Chapter 3

Theory of Equations

Ex 3.1

Question 1.
If the sides of a cubic box are increased by 1, 2, 3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid.

Solution:
Let the side of the cubic box are x, x, x.
Then its volume is x^3
Sides are by 1, 2, 3 units respectively.
Cuboid sides be $(x + 1)(x + 2)(x + 3)$.
volume of cuboid is
$\Rightarrow (x + 1)(x + 2)(x + 3) = x^3 + 52$
$\Rightarrow (x^2 + 3x + 2)(x + 3) = x^3 + 52$
$\Rightarrow x^3 + 3x^2 + 2x + 3x^2 + 9x + 6 = x^3 + 52$
$\Rightarrow 6x^2 + 11x + 6 - 52 = 0$
$\Rightarrow 6x^2 + 11x - 46 = 0$
$\Rightarrow 6x^2 - 12x + 23x - 46 = 0$
$\Rightarrow 6x(x - 2) + 23(x - 2) = 0$
$\Rightarrow x - 2 = 0, 6x + 23 = 0$
$\Rightarrow x = 2, 6x = -23$
$\therefore x = 2, x = -236$ (is not possible)
volume of cube $= x^3 = 2^3 = 8$
Volume of cuboid $= 52 + 8 = 60$

Question 2.
Construct a cubic equation with roots
(i) 1, 2 and 3
(ii) 1, 1 and -2
(iii) 2, 12 and 1

Solution:
(i) Given roots are $\alpha = 1, \beta = 2, \gamma = 3$
The cubic equation is
$x^3 - x^2(\alpha + \beta + \gamma) + x(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma = 0$
$\Rightarrow x^3 - x^2(1 + 2 + 3) + x(2 + 6 + 3) - (1)(2)(3) = 0$
$\Rightarrow x^3 - 6x^2 + 11x - 6 = 0$
(ii) $\alpha = 1, \beta = 1, \gamma = -2$
The cubic equation is

$x^3 - x^2 (\alpha + \beta + \gamma) + x (\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma = 0$
$\Rightarrow x^3 - x^2 (1 + 1 - 2) + x (1 - 2 - 2) - (1) (1) (-2) = 0$
$\Rightarrow x^3 - 0x^2 - 3x + 2 = 0$
$\Rightarrow x^3 - 3x + 2 = 0$
(iii) $\alpha = 2, \beta = 12, \gamma = 1$
The cubic equation is
$x^3 - x^2 (\alpha + \beta + \gamma) + x (\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma = 0$

$$x^3 - x^2\left(2+\frac{1}{2}+1\right)+x\left(1+\frac{1}{2}+2\right)-(2)\left(\frac{1}{2}\right)(1) = 0$$

$$x^3 - x^2\left(\frac{4+1+2}{2}\right)+x\left(\frac{2+1+4}{2}\right)-1 = 0$$

$$x^3 - x^2\left(\frac{7}{2}\right)+x\left(\frac{7}{2}\right)-1 = 0$$

$2x^3 - 7x^2 + 7x - 2 = 0$

Question 3.
If α, β and γ are the roots of the cubic equation x3 + 2x2 + 3x + 4 = 0, form a cubic equation whose roots are
(i) 2α, 2β, 2γ
(ii) 1α,1β,1γ
(iii) -α, -β, -γ

Solution:
(i) 2α, 2β, 2γ

$x^3 + 2x^2 + 3x + 4 = 0$

$\therefore a = 1, b = 2, c = 3, d = 4$

$\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-2}{1} = -2$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{3}{1} = 3$

$\alpha\beta\gamma = \frac{-d}{a} = -4$

$2\alpha + 2\beta + 2\gamma = 2(\alpha + \beta + \gamma)$

$= 2(-2) = -4$

$(2\alpha)(2\beta) + (2\beta)(2\gamma) + (2\gamma)(2\alpha) = 4\alpha\beta + 4\beta\gamma + 4\gamma\alpha$

$= 4(\alpha\beta + \beta\gamma + \gamma\alpha)$

$= 4(3) = 12$

(2α) (2β) (2γ) = 8(αβγ)

= 8(-4) = -32

$x^3 - (-4)x^2 + 12x - (-32) = 0$

$x^3 + 4x^2 + 12x + 32 = 0$

(ii) The given roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$

The cubic equation is

$$x^3 - x^2\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right) + x\left(\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}\right) - \frac{1}{\alpha\beta\gamma} = 0$$

$$x^3 - x^2\left(\frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}\right) + x\left(\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}\right) - \frac{1}{\alpha\beta\gamma} = 0$$

$$x^3 - x^2\left(\frac{3}{-4}\right) + x\left(\frac{-2}{-4}\right) - \left(\frac{1}{-4}\right) = 0$$

$$x^3 + \frac{3x^2}{4} + \frac{2x}{4} + \frac{1}{4} = 0$$

$4x^3 + 3x^2 + 2x + 1 = 0$ (Multiply by 4)

(iii) The new roots are -α, -β, -γ

∑1 = -(α + β + γ) = -(-2) = 2

∑2 = αβ + βγ + γα = 3

∑3 = -(αβγ) = -(-4) = 4

The required equation is

$x^3 - 2x^3 + 3x - 4 = 0$

Question 4.

Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.

Solution:

The given equation is $3x^3 - 16x^2 + 23x - 6 = 0$

$\Rightarrow x^3 - \frac{16}{3}x^2 + \frac{23}{3}x - 2 = 0$ (÷3)

Let the roots be α, β, γ

$\alpha + \beta + \gamma = -b = \frac{16}{3}$ .... (1)

$\alpha\beta + \beta\gamma + \gamma\alpha = c = \frac{23}{3}$ .... (2)

$\alpha\beta\gamma = -d = 2$ .... (3)

Given that $\alpha\beta = 1$

from (3), $\gamma = 2$

Substitute $\beta = \frac{1}{\alpha}$, $\gamma = 2$ in (1)

$\Rightarrow \alpha + \frac{1}{\alpha} + 2 = \frac{16}{3}$

$\Rightarrow \frac{\alpha^2+1}{\alpha} = \frac{16}{3} - 2$

$\Rightarrow \frac{\alpha^2+1}{\alpha} = \frac{10}{3}$

$\Rightarrow 3\alpha^2 + 3 = 10\alpha$

$\Rightarrow 3\alpha^2 - 10\alpha + 3 = 0$

$\Rightarrow (3\alpha - 1)(\alpha - 3) = 0$

$\Rightarrow \alpha = \frac{1}{3}, 3$

$\alpha = \frac{1}{3}, \beta = 3$ (or) when $a = 3$, $\beta = \frac{1}{3}$

$\therefore$ The roots are $3, \frac{1}{3}, 2$

(or) when $\gamma = 2$, by synthetic division method.

