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Chapter

TRIGONOMETRICAL RATIOS OF STANDARD ANGLES

KEY FACTS

1. T-ratios of 0°

Consider the right angled $\triangle OMP$, rt. $\angle$ at M . Here, for acute $\angle O$, $OP = \text{hyp.} = h$, $PM = \text{perp.} = p$, $OM = \text{base} = b$.

When $\theta = 0^\circ$, P coincides with M , so that

$$p = 0 \text{ and } b = h$$

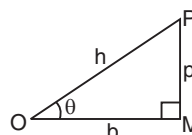
Then,

$$\sin \theta = \frac{p}{h} = \frac{0}{h} = 0$$

$$\cos \theta = \frac{b}{h} = \frac{h}{h} = 1$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{0}{1} = 0$$

Hence, $\sin 0^\circ = 0$, $\cos 0^\circ = 1$, $\tan 0^\circ = 0$.



2. T-ratios of 90°

Again in the rt $\angle$ $\triangle OMP$, $\angle O$ is increased so that it becomes 90° . Hence $\angle P$ reduces, thus making side OM reduce such that the point O coincides with point M and $\angle P = 0^\circ$.

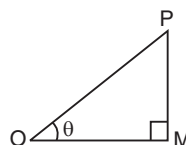
When $\theta = 90^\circ$, base = 0, perp. = hyp.

$$\therefore \sin \theta = \frac{\text{perp.}}{\text{hyp.}} = \frac{\text{hyp.}}{\text{hyp.}} = 1$$

$$\cos \theta = \frac{\text{base}}{\text{hyp.}} = \frac{0}{\text{hyp.}} = 0$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{1}{0} = \text{not defined (approaches } \infty)$$

Hence, $\sin 90^\circ = 1$, $\cos 90^\circ = 0$, $\tan 90^\circ$ is not defined.



Note . $\sin \theta$ increases from 0 to 1
 $\cos \theta$ decrease from 1 to 0
 $\tan \theta$ increases from 0 to ∞

3. T-ratios of 45°

Draw $\triangle OPM$ right angled at M such that $\angle POM = 45^\circ$. Then $\angle MPO = 45^\circ$.

$\therefore \triangle OPM$ is an isosceles triangle with $MP = MO = a$ (say)

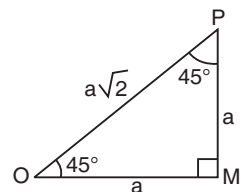
Then, by Pythagoras' Theorem,

$$\begin{aligned} OP^2 &= OM^2 + PM^2 \\ &= a^2 + a^2 = 2a^2 \end{aligned}$$

$$\Rightarrow OP = \sqrt{2}a$$

$$\text{Hence, } \sin 45^\circ = \frac{PM}{OP} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}; \cos 45^\circ = \frac{OM}{OP} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\tan 45^\circ = \frac{PM}{OM} = \frac{a}{a} = 1$$



4. T-ratios of 30° and 60°

Let ABC be an equilateral triangle. Draw $AD \perp$ to BC . Then, D is the mid-point of BC . (The altitude of an equilateral $\triangle$ co-incides with the median and bisects the vertical angle)

Let $AB = AC = BC = 2a$. Then, $BD = a$

$$\begin{aligned} \therefore \text{In } \triangle ADB, \quad AD^2 &= AB^2 - BD^2 \\ &= 4a^2 - a^2 = 3a^2 \end{aligned}$$

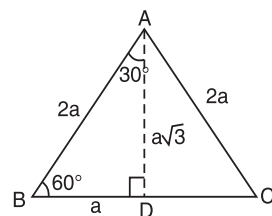
$$AD = \sqrt{3}a$$

Here, $\angle A = 30^\circ$, $\angle B = 60^\circ$

$\therefore$ In $\triangle ABD$,

$$\sin 30^\circ = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}; \quad \cos 30^\circ = \frac{AD}{AB} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}; \quad \tan 30^\circ = \frac{BD}{AD} = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}};$$

$$\sin 60^\circ = \frac{AD}{AB} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}; \quad \cos 60^\circ = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}; \quad \tan 60^\circ = \frac{AD}{BD} = \frac{a\sqrt{3}}{a} = \sqrt{3}$$



