Set and Relation

SET

A set is a well defined collection of distinct objects. Sets are usually denoted by capital letters A, B, …, X, Y, Z. The elements of the set are denoted by small letters. There are two methods for representing a set.

(i) Tabulation method or Roster form

All the elements belonging to the set are written in curly brackets.
If A is the set of days of a week, then
A = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

(ii) Set Builder Method or Set rule method

In this method, we use the definition, which is satisfied by all the elements of set.
In above example set A may be written as
A = $\{x : x$ is a day of week$\}$

(a) Some well known sets and their notations

(i) Set of all natural numbers $N = \{1, 2, 3, \ldots\}$

(ii) Set of all integers Z or $I = \{0, \pm 1, \pm 2, \ldots\}$

(iii) Set of non zero integers Z_0 or $I_0 = \{\pm 1, \pm 2, \pm 3, \ldots\}$

(iv) Set of all rational numbers

$$Q = \left\{x : x = \frac{p}{q}, \text{where p and q are relatively prime integers and } q \neq 0\right\}$$

Q_0 denotes the set of all non–zero rational numbers.
(v) Set of real numbers is denoted by R
(vi) Set of complex numbers is denoted by C

(b) Subset & Superset

If A and B are two sets such that every element of A is also an element of B, then A is a subset of B and B is superset of A. We write $A \subseteq B$.

- If $A \subseteq B$ & $B \subseteq A$ then A and B said to be equal sets, i.e. A = B.
- If $A \subseteq B$ and $A \neq B$, there A is called as proper subset of B and denoted by $A \subset B$.
- If a set A has n elements, then the number of subsets of $A = 2^n$.

(c) Universal Set

The universal set is the superset for all the sets under the consideration.
The set of complex numbers is the universal set for all possible sets related numbers.

(d) Null set or Void set

A set having no element is called as null set or empty set or void set. It is denoted by ϕ or { }. The null set is unique and is the subset of every set.

(e) Disjoint sets

Two sets A & B are said to be disjoint, if $A \cap B = \phi$. If $A \cap B \neq \phi$, then A & B are said to be intersecting or overlapping sets.

(f) Singleton Set

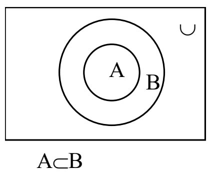
A set having one and only one element is called singleton set or unit set.

(g) Power Set

Power set of a set A is the collection of all subsets of A and is denoted by P(A) or 2^A.

VENN - EULER DIAGRAMS

Venn – diagram is a systematic representation of sets in pictorial form. A set is represented by circle inside the universal set which itself represented by rectangular region.



$A \subset B$

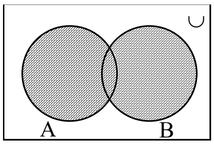
BASIC OPERATIONS

(a) Union of sets

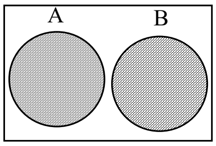
If A and B are two sets, then the union of two sets is denoted by $A \cup B$ and defined as
$A \cup B = \{x : x \in A \text{ or } x \in B\}$
Union of n sets $A_1, A_2, \ldots, A_n$ is denoted and defined as

$$\bigcup_{i=1}^{n} A_i = \{x : x \in A_i \text{ for at least one } i\}$$

Union is also known as join or "logical sum" of A & B.



A∪B



A∪B

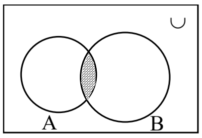
(b) Intersection of sets

The intersection of sets A & B is denoted by $A \cap B$ & defined as

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

Intersection of n sets $A_1, A_2, \ldots, A_n$ is denoted & defined by

$\bigcap_{i=1}^{n} A_i = \{x : x \in A_i \text{ for all } i\}$



A∩B

(c) Complement

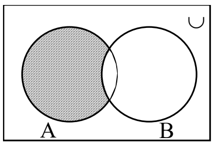
The complement of a set B relative to another set A is denoted & defined as

$A - B = \{x : x \in A \text{ and } x \notin B\}$

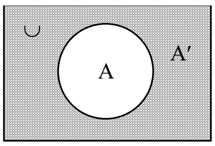
i.e. the set of all points (elements) which belong to A but which do not belong to B.

The complement of set A relative to universal set U is denoted by U – A or by ~ A or A^c or by A′.

Thus $A' = \{x : x \in \cup \text{ and } x \notin A\}$ OR $A' = \{x : x \notin A\}$



A–B



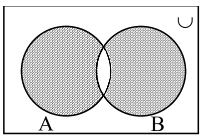
A′

(d) Symmetric difference of two sets

The symmetric difference of sets A & B is the set $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$

Thus,

$A \Delta B = (A - B) \cup (B - A) = \{x : x \notin A \cap B\}$



AΔB

CARTESIAN PRODUCT

The Cartesian product of two sets A and B is the set $\{(a, b): a \in A \text{ and } b \in B\}$ and is denoted by $A \times B$. If A has m elements and B has n elements, then $A \times B$ has mn elements.

