CIRCULAR MOTION

KEY CONCEPT

Cross product (Vector Product) of two vectors :

Vector product of two vectors $\vec{A}$ and $\vec{B}$, also called the cross product, is denoted by $\vec{A}\times\vec{B}$. As the name suggests, the vector product is itself a vector. We will use this product to describe torque and angular momentum and extensively to describe magnetic fields forces.

Vector product of two vectors $\vec{A}$ and $\vec{B}$ which are at an angle ϕ is defined as

$$\vec{C} = \vec{A} \times \vec{B},$$

then $\quad C = AB \sin\phi\ \hat{n}$

where $\hat{n}$ is an unit vector perpendicular to plane containing vector $\vec{A}$ and $\vec{B}$. Direction of $\hat{n}$ is given by right hand rule as described below.

Note: the vector product of two parallel or antiparallel vectors is always zero. In particular, the vector product of any vector with itself is zero.

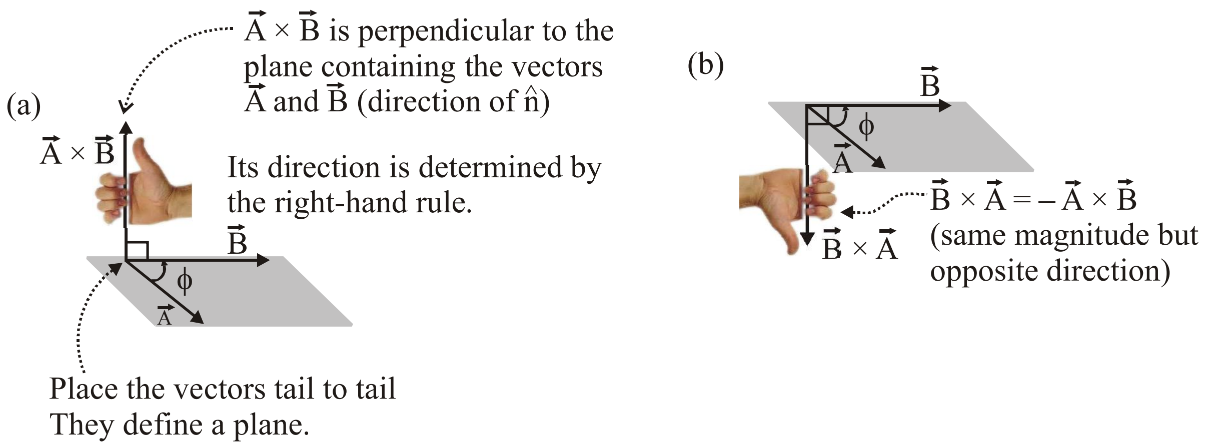


Figure : (a) The vector product $\vec{A}\times\vec{B}$. determined by the right-hand rule. (b) $\vec{B}\times\vec{A} = -\vec{A}\times\vec{B}$

Note: The vector product is not commutative! In fact, for any two vectors $\vec{A}$ and $\vec{B}$,

$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

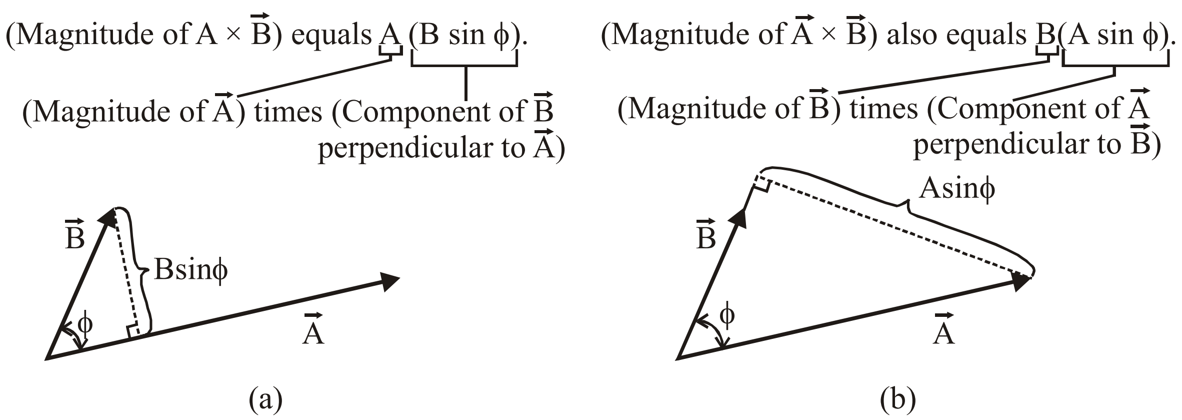


Figure: Calculating the magnitude AB sin ϕ of the vector product of two vector, $\vec{A}\times\vec{B}$.

Calculating the Vector Product Using Components

$$\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$$

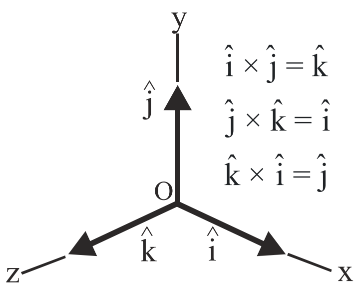
Using the right-hand rule, we find

$$\hat{i}\times\hat{j}=-\hat{j}\times\hat{i}=\hat{k};$$

$$\hat{j}\times\hat{k}=-\hat{k}\times\hat{j}=\hat{i};$$

$$\hat{k}\times\hat{i}=-\hat{i}\times\hat{k}=\hat{j}$$

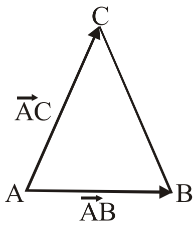
A right-handed coordinate system



Ex. Find the area of the triangle formed by the points having position vectors

$\vec{A}=\hat{i}-\hat{j}-3\hat{k}$, $\vec{B}=4\hat{i}-3\hat{j}+\hat{k}$ and $\vec{C}=3\hat{i}-\hat{j}+2\hat{k}$

Sol.



Let ABC is the triangle formed by the points A, B and C. Then

$$\overrightarrow{AB}=\vec{B}-\vec{A}=\left(4\hat{i}-3\hat{j}+\hat{k}\right)-\left(\hat{i}-\hat{j}-3\hat{k}\right)=3\hat{i}-2\hat{j}+4\hat{k}$$

$$\overrightarrow{AC}=\vec{C}-\vec{A}=\left(3\hat{i}-\hat{j}+2\hat{k}\right)-\left(\hat{i}-\hat{j}-3\hat{k}\right)=2\hat{i}+5\hat{k}$$

Now, $\overrightarrow{AB}\times\overrightarrow{AC}=\left(3\hat{i}-2\hat{j}+4\hat{k}\right)\times\left(2\hat{i}+5\hat{k}\right)$

$$=\hat{i}\left(-10-0\right)+\hat{j}\left(8-15\right)+\hat{k}\left(0+4\right)=-10\hat{i}-7\hat{j}+4\hat{k}$$

Ex. Find unit vector perpendicular to the plane determined by the points P(1, –1, 2), Q(2, 0, –1) and R(0, 2, 1).

Sol. $\vec{P}Q$ = (Position vector of Q) – (Position vector of P)

$= (2\hat{i} - \hat{k}) - (\hat{i} - \hat{j} + 2\hat{k}) = \hat{i} + \hat{j} - 3\hat{k}$

Similarly, $\vec{P}R = (2\hat{j} + \hat{k}) - (\hat{i} - \hat{j} + 2\hat{k}) = -\hat{i} + 3\hat{j} - \hat{k}$

$\therefore \ |\vec{P}Q \times \vec{P}R| = \sqrt{8^2 + 4^2 + 4^2} = 4\sqrt{6}$

$\therefore$ Required unit vectors

$= \pm \dfrac{1}{4\sqrt{6}}(8\hat{i} + 4\hat{j} + 4\hat{k}) = \pm \dfrac{1}{\sqrt{6}}(2\hat{i} + \hat{j} + 9\hat{k})$

Definition of Circular Motion

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed point.

That fixed point is called centre and the distance is called radius of circular path.

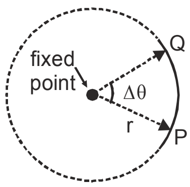
Radius Vector

The vector joining the centre of the circle and the center of the particle performing circular motion is called radius vector. It has constant magnitude and variable direction.

KINEMATICS OF CIRCULAR MOTION

Angular Displacement

- Angle traced by position vector of a particle moving w.r.t. some fixed point is called angular displacement.



$\Delta\theta$ = angular displacement

$\text{Angle} = \dfrac{\text{Arc}}{\text{Radius}}$ or $\Delta\theta = \dfrac{\text{Arc PQ}}{r}$

- Small Angular displacement $d\vec{\theta}$ is a vector quantity, but large angular displacement θ is scalar quantity.

$$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1 \text{ But } \vec{\theta}_1 + \vec{\theta}_2 \neq \vec{\theta}_2 + \vec{\theta}_1$$

- Its direction is perpendicular to plane of rotation and given by right hand screw rule.
- It is dimensionless and has S.I. unit "Radian" while other units are degree or revolution.