Thus, the consolidated table of the t -ratios of some standard angles is

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	not defined

5. Trigonometric ratios of complementary angles

Theorem. If θ is an acute angle

(i) $\sin (90^\circ - \theta) = \cos \theta$

(iii) $\tan (90^\circ - \theta) = \cot \theta$

(v) $\sec (90^\circ - \theta) = \operatorname{cosec} \theta$

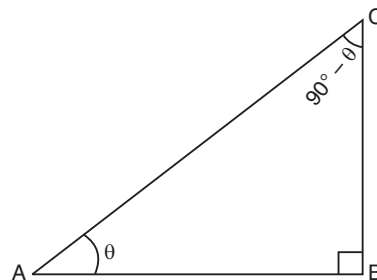
(ii) $\cos (90^\circ - \theta) = \sin \theta$

(iv) $\cot (90^\circ - \theta) = \tan \theta$

(vi) $\operatorname{cosec} (90^\circ - \theta) = \sec \theta$

Ex.

$$\begin{aligned}\sin(90^\circ - 18^\circ) &= \cos 18^\circ \Rightarrow \sin 72^\circ = \cos 18^\circ \\ \cos(90^\circ - 36^\circ) &= \sin 36^\circ \Rightarrow \cos 54^\circ = \sin 36^\circ \\ \tan(90^\circ - 30^\circ) &= \cot 30^\circ \Rightarrow \tan 60^\circ = \cot 30^\circ \\ \cot(90^\circ - 25^\circ) &= \tan 25^\circ \Rightarrow \cot 65^\circ = \tan 25^\circ \\ \sec(90^\circ - 15^\circ) &= \operatorname{cosec} 15^\circ \Rightarrow \sec 75^\circ = \operatorname{cosec} 15^\circ \\ \operatorname{cosec}(90^\circ - 42^\circ) &= \sec 42^\circ \Rightarrow \operatorname{cosec} 48^\circ = \sec 42^\circ\end{aligned}$$



Solved Examples

Ex. 1. Evaluate : $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

Sol. $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ = 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2.$$

Ex. 2. Prove that : $\frac{4}{3} \tan^2 30^\circ + \sin^2 60^\circ - 3 \cos^2 60^\circ + \frac{3}{4} \tan^2 60^\circ - 2 \tan^2 45^\circ = \frac{25}{36}$

Sol. Given exp = $\frac{4}{3} \times \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - 3 \times \left(\frac{1}{2}\right)^2 + \frac{3}{4} \times (\sqrt{3})^2 - 2 \times (1)^2$

$$= \frac{4}{3} \times \frac{1}{3} + \frac{3}{4} - \frac{3}{4} + \frac{9}{4} - 2$$

$$= \frac{4}{9} + \frac{9}{4} - 2 = \frac{16 + 81 - 72}{36} = \frac{25}{36}.$$

Ex. 3. Find the value of x, if $\tan 3x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$

Sol. $\tan 3x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 1$$

$$\Rightarrow \tan 3x = \tan 45^\circ \Rightarrow 3x = 45^\circ \Rightarrow x = 15^\circ$$

Ex. 4. Without using trigonometric tables, show that $\frac{\cos 70^\circ}{\sin 20^\circ} + \cos 49^\circ \sin 41^\circ = 2$

Sol. $\frac{\cos 70^\circ}{\sin 20^\circ} + \cos 49^\circ \operatorname{cosec} 41^\circ = \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} + \cos(90^\circ - 41^\circ) \operatorname{cosec} 41^\circ$

$$= \frac{\sin 20^\circ}{\sin 20^\circ} + \sin 41^\circ \times \frac{1}{\sin 41^\circ} = 1 + 1 = 2.$$

Ex. 5. Evaluate: $\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ$

Sol. $\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ$

$$= \tan 7^\circ \cdot \tan 23^\circ \cdot \sqrt{3} \cdot \tan(90^\circ - 23^\circ) \cdot \tan(90^\circ - 7^\circ) \quad (\because \tan(90^\circ - \theta) = \cot \theta)$$

$$= \tan 7^\circ \cdot \tan 23^\circ \cdot \sqrt{3} \cdot \cot 23^\circ \cdot \cot 7^\circ$$

$$= \tan 7^\circ \cdot \cot 7^\circ \cdot \tan 23^\circ \cdot \cot 23^\circ \cdot \sqrt{3} \quad (\because \tan \theta \cdot \cot \theta = 1)$$

$$= 1 \times 1 \times \sqrt{3} = \sqrt{3}.$$

Ex. 6. Without using trigonometric tables, prove that: $\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \cdot \dots \tan 89^\circ = 1$

