5 Properties of Triangle

Relations between Sides and Angles of a Triangle and the Area of a Triangle

1. Consider the following statements: **[2014-I]**

1. There exists no triangle ABC for which $\sin A + \sin B = \sin C$.
2. If the angle of a triangle are in the ratio 1 : 2 : 3, then its sides will be in the ratio $1 : \sqrt{3} : 2$.

Which of the above statements is/are correct?

(*a*) 1 only (*b*) 2 only
(*c*) Both 1 and 2 (*d*) Neither 1 nor 2

2. If $A + B + C = \pi$, then what is $\cos (A + B) + \cos C$ equal to? **[2014-I]**

(*a*) 0 (*b*) $2 \cos C$
(*c*) $\cos C - \sin C$ (*d*) $2 \sin C$

3. In a triangle ABC, c = 2, A = 45°, a = $2\sqrt{2}$, than what is C equal to? **[2014-II]**

(*a*) 30° (*b*) 15°
(*c*) 45° (*d*) None of these

4. In a triangle ABC, sin A – cos B = cos C, then what is B equal to? **[2014-II]**

(*a*) π (*b*) $\pi/3$
(*c*) $\pi/2$ (*d*) $\pi/4$

5. In a triangle ABC, a = $(1+\sqrt{3})$ cm, b = 2 cm and angle C = 60°. Then the other two angles are **[2015-I]**

(*a*) 45° and 75° (*b*) 30° and 90°
(*c*) 105° and 15° (*d*) 100° and 20°

6. If a, b, c are the sides of a triangle ABC, then $a^{\frac{1}{p}} + b^{\frac{1}{p}} - c^{\frac{1}{p}}$ where $p > 1$, is **[2015-II]**

(*a*) always negative
(*b*) always positive
(*c*) always zero
(*d*) positive if $1 < p < 2$ and negative if $p > 2$

Consider the following for the next two (02) items that follow:

Consider a ΔABC in which

$\cos A + \cos B + \cos C = \sqrt{3} \sin \frac{\pi}{3}$ **[2016-I]**

7. What is the value of $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$?

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{1}{8}$ (*d*) $\frac{1}{16}$

8. What is the value of

$\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{C+A}{2}\right)$?

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{2}$
(*c*) $\frac{1}{16}$ (*d*) None of the above

9. Consider the following statements: **[2016-I]**

1. If ABC is an equilateral triangle, then 3tan(A + B) tan C =1.
2. If ABC is a triangle in which A = 78°, B =66°, then $\tan\left(\frac{A}{2}+C\right) < \tan A$
3. If ABC is any triangle, then $\tan\left(\frac{A+B}{2}\right)\sin\left(\frac{C}{2}\right) < \cos\left(\frac{C}{2}\right)$

Which of the above statements is/are correct?

(*a*) 1 only (*b*) 2 only
(*c*) 1 and 2 (*d*) 2 and 3

10. Consider the following for triangle ABC: **[2017-I]**

1. $\sin\left(\frac{B+C}{2}\right) = \cos\left(\frac{A}{2}\right)$
2. $\tan\left(\frac{B+C}{2}\right) = \cot\left(\frac{A}{2}\right)$
3. sin (B + C) = cos A
4. tan (B + C) = –cot A

Which of the above are correct?

(*a*) 1 and 3 (*b*) 1 and 2
(*c*) 1 and 4 (*d*) 2 and 3

11. In a triangle ABC, $a - 2b + c = 0$. The value of $\cot\left(\frac{A}{2}\right)\cot\left(\frac{C}{2}\right)$ is **[2017-II]**

(*a*) $\frac{9}{2}$ (*b*) 3
(*c*) $\frac{3}{2}$ (*d*) 1

12. In triangle ABC, if $\frac{\sin^2 A+\sin^2 B+\sin^2 C}{\cos^2 A+\cos^2 B+\cos^2 C}=2$ then the triangle is **[2017-II]**

(a) right-angled (b) equilateral
(c) isosceles (d) obtuse-angled

13. In a triangle ABC if $a = 2$, $b = 3$ and $\sin A = \frac{2}{3}$, then what is angle B equal to? **[2018-I]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

14. If x, $x-y$ and $x+y$ are the angles of a triangle (not an equilateral triangle) such that $\tan(x-y)$, $\tan x$ and $\tan(x+y)$ are in GP, then what is x equal to? **[2018-I]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

15. ABC is a triangle inscribed in a circle with centre O. Let $\alpha = \angle BAC$, where $45° < \alpha < 90°$. Let $\beta = \angle BOC$. Which one of the following is correct? **[2018-I]**

(a) $\cos\beta = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$ (b) $\cos\beta = \frac{1+\tan^2\alpha}{1-\tan^2\alpha}$
(c) $\cos\beta = \frac{2\tan\alpha}{1+\tan^2\alpha}$ (d) $\sin\beta = 2\sin^2\alpha$

16. If $A + B + C = 180°$, then what is $\sin 2A - \sin 2B - \sin 2C$ equal to? **[2018-II]**

(a) $-4\sin A\sin B\sin C$ (b) $-4\cos A\sin B\cos C$
(c) $-4\cos A\cos B\sin C$ (d) $-4\sin A\cos B\cos C$

