Chapter 22 Figure Partition and Dot Situation

FIGURE PARTITION :

The problems on figure partition are based on counting the number of figures generated due to partition lines.

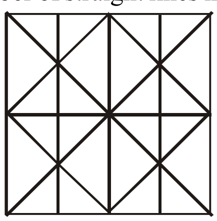
1. If a square is subdivided into n parts on each side, then the total number of squares formed is given by

$$\frac{n(n+1)(2n+1)}{6}$$

2. Total no. of rectangles (including squares) in a rectangular figure of size $n \times m = \frac{n(n+1)}{6}\ \frac{m(m+1)}{2}$

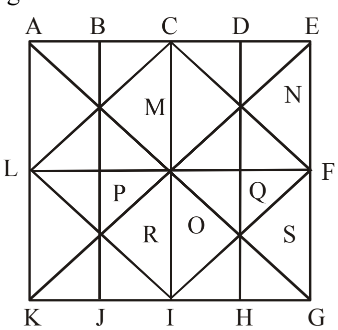
ILLUSTRATION 1 :

What is the number of straight lines in the following figure?



(1) 11 (2) 14 (3) 16 (4) 17

***Sol.* (2)** The figure is labelled as shown.



Clearly, there are 3 horizontal lines namely AE, LF and KG

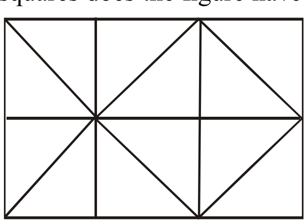
There are 5 vertical lines : AK, BJ, CI, DH and EG.

There are 6 slanting lines : LC, KE, IF, LI, AG and CF.

Thus, there are 3 + 5 + 6 = 14 straight lines in the figure.

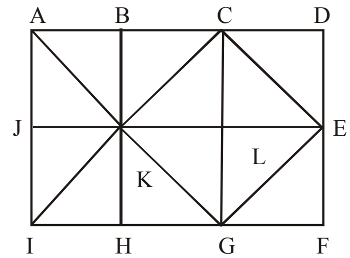
ILLUSTRATION 2 :

How many squares does the figure have ?



(1) 6 (2) 7 (3) 9 (4) 10

***Sol.* (3)** The figure may be labelled as shown :

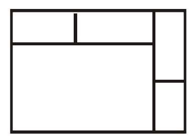


The squares composed to two components each, are ABKJ, BCLK, CDEL, LEFG, KLGH, JKHI. Thus, there are 6 such squares. Only one square, KCEG is composed of four components. Two squares namely, ACGI and BDFH are composed of eight components each. Thus, there are 2 such squares.

∴ There are 6 + 1 + 2 = 9 squares in the figure

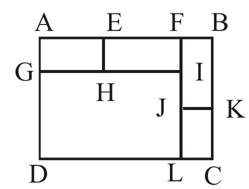
ILLUSTRATION 3 :

What is the number of rectangles in the following figure ?



(1) 5 (2) 7 (3) 8 4) 9

Sol. (4) The figure is labelled as shown :



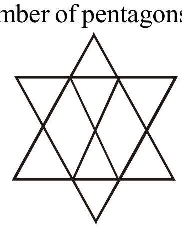
Simplest rectangles are AEHG, EFIH, FBKJ, JKCL and GILD. i.e. there are 5 such rectangles.

The rectangles composed of two components each are AFIG and FBCL. Thus, there are 2 such rectangles. Only one rectangle, namely AFLD is composed of 3 components and only one rectangle, namely ABCD is composed of 5 components.

Thus, there are 5 + 2 + 1 + 1 = 9 rectangles in the figure.

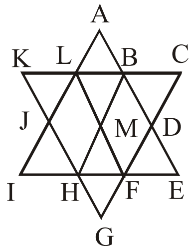
ILLUSTRATION 4 :

Determine the number of pentagons in the following figure:



(1) 5 (2) 6 (3) 8 (4) 10

Sol. **(4)** The figure is labelled as shown. In this case, six pentagons have been formed by the combination of three triangles and two rhombuses-
ADEFHJ, CFHJL, EHJLB, GJLBD, ILBDF and KBDFH.



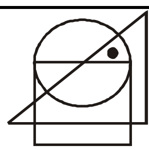
Four other pentagons are formed by the combination of three triangles and one rhombus -LCFHM, LBEHM, BKFHM and BLIFM. Thus, there are 10 pentagons in the figure.

