KTG & THERMODYNAMICS

Contents

JEE (Advance) Syllabus

Heat conduction in one dimension; Elementary concepts of convection and radiation; Newton's law of cooling; Ideal gas laws; Specific heats (Cv and Cp for monatomic and diatomic gases); Isothermal and adiabatic processes, bulk modulus of gases; Equivalence of heat and work; First law of thermodynamics and its applications (only for ideal gases). Blackbody radiation: absorptive and emissive powers; Kirchhoff's law, Wien's displacement law, Stefan's law.

JEE (Main) Syllabus

Equation of state of a perfect gas, work done on compressing a gas. *Kinetic theory of gases*: Assumptions, concept of pressure. Kinetic energy and temperature; *rms* speed of gas molecules; degrees of freedom, law of equipartition of energy (statement only) and application to specific heat capacities of gases; concept of mean free path, Avogadro's number.

Note: ✍ Marked Questions can be used for Revision.

KINETIC THEORY OF GASES AND THERMODYNAMICS

The properties of the gases are entirely different from those of solid and liquid. In case of gases, thermal expansion is very large as compared to solids and liquids .To state the conditions of a gas, its volume, pressure and temperature must be specified.

Intermolecular force Solid > liquid > real gas > ideal gas (zero)

Potential energy Solid < liquid < real gas < ideal gas (zero)

Internal energy, internal kinetic energy, internal potential energy

At a given temperature for solid, liquid and gas:

(i) Internal kinetic energy : Same for all

(ii) Internal potential Energy : Maximum for ideal gas (PE = 0) and Minimum for solids (PE = –ve)

(iii) Internal Energy : Maximum for Ideal gas and Minimum for solid

At a given temperature for rared and compressed gas :

(i) Internal kinetic energy $\rightarrow$ Same

(ii) Internal potential energy $\rightarrow$ $(PE)_{Rared} > (PE)_{compressed}$

(iii) Internal Energy $\rightarrow$ $(U)_{Rared} > (U)_{compressed}$

	N.T.P.	**S.T.P.**
Temperature	0° C = 273.15 K	0.01° C = 273.16K
Pressure	1 atm = 1.01325×10^5 N/m^2 = 1.01325×10^5 pascal	1 atm
Volume	22.4 litre	22.4 litre

IDEAL GAS CONCEPT

- A gas which follows all gas laws and gas equation at every possible temperature and pressure is known as ideal or perfect gas.
- Volume of gas molecules is negligible as compared to volume of container so volume of gas = volume of container (Except 0 K)
- No intermoleculer force act between gas molecules.
- Potential energy of ideal gas is zero so internal energy of ideal gas is perfectly translational K.E. of gas. It is directly proportional to absolute temperature.

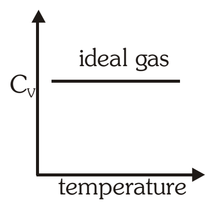
  So, internal energy depends only and only on its temperature.

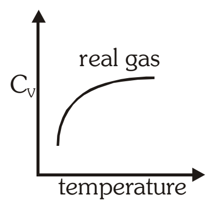
  $E_{trans} \propto T$

  For a substance $U = U_{KE} + U_{PE}$

  U_{KE} : depends only on T, U_{PE} : depends upon intermolecular forces (Always negative)

- Specific heat of ideal gas is constant quantity and it does not change with temperature
- All real gases behaves as ideal gas at high temperature and low pressure.





- Volume expansion coefficient (α) and pressure expansion coefficient (β) is same for a ideal gas and value of each is $\frac{1}{273}$ per °C $\alpha = \beta = \frac{1}{273}$ per °C
- Gas molecule have point mass and negligible volume and velocity is very high (10^7 cm/s). That's why there is no effect of gravity on them.

EQUATION OF STATE FOR IDEAL GAS

PV = μRT where μ = number of moles of gas $\Rightarrow PV = \frac{M}{M_w}RT = \left[\frac{mN}{mN_0}\right]RT = \left[\frac{R}{N_0}\right]NT = NkT$

SOLVED EXAMPLE

Example 1. By increasing temperature of gas by 5° C its pressure increases by 0.5% from its initial value at constant volume then what is initial temperature of gas ?

Solution: $\because$ At constant volume $T \propto P$ $\therefore \frac{\Delta T}{T} \times 100 = \frac{\Delta P}{P} \times 100 = 0.5 \Rightarrow T = \frac{5 \times 100}{0.5} = 1000K$

Example 2. Calculate the value of universal gas constant at STP.

Solution: Universal gas constant is given by $R = \frac{PV}{T}$

One mole of all gases at S.T.P. occupy volume V = 22·4 litre = 22·4 × 10^{-3} m^3

P = 760 mm of Hg = 760 × 10^{-3} × 13·6 × 10^3 × 9.80 N m^{-2} T = 273 K

$\therefore R = \frac{760 \times 10^{-3} \times 13.6 \times 10^3 \times 9.80 \times 22.4 \times 10^{-3}}{273}$ = 8·31 J mol^{-1} K^{-1}

Example 3. A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases at a temperature of 27°C and pressure of 1 × 10^5 Nm^2. The total mass of the mixture is 28 g. If the gram molecular weights of neon and argon are 20 and 40 respectively, find the masses of the individual gases in the container, assuming them to be ideal. Given : R = 8.314 J/mol/K.

Solution: Let m gram be the mass of neon. Then, the mass of argon is (28 – m)g.

Total number of moles of the mixture,

$$\mu = \frac{m}{20} + \frac{28 - m}{40} = \frac{28 + m}{40} \quad ...(i)$$

Now, $\mu = \frac{PV}{RT} = \frac{1 \times 10^5 \times 0.02}{8.314 \times 300} = 0.8$ ...(ii)

By (i) and (ii), $\frac{28 + m}{40} = 0.8 \Rightarrow$

28 + m = 32 $\Rightarrow$ m = 4 gram or mass of argon = (28 – 4)g = 24 g

Example 4. Calculate the temperature of the Sun if density is 1.4 g cm^{-3}, pressure is 1.4 × 10^9 atmosphere and average molecular weight of gases in the Sun in 2 g/mole. [Given R = 8.4 J $mol^{-1}K^{-1}$]

Solution: $PV = \mu RT \Rightarrow T = \frac{PV}{\mu R}$ ...(i)

But $\mu = \frac{M}{M_w}$ and $\rho = \frac{M}{V}$ $\therefore$ $\mu = \frac{\rho V}{M_w}$

From equation (i) $T = \frac{PVM_w}{\rho VR} = \frac{PM_w}{\rho R} = \frac{1.4 \times 10^9 \times 1.01 \times 10^5 \times 2 \times 10^{-3}}{1.4 \times 1000 \times 8.4} = 2.4 \times 10^7$ K

Example 5. At the top of a mountain a thermometer reads 7°C and barometer reads 70 cm of Hg. At the bottom of the mountain they read 27°C and 76 cm of Hg respectively. Compare the density of the air at the top with that at the bottom.

Solution: By gas equation $PV = \frac{M}{M_w}RT \Rightarrow \frac{P}{\rho T} = \frac{R}{M_w}$ $\left[\because \mu = \frac{M}{M_w} \text{ and } \frac{M}{V} = \rho\right]$

Now as M_W and R are same for top and bottom $\left[\frac{P}{\rho T}\right]_T = \left[\frac{P}{\rho T}\right]_B$

So $\frac{\rho_T}{\rho_B} = \frac{P_T}{P_B} \times \frac{T_B}{T_T} = \frac{70}{76} \times \frac{300}{280} = \frac{75}{76}$ = 0.9868

Example 6. During an experiment an ideal gas is found to obey an additional law VP^2 = constant. The gas is initially at temperature T and volume V. What will be the temperature of the gas when it expands to a volume 2V.

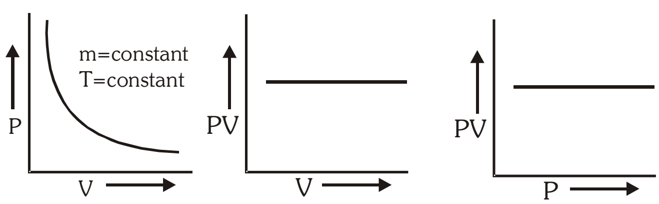
Solution : By gas equation $PV = \mu RT$ and VP^2 = constant on eliminating P

$$\left[\frac{A}{\sqrt{V}}\right]V = \mu RT \Rightarrow \sqrt{V} = \frac{\mu R}{A}T$$

$$\therefore \frac{\sqrt{V_1}}{\sqrt{V_2}} = \left[\frac{T_1}{T_2}\right] \Rightarrow \frac{\sqrt{V}}{\sqrt{2V}} = \frac{T}{T'} \Rightarrow T' = \left(\sqrt{2}\right)T$$

GAS LAWS

- **Boyle's Law**



According to it for a given mass of an ideal gas at constant temperature, the volume of a gas is inversely proportional to its pressure, i.e., $V \propto \frac{1}{P}$ if m and T = Constant

SOLVED EXAMPLE

Example 7. A sample of oxygen with volume of 500 cc at a pressure of 2 atm is compressed to a volume of 400 cc. What pressure is needed to do this if the temperature is kept constant ?

Solution : Temperature is constant, so $P_1 V_1 = P_2V_2$ $\therefore$ $P_2 = P_1 \frac{V_1}{V_2} = 2\left[\frac{500}{400}\right]$ = 2.5 atm

Example 8. An air bubble doubles in radius on rising from bottom of a lake to its surface. If the atmosphere pressure is equal to that due to a column of 10 m of water, then what will be the depth of the lake. (Assuming that surface tension is negligible) ?

Solution: Given that constant temperature, we use $P_1V_1 = P_2V_2$

P_2 = (10) dg (for water column) P_1 = (10+h) dg (where h=depth of lake)

$V_1 = \frac{4\pi}{3}r^3$, $V_2 = \frac{4\pi}{3}(2r)^3 = 8\left(\frac{4\pi}{3}r^3\right)$ = $8V_1$ Thus for $P_2V_2 = P_1V_1$,

We have 10 dg ($8V_1$) = (10 + h) dg $V_1 \Rightarrow$ 80 = 10 + h $\Rightarrow$ h = 70 m

Example 9. A vessel of volume 8.0 × 10^{-3} m^3 contains an ideal gas at 300 K and 200 k Pa. The gas is allowed to leak till the Pressure falls to 125 kPa. Calculate the amount of the gas leaked assuming that the temperature remains constant.

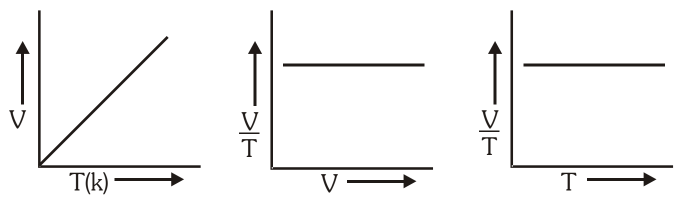
Solution: As the gas leaks out, the volume and the temperature of the remaining gas do not change. The number of moles of the gas in the vessel in given by n = $\frac{PV}{RT}$.

The number of moles in the vessel before the leakage is $n_1 = \frac{P_1V}{RT}$ and that after the leakage is $n_2 = \frac{P_2V}{RT}$.

The amount leaked is $n_1 - n_2 = \frac{(P_1 - P_2)V}{RT} = \frac{(200-125)\times 10^3 \times 8.0 \times 10^{-3}}{8.3 \times 300}$ = 0.24 mole

- **Charle's Law**

According to it for a given mass of an ideal gas at constant pressure, volume of a gas is directly proportional to its absolute temperature, i.e. $V \propto T$ if m and P = Constant



SOLVED EXAMPLE

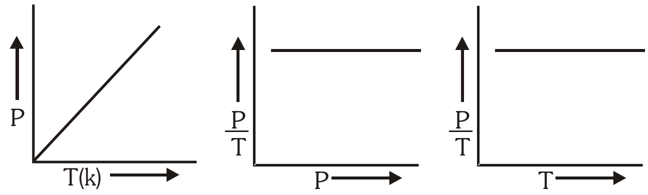
Example 10. 1500 ml of a gas at a room temperature of 23°C is inhaled by a person whose body temperature is 37°C, if the pressure and mass stay constant, what will be the volume of the gas in the lungs of the person ?

Solution: T_1 = 273 + 37 = 310 K; T_2 = 273 + 23 = 296 K. Pressure and amount of the gas are kept constant,

So $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ $\therefore$ $V_2 = V_1 \times \frac{T_2}{T_1}$ = $1500 \times \frac{293}{310}$ = 1417.74 ml

Gay–Lussac's Law

According to it, for a given mass of an ideal gas at constant volume, pressure of a gas is directly proportional to its absolute temperature, i.e., $P \propto T$ if m and V = constant



SOLVED EXAMPLE

Example 11. A sample of O_2 is at a pressure of 1 atm when the volume is 100 ml and its temperature is 27°C. What will be the temperature of the gas if the pressure becomes 2 atm and volume remains 100 ml.

Solution: $T_1 = 273 + 27 = 300$ K

For constant volume $\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow T_2 = T_1 \times \frac{P_2}{P_1} = 300 \times \frac{2}{1}$ = 600 K = 600 – 273 = 327°C

Avogadro's Law

According to it, at same temperature and pressure of equal volumes of all gases contain equal number of molecules, i.e., $N_1 = N_2$ if P,V and T are same.

The kinetic theory of gases

- Rudolph Claussius (1822–88) and James Clark Maxwell (1831–75) developed the kinetic theory of gases in order to explain gas laws in terms of the motion of the gas molecules. The theory is based on following assumptions as regards to the motion of molecules and the nature of the gases.

Basic postulates of Kinetic theory of gases

- Every gas consists of extremely small particles known as molecules. The molecules of a given as are all identical but are different than those another gas.
- The molecules of a gas are identical, spherical, rigid and perfectly elastic point masses.
- The size is negligible in comparision to inter molecular distance (10^{-9} m)

Assumptions regarding motion :

- Molecules of a gas keep on moving randomly in all possible direction with all possible velocities.
- The speed of gas molecules lie between zero and infinity (very high speed).
- The number of molecules moving with most probable speeds is maximum.

Assumptions regarding collision:

- The gas molecules keep on colliding among themselves as well as with the walls of containing vessel. These collision are perfectly elastic. (ie., the total energy before collision = total energy after the collisions.)

Assumptions regarding force:

- No attractive or repulsive force acts between gas molecules.
- Gravitational attraction among the molecules is ineffective due to extremely small masses and very high speed of molecules.

Assumptions regarding pressure:

- Molecules constantly collide with the walls of container due to which their momentum changes. This change in momentum is transferred to the walls of the container. Consequently pressure is exerted by gas molecules on the walls of container.

Assumptions regarding density:

- The density of gas is constant at all points of the container.

PROPERTIES/ASSUMPTIONS OF IDEAL GAS

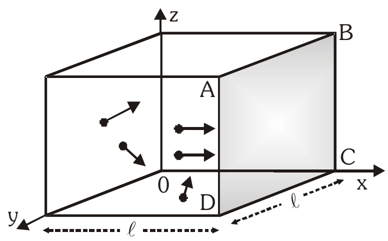
- The molecules of a gas are in a state of continuous random motion. They move with all possible velocities in all possible directions. They obey Newton's law of motion.
- Mean momentum = 0; Mean velocity = 0.$< \vec{v} > = 0$; $< v^2 > \neq 0$ (Non zero); $< v^3 > = < v^5 > = 0$
- The average distance travelled by a molecule between two successive collisions is called as mean free path (λ_m) of the molecule.
- The time during which a collision takes place is negligible as compared to time taken by the molecule to cover the mean free path so NTP ratio of time of collision to free time of motion $10^{-8} : 1$.
- When a gas taken into a vessel it is uniformly distributed in entire volume of vessel such that its density, moleculer density, motion of molecules etc. all are identical for all direction, therefore root mean velocity

  $\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2 \rightarrow$ equal Pressure exerted by the gas in all direction $P_x = P_y = P_z = P \rightarrow$ equal
- All those assumptions can be justified, if number of gas molecules are taken very large

EXPRESSION FOR PRESSURE OF AN IDEAL GAS

Consider an ideal gas enclosed in a cubical vessel of length ℓ. Suppose there are 'N' molecules in a gas which are moving with velocities $\vec{v}_1, \vec{v}_2........\vec{v}_N$.

If we consider any single molecule than its instantaneous velocity $\vec{v}$ can be expressed as $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$



Due to random motion of the molecule $v_x = v_y = v_z$

$$|v| = v_x\sqrt{3} = v_y\sqrt{3} = v_z\sqrt{3} = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Suppose a molecule of mass m is moving with a velocity v_x towards the face ABCD. It strikes the face of the cubical vessel and returns back to strike the opposite face.

Change in momentum of the molecule per collision $\Delta p = - mv_x - mv_x = - 2\,mv_x$

Momentum transferred to the wall of the vessel per molecule per collision $\Delta p = 2\,mv_x$

The distance travelled by the molecule in going to face ABCD and coming back is 2ℓ.

