CHAPTER TWENTY EIGHT

Mathematical Reasoning

SOME DEFINITIONS

1. A *statement* or proposition is a declarative sentence which is either true or false but not both simultaneously.
2. A paradox is a sentence which is both true and false simultaneously.
3. A statement is said to be simple if it cannot be broken further into simple statements otherwise statement is said to be compound.
   Truth value of a compound statement is completely determined by its constituent statements.
4. Table for basic logical connectives.

p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow p$	$\sim p$
T	T	T	T	T	T	F
T	F	F	T	F	F	F
F	T	F	T	T	F	T
F	F	F	F	T	T	T

5. Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$
6. Converse of the conditional statement $p \rightarrow q$ is $q \rightarrow p$
7. Inverse of the conditional statement $p \rightarrow q$ is $\sim p \rightarrow \sim q$.
8. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

TIP

Negation of an if statement does not start with the word if.

Some Definitions

1. A statement is said to be a *tautology* if it is always true.
2. A statement is a *contradiction* if it is always false.
3. Dual of a compound statement not involving logical connective other than $\wedge$ and $\vee$ is obtained by replacing each occurrence of $\wedge$ and $\vee$ by $\vee$ and $\wedge$ respectively, and each occurance of T by F and F by T.

Some Logical Equivalences

Given any statement variables p, q and r, a tautology t and a contradiction c, the following logical equivalances hold:

1. Commutative laws:
   (i) $p \wedge q \equiv q \wedge p$
   (ii) $p \vee q \equiv q \vee p$
2. Associative laws:
   (i) $(p \wedge q) \wedge r = p \wedge (q \wedge r)$
   (ii) $(p \vee q) \vee r = p \vee (q \vee r)$
3. Distributive laws:
   (i) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
   (ii) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (q \vee r)$
4. Identity laws:
   (i) $p \wedge t \equiv p$
   (ii) $p \vee c \equiv p$
5. Negation laws:
   (i) $p \vee (\sim p) \equiv t$
   (ii) $p \wedge (\sim p) \equiv c$
6. Idempotent laws:
   (i) $p \wedge p \equiv p$
   (ii) $p \vee p \equiv p$
7. De Morgan's Laws:
   (i) $\sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$
   (ii) $\sim (p \vee q) \equiv (\sim p) \wedge (\sim q)$
8. Universal bound laws:
   (i) $p \vee t \equiv t$
   (ii) $p \wedge c \equiv c$
9. Absorption laws:
   (i) $p \vee (p \wedge q) \equiv p$
   (ii) $p \wedge (p \vee q) \equiv p$
10. Double negative laws:
    $\sim (\sim p) \equiv p$
11. Negation of t and c
    (i) $\sim t \equiv c$
    (ii) $\sim c \equiv t$

The principle of Duality Let s and t be statements that contain no logical connectives other than $\wedge$ and $\vee$. If $s \leftrightarrow t$, then $s^d \leftrightarrow t^d$ where s^d denotes dual of s etc.

Quantifiers are the phrases like "There exists" and "for every".

SOLVED EXAMPLES

Concept-based Straight Objective Type Questions

Example 1: Which of the following is not a statement?

(a) 17 is a prime number
(b) 22 is an odd number
(c) What a beautiful flower !
(d) New Delhi is Capital of India

Ans. (c)

Solution: (a), (b) and (d) can be assigned values true or false:

Example 2: Let p and q be the statements:

p : Rakshit gets 100% marks in mathematics
q : Rakshit is a good dancer.
The verbal form of $\sim p \to q$ is

(a) If Rakshit does not get 100% marks in mathematics, then Rakshit is a good dancer
(b) If Rakshit gets 100% marks in mathematics then Rakshit is a good dancer
(c) Mathematics is good for dancing
(d) Rakshit is good in mathematics and dance

Ans. (a)

Solution: Verbal interpretation of $\sim p \to q$ is
If Rakshit does not get 100% marks in methematics then Rakshit is a good dancer.

Example 3: If truth value of p is T, q is F, then truth values of $(p \to q)$ and $(q \to p) \vee (\sim p)$ is are respectively

(a) F, F (b) F, T
(c) T, F (d) T, T

Ans. (b)

Solution: If $p = T$, $q = F$, then $p \to q$ is F. and $q \to p$ is T.
$\therefore$ truth value of $(q \to p) \vee (\sim p)$ is T

Example 4: If p, q, r are three statements and truth value of $p \wedge q \to r$ is F, then truth values of p, q, r are respectively

(a) T, F, T (b) T, T, F
(c) F, T, T (d) F, F, T

Ans. (b)

Solution: As $p \wedge q \to r$ has truth value F, $p \wedge q$ is T and r is F
$\therefore p = T, q = T, r = F$

Example 5: If p is any statement, then which of the following is a contradiction?

(a) $p \wedge p$ (b) $p \wedge \sim p$
(c) $p \vee (\sim p)$ (d) $(\sim p) \wedge (\sim p)$

Ans. (b)

Solution: $p \wedge (\sim p) \equiv F$ for both the truth values of p.

Example 6: Let p and q be two statements. If truth value of $p \to (\sim p \wedge q)$ is F, then truth values of p, q are respectively:

(a) F, F (b) T, F
(c) F, T (d) T, T

Ans. (b)

Solution: As $p \to (\sim p \vee q)$ has truth value F, p must be T and $\sim p \vee q$ must be F. As $\sim p$ is F, q must be F. Thus, truth values of p and q are respectively T and F.

Example 6: For integers m and n, both greater than 1, consider the following three statements

p : m divides n
q : m divides n^2
r : m is prime
Then

(a) $q \wedge r \to p$ (b) $p \wedge q \to r$
(c) $q \to r$ (d) $q \to p$

Ans. (a)

Solution: We know that if m is prime and $m|n^2$ the $nm|n$. Thus, $q \wedge r \to p$

Example 7: The statement $\sim (p \leftrightarrow \sim q)$ is

(a) equivalent to $\sim p \leftrightarrow q$ (b) a tautology
(c) a fallacy (d) equivalent to $p \leftrightarrow q$

Ans. (d)

Solution:
$$\begin{aligned} & \sim (p \leftrightarrow \sim q) \\ & \equiv \sim((p \to \sim q) \wedge (\sim q \to p)) \\ & \equiv \sim((\sim p \vee \sim q) \wedge (q \vee p)) \end{aligned}$$

$\equiv \sim(\sim q \vee \sim p) \vee (\sim(q \vee p))$
$\equiv (\sim(q) \wedge \sim(\sim p)) \vee (\sim q \wedge \sim p)$
$\equiv (q \wedge p) \vee (\sim q \wedge \sim p)$
$\equiv [(q \wedge p) \vee (\sim q)] \wedge [(q \wedge p) \vee (\sim p)]$
$\equiv [(q \vee (\sim q)) \wedge (p \vee (\sim q))] \wedge [(q \vee (\sim p)) \wedge (p \vee (\sim p))]$
$\equiv [t \wedge (\sim q \vee p)] \wedge [((\sim p) \vee q)) \wedge t]$
$\equiv (\sim q \vee p) \wedge (\sim p \vee q)$
$\equiv (q \to p) \wedge (p \to q)$
$\equiv p \leftrightarrow q$

Alternative Solution. Use the following table.

p	**q**	**~q**	**p ↔ ~q**	**~(p ↔ ~q)**	**p ↔ q**
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

From the last two colums, we get $\sim(p \leftrightarrow (\sim q)) \equiv p \leftrightarrow q$

Example 8: The statement $\sim(p \wedge q) \vee q$:
(a) is a tautology
(b) is equivalent to $(p \wedge q) \vee (\sim q)$
(c) is equivalent to $p \vee q$
(d) is a contradiction

Ans. (a)

Solution: $\sim(p \wedge q) \vee q$
$\equiv ((\sim p) \vee (\sim q)) \vee q$
$\equiv (\sim p) \vee ((\sim q) \vee q)$
$\equiv (\sim p) \vee t \equiv t$

Thus, $\sim(p \wedge q) \vee q$ is a tautology.