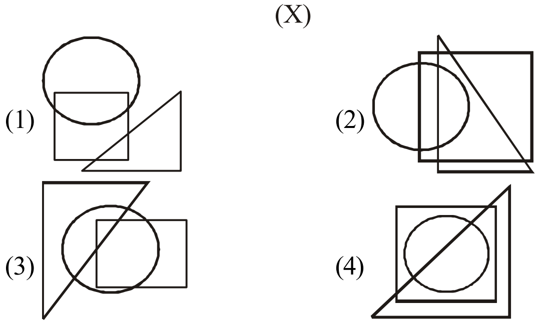
DOT SITUATION :

The problems on dot situation involve the search of similar conditions in the alternative figures as indicated in the problem figure. The problem figure contains dots placed in the spaces enclosed between the combinations of square, triangle, rectangle and circle. Selecting one of these dots we observe the region in which this dot is enclosed i.e. to which of the four figures (circle, square, rectangle and triangle) is this region common. Then we look for such a region in the four alternatives. Once we have found it we repeat the procedure for other dots, if any. The alternative figure which contains all such regions is the answer.

DIRECTIONS (ILLUSTRATION 5-9) : In each of the following examples, there is a diagram marked (X), with one or more dots placed in it. The diagram is followed by four other figures, marked (1), (2), (3) and (4) only one of which is such as to make possible the placement of the dot. Select this alternative as the answer.

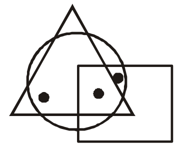
ILLUSTRATION 5 :

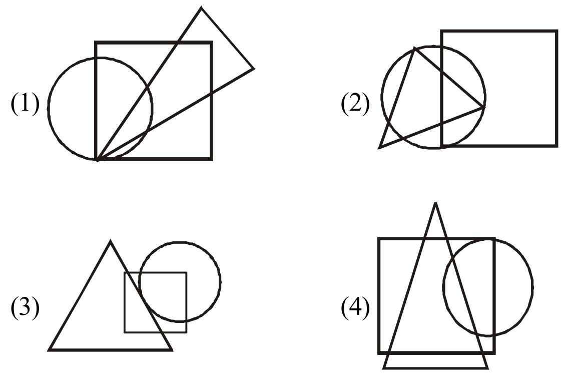

(X)



Sol. **(3)** In figure (X), the dot lies in the region common to the circle and the triangle only. Such a region is present in figure (3) only.

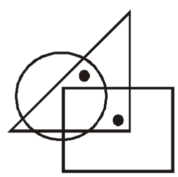
ILLUSTRATION 6 :

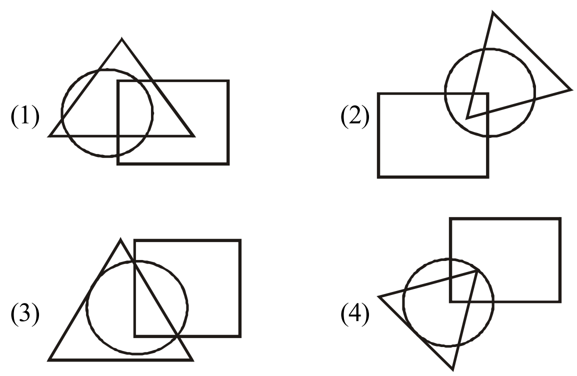

(X)



Sol. **(2)** Figure (X) contains one dot in the region common to the circle and the triangle, another dot in the region common to all the three figures and the third dot in the region common to the square and the circle only. In figures (1) and (4), the region common to the circle and the triangle lies within the square. In figure (3), there is no region common to the circle and the triangle. Only figure (2) contains all the three types of regions.

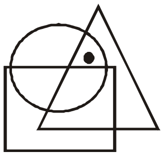
ILLUSTRATION 7 :


(X)

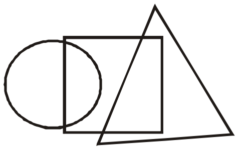
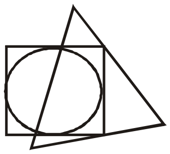


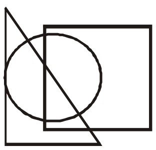
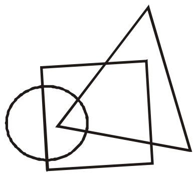
Sol. **(1)** In figure (X), one of the dots is placed in the region common to the circle and the triangle and the other dot is placed in the region common to the triangle and the square. From amongst the figures (1), (2), (3) and (4), only figure (1) has both the regions, one common to circle and triangle and the other common to triangle and square.

ILLUSTRATION 8 :



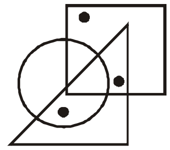
(X)

(1)  (2) 

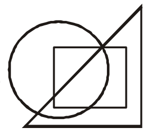
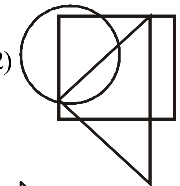
(3)  (4) 

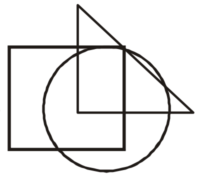
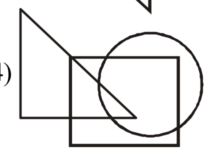
Sol. **(3)** In figure (X), the dot lies in the region common to the circle and the triangle only. Such a region is present in figure (3) only.