So, the time between two successive collision is $\Delta t = \dfrac{2\ell}{v_x}$

Number of collision per sec per molecule is $f_c = \dfrac{v_x}{2\ell} = \dfrac{\text{molecule velocity}}{\text{mean free path}}$, $f_c = \dfrac{v_{rms}}{\lambda_m}$ or $f_c = \dfrac{v_m}{\lambda_m}$

Hence momentum transferred in the wall per second by the molecule is = force on the wall

$$\text{force } F = (2\, mv_x)\frac{v_x}{2\ell} = \frac{mv_x^2}{\ell} = \frac{mv^2}{3\ell}$$

Pressure exerted by gas molecule $P = \frac{F}{A} = \frac{1}{3}\frac{mv^2}{\ell \times A}$ $\Rightarrow P = \frac{1}{3}\frac{mv^2}{V}$ $[\because A \times \ell = V]$

Pressure exerted by gas $P = \sum \frac{1}{3}\frac{mv^2}{V} = \sum \frac{1}{3}\frac{mv^2}{V} \times \frac{N}{N} = \frac{1}{3}\frac{mN}{V}\frac{\sum v^2}{N} = \frac{1}{3}\frac{mN}{V}v_{rms}^2$

$$v_{rms}^2 = \frac{3PV}{M} = \frac{3\mu RT}{\mu M_w} \Rightarrow v_{rms} = \sqrt{\frac{3RT}{M_w}},\ P = \frac{1}{3}\frac{M}{V}v_{rms}^2 = \frac{1}{3}\rho v_{rms}^2$$

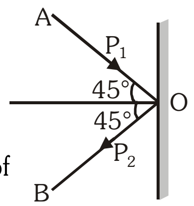
- Average number of molecules for each wall = $\frac{N}{6}$, No. of molecules along each axis = $\frac{N}{3}$ $(N_x = N_y = N_z)$
- $\overline{v}_x^2 = \overline{v}_y^2 = \overline{v}_z^2 = \frac{\overline{v}_{rms}^2}{3}$ Root mean square velocity along any axis for gas molecule is $(v_{rms})_\chi = (v_{rms})_y = (v_{rms})_z = \frac{v_{rms}}{\sqrt{3}}$

All gas laws and gas equation can be obtained by expression of pressure of gas (except Joule's law)

SOLVED EXAMPLE

Example 12. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} molecules are colliding per second on a stationary wall of area 2 cm^2 at an angle of 45° to the normal to the wall and reflected elastically with a speed 10^3 m/s. Find the pressure exerted on the wall (in N/m^2)

Solution : As the impact is elastic $\therefore |\vec{p}_1| = |\vec{p}_2| = p = mv = 3.32 \times 10^{-24}$ kg m/s



The change in momentum along the normal $\Delta p = |\vec{p}_2 - \vec{p}_1| = 2p\cos 45° = \sqrt{2}p$

If f is the collision frequency then force applied on the wall $F = \frac{\Delta p}{\Delta t} = \Delta p \times f = \sqrt{2}pf$

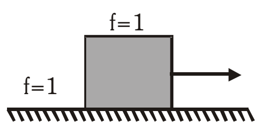
$$\therefore \text{Pressure } P = \frac{F}{A} = \frac{\sqrt{2}pf}{A} = \frac{\sqrt{2} \times 3.32 \times 10^{-24} \times 10^{23}}{2 \times 10^{-4}} = 2.347 \times 10^3\ N/m^2$$

DEGREE OF FREEDOM (f)

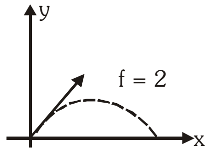
- The number of independent ways in which a molecule or an atom can exhibit motion or have energy is called it's degrees of freedom.

FOR EXAMPLE

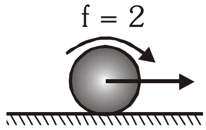
(a) Block has one degree of freedom, because it is confined to move in a straight line and has only one translational degree of freedom.



(b) The projectile has two degrees of freedom becomes it is confined to move in a plane and so it has two translational degrees of freedom.



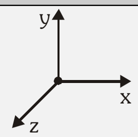
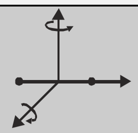
(c) The sphere has two degrees of freedom one rotational and another translational. Similarly a particle free to move in space will have three translational degrees of freedom.



Note : In pure rolling sphere has one degree of freedom as KE = $\frac{1}{2}$ mv^2 (1+ $\frac{K^2}{R^2}$) = $\frac{7}{10}$ mv^2

- The degrees of freedom are of three types :
  (a) **Translational Degree of freedom :** Maximum three degree of freedom are there corresponding to translational motion.
  (b) **Rotational Degree of freedom** : The number of degrees of freedom in this case depends on the structure of the molecule.
  (c) **Vibrational Degree of freedom :** It is exhibited at high temperatures.

Degree of freedom for different gases according to atomicity of gas at low temperature

Atomicity of gas	Translational	Rotational	Total	
Monoatomic **Ex.** Ar, Ne, Ideal gas etc	3	0	3	
Diatomic **Ex.** O_2, Cl_2, N_2 etc.	3	2	5	

At high temperatures a diatomic molecule has 7 degrees of freedom. (3 translational, 2 rotational and 2 vibrational)

SOLVED EXAMPLE

Example 13. Calculate the total number of degrees of freedom possessed by the molecules in one cm^3 of H_2 gas at NTP.

Solution : 22400 cm^3 of every gas constains 6.02 × 10^{23} molecules.

$\therefore$ Number of molecules in 1 cm^3 of H_2 gas = $\frac{6.02 \times 10^{23}}{22400}$ = 0.26875 × 10^{20}

Number of degrees of freedom of a H_2 gas molecule = 5

$\therefore$ Total number of degrees of freedom of 0.26875 × 10^{20} × 5 = 1.34375 × 10^{20}.

MAXWELL'S LAW OF EQUIPARTITION OF ENERGY

The total kinetic energy of a gas molecules is equally distributed among its all degree of freedom and the energy associated with each degree of freedom at absolute temperature T is $\frac{1}{2}kT$

For one molecule of gas

Energy related with each degree of freedom = $\frac{1}{2}kT$

Energy related with all degree of freedom = $\frac{f}{2}kT \because \overline{v}_x^2 = \overline{v}_y^2 = \overline{v}_z^2 = \frac{\overline{v}_{rms}^2}{3} \Rightarrow \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$

So energy related with one degree of freedom = $\frac{1}{2}m\frac{v_{rms}^2}{3} = \frac{3}{2}\frac{kT}{3} = \frac{1}{2}kT$

SOLVED EXAMPLE

Example 14. A cubical box of side 1 meter contains helium gas (atomic weight 4) at a pressure of 100 N/m^2. During an observation time of 1 second, an atom travelling with the root–mean–square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. Take R = $\frac{25}{3}$ J/mol–K and k = 1.38 × 10^{-23} J/K.

(a) Evaluate the temperature of the gas.

(b) Evaluate the average kinetic energy per atom.

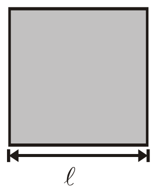
(c) Evaluate the total mass of helium gas in the box.

Solution : Volume of the box = 1m^3, Pressure of the gas = 100 N/m^2. Let T be the temperature of the gas

(a) Time between two consecutive collisions with one wall = $\frac{1}{500}$ sec

This time should be equal to $\frac{2I}{v_{rms}}$,

where ℓ is the side of the cube .



$$2\ell v_{rms} = \frac{1}{500} \Rightarrow v_{rms} = 1000 \text{ m/s}$$

$$\therefore \sqrt{\frac{3RT}{M}} = 1000 \Rightarrow T = \frac{(1000)^2 M}{3R} = \frac{(10)^6(3\times10^{-3})}{3\left(\frac{25}{3}\right)} = 160K$$

(b) Average kinetic energy per atom = $\frac{3}{2}$kT = $\frac{3}{2}$ [(1.38 × 10^{-23}) 160] J = 3.312 ×$^{-21}$ J

(c) From PV = nRT = $\frac{m}{M}$RT,

Mass of helium gas in the box m= $\frac{PVM}{RT}$

Substituting the values, m = $\frac{(100)(1)(4\times10^{-3})}{\left(\frac{25}{3}\right)(160)}$ = 3.0 × 10^{-4} kg

DIFFERENT K.E. OF GAS (INTERNAL ENERGY)

- **Translatory kinetic energy (E_T)** $E_T = \frac{1}{2}Mv_{rms}^2 = \frac{3}{2}PV$

Kinetic energy of volume V is = $\frac{1}{2}Mv_{rms}^2$ Note : Total internal energy of ideal gas is kinetic

- **Energy per unit volume or energy density (E_V)**

$$E_V = \frac{\text{Total energy}}{\text{Volume}} = \frac{E}{V} \;;$$

$$E_V = \frac{1}{2}\left[\frac{M}{V}\right]v_{rms}^2 = \frac{1}{2}\rho v_{rms}^2 \qquad \because\ P = \frac{2}{3}\left[\frac{1}{2}\rho v_{rms}^2\right]$$

$$\therefore\ E_V = \frac{3}{2}P$$

- **Molar K.E. or Mean Molar K.E. (E)**

$E = \frac{1}{2}M_w v_{rms}^2$ for N_0 molecules or M_w (gram) $\qquad E = \frac{3}{2}RT = \frac{3}{2}N_0kT$

- **Molecular kinetic energy or mean molecular K.E. ($\bar{E}$)**

$$E = \frac{1}{2}M_w v_{rms}^2,\ \bar{E} = \frac{E}{N_0} = \frac{3}{2}\frac{RT}{N_0} \Rightarrow \bar{E} = \frac{3}{2}kT$$

SOLVED EXAMPLE

Example 15. Two ideal gases at temperature T_1 and T_2 are mixed. There is no loss of energy. If the masses of molecules of the two gases are m_1 and m_2 and number of their molecules are n_1 and n_2 respectively. Find the temperature of the mixture.

Solution : Total energy of molecules of first gas = $\frac{3}{2}n_1kT_1$, Total energy of molecules of second gas = $\frac{3}{2}n_2kT_2$

Let temperature of mixture be T then total energy of molecules of mixture = $\frac{3}{2}k(n_1 + n_2)T$

$$\therefore\ \frac{3}{2}(n_1 + n_2)kT = \frac{3}{2}k(n_1T_1 + n_2T_2) \Rightarrow T = \frac{n_1T_1 + n_2T_2}{(n_1 + n_2)}$$

Example 16. The first excited state of hydrogen atom is 10.2 eV above its ground state. What temperature is needed to excite hydrogen atoms to first excited level.

Solution : K.E. of the hydrogen atom $\frac{3}{2}kT$ = 10.2 eV = $10.2 \times (1.6 \times 10^{-19})$ J

$$\Rightarrow\ T = \frac{2}{3} \times \frac{10.2 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}} = 7.88 \times 10^4\ \text{K}$$

DIFFERENT SPEEDS OF GAS MOLECULES

- **Average velocity**

Because molecules are in random motion in all possible direction in all possible velocity. Therefore, the average velocity of the gas in molecules in container is zero. $<\vec{v}> = \dfrac{\vec{v}_1 + \vec{v}_2 + \ldots\ldots\ldots\ldots \vec{v}_N}{N} = 0$

rms speed of molecules $v_{rms} = \sqrt{\dfrac{3P}{\rho}} = \sqrt{\dfrac{3RT}{M_w}} = \sqrt{\dfrac{3kT}{m}} = 1.73\sqrt{\dfrac{kT}{m}}$

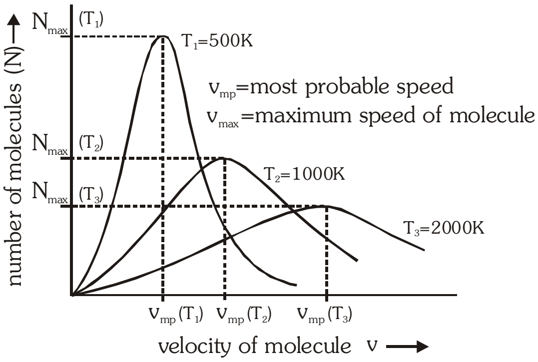
Mean speed of molecules : By maxwell's velocity distribution law v_M or $<|\vec{v}|> = v_{mean}$

$$<|\vec{v}|> = v_{mean} = \frac{|\vec{v}_1| + |\vec{v}_2| + \ldots\ldots\ldots |\vec{v}_n|}{N} = \sqrt{\frac{8}{\pi}\frac{P}{\rho}} = \sqrt{\frac{8RT}{\pi M_w}} = \sqrt{\frac{8kT}{\pi m}} = 1.59\sqrt{\frac{kT}{m}}$$

Most probable speed of molecules (v_{mp})

At a given temperature, the speed to which maximum number of molecules belongs is called as most probable speed (v_{mp}) $\quad v_{mp} = \sqrt{\dfrac{2P}{\rho}} = \sqrt{\dfrac{2RT}{M_w}} = \sqrt{\dfrac{2kT}{m}} = 1.41\sqrt{\dfrac{kT}{m}}$

MAXWELL'S LAW OF DISTRIBUTION OF VELOCITIES



SOLVED EXAMPLE

Example 17. The velocities of ten particles in ms^{-1} are 0, 2, 3, 4, 4, 4, 5, 5, 6, 9. Calculate
(i) average speed and (ii) rms speed (iii) most probable speed.

Solution : (i) average speed, $v_{av} = \dfrac{0+2+3+4+4+4+5+5+6+9}{10} = \dfrac{42}{10} = 4{\cdot}2\ ms^{-1}$

(ii) rms speed, $v_{rms} = \left[\dfrac{(0)^2+(2)^2+(3)^2+(4)^2+(4)^2+(4)^2+(5)^2+(5)^2+(6)^2+(9)^2}{10}\right]^{1/2}$

$= \left[\dfrac{228}{10}\right]^{1/2} = 4.77\ ms^{-1}$

(iii) most probable speed $v_{mp} = 4$ m/s

Example 18. At what temperature, will the root mean square velocity of hydrogen be double of its value at S.T.P., pressure remaining constant ?

Solution : Let v_1 be the r.m.s. velocity at S.T.P. and v_2 be the r.m.s. velocity at unknown temperature T_2.

$$\therefore \quad \frac{v_1^2}{v_2^2} = \frac{T_1}{T_2}$$

or $$T_2 = T_1\left[\frac{v_2}{v_1}\right]^2 = 273 \times (2)^2 = 273 \times 4 = 1092\ K = (1092 - 273) = 819°C$$

Example 19. Calculate rms velocity of oxygen molecule at 27°C

Solution : Temperature, T = 27° C $\Rightarrow$ 273 + 27 = 300 K,

Molecular weight of oxygen = 32×10^{-3} kg

and R = $8{\cdot}31$ J mol^{-1} K^{-1}

rms velocity is $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8{\cdot}31 \times 300}{32 \times 10^{-3}}} = 483{\cdot}5\ ms^{-1}$

Example 20. Calculate the kinetic energy of a gram moelcule of argon at 127°C.

Solution: Temperature, T = 127°C = 273 + 127 = 400 K, R = 8.31 J/mol K

K.E. per gram molecule of argon = $\frac{3}{2}RT = \frac{3}{2} \times 8.31 \times 400 = 4986$ J

INTERNAL ENERGY :

The internal energy of a system is the sum of kinetic and potential energies of the molecules of the system. It is denoted by U. Internal energy (U) of the system is the function of its absolute temperature (T) and its volume (V). i.e. U = f (T, V)

In case of an ideal gas, intermolecular force is zero. Hence its potential energy is also zero. In this case, the internal energy is only due to kinetic energy, which depends on the absolute temperature of the gas. i.e. U = f (T). **For an ideal gas internal energy** $U = \frac{f}{2}nRT$.

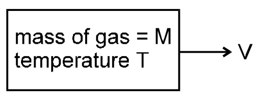
SOLVED EXAMPLE

Example 21. A light container having a diatomic gas enclosed within is moving with velocity V. Mass of the gas is M and number of moles is n.

(i) What is the kinetic energy of gas w.r.t. centre of mass of the system?

(ii) What is K.E. of gas w.r.t. ground?

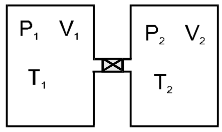
Solution : (i) K.E. = $\frac{5}{2}$ nRT



(ii) Kinetic energy of gas w.r.t. ground = Kinetic energy of gas w.r.t. centre of mass + Kinetic energy of centre of mass w.r.t. ground.

K.E. = $\frac{1}{2}MV^2 + \frac{5}{2}nRT$

Example 22. Two non conducting containers having volume V_1 and V_2 contain mono atomic and diatomic gases respectively. They are connected as shown in figure. Pressure and temperature in the two containers are P_1, T_1 and P_2, T_2 respectively. Initially stop cock is closed, if the stop cock is opened find the final pressure and temperature.



Solution : $n_1 = \dfrac{P_1V_1}{RT_1}$ $\qquad n_2 = \dfrac{P_2V_2}{RT_2}$

$n = n_1 + n_2$ (number of moles are conserved)

Finally pressure in both parts & temperature of the both the gases will become equal.

$$\frac{P(V_1+V_2)}{RT} = \frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2}$$

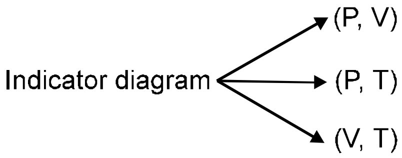
From energy conservation

$$\frac{3}{2}n_1RT_1 + \frac{5}{2}n_2RT_2 = \frac{3}{2}n_1RT + \frac{5}{2}n_2RT$$

$$\Rightarrow \qquad T = \frac{(3P_1V_1+5P_2V_2)T_1T_2}{3P_1V_1T_2+5P_2V_2T_1} \quad \Rightarrow P = \left(\frac{3P_1V_1+5P_2V_2}{3P_1V_1T_2+5P_2V_2T_1}\right)\left(\frac{P_1V_1T_2+P_2V_2T_1}{V_1+V_2}\right)$$

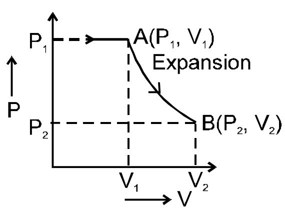
INDICATOR DIAGRAM :

A graph representing the variation of pressure or variation of temperature or variation of volume with each other is called indicator diagram.



(A) Every point of Indicator diagram represents a unique state (P, V, T) of gases.

(B) Every curve on Indicator diagram represents a unique process.



THERMODYNAMICS

Thermodynamics is mainly the study of exchange of heat energy between bodies and conversion of the same into mechanical energy and vice-versa.

THERMODYNAMIC SYSTEM

Collection of an extremely large number of atoms or molecules confined within certain boundaries such that it has a certain value of pressure (P), volume (V) and temperature (T) is called a **thermodynamic system**. Anything outside the thermodynamic system to which energy or matter is exchanged is called its surroundings. Taking into consideration the interaction between a system and its surroundings thermodynamic system is divided into three classes :

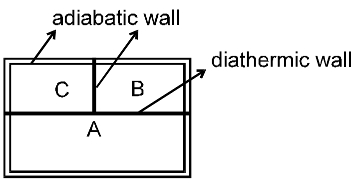
(a) **Open system :** A system is said to be an open system if it can exchange both energy and matter with its surroundings.