- The Cartesian product of n sets $A_1, A_2, \ldots, A_n$ is the set of all ordered n tuples $(a_1, a_2, \ldots, a_n)$, $a_i \in A_i$, $i = 1, 2, \ldots n$ and is denoted by $A_1 \times A_2 \times \ldots \times A_n$ or $\prod_{i=1}^{n} A_i$

CARDINAL NUMBER

The no. of distinct elements in a set A is denoted by n(A) and known as cardinal number of the set A.

- Two finite sets A & B are equivalent if their cardinal number are same.

IMPORTANT PROPERTIES

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B) \Leftrightarrow$ A & B are disjoint non–void sets.

(iii) $n(A - B) = n(A) - n(A \cap B)$

(iv) $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$

(v) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(vi) No. of elements in exactly two of the sets A, B, C $= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(vii) No. of elements in exactly one of the sets A, B, C $= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

(viii) $n(A' \cup B') = n(U) - n(A \cap B)$

(ix) $n(A' \cap B') = n(U) - n(A \cup B)$

LAWS OF ALGEBRA OF SETS

(a) Idempotent laws

(i) $A \cup A = A$ (ii) $A \cap A = A$

(b) Identity laws

(i) $A \cup \phi = A$ (ii) $A \cap U = A$

i.e. ϕ & U are identity elements for union & intersection respectively.

(c) Commutative laws

(i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

i.e. union & intersection are commutative

(d) Associative laws

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $A \cap (B \cap C) = (A \cap B) \cap C$

i.e. union & intersection are associative

(e) Distributive laws

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

i.e. union and intersection are distributive over intersection and union respectively.

(f) De–Morgan's laws

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

(iii) $(A')' = A$

SOLVED EXAMPLE

Example-1

If A and B be two sets containing 3 and 6 elements respectively, what can be the minimum number of elements in $A \cup B$? Find also, the maximum number of elements in $A \cup B$.

Sol. We have $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

This shows that $n(A \cup B)$ is minimum or maximum according as $n(A \cap B)$ is maximum or minimum respectively.

Case 1

When $n(A \cap B)$ is minimum, i.e., $n(A \cap B) = 0$

This is possible only when $A \cap B = \phi$. In this case,

$n(A \cup B) = n(A) + n(B) - 0 = n(A) + n(B) = 3 + 6 = 9$.

So maximum number of elements in $A \cup B$ is 9.

Case 2

When $n(A \cap B)$ is maximum.

This is possible only when $A \subseteq B$.

In this case $n(A \cap B) = 3$.

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B) = (3 + 6 - 3) = 6$

So, minimum number of elements in $A \cup B$ is 6.

Example-2

If A, B and C are three sets and U is the universal set such that n(U) = 700, n(A) = 200, n(B) = 300 and $In(A \cap B) = 100$. Find $n(A' \cap B')$.

Sol. We have $A' \cap B' = (A \cup B)'$

$\therefore n(A' \cap B') = n((A \cup B)') = n(U) - n(A \cup B)$

$= n(U) - [n(A) + n(B) - n(A \cap B)]$

$= 700 - (200 + 300 - 100) = 300$ **Example-3**

Prove that for non empty sets A and B.

$A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

Sol. Consider

$(A \cup B) - (A \cap B) = (A \cup B) \cap (A \cap B)'$

$= (A \cap B) \cap (A' \cup B')$

$= [(A \cup B) \cap A'] \cup [(A \cup B) \cap B']$

$= [(A \cap A') \cup (B \cap A')] \cup [(A \cap B') \cup (B \cap B')]$

$= [\phi \cup (B \cap A')] \cup [(A \cap B') \cup \phi]$

$= (B \cap A') \cup (A \cap B')$

$= (B - A) \cup (A - B)$ (1)

Now R.H.S. $= (A - B) \cup (B - A) \cup (A \cap B)$

$= [(A \cup B) - (A \cap B)] \cup (A \cap B)$ using (1)

$= [(A \cup B) \cap (A \cap B') \cup (A \cap B)$

$= [(A \cup B) \cup (A \cap B)] \cap [(A \cap B)' \cup (A \cap B)]$

$= (A \cup B) \cap U$, where U is universal set.

$= A \cup B =$ L.H.S.

Example-3

If $aN = \{ax : x \in N\}$ describe the set $3N \cap 7N$.