Example 9: If p is any logical statement, then:
(a) $p \wedge (\sim p)$ is a tautology.
(b) $p \vee (\sim p)$ is a contradiction
(c) $p \wedge p \equiv p$
(d) $p \vee (\sim p) \equiv p$

Ans. (c)

Solution: Idempotent law $p \wedge p \equiv p$ is always true.

Example 10: If p, q are two statements, then $\sim(\sim p \wedge q) \wedge (p \wedge q)$ is logically equivalent to
(a) p (b) q
(c) $p \wedge q$ (d) $(\sim p) \vee q$

Ans. (a)

Solution: $\sim(\sim p \wedge q) \wedge (p \vee q)$
$\equiv ((\sim(\sim p)) \vee (\sim q)) \wedge (p \vee q)$
$\equiv (p \vee (\sim q)) \wedge (p \vee q)$
$\equiv p \vee (\sim q \wedge q) \equiv p \vee c \equiv p$

LEVEL 1

Straight Objective Type Questions

Example 11: Which of the following is not a negation of the statement p: $\sqrt{5}$ is rational?
(a) It is not the case that $\sqrt{5}$ is rational
(b) $\sqrt{5}$ is not rational
(c) $\sqrt{5}$ is an irrational number
(d) none of these

Ans. (d)

Solution: (a), (b) and (c) are negation of p.

Example 12: Negation of $\sqrt{5}$ is irrational or 3 is rational is
(a) $\sqrt{5}$ is rational or 3 is irrational
(b) $\sqrt{5}$ is rational and 3 is rational
(c) $\sqrt{5}$ is rational and 3 is irrational
(d) none of these

Ans. (c)

Solution: Use De Morgan's Law.

Example 13: Contrapositive of the statement.
If a number is divisible by 9, then it is divisible by 3, is
(a) If a number is not divisible by 3, it is not divisible by 9.
(b) If a number is not divisible by 3, it is divisible by 9.
(c) If a number is not divisible by 9, it is not divisible by 3.
(d) none of these

Ans. (a)

Solution: Contrapositive of $p \to q$ is $\sim q \to \sim p$.

Example 14: Converse of the statement:
If a number n is even, then n^2 is even, is
(a) If a number n^2 is even, then n is even
(b) If n is not even, then n^2 is not even

(c) Neither n nor n^2 is even
(d) none of these

Ans. (a)

◎ **Solution:** Converse of $p \to q$ is $q \to p$.

Example 15: Which of the following statement is not equivalent to $p \leftrightarrow q$?

(a) p if and only if q
(b) q if and only if p
(c) p is necessary and sufficient for q
(d) none of these

Ans. (d)

◎ **Solution:** Each of (a), (b), (c) is equivalent to $p \leftrightarrow q$.

Example 16: Negation of the statement

p: for every real number, either $x > 1$ or $x < 1$ is

(a) There exist a real number x such that neither $x > 1$ nor $x < 1$
(b) There exist a real number x such that $0 < x < 1$
(c) There exist a real number x such that neither $x \geq 1$ nor $x \leq 1$
(d) none of these

Ans. (a)

◎ **Solution:** Negation of $p \vee q$ is $(\sim p) \wedge (\sim q)$.

Example 17: Contrapositive of

p: "If x and y are integers such that xy is odd then both x and y are odd" is

(a) If both x and y are odd, then xy is odd
(b) If both x and y are even, then xy is even
(c) If x or y is odd, then xy is odd
(d) If it is false that both x and y are odd then the product xy is odd

Ans. (d)

◎ **Solution:** Contrapositive of $p \to q$ is $\sim q \to \sim p$.

Example 18: The statement $p \to (q \to p)$ is equivalent to

(a) $p \to (p \to q)$ (b) $p \to (p \vee q)$
(c) $p \to (p \wedge q)$ (d) $p \to (p \leftrightarrow q)$

Ans. (b)

◎ **Solution:** $p \to (q \to p) \equiv \sim p \vee (q \to p)$

$$\equiv (\sim p) \vee (\sim q \vee p)$$
$$\equiv (\sim q) \vee (p \vee \sim p)$$
$$\equiv (\sim q) \vee T = T$$

$\therefore$ $p \to (q \to p)$ is a tautology.

Also $p \to (p \vee q) \equiv \sim p \vee (p \vee q)$

$$\equiv (\sim p \vee p) \vee q \equiv T \vee q = T$$

$\therefore$ $p \to (p \vee q)$ is also a tautology.

Thus, $p \to (q \to p)$ is equivalent to $p \to (p \vee q)$.

Example 19: The statement $p \to (q \vee r)$ is **not** equivalent to

(a) $(p \to q) \vee (p \to r)$
(b) $p \wedge (\sim q) \to r$
(c) $p \wedge (\sim r) \to q$
(d) $p \wedge q \to (p \wedge r) \vee (q \wedge r)$

Ans. (d)

◎ **Solution:** $p \to (q \vee r) \equiv (\sim p) \vee (q \vee r)$

$$\equiv (\sim p \vee q) \vee (\sim p \vee r)$$
$$\equiv (p \to q) \vee (p \to r)$$

Also, $p \to (q \vee r) \equiv (\sim p) \vee (q \vee r) \equiv (\sim p \vee q) \vee r$

$$\equiv \sim (p \wedge (\sim q)) \vee r$$
$$\equiv p \wedge (\sim q) \to r$$

Interchanging the roles of q and r in the above paragraph, we find

$$p \to (q \vee r) \equiv p \wedge (\sim q) \to r \equiv p \wedge (\sim r) \to q$$

For $p = T$, $q = F$, $r = F$, $p \to (q \vee r)$ is F, but

$(p \wedge q) \to (p \vee r) \vee (q \wedge r)$ is T.

Therefore, $p \to (q \vee r)$ and

$$p \wedge q \to (p \wedge r) \vee (q \wedge r)$$

are not equivalent.

Example 20: Negation of $p \to q$ is

(a) $p \wedge (\sim q)$ (b) $\sim p \vee q$
(c) $\sim q \to \sim p$ (d) $p \vee (\sim q)$

Ans. (a)

◎ **Solution:** $\sim (p \to q) \equiv \sim (\sim p \vee q) \equiv \sim (\sim p) \wedge (\sim q)$ [De Morgan's Laws]

$\therefore$ $\sim (p \to q) \equiv p \wedge (\sim q)$

Example 21: Contrapositive of $p \to (q \to r)$ is logically equivalent to

(a) $p \to (q \to r)$ (b) $(q \to r) \to \sim p$
(c) $p \vee q \to r$ (d) $(q \to r) \to p$

Ans. (a)

◎ **Solution:** Contrapositive of $p \to (q \to r)$ is

$$\sim (q \to r) \to \sim p$$
$$\equiv \sim [\sim (q \to r)] \vee (\sim p)$$
$$\equiv (q \to r) \vee (\sim p) \equiv (\sim p) \vee (q \to r)$$
$$\equiv p \to (q \to r)$$

Example 22: Let S be a non-empty subset of **R**. Consider the following statement:

P : There is a rational number $x \in S$ such that $x > 0$.

Which of the following statements is the negation of the statement P ?

(a) Every rational number $x \in S$ satisfies $x \leq 0$.
(b) $x \in S$ and $x \leq 0 \Rightarrow x$ is not a rational number.

(c) There is a rational number $x \in S$ such that $x \leq 0$.
(d) There is no rational number $x \in S$ such that $x \leq 0$.