Sol. Given exp. = $\tan (90^\circ - 89^\circ) \tan (90^\circ - 88^\circ) \tan (90^\circ - 87^\circ) \dots \tan 87^\circ \cdot \tan 88^\circ \cdot \tan 89^\circ$
 $= \cot 89^\circ \cdot \cot 88^\circ \cdot \cot 87^\circ \dots \tan 87^\circ \cdot \tan 88^\circ \cdot \tan 89^\circ$
 $= (\cot 89^\circ \cdot \tan 89^\circ) (\cot 88^\circ \cdot \tan 88^\circ) \dots (\cot 44^\circ \cdot \tan 44^\circ) \cdot \tan 45^\circ$
 $= 1 \times 1 \times \dots \times 1 \times 1 = 1 \quad (\because \tan 45^\circ = 1, \cot \theta \cdot \tan \theta = 1)$

Ex. 7. If $\sin 3\theta = \cos (\theta - 6^\circ)$, where 3θ and $\theta - 6^\circ$ are acute angles, find the value of θ .

Sol. $\sin 3\theta = \cos (\theta - 6^\circ)$
 $\Rightarrow \cos (90^\circ - 3\theta) = \cos (\theta - 6^\circ)$
 $\Rightarrow 90^\circ - 3\theta = \theta - 6^\circ \Rightarrow 4\theta = 96^\circ \Rightarrow \theta = 24^\circ$.

Ex. 8. Show that $\frac{1}{1 + \cos (90^\circ - \theta)} + \frac{1}{1 - \cos (90^\circ - \theta)} = 2 \operatorname{cosec}^2 (90^\circ - \theta)$

Sol. L.H.S. = $\frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta} = \frac{1 - \sin \theta + 1 + \sin \theta}{(1 + \sin \theta)(1 - \sin \theta)}$
 $= \frac{2}{1 - \sin^2 \theta} = \frac{2}{\cos^2 \theta} = 2 \sec^2 \theta$
 R.H.S. = $2 \operatorname{cosec}^2 (90^\circ - \theta) = 2 \sec^2 \theta$.
 $\therefore$ L.H.S. = R.H.S

Question Bank-33

- $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$
 (a) $\tan 90^\circ$ (b) 1
 (c) $\sin 45^\circ$ (d) 0
- If $x \tan 30^\circ = \frac{\sin 30^\circ + \cos 60^\circ}{\tan 60^\circ + \sin 60^\circ}$, then the value of x is :
 (a) $\frac{2}{3}$ (b) $\frac{2}{\sqrt{3}}$
 (c) $\frac{2}{3\sqrt{3}}$ (d) $\frac{3}{2}$
- The value of $\sin 0^\circ + \cos 30^\circ - \tan 45^\circ + \operatorname{cosec} 60^\circ + \cot 90^\circ$ is equal to
 (a) $\frac{5\sqrt{3}-6}{6}$ (b) $\frac{-6+7\sqrt{3}}{6}$
 (c) 0 (d) 2
- If $2 \sin^2 x + \cos^2 45^\circ = \tan 45^\circ$ and x is an acute angle, then the value of $\tan x$ is:
 (a) 1 (b) $\sqrt{3}$
 (c) $\frac{1}{\sqrt{3}}$ (d) 3

5. The value of $a \sin 0^\circ + b \cos 90^\circ + c \tan 45^\circ$ is:

- (a) $a + b + c$ (b) $b + c$
 (c) $a + c$ (d) c

6. The value of $\frac{\sin 30^\circ - \cos 60^\circ + \tan 45^\circ}{\cos 90^\circ - \tan 45^\circ + \sin 90^\circ}$ is:

- (a) $\frac{1}{2}$ (b) 1
 (c) $\sqrt{3}$ (d) ∞

7. The value of $\frac{1}{2} \sin^2 90^\circ \sin^2 30^\circ \cos^2 45^\circ + 4 \tan^2 30^\circ + \frac{1}{2} \sin^2 90^\circ - 2 \cos^2 90^\circ$ is:

- (a) $\frac{45}{24}$ (b) $\frac{46}{24}$
 (c) $\frac{47}{24}$ (d) $\frac{49}{24}$

8. The value of $(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ) (\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$ is:

- (a) 0 (b) 1
 (c) $\frac{7}{4}$ (d) $\frac{1}{2}$

9. $\frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$ equals

- (a) $\tan 60^\circ$ (b) $\tan 0^\circ$
(c) $\tan 30^\circ$ (d) $\frac{1}{3}$

10. Find the value of x , if
 $\sin 2x = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$
 (a) 20° (b) 15°
 (c) 30° (d) 45°

All the questions from Q.No. 11 – Q.No. 18. Should be attempted without using the trigonometric tables.

11. $\tan 26^\circ - \cot 64^\circ$ equals

- (a) -1 (b) 1
(c) 0 (d) 2

12. The value of $\frac{\sin 19^\circ}{\cos 71^\circ} + \frac{\cos 73^\circ}{\sin 17^\circ}$ is equal to :

- (a) 0 (b) 1
(c) 2 (d) $\frac{1}{2}$

13. Consider the following equations:

1. $\frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\operatorname{cosec} 18^\circ}{\sec 72^\circ} = 1$

2. $\frac{\tan 50^\circ + \sec 50^\circ}{\cot 40^\circ + \operatorname{cosec} 40^\circ} + \cos 40^\circ \operatorname{cosec} 50^\circ = 2$

3. $\frac{\sin 80^\circ}{\cos 10^\circ} - \sin 59^\circ \sec 31^\circ = 0$

Which of these statements given below is correct?

- (a) 1 only is correct
 (b) 3 only is correct
 (c) All 1, 2 and 3 are correct
 (d) 2 and 3 are correct

14. $\sin^2 25^\circ + \sin^2 65^\circ$ is equal to

- (a) 0 (b) $2 \sin^2 25^\circ$
(c) $2 \cos^2 65^\circ$ (d) 1

15. If $\sin (30^\circ - \theta) = \cos (60^\circ + \phi)$, then

- (a) $\phi - \theta = 30^\circ$ (b) $\phi - \theta = 0^\circ$
 (c) $\phi + \theta = 60^\circ$ (d) $\phi - \theta = 60^\circ$

16. The value of $\cot 15^\circ \cot 16^\circ \cot 17^\circ \dots \cot 73^\circ \cot 74^\circ \cot 75^\circ$ is:

- (a) $\frac{1}{2}$ (b) 0
(c) 1 (d) -1

17. If $\sin \theta = \cos \theta$, then value of θ is:

- (a) 60° (b) 0°
(c) 45° (d) 90°

18. Value of $\cos^2 5^\circ + \cos^2 10^\circ + \cos^2 80^\circ + \cos^2 85^\circ$ is:

- (a) 1 (b) 0
(c) 2 (d) 3

19. If $\sin 3\theta = \cos (\theta - 2^\circ)$ where 3θ and $(\theta - 2^\circ)$ are acute angles, what is the value of θ ?

- (a) 22° (b) 23°
(c) 24° (d) 25°

20. If $\tan \theta = 1$ and $\sin \theta = \frac{1}{\sqrt{2}}$, then the value of $\cos (\theta + \phi)$ is:

- (a) -1 (b) 0
(c) 1 (d) $\frac{\sqrt{3}}{2}$

21. If $x \cos 60^\circ + y \cos 0^\circ = 3$ and $4x \sin 30^\circ - y \cot 45^\circ = 2$, then what is the value of x ?

- (a) -1 (b) 0
(c) 1 (d) 2

22. Which one of the following is true?

- (a) $\tan x > 1$; $45^\circ < x < 90^\circ$
 (b) $\sin x > \frac{1}{2}$; $0^\circ < x < 30^\circ$
 (c) $\cos x > \frac{1}{2}$; $60^\circ < x < 90^\circ$
 (d) $\sin x = \cos x$ for some value of x ; $30^\circ < x < 45^\circ$