Sol. $3N = \{3x : x \in N\} = \{3, 6, 9, \ldots\}$

$7N = \{7x : x \in N\} = \{7, 14, 21, 28, \ldots\}$

$\therefore 3N \cap 7N = \{y : y$ is a multiple of 3 and y is a multiple of $7\}$

$= \{y : y$ is a multiple of $21\}$

$= \{21, 42, 63, \ldots\} = 21\,N$

Example-4

Solve $3x^2 - 12x = 0$.

when (i) $x \in N$ (ii) $x \in Z$

(iii) $x \in S$ where $S = \{a + ib : b \neq 0; a, b \in R\}$

Sol. $3x^2 - 12x = 0 \Leftrightarrow 3x(x - 4) = 0 \Leftrightarrow x = 0$ or $x = 4$

(i) when $x \in N \Rightarrow x = 4$

(ii) when $x \in Z \Rightarrow x = 0$ or $x = 4$

(iii) when $x \in S \Rightarrow$ No solution

Example-5

In a certain city, only two newspapers A and B are published. It is known that 25% of the city population reads A and 20% reads B, while 8% reads both A and B. It is also known that 30% of those who read A but not B, look into advertisements and 40% of those who read B but not A, look into advertisements, while 50% of those who read both A and B, look into advertisements, What % of the population read an advertisement?

Sol. Let L = Set of people who read paper A

M = Set of people who read paper B

Then $n(L) = 25$, $n(M) = 20$, $n(L \cap M) = 8$

$n(L - M) = n(L) - n(L \cap M) = 25 - 8 = 17$

$n(M - L) = n(M) - n(L \cap M) = 20 - 8 = 12$

$\therefore$ % of people reading an advertisement

$= (30\% \text{ of } 17) + (40\% \text{ of } 12) + (50\% \text{ of } 8)$

$= \frac{51}{10} + \frac{24}{5} + 4 = 13.9\%$

RELATIONS

A relation between two sets A and B is a sub–set of $A \times B$.

R is a relation from A to B iff $R \subset A \times B$, we write aRb iff $(a, b) \in R$ and say that a is R–related to b or b is R–relative of a. We also write a (~R)b if a is not R–related to b.

- If A consists of m elements and B consists of n elements, then the total no. of different relations from A to B is 2^{mn}.
- Any subset of $A \times A$ is said to be a relation on A.

DOMAIN AND RANGE

If R is relation from A to B, then domain and range are defined as

Domain $= \{x : (x, y) \in R\}$

Range $= \{y : (x, y) \in R\}$

SOLVED EXAMPLE

Example-6

Let $A = \{1, 2, 3\}$ & $B = \{2, 4, 6, 8\}$. Let $R_1 = \{(1, 2), (2, 4), (3, 6)\}$ and $R_2 = \{(2, 4), (2, 6), (3, 8), (1, 6)\}$ then find domains and range of relation R_1 and R_2.

Sol. Domain $R_1 = \{1, 2, 3\}$

Range $R_1 = \{2, 4, 6\}$

Domain $R_2 = \{2, 3, 1\}$

Range $R_2 = \{4, 6, 8\}$

TYPES OF RELATION

(i) Null Relation

If A is non empty set, then any subset of $A \times A$ is said to be Null relation on A having no elements.

Number of null relations = 1

(ii) Universal Relation

If A is non empty set, then any subset of $A \times A$ is said to be Universal relation on A having all of elements of $A \times A$.

Number of universal relations = 1

(iii) Binary Relation

If A is non empty set, then any subset of $A \times A$ is said to be Binary relation on A or a relation on A.

(iv) Reflexive relation

A relation R on a set A is said to be a reflexive relation on A if $x R x$ i.e. $(x, x) \in R; \forall x \in A$

Number of Reflexive relations = 2^{n^2-n}

where n is the number of elements of set A.

(v) Symmetric relation

A relation R on a set A is said to be a symmetric relation on A if

$xRy \Rightarrow yRx$

i.e. $(x, y) \in R \Rightarrow (y, x) \in R \; \forall x, y \in A$

Number of Symmetric relations = $2^{n\left(\frac{n+1}{2}\right)}$

where n is the number of elements of set A.

(vi) Anti – Symmetric relation

A relation R on a set A is said to be an anti–symmetric relation on A if

xRy and $yRx \Rightarrow x = y$

i.e. $(x, y) \in R$ and $(y, x) \in R \Rightarrow x = y; \;\; \forall x, y \in A$

(vii) Transitive relation

A relation R on a set A is said to be a transitive relation on A if

$(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow (x, z) \in R; \forall x, y, z \in A$

i.e. xRy and $yRz \Rightarrow xRz$

No formula is given for finding number of transitive relation.

(viii) Identity relation

A relation R on a set A is said to be an identity relation on A if

$R = \{(x, y): x \in A, y \in A, x = y\}$

This is denoted by I_A

$\therefore I_A = \{(x, x) : x \in A\}$

(ix) $A \times A$ is said to be the universal relation on A.