17. If the angles of a triangle ABC are in the ratio 1: 2: 3, then the corresponding sides are in the ratio **[2019-I]**

(a) 1: 2: 3 (b) 3: 2: 1 (c) $1:\sqrt{3}:2$ (d) $1:\sqrt{3}:\sqrt{2}$

18. If the angles of a triangle ABC are in AP and $b : c = \sqrt{3}:\sqrt{2}$, then what is the measure of angle A? **[2019-II]**

(a) 30° (b) 45° (c) 60° (d) 75°

19. If angle C of a triangle ABC is a right angle, then what is $\tan A + \tan B$ equal to? **[2019-II]**

(a) $\frac{a^2-b^2}{ab}$ (b) $\frac{a^2}{bc}$ (c) $\frac{b^2}{ca}$ (d) $\frac{c^2}{ab}$

Consider the following for the next two (02) items that follows:

ABCD is a trapezium such that *AB* and *CD* are parallel and *BC* is perpendicular to them. Let $\angle ADB = \theta$, $\angle ABD = \alpha$, $BC = p$ and $CD = q$

20. Consider the following : **[2020-I & II]**

(1) $AD\sin\theta = AB\sin\alpha$ (2) $BD\sin\theta = AB\sin(\theta+\alpha)$

Which of the above is/are correct?

(a) (1) only (b) (2) only
(c) Both (1) and (2) (d) Neither (1) nor (2)

21. What is AB equal to? **[2020-I & II]**

(a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$ (b) $\frac{(p^2-q^2)\cos\theta}{p\cos\theta+q\sin\theta}$
(c) $\frac{(p^2+q^2)\sin\theta}{q\cos\theta+p\sin\theta}$ (d) $\frac{(p^2-q^2)\cos\theta}{q\cos\theta+p\sin\theta}$

22. Consider the following statements : **[2020-I & II]**

(1) If ABC is a right-angled triangle, right-angled at A and if $\sin B = \frac{1}{3}$, then cosec $C = 3$

(2) If $b\cos B = c\cos C$ and if the triangle ABC is not right-angled, then ABC must be isosceles.

Which of the above statements is/are correct?

(a) (1) only (b) (2) only
(c) Both (1) and (2) (d) Neither (1) nor (2)

23. Consider the following statements : **[2020-I&II]**

(1) If in a triangle ABC, $A = 2B$ and $b = c$, then it must be an obtuse-angled triangle.

(2) There exists no triangle ABC with $A = 40°$, $B = 65°$ and $\frac{a}{c} = \sin 40° \operatorname{cosec} 15°$

Which of the above statements is/are correct?

(a) (1) only (b) (2) only
(c) Both (1) and (2) (d) Neither (1) nor (2)

24. The sides of a triangle are m, n and $\sqrt{m^2+n^2+mn}$. What is the sum of the acute angles of the triangle? **[2021-I]**

(a) 45° (b) 60° (c) 75° (d) 90°

25. What is the area of the triangle ABC with sides $a = 10$ cm, $c = 4$ cm and angle $B = 30°$? **[2021-I]**

(a) 16 cm² (b) 12 cm²
(c) 10 cm² (d) 8 cm²

26. In a triangle ABC, $\sin A - \cos B - \cos C = 0$. **[2022-I]**

What is angle B equal to?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

27. Let a, b, c be the lengths of sides BC, CA, AB respectively of a triangle ABC. If p is the perimeter and q is the area of the triangle, then what is $p(p-2a)\tan\left(\frac{A}{2}\right)$ equal to? **[2022-I]**

(a) q (b) $2q$ (c) $3q$ (d) $4q$

28. In a triangle ABC, $a = 4$, $b = 3$, $c = 2$. **[2022-II]**

What is $\cos 3C$ equal to?

(a) $\frac{7}{128}$ (b) $\frac{11}{128}$ (c) $\frac{7}{64}$ (d) $\frac{11}{64}$

Consider the following for the next two (02) items that follow:

The perimeter of a triangle ABC is 6 times the AM of sine of angles of the triangle. Further $BC = \sqrt{3}$ and $CA = 1$.

29. What is the perimeter of the triangle? **[2023-I]**

(a) $\sqrt{3}+1$ (b) $\sqrt{3}+2$ (c) $\sqrt{3}+3$ (d) $2\sqrt{3}+1$

30. Consider the following statements: **[2023-I]**

1. *ABC* is right angled triangle
2. The angles of the triangle are in AP

Which of the statements given above is/are correct?