Ans. (a)

◎ **Solution:** The statement P can be written as follows :

$$P : \exists\, x \in Q \cap S \text{ such that } x > 0$$

Negation of P is $\sim P : \forall\, x \in Q \cap S$ satisfies $x \leq 0$.

Example 23: The negation of the following statement :

P : Neha lives in Ludhiana or she lives in Gurudaspur.

(a) Neha does not live in Ludhiana or she does not live in Gurudaspur.
(b) Neha does not live in Ludhiana and she does not live in Gurudaspur
(c) Neha does not live in Punjab.
(d) None of these.

Ans. (b)

◎ **Solution:** Let p and q be the statements defined as follows :

p : Neha lives in Ludhiana

q : Neha lives in Gurudaspur.

The statement P is

$$P : p \vee q$$

Negation of p is

$$\sim P : \sim (p \vee q)$$

or $\quad \sim P : (\sim p) \wedge (\sim q) \quad [\because \sim (p \vee q) \equiv (\sim p) \wedge (\sim q)]$

Thus, negation of P is given by (b)

Example 24: The converse of the statement "If $a < b$ then $x + a < x + b$", is

(a) If $a > b$ then $x + a > x + b$
(b) If $a \geq b$ then $x + a \geq x + b$
(c) If $x + a < x + b$ then $a < b$
(d) If $x + a \geq x + b$ then $a \geq b$

Ans. (c)

◎ **Solution:** See theory.

Example 25: Which of the following is the conditional statement $p \rightarrow q$?

(a) p is necessary for q
(b) p is sufficient for q
(c) p only if q
(d) if q then p

Ans. (b)

◎ **Solution:** As $p \rightarrow q$, therefore truth of p is sufficient for truth of q.

Example 26: Converse of the statement

"if x^2 is odd then x is odd" is

(a) if x^2 is even then x is even
(b) if x is odd then x^2 is odd
(c) if x is odd then x^2 is even
(d) if x is even then x^2 is odd

Ans. (b)

◎ **Solution:** See theory.

Example 27: Consider the following statements :

P: Suman is brilliant

Q: Suman is rich

R: Suman is honest

The negation of the "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as

(a) $\sim (P \wedge \sim R) \leftrightarrow Q$ (b) $\sim p \wedge (Q \leftrightarrow \sim R)$
(c) $\sim (Q \leftrightarrow (P \wedge \sim R))$ (d) $\sim Q \leftrightarrow \sim p \wedge R$

Ans. (c)

◎ **Solution:** $P \wedge \sim R$ stands for Suman is brilliant and dishonest. Thus, $P \wedge \sim R \leftrightarrow Q$ stands for Suman is brilliant and dishonest if and only if Suman is rich.

Its negation is $\sim (P \wedge \sim R \leftrightarrow Q)$ or $\sim (Q \leftrightarrow P \wedge \sim R)$

Example 28: The only statement among the following that is a tautology is:

(a) $A \wedge (A \vee B)$ (b) $A \vee (A \wedge B)$
(c) $[A \wedge (A \rightarrow B)] \rightarrow B$ (d) $B \rightarrow [A \wedge (A \rightarrow B)]$

Ans. (c)

◎ **Solution:** Note that

$A \wedge (A \vee B)$ is F when $A = F$,

$A \vee (A \wedge B)$ is F when $A = F$, $B = F$,

and $B \rightarrow [A \wedge (A \rightarrow B)]$ is F when $A = F$, $B = T$

$\therefore$ We check only (c)

$$\begin{aligned}
&[A \wedge (A \rightarrow B)] \rightarrow B \\
\equiv\ &[A \wedge (\sim A \vee B)] \rightarrow B \\
\equiv\ &[(A \wedge (\sim A))] \vee (A \wedge B)] \rightarrow B \\
\equiv\ &A \wedge B \rightarrow B \\
\equiv\ &\sim (A \wedge B) \vee B \equiv \sim [(A \wedge B) \wedge (\sim B)] \\
\equiv\ &\sim [A \wedge (B \wedge \sim B)] \equiv \sim [A \wedge F] \equiv \sim F \equiv T
\end{aligned}$$

Thus,

$$[A \wedge (A \rightarrow B)] \rightarrow B \text{ is a tautology.}$$

Example 29: The negation of the statement

"If I become teacher, then I will open a school."

(a) Either I will not become a teacher or I will not open a school.
(b) Neither I will become a teacher nor I will open a school
(c) I will not become a teacher or I will not open a school
(d) I will become a teacher and I will not open a school

Ans. (d)

Solution: Let p : I become a teacher

q : I will open a school.

The given statement is $p \to q \equiv (\sim p) \vee q$

It negation is $\sim (\sim p) \vee q) \equiv p \wedge (\sim q)$

Thus, negation of the given statement is I will become a teacher and I will not open a school.

Assertion-Reason Type Questions

Example 30:

Statement-1 $\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$

Statement-2 $\sim (p \leftrightarrow \sim q)$ is a tautology

Ans. (c)

Solution:

Table for basic logical connectives.

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim (p \to \sim q)$	$(p \leftrightarrow q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

Note that $\sim (p \leftrightarrow \sim q)$ is not tautology.

$\therefore$ Statement-2 is false.

From table $\sim (p \leftrightarrow \sim q)$ is equivalent to $(p \leftrightarrow q)$

Thus, Statement-1 is true.

Example 31: Let p, q and r be the statements.

p : X is a rectangle

q : X is a square

r : $p \to q$

Statement-1: $p \to r$ is a tautology.

Statement-2: A tautology is equivalent to T.

Ans. (d)

Solution: $p \to r \equiv p \to (p \to q)$

$$\equiv (\sim p) \vee (p \to q) \equiv (\sim p) \vee [(\sim p) \vee q]$$

$$\equiv [(\sim p) \vee (\sim p)] \vee q$$

$$\equiv (\sim p) \vee q \equiv p \to q \equiv r$$

Thus, Statement-1 is not a tautology.

Example 32: Let p, q, r be three statements.

Statement-1: If $p \to q$ and $q \to r$ then $p \to r$ is a tautology.

Statement-2: $(p \to q) \wedge (q \to r) \leftrightarrow (p \to r)$

Ans. (c)

Solution: Statement-2 is not true.

It can be checked by taking

$$p = T, q = F, r = T$$

We can write Statement-1 as

$$(p \to q) \wedge (q \to r) \to (p \to r)$$

Assume it to be F then

$$p \to r \text{ is } F \text{ and } (p \to q) \wedge (q \wedge r) \text{ is } T.$$

But $p \to r$ is $F \Leftrightarrow p = T$ and $r = F$.

Now, $(T \to q) \wedge (q \to F)$ must be T

$\Rightarrow$ $T \to q$ is T and $q \to F$ is T

$\Rightarrow$ q is T and q is F.

A contradiction.

$\therefore$ $(p \to q) \wedge (q \to r) \to (p \to r)$

cannot be F for any assignment of T and F to p, q, r.

$\therefore$ $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.

We may abbreviate the above procedure as follows:

1. $(p \to q) \wedge (q \to r) \to (p \to r)$
   F
2. $(p \to q) \wedge (q \to r) \to (p \to r)$
   T F F
3. $(p \to q) \wedge (q \to r) \to (p \to r)$
   T T T T F F T F F
4. $(p \to q) \wedge (q \to r) \to (p \to r)$
   T T T T F T F F T F F

This assignment is not possible.

Example 33: Let p, q, r and s be three statements.

Statement-1: $(p \to r) \wedge (\sim p \to q) \wedge (q \to s) \wedge (\sim r \to s)$ is a tautology.

Statement-2: $(p \to q) \wedge (\sim p \to q) \to q$ is a tautology.

