Polynomials

Exercise 3.1

Q. 1. If $p(x) = 5x^7 – 6x^5 + 7x - 6$, find

(i) coefficient of x^5
(ii) degree of p(x)
(iii) constant term.

Answer : (i) A coefficient is a multiplicative factor in some term of a polynomial. It is the constant (-6 here) written before the variable (x^5 here).

∴ The coefficient of x^5 is -6.

(ii) Degree of p(x) is the highest power of x in p(x).

∴ The degree of p(x) is 7.

(iii) The term that is not attached to a variable (i.e., x) is -6.

∴ The constant term is -6.

Q. 2. State which of the following statements are true and which are false? Give reasons for your choice.

(i) The degree of the polynomial $\sqrt{2}x^2 - 3x + 1$ is $\sqrt{2}$.
(ii) The coefficient of x2 in the polynomial $p(x) = 3x^3 – 4x^2 + 5x + 7$ is 2.
(iii) The degree of a constant term is zero.
(iv) $\frac{1}{x^2 - 5x + 6}$ is a quadratic polynomial.
(v) The degree of a polynomial is one more than the number of terms in it.

Answer : (i) False

Since, Degree of a polynomial is the highest power of x in the polynomial which is 2 in $\sqrt{2}x^2 - 3x + 1$.

(ii) False

A coefficient is a multiplicative factor in some term of a polynomial. It is the constant written before the variable.

∴ The coefficient of x^2 is -4.

(iii) True

Since, the power of x of a constant term is 0.

∴ the degree of a constant term is 0.

(iv) False

For an expression to be a polynomial term, any variables in the expression must have whole-number powers (i.e., x^0, x^1, x^2,……)

Since, the power in the expression $\frac{1}{x^2-5x+6}$ has powers -2 and -1 which are not whole numbers, therefore, $\frac{1}{x^2-5x+6}$ is not a polynomial.

(v) False

Degree of a polynomial is the highest power of x in the polynomial. The degree of polynomial is not related to the number of terms in the polynomial.

Therefore, the statement is false.

Q. 3. If p(t) t^3 – 1, find the values of p(1),p(-1),P(0),p(2),p(-2).

Answer : $P(t) = t^3 – 1$

Therefore,

$P(1) = 1^3 – 1$

$\Rightarrow P(1) = 1 – 1$

$\Rightarrow P(1) = 0$

$P(-1) = (-1)^3 – 1$

$\Rightarrow P(-1) = -1 – 1$

$\Rightarrow P(-1) = -2$

$P(0) = 0^3 – 1$

⇒ $P(0) = 0 – 1$

⇒ $P(0) = -1$

$P(2) = 2^3 – 1$

⇒ $P(2) = 8 – 1$

⇒ $P(2) = 7$

$P(-2) = (-2)^3 – 1$

⇒ $P(2) = -8 – 1$

⇒ $P(2) = -9$

Q. 4. Check whether -2 and 2 are the zeroes of the polynomial x^4 - 16

Answer : A zero or root of a polynomial function is a number that, when plugged in for the variable, makes the function equal to zero.

Now,

$P(x) = x^4 – 16$

Therefore,

$P(-2) = (-2)^4 – 16$

⇒ $P(-2) = 16 – 16$

⇒ $P(-2) = 0$

$P(2) = 2^4 – 16$

⇒ $P(2) = 16 – 16$

⇒ $P(2) = 0$

Hence, Yes, -2 and -2 are zeroes of the polynomial $x^4 – 16$.

Q. 5. Check whether 3 and -2 are the zeroes of the polynomial when $p(x) = x^2 – x - 6$

Answer : A zero or root of a polynomial function is a number that, when plugged in for the variable, makes the function equal to zero.

Now,

$P(x) = x^2 – x – 6$

Therefore,

$P(-2) = (-2)^2 – (-2) – 6$

$\Rightarrow P(-2) = 4 + 2 – 6$

$\Rightarrow P(-2) = 0$

$P(3) = 3^2 – 3 – 6$

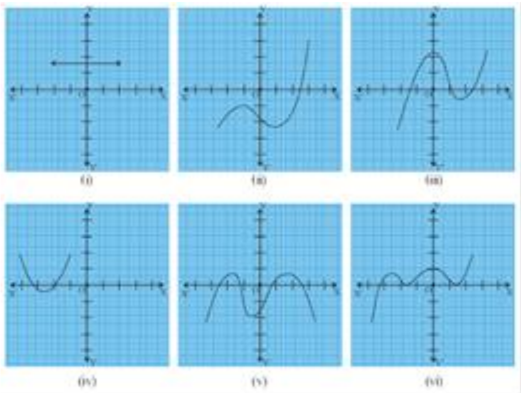
$\Rightarrow P(3) = 9 – 3 – 6$

-$\Rightarrow P(3) = 0$

Hence, Yes, 3 and -2 are zeroes of the polynomial $x^2 – x – 6$.

Exercise 3.2

Q. 1. The graphs of y = p(x) are given in the figure below, for some polynomials p(x) In each case, find the number of zeroes of p(x)



Answer : (i) Since, the graph does not intersect with x-axis at any point therefore, it has no zeroes.

(ii) Since, the graph intersects with x-axis at only one point therefore, it has 1 number of zeroes.

(iii) Since, the graph intersects with x-axis three points therefore, it has 3 number of zeroes.

(iv) Since, the graph intersects with x-axis two points therefore, it has 2 number of zeroes.

(v) Since, the graph intersects with x-axis at four points therefore, it has 4 number of zeroes.

(vi) Since, the graph intersects with x-axis at three points therefore, it has 3 number of zeroes.

Q. 2 A. Find the zeroes of the given polynomials.

p(x) = 3x

Answer : A zero or root of a polynomial function is a number that, when plugged in for the variable, makes the function equal to zero.

Therefore, to find zeroes put p(x)=0.

$p(x) = 0$

$\Rightarrow 3x = 0$

$\Rightarrow x = 0$

Hence, x=0 is the zero of the polynomial.