(*a*) 1 only (*b*) 2 only
(*c*) Both 1 and 2 (*d*) Neither 1 nor 2

Consider the following for the next two (02) items that follow:

In the triangle ABC, $a^2 + b^2 + c^2 = ac + \sqrt{3}bc$

31. What is the nature of the triangle? **[2023-I]**

(*a*) Equilateral
(*b*) Isosceles
(*c*) Right angled triangle
(*d*) Scalene but not right angled

32. If $c = 8$, what is the area of the triangle? **[2023-I]**

(*a*) $4\sqrt{3}$ (*b*) $6\sqrt{3}$ (*c*) $8\sqrt{3}$ (*d*) $12\sqrt{3}$

Consider the following for the next two (02) items that follow:

In a triangle *PQR*, *P* is the largest angle and $\cos P = \frac{1}{3}$. Further the in-circle of the triangle touches the sides *PQ*, *QR* and *RP* at *N*, *L* and *M* respectively such that the lengths *PN*, *QL* and *RM* are n, $n+2$, $n+4$ respectively where n is an integer.

33. What is the value of *n*? **[2023-I]**

(*a*) 4 (*b*) 6 (*c*) 8 (*d*) 10

34. What is the length of the smallest side? **[2023-I]**

(*a*) 12 (*b*) 14 (*c*) 16 (*d*) 18

Consider the following for the next two (02) items that follow:

The angles *A*, *B* and *C* of a triangle *ABC* are in the ratio 3 : 5 : 4.

35. What is the value of $a + b + \sqrt{2}c$ equal to ? **[2023-II]**

(*a*) 3*a* (*b*) 2*b* (*c*) 3*b* (*d*) 2*c*

36. What is the ratio of $a^2 : b^2 : c^2$? **[2023-II]**

(*a*) $2 : 2+\sqrt{3} : 3$ (*b*) $2 : 2-\sqrt{3} : 2$
(*c*) $2 : 2+\sqrt{3} : 2$ (*d*) $2 : 2-\sqrt{3} : 3$

37. *ABC* is a triangle such that angle $C = 60°$, then what is $\frac{\cos A + \cos B}{\cos\left(\frac{A-B}{2}\right)}$ equal to? **[2024-I]**

(*a*) 2 (*b*) $\sqrt{2}$ (*c*) 1 (*d*) $\frac{1}{\sqrt{2}}$

38. In a triangle *ABC*, *AB* = 16 cm, *BC* = 63 cm and *AC* = 65 cm. What is the value of cos 2*A* + cos 2*B* + cos 2*C*? **[2024-I]**

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) $\frac{76}{65}$

39. Consider the following statements: **[2024-I]**

1. In a triangle *ABC*, if $\cot A \cdot \cot B \cdot \cot C > 0$, then the triangle is an acute angled triangle.
2. In a triangle *ABC*, if $\tan A \cdot \tan B \cdot \tan C > 0$, then the triangle is an obtuse angled triangle.

Which of the statements given above is/are correct?

(*a*) 1 only (*b*) 2 only
(*c*) Both 1 and 2 (*d*) Neither 1 nor 2

ANSWER KEY

1. (*c*)	**2.** (*a*)	**3.** (*a*)	**4.** (*c*)	**5.** (*a*)	**6.** (*b*)	**7.** (*c*)	**8.** (*d*)	**9.** (*b*)	**10.** (*b*)
11. (*b*)	**12.** (*a*)	**13.** (*b*)	**14.** (*b*)	**15.** (*a*)	**16.** (*d*)	**17.** (*c*)	**18.** (*d*)	**19.** (*d*)	**20.** (*c*)
21. (*a*)	**22.** (*b*)	**23.** (*d*)	**24.** (*b*)	**25.** (*c*)	**26.** (*d*)	**27.** (*d*)	**28.** (*a*)	**29.** (*c*)	**30.** (*c*)
31. (*c*)	**32.** (*c*)	**33.** (*c*)	**34.** (*d*)	**35.** (*c*)	**36.** (*a*)	**37.** (*c*)	**38.** (*a*)	**39.** (*a*)	

EXPLANATIONS

1. (*c*)

1. Given, $\sin A + \sin B = \sin C$

$\Rightarrow a + b = c$

$\left(\because \text{By sine law}, \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = K\right)$

Here, the sum of two sides of ΔABC is equal to the third side, but it is not possible

Because by triangle inequality, the sum of the length of two sides of a triangle is always greater than the length of the third side

2. Ratio of angles of a triangle

$A : B : C = 1 : 2 : 3$

$A + B + C = 180°$

$\therefore A = 30°,\ B = 60°$ and $C = 90°$

the ratio in sides according to sine rule

$a : b : c = \sin A : \sin B : \sin C$

$= \sin 30° : \sin 60° : \sin 90°$

$= \frac{1}{2} : \frac{\sqrt{3}}{2} : 1 = 1 : \sqrt{3} : 2$

2. (*a*) Given, $A + B + C = \pi$

$\Rightarrow A + B = \pi - C$

$\Rightarrow \cos(A + B) = \cos(\pi - C)$

$\Rightarrow \cos(A + B) = -\cos C$

$\Rightarrow \cos(A + B) + \cos C = 0$

3. (*a*) According to sine rule,

$$\frac{a}{\sin A}=\frac{c}{\sin C}$$

$$\Rightarrow \sin C=\frac{c.\sin A}{a}=\frac{2.\sin 45^\circ}{2\sqrt{2}}$$

$$=\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}=\frac{1}{2}=\sin 30^\circ$$