Ans. (b)

Solution: Assume Statement-1 is false. Then

1. $(p \to r) \wedge (\sim p \to q) \wedge (q \to s) \to (\sim r \to s)$
   T T F F
2. $(p \to r) \wedge (\sim p \to q) \wedge (q \to s) \to (\sim r \to s)$
   T T T T T F T F F F
3. $(p \to r) \wedge (\sim p \to q) \wedge (q \to s) \to (\sim r \to s)$
   F T F T T F T F T F T F F T F F F

   Look at the encircled part. This means $\sim p \to q$ is both T and F.

   A contradiction.

$\therefore$ Statement-1 is always true.

Next assume Statement-2 is false.

Then

$$(p \to q) \wedge (\sim p \to q) \to q$$
$$\begin{matrix} F & T & F & T & TF & T & T & F & F \end{matrix}$$

Not possible

$\therefore$ Statement-2 is a tautology.

Example 34: Statement-1: $\sim (A \Leftrightarrow \sim B)$ is equivalent to $A \Leftrightarrow B$

Statement-2: $A \vee (\sim(A \wedge \sim B))$ is a tautology.

Ans. (b)

Solution: We have

$$A \vee [\sim (A \wedge \sim B)]$$
$$\equiv A \vee [\sim A \vee B]$$
$$\equiv (A \vee \sim A) \vee B \equiv T \vee B \equiv T$$

Thus, Statement-2 is true.

We have

$$\equiv \sim [A \Leftrightarrow \sim B]$$
$$\equiv \sim [(A \to \sim B) \wedge (\sim B \to A)]$$
$$\equiv \sim [(\sim A \vee \sim B) \wedge (B \vee A)]$$
$$\equiv \sim [\sim(A \wedge B) \wedge (A \vee B)]$$
$$\equiv [(A \wedge B) \vee (\sim A \wedge \sim B)]$$
$$\equiv [(A \wedge B) \vee (\sim A)] \wedge [(A \wedge B) \vee (\sim B)]$$
$$\equiv [(A \vee \sim A) \vee (B \vee \sim A)] \wedge [(A \vee \sim B) \wedge (B \vee \sim B)]$$
$$\equiv [T \wedge (\sim A \vee B)] \wedge [(A \vee \sim B) \wedge T]$$
$$\equiv (A \to B) \wedge (B \to A) \equiv A \Leftrightarrow B$$

Example 35:

Statement-1: Consider the following statements

p : Delhi is in India

q : Mumbai is not in Italy

Then negation of $p \vee q$ is Delhi is not in India and Mumbai is in Italy.

Statement-2: For two statements p and q

$$\sim (p \vee q) \equiv \sim p \wedge \sim q$$

Ans. (a)

Solution: Statement-2 is true. [See Theory]

As $\sim (p \vee q) \equiv \sim p \wedge \sim q$

$\equiv \sim$ (Delhi is in India) and $\sim$(Mumbai is not is Italy)

$\equiv$ Delhi is not in India and Mumbai is in Italy.

LEVEL 2

Straight Objective Type Questions

Example 36: Let p, q and r be the statements:

p: city X is in U.P.

q: city X is in India

$r : p \to q$

Contrapositive of r is

(a) if city X is not in India then X is not in U.P.
(b) city X is neither in U.P. nor in India
(c) city X is in India but not in U.P.
(d) none of these

Ans. (a)

Solution: Contrapositive of $p \to q$ is $\sim q \to \sim p$.

Example 37: If p, q are r are as in Example 31, then which one of the following represents converse of $p \to q$.

(a) If X is a rectangle then X is a square.
(b) If X is a rectangle then X is not a square
(c) X is a rectangle but X is not a square
(d) none of these

Ans. (a)

Solution: Converse of $p \to q$ is $q \to p$.

Example 38: Let p, q, r be three statements, then

$$(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r), \text{ is a}$$

(a) tautology (b) contradiction
(c) fallacy (d) none of these

Ans. (a)

Solution: $p \to (q \to r) \equiv \sim p \vee (q \to r)$

$$\equiv \sim p \vee (\sim q \vee r)$$
$$\equiv [(\sim p) \vee (\sim q)] \vee r$$
$$\equiv \sim (p \wedge q) \vee r$$
$$\equiv p \wedge q \to r$$

$\therefore$ given statement is a tautology.

Example 39: Let p and q be two statements, then $\sim (\sim p \wedge q) \wedge (p \vee q)$ is logically equivalent to

(a) q (b) $p \wedge q$
(c) p (d) $p \vee \sim q$

Ans. (c)

Solution: $\sim (\sim p \wedge q) \wedge (p \vee q) \equiv [\sim (\sim p) \vee (\sim q)] \wedge (p \vee q)$

$$\equiv [p \vee (\sim q) \wedge (p \vee q]$$
$$\equiv p \vee [(\sim q) \wedge q]$$
$$\equiv p \vee F \equiv p$$

EXERCISES

Concept-based Straight Objective Type Questions

1. Which of the following pairs are **not** logically equivalent?
   (a) $p \vee (p \wedge q)$ and p
   (b) $p \vee t$ and p
   (c) $\sim(p \vee q)$ and $(\sim p) \wedge (\sim q)$
   (d) $p \vee c$ and p

2. The statement $(p \wedge q) \vee (\sim p \vee (p \wedge(\sim q)))$ is logically equivalent to
   (a) $p \wedge q$ (b) p
   (c) q (d) t

3. If $(\sim p) \vee q \rightarrow \sim q$ has value F, then p, q are respectively
   (a) F, F (b) T, F
   (c) p is T or F, F (d) none of these

4. The statement $p \wedge \sim q \rightarrow r$ is logically equivalent to
   (a) $\sim p \vee q \vee r$ (b) $(\sim p \wedge q) \vee r$
   (c) $(p \wedge \sim q) \vee r$ (d) $(\sim p \vee q) \wedge r$

5. The statement $(p \vee \sim q) \wedge (\sim p \vee \sim q)$ is logically equivalent to
   (a) p (b) q
   (c) $\sim p$ (d) $\sim q$

6. Let p, q, r be the following:
   p : Rohit is healthy
   q : Rohit is wealthy
   r : Rohit is wise.
   The statement $p \vee q \rightarrow \sim r$ means
   (a) If Rohit is healthy or wealthy then Rohit is not wise
   (b) If Rohit is healthy and wealthy then Rohit is wise
   (c) Rohit is neither healthy nor wealthy nor wise
   (d) none of these

7. Let p, q, r be the following three statements:
   p : n is prime
   q : n is odd
   r : n is 2
   Then $p \rightarrow q \vee r$ means
   (a) If n is prime then n is odd or 2
   (b) If n is prime and odd then n cannot be 2
   (c) If n is odd or 2 then n is prime
   (d) If n is odd then n is prime or 2

8. The contrapositive of the statement "I go to college if its not a holiday"
   (a) If I do not go to the college then its a holiday
   (b) If I go the college then its not a holiday
   (c) I go the college and its not a holiday
   (d) If I go to college then its a holiday

9. Suppose p and q are two statements such that $p \rightarrow q$ is false, then which one of the following **not** true?
   (a) Truth value of $\sim p \vee q$ is F
   (b) Truth values of $p \wedge (\sim q)$ is T
   (c) Truth values of $(\sim p) \wedge (\sim q)$ is T
   (d) Truth values of $p \vee q$ is T

10. Which of the following is equivalent to $p \rightarrow q \vee r$?
    (a) $p \wedge (\sim q) \rightarrow r$ (b) $p \vee (\sim r) \rightarrow q$
    (c) $p \wedge (\sim r) \rightarrow \sim q$ (d) $p \vee (\sim r) \rightarrow q$