2π radian = 360° =1 revolution

Frequency (n) : Number of revolutions describes by particle per second is its frequency. Its unit is revolutions per second (r.p.s.) or revolutions per minute (r.p.m.)

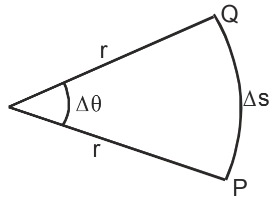
Time Period (T) : It is time taken by particle to complete one revolution. $T = \dfrac{1}{n}$

Angular Velocity (ω) : It is defined as the rate of change of angular displacement of moving particle

$$\omega = \frac{\text{Angle traced}}{\text{Time taken}} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Relation between linear and Angular velocity

$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}} \text{ or } \frac{\Delta s}{r} \qquad \Delta\theta = \frac{\Delta s}{r} \text{ or } \Delta s = r\Delta\theta$$



$$\therefore \frac{\Delta s}{\Delta t} = \frac{r\Delta\theta}{\Delta t} \text{ if } \Delta t \to 0 \text{ then } \frac{ds}{dt} = r\frac{d\theta}{dt} \qquad \boxed{v = \omega r}$$

$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$ (direction of $\vec{v}$ is according to right hand thumb rule)

Average Angular Velocity (ω_{av})

$$\omega_{av} = \frac{\text{total angle of rotation}}{\text{total time taken}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi n$$

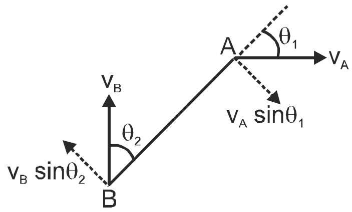
where θ_1 and θ_2 are angular position of the particle at instant t_1 and t_2.

Instantaneous Angular Velocity

The angular velocity at some particular instant $\omega = \lim\limits_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$ or $\vec{\omega} = \frac{d\vec{\theta}}{dt}$

Relative Angular Velocity

- Relative angular velocity of a particle 'A' w.r.t. other moving particle B is the angular velocity of the position vector of A w.r.t. B. That means it is the rate at which position vector of 'A' w.r.t. B rotates at that instant

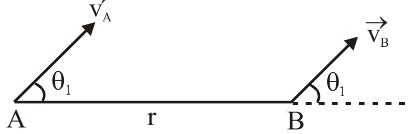


$$\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{\text{Relative velocity of A w.r.t. B perpendicular to line AB}}{\text{seperation between A and B}}$$

here $(v_{AB})_\perp = v_A \sin\theta_1 + v_B \sin\theta_2$ $\qquad \therefore\ \omega_{AB} = \frac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$

- It is an axial vector quantity.
- Its direction is same as that of angular displacement i.e. perpendicular to the plane of rotation and along the axis according to right hand screw rule.
- Its unit is radian/second.

If particles A and B are moving with a velocity $\vec{v}_A$ and $\vec{v}_B$ and separated by a distance r at a given instant then

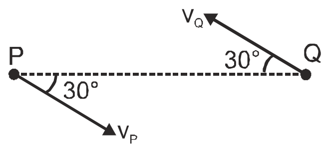


(i) $\frac{dr}{dt} = v_B\cos\theta_2 - v_A\cos\theta_1$ (ii) $\frac{d\theta_{AB}}{dt} = w_{AB} = \frac{v_B \sin\theta_2 - v_A \sin\theta_1}{r}$

Ex. A particle revolving in a circular path completes first one third of circumference in 2 s, while next one third in 1 s. Calculate the average angular velocity of particle.

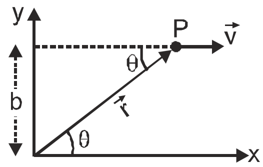
Sol. $\theta_1 = \frac{2\pi}{3}$ and $\theta_2 = \frac{2\pi}{3}$ total time T = 2 + 1 = 3 s $\therefore$ $<\omega_{av}> = \frac{\theta_1 + \theta_2}{T} = \frac{\frac{2\pi}{3}+\frac{2\pi}{3}}{3} = \frac{\frac{4\pi}{3}}{3} = \frac{4\pi}{9}$ rad/s

Ex. Two moving particles P and Q are 10 m apart at any instant. Velocity of P is 8 m/s at 30°, from line joining the P and Q and velocity of Q is 6m/s at 30°.Calculate the angular velocity of P w.r.t. Q

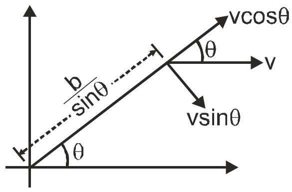


Sol. $\omega_{PQ} = \frac{8\sin 30^\circ - (-6\sin 30^\circ)}{10} = 0.7$ rad/s.

Ex. A particle moving parallel to x–axis as shown in fig. such that at all instant the y–axis component of its position vector is constant and is equal to 'b'. Find the angular velocity of the particle about the origin when its radius vector makes angle θ from the x-axis.



Sol. $\therefore\ \omega_{PO} = \frac{v\sin\theta}{\frac{b}{\sin\theta}} = \frac{v}{b}\sin^2\theta$



Ex. The angular velocity of a particle is given by $\omega = 1.5\,t - 3t^2 + 2$, Find the time when its angular acceleration becomes zero.

Sol. $\alpha = \frac{d\omega}{dt} = 1.5 - 6\,t = 0 \Rightarrow t = 0.25$ s.

Ex. A disc starts from rest and on the application of a torque, it gains an angular acceleration given by $\alpha = 3t - t^2$. Calculate the angular velocity after 2 s.

Sol. $\frac{d\omega}{dt} = 3t - t^2 \Rightarrow \int_0^{\omega} d\omega = \int_0^{t} (3t - t^2)dt \Rightarrow \omega = \frac{3t^2}{2} - \frac{t^3}{3} \Rightarrow$ at t =2 s, $\omega = \frac{10}{3}$ rad/s

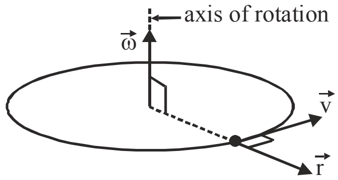
Angular Acceleration (α)

- Rate of change of angular velocity is called angular acceleration. $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$ or $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$
- Its an axial vector quantity. It direction is along the axis according to right hand screw rule.
- Unit $\rightarrow$ rad/s^2

Relation between Angular and Linear Acceleration

$\vec{v} = \vec{\omega} \times \vec{r}$ ($\vec{v}$ is a tangential vector, $\vec{\omega}$ is a axial vector and $\vec{r}$ is a radial vector.)

These three vectors are mutually perpendicular. but $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r}) = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$



or $\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$ ($\frac{d\vec{\omega}}{dt} = \vec{\alpha}$ and $\frac{d\vec{r}}{dt} = \vec{v}$) $\Rightarrow \vec{a} = \vec{a}_T + \vec{a}_C$

($\vec{a}_T = \vec{\alpha} \times \vec{r}$ is tangential acceleration &

$\vec{a}_C = \vec{\omega} \times \vec{v}$ is centripetal acceleration)

$\vec{a} = \vec{a}_T + \vec{a}_C$ ($\vec{a}_T$ and $\vec{a}_C$ are two component of net linear acceleration)

Tangential Acceleration

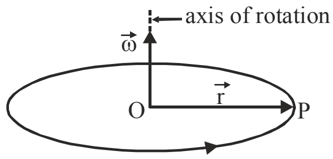
$\vec{a}_T = \vec{\alpha} \times \vec{r}$, its direction is parallel to velocity. $\vec{v} = \vec{\omega} \times \vec{r}$ and $\vec{a}_T = \vec{\alpha} \times \vec{r}$ as $\vec{\omega}$ and $\vec{\alpha}$ both are parallel and along the axis so that $\vec{v}$ and $\vec{a}_T$ are also parallel and along the tangential direction.

Magnitude of tangential acceleration in case of circulation motion.

$a_T = \alpha\, r \sin 90° = \alpha r$ ($\vec{\alpha}$ is axial, $\vec{r}$ is radial so that $\vec{\alpha} \perp \vec{r}$)

As $\vec{a}_T$ is along the direction of motion (in the direction of $\vec{v}$) so that $\vec{a}_T$ is responsible for change in speed of the particle. Its magnitude is rate of change of speed of the particle. If particle is moving on a circular path with constant speed then tangential acceleration is zero.

Centripetal acceleration



$\vec{a}_C = \vec{\omega} \times \vec{v} \Rightarrow \vec{a}_C = \vec{\omega} \times (\vec{\omega} \times \vec{r})$ ($\because\ \vec{v} = \vec{\omega} \times \vec{r}$)

Let $\vec{r}$ is in $\hat{i}$ direction and $\vec{\omega}$ is in $\hat{j}$ direction then direction of

$\vec{a}_C$ is along $\hat{j} \times (\hat{j} \times \hat{i}) = \hat{j} \times (-\hat{k}) = -\hat{i}$

opposite direction of $\vec{r}$ i.e., from P to O and it is

centripetal direction.Magnitude of centripetal acceleration, $a_C = \omega v = \frac{v^2}{r} = \omega^2 r$, $\vec{a}_C = \frac{v^2}{r}(-\hat{r})$

- Centripetal acceleration is always perpendicular to the velocity or displacement at each point.