(b) **Closed system :** A system is said to be closed system if it can exchange only energy (not matter with its surroundings).

(c) **Isolated system :** A system is said to be isolated if it can neither exchange energy nor matter with its surroundings.

ZEROTH LAW OF THERMODYNAMICS :

If two systems (B and C) are separately in thermal equilibrium with a third one (A), then they themselves are in thermal equilibrium with each other.



EQUATION OF STATE (FOR AN IDEAL GASES) :

The relation between the thermodynamic variables (P, V, T) of the system is called equation of state. The equation of state for an ideal gas of n moles is given by

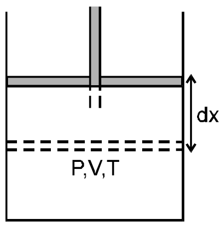
$$PV = nRT,$$

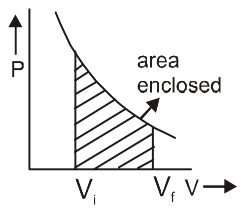
WORK DONE BY A GAS :

Let P and V be the pressure and volume of the gas. If A be the area of the piston, then force exerted by gas on the piston is, $F = P \times A$.

Let the piston move through a small distance dx during the expansion of the gas. Work done for a small displacement dx is $dW = F\,dx = PA\,dx$

Since $A\,dx = dV$, increase in volume of the gas is dV $\Rightarrow$ $dW = P\,dV$





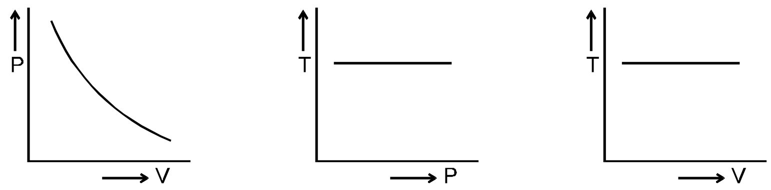
or $W = \int dW = \int P\,dV$

Area enclosed under P-V curve gives work done during process.

DIFFERENT TYPES OF PROCESSES :

(a) Isothermal Process :

T = constant [Boyle's law applicable] PV = constant



There is exchange of heat between system and surroundings. System should be compressed or expanded very slowly so that there is sufficient time for exchange of heat to keep the temperature constant.

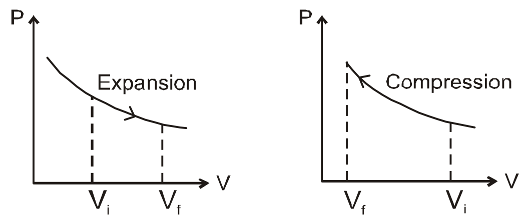
Slope of P–V curve in isothermal process:

PV = constant = C $\Rightarrow$ $\frac{dP}{dV} = -\frac{P}{V}$

Work done in isothermal process:

$W = nRT\ \ell n\ \frac{V_f}{V_i}$ $\begin{bmatrix} \text{If } V_f > V_i \text{ then W is positive} \\ \text{If } V_f < V_i \text{ then W is negative} \end{bmatrix}$

$W = \left[2.303\ nRT\ \log_{10}\frac{V_f}{V_i}\right]$



Internal energy in isothermal process :

U = f (T) $\Rightarrow$ $\Delta U = 0$

(b) Iso- choric Process (Isometric Process) :

V = constant

$\Rightarrow$ change in volume is zero

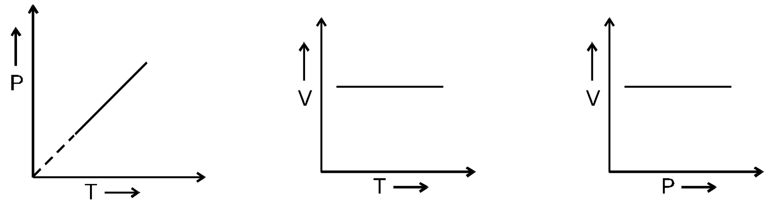
$\Rightarrow$ $\frac{P}{T}$ is constant

$\frac{P}{T} = \text{const.}$ (Gay lussac's law)

Work done in isochoric process :

Since change in volume is zero therefore dW = P dV = 0

Indicator diagram of isochoric process :



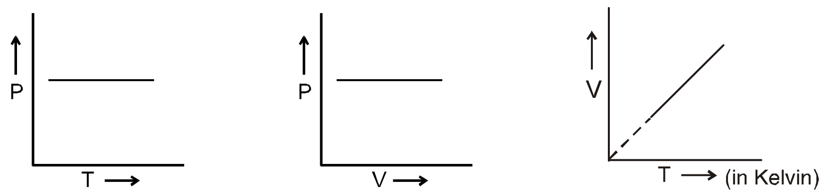
Change in internal energy in isochoric process : $\Delta U = n\frac{f}{2}R\ \Delta T$

Heat given in isochoric process : $\Delta Q = \Delta U = n\frac{f}{2}R\ \Delta T$

(c) **Isobaric Process :** Pressure remains constant in isobaric process

$\therefore$ P = constant $\Rightarrow$ $\frac{V}{T}$ = constant

Indicator diagram of isobaric process :



Work done in isobaric process :

$\Delta W = P\ \Delta V = P\ (V_{final} - V_{initial}) = nR\ (T_{final} - T_{initial})$

Change in internal energy in isobaric process : $\Delta U = n\ C_V\ \Delta T$

Heat given in isobaric process :

$\Delta Q = \Delta U + \Delta W$

$\Delta Q = n\ \frac{f}{2}R\ \Delta T + P\ [V_f - V_i] = n\frac{f}{2}R\ \Delta T + nR\ \Delta T$

Above expression gives an idea that to increase temperature by ΔT in isobaric process heat required is more than in isochoric process.

(d) **Cyclic Process :** In the cyclic process initial and final states are same therefore initial state = final state

Work done = Area enclosed under P-V diagram.

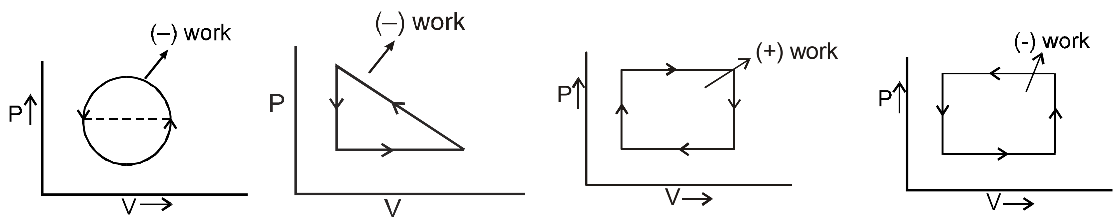
Change in internal Energy $\Delta U = 0$

$\Delta Q = \Delta U + \Delta W$

$\therefore$ $\Delta Q = \Delta W$

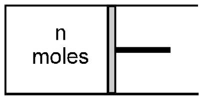
If the process on P-V curve is clockwise , then net work done is (+ve) and vice-versa.

The graphs shown below explains when work is positive and when it is negative



SOLVED EXAMPLE

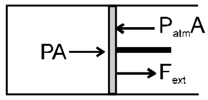
Example 23. The cylinder shown in the figure has conducting walls and temperature of the surrounding is T, the piston is initially in equilibrium, the cylinder contains n moles of a gas. Now the piston is displaced slowly by an external agent to make the volume double of its initial value. Find work done by external agent in terms of n, R, T



Solution : **1st Method :**

Work done by external agent is positive, because F_{ext} and displacement are in the same direction. Since walls are conducting therefore temperature remains constant.

Applying equilibrium condition when pressure of the gas is P



$$PA + F_{ext} = P_{atm}A$$

$$F_{ext} = P_{atm}A - PA$$

$$W_{ext} = \int_0^d F_{ext}\,dx = \int_0^d P_{atm}A\,dx - \int_0^d PA\,dx = P_{atm}A\int_0^d dx - \int_V^{2V}\frac{nRT}{V}\,dV$$

$$= P_{atm}Ad - nRT\ln 2$$

$$= P_{atm}.V_0 - nRT\ln 2 = nRT(1 - \ln 2)$$

2nd Method

Applying work energy theorem on the piston

As $W_{all} = \Delta K.E$

$\Delta K.E = 0$ (given)

$W_{gas} + W_{atm} + W_{ext} = 0$

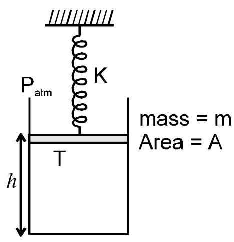
$$nRT\ln\frac{V_f}{V_i} - nRT + W_{ext} = 0$$

$$W_{ext} = nRT(1 - \ln 2)$$

Example 24. A non conducting piston of mass m and area of cross section A is placed on a non conducting cylinder as shown in figure. Temperature, spring constant, height of the piston are given by T, K, h respectively. Initially spring is relaxed and piston is at rest. Find

(i) Number of moles

(ii) Work done by gas to displace the piston by distance d when the gas is heated slowly.



(iii) Find the final temperature

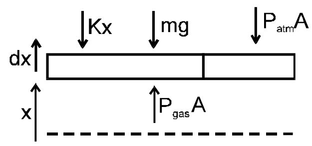
Solution : (i) $PV = nRT \Rightarrow \left(P_{atm} + \frac{mg}{A}\right) Ah = nRT$

$$\Rightarrow \quad n = \frac{\left(P_{atm} + \frac{mg}{A}\right)Ah}{RT}$$

(ii) **1st method**

Applying Newton's law on the piston

$mg + P_{atm} A + Kx = P_{gas}A$



$$W_{gas} = \int_0^d P_{gas}A \, dx$$

$$= \int_0^d (mg + P_{atm} A + Kx)\, dx. \quad \Rightarrow \quad W_{gas} = mgd + P_{atm}dA + \frac{1}{2}Kd^2$$

2nd method

Applying work energy theorem on the piston

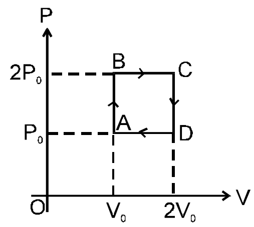
$W_{all} = \Delta KE$

Since piston moves slowly therefore $\Delta KE = 0$

$W_{gravity} + W_{gas} + W_{atm} + W_{spring} = 0$

$$-mgd + W_{gas} + (-P_{atm} Ad) + [-(\frac{1}{2}Kd^2 - 0)] = 0 \Rightarrow W_{gas} = mgd + P_{atm}dA + \frac{1}{2}Kd^2$$

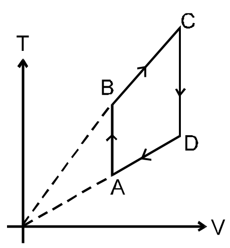
Example 25. Find out the work done in the given graph. Also draw the corresponding T-V curve and P-T curve.

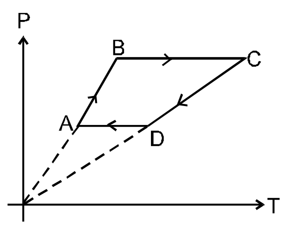


Solution : Since in P-V curves area under the cycle is equal to work done therefore work done by the gas is equal to $P_0 V_0$.

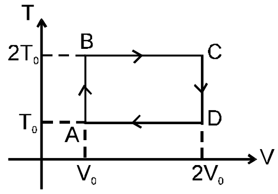
Line A B and CD are isochoric line, line BC and DA are isobaric line.

$\therefore$ the T-V curve and P-T curve are drawn as shown.





Example 26. T-V curve of cyclic process is shown below, number of moles of the gas are n find the total work done during the cycle.



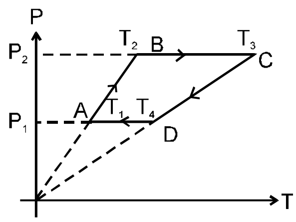
Solution : Since path AB and CD are isochoric therefore work done is zero during path AB and CD. Process BC and DA are isothermal, therefore

$$W_{BC} = nR2T_0 \ln \frac{V_C}{V_B} = 2nRT_0 \ln 2$$

$$W_{DA} = nRT_0 \ln \frac{V_A}{V_D} = -nRT_0 \ln 2$$

Total work done = $W_{BC} + W_{DA} = 2nRT_0 \ln 2 - nRT_0 \ln 2 = nRT_0 \ln 2$

Example 27. P-T curve of a cyclic process is shown. Find out the work done by the gas in the given process if number of moles of the gas are n.

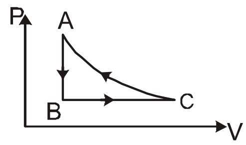


Solution : Since path AB and CD are isochoric therefore work done during AB and CD is zero. Path BC and DA are isobaric.

Hence $W_{BC} = nR\Delta T = nR(T_3 - T_2)$

$W_{DA} = nR(T_1 - T_4)$ Total work done = $W_{BC} + W_{DA} = nR(T_1 + T_3 - T_4 - T_2)$

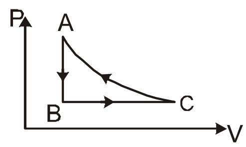
Example 28. In figure, a cyclic process ABCA of 3 moles of an ideal gas is given. The temperatures of the gas at B and C are 500 K and 1000 K respectively. If the work done on the gas in process CA is 2500 J then find the net heat absorbed or released by an ideal gas .



Take R = 25/3 J/mol–K.

Solution : The change in internal energy during the cyclic process is zero. Hence, the heat supplied to the gas is equal to the work done by it. Hence,

$\Delta Q = W_{AB} + W_{BC} + W_{CA}$ ....(i)



The work done during the process AB is zero

$W_{BC} = P_B (V_C - V_B) = nR(T_C - T_B)$

= (3 mol) (25/3 J/mol–K) (500 K) = 12500 J

As $W_{CA} = -2500$ J (given)

$\therefore$ $\Delta Q = 0 + 12500 - 2500$ [from ......(i)]

$\Delta Q = 10$ kJ

FIRST LAW OF THERMODYNAMICS :

The first law of thermodynamics is the law of conservation of energy. It states that if a system absorbs heat dQ and as a result the internal energy of the system changes by dU and the system does a work dW, then **dQ = dU + W**.

But, W = P dV dQ = dU + P dV

which is the mathematical statement of first law of thermodynamics.

Heat gained by a system, work done by a system and increase in internal energy are taken as positive.

Heat lost by a system, work done on a system and decrease in internal energy are taken as negative.

SOLVED EXAMPLE

Example 29. 1 gm water at 100ºC is heated to convert into steam at 100ºC at 1 atm. Find out change in internal energy of water. It is given that volume of 1 gm water at 100ºC = 1 cc, volume of 1 gm steam at 100ºC = 1671 cc. Latent heat of vaporization = 540 cal/g. (Mechanical equivalent of heat J = 4.2J/cal.)

Solution : From first law of thermodynamic $\Delta Q = \Delta U + W$

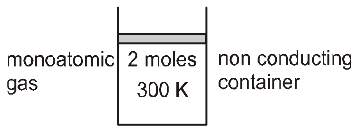
$\Delta Q = mL = 1 \times 540$ cal. = 540 cal.

$$W = P\Delta V = \frac{10^5(1671-1)\times10^{-6}}{4.2} = \frac{10^5\times(1670)\times10^{-6}}{4.2} \approx 40 \text{ cal.}$$

$\Delta U = 540 - 40 = 500$ cal.

Example 30. Two moles of a monoatomic gas at 300 K are kept in a non conducting container enclosed by a piston. Gas is now compressed to increase the temperature from 300 K to 400 K. Find work done by the gas

($R = \frac{25}{3}$ J/mol–K)



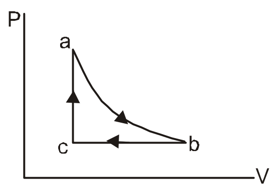
Solution : $\Delta Q = \Delta U + W$

Since container is non conducting therefore

$$\Delta Q = 0 = \Delta U + W \Rightarrow \quad W = -\Delta U = -n\frac{f}{2}R\ \Delta T = -2 \times \frac{3}{2}R\,(400-300)$$

$$= -3 \times \frac{25}{3} \times 100 \text{ J} \quad = -2500 \text{ J}$$

Example 31. In figure, a sample of an ideal gas is taken through the cyclic process abca. 800 J of work is done by the gas during process ab. If gas absorb no heat in process ab, rejects 100 J of heat during bc and absorb 500 J of heat during process ca. Then (a) find the internal energy of the gas at b and c if it is 1000 J at a. (b) Also calculate the work done by the gas during the part bc.



Solution : (a) In process ab

$\Delta Q = \Delta U + W$

$0 = U_B - 100 + 800$

$U_B = 200$ J

for Cyclic process

$\Delta Q = \Delta U + W$

$400 = 0 + 800 + W_{BC}$

$W_{BC} = -400$ J

for process bc

$\Delta Q = \Delta U + W$

$-100 = -400 + U_C - 200$

$\therefore \quad U_C = 500$ J

Example 32. Two moles of nitrogen gas is kept in a cylinder of cross-section area 10 cm^2. The cylinder is closed by a light frictionless piston. Now the gas is slowly heated such that the displacement of piston during process is 50 cm, find the rise in temperature of gas when 200 J of heat is added in it. (Atmospheric pressure = 100 kPa, R = 25/3 J/mol-K)

Solution : The change in internal energy of the gas is

$$\Delta U = 5/2\, nR\, (\Delta T)$$
$$= 5/2 \times 2R \times (\Delta T) = 5R \times \Delta T$$

The heat given to the gas = 200 J

The work done by the gas is

$$W = \Delta Q - \Delta U$$
$$= 200\text{ J} - 5R\,\Delta T \qquad ....(i)$$

As the distance moved by the piston is 50 cm, $\therefore$ the work done is

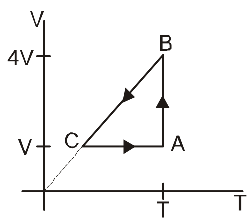
$$\Delta W = P\Delta V = PA\Delta x = 10^5 \times 10 \times 10^{-4} \times 50 \times 10^{-2} \qquad ....(ii)$$

From (i) and (ii)

$\Delta T = 18/5$ K $= 3.6$ K

Example 33. An ideal gas initially has pressure P volume V and temperature T. It is isothermally expanded to four times of its original volume, then it is compressed at constant pressure to attain its original volume V. Finally, the gas is heated at constant volume to get the original temperature T. (a) Draw V-T curve (b) Calculate the total work done by the gas in the process. (given $\ell n2 = 0.693$)

Solution :



(a) V–T curve for all process is shown in figure. The initial state is represented by the point A. In the first step, it is isothermally expanded to a volume 4V. This is shown by AB. Then the pressure is kept constant and the gas is compressed to the initial volume V. From the ideal gas equation, V/T is constant at constant pressure (PV = nRT). Hence, the process is shown by a line BC which passes through the origin. At point C, the volume is V. In the final step, the gas is heated at constant volume to a temperature T. This is shown by CA. The final state is the same as the initial state.