LEVEL 1

Straight Objective Type Questions

11. Let p be the proposition: Mathematics is interesting and q be mathematics is difficult, then $p \wedge q$ means
    (a) Mathematics is interesting or difficult
    (b) Mathematics is interesting and difficult
    (c) Mathematics is interesting implies it is difficult
    (d) Mathematics is interesting is equivalent to saying that it is difficult

12. If $p \rightarrow (\sim p \vee q)$ is false, the truth value of p and q are respectively
    (a) T, T (b) T, F
    (c) F, T (d) F, F

13. The contrapositive of $p \rightarrow q$ is
    (a) $q \rightarrow p$ (b) $\sim p \rightarrow \sim q$
    (c) $\sim q \rightarrow \sim p$ (d) $p \rightarrow q$

14. The contrapositive of $(p \vee q) \rightarrow r$ is
    (a) $r \rightarrow (p \vee q)$ (b) $\sim r \rightarrow (\sim p) \wedge (\sim q)$
    (c) $(\sim p) \vee (\sim q) \rightarrow \sim r$ (d) none of these

15. $\sim p \wedge q$ is logically equivalent to

(a) $p \to q$ (b) $q \to p$
(c) $\sim (p \to q)$ (d) $\sim (q \to p)$

16. Negation of $q \vee \sim (p \wedge r)$ is

(a) $\sim q \vee \sim (p \vee r)$ (b) $\sim q \wedge (p \wedge r)$
(c) $\sim q \wedge (p \wedge r)$ (d) $\sim q \vee (p \wedge r)$

17. Negation of the statement "If a number is prime then it odd", is

(a) A number is not prime but odd
(b) A number is prime but it is not odd
(c) A number is neither prime nor odd
(d) none of these

18. If p, q are two propositions, then $\sim (p \vee q) \equiv \sim p \wedge \sim q$, is

(a) a tautology (b) a contradiction
(c) a simple statement (d) none of these

19. Which one of the following is a tautology?

(a) $(p \to q) \wedge p \to q$ (b) $(p \to q) \vee p \to q$
(c) $(p \to q) \vee p \to q$ (d) $(p \to q) \wedge (\sim q) \to p$

20. Which of the following is not equivalent to $\sim p \wedge q$?

(a) $\sim (q \to p)$ (b) $\sim (p \vee \sim q)$
(c) $\sim p \to \sim q$ (d) $\sim (p \vee q)$

21. Which of the following is equivalent to $p \leftrightarrow q$?

(a) $(\sim p \vee q) \vee (p \vee q)$ (b) $(p \wedge q) \vee (\sim p \wedge \sim q)$
(c) $(p \vee q) \wedge (p \vee \sim q)$ (d) $(p \wedge q) \vee (p \vee q)$

Assertion-Reason Type Questions

22. Let p, q, r be three statements.

Statement-1: $p \leftrightarrow q \equiv (p \to q) \wedge (\sim q \vee p)$ is a tautology.

Statement-2: $p \vee q \to r \equiv (p \to r) \wedge (q \to r)$ is a tautology.

23. Let p and q be two statements.

Statement-1: $(p \vee q) \vee \sim (\sim p \wedge q)$ is logically equivalent to p.

Statement-2: $p \vee T \equiv p$

24. Let p, q and r be three statements.

Statement-1: $[p \vee (q \wedge r)] \vee [\sim (p \vee q)] \equiv p \vee q$ is a tautology.

Statement-2: $[p \vee q \to r] \leftrightarrow [\sim r \to (\sim p) \wedge (\sim q)]$ is a tautology.

25. Let p, q and r be three statements.

Statement-1: Negation of $p \wedge (q \vee r)$ is $\sim p \vee (\sim q \wedge \sim r)$

Statement-2: Negation of $p \vee q$ is $(\sim p) \wedge (\sim q)$; and that of $p \wedge q$ is $(\sim p) \vee (\sim q)$

LEVEL 2

Straight Objective Type Questions

26. Let p and q be two statements, then $q \leftrightarrow (\sim p \vee \sim q)$ is logically equivalent to

(a) p (b) q
(c) $p \to q$ (d) $\sim p \wedge q$

27. Dual of $(p \to q) \to r$ is

(a) $(q \to p) \wedge r$ (b) $p \to (q \to r)$
(c) $(p \vee \sim q) \vee r$ (d) none of these.

28. Statement $(p \vee q) \to (p \wedge q)$ is equivalent

(a) F (b) $p \leftrightarrow q$
(c) T (d) $q \to p \wedge q$

Previous Years' AIEEE/JEE Main Questions

1. The statement $p \rightarrow (q \rightarrow p)$ is equivalent to
   (a) $p \rightarrow (q \leftrightarrow p)$ (b) $p \rightarrow (p \rightarrow q)$
   (c) $p \rightarrow (p \vee q)$ (d) $p \rightarrow (p \wedge q)$ **[2008]**

2. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is a rational number iff y is a transcendental number".
   Statement-1: r is equivalent to either q or p.
   Statement-2: r is equivalent to $\sim (p \leftrightarrow \sim q)$.
   [2008]

3. **Statement-1:** $\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.
   Statement -2: $\sim (p \leftrightarrow \sim q)$ is tautology.
   [2009]

4. Let S be a non-empty subset of **R.** Consider the following statement :
   P : There is a rational number $x \in S$ such that $x > 0$.
   Which of the following statements is the negation of the statement P ?
   (a) Every rational number $x \in S$ satisfies $x \le 0$.
   (b) $x \in S$ and $x \le 0 \Rightarrow x$ is not rational.
   (c) There is a rational number $x \in S$ such that $x \le 0$.
   (d) There is no rational number x such that $x \le 0$.
   [2010]

5. Consider the following statements
   P : Suman is brilliant
   Q : Suman is rich
   R : Suman is honest
   The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as
   (a) $\sim (P \wedge \sim R) \leftrightarrow Q$
   (b) $\sim P \wedge (Q \wedge \sim R)$
   (c) $\sim (Q \leftrightarrow (P \wedge \sim R))$
   (d) $\sim Q \leftrightarrow \sim P \wedge R$ **[2011]**

6. The only statement among the followings that is a tautology is
   (a) $A \wedge (A \vee \mathrm{B})$
   (b) $A \vee (A \wedge B)$
   (c) $[A \wedge (A \rightarrow B)] \rightarrow B$
   (d) $B \rightarrow [A \wedge (A \rightarrow B)]$ **[2011]**

7. The negation of the statement
   "If I become a teacher, then I will open a school", is
   (a) Either I will not become a teacher or I will not open a school.
   (b) Neither I will become a teacher nor I will open a school.
   (c) I will not become a teacher or I will open a school.
   (d) I will become a teacher and I will not open a school.
   [2012]

8. Consider:
   Statement-1: $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy
   Statement-2: $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a tautology.
   [2013]

9. The statement $p \rightarrow (q \rightarrow p)$ is equivalent to
   (a) $p \rightarrow q$ (b) $p \rightarrow (p \vee q)$
   (c) $p \rightarrow (p \rightarrow q)$ (d) $p \rightarrow (p \wedge q)$
   [2013, online]

10. Let p and q be any two logical statements and r : $p \rightarrow (\sim p \vee q)$.
    If r has a truth value F, the truth values of p and q are respectively
    (a) F, F (b) T, T
    (c) T, F (d) F, T **[2013, online]**

11. **Statement-1:** The statement $A \rightarrow (B \rightarrow A)$ is equivalent to $A \rightarrow (A \vee B)$.
    Statement-2: The statement $\sim[(A \wedge B) \rightarrow (\sim A \vee B)]$ is a tautology. **[2013, online]**