2	3	–16	23	–6
	0	6	–20	6
3	3	–10	3	0
	0	9	–3	
	3	–1	0	

The factors are $(x - 2)(x - 3)(3x - 1)$

$\therefore$ The roots are $2, 3, \frac{1}{3}$

Question 5.

Find the sum of squares of roots of the equation 2x4 – 8x3 + 6x2 – 3 = 0.

Solution:

Let the roots be $\alpha, \beta, \gamma, \delta$

$\alpha + \beta + \gamma + \delta = \frac{-b}{a} = \frac{8}{2} = 4$

$\alpha\beta + \beta y + \gamma\alpha + \delta\alpha + \beta\delta + \gamma\delta = \frac{c}{a} = \frac{6}{2} = 3$

$(a + b + c + d)^2 = a^2 + b^2 + c^2 + d^2 + 2(ab + ac + ad + bc + bd + cd)$

To find $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = (\alpha + \beta + \gamma + \delta)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha + \delta\alpha + \beta\delta + \gamma\delta)$

$= (4)^2 - 2(3) = 16 - 6 = 10$

Question 6.
Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio 3 : 2.

Solution:
The given equation is $x^3 - 9x^2 + 14x + 24 = 0$.
Since the two roots are in the ratio 3 : 2.
The roots are $\alpha, 3\lambda, 2\lambda$
$\alpha + 3\lambda + 2\lambda = -b = 9$
$\Rightarrow \alpha + 5\lambda = 9$ …… (1)
$(\alpha)(3\lambda)(2\lambda) = -24$
$6\lambda^2\alpha = -24$
$\Rightarrow \lambda^2\alpha = -4$ …… (2)
(1) $\Rightarrow \alpha = 9 - 5\lambda$
(2) $\Rightarrow \lambda^2(9 - 5\lambda) = -4$
$9\lambda^2 - 5\lambda^3 + 4 = 0$

2	5	–9	0	–4
	0	10	2	4
	5	1	2	0

$5\lambda^3 - 9\lambda^2 - 4 = 0$
$(\lambda - 2)(5\lambda^2 + \lambda + 2) = 0$
$\lambda = 2$, $5\lambda2 + \lambda + 2 = 0$ has only Imaginary roots $\Delta < 0$
when $\lambda = 2$, $\alpha = 9 - 5(2) = 9 - 10 = -1$
The roots are $\alpha, 3\lambda, 2\lambda$ i.e., -1, 6, 4

Question 7.
If α, β and γ are the roots of the polynomial equation $ax^3 + bx^2 + cx + d = 0$, find the value of $\Sigma\frac{\alpha}{\beta\gamma}$ in terms of the coefficients.

Solution:
The given equation is $ax^3 + bx^2 + cx + d = 0$.

$\div a \Rightarrow x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a} = 0$

Let the roots be α, β, γ

$\alpha + \beta + \gamma = -\frac{b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$\alpha\beta\gamma = -\frac{d}{a}$

To find:

$$\sum \frac{\alpha}{\beta\gamma} = \frac{\alpha}{\beta\gamma} + \frac{\beta}{\gamma\alpha} + \frac{\gamma}{\alpha\beta} = \frac{\alpha^2 + \beta^2 + \gamma^2}{\alpha\beta\gamma}$$

$$= \frac{(\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha)}{\alpha\beta\gamma}$$

$$= \frac{\left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{-\frac{d}{a}} = \frac{\left(b^2 - 2ac\right)}{a^2} \times \frac{-a}{d} = \frac{2ac - b^2}{ad}$$

Question 8.

If α, β, γ and δ are the roots of the polynomial equation $2x^4 + 5x^3 - 7x^2 + 8 = 0$, find a quadratic equation with integer coefficients whose roots are $\alpha + \beta + \gamma + \delta$ and $\alpha\beta\gamma\delta$.

Solution:

The given equation is $2x^4 + 5x^3 - 7x^2 + 8 = 0$.

$\div 2 \Rightarrow x^4 + \frac{5}{2}x^3 - \frac{7}{2}x^2 + 4 = 0$

Let the roots be α, β, γ, δ

$\alpha + \beta + \gamma + \delta = -\frac{5}{2}$

$\alpha\beta\gamma\delta = -4$

To form the quadratic equation with the given roots $\alpha + \beta + \gamma + \delta$, $\alpha\beta\gamma\delta$.

$x^2 - x(\text{S.O.R}) + \text{P.O.R} = 0$

$x^2 - x\left(\frac{-5}{2} - 4\right) + \left(\frac{-5}{2}\right)(-4) = 0$

$\Rightarrow x^2 - x\left(\frac{-13}{2}\right) + 10 = 0$

$2x^2 + 13x + 20 = 0$

Question 9.

If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$

that pq−−√+qp−−√+nl−−√=0

Solution:
The given equation is $lx^2 + nx + n = 0$.

$p + q = -\frac{n}{l}$, $pq = \frac{n}{l}$

$$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = \frac{\sqrt{p}}{\sqrt{q}} + \frac{\sqrt{q}}{\sqrt{p}} + \frac{\sqrt{n}}{\sqrt{l}} = \frac{p+q}{\sqrt{pq}} + \frac{\sqrt{n}}{\sqrt{l}} = \frac{\left(\frac{-n}{l}\right)}{\sqrt{\frac{n}{l}}} + \frac{\sqrt{n}}{\sqrt{l}}$$

$$= \frac{-n\sqrt{l}}{l \cdot \sqrt{n}} + \frac{\sqrt{n}}{\sqrt{l}} = \frac{-\sqrt{n}}{\sqrt{l}} + \frac{\sqrt{n}}{\sqrt{l}} = 0$$

Question 10.