ILLUSTRATION 9 :



(X)

(1)  (2) 

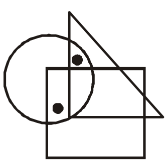
(3)  (4) 

Sol. **(3)** Figure (X), contains one dot in the square only, another dot in the region common to the square and the triangle only and the third dot in the region common to the circle and the triangle. Figure (1) does contain a region which lies in the square alone. Figures (2) and (4) do not contain any region common to the circle and the triangle. Only figure (3) contains all the three types of regions.

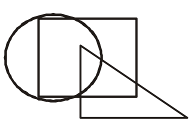
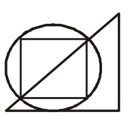
Exercise 1

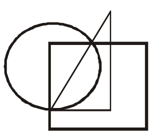
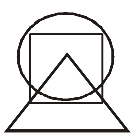
DIRECTIONS (Qs.1-8) : In each of the following examples, there is a diagram marked (X), with one or more dots placed in it. The diagram is followed by four other figures, marked (1), (2), (3) and (4) only one of which is such as to make possible the placement of the alternative in each these.

1.

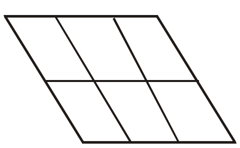


(X)

(1)  (2) 

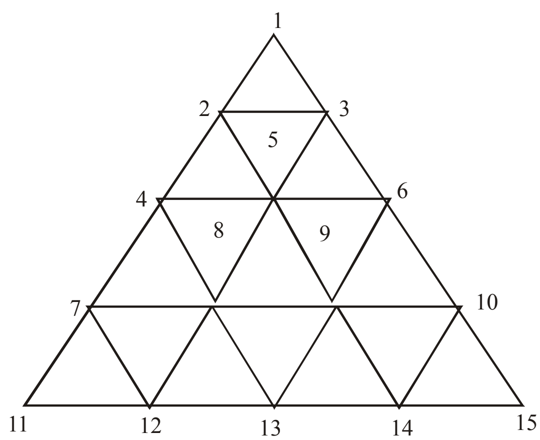
(3)  (4) 

2. How many parallelograms are there in the figure ?



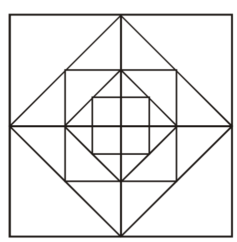
(1) 14 (2) 15 (3) 16 (4) 18

3. Count the number of triangles in the figure.



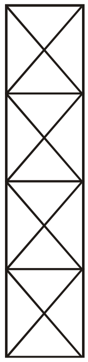
(1) 40 (2) 48 (3) 30 (4) 50

4. What is the number of squares in fig ?



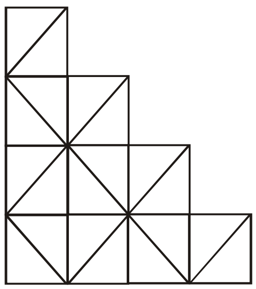
(1) 12 (2) 13 (3) 15 (4) 17

5. What is the number of triangles in figure.



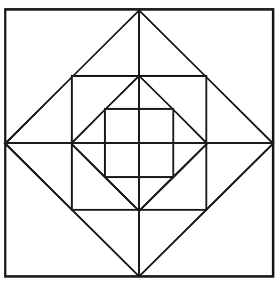
(1) 16 (2) 28 (3) 32 (4) 38

6. What is the number of rectangles (excluding squares) in figure.



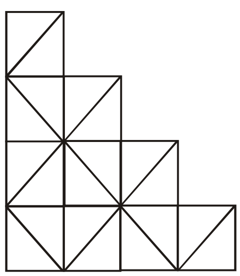
(1) 10 (2) 12 (3) (4) 22

7. What is the number of triangles in figure ?



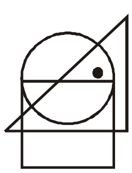
(1) 24 (2) 32 (3) 48 (4) 52

8. What is the number of triangles in figure ?

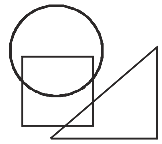


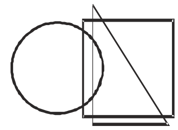
(1) 26 (2) 41 (3) 36 (4) 40

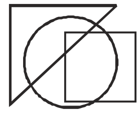
9. From amongst the figures marked (1), (2), (3) and (4), select the figure which satisfies the same conditions of placement of dots as in figure (X).

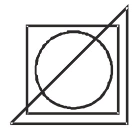


(X)

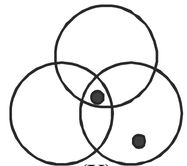
(1) 

(2) 

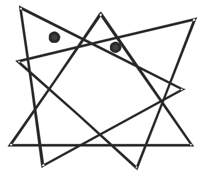
(3) 

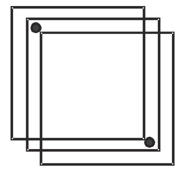
(4) 

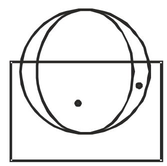
10. In each of the following questions there is a diagram marked in which one or more dots have been placed in certain positions. Examine the placement of these dots carefully. From the four choices given below dots, select the one in which the placement of dots is similar to that in the diagram marked (X).