$$\therefore C=30^\circ$$

4. (*c*) In a ΔABC, we have

$\sin A-\cos B=\cos C$

$\Rightarrow \sin A=\cos B+\cos C$

$$\Rightarrow 2\sin\frac{A}{2}\cdot\cos\frac{A}{2}=2\cos\left(\frac{B+C}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)$$

$$\Rightarrow 2\sin\frac{A}{2}\cdot\cos\frac{A}{2}=2\cos\left(90^\circ-\frac{A}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)$$

$$\left[\because A+B+C=180^\circ\Rightarrow\left(\frac{B+C}{2}\right)=90^\circ-\frac{A}{2}\right]$$

$$\Rightarrow 2\sin\frac{A}{2}\cdot\cos\frac{A}{2}=2\sin\frac{A}{2}\cdot\cos\left(\frac{B-C}{2}\right)$$

$$\Rightarrow \cos\frac{A}{2}=\cos\left(\frac{B-C}{2}\right)$$

$$\Rightarrow \frac{A}{2}=\frac{B-C}{2}$$

$\Rightarrow A+C=B$ ...(i)

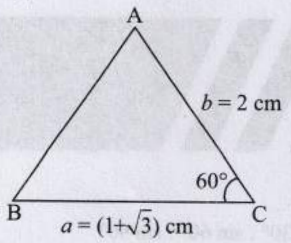
Also, $A+C=180^\circ-B$ ...(ii)

So, $180^\circ-B=B$

$\Rightarrow 2B=180^\circ$

$\therefore B=90^\circ$

5. (*a*)



Now as $a>b$

$\therefore \angle A>\angle B$

Now from Sine Rule,

$$\frac{\sin A}{a}=\frac{\sin B}{b}$$

$$\frac{\sin A}{1+\sqrt{3}}=\frac{\sin B}{2}$$

From option (*a*), $\frac{\sin 75^\circ}{1+\sqrt{3}}=\frac{\sin 45^\circ}{2}$

$$\frac{\sqrt{3}+1}{2\sqrt{2}(1+\sqrt{3})}=\frac{1}{2\sqrt{2}}$$

$\Rightarrow$ L.H.S. = R.H.S.

6. (*b*) Consider any equilateral triangle:

$c=b=a=1$ unit

Take value of p between 1 & 2 *i.e.,* $\frac{3}{2}$ ... (i)

$\therefore a^{1/p}+b^{1/p}-c^{1/p}=(1)^{2/3}+(1)^{2/3}-(1)^{2/3}$

$=1+1-1=1>0$

Take value of p greater than 2 *i.e*; 3.

$\therefore a^{1/p}+b^{1/p}-c^{1/p}=(1)^{1/3}+(1)^{1/3}-(1)^{1/3}$

$=1>0.$

$\therefore$ By considering all the options carefully; we came to a conclusion that opton (*b*) is correct.

7. (*c*) Given,

$$\cos A+\cos B+\cos C=\sqrt{3}\sin\frac{\pi}{3}\quad\text{...(i)}$$

Now, $\cos A+\cos B+\cos C$

$$=2\cos\frac{A+B}{2}\cdot\cos\frac{A-B}{2}+\left(1-2\sin^2\frac{C}{2}\right)$$

$$=2\sin\frac{C}{2}\cos\frac{A-B}{2}+1-2\sin^2\frac{C}{2}$$

$$\left(\because\frac{A+B}{2}=90^\circ-\frac{C}{2}\right)$$

$$=1+2\sin\frac{C}{2}\left[\cos\frac{A-B}{2}-\sin\frac{C}{2}\right]$$

$$=1+2\sin\frac{C}{2}\left[\cos\frac{A-B}{2}-\cos\frac{A+B}{2}\right]$$

$$=1+4\sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}$$

$$\Rightarrow 1+4\sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}=\sqrt{3}\times\frac{\sqrt{3}}{2}$$

[From (i)]

$$\Rightarrow 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\frac{3}{2}-1$$

$$\Rightarrow \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\frac{1}{8}$$

8. (*d*) $\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{C+A}{2}\right)$

$$\Rightarrow \cos\left(90^\circ-\frac{C}{2}\right)\cos\left(90^\circ-\frac{A}{2}\right)\cos\left(90^\circ-\frac{B}{2}\right)$$

$$\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\frac{1}{8}$$

$$\therefore \cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{C+A}{2}\right)=\frac{1}{8}$$

9. (*b*) **Statement-1**

$\because ABC$ is an equilateral triangle.

$\therefore A=B=C=60^\circ$

L.H.S. = $3\tan(A+B)\tan C$

$=3\tan 120^\circ\tan 60^\circ$

$=3(-\sqrt{3})(\sqrt{3})$

$=-9\neq 1$

Hence statement (1) is incorrect.

Statement-2

ABC is a triangle such that $A=78^\circ$ and $B=66^\circ$

$\therefore C=180-(78+66)=180-144=36^\circ$

$$\Rightarrow \frac{A}{2}+C=\frac{78}{2}+36^\circ=75$$

$$\tan\left(\frac{A}{2}+C\right)<\tan A$$

$\Rightarrow \tan 75^\circ<\tan 78^\circ$

Hence statement (2) is correct.

