triangle. The angle of elevation of the top of the lamp post from A is 30°. If the sides AB, BC and CA respectively are 40m, $20\sqrt{6}$ m and 20m, obtain the height of lamp post.

Sol.



Let PD be the lamp post of height h

In ΔPAD, $\dfrac{PD}{AD} = \tan 30°$

$\Rightarrow AD = h\sqrt{3}$

Applying Appollonius theorem in ΔABC, we have

$AB^2 + AC^2 = 2(AD^2 + BD^2)$

$\Rightarrow (40)^2 + (20)^2 = 2[(h\sqrt{3})^2 + (10\sqrt{6})^2]$

$\Rightarrow h = \dfrac{20}{\sqrt{3}}\, m.$

Example-11

A rocket of height h metres is fired vertically upwards. Its velocity at time t seconds is (2t + 3) metres/second. If the angle of elevation of the top of the rocket from a point on the ground after 1 second of firing is π/6 and after 3 seconds it is π/3 then the distance of the point from the rocket is

(A) $14\sqrt{3}$ metres

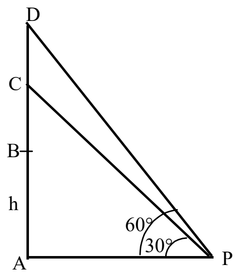
(B) $7\sqrt{3}$ metres

(C) $2\sqrt{3}$ metres

(D) cannot be found without the value of h

Ans. **(B)**

Sol.



$x = \int (2t+3)\, dt \Rightarrow x = t^2 + 3t + c$

$\therefore BC = 1^2 + 3.1 + c = 4 + c$

$BD = 3^2 + 3.3 + c = 18 + c$

$\therefore \dfrac{h+4+c}{AP} = \tan 30°$

$\dfrac{h+18+c}{AP} = \tan 60°$

$\therefore \dfrac{14}{AP} = \tan 60° - \tan 30°$

$= \sqrt{3} - \dfrac{1}{\sqrt{3}} = \dfrac{2}{\sqrt{3}}$

$\therefore AP = 7\sqrt{3}$.

Example-12

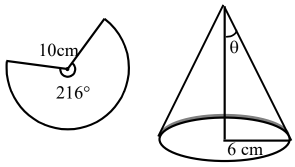
A piece of paper in the shape of a sector of a circle of radius 10 cm and of angle 216° just covers the lateral surface of a right circular cone of vertical angle 2θ. Then sinθ is

(A) 3/5 (B) 4/5

(C) 3/4 (D) none of these

Ans. **(A)**

Sol.



The area of the sector $= \dfrac{\pi.10^2}{360} \times 216 \text{ cm}^2$

= lateral surface area of the cone.

The circumference of the base circle of the cone

$= \dfrac{2\pi \times 10}{360} \times 216 \text{ cm}$

$\therefore\ 2\pi r = \dfrac{2\pi r \times 10}{360} \times 216 \Rightarrow r = 6 \text{ cm}$

The lateral height of the cone = 6cosecθ cm.

$\therefore$ lateral surface area $= \pi r l = \pi.6.6\text{cosec}\theta \text{ cm}^2$

$\therefore\ \pi.36\text{cosec}\ \theta = \dfrac{\pi.10^2.216}{360}$

$\Rightarrow \sin\theta = \dfrac{3}{5}$

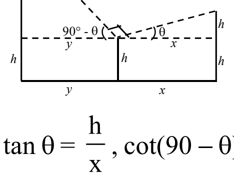
Example-13

A man standing between two vertical posts finds that the angle subtended at his eyes by the tops of the posts is a right angle. If the heights of the two posts are two times and four times the height of the man, and the distance between them is equal to the length of the longer post, then the ratio of the distances of the man from the shorter and the longer post is

(A) 3 : 1 (B) 2 : 3 (C) 3 : 2 (D) 1 : 3

Ans. **(A, D)**

Sol.



$\tan\theta = \dfrac{h}{x}$, $\cot(90 - \theta) = \dfrac{y}{3h}$ $\therefore \tan\theta = \dfrac{y}{3h}$

So, $xy = 3h^2$

Also $x + y = 4h$. Solving these,

$x = 3h, y = h$ or $x = h, y = 3h$.

Example-14

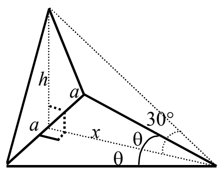
An isosceles triangle of wood is placed in a vertical plane, vertex upwards and faces the sun. If 2a be the base of the triangle, h its height and 30° the altitude of the sun, then the tangent of the angle at the apex of the shadow is

(A) $\dfrac{2ah\sqrt{3}}{3h^2 - a^2}$ (B) $\dfrac{2ah\sqrt{3}}{3h^2 + a^2}$

(C) $\dfrac{ah\sqrt{3}}{h^2 - a^2}$ (D) none of these

Ans. **(A)**

Sol.



$x = h\cot 30° = h\sqrt{3}$

$\tan\theta = \dfrac{a}{x} = \dfrac{a}{h\sqrt{3}}$

$\Rightarrow \tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

$= \dfrac{2a}{h\sqrt{3}} \cdot \dfrac{3h^2}{3h^2 - a^2}$

$= \dfrac{2ah\sqrt{3}}{3h^2 - a^2}$

Example-15

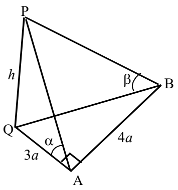
A person standing at the foot of a tower walks a distance 3a away from the tower and observes that the angle of elevation of the top of the tower is α. He then walks a distance 4a perpendicular to the previous direction and observes the angle of elevation to be β. The height of the tower is

(A) 3a tan α (B) 5a tan β

(C) 4a tan β (D) 7a tan β

Ans. **(A, B)**

Sol.



$QB^2 = QA^2 + AB^2$

$\Rightarrow (h\cot\beta)^2 = (3a)^2 + (4a)^2$

$\Rightarrow h^2 = 25a^2\tan^2\beta$

$\Rightarrow h = 5a\tan\beta$

Also, $3a = h\cot\alpha \Rightarrow h = 3a\tan\alpha$

Example-16

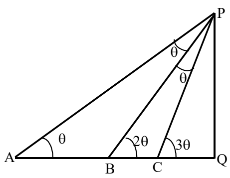
A tower subtends angle θ, 2θ and 3θ at 3 points A, B, C respectively, lying on a horizontal line through the foot of the tower then the ratio AB/BC equals to

(A) $\frac{\sin 3\theta}{\sin\theta}$ (B) $\frac{\sin\theta}{\sin 3\theta}$

(C) $\frac{\cos 3\theta}{\cos\theta}$ (D) $\frac{\tan\theta}{\tan 3\theta}$

Ans. **(A)**

Sol.



$$\frac{AB}{BC} = \frac{BP}{BC} = \frac{\sin(180^\circ - 3\theta)}{\sin\theta}$$

(From isosceles triangle ABP and sine rule in ΔBCP)

$$\Rightarrow \frac{AB}{BC} = \frac{\sin 3\theta}{\sin\theta}$$

Example-17

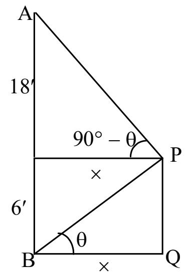
A 6-ft-tall man finds that the angle of elevation of the top of a 24-ft-high pillar and the angle of depression of its base are complementary angles. The distance of the man from the pillar is

(A) $2\sqrt{3}$ ft (B) $8\sqrt{3}$ ft

(C)$6\sqrt{3}$ ft (D) none of these

Ans. **(C)**

Sol.



Here, $\tan\theta = \frac{6}{x}$, $\tan(90^\circ - \theta) = \frac{18}{x}$

$\therefore \frac{6}{x}\cdot\frac{18}{x} = 1$

or $x^2 = 6 \times 18$

or $x = 6\sqrt{3}$

Example-18

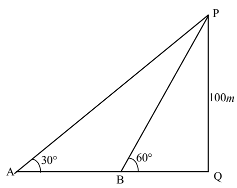
A man from the top of a 100 metres high tower sees a car moving towards the tower at an angle of depression of 30°. After some time, the angle of depression becomes 60°. The distance (in metres) travelled by the car during this time is

(A) $100\sqrt{3}$ (B) $200\sqrt{3}/3$

(C) $100\sqrt{3}/3$ (D) $200\sqrt{3}$

Ans. **(B)**

Sol.



$AB = 100\cot 30^\circ - 100\cot 60^\circ$

$$100\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{200}{\sqrt{3}} = \frac{200\sqrt{3}}{3}$$

Example-19

The angle of elevation of the top of a tower observed from each of the three points A, B, C on the ground, forming an equilateral triangle of side length a, is the same angle α. The height of the tower is

(A) a sinα (B) $\frac{a}{\sqrt{3}}\sin\alpha$

(C) a tanα (D) $\frac{a}{\sqrt{3}}\tan\alpha$

Ans. **(D)**

Sol. Obviously foot of the tower is the circumcentre of the triangle ABC. The circum radius of the circumcircle of

$$\Delta ABC = \frac{a}{2\sin 60^o} = \frac{a}{\sqrt{3}}$$

Hence height of the tower $= \frac{a}{\sqrt{3}} \tan\alpha$

Example-20

Three vertical poles of heights h_1, h_2 and h_3 at the vertices A, B and C of a ΔABC subtend angles α, β and γ respectively at the circumcentre of the triangle. If $\cot\alpha$, $\cot\beta$ and $\cot\gamma$ are in AP then h_1, h_2, h_3 are in

(A) AP (B) GP

(C) HP (D) none of these

Ans. **(C)**

Sol. $\frac{R}{h_1} = \cot\alpha$, etc., where R is the circumradius

$\cot\alpha, \cot\beta, \cot\gamma$ are in A.P.

$\Rightarrow \frac{1}{h_1}, \frac{1}{h_2}, \frac{1}{h_3}$ are in A.P.

$\Rightarrow h_1, h_2, h_3$ are in H.P.

Example-21

The angle of elevation of the top of a hill from each of the vertices A, B, C of a horizontal triangle is α. The height of the hill is

(A) b $\tan\alpha$. cosec β (B) $\frac{1}{2}$ atanatana.cA

(C) $\frac{1}{2}$ c $\tan\alpha$.cosecC (D) none of these

Ans. **(B)**

Sol. The distance of the foot from each vertex = $h\cot\alpha$

$\therefore$ the foot is at the circumcentre of the triangle.

So, $R = h\cot\alpha$

$$\therefore h = R\tan\alpha = \frac{a}{2\sin A}\tan\alpha$$

Example-22

If the angular elevations of the tops of two spires which appear in a straight line is α and the angular depression of their reflections in a lake, h feet below the point of observation are β and γ, show that the distance between the spires is $2h\cos^2\alpha \sin(\gamma-\beta)\,\text{cosec}(\beta-\alpha)\,\text{cosec}(\gamma-\alpha)$ ft where $\gamma > \beta$.

Sol. Let P and Q be the tops of two spires, P′ and Q′ be their reflections. From question, OA = h

Let $BP = BP' = h_1$

$CQ = CQ' = h_2$

Let the distance between the spires be x.

We have to find x.

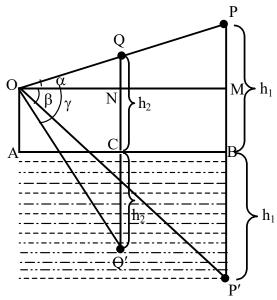
Clearly $x = MN = OM - ON$

From right angled triangle OMP′

$$\tan\beta = \frac{P'M}{OM} = \frac{h_1 + h}{OM} \text{ or,}$$

$OM\tan\beta = h_1 + h = (h + PM) + h$

$[\because h_1 = BP = BM + PM = h + PM]$



or, $OM\tan\beta = 2h + PM = 2h + OM\tan\alpha$

$$\left[\because \text{From triangle PMO, } \tan\alpha = \frac{PM}{OM}\right]$$

or, $OM(\tan\beta - \tan\alpha) = 2h$

$$\Rightarrow OM = = \frac{2h}{\tan\beta - \tan\alpha} \quad (1)$$

$$\text{Similarly } ON = \frac{2h}{\tan\gamma - \tan\alpha}$$

Hence $x = OM - ON$

$$= \frac{2h}{\tan\beta - \tan\alpha} - \frac{2h}{\tan\gamma - \tan\alpha}$$

$$= 2h\left[\frac{1}{\tan\beta - \tan\alpha} - \frac{1}{\tan\gamma - \tan\alpha}\right]$$

$$= 2h\left[\frac{\tan\gamma - \tan\alpha - \tan\beta + \tan\alpha}{(\tan\beta - \tan\alpha)(\tan\gamma - \tan\alpha)}\right]$$

$$= 2h\left[\frac{\tan\gamma - \tan\beta}{(\tan\beta - \tan\alpha)(\tan\gamma - \tan\alpha)}\right]$$

$$= 2h\left[\frac{\sin(\gamma-\beta)\cos\beta\cos\alpha\cos\gamma\cos\alpha}{\cos\gamma.\cos\beta\sin(\beta-\alpha)\sin(\gamma-\alpha)}\right]$$

$= 2h\cos^2\alpha \sin(\gamma - \beta)\,\text{cosec}(\beta - \alpha)\,\text{cosec}(\gamma - \alpha)$ ft.