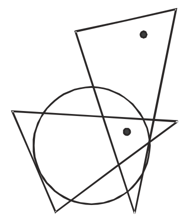


(X)

(1) 

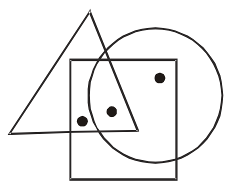
(2) 

(3) 

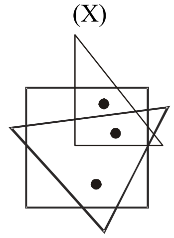
(4) 

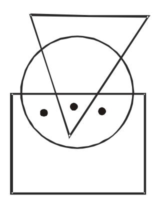
DIRECTIONS (Qs.11-12) : In each of the following questions there is a diagram (X) in which one or more dots have been placed in certain positions. Examine the placement of these dots carefully. From the four choices, select the one in which the placement of dots is similar to that in the diagram.

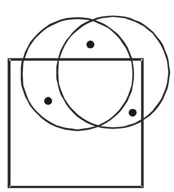
11.

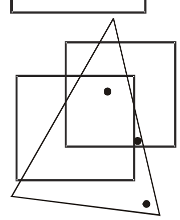


(X)

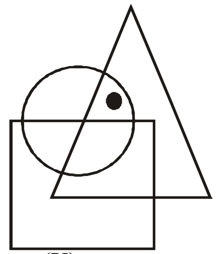
(1) 

(2) 

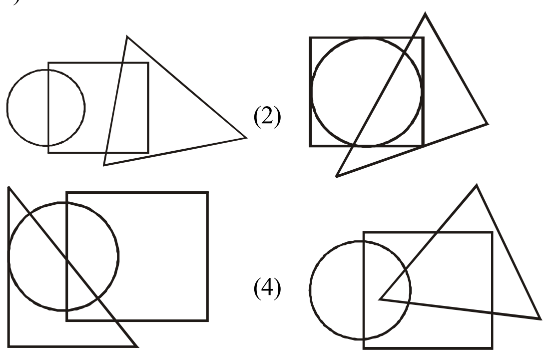
(3) 

(4) 

12.



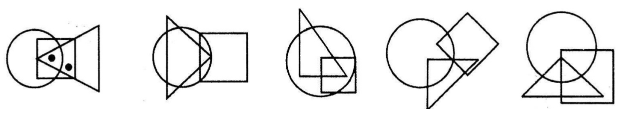
(X)



(1) (2) (3) (4)

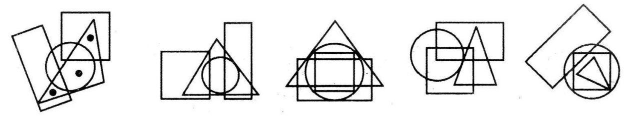
DIRECTIONS (Qs.13-18) : In each of the following questions, from amongst the figures marked (1), (2), (3) and (4), select the one which satisfies the same conditions of placement of the dot as in fig. (X).

13.



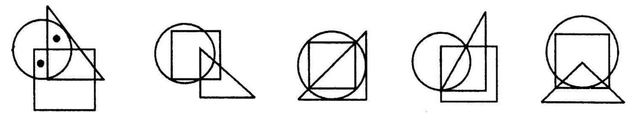
(X) (1) (2) (3) (4)

14.



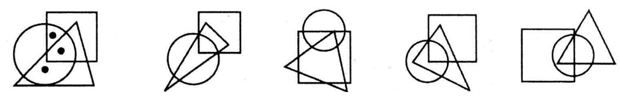
(X) (1) (2) (3) (4)

15.



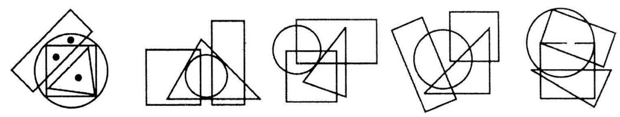
(X) (1) (2) (3) (4)

16.



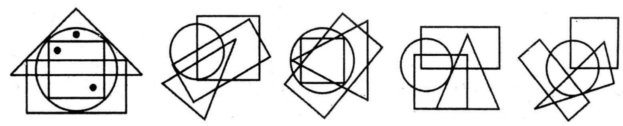
(X) (1) (2) (3) (4)

17.



(X) (1) (2) (3) (4)

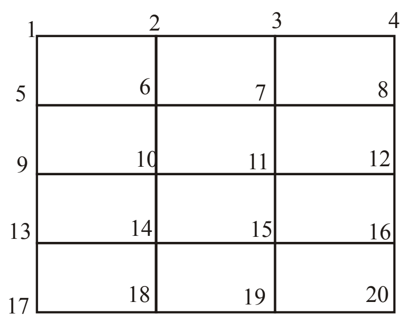
18.