Q. 2 B. Find the zeroes of the given polynomials.

$p(x) = x^2 + 5x + 6$

Answer : $p(x) = 0$

$\Rightarrow x^2 + 5x + 6 = 0$

$\Rightarrow x^2 + 3x + 2x + 6 = 0$

$\Rightarrow x(x + 3) + 2(x + 3) = 0$

$\Rightarrow (x + 3)(x + 2) = 0$

$\Rightarrow x = -3$ and $x = -2$

Hence, x=-3 and x=-2 are the zeroes of the polynomial.

Q. 2 C. Find the zeroes of the given polynomials.

p(x)- (x+2)(x+3)

Answer :

p(x) = 0

⇒ (x + 3)(x + 2)

⇒ x = -3 and x = -2

Hence, x=-3 and x=-2 are the zeroes of the polynomial.

Q. 2 D. Find the zeroes of the given polynomials.

p(x)=x^4 - 16

Answer : p(x) = 0

⇒ $x^4 – 16 = 0$

⇒ $(x^2 – 4)(x^2 + 4)= 0$

⇒ $(x + 2)(x – 2)(x^2 + 4)= 0$

⇒ x = -2, x = 2 and $x^2 = 4$

⇒ x = -2, x = 2 and x = ±2

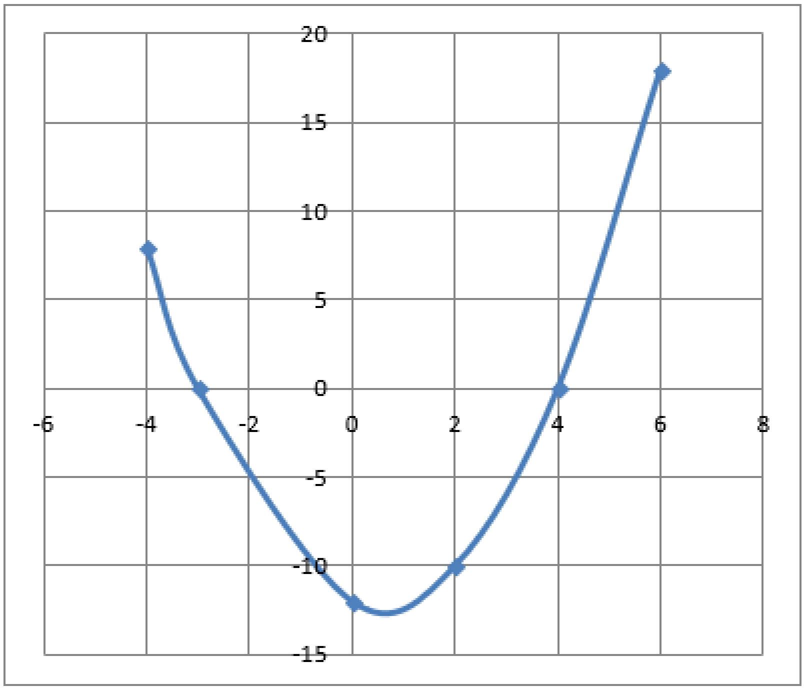
Hence, x=2 and x=-2 are the zeroes of the polynomial.

Q. 3 A. Draw the graphs of the given polynomial and find the zeroes. Justify the answer.

p(x) = x^2 – x - 12

Answer : p(x) = $x^2 – x – 12$

X	0	4	2	-4	-3	6
y = p(x)	-12	0	-10	8	0	18



Clearly, the graph intersects with x – axis at x = -3 and x = 4

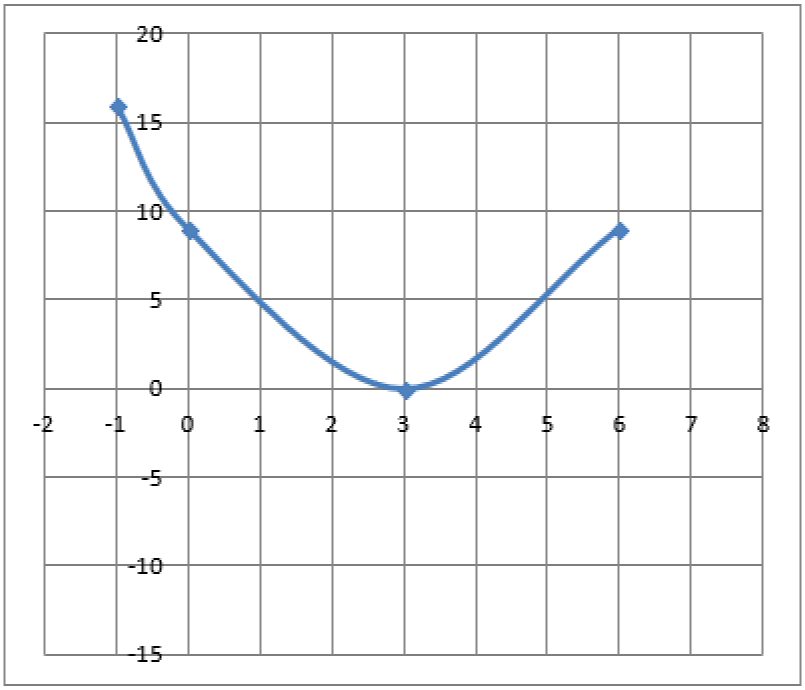
Hence, the zeroes of the polynomial $x^2 - x - 12$ are x = -3 and x=4.