Example-23

A pole stands vertically on the center of a square. When α is the elevation of the sun its shadow just reaches the side of the square and is at a distance x and y from the ends of that side. Show that the height of the pole is $\sqrt{\frac{x^2+y^2}{2}}\tan\alpha$.

Sol. Let O be the center of the square, OP the pole. Shadow of the pole OP is OQ. From question BQ = y and CQ = x.

Then, BC = x + y

Let OR ⊥ BC

$\therefore$ OR = $\frac{x+y}{2}$ and BR = $\frac{x+y}{2}$

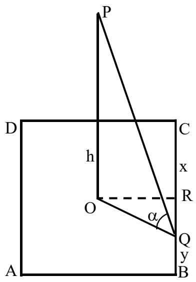
$$RQ = \frac{x+y}{2} - y = \frac{x-y}{2}$$

Let h be the height of the pole.

From right angled triangle POQ,

$\tan\alpha = \frac{h}{OQ}$ $\therefore$ OQ = h$\cot\alpha$

Now, from right angled triangle ORQ.



$OQ^2 = OR^2 + RQ^2$

$$\text{or, } h^2\cot^2\alpha = \left(\frac{x+y}{2}\right)^2 + \left(\frac{x-y}{2}\right)^2$$

$$\text{or, } h^2\cot^2\alpha = \frac{2\left(x^2+y^2\right)}{4}$$

$$\therefore\ h = \sqrt{\frac{x^2+y^2}{2}}.\tan\alpha$$

Example-24

A man observes a tower AB of height h from a point P on the ground. He moves a distance d towards the foot of the tower and finds that the angle of elevation has doubled. He further moves a distance $\frac{3}{4}$d in the same direction and finds that the angle of elevation is three times that at P. Prove that $36h^2 = 35d^2$.

Sol. Let AB = h, PQ = d and QR = $\frac{3}{4}$d.

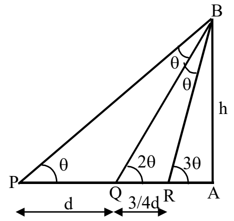
$\because \angle BPQ = \angle PBQ = \theta$

$\therefore$ PQ = QB = d

[Here angles are given, hence we select right angled triangles].

From right angled triangle BAQ.

$\sin 2\theta = \frac{h}{d}$ $\quad\therefore$ h = d $\sin 2\theta$



or, h = 2d $\sin\theta\cos\theta$ (1)

Now, applying sine rule in triangle BQR,

$$\frac{\frac{3}{4}d}{\sin\theta} = \frac{d}{\sin\left(180^\circ - 3\theta\right)}$$

$$\text{or, } \frac{3}{4\sin\theta} = \frac{1}{3\sin\theta - 4\sin^3\theta}$$

or, $9 - 12\sin^2\theta = 4$ $\quad$ [$\because \sin\theta \neq 0$]

or, $\sin^2\theta = \frac{5}{12}$ $\qquad\therefore \cos^2\theta = \frac{7}{12}$

From (1), we have $\quad h^2 = 4d^2\sin^2\theta\cos^2\theta$

or, $h^2 = 4d^2.\frac{5}{12}\cdot\frac{7}{12}$ or, $\quad 36h^2 = 35d^2$

Example-25

A man observes that when he moves up a distance c meters on a slope, the angle of depression of a point on the horizontal plane from the base of the slope is 30°; and when moves up further a distance c metres, then angle of depression of the point is 45°. Obtain the angle of inclination of the slope with the horizontal.

Sol. Let the slope be OB, making an angle θ with the horizontal OQ,

i.e. $\angle BOQ = \theta$.

Let P be the point on the horizontal plane through the base of the slope.

From question OA = c and AB = c.

Also $\angle XAP = 30° = \angle APO$

and $\angle YBP = 45° = \angle BPO$.

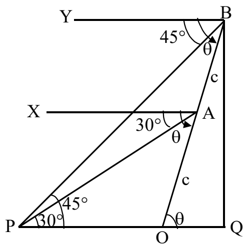
Clearly, $\angle XAO = \angle AOQ = \theta$

$\therefore \angle PAO = \theta - 30°$

and $\angle YBO = \angle BOQ = \theta \quad \therefore \angle PBO = \theta - 45°$

Suppose OP = x. We have to find θ.

Note: since given angles are not contained in right angled triangles we will use sine rule.



Now, applying sine rule in triangle POA.

$$\frac{x}{\sin(\theta - 30^o)} = \frac{c}{\sin 30^o}$$

$$\therefore x = \frac{c\sin(\theta - 30^o)}{\sin 30^o} \qquad (1)$$

Applying sine rule in triangle POB,

$$\frac{x}{\sin(\theta - 45^o)} = \frac{2c}{\sin 45^o}$$

$$\therefore x = \frac{2c\sin(\theta - 45^o)}{\sin 45^o} \qquad (2)$$

From (1) and (2), we get

$$\frac{c\sin(\theta - 30^o)}{\sin 30^o} = \frac{2c\sin(\theta - 45^o)}{\sin 45^o}$$

$$\text{or, } 2\left(\sin\theta\cdot\frac{\sqrt{3}}{2} - \cos\theta\cdot\frac{1}{2}\right)$$

$$= \sqrt{2.2}\left(\sin\theta\cdot\frac{1}{\sqrt{2}} - \cos\theta\frac{1}{\sqrt{2}}\right)$$

$$\tan\theta = \frac{1}{2-\sqrt{3}} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2-\sqrt{3}}\right)$$

Example-26

A tower is observed from two stations A and B, where B is east of A at a distance 100 metres. The tower is due north of A and due north west of B. The angle of elevation of the tower from A and B are complementary. Find the height of the tower.

Sol. Let OP be the tower of height h. A is a point due south of the tower and the angle of elevation of the tower at B will be (90° – θ) (complementary of θ).

i.e. $\angle PAO = \theta$; $\angle PBO = 90° - \theta$

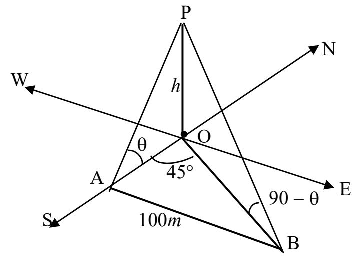
From right angled triangle POA.

$$\tan\theta = \frac{h}{OA} \qquad (1)$$

From right angle triangle POB,

$$\tan(90° - \theta) = \frac{h}{OB} \text{ or, } \tan\theta = \frac{OB}{h} \qquad (2)$$

From (1) and (2), we have



$$\frac{h}{OA} = \frac{OB}{h} \quad \therefore h = \sqrt{OA.OB} \qquad (3)$$

Since BO is north-west, OB will be south-east

$\therefore \angle AOB = 45° = \angle OBA$

$[\because \angle OAB = 90°]$

$\Rightarrow OA = AB = 100m$

Now, From right angled triangle OAB,

$OB^2 = OA^2 + AB^2 = 100^2 + 100^2$

$\therefore$ OB = $100\sqrt{2}$ m From (3)

$$h = \sqrt{100.100\sqrt{2}} = 100(2)^{1/4} \text{ metres}$$

Example-27

A tower stands on the edge of the circular lake ABCD. The foot of the tower is at D and the angle of elevation of the top at A, B, C are respectively α, β, γ. If $\angle ACB = \theta = \angle BAC$. Show that $2\cos\theta\cot\beta = \cot\alpha + \cot\gamma$

Sol. Let A, B, C be there points on the circular lake and DP be the tower of height *h* standing at D.

$\angle PAD = \alpha$, $\angle PBD = \beta$

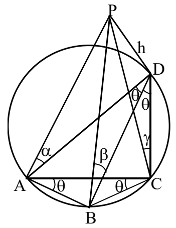
and $\angle PCD = \gamma$

Also $\angle BAC = \theta = \angle ACB$

Since, angles on the same segment of a circle are equal.

$\therefore \angle ADB = \angle ACB = \theta$

[angles on the segment AB]



From right angled ΔPDA,

$\tan\alpha = \dfrac{h}{AD} \quad \therefore AD = h\cot\alpha \qquad (1)$

Similarly, $BD = h\cot\beta \qquad (2)$

and $CD = h\cot\gamma \qquad (3)$

In triangle ABC,

$\angle BAC = \angle ACB = \theta$

$\therefore AB = BC$

or, $AB^2 = BC^2 \qquad (4)$

Applying cosine rule in triangle ADB.

$$\cos\theta = \frac{AD^2 + BD^2 - AB^2}{2AD.BD}$$

or, $AB^2 = AD^2 + BD^2 - 2AD.BD\cos\theta$

Similarly from triangle BDC,

$BC^2 = BD^2 + CD^2 - 2BD.CD\cos\theta$

Hence from (4), we have

$AD^2 + BD^2 - 2AD.BD\cos\theta = BD^2 + CD^2 - 2BD.CD\cos\theta$

or, $2BD\cos\theta\,[CD - AD] = CD^2 - AD^2$

or, $2.h\cot\beta\cos\theta\,(h\cot\gamma - h\cot\alpha] = h^2\cot^2\gamma - h^2\cot^2\alpha$

or, $2\cot\beta\cos\theta = \cot\gamma + \cot\alpha$

Example-28

A spherical ball of diameter d subtends an angle α at a man's eye when the elevation of its centre is β. Prove that the height of the center of the ball is 1/2 d sin β cosec (α/2).

Sol. Let O be the center of the ball and E, the man's eye. EA and EB are the tangents covering the ball and hence

$\angle AEB = \alpha$

Draw $OO' \perp EO'$

$\angle OEO' = \beta$

(From question)

Let $OO' = h$

From right angled triangle OO′E,

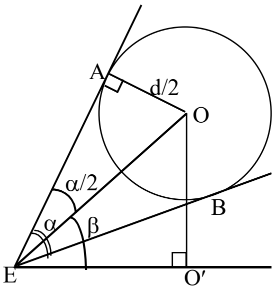
$\sin\beta = \dfrac{h}{EO}$

$\therefore h = EO\sin\beta \qquad (1)$

From geometry, it is clear that

$\angle AEO = \angle BEO = \dfrac{\alpha}{2} \qquad [\angle AEB = \alpha]$

Now, From right angled triangle OAE,



$\sin\dfrac{\alpha}{2} = \dfrac{d/2}{OE}$ or, $OE = \dfrac{1}{2}d\,\text{cosec}\dfrac{\alpha}{2} \qquad (2)$

From (1) and (2), $h = \dfrac{1}{2}d\,\sin\beta\,\text{cosec}\dfrac{\alpha}{2}$

Example-29

A right circular cylindrical tower of height h and radius r stands on a horizontal plane. Let A be a point in the horizontal plane and PQR be the semi-circular edge of the top of the tower such that Q is the point in it nearest to A. The angles of elevation of the points P and Q are 45° and 60° respectively. Show that $\dfrac{h}{r} = \dfrac{\sqrt{3}\left(1+\sqrt{5}\right)}{2}$.

Sol. Let C be the center of the top of the circular tower.

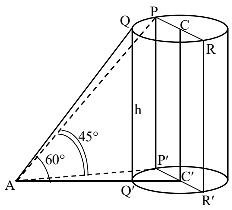
Draw QQ′, PP′ CC′ and RR′ perpendicular to the horizontal plane.

Since A is the point on the horizontal plane nearest to Q, hence A will be on the line Q′A, where $Q'A \perp QQ'$.

From question $QQ' = h$, $C'Q' = r$, $\angle QAQ' = 60°$ and $\angle PAP' = 45°$

From right angled triangle QQ′A

$\tan 60° = \dfrac{h}{AQ'}, \ \therefore AQ' = \dfrac{h}{\sqrt{3}} \qquad (1)$



From right angle triangle PP′A,

$$\tan 45° = \frac{h}{AP'}, \quad \therefore AP' = h \qquad (2)$$

$$\text{Now, } AC' = AQ' + Q'C' = \frac{h}{\sqrt{3}} + r$$

$$C'P' = r \qquad (3)$$

$$\text{and } \angle AC'P' = 90° \qquad (4)$$

From right angled triangle AC′P′, $AP'^2 = AC'^2 + C'P'^2$

$$\text{or, } h^2 = \left(\frac{h}{\sqrt{3}} + r\right)^2 + r^2$$

$$\text{or, } h^2 = \frac{h^2}{3} + 2\frac{h}{\sqrt{3}}.r + r^2 + r^2$$

or, $2h^2 - 2\sqrt{3}\ hr - 6r^2 = 0$ or, $h^2 - \sqrt{3}\ rh - 3r^2 = 0$

$$\therefore h = \frac{\sqrt{3}r \pm \sqrt{3r^2 - 4.1(-3r^2)}}{2}$$

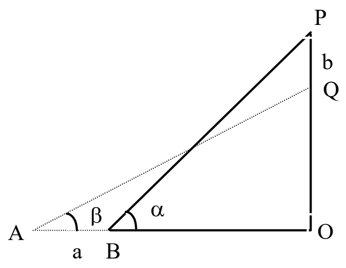
$$\text{or, } h = \frac{\sqrt{3}r(1 \pm \sqrt{5})}{2}$$

$$\text{A sh} > 0 \Rightarrow \frac{h}{r} = \frac{\sqrt{3}(1 + \sqrt{5})}{2}$$

Example-30

A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance a, so that it slides a distance b down the wall making an angle β with the horizontal. Show that a = b $\tan \frac{1}{2}(\alpha + \beta)$

Sol.