Net Linear Acceleration :

$\vec{a} = \vec{a}_T + \vec{a}_C$ and $\vec{a}_T \perp \vec{a}_C$ so that $|\vec{a}| = \sqrt{a_T^2 + a_C^2}$

About uniform circular motion :–

- Position vector $(\vec{r})$ is always perpendicular to the velocity vector $(\vec{v})$ i.e. $\vec{r}.\vec{v} = 0$
- velocity vector is always perpendicular to the acceleration. $\vec{v}.\vec{a} = 0$
- $\because |\vec{v}|$ = constant so tangential acc. a_t=0 $\therefore$ f_t = 0
- Important difference between the projectile motion and uniform circular motion :
  In projectile motion, both the magnitude and the direction of acceleration (g) remain constant, while in uniform circular motion the magnitude remains constant but the direction continuously changes.

DYNAMICS OF CIRCULAR MOTION

If a particle moves with constant speed in a circle, motion is called uniform circular. In uniform circular motion a resultant non-zero force acts on the particle. The acceleration is due to the change in direction of the velocity vector. In uniform circular motion tangential acceleration (a_t) is zero. The acceleration of the particle is towards the centre and its magnitude is $\frac{v^2}{r}$. Here, v is the speed of the particle and r the radius of the circle. The direction of the resultant force F is therefore towards centre and its magnitude is $F = \frac{mv^2}{r} = mr\omega^2 (\text{as } v = r\omega)$

Here, ω is the angular speed of the particle. This force F is called the centripetal force. Thus, a centripetal force of magnitude $\frac{mv^2}{r}$ is needed to keep the particle moving in a circle with constant speed. This force is provided by some external source such as friction, magnetic force, coulomb force, gravitation, tension, etc.

Circular Turning of Roads

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways:

• By friction only. • By banking of roads only. • By friction and banking of roads both.

In real life the necessary centripetal force is provided by friction and banking of roads both.

- **By Friction only**

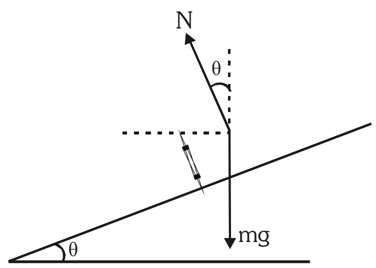
Suppose a car of mass m is moving at a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force to the car will be provided by force of friction f acting towards centre.

Thus, $f = \frac{mv^2}{r}$ $\because f_{max} = \mu N = \mu mg$

Therefore, for a safe turn without sliding $\frac{mv^2}{r} \le f_{max} \Rightarrow \frac{mv^2}{r} \le \mu mg \Rightarrow v \le \sqrt{\mu rg}$

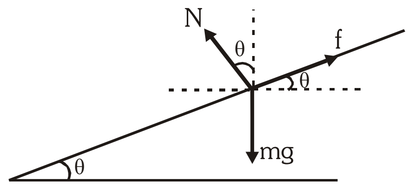
- **By Banking of Roads only**

Friction is not always reliable at circular turns if high speeds and sharp turns are involved. To avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is some what lifted compared to the inner part.



$N \sin\theta = \frac{mv^2}{r}$ and $N \cos\theta = mg \Rightarrow \tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta}$

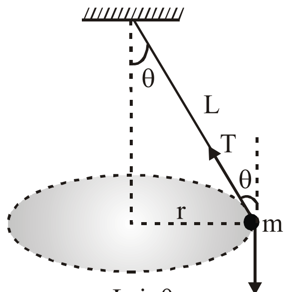
• **Friction and Banking of Road both**



If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed both in magnitude and direction. The direction of second force, i.e., normal reaction N is also fixed (perpendicular to road) while the direction of the third force, i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_{max} = \mu N$). So, the magnitude of normal reaction N and direction plus magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the centre.

Conical Pendulum

If a small particle of mass m tied to a string is whirled in a horizontal circle, as shown in figure. The arrangement is called the 'conical pendulum'. In case of conical pendulum the vertical component of tension balances the weight while its horizontal component provides the necessary centripetal force. Thus,,



$$T\sin\theta = \frac{mv^2}{r} \quad \text{and} \quad T\cos\theta = mg \Rightarrow v = \sqrt{rg\tan\theta}$$

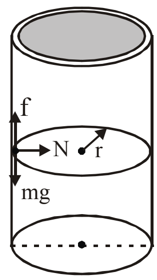
$$\therefore \text{ Angular speed } \omega = \frac{v}{r} = \sqrt{\frac{g\tan\theta}{r}}$$

So, the time period of pendulum is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$

'Death Well' or Rotor

In case of 'death well' a person drives a motorcycle on a vertical surface of a large wooden well while in case of a rotor at a certain angular speed of rotor a person hangs resting against the wall without any support from the bottom. In death well walls are at rest and person revolves while in case of rotor person is at rest and the walls rotates.

In both cases friction balances the weight of person while reaction provides the centripetal force for circular motion,



Death well

Rotor

i.e., $f = mg$ and $N = \frac{mv^2}{r} = mr\omega^2$

Ex. Find the maximum speed at which a car can turn round a curve of 30 m radius on a level road if the coefficient of friction between the tyres and the road is 0.4 [acceleration due to gravity = 10 m/s^2]

Sol. Here centripetal force is provided by friction so

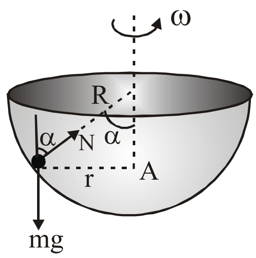
$$\frac{mv^2}{r} \le \mu mg \Rightarrow v_{max} = \sqrt{\mu rg} = \sqrt{120} \approx 11 \text{ ms}^{-1}$$

Ex. For traffic moving at 60 km/hr, if the radius of the curve is 0.1 km, what is the correct angle of banking of the road ? ($g = 10\ m/s^2$)

Sol. In case of banking $\tan\theta = \frac{v^2}{rg}$ Here $v = 60\ km/hr = 60 \times \frac{5}{18}\ ms^{-1} = \frac{50}{3}\ ms^{-1}$ $r = 0.1\ km = 100\ m$

So $\tan\theta = \frac{50/3 \times 50/3}{100 \times 10} = \frac{5}{18} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{18}\right)$

Ex. A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is α. Find the angular speed at which the bowl is rotating.

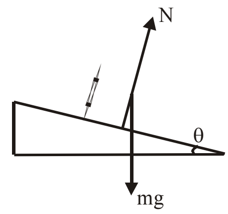


Sol. $N\cos\alpha = mg$ and $N\sin\alpha = mr\omega^2$ but $r = R\sin\alpha$

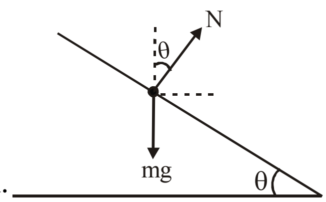
$\Rightarrow N\sin\alpha = mR\sin\alpha\omega^2 \Rightarrow N = mR\omega^2$

$\Rightarrow (mR\omega^2)\cos\alpha = mg \Rightarrow \omega = \sqrt{\frac{g}{R\cos\alpha}}$

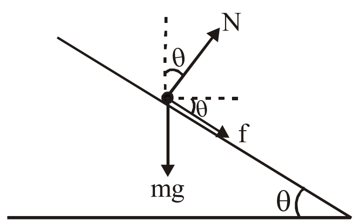
Ex. A car is moving along a banked road laid out as a circle of radius r. (a) What should be the banking angle θ so that the car travelling at speed v needs no frictional force from the tyres to negotiate the turn? (b) The coefficients of friction between tyres and road are $\mu_s = 0.9$ and $\mu_k = 0.8$. At what maximum speed can a car enter the curve without sliding towards the top edge of the banked turn?



Sol. (a) $N\sin\theta = \frac{mv^2}{r}$ and $N\cos\theta = mg \Rightarrow \tan\theta = \frac{v^2}{rg}$



Note : In above case friction does not play any role in negotiating the turn.
(b) If the driver moves faster than the speed mentioned above, a friction force must act parallel to the road, inward towards centre of the turn.