Q. 3 B. Draw the graphs of the given polynomial and find the zeroes. Justify the answer.

$p(x) = x^2 - 6x + 9$

Answer : $p(x) = x^2 - 6x + 9$

X	0	3	-1	6
y = p(x)	9	0	16	9



Clearly, the graph intersects with x – axis at x = 4

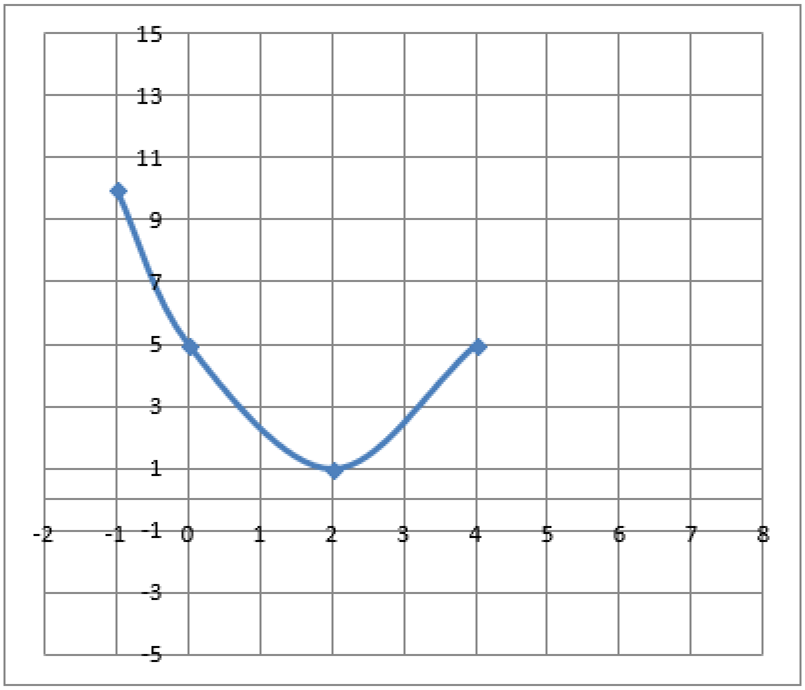
Hence, the zeroes of the polynomial $x^2 - 6x + 9$ are x=3.

Q. 3 C. Draw the graphs of the given polynomial and find the zeroes. Justify the answer.

p(x) = $x^2 - 4x + 5$

Answer : p(x) = $x^2 - 4x + 5$

X	-1	0	2	4
y = p(x)	10	5	1	5



Clearly, the graph does not intersect with x – axis at any point.

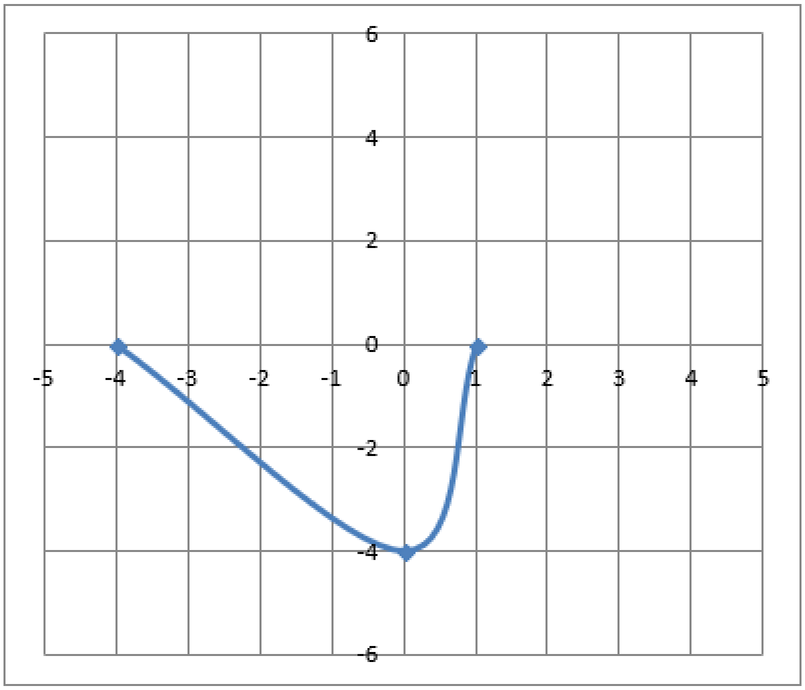
Hence, there are no zeroes of the polynomial $x^2 - 4x + 5$.

Q. 3 D. Draw the graphs of the given polynomial and find the zeroes. Justify the answer.

p(x) = $x^2 + 3x - 4$

Answer : p(x) = $x^2 + 3x - 4$

X	-4	0	1
y = p(x)	0	-4	0



Clearly, the graph intersects with x – axis at x = -4 and x = 1.

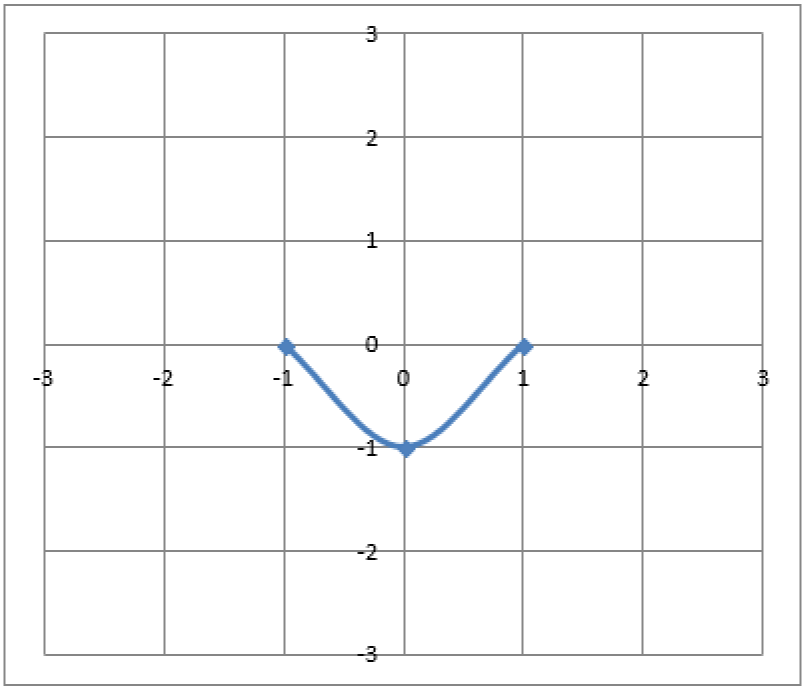
Hence, the zeroes of the polynomial $x^2 + 3x - 4$ are x = -4 and x= 1

Q. 3 E. Draw the graphs of the given polynomial and find the zeroes. Justify the answer.

$p(x) = x^2 - 1$

Answer : $p(x) = x^2 - 1$

X	-1	0	1
y = p(x)	0	-1	0



Clearly, the graph intersects with x – axis at x = -1 and x = 1.