Let l be the length of the ladder.

$a = OA - OB = l \cos\beta - l \cos\alpha$

$b = OP - OQ = l \sin\alpha - l \sin\beta$

$$\Rightarrow \frac{a}{b} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta} = \frac{2\sin\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha-\beta}{2}}{2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}}$$

$$\Rightarrow a = b\tan\frac{\alpha+\beta}{2}$$

EXERCISE-I

Q.1 The angle of elevation of the top of a tower at point on the ground is 30°. If on walking 20 *metres* toward the tower, the angle of elevation become 60°, then the height of the tower is

(1) 10 meter (2) $\frac{10}{\sqrt{3}}$ meter

(3) $10\sqrt{3}$ meter (4) $100\sqrt{3}$ meter

Q.2 The angle of elevation of the top of the tower observed from each of the three points A, B, C on the ground, forming a triangle is the same angle α. If R is the circum-radius of the triangle ABC, then the height of the tower is

(1) R sin α (2) R cos α (3) R cot α (4) R tan α

Q.3 A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°. When he retires 40 *meters* from the bank, he finds the angle to be 30°. The breadth of the river is

(1) 20 m (2) 40 *m* (3) 30 m (4) 60 *m*

Q.4 A vertical pole consists of two parts, the lower part being one third of the whole. At a point in the horizontal plane through the base of the pole and distance 20 *meters* from it, the upper part of the pole subtends an angle whose tangent is $\frac{1}{2}$. The possible heights of the pole are

(1) 20 *m* and $20\sqrt{3}$ m (2) 20 *m* and 60 *m*

(3) 16 *m* and 48 *m* (4) 20 m and 16 mAns.

Q.5 From a 60 meter high tower angles of depression of the top and bottom of a house are α and β respectively. If the height of the house is $\frac{60\sin(\beta-\alpha)}{x}$, then x =

(1) sin α sin β (2) cos α cos β

(3) sin α cos β (4) cos α sin β

Q.6 An observer on the top of a tree, finds the angle of depression of a car moving towards the tree to be 30°°. After 3 minutes this angle becomes 60°. After how much more time, the car will reach the tree

(1) 4 *min.* (2) 4.5 *min.* (3) 1.5 *min.* (4) 2 *min.*

Q.7 A house of height 100 *metres* subtends a right angle at the window of an opposite house. If the height of the window be 64 *metres*, then the distance between the two houses is

(1) 48 *m* (2) 36 *m* (3) 54 *m* (4) 72 *m*

Q.8 The length of the shadow of a pole inclined at 10° to the vertical towards the sun is 2.05 *metres*, when the elevation ofthe sun is 38°. The length of the pole is

(1) $\frac{2.05 \sin 38^o}{\sin 42^o}$ (2) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)30\text{m}$

(3) $\frac{2.05 \cos 38^o}{\cos 42^o}$ (4) $\frac{2.05 \cos 42^o}{\sin 38^o}$

Q.9 From the top of a light house 60 *meters* high with its base at the sea level, the angle of depression of a boat is 15°. The distance of the boat from the foot of light house is

(1) $\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)60$ m (2) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)60$ m

(3) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$ m (4) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)30\text{m}$

Q.10 An observer in a boat finds that the angle of elevation of a tower standing on the top of a cliff is 60° and that of the top of cliff is 30°. If the height of the tower be 60 meters, then the height of the cliff is

(1) 30 m (2) $40\sqrt{3}$ m (3) $20\sqrt{3}$ m (4) $40\sqrt{3}$ m

Q.11 From a point *a metre* above a lake the angle of elevation of a cloud is α and the angle of depression of its reflection is β. The height of the cloud is

(1) $\frac{a\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$ *metre* (2) $\frac{a\sin(\alpha+\beta)}{\sin(\beta-\alpha)}$ *metre*

(3) $\frac{a\sin(\beta-\alpha)}{\sin(\alpha+\beta)}$ *metre* (4) $\frac{2a\sin(\beta-\alpha)}{\sin(\alpha+\beta)}$

Q.12 The shadow of a tower is found to be 60 *metre* shorter when the sun's altitude changes from 30° to 60°. The height of the tower from the ground is approximately equal to

(1) 62*m* (2) 301*m* (3) 101*m* (4) 52*m*

Q.13 If the angles of elevation of two towers from the middle point of the line joining their feet be 60° and 30° respectively, then the ratio of their heights is

(1) 2 : 1 (2) $1:\sqrt{2}$ (3) 3 : 1 (4) $1:\sqrt{3}$

Q.14 Some portion of a 20 meters long tree is broken by the wind and the top struck the ground at an angle of 30°. The height of the point where the tree is broken is

(1) 10 *m* (2) $(2\sqrt{3}-3)20$ m

(3) $\frac{20}{3}$m (4) $10\sqrt{3}$ m

Q.15 The base of a cliff is circular. From the extremities of a diameter of the base the angles of elevation of the top of the cliff are 30^o and 60^o. If the height of the cliff be 500 *metres*, then the diameter of the base of the cliff is

(1) $1000\sqrt{3}\,m$ (2) $2000/\sqrt{3}\,m$

(3) $1000/\sqrt{3}\,m$ (4) $2000\sqrt{2}\,m$

Q.16 The angle of elevation of the top of a tower from the top of a house is 60^o and the angle of depression of its base is 30^o. If the horizontal distance between the house and the tower be 12 *m*, then the height of the tower is

(1) $48\sqrt{3}$ m (2) $16\sqrt{3}$ m

(3) $24\sqrt{3}$ m (4) $16/\sqrt{3}$ m

Q.17 A man whose eye level is 1.5 *metres* above the ground observes the angle of elevation of a tower to be 60^o. If the distance of the man from the tower be 10 meters, the height of the tower is

(1) $(1.5+10\sqrt{3})\,m$ (2) $10\sqrt{3}\,m$

(3) $\left(1.5+\frac{10}{\sqrt{3}}\right)m$ (4) $100\sqrt{3}m$

Q.18 A tower subtends an angle of 30^o at a point distant d from the foot of the tower and on the same level as the foot of the tower. At a second point *h* vertically above the first, the depression of the foot of the tower is 60^o. The height of the tower is

(1) $h/3$ (2) $h/3d$ (3) $3h$ (4) $\frac{3h}{d}$

Q.19 A tree is broken by wind, its upper part touches the ground at a point 10 *metres* from the foot of the tree and makes an angle of 45^o with the ground. The total length of tree is

(1) 15 *metres* (2) 20 *metres*

(3) $10(1+\sqrt{2})$ *metres* (4) $10\left(1+\frac{\sqrt{3}}{2}\right)$ *metres*

Q.20 The angle of elevation of a stationary cloud from a point 2500 *m* above a lake is 15^o and the angle of depression of its reflection In the lake is 45^o. The height of cloud above the lake level is

(1) $2500\sqrt{3}$ metres (2) 2500 *metres*

(3) $500\sqrt{3}$ metres (4) 5000 metres

Q.21 From an aeroplane vertically over a straight horizontally road, the angles of depression of two consecutive mile stones on opposite sides of the aeroplane are observed to be and β, then the height in miles of aeroplane above the road is

(1) $\frac{\tan\alpha.\tan\beta}{\cot\alpha+\cot\beta}$ (2) $\frac{\tan\alpha+\tan\beta}{\tan\alpha.\tan\beta}$

(3) $\frac{\cot\alpha+\cot\beta}{\tan\alpha.\tan\beta}$ (4) $\frac{\tan\alpha.\tan\beta}{\tan\alpha+\tan\beta}$

Q.22 A flag-post 20*m* high standing on the top of a house subtends an angle whose tangent is $\frac{1}{6}$ at a distance 70 *m* from the foot of the house. The height of the house is

(1) 30 *m* (2) 60 *m* (3) 50 *m* (4) 20 m

Q.23 A balloon is coming down at the rate of 4 *m/min.* and its angle of elevation is 45^o from a point on the ground which has been reduced to 30^o after 10 *minutes.* Balloon will be on the ground at a distance of how many meters from the observer

(1) $20\sqrt{3}\,m$ (2) $20(3+\sqrt{3})\,m$

(3) $10(3+\sqrt{3})\,m$ (4) $10\left(3-\sqrt{3}\right)m$

Q.24 *AB* is a vertical pole resting at the end *A* on the level ground. *P* is a point on the level ground such that *AP* = 3 *AB*. If *C* is the mid- point of *AB* and *CB* subtends an angle β at *P*, the value of $\tan\beta$ is

(1) $\frac{18}{19}$ (2) $\frac{3}{19}$ (3) $\frac{1}{6}$ (4) $\frac{1}{64}$

Q.25 Two straight roads intersect at an angle of 60^o. A bus on one road is 2 *km* away from the intersection and a car on the other road is 3 *km* away from the intersection. Then the direct distance between the two vehicles is

(1) 1 *km* (2) $\sqrt{2}$ *km* (3) 4 *km* (4) $\sqrt{7}$ *km*

Q.26 The angle of elevation of a cliff at a point *A* on the ground and a point *B*, 100 *m* vertically at *A* are α and β respectively. The height of the cliff is

(1) $\frac{100\cot\alpha}{\cot\alpha-\cot\beta}$ (2) $\frac{100\cot\beta}{\cot\alpha-\cot\beta}$

(3) $\frac{100\cot\beta}{\cot\beta-\cot\alpha}$ (4) $\frac{100\cot\beta}{\cot\beta+\cot\alpha}$

Q.27 The angular elevation of a tower *CD* at a point *A* due south of it is 60^o and at a point *B* due west of *A*, the elevation is 30^o. If *AB* = 3 *km,* the height of the tower is

(1) $2\sqrt{3}$ km (2) $2\sqrt{6}$ km (3) $\frac{3\sqrt{3}}{2}$ km (4) $\frac{3\sqrt{6}}{4}$ km

Q.28 Two men are on the opposite side of a tower. They measure the angles of elevation of the top of the tower 45° and 30° respectively. If the height of the tower is 40 *m*, find the distance between the men

(1) 40 *m* (2) $40\sqrt{3}$ m (3) 68.280 *m* (4) 109.28 *m*

Q.29 The angles of elevation of the top of a tower (*1*) from the top (*2*) and bottom (*4*) at a building of height *a* are 30° and 45° respectively. If the tower and the building stand at the same level, then the height of the tower is

(1) $a\sqrt{3}$ (2) $\frac{a\sqrt{3}}{\sqrt{3}-1}$ (3) $\frac{a(3+\sqrt{3})}{2}$ (4) $a(\sqrt{3}-1)$

Q.30 A ladder 5 *metre* long leans against a vertical wall. The bottom of the ladder is 3 *metre* from the wall. If the bottom of the ladder is pulled 1 *metre* farther from the wall, how much does the top of the ladder slide down the wall

(1) 1 *m* (2) 7 *m* (3) 2 *m* (4) 3 m

Q.31 The angle of elevation of the top of a pillar at any point *A* on the ground is 15°. On walking 40 *metres* towards the pillar, the angle become 30°. The height of the pillar is

(1) 40 *metres* (2) 20 *metres*

(3) $20\sqrt{3}$metres (4) $\frac{40}{3}\sqrt{3}$metres

Q.32 The top of a hill observed from the top and bottom of a building of height *h* is at the angle of elevation *p* and *q* respectively. The height of the hills is

(1) $\frac{h\cot q}{\cot q-\cot p}$ (2) $\frac{h\cot p}{\cot p-\cot q}$

(3) $\frac{h\tan p}{\tan p-\tan q}$ (4) $\frac{h\cot p}{\cot q+\cot p}$

Q.33 The shadow of a tower standing on a level ground is found to be 60 *m* longer when the sun's altitude is 30° than when it is 45°. The height of the tower is

(1) 60 *m* (2) 30 *m* (3) $60\sqrt{3}$m (4) $30(\sqrt{3}+1)$m

Q.34 For a man, the angle of elevation of the highest point of the temple situated east of him is 60°. On walking 240 *metres* to north, the angle of elevation is reduced to 30°, then the height of the temple is

(1) $60\sqrt{6}$m (2) 60*m* (3) $50\sqrt{3}$m (4) $30\sqrt{6}$m

Q.35 A tower subtends angles α, 2α, 3α respectively at points A, B and C, all lying on a horizontal line through the foot of the tower. Then $AB/BC=$

(1) $\frac{\sin 3\alpha}{\sin 2\alpha}$ (2) $1+2\cos 2\alpha$

(3) $2+\cos 3\alpha$ (4) $\frac{\sin 2\alpha}{\sin\alpha}$

Q.36 Two pillars of equal height stand on either side of a roadway which is 60 *metres* wide. At a point in the roadway between the pillars, the elevation of the top of pillars are 60° and 30°. The height of the pillars is

(1) $15\sqrt{3}$m (2) $\frac{15}{\sqrt{3}}$m (3) 15*m* (4) 20*m*

Q.37 A ladder rests against a wall making an angle α with the horizontal. The foot of the ladder is pulled away from the wall through a distance *x*, so that it slides a distance *y* down the wall making an angle β with the horizontal. The correct relation is