(X) (1) (2) (3) (4)

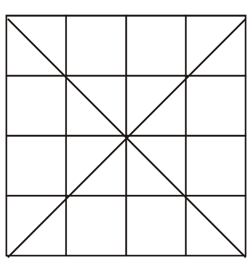
Exercise 2

1. Find the total number of squares formed in figure ?



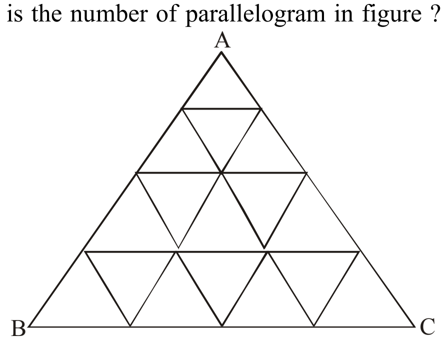
(1) 22 (2) 18 (3) 20 (4) 30

2. What is the number of triangles in figure ?



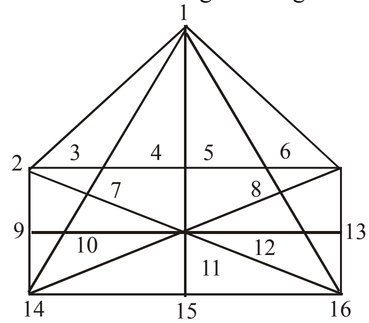
(1) 32 (2) 36 (3) 40 (4) 56

3. What is the number of parallelogram in figure ?



(1) 27 (2) 33 (3) 34 (4) 42

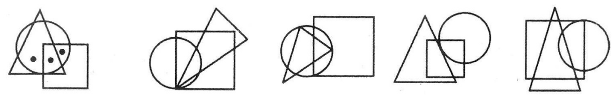
4. What is the number of triangles in figure ?



(1) 32 (2) 39 (3) 46 (4) 60

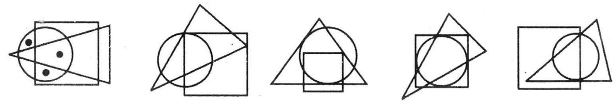
DIRECTIONS (Qs.5-10) : In each of the following questions, from amongst the figures marked (1), (2), (3) and (4), select the one which satisfies the same conditions of placement of the dot as in fig. (X).

5.



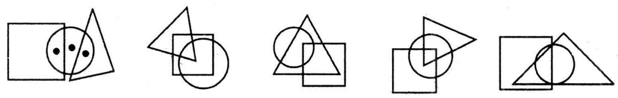
(X) (1) (2) (3) (4)

6.



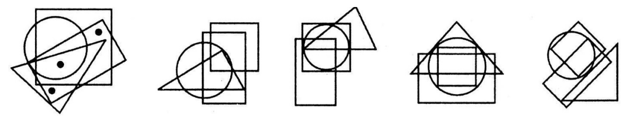
(X) (1) (2) (3) (4)

7.



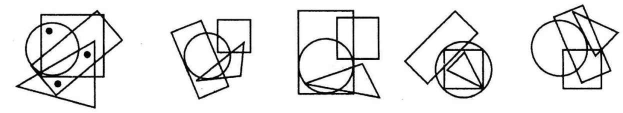
(X) (1) (2) (3) (4)

8.



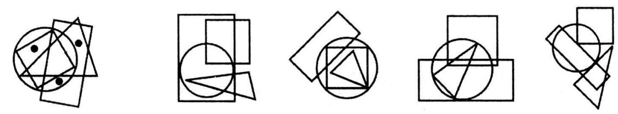
(X) (1) (2) (3) (4)

9.



(X) (1) (2) (3) (4)

10.



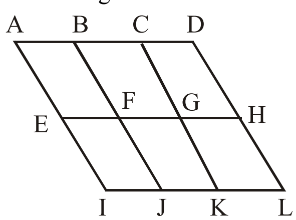
(X) (1) (2) (3) (4)

Hints & SOLUTIONS

Exercise 1

1. **(2)** In figure (X), one of the dots lies in the region common to the circle and the triangle only and the other dot lies in the region common to the circle and the square only. In figures (1), (3) and (4), the region common to the circle and the triangle lies within the square. Only figure (2) contains a region common to the circle and the triangle only and also a region common to the circle and the square.

2. **(3)** We can label the figure as shown.



The simplest parallelogram are ABFE, BCGF, CDHG, EFJI, FGKJ and GHLK. These are 6 in number.

The parallelograms composed of two components each, are ACGE, BDHF, EGKI, FHLJ, ABJI, BCKJ and CDLK. Thus, there are 7 such parallelograms. The parallelogram composed of four components each are ACKI and BDLJ i.e. 2 in number. There is only one parallelogram composed of six components, namely, ADLI. Thus, there are 6 + 7 + 2 + 1 = 16 parallelograms in the figure.