Hence, the zeroes of the polynomial $x^2 – 1$ are x = -1 and x= 1

Q. 4. Why are 1/4 and -1 zeroes of the polynomials p(x) – $4x^2 + 3x – 1$?

Answer : A zero or root of a polynomial function is a number that, when plugged in for the variable, makes the function equal to zero.

Theefore, if p(x)=0, for a given x then the value of x is zero of polynomial.

$p(x) = 4x^2 + 3x – 1$

For 1/4,

$p(1/4) = 4(1/4)^2 + 3(1/4) – 1$

⇒ p(1/4) = 1/4 + 3/4 – 1

⇒ p(1/4) = 1 – 1

⇒ p(1/4) = 0

Therefore, x = 1/4 is a zero of the polynomial $4x^2 + 3x – 1$.

For -1,

$p(-1) = 4(-1)^2 + 3(-1) – 1$

⇒ $p(-1) = 4 – 3 – 1$

⇒ $p(-1) = 4 – 4$

⇒ $p(-1) = 0$

Therefore, $x = -1$ is a zero of the polynomial $4x^2 + 3x – 1$.

Hence, $x = 1/4$ and $x = -1$ are zeroes of the polynomial $4x^2 + 3x – 1$.

Exercise 3.3

Q. 1 A. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$x^2 – 2x – 8$

Answer : $p(x) = 0$

⇒ $x^2 – 2x – 8 = 0$

⇒ $x^2 + 2x – 4x – 8 = 0$

⇒ $x(x + 2) – 4(x + 2) = 0$

⇒ $(x + 2)(x – 4) = 0$

⇒ $x = -2$ and $x = 4$

Hence, -2 and 4 are zeroes of the polynomial $x^2 – 2x – 8$.

Now,

Sum of zeroes = -2 + 4

⇒ Sum of zeroes = 2

$$\frac{-\text{Coefficient of x}}{\text{Coefficient of } x^2} = \frac{-(-2)}{1}$$

$$\Rightarrow \frac{-\text{Coefficient of x}}{\text{Coefficient of } x^2} = 2$$

$$\Rightarrow \frac{-\text{Coefficient of x}}{\text{Coefficient of } x^2} = \text{Sum of zeroes}$$

Product of zeroes = -2 × 4

⇒ Product of zeroes = -8

$$\frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-8}{1}$$

$$\frac{\text{Constant term}}{\text{Coefficient of } x^2} = -8$$

$$\Rightarrow \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \text{Product of zeroes}$$

Q. 1 B. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$4s^2 – 4s + 1$

Answer : p(s) = 0

⇒ $4s^2 – 4s + 1 = 0$

⇒ $4s^2 – 2s – 2s + 1 = 0$

⇒ 2s(2s – 1) – 1(2s – 1) = 0

⇒ (2s – 1) (2s – 1) = 0

⇒ x = 1/2 and x = 1/2

Hence, 1/2 and 1/2 are zeroes of the polynomial $4s^2 – 4s + 1$.

Now,

Sum of zeroes = 1/2 + 1/2

⇒ Sum of zeroes = 1

$$\frac{-\text{Coefficient of s}}{\text{Coefficient of } s^2} = \frac{-(-4)}{4}$$

$$\Rightarrow \frac{-\text{Coefficient of s}}{\text{Coefficient of s}^2} = 1$$

$$\Rightarrow \frac{-\text{Coefficient of s}}{\text{Coefficient of s}^2} = \text{Sum of zeroes}$$

Product of zeroes = 1/2 × 1/2

⇒ Product of zeroes = 1/4

$$\frac{\text{Constant term}}{\text{Coefficient of s}^2} = \frac{1}{4}$$

$$\Rightarrow \frac{\text{Constant term}}{\text{Coefficient of s}^2} = \text{Product of zeroes}$$

Q. 1 C. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$6x^2 – 3 – 7x$

Answer : p(x) = 0

⇒ $6x^2 – 3 – 7x = 0$

⇒ $6x^2 – 7x – 3 = 0$

⇒ $6x^2 – 9x + 2x – 3 = 0$

⇒ $3x(2x – 3) + 1(2x – 3) = 0$

⇒ $(2x – 3) (3x + 1) = 0$

$\Rightarrow x = \frac{3}{2}$ and $x = -\frac{1}{3}$

Hence, $\frac{3}{2}$ and $-\frac{1}{3}$ are zeroes of the polynomial $6x^2 – 3 – 7x$.

Now,

$$\text{Sum of zeroes} = \frac{3}{2} - \frac{1}{3}$$

$\Rightarrow$ Sum of zeroes $= \dfrac{9-2}{6}$

$\Rightarrow$ Sum of zeroes $= \dfrac{7}{6}$

$\dfrac{-\text{Coefficient of x}}{\text{Coefficient of } x^2} = \dfrac{-(-7)}{6}$

$\Rightarrow \dfrac{-\text{Coefficient of x}}{\text{Coefficient of } x^2} = \dfrac{7}{6}$

$\Rightarrow \dfrac{-\text{Coefficient of x}}{\text{Coefficient of } x^2} = $ Sum of zeroes

Product of zeroes $= \dfrac{3}{2} \times -\dfrac{1}{3}$

$\Rightarrow$ Product of zeroes = -1/2

$\dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \dfrac{-3}{6}$

$\Rightarrow \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \dfrac{-1}{2}$

$\Rightarrow \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = $ Product of zeroes

Q. 1 E. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

t^2 - 15

Answer : p(t) = 0

$\Rightarrow t^2 - 15 = 0$

$\Rightarrow (t - \sqrt{15})(t + \sqrt{15}) = 0$

$\Rightarrow t = -\sqrt{15}$ and $t = \sqrt{15}$

Hence, $-\sqrt{15}$ and $\sqrt{15}$ are zeroes of the polynomial $t^2 - 15$.