(1) $x=y\tan\frac{\alpha+\beta}{2}$ (2) $y=x\tan\frac{\alpha+\beta}{2}$

(3) $x=y\tan(\alpha+\beta)$ (4) $y=x\tan(\alpha+\beta)$

EXERCISE-II

Q.1 A pole 25*m* long stands on the top of a tower 225*m* high. If θ is the angle subtended by the pole at a point on the ground which is at a distance of 2.25 km from the foot of the tower, then tanθ is equal to

(1) 1/90 (2) 1/91

(3) 1/10 (4) 1/9

Q.2 The angle of elevation of the top of a vertical pole when observed from each vertex of a regular hexagon is $\pi/3$. If the area of the circle circumscribing the hexagon be A metre^2 then the area of the hexagon is

(1) $\frac{3\sqrt{3}}{8}A$ metre^2 (2) $\frac{\sqrt{3}}{\pi}A$ metre^2

(3) $\frac{3\sqrt{3}}{4\pi}A$ metre^2 (4) $\frac{3\sqrt{3}}{2\pi}A$ metre^2

Q.3 A vertical pole PO is standing at the center O of a square ABCD. If AC subtends an angle 90° at the top P of the pole then the angle subtended by a side of the square at P is

(1) 45° (2) 30° (3) 60° (4) 90°

Q.4 The angles of elevation of the top of a tower standing on a horizontal plane, from two points on a line passing through its foot at distances *a* and *b* respectively, are complementary angles. If the line joining the two points subtends an angle θ at the top of the tower, then

(1) $\sin\theta = \frac{a-b}{a+b}$ (2) $\tan\theta = \frac{2\sqrt{ab}}{a-b}$

(3) $\sin\theta = \frac{a+b}{a-b}$ (4) $\cos\theta = \frac{2\sqrt{ab}}{a-b}$

Q.5 A vertical lamp-post, 6 *m* high, stands at a distance of 2 *m* from a wall, 4 *m* high. A 1.5-*m*-tall man starts to walk away from the wall on the other side of the wall, in line with the lamp-post. The maximum distance to which the man can walk remaining in the shadow is

(1) $\frac{5}{2}m$ (2) $\frac{3}{2}m$ (3) $4m$ (4) $\frac{4}{3}$m

Q.6 A circular ring of radius 3 *cm* is suspended horizontally from a point 4 cm vertically above the center by 4 strings attached at equal intervals to its circumference. If the angle between two consecutive strings be θ then cos θ is

(1) 4/5 (2) 4/25 (3) 16/25 (4) 5/26

Q.7 A flagstaff stands vertically on a pillar, the height of the flagstaff being double the height of the pillar. A man on the ground at a distance finds that both the pillar and the flagstaff subtend equal angles at his eyes. The ratio of the height of the pillar and the distance of the man from the pillar, is

(1) $\sqrt{3} : 1$ (2) $1 : 3$ (3) $1 : \sqrt{3}$ (4) $\sqrt{3} : 2$

Q.8 As seen from *A*, due west of a hill *HL* itself leaning east, the angle of elevation of top *H* of the hill is 60°; and after walking a distance of one kilometer along an incline of 30° to a point *B*, it was seen that the hill *LH* was printed at right angles to *AB*, the height *LH* of the hill is

(1) $\frac{1}{\sqrt{3}}\,km$ (2) $\sqrt{3}$

(3) $2\sqrt{3}\,km$ (4) $\frac{2}{\sqrt{3}}\,km$

Q.9 *ABC* is a triangular park with *AB* = *AC* = 100 metres. A clock tower is situated at the mid point of *BC*. The angles of elevation of the top of the tower at *A* and *B* are $\cot^{-1} 3.2$ and $\operatorname{cosec}^{-1} 2.6$ respectively. The height of the tower is

(1) 16 *m* (2) 25 *m* (3) 50 *m* (4) 75 m

Q.10 The angles of elevation of the top of a tower from the top and bottom of a building of height 'a' are 30° and 45°, respectively. If the tower and the building stand at the same level, the height of the tower is

(1) $a\sqrt{3}$ (2) $\frac{3\sqrt{3}a}{2}$ (3) $\frac{\sqrt{3}a}{\sqrt{3}-1}$ (4) $a(\sqrt{3}+1)$

Q.11 A flag staff of 5 mt high stands on a building of 25 mt high. At an observer at a height of 30 mt. the flag staff and the building subtend equal angles. The distance of the observer from the top of the flag staff is

(1) $\frac{5\sqrt{3}}{2}$ (2) $5\sqrt{\frac{3}{2}}$ (3) $5\sqrt{\frac{2}{3}}$ (4) $5\sqrt{\frac{2}{5}}$

Q.12 If a flag-staff of 6 metres high placed on the top of a tower throws a shadow of $2\sqrt{3}$ metres along the ground then the angle (in degrees) that the sun makes with the ground is

(1) 60° (2) 30° (3) 45° (4) 90°

Q.13 The angle of elevation of the top of an incomplete vertical pillar at a horizontal distance of 100 mt. from its base is 45°. If the angle of elevation of the top of the complete pillar at the same point is to be 60°, then the height of the incomplete pillar is to be increased by

(1) $50\sqrt{2}$ mt (2) 100 mt

(3) $100(\sqrt{3}-1)$mt (4) $100(\sqrt{3}+1)$ mt

Q.14 The angles of elevation of a cliff at a point A on the ground and a point B, 100 meters vertically at A are α and β respectively. The height of the cliff is

(1) $\frac{100\cot\alpha}{\cot\alpha-\cot\beta}$ (2) $\frac{100\cot\beta}{\cot\alpha-\cot\beta}$

(3) $\frac{100\cot\beta}{\cot\beta-\cot\alpha}$ (4) $\frac{100\cot\beta}{\cot\beta+\cot\alpha}$

Q.15 The angle of elevation of a cloud from a point *h* mt. above is θ° and the angle of depression of its reflection in the lake is θ. Then the height is

(1) $\frac{h\sin(\phi-\theta)}{\sin(\phi+\theta)}$ (2) $\frac{h\sin(\phi+\theta)}{\sin(\phi-\theta)}$

(3) $\frac{h\sin(\theta+\phi)}{\sin(\theta-\phi)}$ (4) $\frac{h\sin(\phi-\theta)}{\sin(\theta-\phi)}$

Q.16 On the level ground the angle of elevation of the top of a tower is 30°. On moving 20 m. nearer the tower, the angle of elevation is found to be 60°. The height of the tower is

(1) 10 m (2) 20 m (3) $10\sqrt{3}$ m (4) $100\sqrt{3}$ m

Q.17 Each side of a square subtends an angle of 60° at the top of a tower *h* metres high standing in the centre of the square. If a is the length of each side of the square, then

(1) $2a^2 = h^2$ (2) $2h^2 = a^2$ (3) $3a^2 = 2h^2$ (4) $3h^2 = 2a^2$.

Q.18 From the top of light house 60 metres high with its base at the sea level, the angle of depression of a boat is 15°. The distance of the boat from the foot of the light house is

(1) $\frac{\sqrt{3}-1}{\sqrt{3}+1}.60$ metres (2) $\frac{\sqrt{3}+1}{\sqrt{3}-1}.60$ metres

(3) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ metres (4) $\sqrt{3}+1$ metre

Q.19 A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°, when he retires 40 metres from the bank he finds the angle to be 30°. Then the breadth of the river is

(1) 40 m (2) 60 m (3) 20 m (4) 30 m

Q.20 AB is a vertical pole. The end A is on the level ground. C is the middle point of AB. P is a point on the level ground. The portion BC subtends an angle β at P. If AP = *n* AB, then tan β =

(1) $\frac{n}{2n^2+1}$ (2) $\frac{n}{n^2-1}$ (3) $\frac{n}{n^2+1}$ (4) $\frac{n^2}{n+1}$

Q.21 A tree is broken by wind, its upper part touches the ground at a point 10 metres from the foot of the tree and makes an angle of 45° with the ground. The entire length of the tree is

(1) 15 metres (2) 20 metres

(3) $10(1+\sqrt{2})$ metres (4) $\left(1+\frac{\sqrt{3}}{2}\right)$ metres

Q.22 An aeroplane flying at a height of 300 metres above the ground passes vertically above another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. The height of the lower plane from the ground (in metres) is

(1) $100\sqrt{3}$ (2) $\frac{100}{\sqrt{3}}$

(3) 50 (4) $150(\sqrt{3}+1)$

Q.23 A tower subtends an angle α at a point A in the plane of its base and the angle of depression of the foot of the tower at a point *b* ft. just above A is β. Then height of the tower is

(1) *b* tan α cot β (2) *b* cot α tan β

(3) *b* tan α tan β (4) *b* cot α cot β

EXERCISE-III

JEE-MAIN
PREVIOUS YEAR'S

Q.1 A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 min from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then, the time taken (in min) by him, from B to reach the pillar, is : **[JEE Main-2016]**

(1) 6 (2) 10 (3) 20 (4) 5

Q.2 Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If ∠BPC = β, then tanβ is equal to : **[JEE Main-2017]**

(1) $\frac{4}{9}$ (2) $\frac{6}{7}$ (3) $\frac{1}{4}$ (4) $\frac{2}{9}$

Q.3 PQR is a triangular park with PQ = PR = 200m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is - **[JEE Main-2018]**

(1) 50 (2) $100\sqrt{3}$

(3) $50\sqrt{2}$ (4) 100

Q.4 Consider a triangular plot ABC with sides AB = 7 m, BC = 5m and CA = 6m. A vertical lamp-post at the mid point D of AC subtendes an angle 30° at B. The height (in m) of the lamp-post is: **[JEE Main - 2019 (January)]**

(1) $\frac{3}{2}\sqrt{21}$ (2) $\frac{2}{3}\sqrt{21}$

(3) $2\sqrt{21}$ (4) $7\sqrt{21}$

Q.5 If the angle of elevation of a cloud from apoint P which is 25 m above a lake be 30º and the angle of depressionof reflection of the cloud in the lake from P be 60º, then the height of the cloud (in meters) from the surface of the lake is **[JEE Main - 2019 (January)]**

(1) 60 (2) 50

(3) 45 (4) 42

Q.6 Two vertical poles of heights, 20m and 80m stand a part on a horizontal plane. The height (in meters) of the points of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is : **[JEE Main - 2019(April)]**

(1) 12 (2) 15

(3) 16 (4) 18

Q.7 Two poles standing on a horizontal ground are of heights 5m and 10 m respectively. The line joining their tops makes an angle of 15º with ground. Then the distance **(in m)** between the poles, is :- **[JEE Main - 2019(April)]**

(1) $\frac{5}{2}(2+\sqrt{3})$ (2) $5(\sqrt{3}+1)$

(3) $5(2+\sqrt{3})$ (4) $10(\sqrt{3}-1)$

Q.8 ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}\left(3\sqrt{2}\right)$ and $\operatorname{cosec}^{-1}\left(2\sqrt{2}\right)$ respectively, then the height of the tower (in metres) is : **[JEE Main - 2019(April)]**

(1) $10\sqrt{5}$ (2) $\frac{100}{3\sqrt{3}}$

(3) 20 (4) 25

Q.9 The angle of elevation of the top of vertical tower standing on a horizontal plane is observed to be 45º from a point A on the plane. Let B be the point 30 m vertically above the point . If the angle of elevation of the top of the tower from B be 30º then the distance (in m) of the foot of the tower from the point A is : **[JEE Main - 2019(April)]**

(1) $15\left(3-\sqrt{3}\right)$ (2) $15\left(3+\sqrt{3}\right)$

(3) $15\left(1+\sqrt{3}\right)$ (4) $15\left(5-\sqrt{3}\right)$

Q.10 The angle of elevation of a cloud C from a point P, 200 m above a still lake is 30°. If the angle of depression of the image of C in the lake from the point P is 60°, then PC (in m) is equal to : **[JEE Main-2020 (September)]**

(1) 400 (2) $400\sqrt{3}$

(3) 100 (4) $200\sqrt{3}$

Q.11 Two vertical poles AB = 15 m and CD = 10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is : **[JEE Main-2020 (September)]**

(1) 6 (2) 20/3

(3) 10/3 (4) 5

Q.12 The angle of elevation of the summit of a mountain from a point on the ground is 45°. After climbing up one km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60°. Then the height (in km) of the summit from the ground is : **[JEE Main-2020 (September)]**

(1) $\frac{1}{\sqrt{3}+1}$ (2) $\frac{1}{\sqrt{3}-1}$

(3) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (4) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Q.13 The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45º. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30º to the horizontal plane, the angle of elevation of the top of the hill becomes 75º. Then the height of the hill (in meters) is — **[JEE Main-2020 (September)]**

ANSWER KEY

EXERCISE-I

Q.1 (3)	**Q.2** (4)	**Q.3** (1)	**Q.4** (2)	**Q.5** (4)	**Q.6** (3)	**Q.7** (1)	**Q.8** (1)	**Q.9** (2)	**Q.10** (1)
Q.11 (2)	**Q.12** (4)	**Q.13** (3)	**Q.14** (3)	**Q.15** (2)	**Q.16** (2)	**Q.17** (1)	**Q.18** (1)	**Q.19** (3)	**Q.20** (1)
Q.21 (4)	**Q.22** (3)	**Q.23** (2)	**Q.24** (2)	**Q.25** (4)	**Q.26** (3)	**Q.27** (4)	**Q.28** (4)	**Q.29** (3)	**Q.30** (1)
Q.31 (2)	**Q.32** (2)	**Q.33** (4)	**Q.34** (1)	**Q.35** (2)	**Q.36** (1)	**Q.37** (1)			

EXERCISE-II

Q.1 (2)	**Q.2** (4)	**Q.3** (3)	**Q.4** (4)	**Q.5** (1)	**Q.6** (3)	**Q.7** (3)	**Q.8** (1)	**Q.9** (2)	**Q.10** (3)
Q.11 (2)	**Q.12** (1)	**Q.13** (3)	**Q.14** (3)	**Q.15** (2)	**Q.16** (3)	**Q.17** (2)	**Q.18** (2)	**Q.19** (3)	**Q.20** (1)
Q.21 (3)	**Q.22** (1)	**Q.23** (1)							

EXERCISE-III

JEE-MAIN
PREVIOUS YEAR'S

Q.1 (4)	**Q.2** (4)	**Q.3** (4)	**Q.4** (2)	**Q.5** (2)	**Q.6** (3)	**Q.7** (3)	**Q.8** (3)	**Q.9** (2)	**Q.10** (1)
Q.11 (1)	**Q.12** (2)	**Q.13** (80.00)							

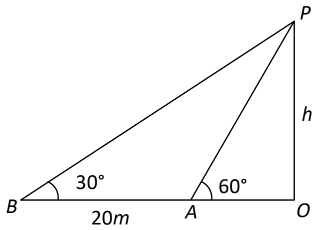
EXERCISE (Solution)

EXERCISE-I

Q.1 (3)

$OA = h\cot 60^o, OB = h\cot 30^o$

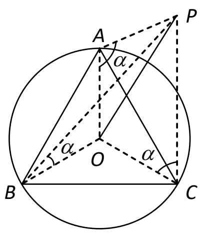
$OB - OA = 20 = h(\cot 30^o - \cot 60^o)$



$$h = \frac{20}{\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)} = \frac{20\sqrt{3}}{2} = 10\sqrt{3}$$

Q.2 (4)

Since the tower makes equal angles at the vertices of the triangle, therefore foot of the tower is at the circumcentre.