If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$

Solution:

If α is the common root, then.

$\alpha^2 + p\alpha + q = 0$ .... (1)

$\alpha^2 + p'\alpha + q' = 0$ .... (2)

Subtracting $\alpha (p - p') = q' - q$

$\alpha = \frac{q'-q}{p-p'} = \frac{q-q'}{p'-p}$ .... (3)

Eliminating α from (1) & (2)

$$\begin{array}{l} p'\alpha^2 + pp'\alpha + p'q = 0 \\ p\alpha^2 + pp' + pq' = 0 \\ (-) \quad (-) \quad\quad (-) \\ \hline \alpha^2 \ (p'-p) + p'q - p'q = 0 \end{array}$$

$$\alpha^2 (p' - p) = pq' - p'q$$

$$\alpha^2 = \frac{pq' - p'q}{p' - p}$$

$$\frac{(4)}{(3)} \Rightarrow \frac{\alpha^2}{\alpha} = \frac{pq' - p'q}{(p' - p)} \times \frac{p' - p}{q - q'}$$

$$\alpha = \frac{pq' - qp'}{q - q'} \text{ (or) } \frac{q - q'}{p' - p}$$

Question 11.
Formulate into a mathematical problem to find a number such that when its cube root is added to it, the result is 6.
Solution:

Let the number be x.

Given that $\sqrt[3]{x}+x=6$

$\Rightarrow \sqrt[3]{x}=6-x$

Cubing on both sides

$x = (6 - x)^3$

$\Rightarrow x = 216 - 3(6)^2(x) + 3(6)(x)^2 - x^3$

$\Rightarrow x = 216 - 108x + 18x^2 - x^3$

$\Rightarrow x^3 - 18x^2 + 109x - 216 = 0$

Question 12.
A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was cut away.
Solution:

Let the two parts be x and (12 – x)

Given that $x=\sqrt[3]{12-x}$

Cubing on both side,

$x^3 = 12 - x$

$\Rightarrow x^3 + x - 12 = 0$

Ex 3.2

Question 1.
If k is real, discuss the nature of the roots of the polynomial equation 2x2 + kx + k = 0, in terms of k.
Solution:
$\Delta = b^2 - 4ac$
$a = 2, b = k, c = k$
$\Delta = k^2 - 4 \times 2(k)$
$\Delta = k^2 - 8k$
when $k < 0$, the polynomial has real roots $(\Delta > 0)$
If $\Delta = 0$ $k^2 - 8k = 0$
$k(k - 8) = 0$
$k = 0$ or $k = 8$
When $k = 0$ or $k = 8$. The roots are real and equal.
When $0 < k < 8$, $(\Delta < 0)$ the roots are imaginary.
when $k > 8$. The roots are real and distinct.

Question 2.
Find a polynomial equation of minimum degree with rational coefficients, having $2 + \sqrt{3}\,i$ as a root.

Solution:
Given roots is $(2 + \sqrt{3}\,i)$
The other root is $(2 - \sqrt{3}\,i)$, since the imaginary roots with real co-efficient occur as conjugate pairs.
$x^2 - x(\text{S.O.R}) + \text{P.O.R} = 0$
$\Rightarrow x^2 - x(4) + (4 + 3) = 0$
$\Rightarrow x^2 - 4x + 7 = 0$

Question 3.
Find a polynomial equation of minimum degree with rational coefficients, having 2i + 3 as a root.
Solution:
Let the root be $3 + 2i$
Another root be $3 - 2i$
Sum of the roots $= 3 + 2i + 3 - 2i = 6$
Product of the roots $= (3 + 2i)(3 - 2i) = 3^2 + 2^2 = 9 + 4 = 13$
Required equation is $x^2 - (SR)x + PR = 0$
$x^2 - 6x + 13 = 0$

Question 4.
Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$ as a root.

Solution:
The given one roots of the polynomial equation are $(\sqrt{5} - \sqrt{3})$
The other roots are $(\sqrt{5} + \sqrt{3})$, $(-\sqrt{5} + \sqrt{3})$ and $(-\sqrt{5} - \sqrt{3})$.
The quadratic factor with roots $(\sqrt{5} - \sqrt{3})$ and $(\sqrt{5} + \sqrt{3})$ is
$= x^2 - x(S.O.R) + P.O.R$
$= x^2 - x(2\sqrt{5}) + (\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})$
$= x^2 - 2\sqrt{5}\,x + 2$
The other quadratic factors with roots $(-\sqrt{5} + \sqrt{3})$ $(-\sqrt{5} - \sqrt{3})$ is
$= x^2 - x\,(S.O.R) + P.O.R$
$= x^2 - x\,(-2\sqrt{5}\,) + (5 - 3)$
$= x^2 + 2\sqrt{5}x + 2$
To rationalize the co-efficients with minimum degree
$(x^2 - 2\sqrt{5}\,x + 2)\,(x2 + 2\sqrt{5}\,x + 2) = 0$
$\Rightarrow (x^2 + 2)2 - (2\sqrt{5}\,x)2 = 0$
$\Rightarrow x^4 + 4 + 4x2 - 20x2 = 0$
$\Rightarrow x^4 - 16x2 + 4 = 0$

Question 5.
Prove that a straight line and parabola cannot intersect at more than two points.
Solution:
let be the equation of a straight line $y = mx + c$
Let be the equation of a parabola $y^2 = 4ax$
$(mx + c)^2 = 4ax$
$m^2x^2 + 2mcx + c^2 - 4ax = 0$
$m^2x^2 + (2x)(mc - 2a) + c^2 = 0$
Which is a quadratic equation.
It can not have more than two solutions.

Ex 3.3

Question 1.

Solve the cubic equation: 2x3 – x2 – 18x + 9 = 0 if sum of two of its roots vanishes.