Now,

Sum of zeroes = -√15 + √15

⇒ Sum of zeroes = -√15 + √15

⇒ Sum of zeroes = 0

$$\frac{-\text{Coefficient of t}}{\text{Coefficient of } t^2} = \frac{0}{1}$$

$$\Rightarrow \frac{-\text{Coefficient of t}}{\text{Coefficient of } t^2} = 0$$

$$\Rightarrow \frac{-\text{Coefficient of t}}{\text{Coefficient of } t^2} = \text{Sum of zeroes}$$

Product of zeroes = -√15 × √15

⇒ Product of zeroes = -15

$$\frac{\text{Constant term}}{\text{Coefficient of } t^2} = \frac{-15}{1}$$

$$\Rightarrow \frac{\text{Constant term}}{\text{Coefficient of } t^2} = -15$$

$$\Rightarrow \frac{\text{Constant term}}{\text{Coefficient of } t^2} = \text{Product of zeroes}$$

Q. 1 F. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$3x^2 – x – 4$

Answer : p(x) = 0

⇒ $3x^2 – x – 4 = 0$

⇒ $3x^2 + 3x – 4x – 4 = 0$

⇒ 3x(x + 1) –4(x + 1) = 0

⇒ (x + 1)(3x – 4)= 0

$$\Rightarrow \; x = \; -1 \text{ and } x = \frac{4}{3}$$

Hence, -1 and $\frac{4}{3}$ are zeroes of the polynomial $3x^2 - x - 4$.

Now,

$$\text{Sum of zeroes} \; = \; -1 \; + \; \frac{4}{3}$$

$$\Rightarrow \text{Sum of zeroes} \; = \; \frac{-3+4}{3}$$

$$\Rightarrow \; \text{Sum of zeroes} \; = \; \frac{1}{3}$$

$$\frac{-\text{Coefficient of x}}{\text{Coefficient of x}^2} = \; \frac{-(-1)}{3}$$

$$\Rightarrow \frac{-\text{Coefficient of x}}{\text{Coefficient of x}^2} \; = \; \frac{1}{3}$$

$$\Rightarrow \frac{-\text{Coefficient of x}}{\text{Coefficient of x}^2} = \; \text{Sum of zeroes}$$

$$\text{Product of zeroes} \; = \; -1 \times \frac{4}{3}$$

$$\text{Product of zeroes} \; = \; -\frac{4}{3}$$

$$\frac{\text{Constant term}}{\text{Coefficient of x}^2} = \frac{-4}{3}$$

$$\Rightarrow \frac{\text{Constant term}}{\text{Coefficient of x}^2} = \frac{-4}{3}$$

$$\Rightarrow \frac{\text{Constant term}}{\text{Coefficient of x}^2} = \; \text{Product of zeroes}$$

Q. 2 A. Find the quadratic polynomial in each case, with the given numbers as the sum and product of its zeroes respectively.

1/4, -1

Answer : Given: $\alpha + \beta = \frac{1}{4}$

$\alpha\beta = -1$

Let the quadratic polynomial be $ax^2 + bx + c$ …(1)

Where, $a \neq 0$

And zeroes of the polynomial are α and β .

Now we know that,

$$\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = \text{Sum of zeroes}$$

$$\Rightarrow \frac{-b}{a} = \alpha + \beta = \frac{1}{4} \text{ …(2)}$$

And,

$$\frac{\text{Constant term}}{\text{Coefficient of } x^2} = \text{Product of zeroes}$$

$$\Rightarrow \frac{c}{a} = \alpha\beta = -1$$

If a = 4

$$\Rightarrow \frac{c}{4} = -1 \Rightarrow c = -4 \text{ ...(3)}$$

From (2) and (3),

a = 4, b = -1 and c = -4

Hence, the polynomial is $4x^2 - x - 4$

Q. 2 B. Find the quadratic polynomial in each case, with the given numbers as the sum and product of its zeroes respectively.

$\sqrt{2}, \frac{1}{3}$

Answer : Given: $\alpha + \beta = \sqrt{2}$

$$\alpha\beta = \frac{1}{3}$$

Let the quadratic polynomial be $ax^2 + bx + c$ …(1)

Where, $a \neq 0$

And zeroes of the polynomial are α and β .

Now we know that,

$$\frac{-\text{Coefficient of x}}{\text{Coefficient of } x^2} = \text{ Sum of zeroes}$$

$$\Rightarrow \frac{-b}{a} = \alpha + \beta = \sqrt{2}$$

And,

$$\frac{\text{Constant term}}{\text{Coefficient of } x^2} = \text{ Product of zeroes}$$

$$\Rightarrow \frac{c}{a} = \alpha\beta = \frac{1}{3}\text{If } a = 3$$

$$\Rightarrow \frac{-b}{3} = \sqrt{2}$$

$$\Rightarrow b = -3\sqrt{2} \ldots(2)$$

$$\Rightarrow \frac{c}{3} = \frac{1}{3} \Rightarrow c = 1 \ldots(3)$$

From (2) and (3),

$a = 3$, $b = -3\sqrt{2}$ and $c = 1$

Hence, the polynomial is $3x^2 – 3\sqrt{2}x + 1$

Q. 2 C. Find the quadratic polynomial in each case, with the given numbers as the sum and product of its zeroes respectively.

$0, \sqrt{5}$

Answer : Given: $\alpha + \beta = 0$

$\alpha\beta = \sqrt{5}$

Let the quadratic polynomial be $ax^2 + bx + c$ …(1)

Where, $a \neq 0$

And zeroes of the polynomial are α and β .