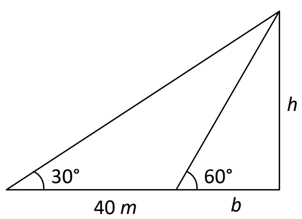


From ΔOAP, we have $\tan\alpha = \frac{OP}{OA}$

$\Rightarrow$ $OP = OA\tan\alpha$ $OP = R\tan\alpha$

Q.3 (1)

$b = h\cot 60^o, b + 40 = h\cot 30^o$

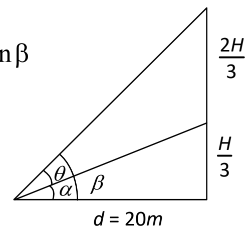


$$\Rightarrow \frac{b}{b+40} = \frac{\cot 60^o}{\cot 30^o} \Rightarrow b = 20m$$

Q.4 (2)

$\frac{H}{3}\cot\alpha = d$ and $H\cot\beta = d$

or $\frac{H}{3d} = \tan\alpha$ and $\frac{H}{d} = \tan\beta$



$$\tan(\beta - \alpha) = \frac{1}{2} = \frac{\frac{H}{d} - \frac{H}{3d}}{1 + \frac{H^2}{3d^2}}$$

$$\Rightarrow 1 + \frac{H^2}{3d^2} = \frac{4H}{3d}$$

$\Rightarrow H^2 - 4dH + 3d^2 = 0$

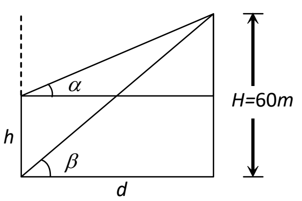
$\Rightarrow H^2 - 80H + 3(400) = 0$

$\Rightarrow H = 20$ or $60m$

Q.5 (4)

$H = d\tan\beta$ and $H - h = d\tan\alpha$

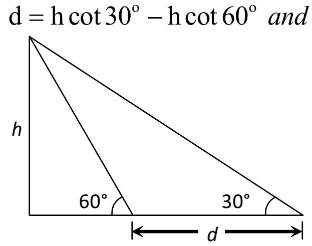
$$\Rightarrow \frac{60}{60-h} = \frac{\tan\beta}{\tan\alpha} \Rightarrow -h = \frac{60\tan\alpha - 60\tan\beta}{\tan\beta}$$



$$\Rightarrow h = \frac{60\sin(\beta-\alpha)}{\cos\alpha\cos\beta\frac{\sin\beta}{\cos\beta}} \Rightarrow x = \cos\alpha\sin\beta$$

Q.6 (3)

$d = h\cot 30^o - h\cot 60^o$ *and time* $= 3$ min.



$$\therefore \text{Speed} = \frac{h(\cot 30^o - \cot 60^o)}{3} \text{ per minute}$$

It will travel distance $h\cot 60^{\circ}$ in

$$\frac{h\cot 60^{\circ}\times 3}{h(\cot 30^{\circ}-\cot 60^{\circ})}=1.5 \text{ minute}$$

Q.7 (1)

$64\cot\theta = d$

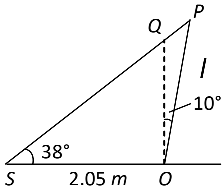
Also $(100-64)\tan\theta = d$

or $(64)(36)=d^2$

$\therefore d = 8\times 6 = 48$ m.

Q.8 (1)

$$\frac{\sin 38^{\circ}}{1}=\frac{\sin(SPO)}{2.05}$$



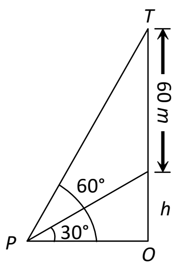
$$=\frac{\sin(180^{\circ}-38^{\circ}-90^{\circ}-10^{\circ})}{2.05}\Rightarrow l=\frac{2.05\sin 38^{\circ}}{\sin 42^{\circ}}$$

Q.9 (2)

Required distance = $60\cot 15^{\circ}=60\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$

Q.10 (1)

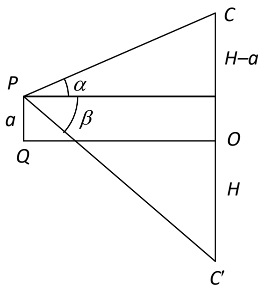
$(60+h)\cot 60^{\circ}=h\cot 30^{\circ}\Rightarrow h=30\text{m}$



Q.11 (2)

$(H+a)\cot\beta=(H-a)\cot\alpha$

$$H=\frac{a\sin(\alpha+\beta)}{\sin(\beta-\alpha)}$$



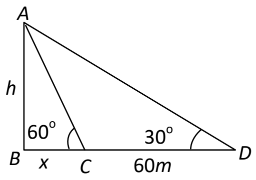
Using $\cot\alpha+\cot\beta=\dfrac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$

and $\cot\alpha-\cot\beta=\dfrac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$

Q.12 (4)

$$\tan 30^{\circ}=\frac{h}{x+60},\ \frac{1}{\sqrt{3}}=\frac{h}{x+60}$$

$$x+60=\sqrt{3}h,\ x=\sqrt{3}h-60$$



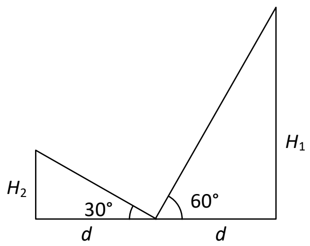
$$\tan 60^{\circ}=\frac{h}{x},\ x=\frac{h}{\sqrt{3}}$$

$$\Rightarrow\sqrt{3}h-60=\frac{h}{\sqrt{3}}\Rightarrow 3h-60\sqrt{3}=h$$

$$\Rightarrow h=\frac{60\sqrt{3}}{2}=30\sqrt{3}=51.96\approx 52\text{m}$$

Q.13 (3)

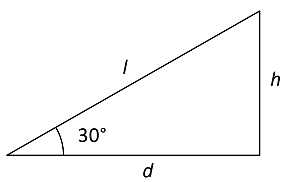
$H_1=d\tan 60^{\circ},\ H_2=d\tan 30^{\circ}$



$$\frac{H_1}{H_2}=\frac{\tan 60^{\circ}}{\tan 30^{\circ}}=\frac{3}{1}$$

Q.14 (3)

$$H=20=l+h,\ l=\frac{d}{\cos 30^{\circ}},\ h=d\tan 30^{\circ}$$

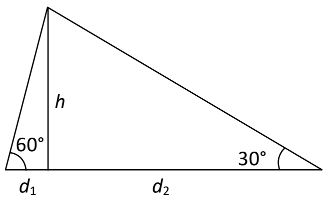


$$\therefore d=\frac{20}{(\sec 30^{\circ}+\tan 30^{\circ})}=\frac{20}{\sqrt{3}}$$

and hence $h=d\tan 30^{\circ}=\dfrac{20}{3}\text{m}$

Q.15 (2)

$$d_2 = h\cot 30^\circ = 500\sqrt{3}, d_1 = \frac{500}{\sqrt{3}}$$

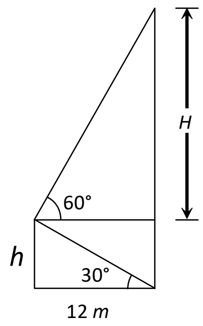


Diameter $D = 500\sqrt{3} + \frac{500}{3}\sqrt{3} = \frac{2000}{\sqrt{3}}$ m

Q.16 (2)

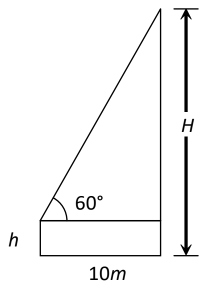
$h = 12\tan 30^\circ = \frac{12}{\sqrt{3}}$ and $H = 12\tan 60^\circ + \frac{12}{\sqrt{3}}$

$$12\sqrt{3} + \frac{12}{\sqrt{3}} = 16\sqrt{3}\text{m}$$



Q.17 (1)

$$H = (10\tan 60^\circ + 1.5) = (10\sqrt{3} + 1.5)\text{m}$$



Q.18 (1)

Trick: From $H = l\tan\alpha.\tan\beta$, the height of tower is

$h\tan 30^\circ \cot 60^\circ$ *or* $\frac{h}{3}$

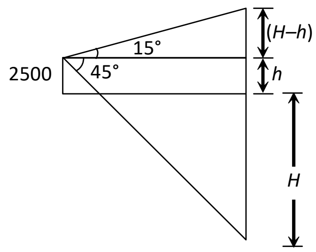
Q.19 (3)

Obviously, the length of the tree is equal to $10 + 10\sqrt{2} = 10\ (1 + \sqrt{2})$m

Q.20 (1)

$$(H - h)\cot 15^\circ = (H + h)\cot 45^\circ$$

or $H = \frac{h(\cot 15^\circ + 1)}{(\cot 15^\circ - 1)}$

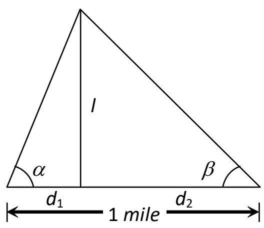


Since $h = 2500$ and substitute $\cot 15^\circ = 2 + \sqrt{3}$, we get, $H = 2500\sqrt{3}$

Q.21 (4)

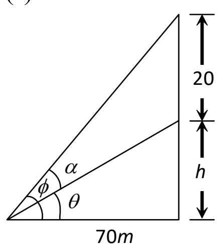
$d_1 = h\cot\alpha$ and $d_2 = h\cot\beta$

$d_1 + d_2 = 1$ *mile* $= h(\cot\alpha + \cot\beta)$



$$\Rightarrow h = \frac{1}{(\cot\alpha + \cot\beta)} = \frac{\tan\alpha.\tan\beta}{\tan\alpha + \tan\beta}$$

Q.22 (3)



$$\tan\alpha = \tan(\varphi - \theta)$$

$$\tan\alpha = \frac{1}{6} = \frac{\frac{20+h}{70} - \frac{h}{70}}{1 + \frac{(20+h)h}{(70)^2}}$$

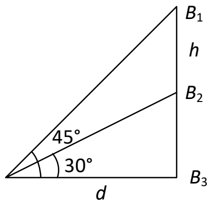
$$\Rightarrow (70)^2 + 20h + h^2 = (6)(70)(20)$$

$$\Rightarrow h^2 + 20h + 70(70 - 120) = 0$$

$$\Rightarrow h^2 + 20h - (50)(70) = 0$$

$$\Rightarrow h = \frac{-20 \pm \sqrt{400 + (4)(50)(70)}}{2} = 50\text{m}$$

Q.23 (2)



$B_1B_2 = h = (d\tan 45^\circ - d\tan 30^\circ)$

Time taken = 10 *min*

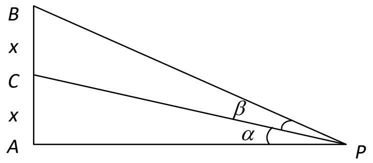
Rate = $4 = \frac{d}{10}\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)$

$\Rightarrow \; d = \frac{40\sqrt{3}}{\sqrt{3}-1} = 20(3+\sqrt{3})\,m$.