23. If $x + y = 90^\circ$, then what is

$\sqrt{\cos x \operatorname{cosec} y - \cos x \sin y}$ equal to

- (a) $\cos x$ (b) $\sin x$
(c) $\sqrt{\cos x}$ (d) $\sqrt{\sin x}$

24. If $0^\circ < \theta < 90^\circ$ and $\cos^2 \theta - \sin^2 \theta = \frac{1}{2}$, then what is the value of θ ?

- (a) 30° (b) 45°
(c) 60° (d) 90°

25. The value of $\sin^2 (90^\circ - \theta) [1 + \cot^2 (90^\circ - \theta)]$ is

- (a) -1 (b) 0
(c) $\frac{1}{2}$ (d) 1

Answers

1. (d)	2. (a)	3. (b)	4. (c)	5. (d)	6. (d)	7. (c)	8. (c)	9. (c)	10. (b)
11. (c)	12. (c)	13. (c)	14. (d)	15. (b)	16. (c)	17. (c)	18. (c)	19. (b)	20. (b)
21. (d)	22. (a)	23. (b)	24. (a)	25. (d)					

Hints and Solutions

$$1. (d) \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \frac{1 - 1}{1 + 1} = \frac{0}{2} = 0 \quad (\because \tan 45^\circ = 1)$$

$$\begin{aligned} 2. (a) \quad x \tan 30^\circ &= \frac{\sin 30^\circ + \cos 60^\circ}{\tan 60^\circ + \sin 60^\circ} \\ &= \frac{\frac{1}{2} + \frac{1}{2}}{\sqrt{3} + \sqrt{3}} = \frac{1}{2\sqrt{3} + \sqrt{3}} = \frac{2}{3\sqrt{3}} \\ x \times \frac{1}{\sqrt{3}} &= \frac{2}{3\sqrt{3}} \Rightarrow x = \frac{2\sqrt{3}}{3\sqrt{3}} = \frac{2}{3} \end{aligned}$$

$$\begin{aligned} 3. (b) \quad \text{Given exp.} &= 0 + \frac{\sqrt{3}}{2} - 1 + \frac{2}{\sqrt{3}} + 0 \\ &= \frac{\sqrt{3}}{2} - 1 + \frac{2}{\sqrt{3}} = \frac{3 - 2\sqrt{3} + 4}{2\sqrt{3}} = \frac{7 - 2\sqrt{3}}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{7\sqrt{3} - 6}{6} \end{aligned}$$

$$\begin{aligned} 4. (c) \quad 2 \sin^2 x + \cos^2 45^\circ &= \tan 45^\circ \\ \Rightarrow 2 \sin^2 x + \left(\frac{1}{\sqrt{2}}\right)^2 &= 1 \\ \Rightarrow 2 \sin^2 x + 1 - \frac{1}{2} &= \frac{1}{2} \\ \Rightarrow 2 \sin^2 x &= \frac{1}{2} \Rightarrow \sin x = \frac{1}{2} = \sin 30^\circ \Rightarrow x = 30^\circ \\ \therefore \tan x &= \tan 30^\circ = \frac{1}{\sqrt{3}} \end{aligned}$$

$$\begin{aligned} 5. (d) \quad a \sin 0^\circ + b \cos 90^\circ + c \tan 45^\circ \\ = a \times 0 + b \times 0 + c \times 1 = c \end{aligned}$$

$$6. (d) \quad \text{Given exp.} = \frac{\frac{1}{2} - \frac{1}{2} + 1}{0 - 1 + 1} = \frac{1}{0} = \infty$$

$$\begin{aligned} 7. (c) \quad \text{Given exp.} \\ &= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4 \times \left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2} \times (1)^2 - 2 \times 0 \\ &= \frac{1}{4} \times \frac{1}{2} + 4 \times \frac{1}{3} + \frac{1}{2} = \frac{1}{8} + \frac{4}{3} + \frac{1}{2} = \frac{3 + 32 + 12}{24} = \frac{47}{24} \end{aligned}$$

8. Do yourself.

9. Do yourself.

$$\begin{aligned} 10. (b) \quad \sin 2x &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} \\ &= \frac{3}{4} - \frac{1}{4} = \frac{1}{2} \end{aligned}$$

$$\therefore \sin 2x = \frac{1}{2} = \sin 30^\circ \Rightarrow 2x = 30^\circ \Rightarrow x = 15^\circ$$

$$\begin{aligned} 11. (c) \quad \tan 26^\circ - \cot 64^\circ \\ = \tan (90^\circ - 64^\circ) - \cot 64^\circ \\ = \cot 64^\circ - \cot 64^\circ = 0 \quad (\because \tan (90^\circ - \theta) = \cot \theta) \end{aligned}$$

$$\begin{aligned} 12. (c) \quad \text{Given exp.} &= \frac{\sin(90^\circ - 71^\circ)}{\cos 71^\circ} + \frac{\cos 73^\circ}{\sin(90^\circ - 73^\circ)} \\ &= \frac{\cos 71^\circ}{\cos 71^\circ} + \frac{\cos 73^\circ}{\cos 73^\circ} \quad (\because \sin(90^\circ - \theta) = \cos \theta) \\ &= 1 + 1 = 2 \end{aligned}$$

