1. LINEAR EQUATION IN TWO VARIABLES

■ Linear equation

- **Equation :** A statement of equality which involves in number (s) is called an equation.

  $4x = 12$, $4 + x = 10$, $7 - 2x = 5$ etc.

  The literal numbers involved in each equation are called its variables (unknowns). Usually the variables are denoted by letters towards the end of English alphabet, e.g., x,y,z,u,v,w, etc.

- **Linear equation :** An equation in which the highest value of the variables involved is one, is called a linear equation. eg., $x + y = 10$, $7x = 21$, $\frac{x}{3} = 8$

- **Linear equation in one variables :** An equation only one variable (literal) with highest power 1 is called a linear equation in one variable. e.g., $17x = 51$, $17x - 30$, $5y = 30$, etc.

- **Linear equation in two variables :** An equation of the form $ax + by = c$, where a, b, c are real numbers is called a linear equation in two variables x and y.

  - The graph of a linear equation $ax + by = c$ is a straight line.

    $3x + 2y = 18$, is an example of a linear equation in two tables.

  - The value of the variables that satisfy the equation is called the solution (or solution set) of the equation.

- **Important points about linear equation in two variables**

  (i) The graph of an equation of the type $x = k$ (where k is a constant) is a straight line parallel to the y-axis at a distance of k units from the y-axis.

  (ii) The graph of an equation of the type $y = k$ (where k is a constant) is a straight line parallel to the x-axis at a distance of k units from the x-axis.

  (iii) The points of intersection of the two lines gives the solution of the two equations.

  (iv) A single linear equation in two variables has infinite no. of solutions.

■ Pair of linear equations in two variables

Two or more linear equations in two variables form a system of linear simultaneous equations.

e.g., $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$.

where a_1, a_2, b_1, b_2, c_1, c_2 are real numbers. Such that $a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$

Clearly a pair of linear equations in two variables is said to be form a system of simultaneous linear equations.

Ex. $x + y - 3 = 0$ and $2x + 3y - 7 = 0$

- **Consistent system :** A system consisting of two simultaneous linear equation is said to be consistent, if it has at least one solution.

- **Inconsistent system :** A system consisting of two simultaneous linear equations is said to be inconsistent, if it has no solution.

Conditions for simultaneous equations

Pair of linear equations $a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	Algebraic conditions	Graphical representation	Algebraic interpretation
Consistent (Independent)	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution (unique solution)
Consistent (Dependent)	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions
In-consistent	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Pair of parallel lines	No solution

- The homogeneous system has a non-zero solution only when $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ and in this case, the system has an infinite number of solution.
- A system of equations has unique solution, when only one variable satisfies the equation.
- For a system of equations a unique solution is possible only when the number of variables is equal to or less than number of independent and consistent equations.

  e.g., $2x + 3y = 5$ and $7x + 5y = 20$

  and $2x + 3y = 5$, $7x + 5y = 12$, $5x + 8y = 13$
- The equation of the type $ax + by = c$ and $kax + kby = kc$ are known as dependent equations.

  e.g., $2x + y = 11$ ..... (i)

  $6x + 3y = 33$ ..... (ii)

  Here equation (ii) is the multiple of equation (i). Thus, there are basically both the equations same, i.e., equation (ii) is dependent on equation (i). In this case there are infinite number of solutions.
- The equation of the type $ax + by = c$ and $kax + kby = \ell c$ are known as inconsistent equations.

  e.g., $3x + 2y = 8$ ..... (i)

  $6x + 4y = 6$ ..... (ii)

  Properties of graphs of $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$

  (i) intersecting, if the system has a unique solution.

  (ii) coincident, if the system has infinte number of solution.

  (iii) parallel, if the system has no solution.
- **Graphical representation of simultaneous equations :**

  (i) intersecting lines (unique solution)

  e.g., $x + y = 4$ ..... (i)

  and $3x + 2y = 11$ ..... (ii)

