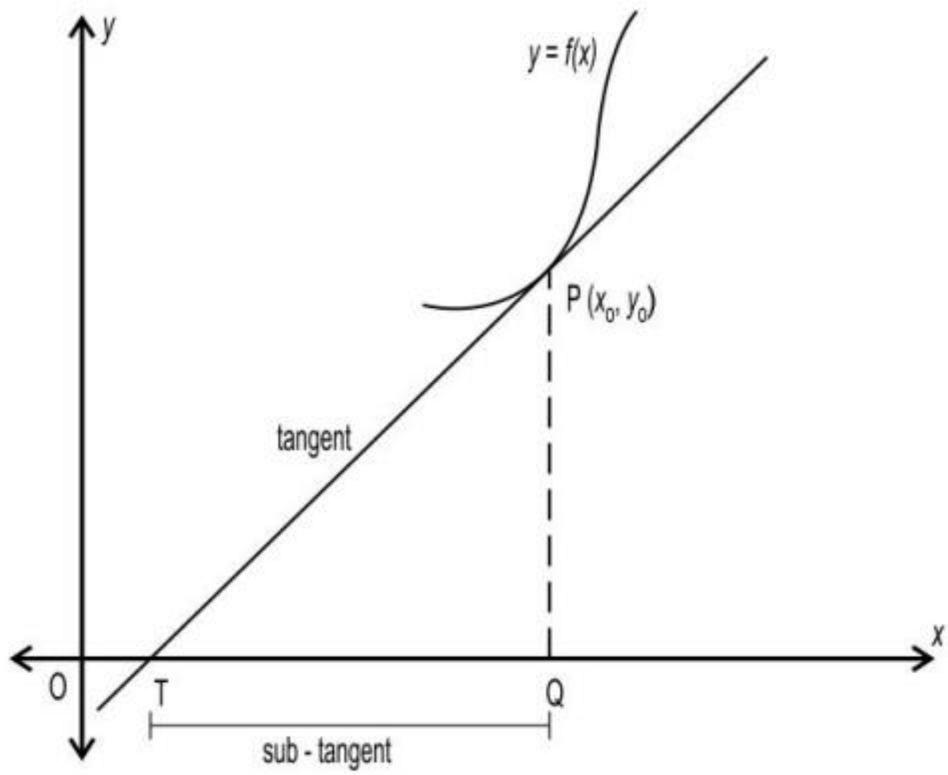
CBSE 12th Maths 2024-2025

Chapter - 6 Application of Derivatives and multiconcepts

Competency-Based Questions

Q.1 The sub-tangent of a curve at a point is the projection on the x -axis of the portion of the tangent to the curve between the x -axis and the point of tangency. The sub-tangent of a curve y = f (x) at a point P(x_0 ,y_0) is illustrated below.

Answer.



Among the given slopes of tangents of a curve at a given point, which will result in the longest sub-tangent?

1. 30°
2. 45°
3. 60°
4. 90°

Answer. (1)

Q: 2 The diagonal of a square, of side 3√2 cm, is increasing at a rate of 2 cm/s.

Which of the following is the rate at which its area is increasing?

1. $\sqrt{2}$ cm^2/s
3. 12 cm^2/s
2. $6\sqrt{2}$ cm^2/s
4. 24 cm^2/s

Answer. (3)

Q: 3 Sameer wants to find the area of his garden. He measures the side of his square garden as 10 m with an error of 0.05 m.

Which of the following is the approximate error that Sam will have in calculating its area?

1. 0.05 m^2
2. 0.5 m^2
3. 1 m^2
4. 10.05 m^2

Answer. (3)

Q: 4 Gagan wants to calculate the capacity of a cylindrical water tank. He precisely measures the height of the tank as 7 m. Next, he measures the radius of the tank as 2 m, with an error of 0.05 m. The error occured while measuring the radius results in an error in calculating the capacity.

What is the approximate error in calculating the capacity of the water tank?

(Note: Take $\pi = \frac{22}{7}$.)

1. 2.2 m^3
2. 2. 4.4 m^3
3. 44 m^3
4. 88 m^3

Answer. (2)

Q: 5 Study the function below.

$f(x) = \frac{2x^3}{3} - 18x + k$, where k is a constant

Which of the following is true about the nature of the function in the interval [-3, 3]?