3. **(2)** Let us count the number of triangles vertex wise.

From vertex 1, we can count 4 triangles.

From vertices 2 and 3, we can count 3 triangles each.

From vertices 4, 5, 6, we have 2 triangles each.

Finally, from vertices, 7, 8, 9, 10, we have 1 triangle each.

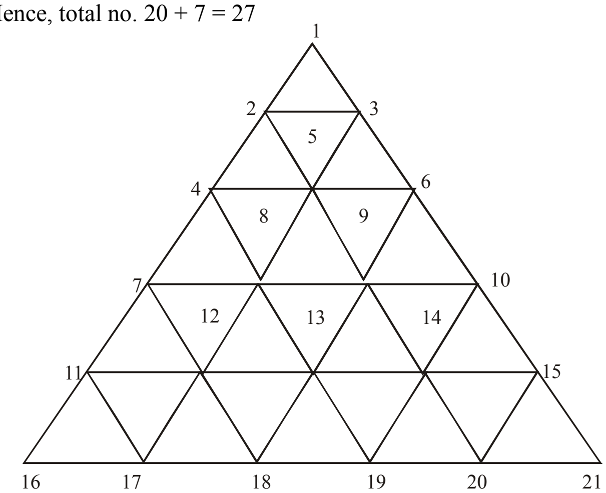
We thus have 4 × 1 + 3 × 2 + 2 × 3 + 1 × 4 = 40 triangles.

These are however, upright triangles only.

From vertices 12, 13, 14 we have 1 + 2+ 1 = 4 triangles in inverted shape.

From 8, 9, we have 1 triangle each and from 5, we have 1 triangle.
Thus, total no. of inverted triangles = 7

Hence, total no. 20 + 7 = 27



Extending this pattern of counting, let us count the triangles in fig.

Upright triangles = 5 × 1 + 4 × 2 + 3 × 3 + 2 × 4 + 1 × 5 = 35

Inverted triangles = (1 + 2 + 2 + 1) + (1 + 2 + 1) + (1 + 1) + 1 = 13

Hence total number = 48

4. **(4)** We have three squares with vertical and horizontal sides. Each such square has $1^2 + 2^2 = 5$ squares in it. Thus there are 15 such squares.

In addition, we have two obliquely placed squares.

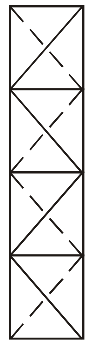
Hence total no. of squares = 17

5. **(4)** Each small square is bisected by its diagonals to give 4 triangles of half the size of the square and 4 triangles of 1/4th the size of the square.

Thus there are 8 × 4 = 32 triangles in the four squares.

Then there are six triangles as shown in the adjoining figure. Total no. of triangles = 32 + 6 = 38

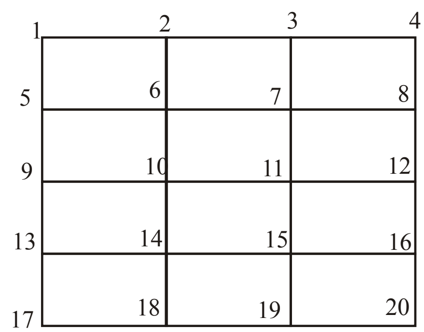
6. **(4)** Consider the longest vertical strip.
It has six rectangles (excluding squares).
In the second vertical strip there are 3 rectangles. In the second vertical strip there are 3 rectangles.
This pattern is repeated with the largest horizontal strip and the second horizontal strip.
Next consider the third vertical and horizontal strips. Each has one rectangle.
Again, consider two vertical and horizontal strips together . Each has one rectangle.
Hence total no. of rectangles
$= 2 (6 + 3 + 1 + 1) = 22$