  $\therefore \quad 3(x + y) = 3 \times 4 \Rightarrow 3x + 3y = 12$

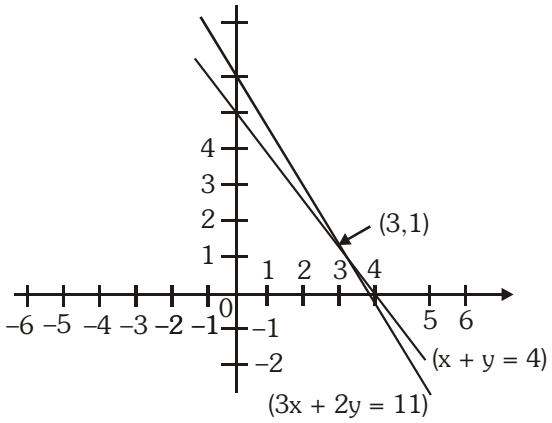
  $\therefore$

  $$\begin{array}{r} 3x + 3y = 12 \\ 3x + 2y = 11 \\ - \quad - \quad - \\ \hline y = 1 \end{array}$$

  $\therefore \quad x + y = 4 \Rightarrow x + 1 = 4 \Rightarrow x = 3$

  $\therefore \quad x = 3$ and $y = 1$

Thus two given lines intersect at x = 3, y = 1.

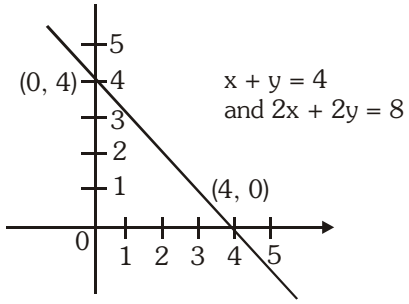


(ii) Coincident lines (infinite number of solutions)

e.g., $x + y = 4$ ..... (i)

$2x + 2y = 8$ ..... (ii)

The two equations are dependent, therefore the graph of two lines will coincide.

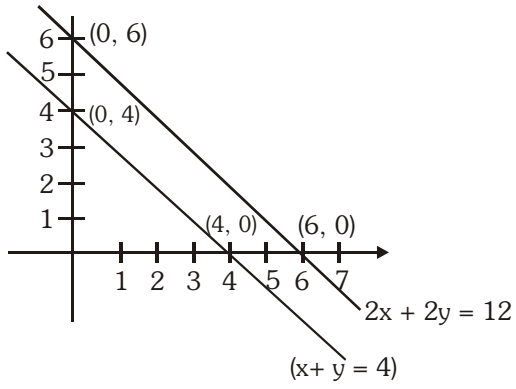


(iii) Parallel lines (no solution)

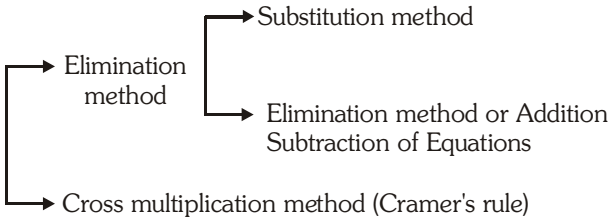
e.g., $x + y = 4$

$2x + 2y = 12$

Since the system of equations is inconsistent then there is no any solution and in this case we obtain two distinct parallel lines.



■ Algebraic methods of solving simultaneous equations in two variables



- **Substitution method**

To solve a pair of linear equations in two variables x and y by substitution method, we follow the following steps:

$a_1x + b_1y + c_1 = 0$ ...(1)

and $a_2x + b_2y + c_2 = 0$ ...(2)

Step-1: Choose one of the two equations and express y in terms of x (or x in terms of y), i.e., express, one variable in terms of the other.

Step-2 : Substitute this value of y obtained in step-I, in the other equation to get a linear equation in x.

Step-3 : Solve the linear equation obtained in step-II and get the value of x.

Step-4 **:** Substitute this value of x in the relation obtained in step-I and find the value of y.

Ex. Solve the following pair of linear equations by the substitution method.

$\sqrt{2}x + \sqrt{3}y = 0$ and $\sqrt{3}x - \sqrt{8}y = 0$

Sol. We have,

$\sqrt{2}x + \sqrt{3}y = 0$ ...(1) and $\sqrt{3}x - \sqrt{8}y = 0$ ...(2)

From (1), we get $y = \dfrac{-\sqrt{2}x}{\sqrt{3}}$ ...(3)

Substituting $y = \dfrac{-\sqrt{2}x}{\sqrt{3}}$ in (2), we get

$$\sqrt{3}x - \sqrt{8}\left(\frac{-\sqrt{2}x}{\sqrt{3}}\right) = 0$$

$$\Rightarrow \sqrt{3}x + \frac{4x}{\sqrt{3}} = 0 \Rightarrow 3x + 4x = 0 \Rightarrow 7x = 0$$

$$\Rightarrow x = 0$$

Substituting x = 0 in (3), we get $y = \dfrac{-\sqrt{2}\times 0}{\sqrt{3}} = 0$

Hence, the solution is x = 0 and y = 0.