(b) Total work done by gas, $W_{Total} = W_{AB} + W_{BC} + W_{CA}$

$$W_{AB} = nRT \ln \frac{4V}{V} = 2nRT \ln 2 = 2PV \ln 2.$$

Also $P_A V_A = P_B V_B$ (As AB is an isothermal process)

or, $P_B = \frac{P_A V_A}{V_B} = \frac{PV}{4V} = \frac{P}{4}$.

In the step BC, the pressure remains constant. Hence the work done is,

$$W_{BC} = \frac{P}{4}(V - 4V) = -\frac{3PV}{4}.$$

In the step CA, the volume remains constant and so the work done is zero. The net work done by the gas in the cyclic process is

$$W = W_{AB} + W_{BC} + W_{CA} = 2PV \ln 2 - \frac{3PV}{4} + 0$$

Hence, the work done by the gas 0.636 PV.

Example 34. A diatomic gas is heated at constant pressure. If 105 J of heat is given to the gas, find (a) the change in internal energy of the gas (b) the work done by the gas.

Solution : Suppose the volume changes from V_1 to V_2 and the temperature changes from T_1 to T_2.
The heat supplied is

$$\Delta Q = \Delta U + P\Delta V = \Delta U + nR\Delta T = \Delta U + \frac{2\Delta U}{f} \quad \left[\Delta U = \frac{nfR\Delta T}{2}\right]$$

(a) The change in internal energy is

$$\Delta Q = \Delta U\left[1 + \frac{2}{f}\right];\ 105 = \Delta U\left[1 + \frac{2}{5}\right],\ \Delta U = 75 \text{ J}$$

(b) The work done by the gas is
$W = \Delta Q - \Delta U = 105 \text{ J} - 75 \text{ J} = 30 \text{ J}.$

Efficiency of a cycle (η) :

$$\eta = \frac{\text{total Mechanical work done by the gas in the whole process}}{\text{Heat absorbed by the gas (only + ve)}}$$

$$= \frac{\text{area under the cycle in P - V curve}}{\text{Heat injected into the system}}$$

$$\eta = \left(1 - \frac{Q_2}{Q_1}\right) \text{ for Heat Engine,}$$

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \text{ for Carnot cycle}$$

SOLVED EXAMPLE

Example 35. n moles of a diatomic gas has undergone a cyclic process ABCA as shown in figure. Temperature at A is T_0. Find

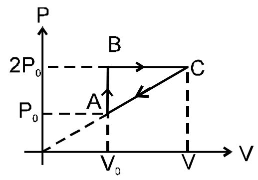
(i) Volume at C ?

(ii) Maximum temperature ?

(iii) Total heat given to gas ?

(iv) Is heat rejected by the gas, if yes how much heat is rejected ?

(v) Find out the efficiency



Solution : (i) For process AC, $P \alpha V$

$$\frac{2P_0}{V_c} = \frac{P_0}{V_0} \quad \Rightarrow V_c = 2V_0$$

(ii) Since process AB is isochoric hence

$$\frac{P_A}{T_A} = \frac{P_B}{T_B} \quad \Rightarrow T_B = 2T_0$$

Since process BC is isobaric therefore $\frac{T_B}{V_B} = \frac{T_C}{V_C}$

$\Rightarrow \quad T_C = 2T_B = 4\,T_0 \qquad \therefore \quad T_{max} = 4T_0$

(iii) Since process is cyclic therefore

$$\Delta Q = W = \text{area under the cycle} = \frac{1}{2} P_0 V_0 .$$

(iv) Since ΔU and ΔW both are negative in process CA

$\therefore$ ΔQ is negative in process CA and heat is rejected in process CA

$$\Delta Q_{CA} = W_{CA} + \Delta U_{CA}$$

$$= -\frac{1}{2}\,[P_0 + 2P_0]\,V_0 - \frac{5}{2} nR\,(T_C - T_A)$$

$$= -\frac{1}{2}\,[P_0 + 2P_0]\,V_0 - \frac{5}{2} nR \left(\frac{4P_0V_0}{nR} - \frac{P_0V_0}{nR}\right)$$

$$= -9P_0V_0 \text{ (Heat rejected)}$$

(v) η = efficiency of the cycle = $\frac{\text{work done by the gas}}{\text{heat injected}} \Rightarrow \quad \eta = \frac{P_0V_0/2}{Q_{injected}} \times 100$

$$\Delta Q_{inj} = \Delta Q_{AB} + \Delta Q_{BC}$$

$$= \left[\frac{5}{2} nR(2T_0 - T_0)\right] + \left[\frac{5}{2} nR(2T_0) + 2P_0(2V_0 - V_0)\right] = \frac{19}{2} P_0V_0.$$

$$\eta = \frac{100}{19}\,\%$$

SPECIFIC HEAT :

The specific heat capacity of a substance is defined as the heat supplied per unit mass of the substance per unit rise in the temperature. If an amount ΔQ of heat is given to a mass m of the substance and its temperature rises by ΔT, the specific heat capacity s is given by equation

$$s = \frac{\Delta Q}{m\Delta T}$$

The molar heat capacity of a gas is defined as the heat given per mole of the gas per unit rise in the temperature. The molar heat capacity at constant volume, denoted by C_V, is :

$$C_v = \left(\frac{\Delta Q}{n\,\Delta T}\right)_{\text{constant volume}} = \frac{f}{2}R$$

and the molar heat capacity at constant pressure, denoted by C_p is,

$$C_P = \left(\frac{\Delta Q}{n\,\Delta T}\right)_{\text{constant pressure}} = \left(\frac{f}{2}+1\right)R$$

where n is the amount of the gas in number of moles and f is degree of freedom. Quite often, the term specific heat capacity or specific heat is used for molar heat capacity. It is advised that the unit be carefully noted to determine the actual meaning. The unit of specific heat capacity is J/kg-K whereas that of molar heat capacity is J/mol–K.

MOLAR HEAT CAPACITY OF IDEAL GAS IN TERMS OF R :

(i) For a monoatomic gas f = 3

$$C_V = \frac{3}{2}R \quad , \quad C_P = \frac{5}{2}R \qquad \Rightarrow \gamma = \frac{C_P}{C_V} = \frac{5}{3} = 1.67$$

(ii) For a diatmoc gas f = 5

$$C_V = \frac{5}{2}R,\; C_P = \frac{7}{2}R\,, \qquad \gamma = \frac{C_P}{C_V} = 1.4$$

(iii) For a Triatomic gas f = 6

$$C_V = 3R,\; C_P = 4R$$

$$\gamma = \frac{C_P}{C_V} = \frac{4}{3} = 1.33$$ [Note for CO_2; f = 5, it is linear]

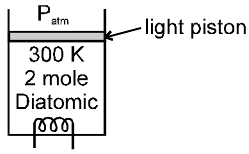
In general if f is the degree of freedom of a molecule , then ,

$$C_V = \frac{f}{2}R,\quad C_P = \left(\frac{f}{2}+1\right)R,$$

$$\gamma = \frac{C_P}{C_V} = \left[1+\frac{2}{f}\right]$$

SOLVED EXAMPLE

Example 36. Two moles of a diatomic gas at 300 K are enclosed in a cylinder as shown in figure. Piston is light. Find out the heat given if the gas is slowly heated to 400 K in the following three cases.



(i) Piston is free to move

(ii) If piston does not move

(iii) If piston is heavy and movable.

Solution : (i) Since pressure is constant

$$\therefore \Delta Q = nC_P \Delta T = 2 \times \frac{7}{2} \times R \times (400 - 300) = 700\ R$$

(ii) Since volume is constant

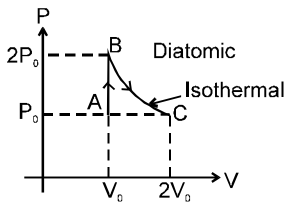
$\therefore$ W = 0 and $\Delta Q = \Delta U$ (from first law)

$$\Delta Q = \Delta U = nC_V \Delta T = 2 \times \frac{5}{2} \times R \times (400 - 300) = 500\ R$$

(iii) Since pressure is constant

$$\therefore \Delta Q = nC_P \Delta T = 2 \times \frac{7}{2} \times R \times (400 - 300) = 700\ R$$

Example 37. P-V curve of a diatomic gas is shown in the figure. Find the total heat given to the gas in the process AB and BC



Solution : From first law of thermodynamics

$\Delta Q_{ABC} = \Delta U_{ABC} + W_{ABC}$

$$W_{ABC} = W_{AB} + W_{BC} = 0 + nR\,T_B \ln \frac{V_C}{V_B} = nR\,T_B \ln \frac{2V_0}{V_0}$$

$$= nRT_B \ln 2 = 2P_0 V_0 \ln 2$$

$$\Delta U = nC_V \Delta T = \frac{5}{2}(2P_0V_0 - P_0V_0) \quad \Rightarrow \quad \Delta Q_{ABC} = \frac{5}{2}P_0V_0 + 2P_0V_0 \ln 2.$$

Example 38. From given data, calculate the value of mechanical equivalent of heat. The specific heat capacity of air at constant volume 170 cal/kg-K, $\gamma = C_p/C_v = 1.4$ and the density of air at STP is 1.29 kg/m^3. Gas constant R = 8.3 J/mol-K.

Solution : Using pV = nRT, the volume of 1 mole of air at STP is

$$V = \frac{nRT}{p} = \frac{(1\,\text{mol}) \times (8.3\ \text{J/mol}-\text{K}) \times (273\text{K})}{1.01 \times 10^5\ \text{N/m}^2} = 0.0224\text{m}^3.$$

The mass of 1 mole is, therefore,

$(1.29 \text{ kg/m}^3) \times (0.0224 \text{ m}^3) = 0.029 \text{ kg}$.

The number of moles in 1 kg is $\frac{1}{0.029}$. The molar heat capacity at constant volume is

$$C_V = \frac{170 \text{ cal}}{(1/0.029) \text{ mol} - \text{K}} = 4.93 \text{ cal/mol-K}.$$

Hence, $C_p = \gamma C_v = 1.4 \times 4.93$ cal/mol-K

or, $C_p - C_v = 0.4 \times 4.93$ cal/mol-K

$= 1.97$ cal/mol-K.

Also, $C_p - C_v = R = 8.3$ J/mol-K.

Thus, 8.3 J = 1.97 cal.

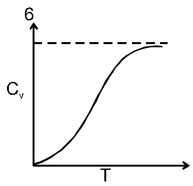
The mechanical equivalent of heat is $\frac{8.3 \text{ J}}{1.97 \text{ cal}} = 4.2$ J/cal.

AVERAGE MOLAR SPECIFIC HEAT OF METALS :

[Dulong and Petit law]

At room temperature average molar specific heat of all metals are same and is nearly equal to 3R (6 cal. mol^{-1} K^{-1}).

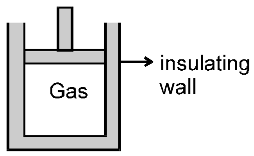
[Note : Temp. above which the metals have constant C_V is called Debye temp.]



MAYER'S EQUATION : $C_P - C_V = R$ (for ideal gases only)

Adiabatic process :

When no heat is supplied or extracted from the system the process is called adiabatic. Process is sudden so that there is no time for exchange of heat. If walls of a container are thermally insulated no heat can cross the boundary of the system and process is adiabatic.



Equation of adiabatic process is given by

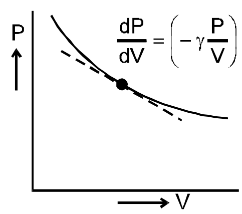
PV^{γ} = constant [Poisson Law]

$T^{\gamma} P^{1-\gamma}$ = constant

$T V^{\gamma-1}$ = constant

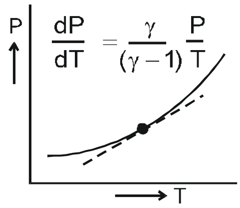
Slope of P–V–curve in adiabatic process : Since PV^{γ} is a constant

$$\therefore \quad \frac{dP}{dV} = -\gamma \left(\frac{P}{V}\right)$$



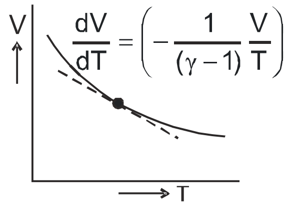
Slope of P–T–curve in adiabatic process : Since $T^{\gamma} P^{1-\gamma}$ is a constant

$$\therefore \quad \frac{dP}{dT} = -\frac{\gamma}{(1-\gamma)}\frac{P}{T} = \frac{(\gamma)}{(\gamma-1)}\frac{P}{T}$$



Slope of T–V–curve :

$$\frac{dV}{dT} = -\frac{1}{(\gamma-1)}\frac{V}{T}$$



Work done in adiabatic Process :

$$\Delta W = -\Delta U = nC_V(T_i - T_f) = \frac{P_iV_i - P_fV_f}{(\gamma-1)} = \frac{nR(T_i - T_f)}{\gamma - 1}$$

work done by system is (+ve) , if $T_i > T_f$ (For expansion)

work done on the system is (–ve) if $T_i < T_f$ (For compression)

SOLVED EXAMPLE

Example 39. A container having slightly conducting walls contains air. The initial temperature and volume are 47ºC (equal to the temperature of the surrounding) and 400cm³ respectively. Find the rise in the temperature if the gas is compressed to 200cm³ (a) in a short time (b) in a long time. Take γ = 1.4. [$2^{0.4}$ = 1.3]

Solution : (a) When the gas is compressed in a short time, the process is adiabatic. Thus,

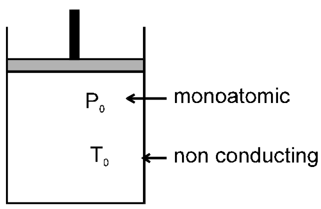
$$T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$$

or $$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = (320\text{ K}) \times \left[\frac{400}{200}\right]^{0.4} = 416\text{ K}.$$

Rise in temperature = $T_2 - T_1$ = 96 K.

(b) When the gas is compressed in a long time, the process is isothermal. Thus, the temperature remains same that is 47ºC $\therefore$ The rise in temperature = 0.

Example 40. An ideal monoatomic gas is enclosed in a non conducting cylinder having a piston which can move freely. Suddenly gas is compressed to 1/8 of its initial volume. Find the final pressure and temperature if initial pressure and temperature are P_0 and T_0 respectively.



Solution : Since process is adiabatic therefore

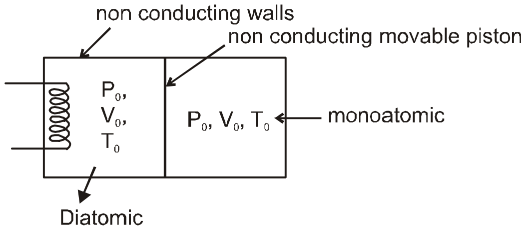
$$P_0 V^{\frac{5}{3}} = P_{final}\left(\frac{V}{8}\right)^{5/3}. \qquad \left[\gamma = \frac{C_P}{C_V} = \frac{5R}{2}/\frac{3R}{2} = \frac{5}{3}\right]$$

$$P_{final} = 32\ P_0 .$$

Since process is adiabatic therefore

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_0 V_0^{2/3} = T_{final}\left(\frac{V_0}{8}\right)^{2/3} \Rightarrow T = 4T_0$$

Example 41. A cylindrical container having non conducting walls is partitioned in two equal parts such that the volume of each part is V_0 . A movable non conducting piston is kept between the two parts. Gas on left is slowly heated so that the gas on right is compressed upto volume $\frac{V_0}{8}$. Find pressure and temperature on both sides if initial pressure and temperature, were P_0 and T_0 respectively. Also find heat given by the heater to the gas. (number of moles in each part is n)



Solution : Since the process on right is adiabatic therefore

$$PV^{\gamma} = \text{constant}$$

$$\Rightarrow \quad P_0 V_0^{\gamma} = P_{final}(V_0/8)^{\gamma} \quad \Rightarrow \quad P_{final} = 32\ P_0$$

$$T_0 V_0^{\gamma-1} = T_{final}(V_0/8)^{\gamma-1} \quad \Rightarrow \quad T_{final} = 4T_0$$

Let volume of the left part is V_1

$$\Rightarrow \quad 2V_0 = V_1 + \frac{V_0}{8} \quad \Rightarrow \quad V_1 = \frac{15V_0}{8}.$$

Since number of moles on left part remains constant therefore for the left part $\frac{PV}{T}$ = constant.

Final pressure on both sides will be same

$$\Rightarrow \quad \frac{P_0 V_0}{T_0} = \frac{P_{final} V_1}{T_{final}} \quad \Rightarrow \quad T_{final} = 60\ T_0$$

$$\Delta Q = \Delta U + W$$

$$\Delta Q = n\frac{5R}{2}(60T_0 - T_0) + n\frac{3R}{2}(4T_0 - T_0)$$

$$\Delta Q = \frac{5nR}{2} \times 59T_0 + \frac{3nR}{2} \times 3T_0 = 152\ nRT_0$$

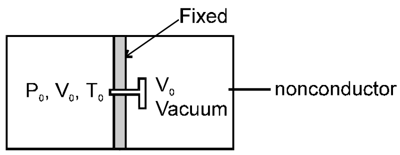
FREE EXPANSION

If a system, say a gas expands in such a way that no heat enters or leaves the system and also no work is done by or on the system , then the expansion is called the "free expansion".