12. For integers m and n, both greater than 1, consider the following three statements:
    P : m divides n
    Q : m divides n^2
    R : m is prime
    then
    (a) $Q \wedge R \rightarrow P$ (b) $P \wedge Q \rightarrow R$
    (c) $Q \rightarrow R$ (d) $Q \rightarrow P$ **[2013, online]**

13. The statement $\sim(p \leftrightarrow \sim q)$ is
    (a) equivalent to $\sim p \leftrightarrow q$ (b) a tautology
    (c) a fallacy (d) equivalent to $p \leftrightarrow q$
    [2014]

14. The contrapositive of the statement " if I am not feeling well, then I will go to the doctor" is

(a) If I am feeling well, then I will not go to the doctor
(b) If I will go to the doctor, then I am feeling well
(c) If I will not go to the doctor, then I am feeling well
(d) If I will go to the doctor, then I am not feeling well **[2014, online]**

15. The proposition $\sim(p \vee \sim q) \vee \sim(p \vee q)$ is logically equivalent to
(a) p (b) q
(c) $\sim p$ (d) $\sim q$ **[2014, online]**

16. Let p, q, r denote three arbitrary statements. The logically equivalent of the statement $p \to (q \vee r)$ is
(a) $(p \to \sim q) \wedge (p \to r)$ (b) $(p \to q) \vee (p \to r)$
(c) $(p \to q) \wedge (p \to \sim r)$ (d) $p \vee q \to \rho$ **[2014, online]**

17. The contrapositive of the statement "I go to school if it does not rain" is
(a) if it rains, I do not go to school
(b) if I do not go to school, it rains
(c) if it rains, I go to school
(d) if I go to school, it rains **[2014, online]**

18. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to:
(a) $s \wedge \sim r$ (b) $s \wedge (r \wedge \sim)$
(c) $s \vee (r \vee \sim s)$ (d) $s \wedge r$ **[2015]**

19. Contrapositive of the statement "If it is raining then I will not come", is
(a) if I will come, then it is not raining
(b) if I will not come, then it is raining
(c) if I will not come, then it is not raining
(d) if I will come, then it is raining **[2015, online]**

20. Consider the following statements:
P : Suman is brilliant
Q : Suman is rich
R : Suman is honest
The negation of the statement,
"Suman is brilliant and dishonest if and only if Suman is rich" can be equivalently expressed as:
(a) $\sim Q \leftrightarrow P \wedge R$ (b) $\sim Q \leftrightarrow \sim P \vee R$
(c) $\sim Q \leftrightarrow P \vee \sim R$ (d) $\sim Q \leftrightarrow P \wedge \sim R$ **[2015, online]**

21. The Boolean Expression $(p \wedge -q) \vee q \vee (\sim p \wedge q)$ is equivalent to
(a) $\sim p \wedge q$ (b) $p \wedge q$
(c) $p \vee q$ (d) $p \vee \sim q$ **[2016]**

22. Consider the following two statements:
P: If 7 is an odd number, then 7 is divisible by 2
Q: If 7 is a prime number, then 7 is an odd number
If V_1 is the truth value of the contrapositive of P and V_2 is the truth value of contrapositive of Q, then the ordered pair (V_1, V_2) equals:
(a) (F, F) (b) (F, T)
(c) (T, F) (d) (T, T) **[2016, online]**

23. The contrapositive of the following statement,
"If the side of a square doubles, then its area increases four times", is
(a) if the area of a square increases four times, then its side is not doubled
(b) if the area of a square increases four times, then its side is doubled
(c) if the area of a square does not increase four times, then its side is not doubled
(d) if the side of a square is not doubled, then its area does not increase four times **[2016, online]**

Previous Years' B-Architecture Entrance Examination Questions

1. The statement $\sim(p \wedge q) \vee q$:
(a) is a tautology
(b) is equivalent to $(p \wedge q) \vee \sim q$
(c) is equivalent to $p \vee q$
(d) is a contradiction **[2009]**

2. The contrapositive of the statement, "If x is a prime number and x divides ab then x divides a or x divides b", can be symbolically represented using logical connectives, on appropriately defined statements p, q, r, s, as
(a) $(\sim r \vee \sim s) \to (\sim p \wedge \sim q)$
(b) $(r \wedge s) \to (\sim p \wedge \sim q)$
(c) $(\sim r \wedge \sim s) \to (\sim p \vee \sim q)$
(d) $(r \vee s) \to (\sim p \vee \sim q)$ **[2010]**

3. **Statement-1:**
$\sim (A \Leftrightarrow \sim B)$ is equivalent to $A \Leftrightarrow B$.
Statement-2:
$A \vee (\sim(A \wedge \sim B))$ a tautology. **[2011]**

4. **Statement-1:** Consider the statements
p : Delhi is in India
q : Mumbai is not in Italy

Then the negation of the statement $p \vee q$, is Delhi is not in India and Mumbai is in Italy.

Statement-2: For any two statements p and q

$\sim(p \vee q) = \sim p \vee \sim q$ **[2012]**

5. If p is any logical statement, then
   (a) $p \wedge (\sim p)$ is a tautology
   (b) $p \vee (\sim p)$ is a contradiction
   (c) $p \wedge p = p$ (d) $p \vee (\sim p) = p$ **[2013]**
6. Let p and q be any two propositions.
   Statement-1: $(p \to q) \leftrightarrow q \vee \sim p$ is a tautology.
   Statement-2: $\sim(\sim p \wedge q) \wedge (p \vee q) \leftrightarrow p$ is a fallacy.
   (a) Both statements 1 and statements 2 are true
   (b) Both statements 1 and statement 2 are false
   (c) Statement 1 is true and statement 2 is false
   (d) Statement 1 is false and statement 2 is true **[2014]**
7. The statement
   $[p \wedge (p \to q)] \to q$, is
   (a) a fallacy
   (b) a tautology
   (c) neither a fallacy nor a tautology
   (d) not a compound statement **[2015]**
8. The negation of $A \to (A \vee \sim B)$ is
   (a) a tautology
   (b) equivalent to $(A \vee \sim B) \to A$
   (c) equivalent to $A \to (A \wedge \sim B)$
   (d) a fallacy **[2016]**

Answers

Concept-based

1. (b)	**2.** (d)	**3.** (c)	**4.** (a)
5. (d)	**6.** (a)	**7.** (a)	**8.** (a)
9. (c)	**10.** (a)		

Level 1

11. (b)	**12.** (b)	**13.** (c)	**14.** (b)
15. (d)	**16.** (b)	**17.** (b)	**18.** (a)
19. (a)	**20.** (d)	**21.** (b)	**22.** (b)
23. (c)	**24.** (d)	**25.** (a)	

Level 2

26. (d)	**27.** (a)	**28.** (b)

Previous Years' AIEEE/JEE Main Questions

1. (c)	**2.** None of the answer matches	**3.** (c)	
4. (a)	**5.** (c)	**6.** (c)	**7.** (d)
8. (b)	**9.** (b)	**10.** (c)	**11.** (c)
12. (a)	**13.** (d)	**14.** (c)	**15.** (c)
16. (b)	**17.** (b)	**18.** (d)	**19.** (a)
20. (b)	**21.** (c)	**22.** (b)	**23.** (a)

Previous Years' B-Architecture Entrance Examination Questions

1. (a)	**2.** (c)	**3.** (b)	**4.** (c)
5. (c)	**6.** (c)	**7.** (b)	**8.** (d)