- **Elimination method**

To solve a pair of linear equations in two variables x and y by elimination method, we follow the following steps:

$a_1x + b_1y + c_1 = 0$ ...(1)

and $a_2x + b_2y + c_2 = 0$ ...(2)

Step-1 : Multiply the given equations by suitable numbers so that the coefficient of one of the variables are numerically equal.

Step-2 : If the numerically equal coefficients are opposite in sign, then add the new equations otherwise subtract.

Step-3 : Solve the linear equations in one variable obtained in step-II and get the value of one variable.

Step-4 : Substitute this value of the variable obtained in step-III in any of the two equations and find the value of the other variable.

Ex. Solve the following pair of equation by the elimination method.

$x + y = 5$ and $2x - 3y = 4$

Sol. Solution By Elimination Method:

$x + y = 5$ ...(i)

$2x - 3y = 4$ ...(ii)

Multiplying (i) by 3 and (ii) by 1 and adding we get $3(x + y) + 1(2x - 3y) = 3 \times 5 + 1 \times 4$

$\Rightarrow \quad 3x + 3y + 2x - 3y = 19$

$\Rightarrow \quad 5x = 19 \Rightarrow x = \frac{19}{5}$

From (i), substituting $x = \frac{19}{5}$, we get

$\frac{19}{5} + y = 5 \Rightarrow y = 5 - \frac{19}{5} \Rightarrow y = \frac{6}{5}$

Hence, $x = \frac{19}{5}, y = \frac{6}{5}$

- **Cross-multiplication method**

Consider the system of linear equations

$a_1x + b_1y + c_1 = 0$ ...(1) ; $a_2x + b_2y + c_2 = 0$ ...(2)

To solve it by cross multiplication method, we follow the following steps :

Step-1 **:** Write the coefficients as follows :

$$\frac{x}{b_1 \quad c_1} = \frac{y}{c_1 \quad a_1} = \frac{1}{a_1 \quad b_1} \quad \text{or} \quad \overbrace{\begin{matrix} b_1 & c_1 \\ b_2 & c_2 \end{matrix}}^{x} \quad \overbrace{\begin{matrix} c_1 & a_1 \\ c_2 & a_2 \end{matrix}}^{y} \quad \overbrace{\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \end{matrix}}^{1}$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$\Rightarrow \quad x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

Case-1 **:** If $a_1b_2 - a_2b_1 \neq 0 \Rightarrow$ x and y have some finite values, with unique solution for the system of equations.

Case-2 **:** If $a_1b_2 - a_2b_1 = 0 \quad \Rightarrow \quad \frac{a_1}{a_2} = \frac{b_1}{b_2}$

Here two cases arise :

(a) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \lambda$ $(\lambda \neq 0)$. Then $a_1 = a_2\lambda$, $b_1 = b_2\lambda$, $c_1 = c_2\lambda$

Put these values in equation $a_1x + b_1y + c_1 = 0$ ...(i)

$\Rightarrow \quad a_2\lambda x + b_2\lambda y + c_2\lambda = 0 \quad \Rightarrow \quad \lambda(a_2x + b_2y + c_2) = 0$ but $\lambda \neq 0$

$\Rightarrow \quad a_2x + b_2y + c_2 = 0$ ...(ii)

So (i) and (ii) are dependent, so there are infinite number of solutions.

(b) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow a_1b_2 - b_1a_2 = 0$

But $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow$ $x = \frac{\text{Finite value}}{0} = \text{does not exist and } y = \frac{\text{Finite value}}{0} = \text{does not exist}$

So system of equations is inconsistent.

Ex. Solve $3x + 2y = -25, -2x - y = 10$.

Sol. The two equations are

$3x + 2y = -25$

$-2x - y = 10$.