$\Rightarrow F\cos\theta + N\sin\theta = \frac{mv^2}{r}$ and $N\cos\theta = mg + f\sin\theta$

For maximum speed of $f = \mu N$

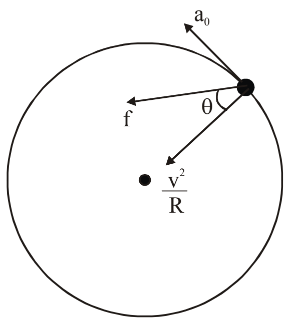
$\Rightarrow N(\mu\cos\theta + \sin\theta) = \frac{mv^2}{r}$ and $N(\cos\theta - \mu\sin\theta) = mg$

$\Rightarrow \frac{v^2}{rg} = \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} \Rightarrow v = \sqrt{\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)rg}$

Ex. A car starts from rest with a consant tangential acceleration a_0 in a circular path of radius r. At time t, the car skids, find the value of coefficient of friction.

Sol. The tangential and centripetal acceleration is provided only by the frictional force.

Thus, $f\sin\theta = ma_0$, $f\cos\theta = \dfrac{mv^2}{r} = \dfrac{m(a_0t)^2}{r}$



$$\Rightarrow f = m\sqrt{a_0^2 + \frac{(a_0t)^4}{r^2}} = ma_0\sqrt{1+\frac{a_0^2t^4}{r^2}} = f_{max}$$

$$\mu mg = ma_0\sqrt{1+\frac{a_0^2t^4}{r^2}} \Rightarrow \mu = \frac{a_0}{g}\sqrt{1+\frac{a_0^2t^4}{r^2}}$$

Centrifugal force :-

Centrifugal force is a pseudo force which an observer needs to consider while making observations in a rotating frame. This force is non physical and arise from kinematics and not due to physical interactions. Centrifugal force is directed away from axis of rotation of rotating frame and its value is $m\omega^2r$, where ω is angular speed of rotating frame where observer has kept himself fixed and r is distance of object of mass m from axis of rotation.

EXERCISE (S-1)

Kinematics of circular motion

1. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone ?

CR0001

2. A particle is revolving in a circle of radius 1m with an angular speed of 12 rad/s. At t = 0, it was subjected to a constant angular acceleration α and its angular speed increased to $(480/\pi)$ rotation per minute (rpm) in 2 sec. Particle then continues to move with attained speed. Calculate

(i) angular acceleration of the particle,

(ii) tangential velocity of the particle as a function of time.

(iii) acceleration of the particle at t = 0.5 second and at t = 3 second

(iv) angular displacement at t = 3 second.

CR0002

3. A particle moves in a circle of radius 1.0 cm at a speed given by v = 2.0 t where v is in cm/s and t in seconds.

(a) Find the radial acceleration of the particle at t = 1s.

(b) Find the tangential acceleration at t = 1s.

(c) Find the magnitude of the acceleration at t = 1 s.

CR0003

4. A particle is travelling in a circular path of radius 4m. At a certain instant the particle is moving at 20m/s and its acceleration is at an angle of 37^{o} from the direction to the centre of the circle as seen from the particle

(i) At what rate is the speed of the particle increasing?

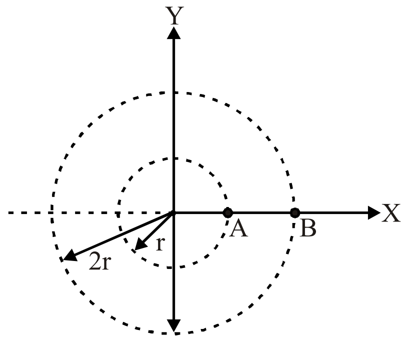
(ii) What is the magnitude of the acceleration?

CR0004

5. A disk rotates about its central axis starting from rest and accelerates with constant angular acceleration. At one instant it is rotating at 12 rad/s and after 80 radian of more angular displacement, its angular speed becomes 28 rad/s. How much time (seconds) does the disk takes to complete the mentioned angular displacement of 80 radians.

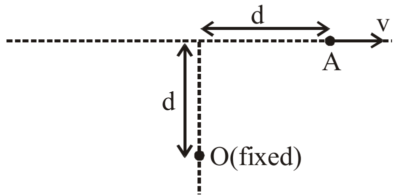
CR0005

6. Two particles A and B are moving in a horizontal plane anticlockwise on two different concentric circles with different constant angular velocities 2ω and ω respectively. Find the relative velocity (in m/s) of B w.r.t. A after time $t = \pi/\omega$. They both start at the position as shown in figure (Take ω = 3rad/sec, r = 2m)



CR0006

7. Find angular velocity of A with respect to O at the instant shown in the figure.



CR0007

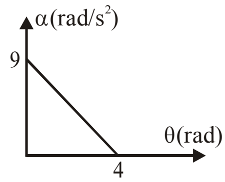
8. A stone is thrown horizontally with the velocity 15m/s. Determine the tangential and normal accelerations of the stone in 1 second after it begins to move.

CR0008

9. A particle moves in the x-y plane with the velocity $\vec{v} = a\hat{i} + bt\hat{j}$. At the instant $t = a\sqrt{3}/b$ the magnitude of tangential, normal and total acceleration are _____, _______, & _________.

CR0009

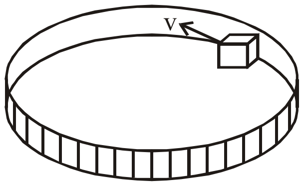
10. A particle starts moving in a non-uniform circular motion, has angular acceleration as shown in figure. The angular velocity at the end of 4 radian is given by ω rad/s then find the value of ω.



CR0010

Dynamics of circular motion

11. A block of mass m moves with speed v against a smooth, fixed vertical circular groove of radius r kept on smooth horizontal surface.



Find :

(i) normal reaction of the floor on the block.

(ii) normal reaction of the vertical wall on the block.

CR0011

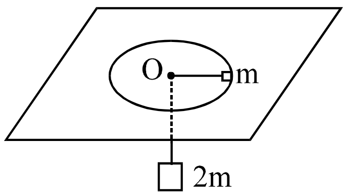
12. A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The co~efficient of static- friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn?

CR0012

13. A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of 40 rev/min in a horizontal plane. What is the tension in the string ? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N ?

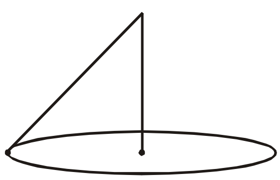
CR0013

14. A mass m rotating freely in a horizontal circle of radius 1 m on a frictionless smooth table supports a stationary mass 2m, attached to the other end of the string passing through smooth hole O in table, hanging vertically. Find the angular velocity of rotation.



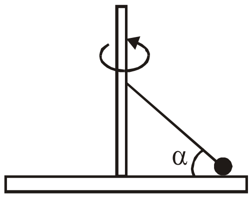
CR0014

15. Consider a conical pendulum having bob of mass m is suspended from a ceiling through a string of length L. The bob moves in a horizontal circle of radius r. Find (a) the angular speed of the bob and (b) the tension in the string.



CR0015

16. A circular platform rotates around a vertical axis with angular velocity $\omega = 10$ rad/s. On the platform is a ball of mass 1 kg, attached to the long axis of the platform by a thin rod of length 10 cm ($\alpha = 30°$). Find normal force exerted by the ball on the platform (in newton). Friction is absent.



CR0016

17. An aircraft executes a horizontal loop at a speed of 720 km/h with its wings banked at 15°. What is the radius of the loop ?

CR0017

18. A block of mass m = 20 kg is kept at a distance R = 1m from central axis of rotation of a round turn table (A table whose surface can rotate about central axis). Table starts from rest and rotates with constant angular acceleration, $\alpha = 3$ rad/sec^2. The friction coefficient between block and table is $\mu = 0.5$. At time $t = \frac{x}{3}$ sec from starting of motion (i.e. t = 0 sec) the block is just about to slip. Find the value of x.

CR0018

EXERCISE (S-2)

1. A stone is launched upward at 45° with speed v_0. A bee follows the trajectory of the stone at a constant speed equal to the initial speed of the stone.

(i) Find the radius of curvature at the top point of the trajectory.

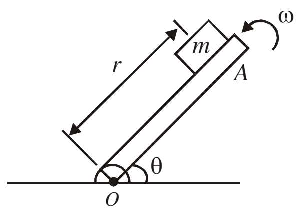
(ii) What is the acceleration of the bee at the top point of the trajectory? For the stone, neglect the air resistance.

CR0019

2. A particle is moving along a circular path of radius R in such a way that at any instant magnitude of radial acceleration & tangential acceleration are equal. If at t = 0 velocity of particle is V_0. Find the speed of the particle after time $t = \dfrac{R}{2V_0}$

CR0020

3. The member OA rotates in vertical plane about a horizontal axis through O with a constant counter clockwise velocity $\omega = 3$ rad/s. As it passes the position $\theta = 0$, a small mass m is placed upon it at a radial distance r = 0.5m. If the mass is observed to slip at $\theta = 37^0$, find the coefficient of friction between the mass & the member.



CR0021

4. Two blocks of mass m_1=10kg and m_2=5kg connected to each other by a massless inextensible string of length 0.3m are placed along a diameter of a turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10rad/sec about a vertical axis passing through its centre. The masses are placed along the diameter of the table on either side of the centre O such that m_1 is at a distance of 0.124 m from O. The masses are observed to be at rest with respect to an observer on the turn table.