$\Delta Q = 0$, $\Delta U = 0$ and $\Delta W = 0$. Temperature in the free expansion remains constant.

SOLVED EXAMPLE

Example 42. A non conducting cylinder having volume $2V_0$ is partitioned by a fixed non conducting wall in two equal parts. Partition is attached with a valve. Right side of the partition is a vacuum and left part is filled with a gas having pressure and temperature P_0 and T_0 respectively. If valve is opened find the final pressure and temperature of the two parts.



Solution : From the first law thermodynamics $\Delta Q = \Delta U + W$

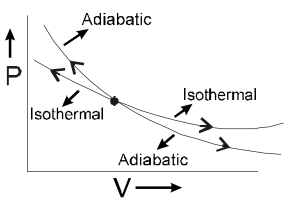
Since gas expands freely therefore W = 0 , since no heat is given to gas $\Delta Q = 0$

$\Rightarrow$ $\Delta U = 0$ and temperature remains constant.

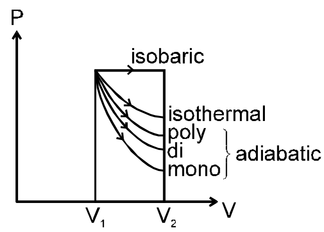
$T_{final} = T_0$

Since the process is isothermal therefore $P_0 \times V_0 = P_{final} \times 2V_0$ $\Rightarrow$ $P_{final} = P_0/2$

Comparison of slopes of an Iso-thermal and Adiabatic Curve



$$\left|\frac{dP}{dV}\right|_{adia} > \left|\frac{dP}{dV}\right|_{isothermal}$$



$W_{mono} < W_{di} < W_{poly} < W_{isothermal} < W_{isobaric}$

In compression up to same final volume: $|W_{adia}| > |W_{isothermal}|$

In Expansion up to same final volume: $W_{isothermal} > W_{adia}$

Limitations of Ist Law of Thermodynamics :

The first law of thermodynamics tells us that heat and mechanical work are interconvertible. However, this law fails to explain the following points :

(i) It does not tell us about the direction of transfer of heat.

(ii) It does not tell us about the conditions under which heat energy is converted into work.

(iii) It does not tell us whether some process is possible or not.

Mixture of non-reacting gases:

(a) Molecular weight = $\dfrac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$,

M_1 & M_2 are molar masses.

(b) Specific heat $C_V = \dfrac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$,

$$C_P = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 + n_2}$$

(c) for mixture, $\gamma = \dfrac{C_{p_{mix}}}{C_{v_{mix}}} = \dfrac{n_1 C_{p_1} + n_2 C_{p_2} + ...}{n_1 C_{v_1} + n_2 C_{v_2} + ...}$

MISCELLANEOUS SOLVED EXAMPLE

Problem 1. A vessel of volume 2×10^{-2} m^3 contains a mixture of hydrogen and helium at 47°C temperature and 4.15×10^5 N/m^2 pressure. The mass of the mixture is 10^{-2} kg. Calculate the masses of hydrogen and helium in the given mixture.

Solution: Let mass of H_2 is m_1 and He is m_2

$\therefore$ $m_1 + m_2 = 10^{-2}$ kg $= 10 \times 10^{-3}$ kg ....(1)

Let P_1, P_2 are partial pressure of H_2 and He

$P_1 + P_2 = 4.15 \times 10^5$ N/m^2

for the mixture

$$(P_1 + P_2)\,V = \left(\frac{m_1}{M_1} + \frac{m_2}{M_2}\right)RT$$

$$\Rightarrow \quad 4.15 \times 10^5 \times 2 \times 10^{-2} = \left(\frac{m_1}{2 \times 10^{-3}} + \frac{m_2}{4 \times 10^{-3}}\right) 8.31 \times 320$$

$$\Rightarrow \quad \frac{m_1}{2} + \frac{m_2}{4} = \frac{4.15 \times 2}{8.31 \times 320} = 0.00312 = 3.12 \times 10^{-3}$$

$\Rightarrow$ $2m_1 + m_2 = 12.48 \times 10^{-3}$ kg .....(2)

Solving (1) and (2)

$m_1 = 2.48 \times 10^{-3}$ kg $\cong$ $\mathbf{2.5 \times 10^{-3}}$ **kg**

and $\mathbf{m_2 = 7.5 \times 10^{-3}}$ **kg.**

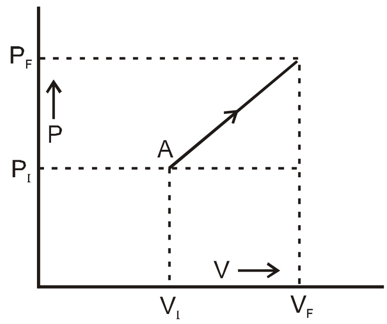
Problem 2. The pressure in a monoatomic gas increases linearly from 4×10^5 N m^{-2} to 8×10^5 N m^{-2} when its volume increases from 0.2 m^3 to 0.5 m^3. Calculate the following:

(a) work done by the gas.

(b) increase in the internal energy.

Solution : **(a)** As here pressure is varying linearly with volume, work done by the gas

$$W = \int P\,dV = \text{area under P-V curve}$$



$$W = P_I\left(V_F - V_I\right) + \frac{1}{2}\left(P_F - P_I\right) \times \left(V_F - V_I\right)$$

i.e., $W = 4 \times 10^5 \times 0.3 + \frac{1}{2} \times 4 \times 10^5 \times 0.3$

i.e., $\mathbf{W = 1.8 \times 10^5\ J}$

(b) The change in internal energy of a gas is given by

$$\Delta U = nC_V\,\Delta T = \frac{nR\Delta T}{(\gamma-1)} = \frac{(P_F V_F - P_I V_I)}{(\gamma-1)}$$

As the gas is monoatomic $\gamma = (5/3)$

So, $\Delta U = \dfrac{10^5(8\times0.5-4\times0.2)}{[(5/3)-1]} = \dfrac{3}{2} \times 10^5(4-0.8).$

i.e., $\mathbf{\Delta U = 4.8 \times 10^5\ J}$

Problem 3. There are two vessels. Each of them contains one mole of a monoatomic ideal gas. Initial volume of the gas in each vessel is $8.3 \times 10^{-3}\ m^3$ at 27^o C. Equal amount of heat is supplied to each vessel. In one of the vessels, the volume of the gas is doubled without change in its internal energy, whereas the volume of the gas is held constant in the second vessel. The vessels are now connected to allow free mixing of the gas. Find the final temperature and pressure of the combined gas system.

Solution : According to Ist law of thermodynamics,

$$\Delta Q = \Delta U + W$$

So for the vessel for which internal energy (and hence, temperature) remains constant.

$$\Delta Q_1 = W = nRT\log_e(V_F/V_I)$$

$$\Delta Q_1 = 1 \times R \times 300\log_e(2) = 0.693 \times 300\,R = 207.9\,R$$

and for the vessel for which volume is kept constant.

$$\Delta Q_2 = \Delta U = nC_V\,\Delta T \qquad [\text{as } W = 0]$$

i.e., $\Delta Q_1 = 1(3/2)R\,\Delta T$

According to given problem $\Delta Q_1 = \Delta Q_2$ i.e.,

$207.9R = (3/2)R\Delta T,$ i.e. $\Delta T = 138.6$

i.e., $T_F - T_I = 138.6$ with $T_I = 300$ K

So, $T_F = 300 + 138.6 =$ **438.6 K**

Now when the free mixing of gases is allowed

$$U_1 + U_2 = U$$

$$n_1(C_V)_1T_1 + n_2(C_V)_2T_2 = nC_VT$$

with $n = n_1 + n_2$

Here $n_1 = n_2 = 1$

and $(C_V)_1 = (C_V)_2 = C_V$

So $1 \times 300 + 1 \times 438.6 = 2T,$

i.e., $T = 369.3$ K

Further for the mixture from $PV = nRT$ with $V = V + 2V = 3V$ and $n = n_1 + n_2 = 2$, we have

$$P = \frac{nRT}{3V} = \frac{2\times8.3\times369.3}{3\times8.3\times10^{-3}} = \mathbf{2.462 \times 10^5\ N/m^2}$$

Problem 4. A gaseous mixture enclosed in a vessel of volume V consists of one gram mole of a gas A with $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$ and another gas B with $\gamma = \frac{7}{5}$ at a certain temperature T. The gram molecular weights of the gases A and B are 4 and 32 respectively. The gases A and B do not react with each other and are assumed to be ideal. The gaseous mixture follows the equation; $PV^{19/13}$ = constant in adiabatic processes.

(a) Find the number of gram moles of the gas B in the gaseous mixture.

(b) Compute the speed of sound in the gaseous mixture at T = 300 K.

(c) If T is raised by 1 K from 300 K, find the percentage change in the speed of sound in the gaseous mixture.

Solution : **(a)** As for ideal gas $C_P - C_V = R$ and $\gamma = (C_P/C_V)$,

So $$\gamma - 1 = \frac{R}{C_V}$$

or $$C_V = \frac{R}{(\gamma - 1)}$$

$\therefore$ $$(C_V)_1 = \frac{R}{(5/3)-1} = \frac{3}{2}R;$$

$$(C_V)_2 = \frac{R}{(7/5)-1} = \frac{5}{2}R$$

and $$(C_V)_{mix} = \frac{R}{(19/13)-1} = \frac{13}{6}R$$

Now from conservation of energy, i.e., $\Delta U = \Delta U_1 + \Delta U_2$,

$$(n_1 + n_2)(C_V)_{mix}\Delta T = [n_1(C_V)_1 + n_2(C_V)_2]\Delta T$$

i.e., $$(C_V)_{mix} = \frac{n_1(C_V)_1 + n_2(C_V)_2}{n_1 + n_2}$$

We have $$\frac{13}{6}R = \frac{1\times\frac{3}{2}R + n\times\frac{5}{2}R}{1+n} = \frac{(3+5n)}{2(1+n)}$$

or, $13 + 13n = 9 + 15n$, **n = 2 mole.**

(b) Molecular weight of the mixture will be given by

$$M = \frac{n_A M_A + n_B M_B}{n_A + n_B} = \frac{(1)(4) + 2(32)}{1+2}$$

$M = 22.67$

Speed of sound in a gas is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Therefore, in the mixture of the gas

$$v = \sqrt{\frac{(19/13)(8.31)(300)}{22.67\times10^{-3}}} \text{ m/s}; \quad \mathbf{v \approx 401\ m/s}$$

(c) $v \propto \sqrt{T}$

or $v = KT^{1/2}$ ......(2)

$\Rightarrow \quad \frac{dv}{dT} = \frac{1}{2} KT^{-1/2}$

$\Rightarrow \quad dv = K\left(\frac{dT}{2\sqrt{T}}\right)$

$\Rightarrow \quad \frac{dv}{v} = \frac{K}{v}\left(\frac{dT}{2\sqrt{T}}\right)$

$\Rightarrow \quad \frac{dv}{v} = \frac{1}{\sqrt{T}}\left(\frac{dT}{2\sqrt{T}}\right) = \frac{1}{2}\left(\frac{dT}{T}\right)$

$\Rightarrow \quad \frac{dv}{v} \times 100 = \frac{1}{2}\left(\frac{dT}{T}\right) \times 100 = \frac{1}{2}\left(\frac{1}{300}\right) \times 100 = \mathbf{0.167} = \frac{1}{6}$

Therefore, percentage change in speed is 0.167%.

*****

Exercise # 1

PART - I : SUBJECTIVE QUESTIONS

SECTION (A) : KINETIC THEORY OF GASES

A-1. An insulated container containing monoatomic gas of molar mass m is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature. **[JEE 2003]**

A-2. A cubical container having each side as ℓ is filled with a gas having N molecules in the container. Mass of each molecule is m. If we assume that at every instant half of the molecules are moving towards the positive x-axis and half of the molecules are moving towards the negative x-axis. Two walls of the container are perpendicular to the x-axis. Find the force acting on the two walls given? Assume that all the molecules are moving with speed v_0 .

SECTION (B) : ROOT MEAN SQUARE SPEED, KINETIC ENERGY AND EQUATION OF STATE

B-1. A mixture of 4 gm helium and 28 gm of nitrogen is enclosed in a vessel of constant volume 300 K. Find the quantity of heat absorbed by the mixture to doubled the root mean velocity of its molecules.

(R = Universal gas constant)

B-2. At room temperature (300 K) , the rms speed of the molecules of a certain diatomic gas is found to be 1930 m/s. Can you guess name of the gas ? Find the temperature at which the rms speed is double of the speed in part one (R = 25/3 J/mol – k)

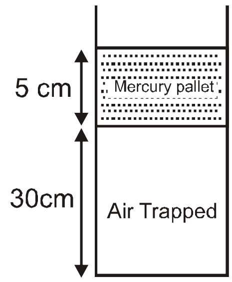
B-3. The speeds of three molecules are 3V, 4V and 5V respectively. Find their rms speed.

B-4. Butane gas burns in air according to the following reaction,

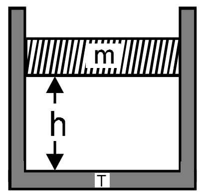
$$2\,C_4H_{10} + 13\,O_2 \longrightarrow 10\,H_2O + 8\,CO_2.$$

Suppose the initial and final temperatures are equal and high enough so that all reactants and products act as perfect gases. Two moles of butane are mixed with 13 moles of oxygen and then completely reacted. Find the final pressure (if the volume remains unchanged and the pressure before reaction is P_0)?

B-5. In given figure, A uniform cylindrical tube closed at one end, contains a pallet of mercury 5 cm long. When the tube is kept vertically with the closed end downward, the length of the air column trapped is 30 cm. Now the tube is inverted so that the closed end goes up find the new length of the air column trapped. Atmospheric pressure = 75 cm of mercury. (Assume temperature remains constant in this process)



B–6. (i) A conducting cylinder whose inside diameter is 4.00 cm contains air compressed by a piston of mass m = 13.0 kg, which can slide freely in the cylinder shown in the figure. The entire arrangement is immersed in a water bath whose temperature can be controlled. The system is initially in equilibrium at temperature t_i = 20°C. The initial height of the piston above the bottom of the cylinder is h_i = 4.00 cm. $P_{atm} = 1 \times 10^5$ N/m^2 and g = 10 m/s^2.



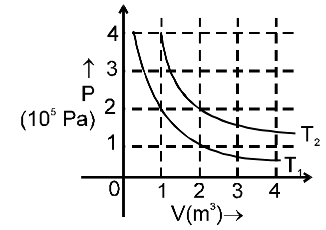
If the temperature of the water bath is gradually increased to a final temperature t_f = 100°C. Find the height h_f of the piston (in cm) at that instant?

(ii) In the above question, if we again start from the initial conditions and the temperature is again gradually raised, and weights are added to the piston to keep its height fixed at h_i. Find the value of the added mass when the final temperature becomes t_f = 100°C?

SECTION (C) : MAXWELL'S DISTRIBUTION OF SPEED

C-1. The mean speed of the molecules of a hydrogen sample equals the mean speed of the molecules of helium sample. Calculate the ratio of the temperature of the hydrogen sample to the temperature of the helium sample.

C-2. The following graph shows two isotherms for a fixed mass of an ideal gas. Find the ratio of r.m.s. speed of the molecules at temperatures T_1 and T_2 ?



C-3. Find the temperature at which average speed of oxygen molecule be sufficient so as to escape from the earth? (Escape speed from the earth is 11.0 km/sec, R = 25/3 J-mol^{-1}K^{-1}) .

C-4. Find the average of magnitude of linear momentum of a helium molecule in a sample of helium gas at 150 π K. Mass of a helium molecule = (166/3) × 10^{-27} kg and R = 25/3 J-mol^{-1} K^{-1}

SECTION (D) : LAW OF EQUIPARTITION AND INTERNAL ENERGY

D-1. 0.040 g of He is kept in a closed container initially at 100.0ºC. The container is now heated. Neglecting the expansion of the container, calculate the temperature at which the internal energy is increased by 12 J.

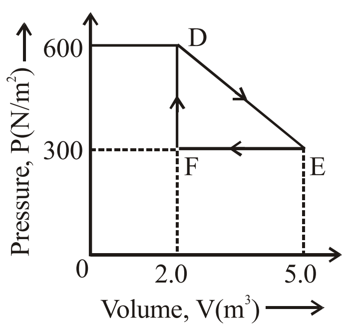
$$\left[R = \frac{25}{3} J-mol^{-1}-k^{-1}\right]$$

D-2. 16 g of oxygen at 37ºC is mixed with 14 g of nitrogen at 27ºC. Find the temperature of the mixture?

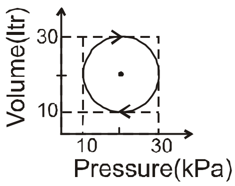
D-3. Show that the internal energy of the air (treated as an ideal gas) contained in a room remains constant as the temperature changes between day and night. Assume that the atmospheric pressure around remains constant and the air in the room maintains this pressure by communicating with the surrounding through the windows etc.

SECTION (E) : CALCULATION OF WORK

E-1. A thermodynamic system is taken from an original state to an intermediate state by the linear process shown in figure. Its volume is then reduced to the original value from E to F by an isobaric process. Calculate the total work done by the gas from D to E to F

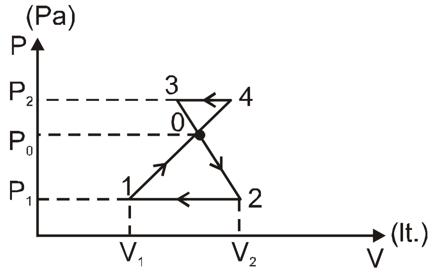


E-2. Find the work done by gas going through a cyclic process shown in figure?



E-3. An ideal gas is compressed at constant pressure of 10^5 Pa until its volume is halved. If the initial volume of the gas as 3.0×10^{-2} m^3, find the work done on the gas?