The solution is given by

$$\frac{x}{2\times10-(-1)\times(-25)} = \frac{y}{(-25)\times(-2)-3\times10} = \frac{-1}{3\times(-1)-(-2)\times2}$$

or $$\frac{x}{-5} = \frac{y}{20} = \frac{-1}{1}$$

or $$x = 5, y = -20.$$

1. 2 men and 3 boys together can do a piece of work in 8 days. The same work is done in 6 days by 3 men and 2 boys together. How long would 1 boy alone and 1 man alone take to complete the work independently.

(1) 40 days, 120 days (2) 120 days, 40 days
(3) 20 days, 120 days (4) 120 days, 20 days

2. A sailor goes 12 km downstream in 2 hours and return to the starting point in 3 hours. Find the speed of the sailor in still water and the speed of the current.

(1) 5 km/hr, 1 km/hr (2) 1 km/hr, 5 km/hr
(3) 4 km/hr, 1 km/hr (4) 3 km/hr, 1 km/hr

3. 5 pens and 6 pencils together cost Rs. 9 and 3 pens and 2 pencils cost Rs. 5. Find the cost of 1 pen and 1 pencil.

(1) Rs. 2, Rs. 0.50 (2) Rs.1.50, Rs.0.25
(3) Rs.1.50, Rs.0.50 (4) Rs.2, Rs.0.25

4. A half-ticket issued by railway costs half the full fare but the reservation charge is the same on half ticket as on full-ticket. One full reserved first class ticket for a journey between two stations costs Rs. 362 and one full and one half reserved first class ticket costs Rs. 554. The reservation charge is.

(1) Rs.18 (2) Rs.22 (3) Rs.38 (4) None

5. A test has 50 questions. A student scores 1 mark for a correct answer, – 1/3 for a wrong answer and – 1/6 for not attempting a question. If the net score of a student is 32, the number of questions answered wrongly by that student cannot be less than

(1) 6 (2) 12 (3) 3 (4) 9

6. A three digits number abc is 459 more than the sum of its digits. What is the sum of the 2 digit number ab and the 1-digit number a ?

(1) 71 (2) 61
(3) 51 (4) Cannot be determine

7. For what value of 'k' will the equations $x + 2y + 7 = 0$ and $2x + ky + 14 = 0$ represent coincident lines ?

(1) 2 (2) 4 (3) 6 (4) 3

8. If $2x + 3y = 7$ and $(p + q)x + 2(p - q)y = 21$, then the values of p and q are (given that the system has infinite solutions).

(1) $p = 10, q = 3$ (2) $p = 5, q = 2$

(3) $p = \frac{21}{4}, q = \frac{3}{4}$ (4) $p = 10, q = 2$

9. If $(3k+1)x + 3y - 2 = 0$
and $(k^2+1)x + (k - 2)y - 5 = 0$
then the value of 'k' for the given equations having no solution is

(1) 1 (2) 0
(3) –2 (4) – 1

10. In a fraction if the numerator is multiplied by 3 and the denominator is reduced by 3, is equal to $\frac{18}{11}$, but if the numerator is increased by 8 and the denominator is doubled then it is equal to $\frac{2}{5}$. So the fraction is

(1) $\frac{12}{25}$ (2) $\frac{13}{12}$ (3) $\frac{8}{5}$ (4) $\frac{7}{19}$

11. The length of the sides of a triangle are $3x + 2y$, $4x + \frac{4}{3}y$ and $3(x + 1) + \frac{3}{2}(y - 1)$. If the triangle is equilateral, then its side is

(1) 8 (2) 10
(3) 12 (4) 16

12. The solution of the equations :

$\frac{x}{4} = \frac{y}{3} = \frac{z}{2}$, $7x + 8y + 5z = 62$ is :

(1) (4,3,2) (2) (2,3,4)
(3) (3,4,2) (4) (4,2,3)

13. If (p,p) is the solution of system of equations $ax + by + (t - s) = 0$ and $bx + ay + (s - r) = 0$, $(a \neq b)$, then which of the following must be true ?

(1) $2r = s + t$ (2) $2t = r + s$
(3) $2s = r + t$ (4) $r + s + t = 0$

14. If $173x + 197y = 149$ and $197x + 173y = 221$, then find (x,y).

(1) (3, – 2) (2) (2,1)
(3) (1, – 2) (4) (2,–1)

15. Mallesh has some cows and some hens in his shed. The total number of legs is 92 and the total number of heads is 29. Find the number of cows in his shed.