Statement-3

In a triangle ABC

$A+B+C=180^\circ$

$A+B=180^\circ-C$

$$\frac{A+B}{2}=\frac{180^\circ-C}{2}$$

$$\Rightarrow \frac{A+B}{2}=90^\circ-\frac{C}{2}$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right)=\tan\left(90-\frac{C}{2}\right)$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right)=\cot\frac{C}{2}\quad\text{...(1)}$$

$$\therefore \tan\left(\frac{A+B}{2}\right)\cdot\sin\frac{C}{2}$$

$$=\cot\frac{C}{2}\cdot\sin\frac{C}{2}=\cos\frac{C}{2}$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right)\cdot\sin\frac{C}{2}=\cos\frac{C}{2}$$

We can see that statement (3) is not correct.

Hence only 2nd statement is correct.

10. (*b*) In triangle ABC, $A+B+C=\pi$

$$\Rightarrow \frac{A+B+C}{2}=\frac{\pi}{2}$$

$$\Rightarrow \frac{A}{2}+\frac{B}{2}+\frac{C}{2}=\frac{\pi}{2}$$

$\Rightarrow B/2+C/2=\pi/2-A/2$ ...(i)

1. $\sin\left(\frac{B+C}{2}\right)=\sin\left(\frac{\pi}{2}-\frac{A}{2}\right)=\cos\frac{A}{2}$

   So, statement 1 is correct

2. Also, from (i),

   $\tan\left(\frac{B+C}{2}\right)=\tan\left(\frac{\pi}{2}-\frac{A}{2}\right)=\cot\frac{A}{2}$

   So, statement 2 is also correct

3. $B + C = 180° - A°$

$\Rightarrow \sin(B + C) = \sin(180° - A)$

$\Rightarrow \sin(B + C) = \sin A$

So, 3 is not correct.

4. $B + C = 180° - A$

$\Rightarrow \tan(B + C) = \tan(180° - A)$

$\Rightarrow \tan(B + C) = -\tan A$

So, 4 is not correct.

11. (*b*) Given, $a - 2b + c = 0$

$\Rightarrow a + c = 2b$ ... (i)

Now, $\cot\left(\frac{A}{2}\right)\cot\left(\frac{C}{2}\right)$

$= \sqrt{\frac{s(s-a)}{(s-b)(s-c)} \times \frac{s(s-c)}{(s-a)(s-b)}}$

$= \sqrt{\frac{s^2}{(s-b)^2}} = \frac{s}{s-b}$

$= \frac{2s}{2s-2b} = \frac{a+b+c}{a+b+c-2b}$

$= \frac{2b+b}{2b-b} = \frac{3b}{b} = 3.$ [From (i)]

12. (*a*) Given, $\frac{\sin^2 A + \sin^2 B + \sin^2 C}{\cos^2 A + \cos^2 B + \cos^2 C} = 2.$

Let us take $A = 30°$, $B = 60°$, $C = 90°$

$\frac{\sin^2 30° + \sin^2 60° + \sin^2 90°}{\cos^2 30° + \cos^2 60° + \cos^2 90°}$

$\frac{\frac{1}{4} + \frac{3}{4} + 1}{\frac{3}{4} + \frac{1}{4} + 0} = \frac{1+1}{1} = \frac{2}{1} = 2.$

So, the given triangle is right-angled triangle.

13. (*b*) In ΔABC, $a = 2$, $b = 3$ and $\sin A = \frac{2}{3}$.

We know, $\frac{\sin A}{a} = \frac{\sin B}{b}$

$\Rightarrow \frac{\frac{2}{3}}{2} = \frac{\sin B}{3}$

$\Rightarrow \frac{2}{6} = \frac{\sin B}{3} \Rightarrow \sin B = \frac{6}{6} = 1$

$\therefore B = \frac{\pi}{2}.$

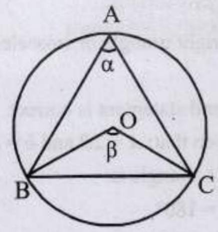
14. (*b*) $x, x - y, x + y$ are angles of a triangle.

Now, $x + x - y + x + y = \pi$ (Sum of angles in triangle $= 180° = \pi$)

$\Rightarrow 3x = \pi$

$\Rightarrow x = \frac{\pi}{3}.$

15. (*a*) $\angle BAC = \alpha$, $\angle BOC = \beta$



We know, from figure,

$\beta = 2\alpha$

$\therefore \cos\beta = \cos 2\alpha$

$= \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$

16. (*d*) $\sin 2A - \sin 2B - \sin 2C$

$= 2\cos\left(\frac{2A+2B}{2}\right)\sin\left(\frac{2A-2B}{2}\right)$

$- \sin 2(180° - A - B)$

$= 2\cos(A + B) \times \sin(A - B) + \sin(2A + 2B)$

$= 2\cos(A + B)\sin(A - B) + 2\sin(A + B)\cos(A + B)$

$= 2\cos(A + B)[\sin(A - B) + \sin(A + B)]$

$= -2\cos C[2\sin A \times \cos B]$

$= -4\sin A\cos B\cos C$

17. (*c*) Given the angles of a triangle ABC are in the ratio 1: 2: 3

Let the angles of triangles be $p°, 2p° 3p°$

$\therefore p° + 2p° + 3p° = 180°$

$\Rightarrow 6p° = 180° \Rightarrow p° = 30°$

Hence the angles of triangles are 30°,60° and 90°.