Q.24 (2)

Let $AC = x = CB, AP = 3AB = 6x$. Let $\angle CPA = \alpha$

In $\Delta ACP, \tan\alpha = \frac{x}{6x} = \frac{1}{6}$



In ΔABP, $\tan(\alpha+\beta) = \frac{2x}{6x} = \frac{1}{3}$

Now $\tan\beta = \tan\{(\alpha+\beta)-\alpha\} = \frac{\tan(\alpha+\beta)-\tan\alpha}{1+\tan(\alpha+\beta)\tan\alpha}$

$$= \frac{\frac{1}{3}-\frac{1}{6}}{1+\frac{1}{3}.\frac{1}{6}} = \frac{1}{6}\times\frac{18}{19} = \frac{3}{19}$$

Q.25 (4)

Let the two roads intersect at A. If the bus and the car are at B and C on the two roads respectively, then $c ,= AB = 2km, b = AC = 3$ km. The distance between the two vehicles = BC = a km

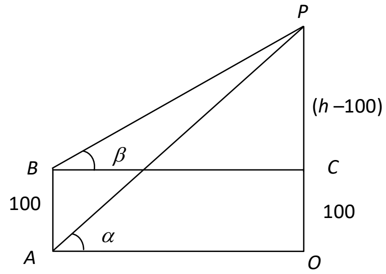
Now $\cos A = \cos 60^\circ = \frac{b^2+c^2-a^2}{2bc}$

$\Rightarrow \; \frac{1}{2} = \frac{3^2+2^2-a^2}{2.3.2} \Rightarrow a = \sqrt{7}$ km.

Q.26 (3)

If OP = h, then CP = h – 100

Now equate the values of OA and BC.



$h\cot\alpha = (h-100)\cot\beta$

$\therefore h = \frac{100\cot\beta}{\cot\beta - \cot\alpha}$

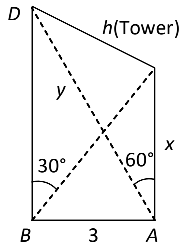
Q.27 (4)

From Δ CDA, $x = h\cot 60^\circ = \frac{h}{\sqrt{3}}$

From ΔCDB, $y = h\cot 30^\circ = \sqrt{3}h$

From ΔABC, by Pythagoras theorem

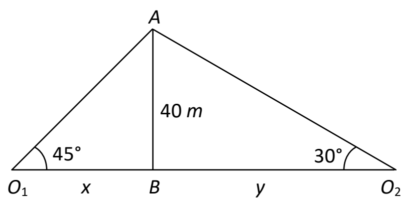
$x^2 + 3^2 = y^2$



$\Rightarrow \left(\frac{h}{\sqrt{3}}\right) + 3^2 = (\sqrt{3}h)^2 \Rightarrow h = \frac{3\sqrt{6}}{4}$ km.

Q.28 (4)

From Δ O_1AB, $\tan 45^\circ = \frac{40}{x} \Rightarrow x = 40m$



From ΔAO_2B, $\cot 30^\circ = \frac{y}{40}$

$\Rightarrow \; y = 40 \; \cot \; 30^\circ = 40\sqrt{3}$

Distance between the men = $40 + 40\sqrt{3} = 109.28$ *m*.

Q.29 (3)

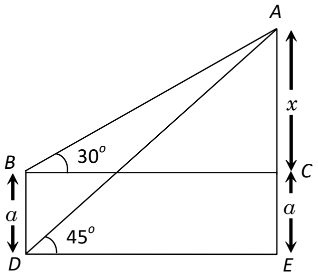
In ΔABC, $\tan 30^\circ = \frac{AC}{BC}$ *or* $\frac{1}{\sqrt{3}} = \frac{x}{BC}$,

where $AC = x$ or $BC = x\sqrt{3}$ and in ΔADE,

$\tan 45^\circ = \frac{a+x}{DE}$

or $1 = \frac{a+x}{x\sqrt{3}}$ or $x\sqrt{3} = a + x$, $x(\sqrt{3}-1) = a$ or

$x = \frac{a}{\sqrt{3}-1}$.

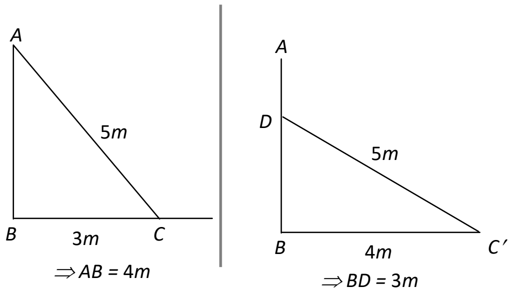


Therefore height of the tower, $a + x = a + \frac{a}{\sqrt{3}-1}$

$$= a\left[\frac{\sqrt{3}-1+1}{\sqrt{3}-1}\right] = \frac{a\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{a(3+\sqrt{3})}{2}.$$

Q.30 (1)

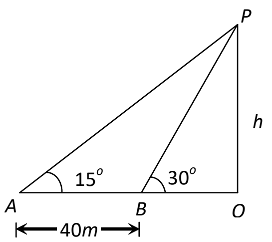
(1) From first case, From second case,



$\therefore AD = 4 - 3 = 1m.$

Q.31 (2)

Let *h* be the height of pillar

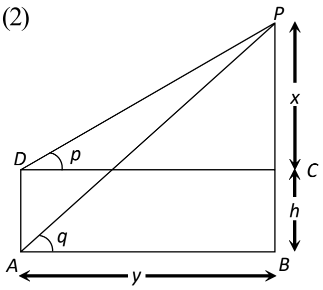


$OB = h\cot 30^\circ$ and $OA = h\cot 15^\circ$

$\Rightarrow AB = OA - OB = h(\cot 15^\circ - \cot 30^\circ)$

$\Rightarrow h = \frac{40}{\cot 15^\circ - \cot 30^\circ} = 20\ metre.$

Q.32 (2)



Let *AD* be the building of height *h* and *BP* be the hill

then $\tan q = \frac{h+x}{y}$ and $\tan p = \frac{x}{y}$

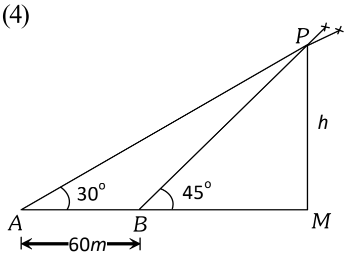
$\Rightarrow \tan q = \frac{h+x}{x\cot p}$

$\Rightarrow x\cot p = (h+x)\cot q$

$\Rightarrow x = \frac{h\cot q}{\cot p - \cot q}$

$\Rightarrow h + x = \frac{h\cot p}{\cot p - \cot q}$

Q.33 (4)



$\because AB = AM - BM \Rightarrow \frac{AB}{h} = \frac{AM}{h} - \frac{BM}{h}$

$\frac{AB}{h} = \cot 30^\circ - \cot 45^\circ \Rightarrow h = \frac{60}{\sqrt{3}-1} = \frac{60(\sqrt{3}+1)}{3-1}$

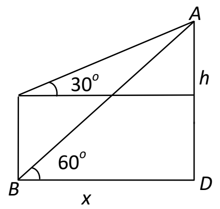
$\Rightarrow h = 30(\sqrt{3}+1)\,m$

Q.34 (1)

Total distance from temple = $\sqrt{x^2 + (240)^2}$ where

$x = \frac{h}{\tan 60^\circ} = \frac{h}{\sqrt{3}}$

So distance $= \sqrt{\frac{h^2}{3} + (240)^2}$

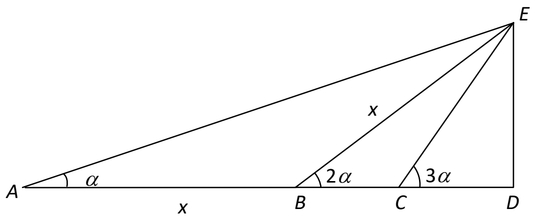


but $\dfrac{h}{\sqrt{\dfrac{h^2}{3}+(240)^2}}=\dfrac{1}{\sqrt{3}} \Rightarrow \dfrac{h^2}{\dfrac{h^2}{3}+(240)^2}=\dfrac{1}{3}$

After solving, $h=60\sqrt{6}$ m.

Q.35 (2)

From sine rule,



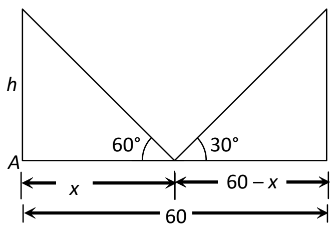
$\Rightarrow \dfrac{BE}{\sin(180^o-3\alpha)}=\dfrac{BC}{\sin\alpha}$

$\Rightarrow \dfrac{AB}{\sin 3\alpha}=\dfrac{BC}{\sin\alpha}$ (Since *BE* = *AB*)

$\Rightarrow \dfrac{AB}{BC}=\dfrac{\sin 3\alpha}{\sin\alpha}=3-4\sin^2\alpha$

$=3-2(1-\cos 2\alpha)=1+2\cos 2\alpha.$

Q.36 (1)



$\tan 60°=\dfrac{h}{x}\Rightarrow\dfrac{\sqrt{3}}{1}=\dfrac{h}{x}\Rightarrow h=\sqrt{3}x$ .....(i)

$\tan 30°=\dfrac{h}{60-x}\Rightarrow\dfrac{1}{\sqrt{3}}=\dfrac{h}{60-x}\Rightarrow 60-x=\sqrt{3}h$ .....(ii)

From equation (i) and (ii), $60-x=\sqrt{3}(\sqrt{3}x)$

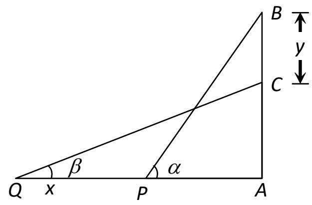
$\dfrac{60}{4}=x\Rightarrow x=15$

Then $h=\sqrt{3}x\Rightarrow h=15\sqrt{3}$ *metre.*

Q.37 (1)

$PB=QC=l$ (Length of ladder)

$\Rightarrow PA=l\cos\alpha, QA=l\cos\beta$



$\Rightarrow AC=l\sin\beta, AB=l\sin\alpha$

$\Rightarrow CB=AB-AC=l(\sin\alpha-\sin\beta)$

$\Rightarrow y=l(\sin\alpha-\sin\beta)$

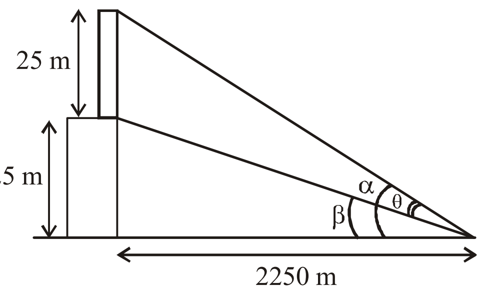
and $QP=x=AQ-AP=l$, $(\cos\beta-\cos\alpha)$

$\Rightarrow \dfrac{CB}{QP}=\dfrac{\sin\alpha-\sin\beta}{\cos\beta-\cos\alpha}=\dfrac{y}{x}=\dfrac{2\sin\left(\dfrac{\alpha-\beta}{2}\right)\cos\left(\dfrac{\alpha+\beta}{2}\right)}{2\sin\left(\dfrac{\alpha+\beta}{2}\right)\sin\left(\dfrac{\alpha-\beta}{2}\right)}$

$\Rightarrow \dfrac{y}{x}=\cot\left(\dfrac{\alpha+\beta}{2}\right)\Rightarrow x=y\tan\left(\dfrac{\alpha+\beta}{2}\right)$

EXERCISE-II

Q.1 **(2)**



$\tan\alpha=\dfrac{250}{2250}=\dfrac{1}{9}$

$\tan\beta=\dfrac{225}{2250}=\dfrac{1}{10}$

$\theta=\alpha-\beta$

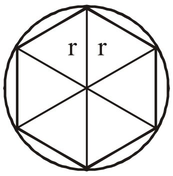
$\tan\theta=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

$=\dfrac{\dfrac{1}{9}-\dfrac{1}{10}}{1+\dfrac{1}{90}}$

$\tan\theta = \frac{1}{91}$

$\Rightarrow \boxed{\tan\theta = \frac{1}{91}}$

Q.2 (4)



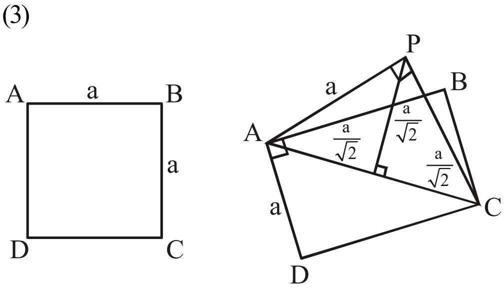
$\tan\left(\frac{\pi}{3}\right) = \frac{h}{r}$

$\text{Area of hexagen} = \frac{3\sqrt{3}}{2}r^2$

$\text{Area of circle} = \pi r^2 = A$

$\Rightarrow \frac{3\sqrt{3}}{2}\cdot\left(\frac{A}{\pi}\right) = \frac{3\sqrt{3}\,\pi}{2\pi}\ m^2$

Q.3 (3)



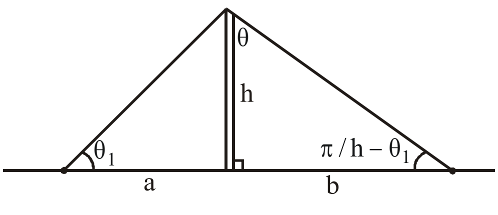
$\Rightarrow AP = AD = PD = a$

$\Rightarrow$ angle subtended

by aside $= \frac{\pi}{3}$

($\because$ equilateral Δu)