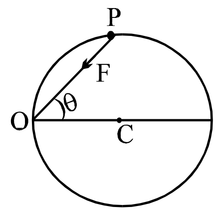
(i) Calculate the frictional force on m_1

(ii) What should be the minimum angular speed of the turn table so that the masses will slip from this position.

(iii) How should the masses be placed with the string remaining taut, so that there is no frictional force acting on the mass m_1.

CR0022

5. A particle P is moving on a circle under the action of only one force acting always towards fixed point O on the circumference. Find ratio of $\frac{d^2\theta}{dt^2}$ & $\left(\frac{d\theta}{dt}\right)^2$.



CR0023

6. A thin circular loop of radius R rotates about its vertical diameter with an angular frequency ω. Show that a small bead on the wire loop remains at its lowermost point for $\omega \le \sqrt{g/R}$. What is the angle made by the radius vector joining the centre to the bead with the vertical downward direction for $\omega = \sqrt{2g/R}$? Neglect friction.

CR0024

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Cross product of vectors

1. The area of parallelogram represented by the vectors $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = \hat{i} + 4\hat{j}$ is :-

(A) 14 unit (B) 7.5 unit (C) 10 unit (D) 5 unit

CR0025

2. What is the angle between $(\vec{P} + \vec{Q})$ and $(\vec{P} \times \vec{Q})$?

(A) 0 (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) π

CR0026

3. The value of n so that vectors $2\hat{i} + 3\hat{j} - 2\hat{k}$, $5\hat{i} + n\hat{j} + \hat{k}$ and $-\hat{i} + 2\hat{j} + 3\hat{k}$ may be coplanar, will be :-

(A) 18 (B) 28 (C) 9 (D) 36

CR0027

4. If $\vec{a}$ and $\vec{b}$ are two vectors then the value of $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$ is :-

(A) $2(\vec{b} \times \vec{a})$ (B) $-2(\vec{b} \times \vec{a})$ (C) $\vec{b} \times \vec{a}$ (D) $\vec{a} \times \vec{b}$

CR0028

5. If $|\vec{a} \cdot \vec{b}| = \sqrt{3}|\vec{a} \times \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is :-

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

CR0029

Kinematics of circular motion

6. A body is executing circular motion in the vertical plane containing directions. If the direction of velocity $(\vec{v})$ at the top most point is towards west, what is the direction of angular velocity $(\vec{\omega})$?

(A) east (B) west (C) north (D) south

CR0030

7. If the magnitude of velocity in the previous question is decreasing with time, what is the direction of angular acceleration $(\vec{\alpha})$?

(A) east (B) west (C) north (D) south

CR0031

8. A mass is revolving in a circle which lies in a plane of paper. The direction of angular acceleration can be :-

(A) perpendicular to radius and velocity (B) towards the radius

(C) tangential (D) at right angle to angular velocity

CR0032

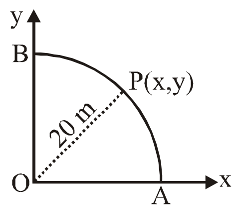
9. The second's hand of a watch has length 6 cm. Speed of end point and magnitude of difference of velocities at two perpendicular positions will be

(A) 2π & 0 mm/s (B) $2\sqrt{2}\,\pi$ & 4.44 mm/s

(C) $2\sqrt{2}\,\pi$ & 2π mm/s (D) 2π & $2\sqrt{2}\,\pi$ mm/s

CR0033

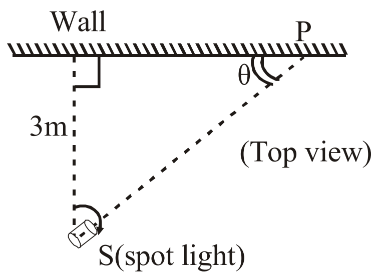
10. A point P moves in counter clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^2 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when $t = 5\sqrt{\frac{3}{10}}$ seconds is nearly :



(A) 2 m/s^2 (B) 1.5 m/s^2 (C) 2.5 m/s^2 (D) 3 m/s^2

CR0034

11. A spot light S rotates in a horizontal plane with a constant angular velocity of 0.1 rad/s. The spot of light P moves along the wall at a distance 3m. What is the velocity of the spot P when $\theta = 45°$?



(A) 0.6 m/s (B) 0.5 m/s (C) 0.4 m/s (D) 0.3 m/s

CR0035

12. Which of the following statements is false for a particle moving in a circle with a constant angular speed ? **[AIEEE - 2004]**

(A) The velocity vector is tangent to the circle

(B) The acceleration vector is tangent to the circle

(C) the acceleration vector points to the centre of the circle

(D) The velocity and acceleration vectors are perpendicular to each other

CR0036

Dynamics of circular motion

13. A particle is moving in a circle :

(A) the resultant force on the particle must be towards the centre

(B) the cross product of the tangential acceleration and the angular velocity will be zero

(C) the direction of the angular acceleration and the angular velocity must be the same

(D) the resultant force may be towards the centre

CR0037

14. A particle of mass m is tied to a light string and rotated with a speed v along a circular path of radius r. If T = tension in the string and mg = gravitational force on the particle then the actual forces acting on the particle are :

(A) mg and T only

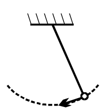
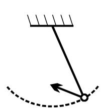
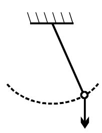
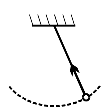
(B) mg, T and an additional force of $\frac{mv^2}{r}$ directed inwards.

(C) mg, T and an additional force of $\frac{mv^2}{r}$ directed outwards.

(D) only a force $\frac{mv^2}{r}$ directed outwards.

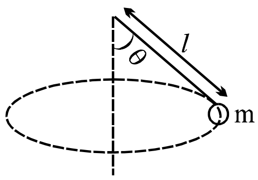
CR0038

15. Which vector in the figures best represents the acceleration of a pendulum mass at the intermediate point in its swing?

(A)  (B)  (C)  (D) 

CR0039

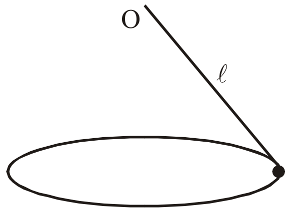
16. A conical pendulum is moving in a circle with angular velocity ω as shown. If tension in the string is T, which of following equations are correct ?

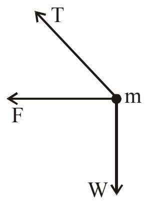


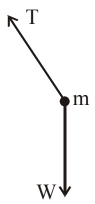
(A) $T = m\omega^2 l$ (B) $T \sin\theta = m\omega^2 l$ (C) $T = mg \cos\theta$ (D) $T = m\omega^2 l \sin\theta$

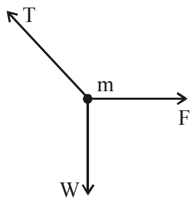
CR0040

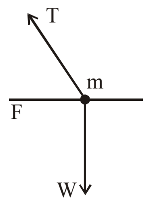
17. A point mass m is suspended from a light thread of length ℓ, fixed at O, is whirled in a horizontal circle at constant speed as shown. From your point of view, stationary with respect to the mass, the forces on the mass are :



(A) 

(B) 

(C) 

(D) 

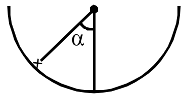
CR0041

18. A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration, α. If the coefficient of friction between the rod and bead is μ, and gravity is neglected, then the time after which the bead starts slipping is :– **[IIT-JEE 2000]**

(A) $\sqrt{\frac{\mu}{\alpha}}$ (B) $\frac{\mu}{\sqrt{\alpha}}$ (C) $\frac{1}{\sqrt{\mu\alpha}}$ (D) infinitesimal

CR0042

19. A insect crawls up a hemispherical surface very slowly (see the figure). The coefficient of friction between the surface and the insect is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given **[IIT-JEE 2001]**



(A) $\cot \alpha = 3$ (B) $\tan \alpha = 3$ (C) $\sec \alpha = 3$ (D) $\operatorname{cosec} \alpha = 3$

CR0043

20. The maximum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is - **[AIEEE - 2002]**

(A) 60 (B) 30 (C) 15 (D) 25

CR0044

21. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane, it follows that- **[AIEEE - 2004]**

(A) Its velocity is constant (B) Its acceleration is constant

(C) Its kinetic energy is constant (D) It moves in a straight line

CR0045

MULTIPLE CORRECT TYPE QUESTIONS

22. A car runs around a curve of radius 10 m at a constant speed of 10 ms^{-1}. Consider the time interval for which car covers a curve of 120° arc :-

(A) Resultant change in velocity of car is $10\sqrt{3}$ ms^{-1}

(B) Instantaneous acceleration of car is 10 ms^{-2}

(C) Average acceleration of car is $\frac{5}{24}$ ms^{-2}

(D) Instantaneous and average acceleration are same for the given period of motion.