using sine rule we have

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$

a:b:c: = $k\sin A$: $k\sin B$: $k\sin C$

$\Rightarrow k\sin 30° : k\sin 60° : k\sin 90°$

$\Rightarrow k\left(\frac{1}{2}\right) : k\left(\frac{\sqrt{3}}{2}\right) : k(1) \Rightarrow 1 : \sqrt{3} : 2$

18. (*d*) Let us consider ΔABC with A, B and C as its angles and a, b and c as its sides.

Then, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

It is given that angles are in *AP* i.e., *A*, *B* and *C* are in *AP*.

$\Rightarrow 2B = A + C$

As we know that, $A + B + C = 180°$

$\Rightarrow 3B = 180°$

$\Rightarrow B = 60°$

Now, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \frac{b}{c} = \frac{\sin B}{\sin C}$

It is given that $b : c = \sqrt{3} : \sqrt{2}$

$\Rightarrow \sin 60° = \frac{\sqrt{3}}{\sqrt{2}}\sin C$

$\Rightarrow \frac{\sqrt{3}}{\sqrt{2}} = \frac{\frac{\sqrt{3}}{2}}{\sin C} \Rightarrow \sin C = \frac{1}{\sqrt{2}}$

$\Rightarrow C = 45°$

Now, $A + B + C = 180°$

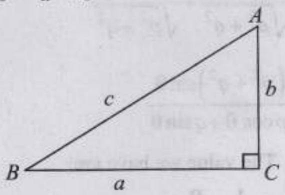
$\Rightarrow A + 60° + 45° = 180°$

$\Rightarrow A = 75°$

19. (*d*) Let $AB = c$, $AC = b$ and $BC = a$

By Pythagoras theorem,

$\Rightarrow c^2 = a^2 + b^2$ ...(i)



$\Rightarrow \tan A = \frac{BC}{AC} = \frac{a}{b}$ ...(ii)

$\Rightarrow \tan B = \frac{AC}{BC} = \frac{b}{a}$ ...(iii)

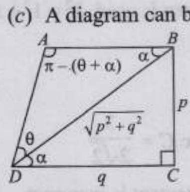
$\Rightarrow \tan A + \tan B = \frac{a}{b} + \frac{b}{a}$

[from (ii) and (iii)]

$\Rightarrow \tan A + \tan B = \frac{a^2 + b^2}{ab}$

$\Rightarrow \tan A + \tan B = \frac{c^2}{ab}$ from (i)

20. (*c*) A diagram can be drawn as:



The sine rule is used in the ΔABD as:

We get: $\frac{\sin\theta}{AB} = \frac{\sin\alpha}{AD}$

$AD\sin\theta = AB\sin\alpha$

So, the first statement is correct.

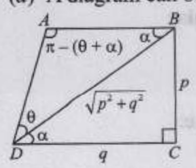
The sine rule is used in the ΔABD as:

We get: $\frac{\sin\theta}{AB} = \frac{\sin(180 - (\alpha + \theta))}{BD}$

$BD\sin\theta = AB\sin(\theta + \alpha)$

So, the second statement is also correct.

21. (*a*) A diagram can be drawn as:



Let us take ΔBCD, then we get:

$\cos\alpha = \dfrac{q}{\sqrt{p^2+q^2}}$ and $\sin\alpha = \dfrac{p}{\sqrt{p^2+q^2}}$

The sine rule is used in the ΔABD as:

$$\frac{AB}{\sin\theta} = \frac{BD}{\sin(\theta+\alpha)}$$

Then, we get:

$$AB = \frac{\sqrt{p^2+q^2}\sin\theta}{\sin\theta\cos\alpha + \cos\theta\sin\alpha}$$

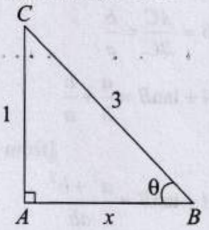
$$= \frac{\sqrt{p^2+q^2}\sin\theta}{\dfrac{\sin\theta \times q}{\sqrt{p^2+q^2}} + \dfrac{\cos\theta\times p}{\sqrt{p^2+q^2}}}$$

$$= \frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$$

22. (*b*) The value we have are:

$\sin B = \dfrac{1}{3} = \dfrac{P}{H}$

A right angle triangle can be drawn as:



The values we have are: $P = 1$, $H = 3$ and $B = x$

Pythagoras Theorem is used: $3^2 = 1^2 + x^2$

$\Rightarrow x^2 = 8$

$\Rightarrow x = 2\sqrt{2}$

The value of cosec $C = \dfrac{3}{2\sqrt{2}}$

Then, the first statement is incorrect.