Now we know that,

$$\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = \text{Sum of zeroes}$$

$$\Rightarrow \frac{-b}{a} = \alpha + \beta = 0 \quad \ldots(2)$$

And,

$$\frac{\text{Constant term}}{\text{Coefficient of } x^2} = \text{Product of zeroes}$$

$$\Rightarrow \frac{c}{a} = \alpha\beta = \sqrt{5} \text{If } a = 1$$

$$\Rightarrow \frac{c}{1} = \sqrt{5} \Rightarrow c = \sqrt{5} \ldots(3)$$

From (2) and (3),

$a = 1$, $b = 0$ and $c = \sqrt{5}$

Hence, the polynomial is $x^2 + \sqrt{5}$

Q. 2 D. Find the quadratic polynomial in each case, with the given numbers as the sum and product of its zeroes respectively.

1, 1

Answer : Given: $\alpha + \beta = 1$

$\alpha\beta = 1$

Let the quadratic polynomial be $ax^2 + bx + c$ …(1)

Where, $a \neq 0$

And zeroes of the polynomial are α and β .

Now we know that,

$$\frac{-\text{Coefficient of x}}{\text{Coefficient of } x^2} = \text{ Sum of zeroes}$$

$$\Rightarrow \frac{-b}{a} = \alpha + \beta = 1$$

And,

$$\frac{\text{Constant term}}{\text{Coefficient of } x^2} = \text{ Product of zeroes}$$

$$\Rightarrow \frac{c}{a} = \alpha\beta = 1 \text{ If } a = 1$$

$$\Rightarrow \frac{-b}{1} = 1$$

$$\Rightarrow b = -1 \text{ ...(2)}$$

$$\Rightarrow \frac{c}{1} = 1 \Rightarrow c = 1 \text{ ...(3)}$$

From (2) and (3),

$a = 1$, $b = -1$ and $c = 1$

Hence, the polynomial is $x^2 – x + 1$

Q. 2 E. Find the quadratic polynomial in each case, with the given numbers as the sum and product of its zeroes respectively.

1/4, 1/4

Answer : Given: $\alpha + \beta = -1/4$

$\alpha\beta = 1/4$

Let the quadratic polynomial be $ax^2 + bx + c$ …(1)

Where, $a \neq 0$

And zeroes of the polynomial are α and β .

Now we know that,

$$\frac{-\text{Coefficient of x}}{\text{Coefficient of } x^2} = \text{Sum of zeroes}$$

$$\Rightarrow \frac{-b}{a} = \alpha + \beta = -\frac{1}{4}$$ And,

$$\frac{\text{Constant term}}{\text{Coefficient of } x^2} = \text{Product of zeroes}$$

$$\Rightarrow \frac{c}{a} = \alpha\beta = \frac{1}{4} \text{If } a = 4$$

$$\Rightarrow \frac{-b}{4} = -\frac{1}{4}$$

$$\Rightarrow b = 1 \ldots(2)$$

$$\Rightarrow \frac{c}{4} = \frac{1}{4} \Rightarrow c = 1 \ldots(3)$$

From (2) and (3),

a = 4, b = 1 and c = 1

Hence, the polynomial is $4x^2 + x + 1$

Q. 2 F. Find the quadratic polynomial in each case, with the given numbers as the sum and product of its zeroes respectively.

4, 1

Answer : Given: α + β = 4

αβ = 1

Let the quadratic polynomial be $ax^2 + bx + c$ …(1)

Where, a≠0

And zeroes of the polynomial are α and β.

Now we know that,

$$\frac{-\text{Coefficient of x}}{\text{Coefficient of x}^2} = \text{Sum of zeroes}$$

$$\Rightarrow \frac{-b}{a} = \alpha + \beta = 4$$ And,

$$\frac{\text{Constant term}}{\text{Coefficient of x}^2} = \text{Product of zeroes}$$

$$\Rightarrow \frac{c}{a} = \alpha\beta = 1$$

If a = 1

$$\Rightarrow \frac{-b}{1} = 4$$

⇒ b = -4 …(2)

$$\Rightarrow \frac{c}{1} = 1 \Rightarrow c = 1 \text{ ...(3)}$$

From (2) and (3),

a = 1, b = -4 and c = 1

Hence, the polynomial is x^2 - 4x + 1

Q. 3 A. Find the quadratic polynomial, for the zeroes given in each case.

2,-1

Answer : Let the quadratic polynomial be $ax^2 + bx + c$

And, its zeroes be α and β

α = 2

β = -1

$$-\frac{b}{a} = \alpha + \beta$$

$$\Rightarrow -\frac{b}{a} = 2 + (-1)$$

$$\Rightarrow -\frac{b}{a} = 1$$

$$\frac{c}{a} = \alpha\beta$$

$$\Rightarrow \frac{c}{a} = 2(-1)$$

$$\Rightarrow \frac{c}{a} = -2$$

If a = 1,

$$\Rightarrow -\frac{b}{1} = 1$$

⇒ b = -1

$$\Rightarrow \frac{c}{1} = -2$$

⇒ c = -2

Hence, the polynomial is $x^2 - x - 2$

Q. 3 B. Find the quadratic polynomial, for the zeroes given in each case.

$\sqrt{3}, \sqrt{3}$

Answer : Let the quadratic polynomial be $ax^2 + bx + c$

And, its zeroes be α and β

$\alpha = \sqrt{3}$

$\beta = -\sqrt{3}$

$$-\frac{b}{a} = \alpha + \beta$$

$$-\frac{b}{a} = \sqrt{3} + (-\sqrt{3})$$

$$\Rightarrow -\frac{b}{a} = 0$$

$$\frac{c}{a} = \alpha\beta$$

$$\Rightarrow \frac{c}{a} = \sqrt{3}(-\sqrt{3})$$

$$\Rightarrow \frac{c}{a} = -3$$

If a = 1,

$$\Rightarrow -\frac{b}{1} = 0$$

⇒ b = 0

$$\Rightarrow \frac{c}{1} = -3$$

⇒ c = -3

Hence, the polynomial is $x^2 - 3$

Q. 3 C. Find the quadratic polynomial, for the zeroes given in each case.

1/4, -1

Answer : Let the quadratic polynomial be $ax^2 + bx + c$

And, its zeroes be α and β

α = 1/4

β = -1

$$-\frac{b}{a} = \alpha + \beta$$

$$-\frac{b}{a} = \frac{1}{4} + (-1)$$

$$\Rightarrow -\frac{b}{a} = -\frac{3}{4}$$

$$\frac{c}{a} = \alpha\beta$$

$$\Rightarrow \frac{c}{a} = \frac{1}{4}(-1)$$

$$\Rightarrow \frac{c}{a} = -\frac{1}{4}$$

If a = 4,

$$\Rightarrow -\frac{b}{4} = -\frac{3}{4}$$

⇒ b = 3

$$\Rightarrow \frac{c}{4} = -\frac{1}{4}$$

⇒ c = -1

Hence, the polynomial is $4x^2 + 3x – 1$.