(x) $\because \phi \subset A \times A$. So ϕ is a relation on A, called void relation on A

- Identity relation is always reflexive but a reflexive relation need not to be identity relation.
- A relation which is not symmetric is not necessarily anti–symmetric.

(xi) Inverse Relation

Let $R \subset A \times B$ be a relation from A to B, then the inverse relation of R, denoted by R^{-1}, is a relation from B to A defined by

$R^{-1} = \{(y, x) : (x, y) \in R\}$

Thus $(x, y) \in R \Leftrightarrow (y, x) \in R^{-1}, \forall x \in A, y \in B$

Clearly Domain R^{-1} = Range R

Range R^{-1} = Domain R also $(R^{-1})^{-1} = R$

Let $A = \{1, 2, 4\}$, $B = \{3, 0\}$ and let $R = \{(1, 3), (4, 0), (2, 3)\}$ be a relation from A to B then $R^{-1} = \{(3, 1), (0, 4) (3, 2)\}$

(xii) Equivalence Relation

Let A be a non empty set, then a relation R on A is said to be equivalence relation if

(i) R is reflexive

(ii) R is symmetric

(iii) R is transitive

(xiii) Partial Order Relation

A relation R defined on a set A is said to be a ‘partial order relation’ on A if it is simultaneously reflexive, transitive and antisymmetric on A.

SOLVED EXAMPLE

Example-7

Let N be the set of all natural numbers. Let a relation R be defined on N by

R = {(a, b) : a, b ∈ N and a ≤ bi}. Show that R is a partial order relation.

Sol. R is reflexive because $a \le a \ \forall \ a \in$ N

R is transitive because $a \le b$ & $b \le c$

$\Rightarrow a \le c, \forall \ a, b, c \in$ N

R is anti–symmetric because $a \le b$ & $b \le a$

$\Rightarrow a = b \ \forall \ a, b \in$ N

Thus R is a partial order relation.

(xii) Total Order Relation

A relation R on a set A is said to be a total order relation on A if R is a partial order relation on A such that given any $x, y \in$ A, we must have either xRy or yRx.

(xiii) Composition of Relations

Let R and S be two relations from sets A to B and B to C respectively. Then we can define a relation SoR from A to C such that

$(a, c) \in SoR \Leftrightarrow \exists$ b ∈ B s.t. $(a, b) \in$ R and (b, c) ∈ S.

This relation is called the composition of R and S.

• In general RoS ≠ SoR. Also $(\text{SoR})^{-1} = \text{R}^{-1}\text{oS}^{-1}$

Example-8

Let T be the set of triangles in a plane and a relation r be defined by xry ⇔ x is similar to y; ∀ x, y ∈ T. Show that r is an equivalence relation on T.

Sol.

1. Every triangle is similar to itself.
   ∴ x is similar to x, $\forall \ x \in$ T
   i.e. xrx. So r is reflexive on T
2. xry $\Rightarrow$ x is similar to y
   $\Rightarrow$ y is similar to x
   $\Rightarrow yrx$
   ∴ r is symmetric relation on T.
3. xry and $yrz \Rightarrow x$ is similar to y and y is similar to z
   $\Rightarrow$ x is similar to z $\Rightarrow xrz$

∴ r is transitive relation. Thus r is an equivalence relation on T.

Example-9

Let N be the set of all natural numbers. A relation R be defined on

N × N by (a, b) R(c, d) ⇔ a + d = b + c. Show that R is an equivalence relation.

Sol.

1. (a, b) R(a, b). For $a + b = b + a$
   ∴ R is reflexive

2. $(a, b)\, R(c, d) \Rightarrow a + d = b + c \Rightarrow c + b = d + a$
   $\Rightarrow (c, d)\, R(a, b)$
   ∴ R is symmetric

3. $(a, b)\, R(c, d)$ and (c, d) R(e, f)
   $\Rightarrow a + d = b + c$ and $c + f = d + e$
   $\Rightarrow a + d + c + f = b + c + d + e$
   $\Rightarrow a + f = b + e \Rightarrow (a, b)\, R(e, f)$
   ∴ R is transitive.
   Thus R is an equivalence relation on N × N.