1. f(x) is increasing
2. f(x) is decreasing
3. f(x) is neither increasing nor decreasing
4. (cannot say without knowing the value of k)

Answer. (2)

Q: 6The normal to the curve y = f (x) at the point (5, 7) makes an angle of π/4 with the x -axis in the positive direction.

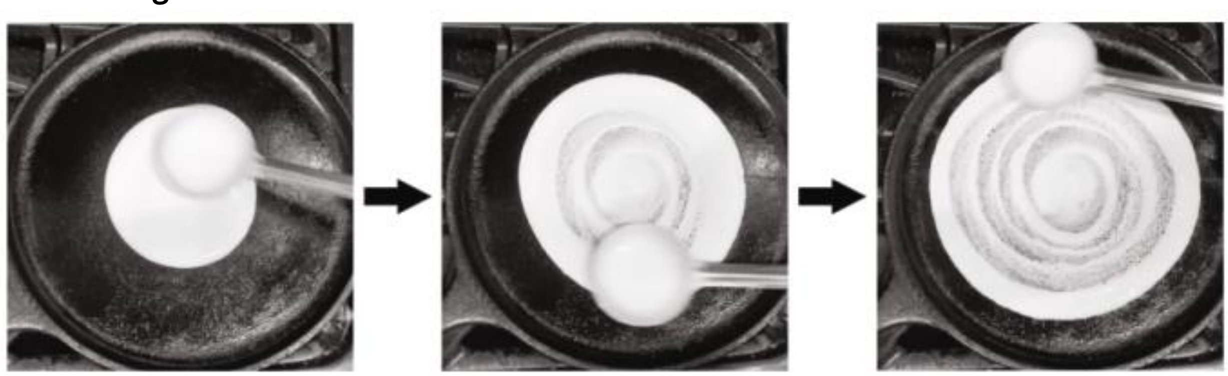
Find f'(5). Show your work.

Answer.

Finds the slope of the normal to the curve $y = f(x)$ at the point (5, 7) as tan $\frac{\pi}{4}$ = 1.

Finds the slope of the tangent to the curve $y = f(x)$ at the point (5, 7) as $\left(\frac{-1}{1}\right)$ = (-1) and concludes that $f'(5)$ = (-1).

Q: 7 Aditi is making a circular dosa. She is spreading the dosa batter such that its radius is increasing at a rate of 2 cm/s.



Find the rate of change of the area of the dosa, in terms of π, when its radius is 9 cm. Show your steps.

Answer. Finds the expression for the rate of change of the area of the dosa as:

$\frac{dA}{dt} = \pi \times 2r \times \frac{dr}{dt}$ cm^2/s

Uses the given information and finds the rate of change of the area of the dosa when its radius is 9 cm as $\frac{dA}{dt} = \pi \times 2 \times 9 \times 2 = 36\pi$ cm^2/s.

Q.8

Find the critical points of the function $y = \tan^{-1}(\sec x)$ where $x \in [\frac{-\pi}{2}, \frac{\pi}{2}]$. Show your steps.

Answer.

Differentiates the function as:

$$\frac{dy}{dx} = \frac{\sec x \tan x}{1+\sec^2 x}$$

Writes that 0 is a critical point as $\frac{dy}{dx}$ is 0 when $x = 0$.

Writes that $\frac{-\pi}{2}$ and $\frac{\pi}{2}$ are critical points as $\frac{dy}{dx}$ is not differentiable at these two points.

Q.9

The maximum value of the function $f(x) = x^{\frac{1}{x}}, x > 0$, is obtained at $x = e$.

Use the above fact and show that $e^\pi > \pi^e$.

Answer. Writes that, as the maximum value of f (x) is obtained at x = e, f (e) > f (x), for every x > 0.

Uses the above step to write:

For x = π, f(e) > f(π)

$$\Rightarrow e^{\frac{1}{e}} > \pi^{\frac{1}{\pi}}$$

Raises the power on both sides of the above inequality by (e π) and simplifies the same to get:

$$\left(e^{\frac{1}{e}}\right)^{e\pi} > \left(\pi^{\frac{1}{\pi}}\right)^{e\pi}$$

$$\Rightarrow e^{\pi} > \pi^{e}$$

Q: 10 A circular metallic plate is expanding such that its area is constantly increasing with respect to time.

Milind claims that the rate of increase of its perimeter with respect to time is inversely proportional to its radius.

Is Milind's claim correct? Justify your answer.