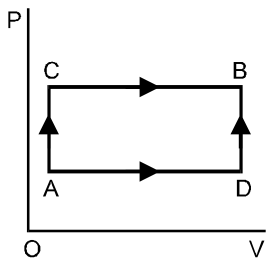
E-4. Find the work done by an ideal gas during a closed cycle $1 \rightarrow 4 \rightarrow 3 \rightarrow 2 \rightarrow 1$ shown in figure if $P_1 = 10^5$ Pa, $P_0 = 3 \times 10^5$ Pa, $P_2 = 4 \times 10^5$ Pa, $V_2 - V_1 = 10$ litre, and segments 4-3 and 2-1 of the cycle are parallel to the V-axis ?



SECTION (F) : FIRST LAW OF THERMODYNAMICS

F-1. In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 22.3 J is done on the system. If the gas is taken from state A to B via a process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the latter case ? (Take 1 cal = 4.19 J)

F-2. In given figure, when a thermodynamic system is taken from state A to state B via path ACB, 100 cal of heat given to the system and 60 cal work is done by the gas. Along the path ADB, the work done by the gas is 20 cal. Find the heat flowing into the system in this case?



F-3. A cylinder fitted with a piston contains an ideal monoatomic gas at a temperature of 400 K. The piston is held fixed while heat ΔQ is given to the gas, It is found the temperature of the gas has increased by 20 K. In an isobaric process the same ΔQ heat is supplied slowly to it. Find the change in temperature in the second process?

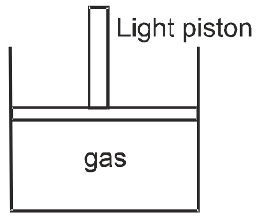
F-4. When 1 g of water at 0ºC and 1×10^5 N m^{-2} pressure is converted into ice of volume 1.091 cm^3, find the work done by water? (ρ_w = 1 gm/cm^3)

F-5. An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are Q_1 = 5960 J, Q_2 = – 5585 J, Q_3 =– 2980 J and Q_4 = 3645 J respectively. The corresponding works involved are W_1 = 2200 J, W_2 = – 825 J, W_3 = – 1100 J and W_4 respectively.

(i) Find the value of W_4.

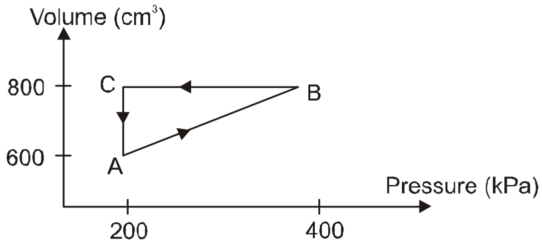
(ii) What is the efficiency of the cycle ?

F-6. In given figure, gas is slowly heated for sometime. During the process , the increase in internal energy of the gas is 10 J and the piston is found to move out by 25 cm, then find the amount of heat supplied. The area of cross-section of cylinder = 40 cm^2 and atmospheric pressure = 100 kPa



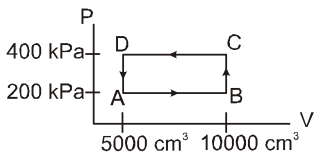
F-7. Find the change in the internal energy of 2kg of water as it is heated from 0ºC to 4ºC. The specific heat capacity of water is 4200 J/kg-K and its densities at 0ºC and 4ºC are 999.9kg/m^3 and 1000 kg/m^3 respectively. Atmospheric pressure = 10^5 Pa.

F-8. In given figure, An ideal gas a gas is taken through a cyclic process ABCA, calculate the value of mechanical equivalent of heat (J) when 4.8 cal of heat is given in the process ?



F-9. In given figure, one mole of an ideal gas (γ = 7/5) is taken through the cyclic process ABCDA.

Take R = $\frac{25}{3}$ J/mol–K



(a) Find the temperature of the gas in states A, B, C and D.

(b) Find the amount of heat supplied/released in processes AB, BC, CD and DA.

(c) Find work done by gas during cyclic process.

SECTION (G) : SPECIFIC HEAT CAPACITIES OF GASES

G-1. Find the molecular mass of a gas if the specific heats of the gas are C_P=0.2 cal/gm°C and C_V = 0.15 cal/gm°C. [Take R = 2 cal/mole°C]

G-2. If γ be the ratio of specific heats (C_p & C_v) for a perfect gas, Find the number of degrees of freedom of a molecule of the gas?:

G-3. Internal energy of two moles of an ideal gas at a temperature of 127ºC is 1200 R. Then find the molar specific heat of the gas at constant pressure?

G-4. Ideal monoatomic gas is taken through a process dQ = 2dU. Find the molar heat capacity (in terms of R) for the process ? (where dQ is heat supplied and dU is change in internal energy)

G-5. When 100 J of heat is given to an ideal gas it expands from 200 cm^3 to 400 cm^3 at a constant pressure of 3×10^5 Pa. Calculate (a) the change in internal energy of the gas, (b) the number of moles in the gas if the initial temperature is 400 K, (c) the molar heat capacity C_p at constant pressure and (d) the molar heat capacity C_v at constant volume. $\left[R = \frac{25}{3} J/mol-k\right]$

G-6. The temperature of 5 mol of a gas which was held at constant volume was changed from 100ºC to 120ºC. The changes in internal energy was found to be 80 J. Find the molar heat capacity of the gas at constant volume?

G-7. For a gas, $\gamma = 9/7$. What is the number of degrees of freedom of the molecules of this gas ?

SECTION (H) : ADIABATIC PROCESS AND FREE EXPANSION

H-1. Two cylinders A and B of equal capacity are connected to each other via a stopcock. A contains a gas at standard temperature and pressure. B is completely evacuated. The entire system is thermally insulated. The stopcock is suddenly opened. Answer the following :

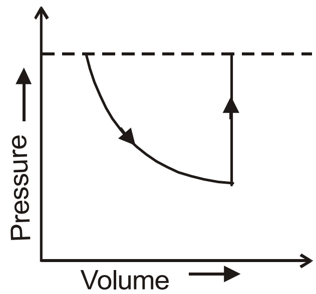
(a) What is the final pressure of the gas in A and B ?

(b) What is the change in internal energy of the gas ?

(c) What is the change in the temperature of the gas ?

(d) Do the intermediate states of the system (before settling to the final equilibrium state) lie on its P-V-T surface ?

H-2. In given figure, a sample of an ideal gas initially having internal energy U_1 is allowed to expand adiabatically performing work W. Heat Q is then supplied to it, keeping the volume constant at its new value, until the pressure rised to its original value. The internal energy is then U_2.



Find the increase in internal energy $(U_2 - U_1)$?

H-3. One mole of an ideal monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$. ($\gamma$ denotes the ratio of specific heat at constant pressure, to that at constant volume) find γ for the mixture?

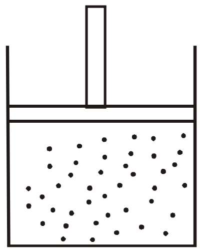
H-4. The pressure and density of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$ change adiabatically from (P, d) to (P′, d′). If $\frac{d'}{d} = 32$, then find the value of $\frac{P'}{P}$?

H-5. An ideal gas ($\gamma = \frac{5}{3}$) is adiabatically compressed from 640 cm^3 to 80 cm^3. If the initial pressure is P then find the final pressure?

H-6. In an adiabatic process, the pressure is increased by $\frac{2}{3}\%$. If $\gamma = \frac{3}{2}$, then find the decreases in volume (approximately)?

H-7. An ideal gas at pressure 4×10^5 Pa and temperature 400 K occupies 100 cc. It is adiabatically expanded to double of its original volume. Calculate (a) the final pressure, (b) final temperature and (c) work done by the gas in the process ($\gamma = 1.5$) :

H-8. In fig, the walls of the container and the piston are weakly conducting. The initial pressure, volume and temperture of the gas are 200 K Pa, 800 cm^3 and 100 K respectively. Find the pressure and the temperature of the gas if it is (a) slowly compressed (b) suddenly compressed to 200 cm^3 ($\gamma = 1.5$).



SECTION (I) : POLYTROPIC PROCESS

I-1. If Q amount of heat is given to a diatomic ideal gas in a process in which the gas perform a work $\frac{2Q}{3}$ on its surrounding. Find the molar heat capacity (in terms of R) for the process.

I-2. An ideal gas is taken through a process in which pressure and volume vary as $P = kV^2$. Show that the molar heat capacity of the gas for the process is given by $C = C_v + \frac{R}{3}$.

I-3. Find the molar heat capacity (in terms of R) of a monoatomic ideal gas undergoing the process : $PV^{1/2}$ = constant ?

SECTION (J) : FOR JEE-MAIN

J-1. A Carnot engine takes 10^3 kilocalories of heat from a reservoir at 627ºC and exhausts it to a sink at 27ºC. What will be the efficiency of the engine ?.

J-2. In the above problem, what will be the work performed by the engine ?

J-3. The efficiency of Carnot's engine is 50%. The temperature of its sink is 7ºC. To increase its efficiency to 70%. What is the increase in temperature of the source ?

J-4. A Carnot engine work as refrigerator in between 0ºC and 27ºC. How much energy is needed to freeze 10 kg ice at 0ºC.

J-5. What is the work efficiency coefficient in above question ?

J-6. A Carnot engine works as a refrigerator in between 250K and 300K. If it acquires 750 calories from heat source at low temperature, then what is the heat generated at higher temperature. (in calories)?

PART - II : OBJECTIVE QUESTIONS

*** Marked Questions are having more than one correct option.**

SECTION (A) : KINETIC THEORY OF GASES

A-1. Which of the following is correct for the molecules of a gas in thermal equilibrium ?

(A) All have the same speed

(B) All have different speeds which remain constant

(C) They have a certain constant average speed

(D) They do not collide with one another.

A-2. When an ideal gas is compressed isothermally then its pressure increases because :

(A) its potential energy decreases

(B) its kinetic energy increases and molecules move apart

(C) its number of collisions per unit area with walls of container increases

(D) molecular energy increases

A-3. In a sample of an ideal gas the average momentum of a molecule depends on

(A) pressure (B) mass of gas (C) number of moles (D) none of these

SECTION (B) : ROOT MEAN SQUARE SPEED, KINETIC ENERGY AND EQUATION OF STATE

B-1. Three particles have speeds of 2u, 10u and 11u. Which of the following statements is correct?

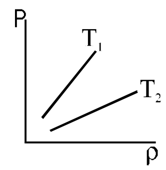
(A) The r.m.s. speed exceeds the mean speed by about u.

(B) The mean speed exceeds the r.m.s. speed by about u.

(C) The r.m.s. speed equals the mean speed.

(D) The r.m.s. speed exceeds the mean speed by more than 2u.

B-2. Figure shows graphs of pressure vs density for an ideal gas at two temperatures T_1 and T_2.



(A) $T_1 > T_2$

(B) $T_1 = T_2$

(C) $T_1 < T_2$

(D) any of the three is possible

B-3. Suppose a container is evacuated to leave just one molecule of a gas in it. Let ν_{mp} and ν_{av} represent the most probable speed and the average speed of the gas, then

(A) $\nu_{mp} > \nu_{av}$ (B) $\nu_{mp} < \nu_{av}$ (C) $\nu_{mp} = \nu_{av}$ (D) none of these

B-4. The temperature at which the r.m.s velocity of oxygen molecules equal that of nitrogen molecules at 100ºC is nearly:

(A) 426.3 K (B) 456.3 K (C) 436.3 K (D) 446.3 K

B-5. Four containers are filled with monoatomic ideal gases. For each container, the number of moles, the mass of an individual atom and the rms speed of the atoms are expressed in terms of n, m and v_{rms} respectively. If T_A, T_B, T_C and T_D are their temperatures respectively then which one of the options correctly represents the order?

	A	B	C	D
Number of moles	n	3n	2n	n
Mass	4m	m	3m	2m
Rms speed	V_{rms}	$2V_{rms}$	V_{rms}	$2V_{rms}$
Temperature	T_A	T_B	T_C	T_D

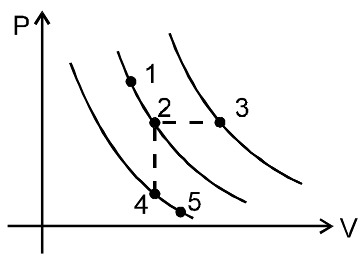
(A) $T_B = T_C > T_A > T_D$ (B) $T_D > T_A > T_C > T_B$ (C) $T_D > T_A = T_B > T_C$ (D) $T_B > T_C > T_A > T_D$

B-6. For a gas sample with N_0 number of molecules, function N(V) is given by : $N(V) = \frac{dN}{dV} = \left(\frac{3N_0}{V_0^3}\right)V^2$ for $0 < V < V_0$ and N(V) = 0 for $V > V_0$. Where dN is number of molecules in speed range V to V+ dV. The rms speed of the molecules is :

(A) $\sqrt{\frac{2}{5}}V_0$ (B) $\sqrt{\frac{3}{5}}V_0$ (C) $\sqrt{2}V_0$ (D) $\sqrt{3}V_0$

SECTION (C) : MAXWELL'S DISTRIBUTION OF SPEED

C-1. A certain gas is taken to the five states represented by dots in the graph. The plotted lines are isotherms. Order of the most probable speed v_p of the molecules at these five states is :



(A) $V_{P\,at\,3} > V_{P\,at\,1} = V_{P\,at\,2} > V_{P\,at\,4} = V_{P\,at\,5}$

(B) $V_{P\,at\,1} > V_{P\,at\,2} = V_{P\,at\,3} > V_{P\,at\,4} > V_{P\,at\,5}$

(C) $V_{P\,at\,3} > V_{P\,at\,2} = V_{P\,at\,4} > V_{P\,at\,1} > V_{P\,at\,5}$

(D) Insufficient information to predict the result.

C-2. Three closed vessels A, B, and C are at the same temperature T and contain gases which obey the Maxwell distribution of speed . Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of O_2 molecules in vessel A is V_1, that of the N_2 molecules in vessel B is V_2, the average speed of the O_2 molecules in vessel C will be :

(A) $(V_1 + V_2)/2$ (B) V_1 (C) $(V_1V_2)^{1/2}$ (D) $\frac{V_1}{2}$

SECTION (D) : LAW OF EQUIPARTITION AND INTERNAL ENERGY

D-1. The pressure of an ideal gas is written as $E = \frac{3PV}{2}$. Here E stands for

(A) average translational kinetic energy (B) rotational kinetic energy

(C) total kinetic energy. (D) None of these

D-2. The quantities which remain same for all ideal gases at the same temperature is/are ?

(A) the kinetic energy of equal moles of gas

(B) the kinetic energy of equal mass of gas

(C) the number of molecules of equal moles of gas

(D) the number of molecules of equal mass of gas

D-3. The quantity $\frac{2U}{fkT}$ represents (where U = internal energy of gas)

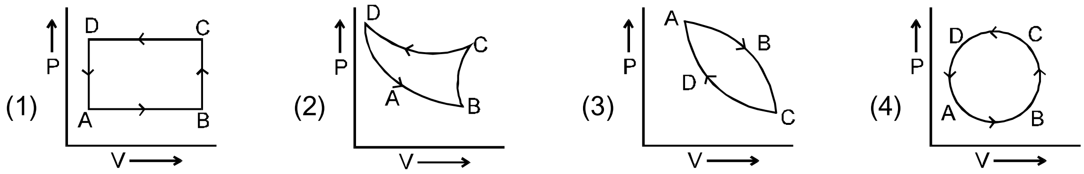
(A) mass of the gas (B) kinetic energy of the gas
(C) number of moles of the gas (D) number of molecules in the gas

SECTION (E) : CALCULATION OF WORK

E-1. One mole of an ideal gas is contained with in a cylinder by a frictionless piston and is initially at temperature T. The pressure of the gas is kept constant while it is heated and its volume doubles. If R is molar gas constant, the work done by the gas in increasing its volume is :-

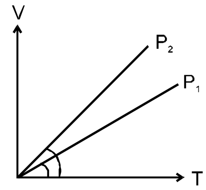
(A) RT ℓn2 (B) 1/2 RT (C) RT (D) 3/2 RT

E-2. In the following figures (1) to (4), variation of volume by change of pressure is shown. A gas is taken along the path ABCDA. The change in internal energy of the gas will be:



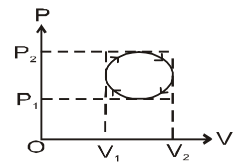
(A) positive in all cases from (1) to (4)
(B) positive in cases (1), (2) and (3) but zero in case (4)
(C) negative in cases (1), (2) and (3) but zero in case (4)
(D) zero in all the four cases.

E-3. In the following V-T diagram what is the relation between P_1 and P_2 :



(A) $P_2 = P_1$
(B) $P_2 > P_1$
(C) $P_2 < P_1$
(D) cannot be predicted

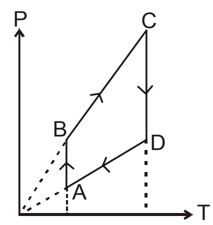
E-4. In a cyclic process shown on the P – V diagram the magnitude of the work done is :



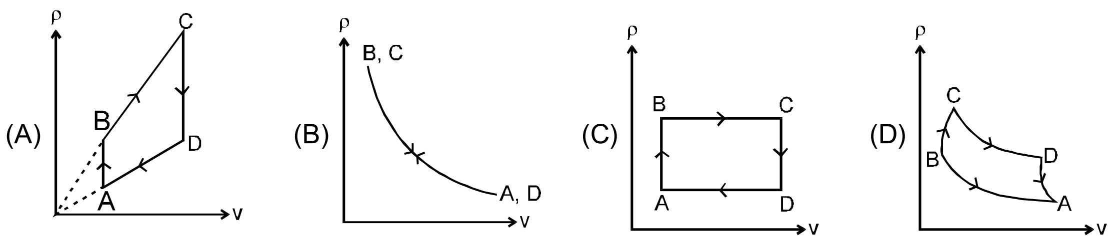
(A) $\pi\left(\frac{P_2 - P_1}{2}\right)^2$ (B) $\pi\left(\frac{V_2 - V_1}{2}\right)^2$

(C) $\frac{\pi}{4}(P_2 - P_1)(V_2 - V_1)$ (D) $\pi(P_2V_2 - P_1V_1)$

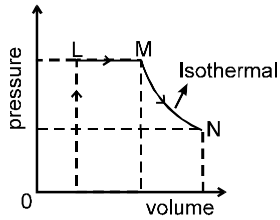
E-5. Pressure versus temperature graph of an ideal gas is as shown in figure.