CR0046

23. A car is moving with constant speed on a rough banked road.

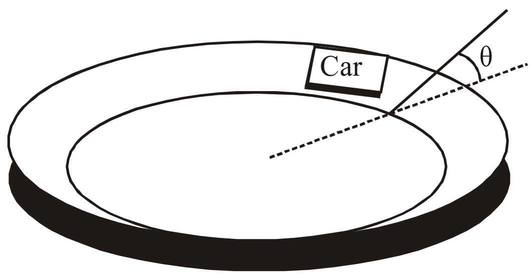
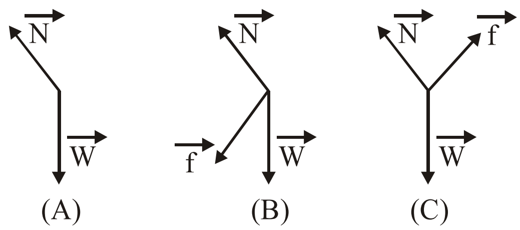


Figure shows the free body diagram of car in three situation A, B & C respectively:-



(A) Car in A has more speed than car in C

(B) Car in A has less speed than car in B

(C) FBD for car in A is not possible

(D) If $\mu > \tan\theta$ the FBD for car C is not possible

CR0047

24. A heavy particle is tied to the end A of a string of length 1.6 m. Its other end O is fixed. It revolves as a conical pendulum with the string making 60° with the vertical. Then

(A) its period of revolution is $\frac{4\pi}{7}$ sec.

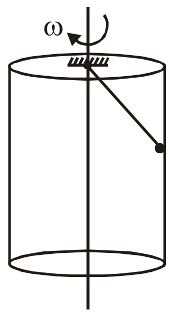
(B) the tension in the string is double the weight of the particle

(C) the velocity of the particle = $2.8\sqrt{3}$ m/s

(D) the centripetal acceleration of the particle is $9.8\sqrt{3}$ m/s^2.

CR004

25. In the shown figure inside a fixed hollow cylinder with vertical axis a pendulum is rotating in a conical path with its axis same as that of the cylinder with uniform angular velocity. Radius of cylinder is 30 cm, length of string is 50 cm and mass of bob is 400 gm. The bob makes contact with the inner frictionless wall of the cylinder while moving :-



(A) The minimum value of angular velocity of the bob so that it does not leave contact is 5 rad/s

(B) Tension in the string is 5N for all values of angular velocity

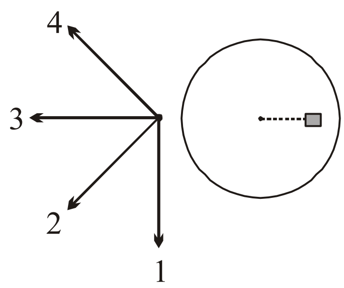
(C) For angular velocity of 10 rad/s the bob pushes the cylinder with a force of 9N

(D) For angular velocity of 10 rad/s, tension in the string is 20N

CR0049

MATRIX MATCH TYPE QUESTION

26. A block is placed on a horizontal table which can rotate about its axis.The block is placed at a certain distance from centre as shown in figure. Table rotates such that particle does not slide. Select possible direction of net acceleration of block at the instant shown in figure.



	Column-I		**Column-II**
(A)	When rotation is clockwise with constant ω	(P)	1
(B)	When rotation is clock wise with decreasing ω	(Q)	2
(C)	When rotation is clockwise with increasing ω	(R)	3
(D)	Just after clockwise rotation begins from rest	(S)	4

CR0050

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. A ring of radius r and mass per unit length m rotates with an angular velocity ω in free space. The tension in the ring is :

(A) zero (B) $\frac{1}{2}m\omega^2 r^2$ (C) $m\omega^2 r^2$ (D) $mr\omega^2$

CR0051

2. A uniform rod of mass m and length ℓ rotates in a horizontal plane with an angular velocity ω about a vertical axis passing through one end. The tension in the rod at a distance x from the axis is :

(A) $\frac{1}{2}m\omega^2 x$ (B) $\frac{1}{2}m\omega^2 \frac{x^2}{\ell}$ (C) $\frac{1}{2}m\omega^2\ell\left(1-\frac{x}{\ell}\right)$ (D) $\frac{1}{2}\frac{m\omega^2}{\ell}[\ell^2 - x^2]$

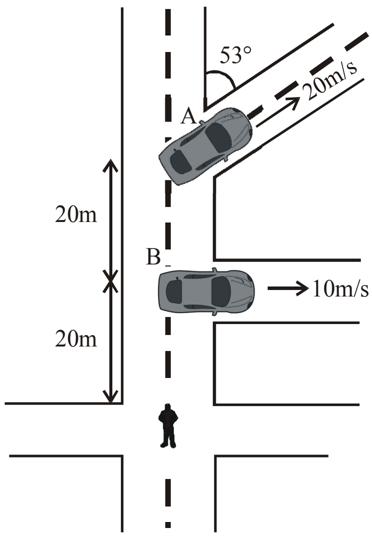
CR0052

3. The magnitude of displacement of a particle moving in a circle of radius a with constant angular speed ω varies with time t as :-

(A) $2a\sin\omega t$ (B) $2a\sin\frac{\omega t}{2}$ (C) $2a\cos\omega t$ (D) $2a\cos\frac{\omega t}{2}$

CR0053

4. A traffic policeman standing at the intersection sees 2 cars A & B turning at angles 53° & 90° respectively (as shown in the figure). Their velocities are $V_A = 20$ m/s, $V_B = 10$ m/s. Then, which car appears to be moving faster to the traffic policeman :-

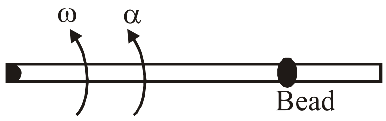


(A) A (B) B (C) Both equally fast (D) Insufficient info.

CR0054

5. A bead is constrained to move on rod in gravity free space as shown in figure. The rod is rotating with angular velocity ω and angular acceleration α about its end. If μ is coefficient of friction. Mark the correct option. (Rod rotates in the plane of paper.)

(A) If $\mu = \frac{\omega^2}{\alpha}$ friction on bead is static in nature



(B) If $\mu > \frac{\omega^2}{\alpha}$ friction on bead is kinetic in nature

(C) If $\mu < \frac{\omega^2}{\alpha}$ friction is static

(D) If bead does not slide relative to rod. Friction will not exist between bead and rod.

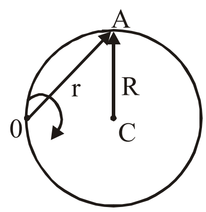
CR0055

6. A particle is moving in a circular path. The acceleration and momentum of the particle at a certain moment are $\vec{a} = \left(4\hat{i} + 3\hat{j}\right)$ m/s^2 and $\vec{p} = \left(8\hat{i} - 6\hat{j}\right)$kg–m/s. The motion of the particle is

(A) uniform circular motion
(B) accelerated circular motion
(C) de–accelerated circular motion
(D) we can not say anything with $\vec{a}$ and $\vec{p}$ only

CR0056

7. A particle A moves along a circle of radius R=50 cm so that its radius vector r relative to the point O (figure) rotates with the constant angular velocity ω=0.40 rad/s. Then modulus of the velocity of the particle, and the modulus of its total acceleration will be



(A) v= 0.4 m/s, a = 0.4 m/s^2
(B) v = 0.32 m/s, a = 0.32 m/s^2
(C) v = 0.32 m/s, a = 0.4 m/s^2
(D) v = 0.4 m/s, a = 0.32 m/s^2

CR0057

MULTIPLE CORRECT TYPE QUESTIONS

8. For a curved track of radius R, banked at angle θ (Take $v_0 = \sqrt{Rg\tan\theta}$)

(A) a vehicle moving with a speed v_0 is able to negotiate the curve without calling friction into play at all

(B) a vehicle moving with any speed $v > v_0$ is always able to negotiate the curve, with friction called into play

(C) a vehicle moving with any speed $v < v_0$ must have the force of friction into play

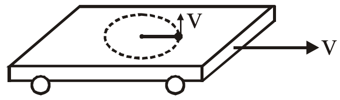
(D) the minimum value of the angle of banking for a vehicle parked on the banked road can stay there without slipping, is given by $\theta = \tan^{-1}\mu_0$ (μ_0 = coefficient of static friction)

CR0058

9. On a train moving along east with a constant speed v, a boy revolves a bob with string of length ℓ on smooth surface of a train, with equal constant speed v relative to train. Mark the correct option(s).

(A) Maximum speed of bob is 2 v in ground frame.

(B) Tension in string connecting bob is $\frac{4mv^2}{\ell}$ at an instant.



(C) Tension in string is $\frac{mv^2}{\ell}$ at all the moments.

(D) Minimum speed of bob is zero in ground frame.