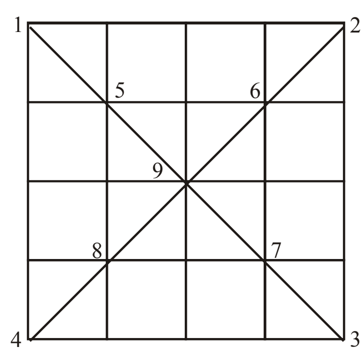
7. **(3)** Name the squares as square 1, 2, 3, 4 5 beginning with the largest.
The space between square 1 and 2 is subdivided into 4 triangles.
The space between square 2 and 3 has 12 triangles as below.
From each vertex of square no. 2, there are two small triangles and one large triangle consisting of these two triangles.
Thus from four vertices, there are 12 triangles.
Each diagonal of square no. 2 divides it into two triangles, making 4 triangles in this manner, each half the size of the square.
The two diagonals together sub- divide the square into four triangles, each 1/4th the size of the square .
Thus there are 20 triangles in square no.
2 Pattern of square no. 1 and 2 is repeated in no. 3 and 4 respectively.
There are no triangles in square no.
Hence, total no. of triangles $= 4 + 20 + 4 + 20 = 48$
8. **(2)** Each square is bisected by one diagonal giving $10 \times 2 = 20$ triangles.
As shown in question 8 above. First second and third vertical strips have $3 + 2 + 1$ triangles.
This is repeated in horizontal strips. There are two squares of size 2×2 which are bisected by two diagonals giving rise to $2 \times 4 = 8$ triangles.
Finally, there is a triangles consisting of 9 small triangles.
Hence, total no. of triangles $= 20 + 2 \times 6 + 2 \times 4 + 1 = 41$
9. **(3)** In figure (X) the dot lies in the region common to the circle and the triangle only.
Such a region is present in fig. (3).
10. **(4)** One point in all three figures and the second point only in one figure.
11. **(1)** One point lies in all three figures. Two points lie in between two figures.
12. **(3)** In fig. (X), the dot is placed in the region common to the circle and the triangle. Amongst the four alternatives only in figure (3), we have a region common to circle and triangle only. Hence figure, (3) is the answer.
13. **(4)** In fig. (X), one of the dots lies in the region common to the square and the triangle only and the other dot lies in the region common to all the three figures—the circle, the square and the triangle. In each of the alternatives (1) and (2), there is no region common to the square and the triangle only. In alternative (3), there is no region common to all the three figures. Only, alternative (4) consists of both the types of regions.
14. **(1)** In fig. (X), one of the dots lies in the region common to the square and the triangle only, another dot lies in the region common to the circle and the triangle only and the third dot lies in the region common to the triangle and the rectangle only. In fig. (2), there is no region common to the square and the triangle only. In fig. (3), there is no region common to the circle and the triangle only. In fig. (4) there is no region common to the triangle and the rectangle only. Only fig. (1) consists of all the three types of regions.
15. **(2)** In fig. (X), one of the dots lies in the region common to the circle and the triangle only and the other dot lies in the region common to the circle and the square only. In each of the figures (1), (3) and (4), there is no region common to the circle and the triangle only. Only fig. (2) consists of both the types of regions.
16. **(4)** In fig. (X), one of the dots lies in the region common to the circle and the triangle only, another dot lies in the region common to all the three figures–the circle, the square and the triangle and the third dot lies in the region common to the circle and the square only. In each of the figures (1) and (3), there is no region common to the circle and the square only and in fig. (2), there is no region common to the circle and the triangle only. Only fig. (4) consists of all the three types of regions.
17. **(4)** In fig. (X), one of the dots lies in the region common to the circle and the rectangle only, another dot lies in the region common to the circle, the square and the rectangle only and the third dot lies in the region common to the circle, the square and the triangle only. In each of the figures (1) and (3) there is no region common to the circle, the square and the rectangle only and in fig. (2), there is no region common to the circle, the square and the triangle only. Only fig. (4) consists of all the three types of regions.
18. **(2)** In fig. (X), one of the dots lies in the region common to the circle and the triangle only, another dot lies in the region common to the circle, the square and the triangle only and the third dot lies in the region common to the circle, the square and the rectangle only. In each of the figures (1) and (3) there is no region common to the circle and the triangle only. In fig. (4) there is no region common to the circle, the square and the rectangle only. Only fig. (2) consists of all the three types of regions.

Exercise 2

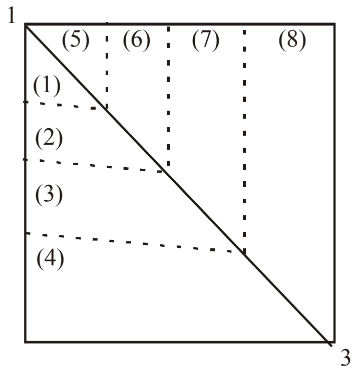
1. **(1)** Largest size square formed in this diagram is of size 3×3.
No. of 3×3 squares $= 2 = 1 \times 2$ (from points 1, 5)
No . of 2×2 squares $= 6 = 2 \times 3$ (from points 1, 2, 5, 6, 9, 10)



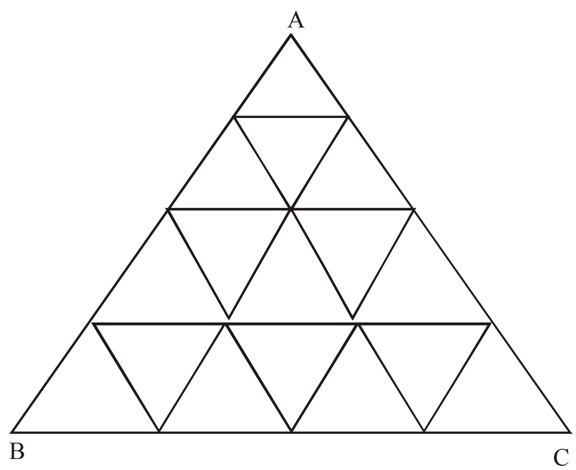
No. of 1×1 squares $= 12 = 3 \times 4$
Hence , total $= 1 \times 2 + 2 \times 3 + 3 \times 4$
$= 3 \times 4 + 2 \times 3 + 1 \times 2 = 20$
2. **(4)** With vertex no. 1 we have four triangles on one side of the diagonal and four triangles on the other side.