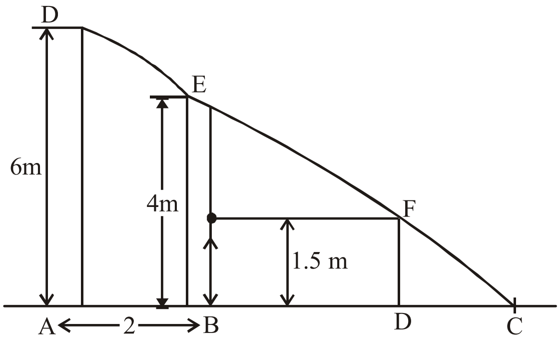
Q.4 (4)



$\tan\theta_1 \tan\left(\frac{\pi}{2} - \theta_1\right) = 1$

$\frac{h}{a} - \frac{h}{b} = 1 \Rightarrow h = \sqrt{ab}$

Q.5 (1)



$BD \Rightarrow \frac{BE}{BC} = \frac{FD}{CD} \qquad \frac{AD}{AC} = \frac{BE}{BC}$

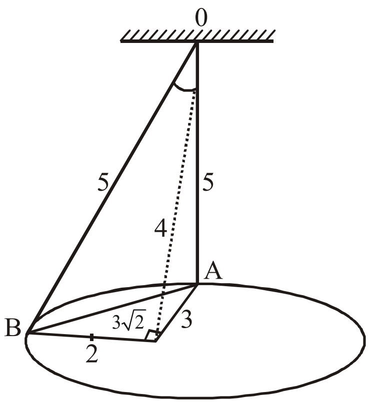
$\frac{4}{2_1} = \frac{1.5}{4-y} \qquad \frac{6}{2+x} = \frac{y}{x}$

$4 - y = 1.5 \Rightarrow y = 2.5 = \boxed{\frac{5}{2}} \qquad 6x = 8 + 4x$

$8x = 8$

$n = 4m$

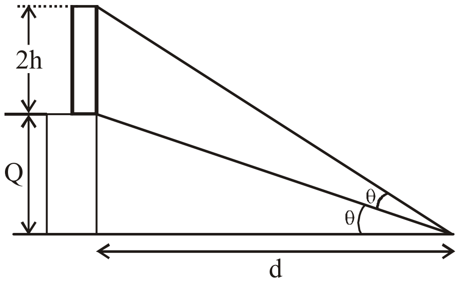
Q.6 (3)



$\cos\theta = \frac{5^2 + 5^2 - (3\sqrt{2})^2}{2(5)(5)} = \frac{25 + 25 - 18}{50}$

$\cos\theta = \frac{32}{50} = \frac{16}{25}$

Q.7 (3)



Sol.

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\tan 2\theta = \frac{3b}{d}$$

$$\tan\theta = \frac{h}{d}$$

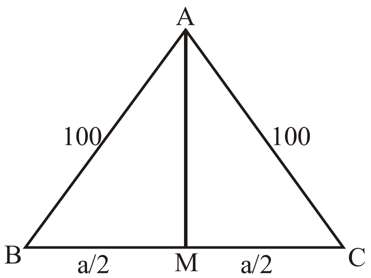
$$\Rightarrow \frac{3h}{d} = \frac{\frac{2h}{d}}{1-\frac{h^2}{d^2}} \quad \Rightarrow \quad 1-\left(\frac{h}{d}\right)^2 = \frac{2}{3}$$

$$\Rightarrow \boxed{\frac{1}{\sqrt{3}} = \frac{h}{d}}$$

Q.8 (1)

$\tan 30^\circ = h \Rightarrow h = {}^{1}\!/\!{}_{\sqrt{3}}$ km

Q.9 (2)

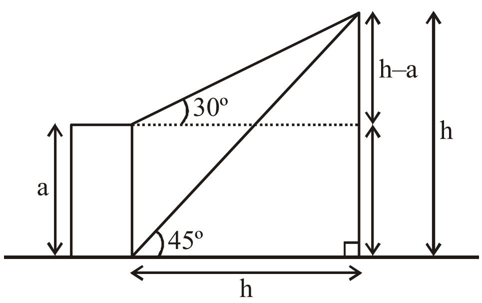


$$AM = \sqrt{2(100)^2 + 2(100)^2 - a^2}$$

given $\frac{AM}{h} = \frac{16}{5}$ ......(1)

$\frac{h}{a/2} = \frac{5}{12}$ ......(2)

Q.10 (3)



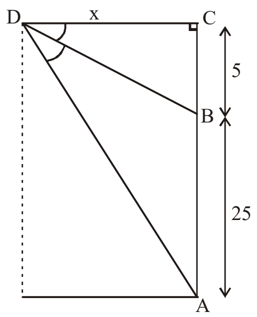
$$\frac{1}{\sqrt{3}} = \frac{h-a}{h}$$

$h = h\sqrt{3} - a\sqrt{3} \Rightarrow a\sqrt{3} - h(\sqrt{3} - 1)$

$$\Rightarrow h = \frac{a\sqrt{3}}{(\sqrt{3}-1)} = a(3-\sqrt{3})$$

$$\boxed{h = \frac{a\sqrt{3}}{(\sqrt{3}-1)}}$$

Q.11 (2)



In ΔBCD In ΔACD

$\tan\alpha = \frac{5}{x}$; $\tan 2\alpha = \frac{30}{x}$

$$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{30}{x}$$

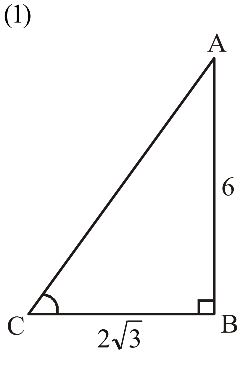
$$\frac{2\times\frac{5}{x}}{1-\frac{25}{x^2}} = \frac{30}{x}$$

$$\frac{10}{x^2}\cdot\frac{x^2}{(x^2-25)} = \frac{30}{x} \Rightarrow x^2 = 3c^2 - 75$$

$75 = 2x^2$

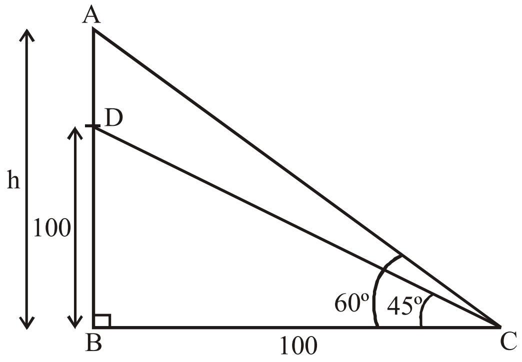
$$x = 5\sqrt{\frac{3}{2}}$$

Q.12 (1)



$$\tan\theta = \frac{6}{2\sqrt{3}} \Rightarrow \theta = 60^\circ$$

Q.13 (3)

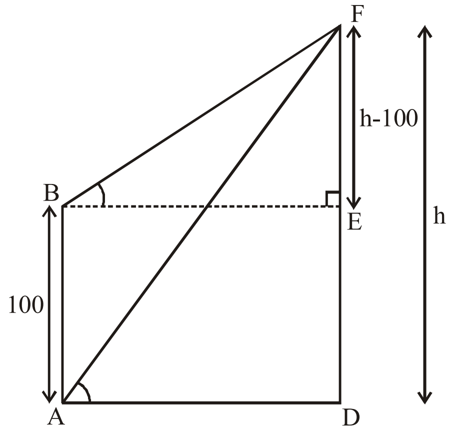


$\frac{h}{100} = \tan 60^o \quad \Rightarrow \quad h = 100\sqrt{3}$

Height increased by $(100\sqrt{3} - 100)$

$= 100(\sqrt{3} - 1)$

Q.14 (3)



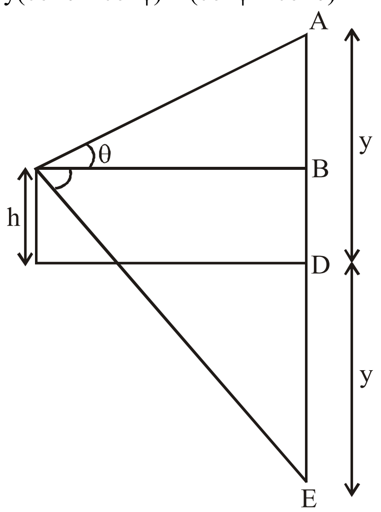
Same as previous question.

Q.15 (2)

In ΔABC at BCE

$(y - h) \cot\theta = (y + h) \cot\phi$

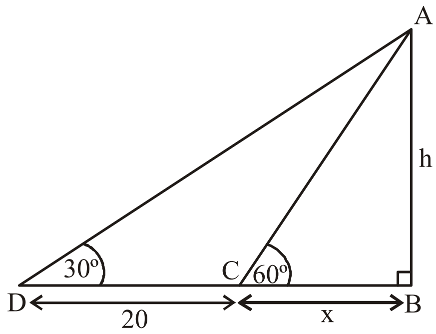
$y(\cot\theta - \cot\phi) = (\cot\phi + \cot\theta)h$



$\Rightarrow y = \frac{(\cot\phi + \cot\theta)h}{\cot\theta - \cot\phi}$

$y = \frac{\sin(\theta + \phi)}{\sin(\phi - \theta)} h$

Q.16 (3)



In ΔABC

$\frac{h}{x} = \tan 60^o$

$\boxed{h = \sqrt{3}x}$ ....(1)

2 ΔABD

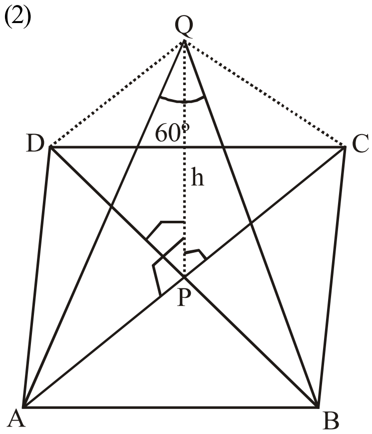
$\frac{h}{x + 20} = \tan 30^o \Rightarrow \sqrt{3}g = x + 20$ ......(2)

from (1) & (2)

$h = \sqrt{3}(\sqrt{3}h - 20) \Rightarrow 2h = 20\sqrt{3}$

$\Rightarrow \boxed{h = 10\sqrt{3}}$

Q.17 (2)



$\angle AQB = 60^o$ and $AQ = BQ$

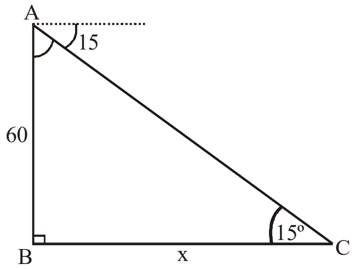
$\Rightarrow$ ABQ is equilateral triangle

In ΔDPQ

$AQ^2 = h^2 + AP^2$

$Q^2 = h^2 + \frac{a^2}{2} \Rightarrow \boxed{2h^2 = a^2}$

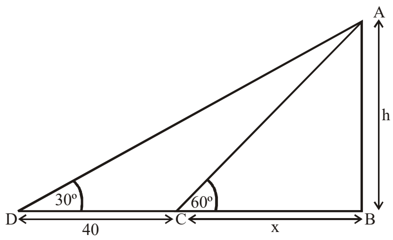
Q.18 (2)



If ΔABC $\frac{x}{60} = \cot 15^\circ$

$\Rightarrow x = (2+\sqrt{3})\,60 = \frac{\sqrt{3}+1}{\sqrt{3}-1}.60$ metres

Q.19 (3)



Total x

If ΔABC ; If ΔABD

$h = \sqrt{3}\,x$ ......(1) $\sqrt{3}\,h = 40 + x$ .....(2)

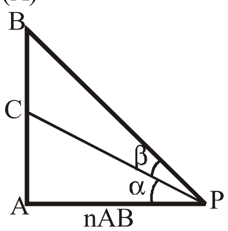
$h = \sqrt{3}\,x = \frac{40+x}{\sqrt{3}}$

Simplify.