EXERCISE-I

Q.1 Let $A = \{x : x \in R, x \geq 2\}$ and $B = \{x : x \in R, x < 4\}$. Then $A \cap B =$
(1) $\{x : x \in R, 2 < x < 4\}$,
(2) $\{x : x \in R, 2 \leq x < 4\}$
(3) B
(4) A

Q.2 Consider the set of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with value 1. Let C be the subset of the of all determinants with value –1. Then
(1) C is empty
(2) B has as many element as C
(3) $A = B \cup C$
(4) B has twice as many elements as C

Q.3 If $A = \{\phi, \{(\phi)\}\}$, then the power set P (1) of A is
(1) A
(2) $\{\phi, \{\phi\}, A\}$
(3) $\{\phi, \{\phi\}, \{\{\phi\}\}, A\}$
(4) $\{\phi, \{\phi\}, \{\{\phi\}\}\}$

Q.4 Let ρ be the relation on the set R of all real numbers defined by setting a ρ b iff $|a - b| \leq \frac{1}{2}$. Then ρ is
(1) Reflexive and symmetric but not transitive
(2) Symmetric and transitive but not reflexive
(3) Transitive but neither reflexive nor symmetric
(4) Reflexive and transitive but not symmetric

Q.5 If $A = \{2, 3\}$ and $B = \{x \mid x \in N \text{ and } x < 3\}$, then $A \times B$ is
(1) $\{(2, 1), (2, 2), (3, 1), (3, 2)\}$
(2) $\{(1, 2), (1, 3), (2, 2), (2, 3)\}$
(3) $\{(1, 2), (2, 2), (3, 3), (3, 2)\}$
(4) $\{(1, 1), (2, 2), (3, 3), (3, 2)\}$

Q.6 Let U be the universal set containing 700 elements. If A, B are sub–sets of U such that $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Then $n(A' \cap B') =$
(1) 400 (2) 600 (3) 300 (4) 200

Q.7 The number of subsets of a set containing n elements is
(1) n (2) $2^n - 1$ (3) n^2 (4) 2^n

Q.8 Let R be a reflexive relation on a set A and I be the identity relation on A. Then
(1) $R \subset I$ (2) $I \subset R$ (3) $R = I$ (4) $R = 2I$

Q.9 The minimum number of elements that must be added to the relation $R = \{(1, 2), (2, 3)\}$ on the set N so that it is an equivalence relation is
(1) 4 (2) 7 (3) 6 (4) 5

Q.10 Let R be a relation on N defined by $x + 2y = 8$. The domain of R is
(1) $\{2, 4, 8\}$
(2) $\{2, 4, 6, 8\}$
(3) $\{2, 4, 6\}$
(4) $\{1, 2, 3, 4\}$

Q.11 Let $R = \{(1, 3), (4, 2), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is
(1) a function
(2) transitive
(3) not symmetric
(4) reflexive

EXERCISE-II

JEE-MAIN
PREVIOUS YEAR'S

Set

Q.1 Two sets A and B are as under
$A = \{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\}$;
$B = \{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \leq 36\}$
Then - **[JEE Main - 2018]**
(1) $A \subset B$
(2) $A \not\subset B = \phi$ (an empty set)
(3) neither $A \subset B$ nor $B \subset A$
(4) $B \subset A$

Q.2 In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is:
[JEE Main - 2019 (January)]
(1) 102
(2) 42
(3) 1
(4) 38

Q.3 Let S = {1,2,3,......, 100}. The number of non-empty subsets A of S such that the product of elements in A is even is : **[JEE Main - 2019 (January)]**

(1) $2^{100}-1$ (2) $2^{50}(2^{50}-1)$
(3) $2^{50}-1$ (4) $2^{50}+1$

Q.4 Let Z be the set of integers. If $A=\{X \in Z : 2^{(x+2)(x^2-5x+6)}=1\}$ and $B=\{x \in Z : -3 < 2x-1 < 9\}$, then the number of subsets of the set A×B, is :

[JEE Main - 2019 (January)]

(1) 2^{15} (2) 2^{18} (3) 2^{12} (4) 2^{10}

Q.5 Two newspapers A and B are published in a city. It is known that 25% of the city populations reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisement is :-

[JEE Main - 2019(April)]

(1) 12.8 (2) 13.5 (3) 13.9 (4) 13

Q.6 Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true?

[JEE Main - 2019(April)]

(1) If $(A-C) \subseteq B$ then $A \subseteq B$

(2) $(C \cup A) \cap (C \cup B) = C$

(3) If $(A-B) \subseteq C$, then $A \subseteq C$

(4) $B \cap C \neq \phi$

Q.7 Let $X=\{n \in N : 1 \le n \le 50\}$. If $A=\{n \in X : n$ is a multiple of 2); $B=\{n \in X : n$ is a multiple of 7$\}$, then the number of elements in the smallest subset of X containing both A and B is ___________.

[JEE Main-2020 (January)]

Q.8 If $A=\{x \in R : |x|<2\}$ and $B=\{|x-2| \ge 3\}$: then :

[JEE Main-2020 (January)]

(1) $A-B=[-1, 2)$ (2) $A \cup B = R-(2, 5)$
(3) $B-A=R-(-2, 5)$ (4) $A \cap B = (-2, -1)$

Q.9 Let S be the set of all integer solutions, (x, y, z), of the system of equations

[JEE Main-2020 (September)]

$x-2y+5z=0$
$-2x+4y+z=0$
$-7x+14y+9z=0$

such that $15 \le x^2+y^2+z^2 \le 150$. Then, the number of elements in the set S is equal to ______ .