Thus from four vertices. We have in all 4 × 8 = 32 triangles.
Next consider square (5 – 6 – 7 – 8).
There are four triangles from each vertex.
Thus we have another 4 × 4 = 16 triangles.
Lastly, we have oblique triangles with vertex 9 such as (9 – 6 – 7), (9 – 2 – 3) and so on.
There are 8 such triangles.
Hence, total no. of triangles = 32 + 16 + 8 = 56

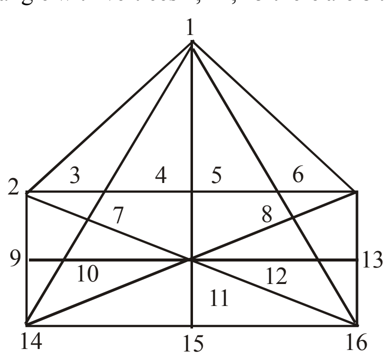


3. **(4)** There are 3 trips parallel to AB and 3 strips parallel to AC. There are 6 horizontal parallelograms in the first. 3 in the second and 1 in the third strip parallel to AB. Kite shaped parallelograms in these strips are 6 + 3 + 1 = 10.
The pattern is repeated for strips parallel to AC except that top-most kite is common to both type of strips.
There are 3 parallelograms each made up of 4 small parallelograms (horizontal and kite shaped.)



Hence total no. of parallelograms = [2 (10 + 10) – 1] + 3 = 42

4. **(4)** Within the triangle with vertices 1, 2, 6, there are 4 + 3 + 2 + 1 = 10 triangles. In the triangle with vertices 1, 2, 14 there are 6 triangles.
In the triangle with vertices 1, 14, 15 there are 8 triangles.



This pattern is repeated for triangle 1, 6, 16 and for triangle 1, 15, 16.
In the triangle with vertices 1, 6, 14 there are 3 triangles and the pattern is repeated for the triangle with vertices 1, 2, 16.
In the parallelogram, there are 4 triangles each half the size, 4 triangles of quarter size and 6 triangles each made up of two small triangles.
Finally, there two triangles with vertices 1, 10, 12 and 1, 14, 16 respectively
Hence total no. of triangles.
= 10 + 2 (6 + 8) + 2 × 3 + (4 + 4 + 6) + 2 = 60

5. **(2)** In fig. (X), one of the dots lies in the region common to the circle and the square only, another dot lies in the region common to all the three figures–the circle, the square and the triangle and the third dot lies in the region common to the circle and the triangle only. In each of the alternatives (1), (3) and (4), there is no region common to the circle and the triangle only. Only fig. (2) consists of all the three types of regions.

6. **(1)** In fig. (X), one of the dots lies in the region common to the circle and the square only, another dot lies in the region common to all the three figures—the circle, the square and the triangle and the third dot lies in the circle alone. In fig. (2) there is no region common to the circle and the square only and in each of the figures (3) and (4) there are regions which lie in the circle alone. Only fig. (1) consists of all the three types of regions.

7. **(3)** In fig. (X), one of the dots lies in the region common to the circle and the triangle only, another dot lies in the circle alone and the third dot lies in the region common to the circle and the square only. In fig. (1) there is no region common to the circle and the triangle only, in fig. (2), there is no region common to the circle and the square and in fig. (4), there is no region which lies in the circle alone. Only, fig. (3) consists of all the three types of regions.

8. **(1)** In fig. (X), one of the dots lies in the region common to the square and the rectangle only, another dot lies in the region common to all the four elements—the circle, the square, the triangle and the rectangle and the third dot lies in the region common to the triangle and the rectangle only. In fig. (2) there is no region common to the triangle and the rectangle only. In fig. (3) there is no region common to the square and the rectangle only. In fig. (4) there is no region common to all the four elements—the circle, the square, the triangle and the rectangle. Only fig. (1) consists of all the three types of regions.

9. **(4)** In fig. (X), one of the dots lies in the region common to the circle and the square only, another dot lies in the region common to the square, the triangle and the rectangle only and the third dot lies in the region common to the triangle and the rectangle only. In each of the figures (1), (2) and (3) there is no region common to the square, the triangle and the rectangle only. Only fig. (4) consists of all the three types of regions.

10. **(4)** In fig. (X), one of the dots lies in the region common to the circle and the square only, another dot lies in the region common to the circle and the rectangle only and the third dot lies in the region common to the triangle and the rectangle only. In fig. (1) there is no region common to the circle and the square only. In figures (2) and (3) there are no regions common to the triangle and the rectangle only. Only fig. (4) consists of all the three types of regions.