13. (c) 1. Do yourself.

$$\begin{aligned} 2. \quad \text{Given exp.} &= \frac{\tan(90^\circ - 40^\circ) + \sec(90^\circ - 40^\circ)}{\cot 40^\circ + \operatorname{cosec} 40^\circ} \\ &\quad + \cos(90^\circ - 50^\circ) \operatorname{cosec} 50^\circ \\ &= \frac{\cot 40^\circ + \operatorname{cosec} 40^\circ}{\cot 40^\circ + \operatorname{cosec} 40^\circ} + \sin 50^\circ \operatorname{cosec} 50^\circ \\ &= 1 + 1 = 2 \end{aligned}$$

3. Do yourself.

$$\begin{aligned} 14. (d) \quad \sin^2 25^\circ + \sin^2 65^\circ &= \sin^2(90^\circ - 65^\circ) + \sin^2 65^\circ \\ &= \cos^2 65^\circ + \sin^2 65^\circ = 1 \end{aligned}$$

$$\begin{aligned} 15. (b) \quad \sin(30^\circ - \theta) &= \cos(60^\circ + \phi) \\ \Rightarrow \cos(90^\circ - (30^\circ - \theta)) &= \cos(60^\circ + \phi) \\ (\because \sin \theta &= \cos(90^\circ - \theta)) \\ \Rightarrow \cos(60^\circ + \theta) &= \cos(60^\circ + \phi) \\ \Rightarrow 60^\circ + \theta &= 60^\circ + \phi \Rightarrow \theta - \phi = 0 \end{aligned}$$

16. Type Solved Example 5.

$$17. (c) \quad \sin \theta = \cos \theta \Rightarrow \cos(90^\circ - \theta) = \cos \theta$$

Now do yourself.

18. Do yourself.

$$\begin{aligned} 19. (b) \quad \sin 3\theta &= \cos(\theta - 2^\circ) \\ \Rightarrow \cos(90^\circ - 3\theta) &= \cos(\theta - 2^\circ) \\ \Rightarrow 90^\circ - 3\theta &= \theta - 2^\circ \\ \Rightarrow 4\theta &= 92^\circ \Rightarrow \theta = 23^\circ \end{aligned}$$

$$20. (b) \quad \tan \theta = 1 \Rightarrow \tan \theta = \tan 45^\circ \Rightarrow \theta = 45^\circ$$

$$\begin{aligned} \sin \phi &= \frac{1}{\sqrt{2}} \Rightarrow \sin \phi = \sin 45^\circ \Rightarrow \phi = 45^\circ \\ \therefore \cos(\theta + \phi) &= \cos(45^\circ + 45^\circ) \\ &= \cos 90^\circ = 0 \end{aligned}$$

$$21. (d) \quad x \cos 60^\circ + y \cos \theta^\circ = 3$$

$$\Rightarrow x \times \frac{1}{2} + y \times 1 = 3$$

$$\Rightarrow x + 2y = 6 \quad \dots(i)$$

and $4x \sin 30^\circ - y \cot 45^\circ = 2$

$$\Rightarrow 4x \times \frac{1}{2} - y \times 1 = 2$$

$$\Rightarrow 2x - y = 2 \quad \dots(ii)$$

Now solve for x and y .

22. (a) $\tan 45^\circ = 1$ and $\tan 90^\circ = \text{undefined } (\infty)$
 $\therefore$ when x lies between 45° and 90° , $\tan x > 1$.