Q. 3 D. Find the quadratic polynomial, for the zeroes given in each case.

1/2, 3/2

Answer : Let the quadratic polynomial be $ax^2 + bx + c$

And, its zeroes be α and β

$$\alpha = \frac{1}{2}$$

$$\beta = \frac{3}{2}$$

$$-\frac{b}{a} = \alpha + \beta$$

$$-\frac{b}{a} = \frac{1}{2} + \frac{3}{2}$$

$$\Rightarrow -\frac{b}{a} = 2$$

$$\frac{c}{a} = \alpha\beta$$

$$\Rightarrow \frac{c}{a} = \frac{1}{2} \times \frac{3}{2}$$

$$\Rightarrow \frac{c}{a} = \frac{3}{4}$$

If a = 4,

$$\Rightarrow -\frac{b}{4} = 2$$

⇒ b = -8

$$\Rightarrow \frac{c}{4} = \frac{3}{4}$$

⇒ c = 3

Hence, the polynomial is $4x^2 - 8x + 3$.

Q. 4. Verify that 1, -1 and -3 are the zeroes of the cubic polynomial $x^3 + 3x^2 - x - 3$ and check the relationship between zeroes and the coefficients.

Answer : $P(x) = x^3 + 3x^2 - x - 3$

For x = 1,

$\Rightarrow P(1) = 1^3 + 3(1)^2 - 1 - 3$

$\Rightarrow P(1) = 1 + 3 - 1 - 3$

$\Rightarrow P(1) = 0$

For x = -1,

$\Rightarrow P(-1) = (-1)^3 + 3(-1)^2 - (-1) - 3$

$\Rightarrow P(-1) = -1 + 3 + 1 - 3$

$\Rightarrow P(-1) = 0$

For x = -3,

$\Rightarrow P(-3) = (-3)^3 + 3(-3)^2 - (-3) - 3$

$\Rightarrow P(-3) = -27 + 27 + 3 - 3$

$\Rightarrow P(-3) = 0$

Now,

$$\frac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = \frac{-3}{1}$$

Sum of zeroes = 1 + (-1) + (-3) = -3

Hence,

$$\frac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = \text{Sum of zeroes}$$

And,

$$\frac{\text{Constant term}}{\text{Coefficient of } x^3} = \frac{-3}{1}$$

Product of zeroes = 1 × (-1) × (-3) = -3

$$\frac{\text{Constant term}}{\text{Coefficient of } x^3} = \text{Product of zeroes}$$

Exercise 3.4

Q. 1 A. **Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:**

$p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

Answer :

$p(x) = x^3 - 3x^2 + 5x - 3$

$g(x) = x^2 - 2$

$$\begin{array}{r|l} & x - 3 \\ \hline x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\ & x^3 \qquad\ \ - 2x \\ & - \qquad\quad\ + \\ \hline & -3x^2 + 7x - 3 \\ & -3x^2 \qquad + 6 \\ & + \qquad\quad\ - \\ \hline & \qquad 7x - 9 \end{array}$$

On dividing them,

The quotient is x – 3.

And,

The remainder is 7x – 9.

Q. 1 B. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:
$p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

Answer : $p(x) = x^4 - 3x^2 + 4x + 5$

$g(x) = x^2 + 1 – x$

On dividing them,

$$x^2 + 1 - x \overline{) \; x^4 - 3x^2 + 4x + 5} \; (x^2 + x - 3$$

$$x^4 + x^2 - x^3$$

$$- \quad - \quad +$$

$$x^3 - 4x^2 + 4x$$

$$x^3 - x^2 + x$$

$$- \quad + \quad -$$

$$-3x^2 + 3x + 5$$

$$-3x^2 + 3x - 3$$

$$+ \quad - \quad +$$

$$8$$

The quotient is $x^2 + x - 3$

And,

The remainder is 8

Q. 1 C. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

$p(x) = x^4 – 5x + 6$, $g(x) = 2 - x^2$

Answer :

$p(x) = x^4 – 5x + 6$

$g(x) = 2 – x^2$

On dividing them,

$$
\begin{array}{r|l}
 & -x^2-2 \\
\hline
-x^2+2 & x^4+0.x^2-5x+6 \\
 & x^4-2x^2 \\
 & -\quad + \\
\hline
 & 2x^2-5x+6 \\
 & 2x^2\quad -4 \\
 & -\quad\quad + \\
\hline
 & -5x+10 \\
\hline
\end{array}
$$

The quotient is $-x^2 - 2$

And,

The remainder is -5x + 10

Q. 2 A. Check in which case the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial.

$t^2 - 3$, $2t^4 + 3t^3 - 2t^2 - 9t - 12$

Answer : $p(x) = 2t^4 + 3t^3 - 2t^2 - 9t - 12$

$g(x) = t^2 - 3$

On dividing them,

$$
\begin{array}{r|l}
 & 2t^2+3t+4 \\
\hline
t^2+0.t-3 & 2t^4+3t^3-2t^2-9t-12 \\
 & 2t^4+0.t^3-6t^2 \\
 & -\quad -\quad + \\
\hline
 & 3t^3+4t^2-9t-12 \\
 & 3t^3+0.t^2-9t \\
 & -\quad -\quad + \\
\hline
 & 4t^2+0.t-12 \\
 & 4t^2+0.t-12 \\
 & -\quad -\quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