Hints and Solutions

Concept-based

1. $p \vee t \equiv t$ not p. [see Results on Logical equivalences]
2. $(p \wedge q) \vee (\sim p \vee (p \wedge (\sim q)))$
   $\equiv (p \wedge q) \vee ((\sim p \vee p) \wedge (\sim p \vee \sim q))$
   $\equiv (p \wedge q) \vee (t \wedge \sim(p \wedge q))$
   $\equiv (p \wedge q) \vee (\sim(p \wedge q)) \equiv t$
3. $(\sim p \vee q) \to \sim q$ is F if $\sim p \vee q$ is T but $\sim q$ is F or q is T. But then $\sim p \vee q$ is T irrespective of value of p.
4. Use $p \to q$ is logically equivalent to $\sim p \vee q$.
5. $(p \vee \sim q) \wedge (\sim p \vee \sim q)$
   $\equiv (p \wedge (\sim p)) \vee (\sim q) \equiv c \vee (\sim q) \equiv \sim q$
6. Use $p \vee q$ mean Rohit is healthy or wealthy
7. If n is prime then n is odd or 2.
8. Let p : It is a holiday
   q : I go the college.
   The given statement is
   $\sim p \to q$
   Its contrapositive is $\sim q \to p$.
9. $p \to q$ is false mean p is T and q is F.
   $\sim p \vee q = (F) \vee (F) = F$, $p \wedge (\sim q) = T$
   $(\sim p) \wedge (\sim q) = F$, $p \vee q = F$
   $\therefore$ (c) is incorrect.
10. $p \to q \vee r \equiv \sim p \vee (q \vee r) \equiv (\sim p \vee q) \vee r$
    $\equiv p \wedge (\sim q) \to r$

Level 1

11. Mathematics is interesting and difficult
12. $p \to (\sim p \vee q)$ is F if $p = T$ and $\sim p \vee q$
    $= F$ or $F \vee q = F$ or if $q = F$.

13. Contrapositive of $p \to q$ is $\sim q \to \sim p$.

14. Contrapositive of $p \vee q \to r$ is $\sim r \to \sim (p \vee q)$ i.e. $\sim r \to (\sim p) \wedge (\sim q)$

15. See Theory

16. $\sim [q \vee \sim (p \wedge r)] \equiv \sim q \wedge (p \wedge r)$

17. Negation of $p \to q = \sim p \vee q$ is $\sim (\sim p \vee q) = p \wedge \sim q$, that is, a number is prime and but it is not odd.

18. It is De Morgan's law.

19. $(p \to q) \wedge p \to q$.

    F T F T T F F

    Contradiction.

    Thus, $(p \to q) \wedge p \to q$ can never take value F.

20. $\sim p \wedge q \equiv \sim (p \vee \sim q)$ by De Morgan's Law.

21. Use $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$$
$$\equiv (p \wedge q) \vee (\sim p \wedge \sim q)$$

22. $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$$\equiv (p \to q) \wedge (\sim q \vee p)$$

    and $p \vee q \to r \equiv \sim (p \vee q) \vee r$

$$\equiv (\sim p \wedge \sim q) \vee r$$
$$\equiv (\sim p \vee r) \wedge (\sim q \vee r)$$
$$\equiv (p \to r) \wedge (q \to r)$$

23. $(p \vee q) \wedge \sim (\sim p \wedge q)$

$$\equiv (p \vee q) \wedge [\sim (\sim p) \vee (\sim q)]$$
$$\equiv (p \vee q) \wedge [p \vee (\sim q)]$$
$$\equiv p \vee [q \wedge (\sim q)] \equiv p \vee F \equiv p$$

    Note that $p \vee T = T$ not p.

24. $[p \vee (q \wedge r)] \vee [\sim (p \vee q)]$

$$= [(p \vee q) \wedge (p \vee r)] \vee [\sim (p \vee q)]$$
$$= [\sim (p \vee q)] \vee [(p \vee q) \wedge (p \vee r)]$$
$$= T \wedge (p \vee r) = p \vee r$$

    Next, $\sim r \to (\sim p) \wedge (\sim q)$

$$\equiv \sim r \to \sim (p \vee q)$$
$$\equiv p \vee q \to r.$$

25. $\sim [p \wedge (q \vee r)]$

$$\equiv \sim p \vee [\sim (q \vee r)]$$
$$\equiv \sim p \vee (\sim q \wedge \sim r)$$

Level 2

26. $q \leftrightarrow (\sim p \vee \sim q)$

$$\equiv q \leftrightarrow \sim (p \wedge q)$$
$$\equiv [q \to \sim (p \wedge q)] \wedge [\sim (p \wedge q) \to q]$$
$$\equiv [\sim q \vee \sim (p \wedge q)] \wedge [(p \wedge q) \vee q]$$
$$\equiv \sim [q \wedge (p \wedge q)] \wedge q$$
$$\equiv \sim (p \wedge q) \wedge q$$
$$\equiv (\sim p \vee \sim q) \wedge q$$
$$\equiv (\sim p \wedge q) \vee (\sim q \wedge q)$$
$$\equiv (\sim p \wedge q)$$

27. $(p \to q) \to r$

$$\equiv (\sim p \vee q) \to r$$
$$\equiv \sim (\sim p \vee q) \vee r$$
$$\equiv [p \wedge (\sim q)] \vee r.$$

    Dual of $(p \to q) \to r$ is

    $[p \vee (\sim q)] \wedge r \equiv (q \to p) \wedge r$

28. $(p \vee q) \to (p \wedge q)$

$$\equiv \sim (p \vee q) \vee (p \wedge q)$$
$$\equiv (\sim p \wedge \sim q) \vee (p \wedge q)$$
$$\equiv [\sim p \vee (p \wedge q)] \wedge [\sim q \vee (p \wedge q)]$$
$$\equiv [\sim p \vee q] \wedge [\sim q \vee p] \equiv (p \to q) \wedge (q \to p)$$
$$\equiv p \leftrightarrow q$$

Previous Years' AIEEE/JEE Main Questions

1. $p \to (q \to p) \equiv \sim p \vee (q \to p)$

$$\equiv (\sim p) \vee (\sim q \vee p)$$
$$\equiv (\sim q) \vee (p \vee \sim p)$$
$$\equiv (\sim q) \vee T = T$$

    $\therefore$ $p \to (q \to p)$ is a tautology.

    Also $p \to (p \vee q) \equiv \sim p \vee (p \vee q)$

$$\equiv (\sim p \vee q) \vee q \equiv T \vee q = T$$

    $\therefore$ $p \to (p \vee q)$ is also an tautology.

    Thus, $p \to (q \to p)$ is equivalent to $p \to (p \vee q)$.

2. Note that the statement r is $\sim p \leftrightarrow q$.

    Now, $\sim p \leftrightarrow q \equiv (\sim p \to q) \wedge (q \to \sim p)$

$$\equiv [\sim(\sim p) \vee q \wedge (\sim q \vee \sim p)]$$
$$\equiv (p \vee q) \wedge [\sim(p \wedge q)]$$
$$\neq p \vee q$$

    Next, $p \leftrightarrow \sim q \equiv \sim q \leftrightarrow p$

$$\equiv (p \vee q) \wedge [\sim(p \wedge q)]$$

    $\Rightarrow \sim(p \leftrightarrow \sim q) \equiv \sim(p \vee q) \vee (p \wedge q)$

    Thus, neither Statement-1 nor Statement-2 is true.

3.

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$	$p \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

Note that $\sim(p \leftrightarrow \sim q)$ is not a tautology.

$\therefore$ Statement-2 is false.

From table $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.

Thus, Statement-1 is true.

4. Negation of P is

"for each rational number $x \in S$, $x \leq 0$".

5. $P \wedge \sim R$ stands for

Suman is brilliant and dishonest.

Thus $P \wedge \sim R \leftrightarrow Q$ stands for Suman is brilliant and dishonest if and only if Suman is rich.