Solution:

$2x^3 - x^2 - 18x + 9 = 0$

Let the roots be α, β and γ

given $\alpha + \beta = 0$

$\alpha + \beta + \gamma = 12$

$\gamma = 12$ ($\because \alpha + \beta = 0$)

$x^2 + (\alpha + \beta)x + \alpha\beta = 0, \gamma = \frac{1}{2}$

$x^2 - 0x + p = 0, (x - \frac{1}{2}) = 0$

$(x^2 + p) = 0, 2x - 1 = 0$

$\therefore (x^2 + p)(2x - 1) = 2x^3 - x^2 - 18x + 9$

$2x^3 - x^2 + 2px - p = 2x^3 - x^2 - 18x + 9$

$-p = 9$

$p = -9$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

The roots are 3, -3, and $\frac{1}{2}$

Question 2.

Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an arithmetic progression.

Solution:

The given equation is $9x^3 - 36x^2 + 44x - 16 = 0$

$x^3 - 4x^2 + \frac{44}{9}x - \frac{16}{9} = 0$

Let the roots be α – d, α, α + d

As they are in A.P

$\alpha - d + \alpha + \alpha + d = -(-4) \quad \Rightarrow 3\alpha = 4 \quad \Rightarrow \alpha = \frac{4}{3}$

$(\alpha - d)(\alpha)(\alpha + d) = -\left(\frac{-16}{9}\right) \quad \Rightarrow \alpha(\alpha^2 - d^2) = \frac{16}{9}$

$\frac{4}{3}\left(\frac{16}{9} - d^2\right) = \frac{16}{9} \quad \Rightarrow \frac{16}{9} - d^2 = \frac{16}{9} \times \frac{3}{4}$

$$\frac{16}{9} - \frac{4}{3} = d^2$$

$$d^2 = \frac{16-12}{9} = \frac{4}{9} \qquad \Rightarrow d = \pm\frac{2}{3}$$

The roots are $\alpha - d, \alpha, \alpha + d$, when $d = \pm\frac{2}{3}, \alpha = \frac{4}{3}$

	$\alpha - d,$	$\alpha,$	$\alpha + d$
(i) $d = \frac{2}{3}, \alpha = \frac{4}{3}$	$\frac{4}{3} - \frac{2}{3} = \frac{2}{3}$	$\frac{4}{3}$	$\frac{4}{3} + \frac{2}{3} = \frac{6}{3} = 2$
(ii) $d = \frac{-2}{3}, \alpha = \frac{4}{3}$	$\alpha - d,$	$\alpha,$	$\alpha + d$
	$\frac{4}{3} - \left(\frac{-2}{3}\right) = \frac{6}{3} = 2$	$\frac{4}{3}$	$\frac{4}{3} - \frac{2}{3} = \frac{2}{3}$

Question 3.
Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if its roots form a geometric progression.
Solution:

The given equation is $3x^3 - 26x^2 + 52x - 24 = 0$

$$x^3 - \frac{26}{3}x^2 + \frac{52}{3}x - 8 = 0$$

Given that the root are GP

$\therefore$ The roots are $\frac{\alpha}{r}, \alpha, \alpha r$

$$\frac{\alpha}{r} \times \alpha \times \alpha r = -(-8)$$

$$\alpha^3 = 8 \qquad \Rightarrow \alpha = 2$$

$$\frac{\alpha}{r} + \alpha + \alpha r = -\left(\frac{-26}{3}\right) \qquad \Rightarrow \alpha\left(\frac{1}{r} + 1 + r\right) = \frac{26}{3}$$

$$2\left(\frac{1}{r} + 1 + r\right) = \frac{26}{3}$$

$$3\left(\frac{1 + r + r^2}{r}\right) = 13 \qquad \Rightarrow 3 + 3r + 3r^2 = 13r$$

$$3r^2 - 10r + 3 = 0 \quad \Rightarrow (3r - 1) + (r - 3) = 0$$

$$r = \frac{1}{3}, r = 3$$

(*i*) $\alpha = 2, r = 3 \quad \Rightarrow$ The roots are $\frac{2}{3}, 2, 6$

(*ii*) $\alpha = 2, r = \frac{1}{3} \quad \Rightarrow$ The roots are $6, 2, \frac{2}{3}$

Question 4.
Determine ft and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of the other two roots.

Solution:
The given equation is $2x^3 - 6x^2 + 3x + k = 0$

$$\div 2 \Rightarrow x^3 - 3x^2 + \frac{3}{2}x + \frac{k}{2} = 0$$

Given that $\alpha = 2(\beta + \gamma) = \frac{\alpha}{2} = \beta + \gamma$

$$\alpha + \beta + \gamma = -(-3)$$

$$\frac{3\alpha}{2} = 3$$

$$\alpha = 2 \quad \Rightarrow \beta + \gamma = 1$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{3}{2}$$

$$2\beta + \beta\gamma + 2\gamma = \frac{3}{2} \quad \Rightarrow 2(\beta + \gamma) + \beta\gamma = \frac{3}{2}$$

$$2(1) + \beta\gamma = \frac{3}{2} \quad \Rightarrow \beta\gamma = \frac{3}{2} - 2 = \frac{-1}{2}$$

$$\beta\gamma = \frac{-1}{2}$$

$$\alpha\beta\gamma = -\frac{k}{2}$$

$$2\left(\frac{-1}{2}\right) = -\frac{k}{2} \qquad \Rightarrow k = +2$$

(1) $\Rightarrow \beta = 1 - \gamma$ substitute in (2)

$$\gamma(1-\gamma) = \frac{-1}{2} \qquad \Rightarrow \gamma - \gamma^2 = \frac{-1}{2}$$

$$2\gamma - 2\gamma^2 = -1 \qquad \Rightarrow 2\gamma^2 - 2\gamma - 1 = 0$$

$$\gamma = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4 - 4(2)(-1)}}{2(2)} = \frac{2 \pm \sqrt{4+8}}{4}$$

$$\gamma = \frac{2 \pm 2\sqrt{3}}{4} = \frac{1 \pm \sqrt{3}}{2}$$

The roots are 2, $\frac{1 \pm \sqrt{3}}{2}$ and $k = 2$

Question 5.

Find all zeros of the polynomial x6 – 3x5 – 5x4 + 22x3 – 39x2 – 39x + 135, if it is known that 1 + 2i and √3 are two of its zeros.