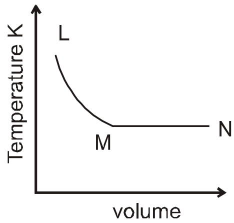
Corresponding density (ρ) versus volume (v) graph will be :

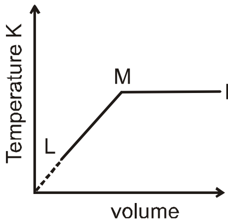


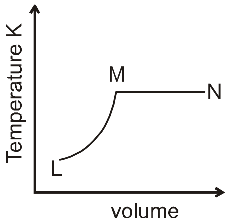
E-6. A fixed mass of an ideal gas undergoes changes of pressure and volume starting at L, as shown in Figure.

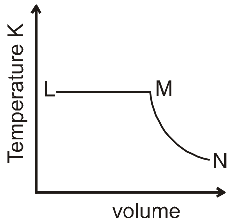


Which of the following is correct :

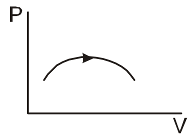
(A) 

(B) 

(C) 

(D) 

E-7.. In figure, P-V curve of an ideal gas is given. During the process, the cumulative work done by the gas



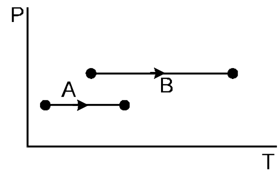
(A) continuously increases

(B) continuously decreases

(C) first increases then decreases

(D) first decreases then increases

E-8. In given figure, let ΔW_1 and ΔW_2 be the work done by the gas in process A and B respectively then (given change in volume is same in both process)



(A) $\Delta W_1 > \Delta W_2$

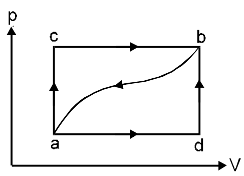
(B) $\Delta W_1 = \Delta W_2$

(C) $\Delta W_1 < \Delta W_2$

(D) Nothing can be said about the relation between ΔW_1 and ΔW_2

SECTION (F) : FIRST LAW OF THERMODYNAMICS

F-1. When a system is taken from state 'a' to state 'b' along the path 'acb', it is found that a quantity of heat Q = 200 J is absorbed by the system and a work W = 80J is done by it. Along the path 'adb', Q = 144J. The work done along the path 'adb' is



(A) 6J (B) 12 J (C) 18 J (D) 24 J

F-2. In above question, if U_a = 40J, value of U_b will be

(A) – 50 J (B) 100 J (C) – 120 J (D) 160 J

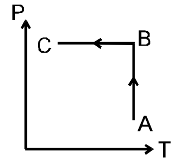
F-3. In above question, if U_d = 88 J, heat absorbed for the path 'db' is

(A) – 72 J (B) 72 J (C) 144 J (D) – 144 J

F-4. In the above question, if the work done on the system along the curved path 'ba' is 52J, heat abosrbed is

(A) – 140 J (B) – 172 J (C) 140 J (D) 172 J

F-5. Ideal gas is taken through process shown in figure:



(A) In process AB, work done by system is positive

(B) In process AB, heat is rejected out of the system.

(C) In process AB, internal energy increases

(D) In process AB internal energy decreases and in process BC internal energy increases.

F-6. In isothermal process if heat is released from an ideal gas then,

(A) the internal energy of the gas will increase

(B) the gas will do positive work

(C) the gas will do negative work

(D) the given process is not possible

F-7. In an isothermal expansion of an ideal gas. Select wrong statement:

(A) there is no change in the temperature of the gas

(B) there is no change in the internal energy of the gas

(C) the work done by the gas is equal to the heat supplied to the gas

(D) the work done by the gas is equal to the change in its internal energy

SECTION (G) : SPECIFIC HEAT CAPACITIES OF GASES

G-1. When an ideal diatomic gas is heated at constant pressure , the fraction of the heat energy supplied which increases the internal energy of the gas is .

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{3}{7}$ (D) $\frac{5}{7}$

G-2. The value of the ratio C_p/C_v for hydrogen is 1.67 at 30 K but decreases to 1.4 at 300 K as more degrees of freedom become active. During this rise in temperature,

(A) C_p remains constant but C_v increases

(B) C_p decreases but C_v increases

(C) Both C_p and C_v decrease by the same amount

(D) Both C_p and C_v increase by the same amount

G-3. For an ideal gas, the heat capacity at constant pressure is larger than than that at constant volume because

(A) positive work is done during expansion of the gas by the external pressure

(B) positive work is done during expasion by the gas against external pressure

(C) positive work is done during expansion by the gas against intermolecular forces of attraction

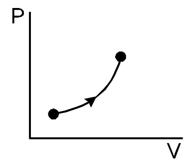
(D) more collisions occur per unit time when volume is kept constant.

G-4. A gas has :

(A) one specific heat only (B) two specific heats only

(C) infinite number of specific heats (D) no specific heat

G-5. If molar heat capacity of the given process (as shown in figure) is C, then



(A) $C < C_V$

(B) $C = 0$

(C) $C > C_v$

(D) $C = C_v$

G-6. For small positive coefficient of expansion in case of solid,

(A) $C_p - C_v = R$ (B) $C_p - C_v = 2R$

(C) C_p is slightly greater than C_v (D) C_p is slightly less than C_v

SECTION (H) : ADIABATIC PROCESS AND FREE EXPANSION

H-1. Monoatomic, diatomic and triatomic gases whose initial volume and pressure are same, are compressed till their volume becomes half the initial volume.

(A) If the compression is adiabatic then monoatomic gas will have maximum final pressure.

(B) If the compression is adiabatic then triatomic gas will have maximum final pressure.

(C) If the compression is adiabatic then their final pressure will be same.

(D) If the compression is isothermal then their final pressure will be different.

H-2. A gas is contained in a metallic cylinder fitted with a piston. The gas is suddenly compressed by pushing piston downward and is maintained at this position. After this process, as time passes the pressure of the gas in the cylinder

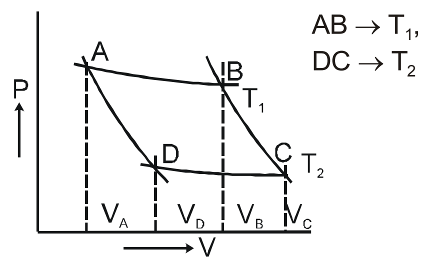
(A) increases

(B) decreases

(C) remains constant

(D) increases or decreases depending on the nature of the gas.

H-3. In the following P–V diagram of an ideal gas, AB and CD are isothermal where as BC and DA are adiabatic process. The value of V_B/V_C is



(A) $= V_A/V_D$

(B) $< V_A/V_D$

(C) $> V_A/V_D$

(D) cannot say

H-4. Let ΔW_1 and ΔW_2 be the work done by the systems 1 and 2 respectively in the previous question then :

(A) $\Delta W_1 > \Delta W_2$ (B) $\Delta W_1 = \Delta W_2$ (C) $\Delta W_1 < \Delta W_2$

(D) The relation between W_1 and W_2 cannot be deduced.

H-5. When an ideal gas undergoes an adiabatic change causing a temparture change ΔT

(i) there is no heat gained or lost by the gas

(ii) the work done by the gas is equal to change in internal energy

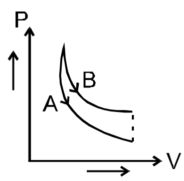
(iii) the change in internal energy per mole of the gas is $C_v \Delta T$, where C_v is the molar heat capacity at constant volume.

(A) (i), (ii), (iii) correct (B) (i), (ii) correct (C) (i), (iii) correct (D) (i) correct

H-6. A given quantity of a gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is:

(A) $\frac{2}{3}P$ (B) P (C) $\frac{3}{2}P$ (D) 2P

H-7. In figure, A and B are two adiabatic curves for two different gases. Then A and B corresponds to :



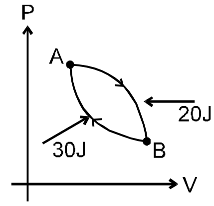
(A) Ar and He respectively

(B) He and H_2 respectively

(C) O_2 and H_2 respectively

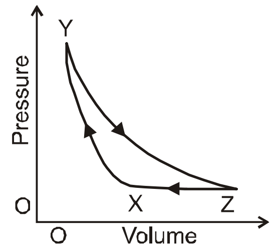
(D) H_2 and He respectively

H-8. In a cyclic process shown in the figure an ideal gas is adiabatically taken from B and A, the work done on the gas during the process $B \rightarrow A$ is 30 J, when the gas is taken from $A \rightarrow B$ the heat absorbed by the gas is 20 J. The change in internal energy of the gas in the process $A \rightarrow B$ is :



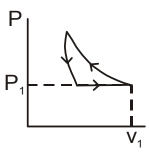
(A) 20 J (B) – 30 J (C) 50 J (D) – 10 J

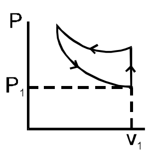
H-9. In given figure, a fixed mass of an ideal gas undergoes the change represented by XYZX below. Which one of the following sets could describe these of changes ?

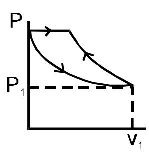


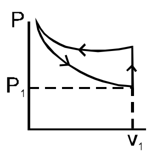
	XY	YZ	ZX
(A)	isothermal expansion	adiabatic compression	compression at constant pressure
(B)	adiabatic expansion	isothermal compression	pressure reduction constant volume
(C)	isothermal compression	adiabatic expansion	compression at constant pressure
(D)	adiabatic compression	isothermal expansion	compression at constant pressure

H-10. A certain mass of an ideal gas is at pressure P_1 and volume V_1. It is compressed isothermally and then allowed to expand adiabatically until its pressure returns to P_1. The gas is then allowed to expand its original volume. Which of the following P-V graphs are these process correctly shown?

(A) 

(B) 

(C) 

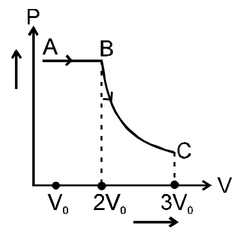
(D) 

SECTION (I) : POLYTROTIC PROCESS

I-1. A gas undergoes a process in which its pressure P and volume V are related as VP^n = constant. The bulk modulus of the gas in the process is :

(A) nP (B) $P^{1/n}$ (C) P/n (D) P^n

I-2. One mole of a gas is subjected to two process AB and BC, one after the other as shown in the figure. BC is represented by PV^n = constant. We can conclude that (where T = temperature, W = work done by gas, V = volume and U = internal energy).



(A) $T_A = T_B = T_C$ (B) $V_A < V_B$, $P_B < P_C$

(C) $W_{AB} < W_{BC}$ (D) $U_A < U_B$

I-3. The molar heat capacity C for an ideal gas going through a process is given by $C = \frac{a}{T}$, where 'a' is a constant. If $\gamma = \frac{C_p}{C_v}$, the work done by one mole of gas during heating from T_0 to ηT_0 will be:

(A) $a \,\ell n\, \eta$ (B) $\frac{1}{a \ell n \eta}$ (C) $a \ell n\eta - \left(\frac{\eta - 1}{\gamma - 1}\right) RT_0$ (D) $a \ell n\eta - (\gamma - 1)RT_0$

I-4. One mole of an ideal gas undergoes a process in which $T = T_0 + aV^3$, where T_0 and 'a' are positive constants and V is volume. The volume for which pressure will be minimum is

(A) $\left(\frac{T_0}{2a}\right)^{1/3}$ (B) $\left(\frac{T_0}{3a}\right)^{1/3}$ (C) $\left(\frac{a}{2T_0}\right)^{2/3}$ (D) $\left(\frac{a}{3T_0}\right)^{2/3}$

I-5. In a certain gas, the ratio of the speed of sound and root mean square speed is $\sqrt{\frac{5}{9}}$. The molar heat capacity of the gas in a process given by PT = constant is: (Take R = 2 cal/mole K). Treat the gas as ideal.

(A) $\frac{R}{2}$ (B) $\frac{3R}{2}$ (C) $\frac{5R}{2}$ (D) $\frac{7R}{2}$

I-6. A polytropic process for an ideal gas is represented by equation PV^n = constant. If γ is ratio of specific heats $\left(\frac{C_p}{C_v}\right)$, then value of n for which molar heat capacity of the process is negative, is given as :

(A) $\gamma > n$ (B) $\gamma > n > 1$ (C) $n > \gamma$ (D) none, as it is not possible

I-7. One mole of an ideal gas at a temperature T_1K expands slowly according to the law $\frac{p}{V}$ = constant. Its final temperature is T_2K. The work done by the gas is:

(A) $R(T_2 - T_1)$ (B) $2R(T_2 - T_1)$ (C) $\frac{R}{2}(T_2 - T_1)$ (D) $\frac{2R}{3}(T_2 - T_1)$

SECTION (J) : FOR JEE MAIN

J-1. The coefficient of performance of a carnot refrigerator working between 30° C and 0° C is

(1) 10 (2) 1 (3) 9 (4) 0

J-2. If the door of a refrigerator is kept open then which of the following is ture

(1) Room is cooled

(2) Room is heated

(3) Room is either cooled or heated

(4) Room is neither cooled nor heated

J-3. An Ideal gas heat engine operated in a carnot's cycle between 227° C and 127° C . It absorbs 6×10^4 J at high temperature. The amount of heat converted into work is

(1) 4.8×10^4 J (2) 3.5×10^4 J (3) 1.6×10^4 J (4) 1.2×10^4 J

J-4. A scientist says that the efficiency of his heat engine which operates at source temperature 127°C and sink temperature 27° C is 26% then

(1) It is impossible (2) It is possible but less probable

(3) It is quite probable (4) Data are incomplete

J-5. A Carnot engine takes 3×10^6 cal of heat from a reservoir at 627ºC and gives it to a sink at 27ºC. The work done by the engine is : **[AIEEE - 2003, 4/300]**

(1) 4.2×10^6 J (2) 8.4×10^6 J (3) 16.8×10^6 J (4) zero

PART - III : MATCH THE COLUMN

1. An ideal monoatomic gas undergoes different types of processes which are described in column-I. Match the corresponding effects in column-II. The letters have usual meaning.

Column-I	**Column-II**
(A) $P = 2V^2$	(p) If volume increases then temperature will also increases.
(B) PV^2 = constant	(q) If volume increases then temperature will decreases.
(C) $C = C_V + 2R$	(r) For expansion, heat will have to be supplied to the gas.
(D) $C = C_V - 2R$	(s) If temperature increases then work done by gas is positive.

2. The figures given below show different processes (relating pressure P and volume V) for a given amount for an ideal gas. W is work done by the gas and ΔQ is heat absorbed by the gas.

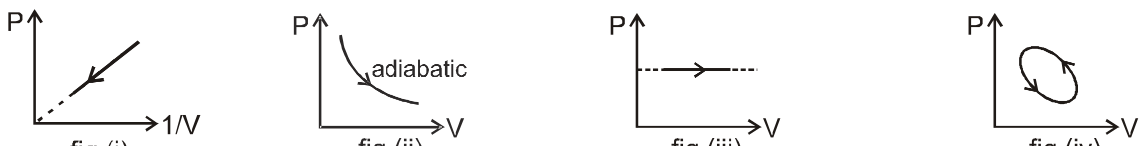


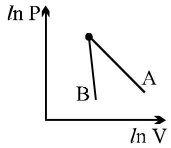
fig (i) fig (ii) fig (iii) fig (iv)

Column-I	**Column-II**
(A) In Figure (i)	(p) $\Delta Q > 0$.
(B) In Figure (ii)	(q) $W < 0$.
(C) In Figure (iii)	(r) $\Delta Q < 0$.
(D) In Figure (iv) (for complete cycle)	(s) $W > 0$.

Exercise # 2

PART - I : ONLY ONE OPTION CORRECT TYPE

1. The figure, shows the graph of logarithmic reading of pressure and volume for two ideal gases A and B undergoing adiabatic process. From figure it can be concluded that



(A) gas B is diatomic
(B) gas A and B both are diatomic
(C) gas A is monoatomic
(D) gas B is monoatomic & gas A is diatomic

2. P_i, V_i are initial pressure and volumes and V_f is final volume of a gas in a thermodynamic process respectively. If PV^n = constant, then the amount of work done by gas is : ($\gamma = C_p/C_v$).

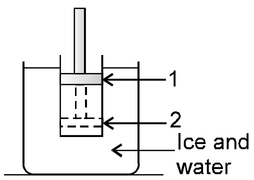
Assume same, initial state & same final volume in all processes.