Its negation is $\sim(P \wedge \sim R \leftrightarrow Q)$ or $\sim(Q \leftrightarrow P \wedge \sim R)$

6. $A \wedge (A \vee B)$ is F when $A = F$

$A \vee (A \wedge B)$ is F when $A = F$, $B = F$

We have

$[A \wedge (A \to B)] \to B$

$\equiv [A \wedge (\sim A \vee B)] \to B$

$\equiv [(A \wedge (\sim A)) \vee (A \wedge B)] \to B$

$\equiv A \wedge B \to B$

$\equiv \sim (A \wedge B) \vee B$

$\equiv [(\sim A) \vee (\sim B)] \vee B$

$\equiv (\sim A) \vee [(\sim B) \vee B]$

$\equiv (\sim A) \vee T \equiv T$

$\therefore$ $[A \wedge (A \to B)] \to B$ is a tautology.

7. Let p : I become a teacher

q : I will open a school.

The given statement is $p \to q \equiv (\sim p) \vee q$

Its negation is $\sim ((\sim p) \vee q) \equiv p \wedge (\sim q)$

Thus, negation of the given Statement is 1 will become a teacher and I will not open a school.

8. As $\sim q \to \sim p \equiv p \to q$, Statement 2 can be written as $(p \to q) \leftrightarrow (p \to q)$

Thus, Statement 2 is a tautology.

Also, $(p \wedge \sim q) \wedge (\sim p \wedge q)$

$\equiv (p \wedge \sim p) \wedge (\sim q \wedge q) = F \wedge F \equiv F$, which is a fallacy. However Statement 2 is not a correct reason for Statement 1.

9. See Solution to Question 1.

10. Statement r is

$p \to (\sim p \vee q)$

If r is false, then

p must be T

and $\sim p \vee q$ must be F

$\Rightarrow p$ is T and $F \vee q$ is F

$\Rightarrow p$ is T and q is F.

11. $A \wedge B \to (\sim A \vee B)$

$\equiv \sim (A \wedge B) \vee (\sim A \vee B)$

$\equiv (\sim A \vee \sim B) \vee (\sim A \vee B)$

$\equiv (\sim A) \vee (\sim B \vee B) \equiv (\sim A) \vee T \equiv \sim A$

$\Rightarrow \sim[(A \wedge B) \to (\sim A \vee B)] \equiv A$

$\therefore$ Statement-2 is false.

Next, $A \to (B \to A)$

$\equiv A \to (\sim B \vee A)$

$\equiv \sim A \vee (\sim B \vee A)$

$\equiv (\sim B) \vee (\sim A \vee A)$

$\equiv (\sim B) \vee T \equiv T$

$\equiv (\sim \mathrm{A} \vee \mathrm{A}) \vee \mathrm{B}$

$\equiv \sim A \vee (A \vee B)$

$\equiv A \to A \vee B$

Thus, Statement-1 is true.

12. $Q \wedge R \to P$

In words it means

m is prime and $m|n^2$

$\Rightarrow m|n$.

13. Sec Solution to Questions 3.

14. Let p, q be statements

p: I am not feeling well

q: I will go to the doctor

Contrapositive of $p \to q$ is $\sim q \to \sim p$

i.e. if I do not go to the docor then I am feeling well.

15. $\sim(p \vee \sim q) \vee \sim(p \vee q)$

$= \sim[(p \vee \sim q) \wedge (p \vee q)]$

$= \sim[p \vee (\sim q \wedge q)]$

$= \sim[p \vee F] = \sim p$

16. $p \to q \vee r$

$\equiv \sim p \vee (q \vee r)$

$\equiv (\sim p \vee q) \vee (\sim p \vee r)$

$\equiv (p \to q) \vee (p \to r)$

17. Let p and q be the statements

p : It does not rain

q : I go to school

The given statement is $p \to q$.

Its contrapositive is $\sim q \to \sim p$ i.e., if I do not go to school it rains.

18. $\sim[\sim s \vee (\sim r \wedge s)]$

$\equiv \sim(\sim s) \wedge \sim(\sim r \wedge s)$

$\equiv s \wedge (r \vee \sim s)$

$\equiv (s \wedge r) \vee (s \wedge \sim s)$

$\equiv (s \wedge r) \vee F \equiv s \wedge r$

19. Let p and q be the statements:

p: It is raining

q: I will not come.

Contrapositive of $p \to q$ is

$\sim q \to \sim p$

that is, if I (will) come then it is not raining.

20. Suman is brilliant and dishonest if and only if Suman is rich, can be expressed as

$P \wedge (\sim R) \leftrightarrow Q$

Its negation is

$\sim Q \leftrightarrow \sim P \vee R$

21. $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

$= (p \wedge \sim q) \vee [q \vee (\sim p \wedge q)]$

$= (p \wedge \sim q) \vee q$ [absorption law]

$= (p \vee q) \wedge (\sim q \vee q)$

$= (p \vee q) \wedge T = p \vee q$

22. As a statement and its counter positive have the same truth values, truth values of (V_1, V_2)

= truth values of (P, Q)

$= (F, T)$

23. Contrapositive of $p \to q$ is $\sim q \to \sim p$. Thus, contrapositive of given statement is (c), that is, if the area of a square does not increase four times, then its side is not doubled.

Previous Years' B-Architecture Entrance Examination Questions

1. $\sim(p \wedge q) \vee q$

$\equiv (\sim p \vee \sim q) \vee q$

$\equiv \sim p \vee (\sim q \vee q)$

$\equiv \sim p \vee T \equiv T$

2. Given statement is

$p \wedge q \to r \vee s$

Its contrapositive is

$\sim(r \vee s) \to \sim(p \wedge q)$

$\Leftrightarrow (\sim r \wedge \sim s) \to (\sim p \vee \sim q)$

3. $A \vee (\sim(A \wedge \sim B))$

$\equiv A \vee (\sim A \vee B)$

$\equiv (A \vee \sim A) \vee B$

$\equiv T \vee B \equiv T$

For Statement-1, see to Question 3 in the previous year AIEEE/JEE Questions.

Thus, Statement-1 is true. However Statement-2 is not a correct reason for statement-1.

4. Statement-2 is true. [See theory]

As $\sim(p \vee q) \equiv \sim p \wedge \sim q$

$\equiv$ ~(Delhi is in India) and ~(Mumbai is not is Italy)

$\equiv$ Delhi is not in India and Mumbai is in Italy.

5. $p \wedge (\sim p) \equiv F$;

$p \vee (\sim p) \equiv T$

and $p \wedge p \equiv p$ is true for each logical statement p.

6. $\sim(\sim p \wedge q) \wedge (p \vee q)$

$\equiv (p \vee \sim q) \wedge (p \vee q)$

$\equiv p \vee (\sim q \wedge q) \equiv p \vee F \equiv p$.

As, $\sim(\sim p \wedge q) \wedge (p \vee q) \leftrightarrow p$

$p \leftrightarrow p$

is a tautology, Statement-2 is false.

For truth Statement-1 see theory.

7. $p \wedge (p \to q) \to q$

$\equiv [p \wedge (\sim p \vee q)] \to q$

$\equiv [(p \wedge (\sim p)) \vee (p \wedge q)] \to q$

$\equiv [F \vee (p \wedge q)] \to q$

$\equiv (p \wedge q) \to q$

$\equiv \sim(p \wedge q) \vee q$

$\equiv (\sim p \vee \sim q) \vee q$

$\equiv \sim p \vee (\sim q \vee q)$

$\equiv (\sim p) \vee T \equiv T$

8. $A \to (A \vee \sim B)$

$\equiv \sim A \vee (A \vee \sim B)$

$\equiv (\sim A \vee A) \vee (\sim B)$

$\equiv T \vee (\sim B) \equiv T$

$\therefore$ Negative of $A \to A \vee \sim B$ is $\sim T = F$.