CR0059

10. Let $\vec{v}(t)$ be the velocity of a particle at time t. Then :

(A) $|d\vec{v}(t)/dt|$ and $d|\vec{v}(t)|/dt$ are always equal

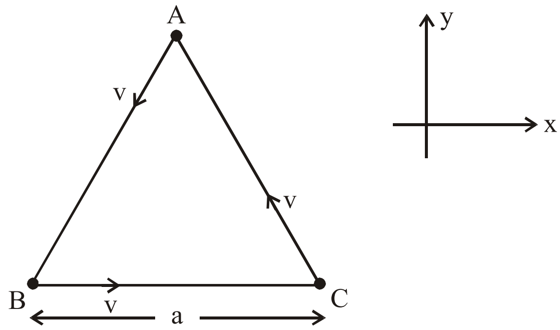
(B) $|d\vec{v}(t)/dt|$ and $d|\vec{v}(t)|/dt$ may be equal

(C) $d|\vec{v}(t)|/dt$ can be zero while $|d\vec{v}(t)/dt|$ is not zero

(D) $d|\vec{v}(t)|/dt \neq 0$ implies $|d\vec{v}(t)/dt| \neq 0$

CR0060

11. Three particles A, B, C are located at the corners of an equilateral triangle as shown in figure. Each of the particle is moving with velocity v. Then at the instant shown, the relative angular velocity of



(A) A wrt B is $\frac{v\cos 30°}{a}$ in z-direction

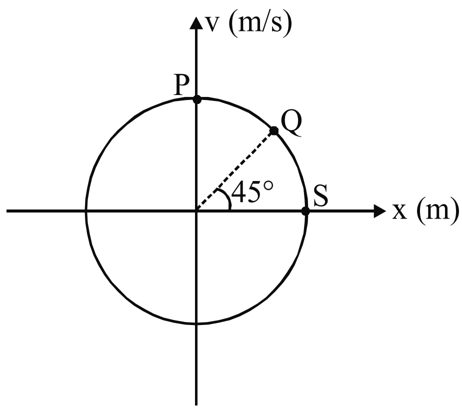
(B) B wrt C is $\frac{v\cos 30°}{a}$ in z-direction

(C) A wrt C is $\frac{v\cos 30°}{a}$ in z-direction

(D) B wrt A is $\frac{v\cos 30°}{a}$ in –z direction

CR0061

12. A particle is in motion on the x-axis. The variation of its velocity with position is as shown. The graph is circle and its equation is $x^2 + v^2 = 1$, where x is in m and v in m/s. The **CORRECT** statement(s) is/are :-



(A) When x is positive, acceleration is negative.

(B) When x is negative, acceleration is positive.

(C) At Q, acceleration has magnitude $\frac{1}{\sqrt{2}}$ m/s^2

(D) At S, acceleration is infinite.

CR0062

13. An ant travels along a long rod with a constant velocity $\vec{u}$ relative to the rod starting from the origin. The rod is kept initially along the positive x-axis. At t = 0, the rod also starts rotating with an angular velocity ω (anticlockwise) in x-y plane about origin. Then

(A) the position of the ant at any time t is $\vec{r} = ut[\cos\omega t\hat{i} + \sin\omega t\hat{j}]$

(B) the speed of the ant at any time t is $u\sqrt{1+\omega^2t^2}$

(C) the magnitude of the tangential acceleration of the ant at any time t is $\frac{\omega^2 tu}{\sqrt{1+\omega^2t^2}}$

(D) the speed of the ant at any time t is $\sqrt{1+2\omega^2t^2u}$

CR0063

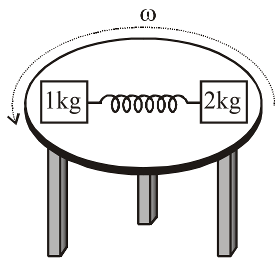
14. On a circular turn table rotating about its center horizontally with uniform angular velocity ω rad/s placed two blocks of mass 1 kg and 2 kg, on a diameter symmetrically about center. Their separation is 1m and friction is sufficient to avoid slipping. The spring between them as shown is stretched and applied force of 5N. If f_1 and f_2 are values of friction on 1 kg & 2kg block respectively:-

(A) For ω = 2 rad/s, f_1 = 3N & f_2 = 1N

(B) For ω = 3 rad/s, f_1 = 0.5 N & f_2 = 4N

(C) For ω = $\sqrt{10}$ rad/s, f_1 = 0 & f_2 = 5N

(D) For ω = $\sqrt{10}$ rad/s, f_1 = 0 & f_2 = 0N



CR0064

15. Two particles move on a circular path (one just inside and the other just outside) with angular velocities ω and 5ω starting from the same point. Then

(A) they cross each other at regular intervals of time $\frac{2\pi}{4\omega}$ when their angular velocities are oppositely directed.

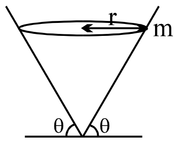
(B) they cross each other at points on the path subtending an angle of 60° at the centre if their angular velocities are oppositely directed.

(C) they cross at intervals of time $\frac{\pi}{3\omega}$ if their angular velocities are oppositely directed.

(D) they cross each other at points on the path subtending 90° at the centre if their angular velocities are in the same sense.

CR0065

16. A ball of mass 'm' is rotating in a circle of radius 'r' with speed v inside a smooth cone as shown in figure. Let N be the normal reaction on the ball by the cone, then choose the correct option.



(A) $N = mg\cos\theta$

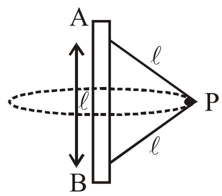
(B) $g\sin\theta = \frac{v^2}{r}\cos\theta$

(C) $N\sin\theta - \frac{mv^2}{r} = 0$

(D) none of these

CR0066

17. A particle P of mass m is attached to a vertical axis by two strings AP and BP of length l each. The separation AB=l. P rotates around the axis with an angular velocity ω. The tensions in the two strings are T_1 and T_2



(A) $T_1=T_2$

(B) $T_1+T_2=m\omega^2 l$

(C) $T_1-T_2=2mg$

(D) BP will remain taut only if $\omega \geq \sqrt{\frac{2g}{\ell}}$

CR0067

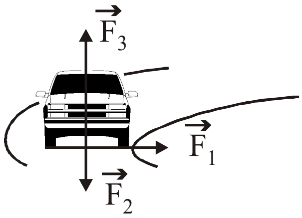
MATRIX MATCH TYPE QUESTION

18. Column-I shows certain situations and column-2 shows information about forces.

Column - I **Column - II**

Situation

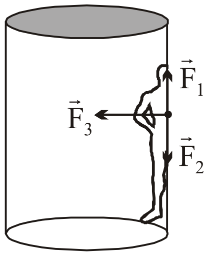
(A)



Front view of a car rounding a curve with constant speed.

(P) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3$ is centripetal force.

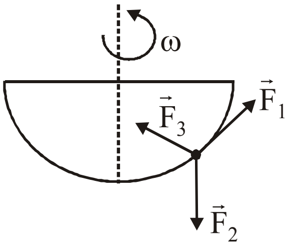
(B)



Passengers in a rotor not sliding relative to rotor wall cylindrical rotor is rotating with constant angular velocity about its symmetry axis.

(Q) $\vec{F}_1$ is static friction.

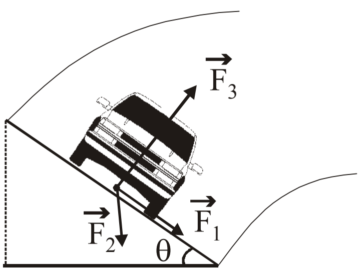
(C)



Particle kept on rough surface of a bowl, no relative motion of particle in bowl, bowl has constant angular velocity

(R) $\vec{F}_1$ can be in direction opposite to that shown in figure.

(D)



Car moving on a banked road with constant speed, no sideways skidding

(S) $\vec{F}_1 + \vec{F}_2 = \vec{0}$

(T) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$

EXERCISE (J-M)

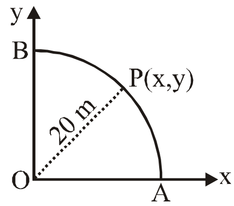
1. For a pacticle in uniform circular motion, the acceleration $\vec{a}$ at a point P(R, θ) on the circle of radius R is (Here θ is measured from the x-axis). **[AIEEE - 2010]**

(1) $\frac{v^2}{R}\hat{i}+\frac{v^2}{R}\hat{j}$ (2) $-\frac{v^2}{R}\cos\theta\,\hat{i}+\frac{v^2}{R}\sin\theta\,\hat{j}$

(3) $-\frac{v^2}{R}\sin\theta\,\hat{i}+\frac{v^2}{R}\cos\theta\,\hat{j}$ (4) $-\frac{v^2}{R}\cos\theta\,\hat{i}-\frac{v^2}{R}\sin\theta\,\hat{j}$

CR0069

2. A point P moves in counter clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when t = 2s is nearly : **[AIEEE - 2010]**