(1) 12 (2) 14
(3) 17 (4) 19

16. A mother said to her son, "the sum of our present ages is twice my age 12 years ago and nine years hence, the sum of our ages will be thrice my age 14 years ago". What is her son's present age ? (in years)

(1) 8 (2) 12
(3) 15 (4) 10

17. A told B, "when I was as old as you are now, then your age was four years less than half of my present age". If the sum of the present ages of A and B is 61 years, what is B's present age? (in years).

(1) 9 (2) 25 (3) 43 (4) 36

18. Dheeraj has twice as many sisters as he has brothers. If Deepa, Dheeraj's sister has the same number of brothers as she has sisters, then Deepa has how many brothers ?

(1) 2

(2) 3

(3) 4

(4) Cannot be determined

19. Swati starts her job with certain monthly salary and earns a fixed increment every year. If her salary was Rs. 22500 per month after 6 years of service and Rs.30000 per month after 11 years of service. find her salary after 8 years of service (in Rs.).

(1) 24000 (2) 25500

(3) 26000 (4) 24500

20. If the demand for fertilizer product is given by $p + 5q = 21$ and the supply is determined by $p - 2q = 7$, where p and q denote the price of the commodity and q is the number of units of fertilizer product supplied. If a man wants to buy q units of fertilizer product, then the amount paid by him is

(1) Rs.22 (2) Rs.30 (3) Rs.32 (4) Rs.24

21. The quantity of fat in a kilogram of food A plus the quantity of protein in a kilogram of food A is 100 g. The quantity of protein in a kilogram of food A minus twice the quantity of fat in a kilogram of food A is 10 g. How many grams of protein are there in a kilogram of food A ?

(1) 30 (2) 45 (3) 50 (4) 70

22. Ram and Mohan are friends. Each has some money. If Ram gives Rs.30 to Mohan, then Mohan will have twice the money left with Ram. But, If Mohan gives Rs.10 to Ram, then Ram will have thrice as much as is left with Mohan. How much money does each have ?

(1) Rs.62, Rs.34 (2) Rs.6, Rs.2

(3) Rs.170, Rs.124 (4) Rs.43, Rs.26

23. In a examination, a student attempted 15 questions correctly and secured 40 marks. If there were two types of questions (2 marks and 4 marks question), how many questions of 2 marks did he attempt correctly ?

(1) 5 (2) 10 (3) 20 (4) 40

24. In a zoo, there are rabbits and pigeons. If their heads are counted, there are 90 while their legs are 224. Find the number of pigeons in the zoo.

(1) 70 (2) 68

(3) 72 (4) 22

25. Shyam had 85 currency notes in all, some of which were of Rs.100 denomination and the remaining of Rs.50 denomination. The total amount of all these currency notes was Rs.5000. How much amount in rupees did he have in the denomination of Rs.50 ?

(1) 3500 (2) 70

(3) 15 (4) 1500

26. A florist was asked to make a bouquet with exactly Rs.1000 with 100 sticks of roses of three colours-Pink, Yellow and Red. While Pink roses cost Rs. 0.50 per stick, Red roses cost Rs.10.00 per stick and Yellow roses cost Rs. 50.00 per stick. How many Red roses did the florist use in the bouquet ?

(1) 1

(2) 5

(3) 80

(4) Several combinations are possible

27. A student was asked to divide a number by 17/8. Instead, he actually multiplied it by 17/8 and hence got 225 more than the expected answer. What was the expected answer ?

(1) 126 (2) 136

(3) 64 (4) None of these

28. A woman sells to the first customer half her stock and half an apple, to the second customer she sells half her remaining stock and half an apple, and so on to the third, and to a fourth customer. She finds that she has now 15 apples left. How many apples did she have before she started selling ?

(1) 63 (2) 127

(3) 240 (4) None of these

29. The equations $3x - 4y = 5$ and $12x - 16y = 20$ have

(1) No common solution

(2) Exactly one common solution

(3) Exactly two common solutions

(4) More than two common solutions

30. To a proper fraction, when six is added to the numerator and the denominator is increased by it's 50%, the ratio becomes $\frac{1}{2}$ and when the numerator is multiplied by 4 and denominator is reduced by 8, then the fraction becomes 3. The fraction (simplified) is

(1) $\frac{1}{3}$

(2) $\frac{15}{28}$

(3) $\frac{33}{52}$

(4) Cannot be determined

31. At the first stop on his route, a driver unloaded 2/5 of the packages in his van. After be unloaded another three pakages at his next stop, 1/2 of the original number of packages remained. How many packages were in the van before the first delivery ?