Answer. Finds the rate of change of area of the circular plate with respect to time and equates it to a constant k as:

$\frac{dA}{dt} = 2\pi r \times \frac{dr}{dt} = k$ cm^2/s, where k is a positive real number, t is the time, A and r are area and radius of the circular plate respectively.

Uses steps 1 and 2 to write:

$\frac{dP}{dt} = 2\pi \times \frac{k}{2\pi r} = \frac{k}{r}$

Concludes that Milind's claim is correct.

Q: 11 A particle is moving such that its distance s from a fixed point at any time t is given by
s = Asin t + Bcos t, where A, B ∈ R.

Show that the particle's acceleration is always numerically equal to its distance from the fixed point.

Answer. Differentiates s to find velocity as:
s' = Acos t - Bsin t

Differentiates s' to find acceleration as:
s" -Asin t - Bcos t

Rewrites acceleration in terms of distance as s" = (-s) and writes that the magnitudes of distance and acceleration are the same.

Hence, concludes that acceleration is always numerically equal to the distance of the particle from the fixed point.

Q: 12 A solid sphere of gold, of radius 5 cm, is being melted in a furnace such that the radius is decreasing uniformly.

When its radius is 1 cm, show that the rate at which its surface area is decreasing with respect to time is twice the rate at which its volume is decreasing with respect to time.

Answer.

Writes that the volume of a sphere, V, is given by $\frac{4}{3}\pi r^3$, where r is the radius of the sphere and differentiates the same with respect to time as:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Writes that the surface area of a sphere, A, is given by $4\pi r^2$ and differentiates the same with respect to time as:

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

Uses steps 1 and 2 to find the relation between $\frac{dV}{dt}$ and $\frac{dA}{dt}$ as:

$$\frac{dV}{dt} = \frac{r}{2} \times \frac{dA}{dt}$$

Substitutes $r = 1$ in the above equation to show that $2\frac{dV}{dt} = \frac{dA}{dt}$.

Q: 13 A basketball has a tiny hole that leads to it getting deflated while maintaining a spherical shape.

Find the ratio of the rate of loss of the volume of air to the rate of loss of the surface area of the ball when the radius of the ball is 8 cm. Show your steps.

Answer.

Writes that the volume of the air in the basketball is $\frac{4}{3}\pi r^3$ and finds the rate of loss of volume of air $\frac{dV}{dt}$ as $4\pi r^2 \frac{dr}{dt}$.

Writes that the surface area of the basketball is $4\pi r^2$ and finds the rate of loss of the surface area $\frac{dA}{dt}$ as $8\pi r \frac{dr}{dt}$.

Uses steps 1 and 2 to find the ratio as r : 2 and evaluates the same at r = 8 cm as 8 : 2 or 4 : 1.

Q: 14 A cylindrical disk of radius R and height H is pressed by a hydraulic press. During the process, the radius and the height of the disk change such that the cylindrical shape is retained and the volume of the disk remains constant.

What is the ratio of the rate of change of height to the rate of change of radius in terms of R? Show your steps and give valid reasons.

Answer.

Writes the volume function of the disk as $V = \pi \times R^2 \times H$.

Writes that since volume remains constant, the derivative of V should be zero.

$\frac{dV}{dt} = 0$

Applies chain rule to get the equation as:

$\pi \times R^2 \times \frac{dH}{dt} + \pi \times H \times 2R \times \frac{dR}{dt} = 0$

Simplifies the equation in step 3 to get the ratio as:

$\frac{dH}{dt} : \frac{dR}{dt} = \frac{-2H}{R}$

Q: 15 Simran cuts a metallic wire of length a m into two pieces. She uses both pieces to create two squares of different side lengths.

Find the side lengths of the squares (in terms of a) for which the combined area will be MINIMUM? Show your steps and give reasons.

Answer.

Assumes the perimeter of one square as x m and the perimeter of the other square as (a - x) m.

Finds the combined area of the two squares as:

$$A = (\frac{x}{4})^2 + (\frac{a-x}{4})^2 \text{ m}^2$$

Differentiates the combined area as:

$$\frac{dA}{dx} = \frac{(4x-2a)}{16}$$

Equates $\frac{dA}{dx}$ to 0 to find the critical point as $x = \frac{a}{2}$ m.

Finds the second derivative as:

$$\frac{d^2A}{dx^2} = \frac{1}{4}$$

Concludes that $x = \frac{a}{2}$ m is a minima.

Writes that the combined area of the two squares will be minimum when the side lengths of both the squares is $\frac{a}{8}$ m.