Solution:

The given equation is $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135 = 0$

The given roots are $1 + 2i$, $\sqrt{3}$

The other roots are $1 - 2i$, $-\sqrt{3}$

The factors are

$= \{x^2 - x(2) + (1 + 4)\}\{(x + \sqrt{3})(x - \sqrt{3})\}$

$= (x^2 - 2x + 5)(x^2 - 3)$

$= x^4 - 3x^2 - 2x^3 + 6x + 5x^2 - 15$

$= x^4 - 2x^3 + 2x^2 + 6x - 15$

To find this roots,

$$\begin{array}{r|l} & x^2 - x - 9 \\ \hline x^4 - 2x^3 + 2x^2 + 6x - 15 & x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135 \\ & x^6 - 2x^5 + 2x^4 + 6x^3 - 15x^2 \\ & \quad (+) \quad (-) \quad (-) \quad (+) \\ \hline & -x^5 - 7x^4 + 16x^3 - 24x^2 - 39x \\ & -x^5 + 2x^4 - 2x^3 - 6x^2 + 15x \end{array}$$

$$\begin{array}{r} (+) \quad (-) \quad (+) \quad (+) \quad (-) \\ \hline - \; 9x^4 + 18x^3 - 18x^2 - 54x + 135 \\ - \; 9x^4 + 18x^3 - 18x^2 - 54x + 135 \\ \hline 0 \\ \hline \end{array}$$

The other factor is $x^2 - x - 9 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 4(1)(-9)}}{2} = \frac{1 \pm \sqrt{37}}{2}$$

$\therefore$ The roots are $1 \pm 2i, \pm \sqrt{3}, \frac{1 \pm \sqrt{37}}{2}$

Question 6.

Solve the cubic equations:

(i) $2x^3 - 9x^2 + 10x = 3$

(ii) $8x^3 - 2x2 - 7x + 3 = 0$

Solution:

(i) Given equation is $2x^3 - 9x^2 + 10x = 3$

Sum of the co-efficients = 0

(x – 1) is a factor.

The other factor is $2x^2 - 7x + 3$

$2x^2 - 7x + 3 = 0$

$(x - 3)(2x - 1) = 0$

The roots are 1, 3, 12

$$\begin{array}{c|cccc} 1 & 2 & -9 & 10 & -3 \\ & 0 & 2 & -7 & 3 \\ \hline & 2 & -7 & 3 & \multicolumn{1}{|c}{0} \end{array}$$

(ii) Given equation is $8x^3 - 2x^2 - 7x + 3 = 0$

Sum of odd co-efficients = Sum of even co-efficients

(x + 1) is a factor.

The other factor is $8x^2 - 10x + 3$

$8x^2 - 10x + 3 = 0$

$(4x - 3)(2x - 1) = 0$

The roots are $\frac{3}{4}, \frac{1}{2}, -1$

-1	8	-2	-7	3
	0	-8	10	-3
	8	-10	3	0

Question 7.
Solve the equation: x4 – 14x2 + 45 = 0.

Solution:
The given equation is $x^4 - 14x^2 + 45 = 0$
Let $x^2 = y$
$y^2 - 14y + 45 = 0$
$(y - 9)(y - 5) = 0$
$y = 9, 5$
$x^2 = 9, x^2 = 5$
$x = \pm 3, x = \pm \sqrt{5}$
The roots are $\pm 3, \pm \sqrt{5}$

Ex 3.4

Question 1.

Solve:

(i) $(x - 5)(x - 7)(x + 6)(x + 4) = 504$

(ii) $(x - 4)(x - 7)(x - 2)(x + 1) = 16$

Solution:

(i) $(x - 5)(x + 4)(x - 7)(x + 6) - 504 = 0$

$(x^2 - x - 20)(x^2 - x - 42) - 504 = 0$

Put $y = x^2 - x$

$(y - 20)(y - 42) - 504 = 0$

$y^2 - 42y - 20y + 840 - 504 = 0$

$y^2 - 62y + 336 = 0$

$(y - 56)(y - 6) = 0$

$y = 56$ or $y = 6$

$x^2 - x = 56$ or $x^2 - x = 6$

$x^2 - x - 56 = 0$ or $x^2 - x - 6 = 0$

$(x - 8)(x + 7) = 0$ or $(x + 2)(x - 3) = 0$

$x = 8, -7$ or $x = 3, x = -2$

The roots are 8, -7, 3, -2

(ii) $(x - 4)(x - 2)(x- 7)(x + 1) - 16 = 0$

$(x^2 - 6x + 8)(x^2 - 6x - 7) - 16 = 0$

Put $y = x^2 - 6x$

$(y + 8)(y + 7) - 16 = 0$

$y^2 - 7y + 8y - 56 - 16 = 0$

$y^2 - y - 72 = 0$

$(y + 9)(y - 8) = 0$

$y = -9$ or $y = 8$

$y = -9 \Rightarrow x^2 - 6x = -9$

$x^2 - 6x + 9 = 0$

$(x - 3)^2 = 0$

$x = 3, 3$

$y = 8 \Rightarrow x^2 - 6x = 8$

$x^2 - 6x - 8 = 0$

$$x = \frac{6 \pm \sqrt{36 - 4(1)(-8)}}{2}$$

$$x = \frac{6 \pm \sqrt{36 + 32}}{2}$$

$$x = \frac{6 \pm \sqrt{68}}{2} = \frac{6 \pm 2\sqrt{17}}{2}$$

$$x = \frac{2\left(3 \pm \sqrt{17}\right)}{2} = 3 \pm \sqrt{17}$$

The roots are 3, 3, 3 $\pm\sqrt{17}$

Question 2.
Solve : (2x – 1) (x + 3) (x – 2) (2x + 3) + 20 = 0.
Solution:
(2x – 1) (2x + 3) (x + 3) (x – 2) + 20 = 0
⇒ ($4x^2$ + 6x – 2x – 3) (x^2 – 2x + 3x – 6) + 20 = 0
⇒ ($4x^2$ + 4x – 3) (x^2 + x – 6) + 20 = 0
⇒ [4(x^2 + x) – 3] [x^2 + x – 6] + 20 = 0
Let y = x^2 + x
⇒ (4y – 3) (y – 6) + 20 = 0
⇒ $4y^2$ – 24y – 3y + 18 + 20 = 0
⇒ $4y^2$ – 27y + 38 = 0
⇒ (4y – 19) (y – 2) = 0
(4y – 19) = 0
4(x^2 + x) – 19 = 0
$4x^2$ + 4x – 19 = 0