23. (b) $x + y = 90^\circ \Rightarrow y = 90^\circ - x$

$$\begin{aligned} \therefore & \sqrt{\cos x \operatorname{cosec} y - \cos x \sin y} \\ &= \sqrt{\cos x \operatorname{cosec} (90^\circ - x) - \cos x \sin (90^\circ - x)} \\ &= \sqrt{\cos x \sec x - \cos x \cos x} \\ &= \sqrt{1 - \cos^2 x} = \sqrt{\sin^2 x} = \sin x \end{aligned}$$

24. (a) $\cos^2 \theta - \sin^2 \theta = \frac{1}{2}$

$$\Rightarrow (1 - \sin^2 \theta) - \sin^2 \theta = \frac{1}{2}$$

$$\Rightarrow 1 - 2 \sin^2 \theta = \frac{1}{2}$$

$$\Rightarrow 2 \sin^2 \theta = \frac{1}{2} \Rightarrow \sin^2 \theta = \frac{1}{4}$$

$$\Rightarrow \sin \theta = \frac{1}{2} = \sin 30^\circ \Rightarrow \theta = 30^\circ$$

25. (d) Given exp. $= \cos^2 \theta [1 + \tan^2 \theta]$
 $= \cos^2 \theta \cdot \sec^2 \theta = 1$

$$\left[\begin{array}{l} \because \sin (90^\circ - \theta) = \cos \theta \\ \cot (90^\circ - \theta) = \tan \theta \end{array} \right]$$

Self Assessment Sheet-32

1. Which of the following is not defined?

- (a) $\sin 90^\circ$ (b) $\tan 0^\circ$
 (c) $\cot 90^\circ$ (d) $\operatorname{cosec} 0^\circ$

2. What is the value of $\frac{\sin 60^\circ}{\cos^2 45^\circ} - 3 \tan 30^\circ + 5 \cos 90^\circ$

- (a) 1 (b) -1
 (c) $\frac{2}{5}$ (d) 0

3. The value of $2\sqrt{2} \cos 45^\circ \cdot \cos 60^\circ + 2\sqrt{3} \sin 30^\circ \cdot \tan 60^\circ - \cos 0^\circ$ is

- (a) $\frac{1}{3}$ (b) 3
 (c) -3 (d) 0

4. If $2 \cos \theta = \sqrt{3}$, evaluate $3 \sin \theta - 4 \sin^3 \theta$

- (a) 3 (b) $\frac{4}{3}$
 (c) 1 (d) 2

5. What is the value of

$$\frac{\sin 2^\circ \sin 4^\circ \sin 6^\circ \dots \sin 88^\circ}{\cos 88^\circ \cos 86^\circ \cos 84^\circ \dots \cos 2^\circ}$$

(Do not use trigonometric tables)

- (a) 0 (b) 1
 (c) 2 (d) 4

6. If $x + y = 90^\circ$, then what is the value of $\left(1 + \frac{\tan x}{\tan y}\right) \sin^2 y$?

- (a) 0 (b) $\frac{1}{2}$

- (c) 1 (d) 2

7. If $\sin A = \cos A$ and A is acute, $\tan A - \cot A$ is equal to :

- (a) 2 (b) 1
 (c) $\frac{1}{2}$ (d) 0

8. Without using trigonometric tables, find the value

$$\text{of: } \frac{2}{3} \left(\frac{\sec 56^\circ}{\operatorname{cosec} 34^\circ} \right) - 2 \cos^2 20^\circ + \frac{1}{2} \cot 28^\circ \cot 35^\circ$$

$$\cot 45^\circ \cot 62^\circ \cot 55^\circ - 2 \cos^2 70^\circ$$

- (a) $\frac{4}{5}$ (b) $-\frac{3}{4}$
 (c) $-\frac{5}{6}$ (d) 1

9. If $11x$ is an acute angle and $\tan 11x = \cot 7x$, then what is the value of x ?

- (a) 5° (b) 6°
 (c) 7° (d) 8°

10. Evaluate $\sin (50^\circ + \theta) - \cos (40^\circ - \theta) + \tan 1^\circ \tan 15^\circ \tan 20^\circ \tan 70^\circ \tan 65^\circ \tan 89^\circ + \sec (90^\circ - \theta) \operatorname{cosec} \theta - \tan (90^\circ - \theta) \cot \theta$.

- (a) 0 (b) 1
 (c) 2 (d) 3

Answers

1. (d) 2. (d) 3. (b) 4. (c) 5. (b) 6. (c) 7. (d) 8. (c) 9. (a) 10. (c)