(1) 14 m/s² (2) 13 m/s² (3) 12 m/s² (4) 7.2 m/s²

CR0070

3. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is :- **[AIEEE - 2012]**

(1) 1 : 1 (2) $m_1 r_1 : m_2 r_2$ (3) $m_1 : m_2$ (4) $r_1 : r_2$

CR0071

4. Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute(rpm) to ensure proper mixing is close to : (Take the radius of the drum to be 1.25 m and its axle to be horizontal): **[JEE Main (Online) - 2016]**

(1) 8.0 (2) 0.4 (3) 1.3 (4) 27.0

CR0072

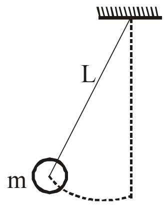
5. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then, **[JEE Main-2018]**

(1) $T\propto R^{\frac{n}{2}+1}$ (2) $T\propto R^{(n+1)/2}$ (3) $T\propto R^{n/2}$ (4) $T\propto R^{3/2}$ for any n

CR0073

EXERCISE (J-A)

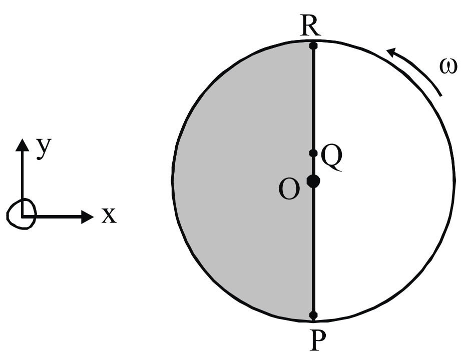
1. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is **[IIT-JEE-2011]**



(A) 9 (B) 18 (C) 27 (D) 36

CR0074

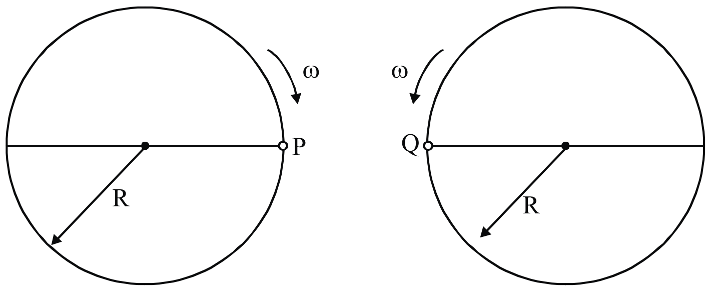
2. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed $\frac{1}{8}$ rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then **[IIT-JEE-2012]**

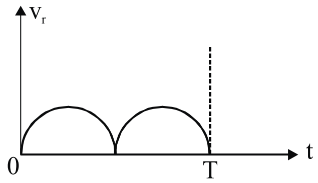


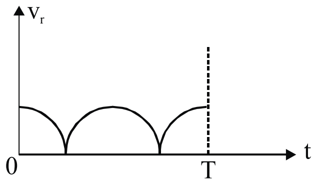
(A) P lands in the shaded region and Q in the unshaded region.
(B) P lands in the unshaded region and Q in the shaded region.
(C) Both P and Q land in the unshaded region.
(D) Both P and Q land in the shaded region.

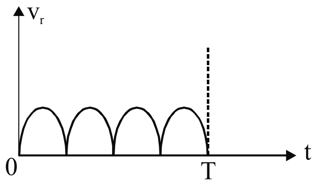
CR0075

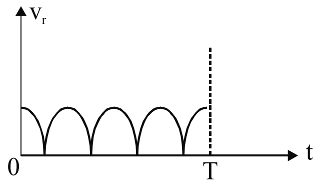
3. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time t =0, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by **[IIT-JEE 2012]**



(A)


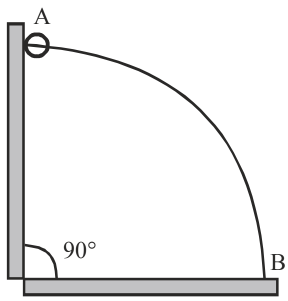
(B)


(C)


(D)


CR0076

4. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is :- **[IIT-JEE Advanced 2014]**



(A) Always radially outwards

(B) Always radially inwards

(C) Radially outwards initially and radially inwards later.

(D) Radially inwards initially and radially outwards later.

CR0077

Paragraph for Question No. 5 and 6

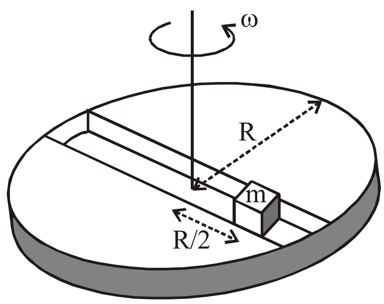
A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is

$$\vec{F}_{rot} = \vec{F}_{in} + 2m(\vec{\upsilon}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega},$$

where $\vec{\upsilon}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the centre of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $(\vec{\omega} = \omega\hat{k})$. A small block of mass m is gently placed in the slot at $\vec{r} = (R/2)\hat{i}$ at t = 0 and is constrained to move only along the slot.

[IIT-JEE Advanced-2016]



5. The distance r of the block at time t is :

(A) $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$ (B) $\frac{R}{2}\cos 2\omega t$ (C) $\frac{R}{2}\cos \omega t$ (D) $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$

CR0078

6. The net reaction of the disc on the block is :

(A) $-m\omega^2 R\cos\omega t\hat{j} - mg\hat{k}$ (B) $m\omega^2 R\sin\omega t\hat{j} - mg\hat{k}$

(C) $\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\hat{k}$ (D) $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t})\hat{j} + mg\hat{k}$

CR0078

ANSWER KEY

EXERCISE (S-1)

1. Ans. 9.9 ms^{-2}, in radial direction towards the centre at all points.

2. Ans. (i) 2 rad/s^2 (ii) 12+2t for $t \le 2s$, 16 for $t \ge 2s$ (iii) a = 169.01 m/s^2 (iv) 44 rad

3. Ans. (a) 4 cm/s^2 (b) 2 cm/s^2 (c) $\sqrt{20}\,cm/s^2$ **4. Ans.** (i) 75m/s^2, (ii) 125m/s^2

5. Ans. 4 **6. Ans. 024** **7. Ans.** $\frac{v}{2d}$ **8. Ans.** $a_t = \frac{20}{\sqrt{13}}$, $a_n = \frac{30}{\sqrt{13}}$

9. Ans. $\sqrt{3}b/2, b/2, b$ **10. Ans. 6** **11. Ans.** (i) mg, (ii) $\frac{mv^2}{r}$

12. Ans. Yes, $a_c = 8$, $\mu g = 1$ **13. Ans.** T = 6.6 N, $v_{max} = 35\ ms^{-1}$

14. Ans. $\sqrt{2g}$ rad/s **15. Ans.** (a) $\sqrt{\frac{g}{\sqrt{L^2 - r^2}}}$ (b) $\frac{mgL}{\sqrt{L^2 - r^2}}$

16. Ans. 5 **17. Ans. 15 km** **18. Ans. 2**

EXERCISE (S-2)

1. Ans. (i) $\frac{V_0^2}{2g}$, (ii) 2g **2. Ans.** $2V_0$ **3. Ans.** $\mu = \frac{3}{16}$

4. Ans. (i) 36N, (ii) 11.66rad/sec, (iii) 0.1m, 0.2m **5. Ans. 2 tanθ** **6. Ans.** θ = 60°

EXERCISE (O-1)

1. Ans. (D) **2. Ans. (B)** **3. Ans. (A)** **4. Ans. (A)** **5. Ans. (A)** **6. Ans. (D)**

7. Ans. (C) **8. Ans. (A)** **9. Ans. (D)** **10. Ans. (C)** **11. Ans. (A)** **12. Ans. (B)**

13. Ans. (D) **14. Ans. (A)** **15. Ans. (B)** **16. Ans. (A)** **17. Ans. (C)** **18. Ans. (A)**

19. Ans. (A) **20. Ans. (B)** **21. Ans. (C)**

22. Ans. (A, B) **23. Ans. (A, B)** **24. Ans. (A,B,C,D)** **25. Ans. (A, B, C)**

26. Ans. (A)-R; (B)-S; (C)-Q; (D)-P

EXERCISE (O-2)

1. Ans. (C) **2. Ans. (D)** **3. Ans. (B)** **4. Ans. (B)**

5. Ans. (A) **6. Ans. (B)** **7. Ans. (D)**

8. Ans. (A, C) **9. Ans. (A,C,D)** **10. Ans. (B,C,D)** **11. Ans. (A,B,C)**

12. Ans. (A, B, C) **13. Ans. (A, B, C)** **14. Ans. (A, B, C)** **15. Ans. (B, C, D)**

16. Ans. (B, C) **17. Ans. (B, C, D)**

18. Ans. (A) P,Q (B) P,Q,S (C) P,Q,R (D) P,Q,R

EXERCISE (J-M)

1. Ans. (4) **2. Ans. (1)** **3. Ans. (4)** **4. Ans. (4)**

5. Ans. (2)

EXERCISE (J-A)

1. Ans. (D) **2. Ans. (C OR D)** **3. Ans. (A)** **4. Ans. (D)** **5. Ans. (D)**

6. Ans. (C)