$$x = \frac{-4 \pm \sqrt{16 + 4(4)19}}{8}$$

$$x = \frac{-4 \pm \sqrt{16 + 304}}{8}$$

$$x = \frac{-4 \pm \sqrt{320}}{8} = \frac{-4 \pm 8\sqrt{5}}{8}$$

$$x = \frac{-1 \pm 2\sqrt{5}}{2}$$

or

$(y - 2) = 0$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2, +1$

The roots are -2, 1, $\frac{-1 \pm 2\sqrt{5}}{2}$

Ex 3.5

Question 1.
i) $\sin^2 x - 5\sin x + 4 = 0$
(ii) $12x^3 + 8x = 29x^2 - 4$

Solution:
(i) $\sin^2 x - 5 \sin x + 4 = 0$
Put $\sin x = t$
$t^2 - 5t + 4 = 0$
$(t - 1)(t - 4) = 0$
$t = 1$ or $t = 4$
$\sin x = 4$ or $\sin x = 1$
(is not possible) $\sin x = \sin \frac{\pi}{2}$
$x = n\pi + (-1)^n \frac{\pi}{2}\ \forall\ n \in z.$

(ii) $12x^3 + 8x = 29x^2 - 4$
$12x^3 - 29x^2 + 8x + 4 = 0$ ..... (1)
By Trail and error method, $(x - 2)$ is a factor of (1)
The other factor is $12x^2 - 5x - 2$
The roots is $12x^2 - 5x - 2 = 0$
$(3x - 2)(4x + 1) = 0$
$x = \frac{2}{3}, x = -\frac{1}{4}$
The roots are $2, \frac{2}{3}, -\frac{1}{4}$

2	12	–29	8	4
	0	24	–10	–4
	12	–5	–2	0

[Here $a_n = 12$, $a_0 = 4$; Let $\frac{p}{q}$ be the root of the equation (1)
The factors of a_0 : ±1, ±2, ±4 (P must divisible by 4)
The factor of a_n : ±1, ±2, ±3, ±4, ±6, ±12
q must divide as (12)
Using these p and q we can form $\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{2}{3}, \pm \frac{1}{6}, \pm \frac{1}{4}, \pm \frac{3}{4}, \pm 3$ are the possible roots of equation. (1)]

Question 2.
Examine for the rational roots of:
(i) $2x^3 - x^2 - 1 = 0$
(ii) $x^8 - 3x + 1 = 0$

Solution:
(i) $2x^3 - x^2 - 1 = 0$
Sum of the co-efficients = 0
∴ $(x - 1)$ is a factor
The other factor is $2x^2 + x + 1$.
The root is $(2x^2 + x + 1) = 0$
Here $\Delta = b^2 - 4ac = (1)^2 - 4(2)(1) = 1 - 8 = -7 < 0$
The remaining roots are imaginary.
The only rational root is $x = 1$

$$\begin{array}{c|cccc} 1 & 2 & -1 & 0 & -1 \\ & 0 & 2 & 1 & 1 \\ \hline & 2 & 1 & 1 & \boxed{0} \end{array}$$

(ii) $x^8 - 3x + 1 = 0$ .... (1)
Here $a_n = 1$, $a_0 = 1$
If $\frac{P}{Q}$ is a rational root of (1)
Then q is a factor a_n, p is a factor of a0
The possible values of p and q are ± 1.
Among the possible values 1, -1, [(p, q) = 1]
None of them satisfies the equation (1)
The above equation has no rational roots.

Question 3.
Solve: $8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$

Solution:
(i)
$$8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$$
Let $y = x^{\frac{3}{2n}}$
$$8y - \frac{8}{y} = 63 \quad \Rightarrow \frac{8y^2 - 8}{y} = 63$$
$$8y^2 - 63y - 8 = 0$$

$$(8y + 1)(y - 8) = 0$$

$$(8y + 1) = 0 \qquad\qquad (y - 8) = 0$$

$$y = \frac{-1}{8} \qquad\qquad y = 8$$

$$x^{\frac{3}{2n}} = \frac{-1}{8} \qquad\qquad x^{\frac{3}{2n}} = 8$$

Squaring on both sides

$$x = \left(\frac{1}{64}\right)^{\frac{n}{3}} \qquad\qquad x = (64)^{\frac{n}{3}}$$

$$x = \left[\left(\frac{1}{4}\right)^3\right]^{\frac{n}{3}} \qquad\qquad x = (4^3)^{\frac{n}{3}}$$

$$x = \frac{1}{4^n} \text{ (not possible)} \qquad\qquad x = 4^n$$

only possible solution is $x = 4^n$

Question 4.

Solve: $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$

Solution:

$$\text{Let } y = \sqrt{\frac{x}{a}} \qquad \Rightarrow 2y + \frac{3}{y} = \frac{b^2 + 6a^2}{ab}$$

$$\frac{2y^2 + 3}{y} = \frac{6a^2 + b^2}{ab}$$

$$(2y^2 + 3)ab = y(6a^2 + b^2)$$

$$2aby^2 - y(6a^2 + b^2) + 3ab = 0$$

$$2aby^2 - 6a^2y - b^2y + 3ab = 0$$

$$2ay(by - 3a) - b(by - 3a) = 0$$

$$(2ay - b)(by - 3a) = 0$$

$$(2ay - b) = 0 \qquad (by - 3a) = 0$$

$$2ay = b \qquad by = 3a$$

$$y = \frac{b}{2a} \qquad y = \frac{3a}{b}$$

$$\sqrt{\frac{x}{a}} = \frac{b}{2a} \qquad \sqrt{\frac{x}{a}} = \frac{3a}{b}$$

Squaring on both sides

$$\frac{x}{a} = \frac{b^2}{4a^2} \qquad \frac{x}{a} = \frac{9a^2}{b^2}$$

$$x = \frac{b^2}{4a} \qquad x = \frac{9a^3}{b^2}$$

Question 5.
Solve the equations:
(i) $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$
(ii) $x^4 + 3x^3 - 3x - 1 = 0$

Solution:
(i) $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$..... (1)