The remainder is 0

Hence, yes $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$

Q. 2 B. Check in which case the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial.

$x^2 + 3x + 1$, $3x^4 + 5x^3 – 7x^2 + 2x + 2$

Answer : p(x) = $3x^4 + 5x^3 – 7x^2 + 2x + 2$

g(x) = $x^2 + 3x + 1$

On dividing them,

$$
\begin{array}{r}
3x^2 - 4x + 2 \\
x^2 + 3x + 1 \,\big)\, 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\
\underline{3x^4 + 9x^3 + 3x^2} \\
-4x^3 - 10x^2 + 2x \\
\underline{-4x^3 - 12x^2 - 4x} \\
2x^2 + 6x + 2 \\
\underline{2x^2 + 6x + 2} \\
0
\end{array}
$$

The remainder is 0

Hence, yes $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 – 7x^2 + 2x + 2$

Q. 2 C. Check in which case the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial.

$x^2 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

Answer : p(x) = $x^5 – 4x^3 + x^2 + 3x + 1$

g(x) = $x^3 – 3x + 1$

On dividing them,

$$
\begin{array}{r|l}
 & x^2 \quad -1 \\
x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\
 & x^5 - 3x^3 + x^2 \\
 & (-) \quad (+) \quad (-) \\
\hline
 & -x^3 \qquad + 3x + 1 \\
 & -x^3 \qquad + 3x - 1 \\
 & (+) \qquad (-) \quad (+) \\
\hline
 & \qquad\qquad\quad 2 \\
\hline
\end{array}
$$

The remainder is $2 \neq 0$

Hence, no $x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$

Q. 3. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$,

if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Answer :

Two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

$\Rightarrow \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right)$ is a factor

$\Rightarrow x^2 - \left(\sqrt{\frac{5}{3}}\right)^2$ is a factor

$\Rightarrow x^2 - \frac{5}{3}$ is a factor

Now,

$$
\begin{array}{r|l}
 & 3x^2 + 6x + 3 \\
\hline
x^2 + 0.x - \frac{5}{3} & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & 3x^4 + 0x^3 - 5x^2 \\
 & - \quad - \quad + \\
\hline
 & 6x^3 + 3x^2 - 10x - 5 \\
 & 6x^3 + 0x^2 - 10x \\
 & - \quad - \quad + \\
\hline
 & 3x^2 + 0x - 5 \\
 & 3x^2 + 0x - 5 \\
 & - \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

Therefore, $3x^2 + 6x + 3$ is also a factor

Dividing $3x^2 + 6x + 3$ by 3,

We get,

$x^2 + 2x + 1$

Factorising $x^2 + 2x + 1$,

$x^2 + 2x + 1 = 0$

$\Rightarrow x^2 + x + x + 1 = 0$

$\Rightarrow x(x + 1) + 1(x + 1) = 0$

$\Rightarrow (x + 1)(x + 1) = 0$

$\Rightarrow x = -1, -1$

Therefore, the other zeroes are -1 and -1.

Q. 4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were x - 2 and – 2x + 4 respectively. Find g(x)

Answer : Dividend = $x^3 - 3x^2 + x + 2$

Quotient = $x - 2$

Remainder = $-2x + 4$

Dividend = Divisor × Quotient + Remainder

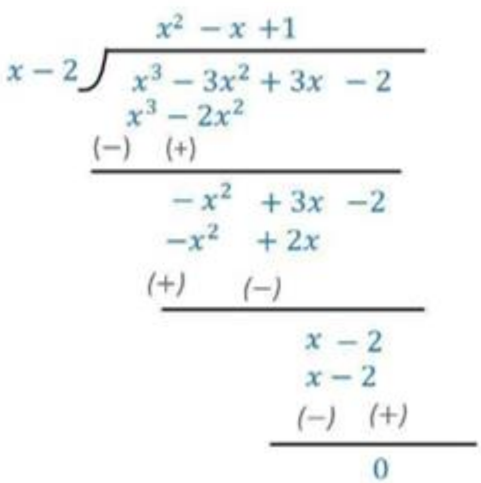
⇒ $x^3 - 3x^2 + x + 2$ = Divisor ×$(x - 2) + (-2x + 4)$

⇒ $x^3 - 3x^2 + x + 2 - (-2x + 4)$ = Divisor ×$(x - 2)$

⇒ $x^3 - 3x^2 + x + 2 + 2x - 4$ = Divisor ×$(x - 2)$

⇒ $x^3 - 3x^2 + 3x - 2$ = Divisor ×$(x - 2)$

$$g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$$



Therefore, $g(x) = x^2 - x + 1$

Q. 5 A. Give examples of polynomials p(x), g(x), q(x) and r(x) which satisfy the division algorithm and

deg p (x) = deg q (x)

Answer : Let g(x) = 2

And $p(x) = 2x^2 - 2x + 14$

Then, dividing p(x) by g(x) gives.

$q(x) = x^2 - x + 7$

and,

r(x) = 0

Deg(p(x)) = Deg(q(x)) = 2

Hence,

g(x) = 2

p(x) = $2x^2 - 2x + 14$

q(x) = $x^2 - x + 7$

r(x) = 0

Q. 5 B. Give examples of polynomials p(x), g(x), q(x) and r(x) which satisfy the division algorithm and

deg q (x) = deg r (x)

Answer : Let g(x) = x + 1

And p(x) = $x^2 + 3$

Then, dividing p(x) by g(x) gives

q(x) = x

and,

r(x) = -x + 3

Deg(r(x)) = Deg(q(x)) = 1

Hence,

g(x) = x + 1

p(x) = $x^2 + 3$

q(x) = x

r(x) = -x + 3

Q. 5 C. Give examples of polynomials p(x), g(x), q(x) and r(x) which satisfy the division algorithm and

deg r (x) = 0

Answer : Let g(x) = 3

And p(x) = 3x + 3

Then, dividing p(x) by g(x) gives

q(x) = x + 1

and,

r(x) = 0

Deg(r(x)) = 0

Hence,

g(x) = 3

p(x) = 3x + 3

q(x) = x + 1

r(x) = 0