It is given that: $b\cos B = c\cos C$

The sine rule is applied here and we get:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{K}$$

$b = K\sin B$ and $c = K\sin C$

$2\sin B\cos B = 2\sin C\cos C$

Now, assuming that: $K = 2$

Then, we get: $\sin 2B = \sin 2C$

$\sin 2B - \sin 2C = 0$

$2\cos(B + C)\sin(B - C) = 0$

Firstly, we will take:

$\cos(B + C) = 0$

$(B + C) = 90°$ ...(i)

Then, we will take:

$\sin(B - C) = 0$

$(B - C) = 0$

$B = C$ (ii)

Either Δ is right triangle or isosceles.

$B = C = 45$

So, the second statement is correct.

23. (*d*) It is given that: $A = 2B$ and $b = c$.

The sum of a triangle is:

$A + B + C = 180°$

$2B + B + B = 180°$

$4B = 180°$

$B = 45°$

A triangle will be formed with sides:

$A = 90°$, $B = 45°$ and $C = 45°$

So, the first statement is incorrect.

It is given that in a triangle: $A = 40°$ and $B = 65°$

So, the value of $C = 75°$

We know that: $\dfrac{a}{c} = \dfrac{\sin A}{\sin C} = \dfrac{\sin 40°}{\sin 75°}$

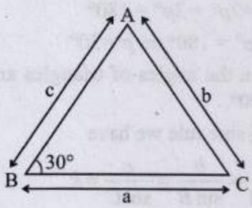
It will become: sin40°cosec75°

So, the second statement is incorrect.

24. (*b*) Let $AB = m$, $AC = n$ and $BC = \sqrt{m^2+n^2+mn}$

Use cosine rule to find cos A

25. (*c*) Given, $a = 10$ cm, $c = 4$ cm, $\angle B = 30°$



$\because$ Area of triangle $= \dfrac{1}{2}ac\sin(\angle B) = 10\,\text{cm}^2$

26. (*d*) $\sin A = \cos B + \cos C$

$$\Rightarrow 2\sin\frac{A}{2}\cos\frac{A}{2}$$

$$= 2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)$$

$$\Rightarrow 2\sin\frac{A}{2}\cos\frac{A}{2}$$

$$= 2\sin\frac{A}{2}\cos\left(\frac{B-C}{2}\right)$$

$$\Rightarrow \cos\frac{A}{2} = \cos\left(\frac{B-C}{2}\right)$$

$$\Rightarrow \frac{A}{2} = \frac{B-C}{2} \Rightarrow A = B - C$$

$$\because A + B + C = \pi \Rightarrow B = \frac{\pi}{2}$$

27. (*d*) $2s = p = a + b + c$ ...(i) &

$q = \sqrt{s(s-a)(s-b)(s-c)}$ ...(ii)

$\therefore p(p - 2a)\tan\left(\dfrac{A}{2}\right)$

$= (a + b + c)(a + b + c - 2a)\tan\left(\dfrac{A}{2}\right)$

$= (a + b + c)(a + b + c - 2a)$

$= \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$

$(a + b + c)(a + b + c - 2a)$

$= \dfrac{\sqrt{s(s-a)(s-b)(s-c)}}{s(s-a)}$

From equation (i) and (ii)

$\Rightarrow 2s \times 2(s-a) \times \dfrac{q}{s(s-a)} = 4q$

28. (*a*) We know, $\cos 3C = 4\cos^3 C - 3\cos C$

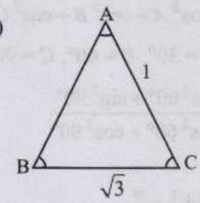
By cosine rule,

$\cos C$

$$= \frac{a^2+b^2-c^2}{2ab} = \frac{4^2+3^2-2^2}{2\times4\times3} = \frac{21}{24}$$

$$\therefore \cos 3C = 4\times\left(\frac{21}{24}\right)^3 - 3\left(\frac{21}{24}\right) = \frac{7}{128}$$

29. (*c*)



Given, $AB + BC + AC$

$$= \frac{\sin A + \sin B + \sin C}{3} \times 6$$

$\Rightarrow AB + BC + AC = 2(\sin A + \sin B + \sin C)$

$\Rightarrow AB + \sqrt{3} + 1 = 2(\sqrt{3}k + k + AB(k))$

(using sine rule)

$\Rightarrow (AB + \sqrt{3} + 1) = 2k(\sqrt{3} + 1 + AB)$

$\Rightarrow k = \dfrac{1}{2}$

$\therefore \sin A = k\sqrt{3} = 60°$

$\sin B = k = 30°$

Hence, $\angle C = 180° - 90° = 90°$

$\sin C = k\,AB \Rightarrow AB = 2$

$\therefore$ Perimeter $= 2 + \sqrt{3} + 1 = 3 + \sqrt{3}$

30. (*c*) Angle *A*, *B* and *C* are 60°, 30° and 90°. Hence, *ABC* is right angled triangle. Also, the angles of a triangle are in *AP*.

Therefore, both statements are correct.