Q.10 Consider the two sets : **[JEE Main-2020 (September)]**

$A=\{m \in R :$ both the roots of $x^2-(m+1)x+m+4=0$ are real$\}$ and $B=[-3, 5)$.

Which of the following is not true?

(1) $A \cap B = \{-3\}$
(2) $B-A=(-3, 5)$
(3) $A \cup B = R$
(4) $A-B=(-\infty, -3) \cup (5, \infty)$

Q.11 Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$ where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i's and exactly 6 of sets Y_i's, then n is equal to

[JEE Main-2020 (September)]

(1) 50 (2) 15 (3) 30 (4) 45

Q.12 A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be : **[JEE Main-2020 (September)]**

(1) 37 (2) 55 (3) 29 (4) 65

Q.13 A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be : **[JEE Main-2020 (September)]**

(1) 63 (2) 36 (3) 38 (4) 54

Q.14 Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than total number of subsets of B, then the value of m · n is —.

[JEE Main-2020 (September)]

Relation

Q.15 14. If $R=\{(x, y) ; x, y \in Z, x^2+3y^2 \le 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is :

[JEE Main-2020 (September)]

(1) {0, 1} (2) {–2, –1, 1, 2}
(3) {–1, 0, 1} (4) {–2, –1, 0, 1, 2}

Q.16 Let R_1 and R_2 be two relation defined as follows :

[JEE Main-2020 (September)]

$R_1=\{(a, b) \in R^2 : a^2+b^2 \in Q\}$ and
$R_2=\{(a,b) \in R^2 : a^2+b^2 \in Q^c\}$, where Q is the set of the rational numbers. Then :

(1) Neither R_1 nor R_2 is transitive.
(2) R_2 is transitive but R_1 is not transitive
(3) R_1 and R_2 are both transitive.
(4) R_1 is transitive but R_2 is not transitive.

ANSWER KEY

EXERCISE-I

Q.1 (2) **Q.2** (2) **Q.3** (3) **Q.4** (1) **Q.5** (1) **Q.6** (3) **Q.7** (4) **Q.8** (2) **Q.9** (2) **Q.10** (3)
Q.11 (3)

EXERCISE-II

JEE-MAIN

PREVIOUS YEAR'S

Q.1 (1) **Q.2** (4) **Q.3** (2) **Q.4** (1) **Q.5** (3) **Q.6** (1) **Q.7** [29] **Q.8** (3) **Q.9** [8] **Q.10** (4)
Q.11 (3) **Q.12** (2) **Q.13** (2) **Q.14** [28.00] **Q.15** (3) **Q.16** (1)

EXERCISE (Solution)

EXERCISE-I

Q.1 (2)
$A = \{2, 3, 4 ....\}$
$B = \{0, 1, 2, 3 ....\}$
$A \cap B = \{2, 3\}$
Then $A \cap B$ is $\{x : x \in R, 2 \le x < 4\}$

Q.2 (2)

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} \forall a_i \in \{0, 1\}$$

This deter minant will take value O, 1 or –1 only & '1' will be taken same no. of times as –1 ; so n(B) = n(C)

Q.3 (3)
$A = \{\phi, \{\phi\}\}$
P(A) = set containing all subsets
$= \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}$
$= \{\phi, \{\phi\}, \{\{\phi\}\}, A\}$

Q.4 (1)

$P : a \rho b$ iff $|a - b| \le \frac{1}{2}$

Reflexive : $a \rho b : |0 - a| \le \frac{1}{2}$ (True)

Symmetric : $a \rho b \Rightarrow b \rho a$

$$|a - b| \le \frac{1}{2} \Rightarrow |b - a| \le \frac{1}{2} \text{ (True)}$$

Transitive : $a \rho b : b \rho a \Rightarrow a \rho c$

$$|a - b| \le \frac{1}{2} ; |b - c| \le \frac{1}{2}$$

$$\Rightarrow |a - c| \le \frac{1}{2}$$

so not transitive

Q.5 (1)
$A = \{2, 3\} ; B = \{1, 2\}$
$A \times B = \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

Q.6 (3)
$n(A \cap B) = n(A) + n(B) - n(A' \cap B')$
$= 200 + 300 - 100$
$n(A \cap B) = 400$
Now $n(A' \cap B') = U - n(A \cup B)$
(De marganistans)
$= 700 - 400 = 300$

Q.7 (4)
conceptual
2^n

Q.8 (2)
Reflexive relation : a R a
but identity relation is $y = x : x \in A$ & $y \in A$
so $I \subset R$