$\sqrt{3}\times\sqrt{3}x = 40 + x$

$2x = 40 \Rightarrow x = 20$ m

Q.20 (A)



In ΔACP

$\tan\alpha = \frac{AC}{AP} = \frac{AB/2}{nAP} = \frac{1}{2n}$

In ΔABP

$\tan(\alpha+\beta) = \frac{AB}{AP} = \frac{AB}{nAB} = \frac{1}{n}$

$\tan(\alpha+\beta) = \frac{1}{n}$

$\frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \frac{1}{n}$

$\frac{\frac{1}{2n}+\tan\beta}{1-\frac{1}{2n}\tan\beta} = \frac{1}{4}$

$\frac{\frac{1+2n\tan\beta}{2n}}{\frac{2n-\tan\beta}{2n}} = \frac{1}{n}$

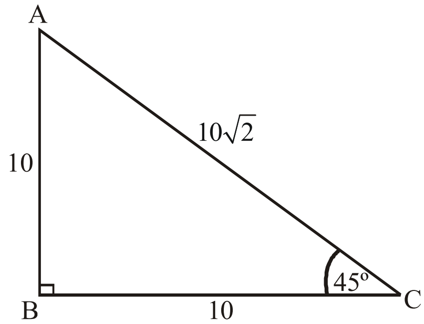
$n(1+2n\tan\beta) = 2n - \tan\beta$

$n + 2n^2\tan\beta = 2n - \tan\beta$

$\tan\beta\,(2n^2+1) = 2n - n$

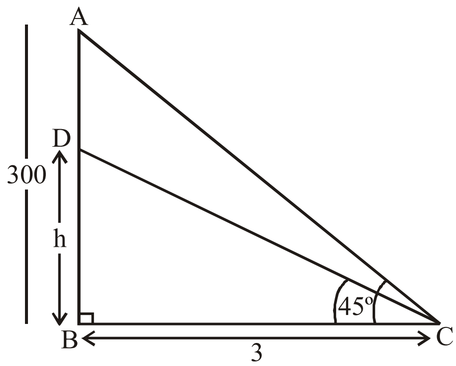
$\tan\beta = \frac{n}{2n^2+1}$

Q.21 (3)



Total height of true $= AB + AC = 10(\sqrt{2}+1)$

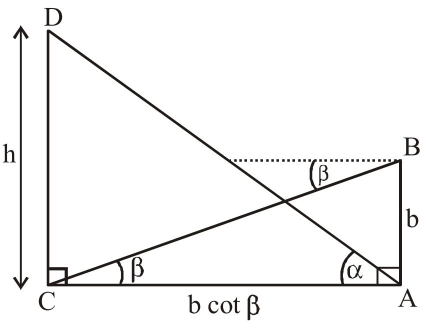
Q.22 (1)



$BC = 300\cot 60^\circ = \frac{100}{\sqrt{3}}$

If ΔBCD $\boxed{h = \frac{300}{\sqrt{3}}} = 100\sqrt{3}$

Q.23 (1)



In ΔABC

$AC = b \cot \beta$

$h\Delta$ ACD

$h = b \cot \beta \tan \alpha.$

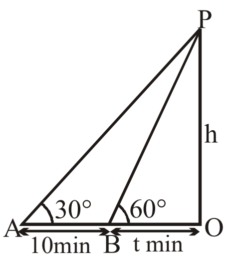
EXERCISE-III

JEE-MAIN

PREVIOUS YEAR'S

Q.1 (4)

In ΔAPO



$\tan 30° = \frac{h}{AO}$

$AO = \sqrt{3}h$

In ΔBPO

$\tan 60° = \frac{h}{t}$

$\sqrt{3} = \frac{h}{t}$

$t = \frac{h}{\sqrt{3}}$

$AO = \sqrt{3}\ h \Rightarrow h = \frac{AO}{\sqrt{3}}$

$OB = \frac{h}{\sqrt{3}} \Rightarrow h = OB\sqrt{3}$

$\frac{AO}{\sqrt{3}} = OB\sqrt{3}$

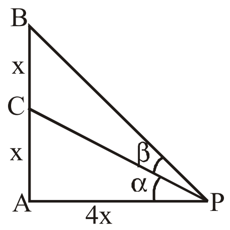
$AO = 3\ OB$

$AOB + BO = 3OB$

$AB = 2OB$

$OB = \frac{AB}{2} = \frac{10}{2} = 5$ min.

Q.2 (4)

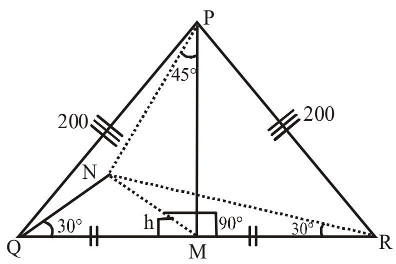


$\tan \beta + \tan\ [(\alpha + \beta) = \alpha]$

$= \frac{\tan(\alpha+\beta) + \tan\alpha}{1 - \tan(\alpha+\beta)\tan\alpha}$

$= \frac{\frac{1}{2} - \frac{1}{4}}{1 + \frac{1}{2}\frac{1}{4}} = \frac{2/8}{9/8} = \frac{2}{9}$

Q.3 (4)



Let height of tower MN is 'h'

In Δ QMN

$\frac{MN}{QM} = \tan 30°$

$\therefore QM = \sqrt{3}h = MR$ . ...(i)

Now in Δ MNP

MN = PM ...(ii)

In Δ PMQ

$MP = \sqrt{(200)^2 - (\sqrt{3}h)^2}$

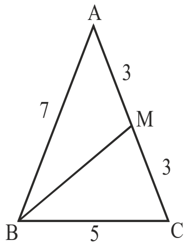
$\therefore$ From (ii)

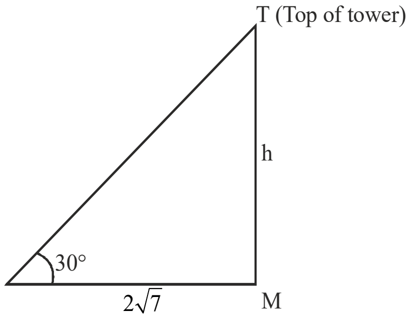
$\sqrt{(200)^2 - (\sqrt{3}h)^2} = h \Rightarrow h = 100m$

Q.4 (2)

Length of mediam $BM = \frac{1}{2}\sqrt{2(BC^2 + BA^2) - (AC)^2}$

$= \frac{1}{2}\sqrt{2(25+49) - 36}$



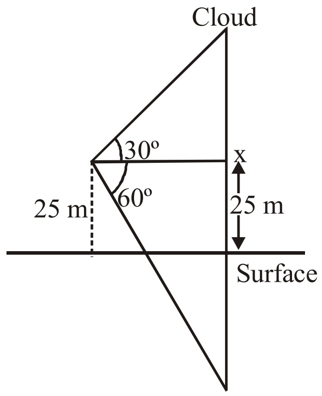


$$=\frac{1}{2}\sqrt{112}=\sqrt{\frac{112}{4}}=\sqrt{28}=2\sqrt{7}$$

Let h be height of tower, given tan 30°

$$=\frac{h}{2\sqrt{h}}\Rightarrow h=2\sqrt{\frac{7}{3}}=\sqrt{\frac{28}{3}}$$

Q.5 (2)



$$\tan 30^{o}=\frac{x}{y}\Rightarrow y=\sqrt{3}x \quad .....(i)$$

$$\tan 60^{o}=\frac{25+x+25}{y}$$

$$\Rightarrow \sqrt{3}y=50+x$$

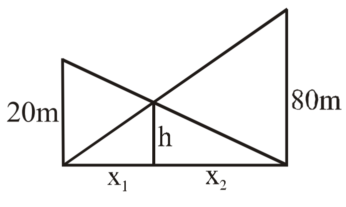
$\Rightarrow 3x=50+x$

$\Rightarrow x=25m$

$\therefore$ Height of cloud from surface

$=25+25=50\text{ m}$

Q.6 (3)



by similar triangle

$$\frac{h}{x_1}=\frac{80}{x_1+x_2} \quad ......(1)$$

$$\text{by } \frac{h}{x_2}=\frac{20}{x_1+x_2} \quad ......(2)$$

by (1) and (2)

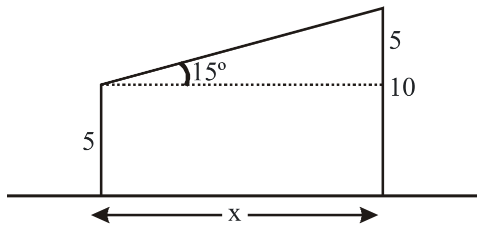
$$\frac{x_2}{x_1}=4 \text{ or } x_2=4x_1$$

$$\Rightarrow \frac{h}{x_1}=\frac{80}{5x_1}$$

or $h=16$ m

Q.7 (3)

$$\tan 15^{o}=\frac{5}{x}$$



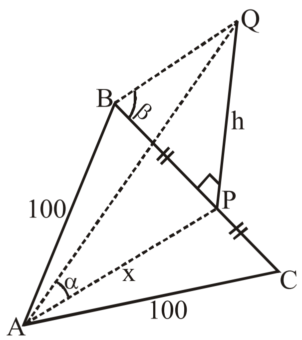
$$2-\sqrt{13}=\frac{5}{x}$$

$$x=5(2+\sqrt{3})$$

Q.8 (3)

$\cot\alpha = 3\sqrt{2}$

& $\operatorname{cosec}\beta = 2\sqrt{2}$



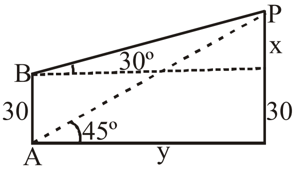
$$\text{So, } \frac{x}{h}=3\sqrt{2} \quad ....(i)$$

$$\text{And } \frac{h}{\sqrt{10^4-x^2}}=\frac{1}{\sqrt{7}} \quad ....(ii)$$

So, from (i) & (ii)

$$\Rightarrow \frac{4}{\sqrt{10^4 - 18h^2}} = \frac{1}{\sqrt{7}}$$
$\Rightarrow 25h^2 = 100 \times 100$
$\Rightarrow h = 20$

Q.9 (2)

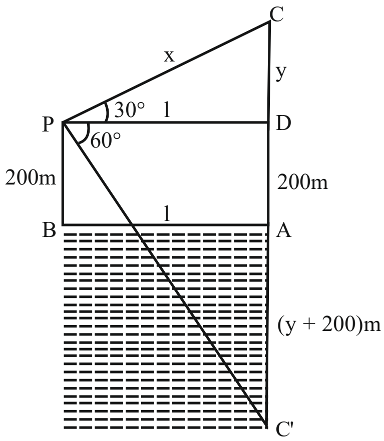


$\tan 45° = 1 = \frac{x+30}{y} \Rightarrow x + 30 = y$ ....(i)

$\tan 45° = \frac{1}{\sqrt{3}} = \frac{x}{y} \Rightarrow x = \frac{y}{\sqrt{3}}$ ....(ii)

from (i) and (ii) $y = 15\left(3+\sqrt{3}\right)$

Q.10 (1)



Here in ΔPCD

$\sin 30° = \frac{y}{x}$

$\Rightarrow x = 2y$

$\Rightarrow y = \frac{x}{2}$

$\cos 30° = \frac{l}{x}$

$\Rightarrow l = \frac{\sqrt{3}x}{2}$

Now, in right $\Delta PC'D$

$\tan 60° = \frac{y+400}{l}$

$\Rightarrow y + 400 = \sqrt{3}l = \sqrt{3} \times \frac{\sqrt{3}x}{2}$

$\Rightarrow \frac{x}{2} - \frac{3x}{2} = -400$

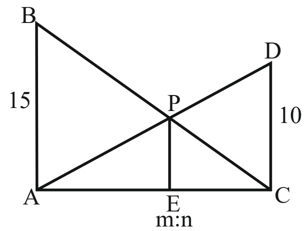
$\Rightarrow x = 400$ m

So, PC = 400 m

Q.11 (1)

Refer to diagram,

Let $PE \perp AC$



and $\frac{AE}{EC} = \frac{m}{n}$

So; $PE = \frac{10m}{m+n}$ ....(i)

(because ΔACD and ΔAEP are similar)

Similarly $PE = \frac{15n}{m+n}$ ...(2)

From (1) and (2)

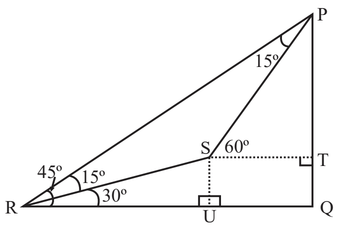
$10\,m = 15\,n \Rightarrow m = \frac{3}{2}n$

So PE = 6

Q.12 (2)

$\because \angle SRP = \angle SPR = 30°$

$\therefore SP = SR = 1$ km



In ΔSPT,

$\frac{PT}{SP} = \sin 60°$

$PT = \frac{\sqrt{3}}{2}$ km

In ΔRSU, $\sin 30° = \frac{SU}{RS} \Rightarrow SU = \frac{1}{2}$ km

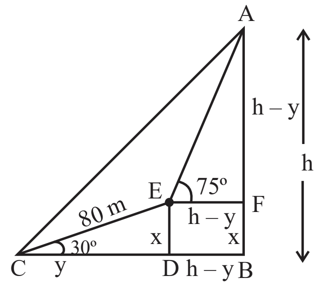
$\therefore$ Height of mountain $= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = \left(\frac{\sqrt{3}+1}{2}\right)$ km

$= \frac{1}{\sqrt{3}-1}$

Q.13 [80.00]

In rt ΔCDE –

Let height = h m



$\sin 30° = \frac{x}{80} \Rightarrow x = 40$

$\cos 30° = \frac{y}{80} \Rightarrow y = 40\sqrt{3}$

Now, In ΔAEF

$\tan 75° = \frac{h-x}{h-y}$

$\left(2+\sqrt{3}\right) = \frac{h-40}{h-40\sqrt{3}}$

$\left(2+\sqrt{3}\right)\left(h-40\sqrt{3}\right) = h-40$

$\Rightarrow 2h + 80\sqrt{3} + \sqrt{3}\,h - 120 = h - 40$

$\Rightarrow h + \sqrt{3}\,h = 80 + 80\sqrt{3}$

$\Rightarrow \left(\sqrt{3}+1\right) h = 80\left(\sqrt{3}+1\right)$

$h = 80$ m