31. (*c*) $a^2 = b^2 + c^2 - 2bc\cos A$ ...(i)

$b^2 = a^2 + c^2 - 2ac\cos B$ ...(ii)

$c^2 = b^2 + a^2 - 2ba\cos C$ ...(iii)

On adding these 3 equations, we get

$2bc \cos A + 2ac \cos B + 2ba \cos C = a^2 + b^2 + c^2$ ...(iv)

On comparing the given equation with equation (iv), we get

$\cos C = 0 \Rightarrow \angle C = 90°$

$\cos A = \frac{\sqrt{3}}{2} \Rightarrow \angle A = 30°$

and $\angle B = 60°$

Hence, the given Δ is right angled Δ.

32. (*c*) $\frac{a}{\sin A} = \frac{c}{\sin C}$

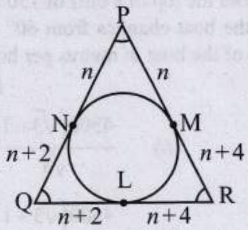
$\Rightarrow a = \frac{8}{1} \times \frac{1}{2} = 4$

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow b = \frac{8}{1} \times \frac{\sqrt{3}}{2}$

$\text{Area of } \Delta = \frac{1}{2}ab = \frac{1}{2} \times 4 \times 8 \times \frac{\sqrt{3}}{2} = 8\sqrt{3}$

33. (*c*)



$\because \cos P = \frac{1}{3}$ [Given]

$\Rightarrow \frac{(2n+4)^2 + (2n+2)^2 - (2n+6)^2}{2 \times (2n+4)(2n+2)} = \frac{1}{3}$

$\Rightarrow n = 8$

34. (*d*) Length of the smallest side $= PQ = 2n + 2$

$= 2 \times 8 + 2 = 18$

35. (*c*) If the angles *A*, *B* and *C* of a triangle *ABC* are in the ratio 3 : 5 : 4.

$\therefore 3x + 5x + 4x = 180°$

$\Rightarrow x = 15°$

$\therefore A = 45°, B = 75° \text{ and } C = 60°$

$\frac{a}{\sin 45°} = \frac{b}{\sin 75°} = \frac{c}{\sin 60°} = k$

$a = \frac{k}{\sqrt{2}}, b = \frac{k(\sqrt{3}+1)}{2\sqrt{2}}, c = \frac{k\sqrt{3}}{2}$

$a + b + \sqrt{2}c = \frac{k}{\sqrt{2}} + \frac{(\sqrt{3}+1)k}{2\sqrt{2}} + \frac{k\sqrt{3}}{\sqrt{2}}$

$= \frac{2k + (\sqrt{3}+1)k + 2k\sqrt{3}}{2\sqrt{2}}$

$= \frac{3k(\sqrt{3}+1)}{2\sqrt{2}} = 3b$

36. (*a*) $a = \frac{k}{\sqrt{2}}, b = \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)k, c = \frac{\sqrt{3}k}{2}$

$a^2 = \frac{k^2}{2}, b^2 = \left(\frac{4+2\sqrt{3}}{8}\right)k^2, c^2 = \frac{3}{4}k^2$

$a^2 = \frac{4}{8}, b^2 = \frac{4+2\sqrt{3}}{8}, c^2 = \frac{6}{8}$

$4 : 4 + 2\sqrt{3} : 6$

$2 : 2 + \sqrt{3} : 3$

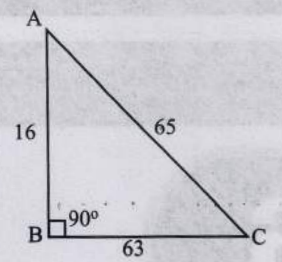
37. (*c*) Given $C = 60°$. Let us asume *ABC* be an equilateral triangle

$\therefore A = B = C = 60°$

$\therefore \frac{\cos A + \cos B}{\cos\left(\frac{A-B}{2}\right)} = \frac{\frac{1}{2} + \frac{1}{2}}{1} = \frac{1}{1} = 1$

38. (*a*) We know that in a right angled triangle

$\cos 2A + \cos 2B + \cos 2C = -1$

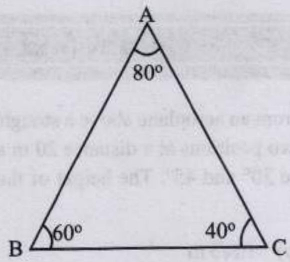


39. (*a*) Let ABC be any acute angled triangle as follow:

Statement 1 is correct as in first quadrant

$\cot 80° \cot 60° \cot 40° > 0$

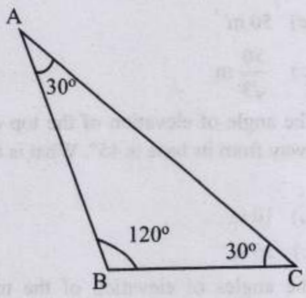
(+) (+) (+)



Also let's take obtuse angled triangle

$\therefore \tan 30°. \tan 120°. \tan 30°.$

$\Rightarrow \frac{1}{\sqrt{3}} \times (-\sqrt{3}) \times \frac{1}{\sqrt{3}} \Rightarrow -\frac{1}{\sqrt{3}} < 0$



Hence statement II is incorrect