It is a even degree reciprocal equation as p(x) = $x^n p\left(\frac{1}{x}\right)$

Dividing equation (1) by x^2,

$$6x^2 - 35x + 62 - \frac{35}{x} + \frac{6}{x^2} = 0 \qquad \Rightarrow 6\left(x^2 + \frac{1}{x^2}\right) - 35\left(x + \frac{1}{x}\right) + 62 = 0$$

$$\text{Let } y = x + \frac{1}{x} \qquad \Rightarrow x^2 + \frac{1}{x^2} = (y^2 - 2)$$

$$6(y^2 - 2) - 35y + 62 = 0 \qquad \Rightarrow 6y^2 - 12 - 35y + 62 = 0$$

$$6y^2 - 35y + 50 = 0 \qquad \Rightarrow (3y - 10)(2y - 5) = 0$$

$$3y - 10 = 0,\ 2y - 5 = 0$$

Case (*i*): $3\left(x+\frac{1}{x}\right)=10 \qquad \Rightarrow 3\left(x^2+\frac{1}{x}\right)=10$

$$3x^2+3=10x \qquad \Rightarrow 3x^2-10x+3=0$$

$$(3x-1)\,(x-3)=0 \qquad \Rightarrow x=\frac{1}{3},\, x=3$$

Case (*ii*): $2y-5=0 \qquad \Rightarrow 2y=5$

$$2\left(x+\frac{1}{x}\right)=5 \qquad \Rightarrow 2(x^2+1)=5x$$

$$2x^2-5x+2=0 \qquad \Rightarrow (2x-1)(x-2)=0$$

$$x=\frac{1}{2},\, x=2$$

The roots are, 3, $\frac{1}{3}$, 2, $\frac{1}{2}$

(ii) $x^4 + 3x^3 - 3x - 1 = 0$ …… (1)
It is an even degree reciprocal function of type II.
1, -1 are the solution of equation (1)
$(x - 1)$, $(x + 1)$ are the factor of (1)
$(x^2 - 1)$ is a factor of (1)
Dividing (1) by $(x^2 - 1)$
we get, $x^2 + 3x + 1 = 0$ is the other factor.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-3\pm\sqrt{9-4}}{2}$$

$$=\frac{-3\pm\sqrt{5}}{2}$$

The roots are 1, –1, $\frac{-3\pm\sqrt{5}}{2}$

$$\begin{array}{r|l} & x^2+3x+1 \\ \hline x^2+0x-1 & x^4+3x^3+0x^2-3x-1 \\ & x^4+0-x^2 \\ & (-)\;(-)(+) \\ \hline \end{array}$$

$$
\begin{array}{r}
+\ 3x^3 + \ x^2 - 3x \\
+\ 3x^3 + 0x^2 - 3x \\
\underline{(-) \quad (-)(+) \qquad} \\
x^2 - 1 \\
\underline{x^2 - 1} \\
\underline{0}
\end{array}
$$

Question 6.
Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$

Solution:

$4^x - 3(2^{x+2}) + 2^5 = 0.$

$(2^x)^2 - 3.(2^x 2^2) + 2^5 = 0$

$(2^x)^2 - 12(2^x) + 32 = 0$

Put $2^x = t$

$(t^2 - 12t + 32 = 0)$

$(y - 4)(y - 8) = 0$

$y = 4$ or $y = 8$

$t = 8$ (or) $t = 4$

$2^x = 8 = 2^3$ (or) $2^x = 4 = (2)^2$

$x = 3$ (or) $x = 2$

Roots are 3, 2

Question 7.
Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

Solution:

$6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ ...... (1)

$x = \frac{1}{3}$ is a Solution

$\therefore$ $(3x - 1)$ is a factor of (1)

(1) is a Reciprocal equation even degree divide (1) by x^2.

$$6x^2 - 5x - 38 - \frac{5}{x} + \frac{6}{x^2} = 0$$

$$6\left(x^2 + \frac{1}{x}\right) - 5\left(x + \frac{1}{x}\right) - 38 = 0$$

$$\text{Let } y = x + \frac{1}{x},\quad x^2 + \frac{1}{x^2} = y^2 - 2$$

$$6(y^2 - 2) - 5y - 38 = 0 \qquad \Rightarrow 6y^2 - 12 - 5y - 38 = 0$$

$$6y^2 - 5y - 50 = 0 \qquad \Rightarrow (2y + 5)\,(3y + 10) = 0$$

$$2y + 5 = 0\ ;\ 3y - 10 = 0$$

Case (*i*) $\quad 2y = -5$

$$2\left(x + \frac{1}{x}\right) = -5 \qquad \Rightarrow 2(x^2 + 1) = -5x$$

$$2x^2 + 5x + 2 = 0 \qquad \Rightarrow (2x + 1)\,(x + 2) = 0$$

$$x = \frac{-1}{2},\ x = -2$$

Case (*ii*) $\quad 3y - 10 = 0$

$$3\left(x + \frac{1}{x}\right) = 10 \qquad \Rightarrow 3x^2 + 3 = 10x$$

$$3x^2 - 10x + 3 = 0 \qquad \Rightarrow (3x - 1)\,(x - 3) = 0$$

$$x = \frac{1}{3},\ x = 3$$

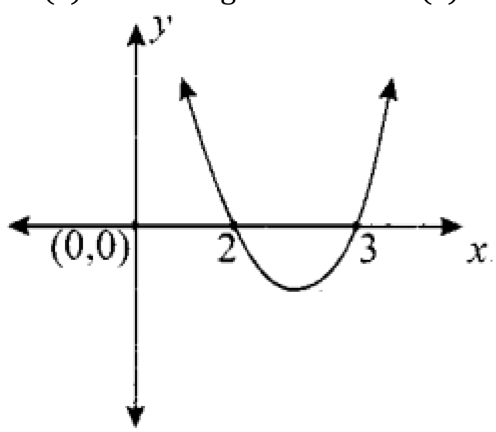
$\therefore$ The roots are $3,\ \frac{1}{3},\ 2,\ \frac{1}{2}$

Ex 3.6

Question 1.
Discuss the maximum possible number of positive and negative roots of the polynomial $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$.
Solution:
Clearly, there are 4 sign changes for the given polynomial P(x) and hence a number of positive roots of P(x) can't be more than four.
$P(x) = 9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$.There are two sign changes.
Hence the number of negative roots can't be more than two.
∴ It has atmost 4 positive roots and atmost two negative roots.