Q.9 (2)
$R = \{(1, 2), (2, 3)\}$
for Reflexive : a R a
for symmetric : a R b $\Rightarrow$ b R a
for transitive : a R b, b R c $\Rightarrow$ a R c
So elements to be added
$\{(1, 1), (2, 2), (3, 3), (2, 1), (3, 2), (1, 3), (3, 1)\}$

Q.10 (3)
for $x = 2, y = 3 \in N$
$x = 4, y = 2 \in N$
$x = 6, y = 1 \in N$

Q.11 (3)
$(4, 2) \in R$ but $(2, 4) \notin R$ &
$(2, 3) \in R$ but $(3, 2) \notin R$

EXERCISE-II

JEE-MAIN
PREVIOUS YEAR'S

Q.1 (1)

$A = \{(a, b) \in R \times R : |a - 5| < 1, |b - 5| < 1\}$

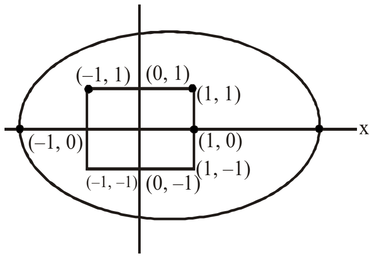
Let $a - 5 = x$, $b - 5 = y$

Set A contains all points inside $|x| < 1, |y < 1|$

$B = \{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \le 36\}$

Set B contains all points inside or on

$$\frac{(x-1)^2}{9} + \frac{y^2}{4} = 1$$



$(\pm 1, \pm 1)$ lies inside the ellipse $\Rightarrow A \subset B$

Q.2 (4)

$$n(P) = \left[\frac{140}{3}\right] = 46$$

$$n(C) = \left[\frac{140}{5}\right] = 28$$

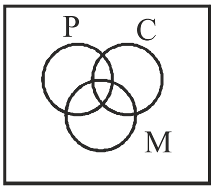
$$n(M) = \left[\frac{140}{2}\right] = 70$$

$n(P \cup C \cup M) = n(P) + n(C) + n(M) - n(P \cap C) - n(C \cap M) - n(M \cap P) + n(P \cap M \cap C)$

$$= 46 + 28 + 70 - \left[\frac{140}{15}\right] - \left[\frac{140}{10}\right] - \left[\frac{140}{6}\right] + \left[\frac{140}{30}\right]$$

$= 144 - 9 - 14 - 23 + 4 = 102$

So requird number os student $= 140 - 102 = 38$



Q.3 (B)

Product is even when atleast one element of subset is even Hence required number of subsets = total subsets –number of subsets all whose elements are odd

$= 2^{100} - 2^{50}$

Q.4 (A)

$$A = \left\{x \in z : 2^{(x+2)(x^2-5x+6)} = 1\right\}$$

$$2^{(x+2)(x^2-5x+6)} = 2^o \Rightarrow x = -2, 2, 3$$

$$A = \{-2, 2, 3\}$$

$$B = \{x \in Z : -3 < 2x - 1 < 9\}$$

$$B = \{0, 1, 2, 3, 4\}$$

Hence, $A \times B$ has is 15 elements.

So number of subsets of $A \times B$ is 2^{15}

Q.5 **(3)**

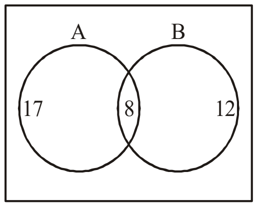
Let population = 100

$n(A) = 25$

$n(B) = 20$

$n(A \cap B) = 8$

$n(A \cap \overline{B}) = 17$

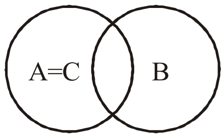


$n(\overline{A} \cap B) = 12$

$$\frac{30}{100} \times 17 + \frac{40}{100} \times 12 + \frac{50}{100} \times 8$$

$5.1 + 4.8 + 4 = 13.9$

Q.6 **(1)**



for $A = C$, $A - C = \phi$

$\Rightarrow \phi \subseteq B$

But $A \not\subseteq B$

$\Rightarrow$ option 1 is NOT true

Let $x \in (C\ x \in (C \cup A) \cap (C \cup B)$

$\Rightarrow x \in (C \cup A)$ and $x \in (C \cup B)$

$\Rightarrow (x \in C$ or $x \in A)$ and $(x \in C$ or $x \in B)$

$\Rightarrow x \in C$ or $x \in (A \cap B)$

$\Rightarrow x \in C$ or $x \in C$ (as $A \cup B \subseteq C$)

$\Rightarrow x \in C$

$\Rightarrow (C \cup A) \cap (C \cup B) \subseteq C$ .....(i)

Now $x \in C \Rightarrow x \in (C \cup A)$ and $x \in (C \cup B)$

$\Rightarrow x \in (C \cup A) \cap (C \cup B)$

$\Rightarrow C \subseteq (C \cup A) \cap (C \cup B)$ ...(ii)