(1) 25 (2) 10

(3) 30 (4) 36

32. The pair of equations $3^{x+y} = 81$; $81^{x-y} = 3$ has

(1) no solution

(2) the solution $x = 2\frac{1}{2}, y = 1\frac{7}{8}$

(3) the solution x = 2, y = 2

(4) the solution $x = 2\frac{1}{8}, y = 1\frac{7}{8}$

33. Find the value of x and y, if $\frac{5x}{8} + \frac{7y}{18} = 6$ and $2(x - y) = -10$.

(1) 4,9 (2) 5,7 (3) 3,12 (4) 10,4

34. If the numerator and the denominator of a fraction are each increased by 4, the fraction becomes 2 and when numerator and denominator of the same fraction are each decreased by 6, the fraction becomes 12. The sum of the numerator and the denominator is

(1) 11 (2) –11 (3) 25 (4) –25

35. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Sanchit paid Rs.45 for a book kept for 7 days, while Karan paid Rs.25 for the book he kept for 5 days. The fixed charge and the charge for each extra day is

(1) Rs.5 and Rs.10

(2) Rs.10 and Rs.5

(3) Rs.15 and Rs.5

(4) Rs.5 and Rs.15

36. How can the relationship between x and y be best defined, if values of x and y are as follows ?

x	2	3	4	5	6
y	0	2	6	12	20

(1) $y = 2x - 4$ (2) $y = x^2 - 3x + 2$

(3) $y = x^2 - 4x$ (4) $y = x^2 - 4$

37. There were 35 students in a hostel. If the number of students increases by 7, the expenses of the mess increase by Rs.42 per day while the average expenditure per head diminished by Rs 1. Find the original expenditure of the mess.

(1) Rs.480 (2) Rs.520

(3) Rs.420 (4) Rs.460

38. In a ΔABC, $\angle A = x°$, $\angle B = y°$ and $\angle C = (y + 20)°$. If $4x - y = 10$, then the triangle is

(1) right angled (2) obstuse angled

(3) equilateral (4) None of these

39. The sum of the two digits of a number is 15. If 9 be added to the number, then the digits are reversed. The number is

(1) 96 (2) 87 (3) 78 (4) 69

40. There are two examination rooms A and B. If 10 candiates are sent from room A to room B, the number of candiates in each room is the same, while if 20 are sent from room B to room A, the number in room A becomes double the number in room B. The number of candidates in each room are respectively

(1) 80 and 100 (2) 100 and 80

(3) 80 and 120 (4) 100 and 60

41. Consider the system of linear equations

$2x + 3y + 4z = 16$

$4x + 4y + 5z = 26$

$ax + by + cz = r$

For r = 5 and a = 1 the system of linear equation will have infinite number of solutions, if c =

(1) 3/2 (2) 1

(3) 1/2 (4) 0

42. In an examination there are 30 questions. 1 mark is given for each correct answer and 0.25 is deducted for every incorrect answer. Ankur attempted all the questions and scored 13.75. How many incorrect answers did he have ?

(1) 10

(2) 11

(3) 12

(4) None of these

43. Which one of the following conditions must a, b and c satisfy so that the following system of linear simultaneous equations has at least one solution, such that

$a + b + c \neq 0$

$m + 2n - 3r = a$

$2m + 6n - 11\ r = b$

$m - 2n + 7r = c$

(1) $5a + 2b + c = 0$ (2) $5a + 2b - c = 0$

(3) $5a - 2b - c = 0$ (4) $5a - 2b + c = 0$

44. If $2x + 3y = 78$ and $3x + 2y = 72$, what is the value of $x + y$?

(1) 36

(2) 32

(3) 30

(4) Cannot be determined

45. If $3Y + 9X = 54$ and $\frac{28X}{13Y} = \frac{140}{39}$ then what is the value of $Y - X$?

(1) –1 (2) –2 (3) 2 (4) 1

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	4	1	2	2	3	3	2	3	4	1	3	1	3	4	3	2	2	2	2	
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	4	1	2	2	1	1	2	4	4	4	3	4	1	3	1	2	3	1	3	
Que.	41	42	43	44	45															
Ans.	3	4	3	3	2															