Question 2.
Discuss the maximum possible number of positive and negative zeros of the polynomials $x^2 - 5x + 6$ and $x^2 - 5x + 16$. Also, draw a rough sketch of the graphs.

Solution:
$P(x) = x^2 - 5x + 6$
The number of sign changes in P(x) is 2.
P(x) has atmost 2 positive roots. $P(-x) = x^2 + 5x + 6$.
The number of sign changes in P(-x) is 0.
∴ P (x) has no negative roots. $P(x) = x^2 - 5x + 16$



Question 3.
Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.

Solution:
P(x) has only one sign change. It has atmost one positive roots.
$P(-x) = -x^9 + 5x^5 + 4x^4 + 2x^2 + 1 = 0$
It has only one sign change.
It has atmost one negative roots.

Clearly 0 is not a root.
So maximum number of real roots is 3 and hence there are atleast six imaginary solutions.

Question 4.
Determine the number of positive and negative roots of the equation $x^9 – 5x^8 – 14x^7 = 0$.

Solution:
$x^9 – 5x^8 – 14x^7 = 0$
$P(x) = x^9 – 5x^8 – 14x^7$. The number of sign changes is P(x) is 1.
The number of positive roots is 1. $P(-x) = -x^9 – 5x^8 + 14x^7$
The number of sign changes is P(-x) is one. The number of negative zero of P(-x) is 1. It is clear that 0 is a root of the equation.
∴ The number of the imaginary roots is at least 6.

Question 5.
Find the exact number of real zeros and imaginary of the polynomial $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$.
Solution:$P(x) = x^9 + 9x^7 + 7x^5 + 5x^3 + 3x = 0$
$P(-x) = -x^9 – 9x^7 – 7x^5 – 5x^3 – 3x$
It has no sign changes.
So it has no negative real roots.
It has no positive real roots and no negative real roots.

Ex 3.7

Question 1.
A zero of $x^3 + 64$ is ______
(a) 0
(b) 4
(c) 4i
(d) -4

Answer:
(d) -4
Hint: $x^3 + 64 = 0$
$\Rightarrow x^3 = -64$
$\Rightarrow x^3 = (-4)^3$
$\Rightarrow x = -4$

Question 2.
If f and g are polynomials of degrees m and n respectively, and if h(x) = (f 0 g) (x), then the degree of h is ______
(a) mn
(b) m + n
(c) m^n
(d) n^m

Answer:
(a) mn

Question 3.
A polynomial equation in x of degree n always has ______
(a) n distinct roots
(b) n real roots
(c) n imaginary roots
(d) at most one root.

Answer:
(c) n imaginary roots (Every real number is also imaginary)

Question 4.
If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\sum\frac{1}{\alpha}$ is ______

(a) $-\frac{q}{r}$
(b) $\frac{q}{p}$
(c) $\frac{q}{r}$
(d) $-\frac{q}{p}$

Answer:

(a) $-\frac{q}{r}$

Hint:

$$\sum\frac{1}{\alpha}=\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}=-\frac{q}{r}$$

Question 5.
According to the rational root theorem, which number is not possible rational zero of $4x^7 + 2x^4 - 10x^3 - 5$?
(a) -1
(b) $\frac{5}{4}$
(c) $\frac{4}{5}$
(d) 5

Answer:
(c) $\frac{4}{5}$
Hint:

$a_n = 4; a_0 = 5$

Let $\frac{p}{q}$ be the root of P (x). P must divide 5, possible values of P are ±1, ±5

q must divide 4, possible values of q are ±1, ±2, ±4

Possible roots are $\pm1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm5, \pm\frac{5}{2}, \pm\frac{5}{4}$

Question 6.
The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies.
(a) $|k| \leq 6$
(b) $k = 0$
(c) $|k| > 6$
(d) $|k| \geq 6$

Answer:

(d) $|k| \geq 6$
Hint:
$x^3 - kx^2 + 9x = 0$
$\Rightarrow x(x^2 - kx + 9) = 0$
x = 0 is one real root. If the remaining roots to be real if the
$b^2 - 4ac \geq 0$
$\Rightarrow k^2 - 36 \geq 0$
$\Rightarrow k^2 \geq 36$
$\Rightarrow |k| \geq 6$

Question 7.
The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is ______
(a) 2
(b) 4
(c) 1
(d) ∞
Answer:
(c) 1
Hint:
$\sin^4 x - 2\sin^2 x + 1 = 0$
$\Rightarrow t^2 - 2t + 1 = 0$
$\Rightarrow (t - 1)^2 = 0$
$\Rightarrow t - 1 = 0$
$\Rightarrow t = 1$
$\Rightarrow \sin^2 x = 1$
$\Rightarrow 1-\cos2x2=1$
$\Rightarrow 1 - \cos 2x = 2$
$\Rightarrow \cos 2x = \cos 0$
$\Rightarrow 2x = 2n\pi$
$\Rightarrow x = n\pi$
$n = 0, x = 0$
$n = 1, x = \pi$
$n = 2, x = 2\pi$

Question 8.
If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if _______
(a) $a \geq 0$
(b) $a > 0$
(c) $a < 0$
(d) $a \leq 0$
Answer:
(c) $a < 0$
Hint:

If $a < 0$, then $P(x) = x^3 + 12x^2 + 10ax + 1999$ has 2 changes of sign.
∴ P (x) has atmost two positive roots. So $a < 0$

Question 9.
The polynomial $x^3 + 2x + 3$ has ______
(a) one negative and two imaginary zeros
(b) one positive and two imaginary zeros
(c) three real zeros
(d) no zeros
Answer:
(a) one negative and two imaginary zeros
Hint:
$P(x) = x^3 + 2x + 3$; No positive root.
$P(-x) = -x^3 - 2x + 3$; Only one change in the sign.
∴ One negative root.

Question 10.

The number of positive zeros of the polynomial $\sum_{j=0}^{n} {}^nC_r (-1)^r x^r$ is _____
(a) 0
(b) n
(c) <n
(d) r
Answer:
(b) n