$\Rightarrow$ from (i) and (ii)

$C = (C \cup A) \cap (C \cup B)$

option 2 is ture

Let $x \in A$ and $x \notin B$

$\Rightarrow x \in (A - B)$

$\Rightarrow x \in C$ (as $A - B \subseteq C$)

Let $x \in A$ and $x \in B$

$\Rightarrow x \in (A \cap B)$

$\Rightarrow x \in C$ (as $A - B \subseteq C$)

Let $x \in A$ and $x \in B$

$\Rightarrow x \in (A \cap B)$

$\Rightarrow x \in C$ (as $A \cap B \subseteq C$)

Hence $x \in A \Rightarrow x \in C$

$\Rightarrow A \subseteq C$

$\Rightarrow$ option 3 is true

as $C \supseteq (A \cap B)$

$\Rightarrow B \cap C \supseteq (A \cap B)$
as $A \cap B \neq \phi$
$\Rightarrow B \cap C \neq \phi$
$\Rightarrow$ Option 4 is true.

Q.7 [29]
$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 25 + 7 - 3 = 29$

Q.8 (3)
$A \{x : x \in (-2, 2)\}$
$B = \{x : x \in (-\infty, -1] \cup [5, \infty)\}$
$A \cap B = \{x : x \in (-2, -1]\}$
$A \cup B = \{x : x \in (-\infty, 2) \cup [5, \infty)\}$
$A - B = \{x : x \in (-1, 2)\}$
$B - A = \{x : x \in (-\infty, -2] \cup [5, \infty)\}$

Q.9 (8)
$x - 2y + 5z = 0$ ...(i)
$-2x + 4y + z = 0$ ...(ii)
$-7x + 14y + 9z = 0$ ...(iii)
From (i) and (ii); $z = 0$ and $x = 2y$
Let $x = 2\alpha$, $y = \alpha$, $z = 0$
Now,
$15 \leq 4\alpha^2 + \alpha^2 \leq 150$
$3 \leq \alpha^2 \leq 30$
$\alpha = \pm 2, \pm 3, \pm 4, \pm 5$
Hence 8 elements are there in set S.

Q.10 (4)
$A = \{m \in R : x^2 - (m+1)x + m + 4 = 0$ has real roots$\}$
$A = \{(-\infty, -3] \cup (5, \infty)\}$ $\{D \geq 0\}$
$B = [-3, 5) \Rightarrow A - B = (-\infty, -3) \cup [5, \infty)$

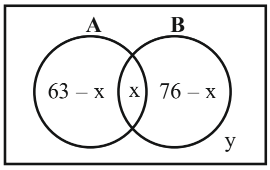
Q.11 (3)

$$\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$$

$$\Rightarrow \frac{10 \times 50}{20} = \frac{5n}{6} \quad \left\{ \begin{array}{l} \because n(x_i) = 10, n(Y_i) = 5 \\ \text{So}, \bigcup_{i=1}^{50} X_i = 500, \bigcup_{i=1}^{n} Y_i = 5n \end{array} \right\}$$

$\Rightarrow n = 30$

Q.12 (2)



Here $63 - x + x + 76 - x + y = 100$
$39 + y = x$
$\therefore 39 \leq x \leq 63$
$\therefore$ Possible value of x is 55.

Q.13 (2)
$n(C) = 73, n(T) = 65$
$\therefore 65 \geq n(C \cap T) \geq 65 + 73 - 100$
$65 \geq x \geq 38$
$x \neq 36$

Q.14 (28.00)
Number of subsets of $A = 2^m$
Number of subsets of $B = 2^n$
Given $= 2^m - 2^n = 112$
$\Rightarrow (m, n) = (7, 4)$
$\Rightarrow mn = 28$

Q.15 (3)
Given $R = \{(x, y) : x, y \in Z, x^2 + 3y^2 \leq 8\}$
So $R = \{(1, 1), (2, 1), (1, -1), (0, 1), (1, 0)\}$
So $D_{R^{-1}} = \{-1, 0, 1\}$

Q.16 (1)
(i) If $(a, b) \in R_1$ and $(b,c) \in R_1$
$\Rightarrow a^2 + b^2 \in Q$ and $b^2 + c^2 \in Q$
then $a^2 + 2b^2 + c^2 \in Q$ but we cannot say anything about $a^2 + c^2$, that it is rational or not. So R_1 is not transitive.
(ii) If $(a, b) \in R_2$ and $(b,c) \in R_2$
$\Rightarrow a^2 + b^2 \notin Q$ and $b^2 + c^2 \notin Q$
but we can't say anything about $a^2 + c^2$
that it is rational or irrational.
So R_2 is not transitive.