(A) minimum for $n = \gamma$
(B) minimum for $n = 1$
(C) minimum for $n = 0$
(D) minimum for $n = \frac{1}{\gamma}$

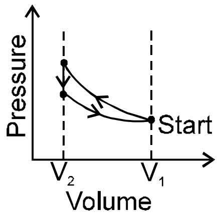
3. The value of $C_p - C_v$ is 1.09 R for a gas sample in state A and is 1.00 R in state B. Let T_A, T_B denote the temperature and p_A and p_B denote the pressure of the states A and B respectively. Then

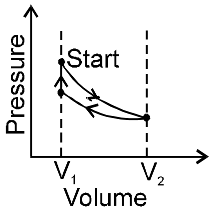
(A) $p_A < p_B$ and $T_A > T_B$ (B) $p_A > p_B$ and $T_A > T_B$ (C) $p_A = p_B$ and $T_A < T_B$ (D) $p_A > p_B$ and $T_A < T_B$

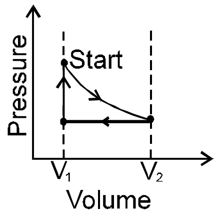
4. Figure shows a conducting cylinder containing gas and closed by a movable piston. The cylinder is submerged in an ice-water mixture. The piston is quickly pushed down from position (1) to position (2). The piston is held at position (2) until the gas is again at 0°C and then is slowly raised back to position

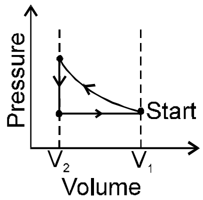


P-V diagram for the above process will be

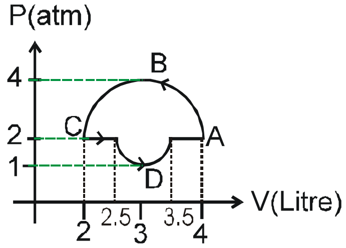
(A) 

(B) 

(C) 

(D) 

5. Find work done by the gas in the process shown in figure :



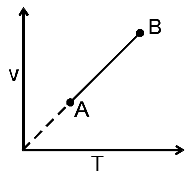
(A) $\frac{5}{2}\pi$ atm L (B) $\frac{5}{2}$ atm L (C) $-\frac{3}{2}\pi$ atm L (D) $-\frac{5}{4}\pi$ atm L

6. Two different ideal diatomic gases A and B are initially in the same state. A and B are then expanded to same final volume through adiabatic and isothermal process respectively. If P_A, P_B and T_A, T_B represents the final pressure and temperatures of A and B respectively then:

(A) $P_A < P_B$ and $T_A < T_B$ (B) $P_A > P_B$ and $T_A > T_B$

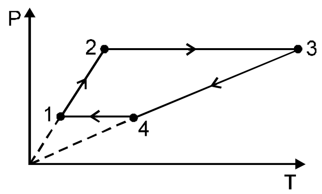
(C) $P_A > P_B$ and $T_A < T_B$ (D) $P_A < P_B$ and $T_A > T_B$

7. An ideal monoatomic gas undergoes the process AB as shown in the figure. If the heat supplied and the work done in the process are ΔQ and W respectively. The ratio ΔQ : W is :



(A) 2.50 (B) 1.67 (C) 0.67 (D) 0.40

8. Three moles of an ideal monoatomic gas perform a cycle shown in figure. The gas temperatures in different states are T_1 = 400 K, T_2 = 800 K, T_3 = 2400 K, and T_4 = 1200 K. The work done by the gas during the cycle is :- $\left[R = \frac{25}{3} J/mol-k\right]$



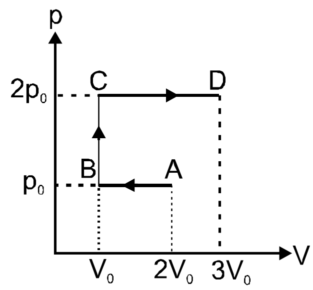
(A) 5 kJ (B) 10 kJ (C) 15 kJ (D) 20 kJ

9. The molar heat capacity at constant presure of nitrogen gas at STP is nearly 3.5 R. Now when the temperature is increased, it gradually increases and approaches 4.5 R. The most appropriate reason for this behaviour is that at high temperatures

(A) nitrogen does not behave as an ideal gas (B) nitrogen molecules dissociate in atoms

(C) the molecules collides more frequently (D) molecular vibration gradually become effective

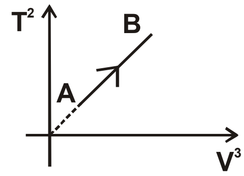
10. P–V indicator diagram for a given sample of monoatomic ideal gas is shown in figure. The average molar specific heat capacity of the system for the process ABCD is : (R is a universal gas constant)



(A) 3R (B) $\frac{3R}{2}$

(C) $\frac{9R}{4}$ (D) $\frac{3R}{4}$

11. If ideal diatomic gas follows the process, as shown in graph, where T is temperature in kelvin and V is volume (m^3), then molar heat capacity for this process will be [in terms of gas constant R] :



(A) $\frac{7R}{2}$ (B) 5R

(C) $\frac{19R}{6}$ (D) $\frac{11R}{2}$

12. An ideal monoatomic gas is initially in state 1 with pressure p_1 = 20 atm and volume v_1 = 1500 cm^3. It is then taken to state 2 with pressure $p_2 = 1.5\ p_1$ and volume $v_2 = 2v_1$. The change in internal energy from state 1 to state 2 is equal to

(A) 2000 J (B) 3000 J (C) 6000 J (D) 9000 J

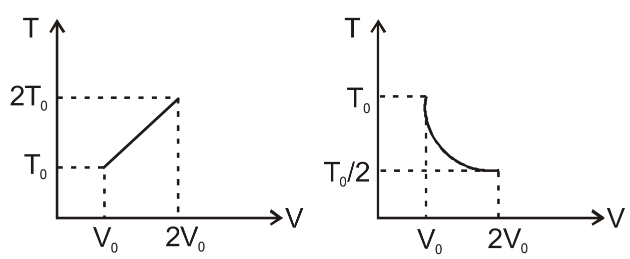
13. A mono–atomic ideal gas is compressed from volume V to V/2 through various process. For which of the following processes final pressure will be maximum :

(A) isobaric (B) isothermal (C) adiabatic (D) PV^2 = constant

14. 4 moles of H_2 at 500 K is mixed with 2 moles of He at 400K .The mixture attains a temperature T and volume V. Now the mixture is compressed adiabatically to a volume V' and temperature T'. If $\frac{T'}{T} = \left(\frac{V}{V'}\right)^n$, find the value of 13n.

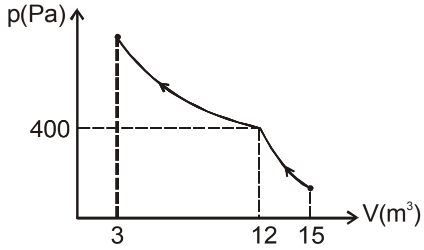
(A) 4 (B) 6 (C) 5 (D) 13

15. For two thermodynamic process temperature and volume diagram are given. In first process, it is a straight line having initial and final coordinates as (V_0, T_0) and $(2V_0, 2T_0)$, where as in second process it is a rectangular hyperbola having initial and final coordinates (V_0, T_0) and $(2V_0, T_0/2)$. Then ratio of work done in the two processes must be



(A) 1 : 2 (B) 2 : 1 (C) 1 : 1 (D) None of these

16. Curve in the figure shows an adiabatic compression of an ideal gas from 15 m^3 to 12 m^3, followed by an isothermal compression to a final volume of 3.0 m^3. There are 2.0 moles of the gas. Total heat supplied to the gas is equal to : ($\ell n2 = 0.693$)



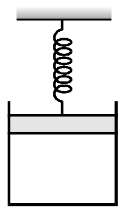
(A) 4521 J (B) –4521 J (C) –6653 J (D) –8476 J

PART - II : INTEGER TYPE QUESTIONS

1. During the expansion process the volume of the gas changes from 4m^3 to 6m^3 while the pressure changes according to p = 30V + 100 where pressure is in Pa and volume is in m^3. Find the work done (in Joule) by gas.

2. A balloon containing an ideal gas has a volume of 10 liter and temperature of 17ºC. If it is heated slowly to 75ºC, the work done (in J) by the gas inside the balloon is (neglect elasticity of the balloon and take atmospheric pressure as 10^5 Pa)

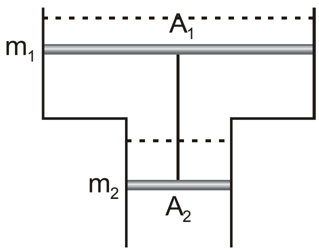
3. One mole of an ideal gas is kept enclosed under a light piston (area=10^{-2} m^2) connected by a compressed spring (spring constant 100 N/m). The volume of gas is 0.83 m^3 and its temperature is 100K. The gas is heated so that it compresses the spring further by 0.1 m. The work done (in J) by the gas in the process is: (Take R = 8.3 J/K-mole and suppose there is no atmosphere).



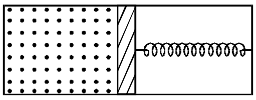
4. Consider a vertical tube open at both ends. The tube consists of two parts, each of different cross-sections and each part having a piston which can move smoothly in respective tubes. The two pistons are joined together by an inextensible wire. The combined mass of the two piston is 5 kg and area of cross-section of the upper piston is 10 cm^2 greater than that of the lower piston. Amount of gas enclosed by the pistons is one mole. When the gas is heated slowly, pistons move by 50 cm as shown in figure.

Find rise in the temperature of the gas, in the form $\frac{X}{R}$K where R is universal gas constant.

[Use g = 10 m/s^2 and outside pressure = 10^5 N/m^2]. Fill value of X.



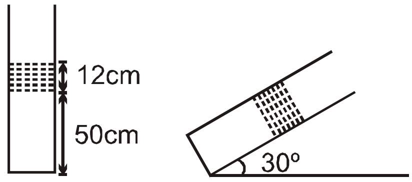
5. A thermally insulated vessel is divided into two equal parts by a heat-insulating piston which can move in the vessel without the friction. The left part of the vessel contains one mole of an ideal monatomic gas, & the right part is empty. The piston is connected to the right wall of the vessel through a spring whose length in free state is equal to the length of the vessel as shown in the figure. If the heat capacity C of the system is xR, find the value of x. (neglect the heat capacities of the vessel, piston and spring).



6. A vessel of volume V = 5 litre contains 1.4 g nitrogen and 0.4 g of He at 1500 K. If 30% of the nitrogen molecules are disassociated into atoms. The gas pressure is found to be n × 10^4 N/m^2. Find the value of n.

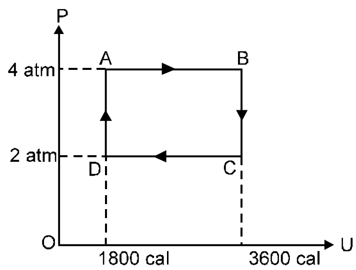
(Assume T constant). $\left[R = \frac{25}{3} J/mol\,K\right]$

7. In given figure, an ideal gas is trapped between a mercury column and the closed end of a uniform vertical tube. The upper end of the tube is open to the atmosphere. Initialy the lengths of the mercury column and the trapped air column are 12 cm and 50 cm respectively. When the tube is tilted slowly in a vertical plane through an angle of 30° with horizontal then find the new length (in cm) of air column. Assuming the temperature to remain constant. (P_{atm} =76 cm of Hg)



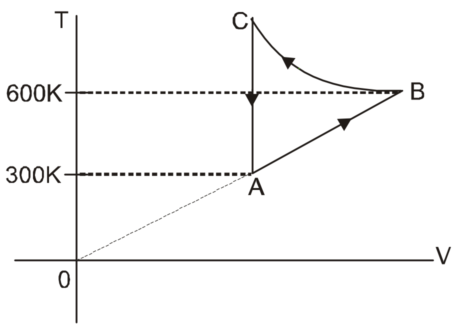
8. Two vessels A and B, thermally insulated, contain an ideal monoatomic gas. A small tube fitted with a valve connects these vessels. Initially the vessel A has 2 litres of gas at 300 K and 2 × 10^5 N m^{-2} pressure while vessel B has 4 litres of gas at 350 K and 4 × 10^5 Nm^{-2} pressure. The valve is now opened and the system reaches equilibrium in pressure and temperature. The new pressure is n × 10^5 N/m^2. Find the value of n.

9. Two moles of an ideal monoatomic gas undergo a cyclic process which is indicated on a P-U diagram, where U is the internal energy of the gas. The work done by the gas in the cycle is x ln 2 cal. Find the value of x.



10. In figure, a sample of 3 moles of an ideal gas is undergoing through a cyclic process ABCA. A total of 1500J of heat is withdrawn from the sample in the process. Find the work done (in J) by the gas during the part BC.

(R = $\frac{25}{3}$ J/mole K)



11. An adiabatic cylindrical tube is fitted with an adiabatic separator as shown in figure. Initially separator is in equilibrium and divides a tube in two equal parts. The separator can be slide into the tube by an external mechanism. An ideal gas (γ = 1.5) is injected in the two sides at equal pressures and temperatures. Now separator is slid to a position where it divides the tube in the ratio 7 : 3. The ratio of the temperatures in the two parts of the vessel is $\sqrt{\frac{x}{7}}$. Find x.

12. Two samples A and B of the same gas have equal volumes and pressures. The gas in sample A is expanded isothermally to four times of its initial volume and the gas in B is expanded adiabatically to double its volume. It work done in isothermal process is twice that of adiabatic process, then γ satisfies the equation $1 - p^{1-\gamma} = (\gamma-1)\ \text{In}q$. Find the value of pq.

PART - III : ONE OR MORE THAN ONE CORRECT OPTIONS

1. In a mixture of nitrogen and helium kept at room tempertaure. As compared to a helium molecule nitrogen molecule hits the wall

(A) With greater average speed (B) with smaller average speed

(C) with greater average kinetic energy (D) with smaller average kinetic energy.

2. An ideal gas of one mole is kept in a rigid container of negligible heat capacity. If 25 J of heat is supplied the gas temperature raises by 2^0C. Then the gas may be

(A) helium (B) argon (C) oxygen (D) carbon dioxide

3. Pick the correct statement (s) :

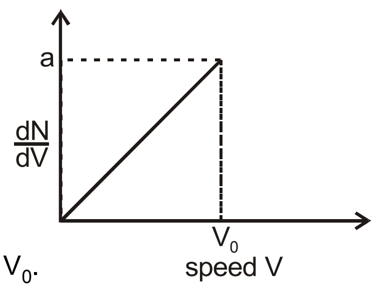
(A) The rms translational speed for all ideal-gas molecules at the same temperature is not the same but it depends on the mass.

(B) Each particle in a gas has average translational kinetic energy and the equation $\frac{1}{2}mv^2_{rms} = \frac{3}{2}kT$ establishes the relationship between the average translational kinetic energy per particle and temperature of an ideal gas. It can be concluded that single particle has a temperature.

(C) Temperature of an ideal gas is doubled from 100°C to 200°C. The average kinetic energy of each particle is also doubled.

(D) It is possible for both the pressure and volume of a monoatomic ideal gas to change simultaneously without causing the internal energy of the gas to change.

4. Graph shows a hypothetical speed distribution for a sample of N gas particle (for V > V_0; $\frac{dN}{dV}$ = 0)



(A) The value of aV_0 is 2N.

(B) The ratio V_{avg}/V_0 is equal to 2/3.

(C) The ratio V_{rms}/V_0 is equal to $1/\sqrt{2}$.

(D) Three fourth of the total particle has a speed between 0.5 V_0 and V_0.

5. During an experiment, an ideal gas is found to obey a condition $\frac{P^2}{\rho}$ = constant [ρ = density of the gas]. The gas is initially at temperature T, pressure P and density ρ. The gas expands such that density changes to $\frac{\rho}{2}$

(A) The pressure of the gas changes to $\sqrt{2}$ P.

(B) The temperature of the gas changes to $\sqrt{2}$ T.

(C) The graph of the above process on the P-T diagram is parabola.

(D) The graph of the above process on the P-T diagram is hyperbola.

6. An ideal gas can be expanded from an initial state to a certain volume through two different processes (i) PV^2 = constant and (ii) $P = KV^2$ where K is a positive constant. Then

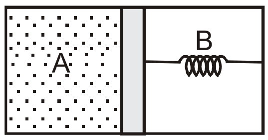
(A) Final temperature in (i) will be greater then in (ii)

(B) Final temperature in (ii) will be greater then in (i)

(C) Total heat given to the gas in (i) case is greater than in (ii)

(D) Total heat given to the gas in (ii) case is greater than in (i)

7. A thermally insulated chamber of volume $2V_0$ is divided by a frictionless piston of area S & mass m into two equal parts A and B.Part A has an ideal gas at pressure P_0 and temperature T_0 and in part B is vacuum. A massless spring of force constant K is connected with the piston and the wall of the container as shown. Initially the spring is undeformed. The gas in chamber A is allowed to expand. Let in equilibrium the spring is compressed by x_0 . Then :



(A) Pressure of the gas at equilibrium is $\frac{Kx_0}{S}$

(B) Work done by the gas is $\frac{1}{2}Kx_0^2$

(C) Increase in internal energy of the gas is $\frac{1}{2}Kx_0^2$

(D) Temperature of the gas is decreased

8. For an ideal gas :

(A) the change in internal energy in a constant pressure process from temperature T_1 to T_2 is equal to $nC_v(T_2 - T_1)$, where C_v is the molar specific heat at constant volume and n the number of moles of the gas.

(B) the change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process.

(C) the internal energy does not change in an isothermal process.

(D) no heat is added or removed in an adiabatic process.

9. A gaseous mixture consists of equal number of moles of two ideal gases having adiabatic exponents γ_1 and γ_2 and molar specific heats at constant volume C_{v_1} and C_{v_2} respectively. Which of the following statements is/are correct?

(A) Adiabatic exponent for gaseous mixture is equal to $\frac{\gamma_1 + \gamma_2}{2}$

(B) Molar specific heat at constant volume for gaseous mixture is equal to $\frac{C_{v_1} + C_{v_2}}{2}$

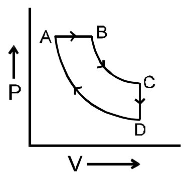
(C) Molar specific heat at constant pressure for gaseous mixture is equal to $\frac{C_{v_1} + C_{v_2} + R}{2}$

(D) Adiabatic exponent for gaseous mixture is $1 + \frac{2R}{C_{v_1} + C_{v_2}}$

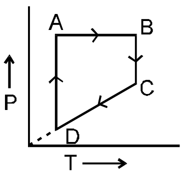
10. A system undergoes a cyclic process in which it absorbs Q_1 heat and gives out Q_2 heat. The efficiency of the process is η and work done is W. Select correct statement:

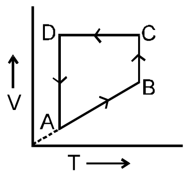
(A) $W = Q_1 - Q_2$ (B) $\eta = \frac{W}{Q_1}$ (C) $\eta = \frac{Q_2}{Q_1}$ (D) $\eta = 1 - \frac{Q_2}{Q_1}$

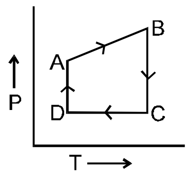
11. A cyclic process ABCD is shown in the P–V diagram. (BC and DA are isothermal)

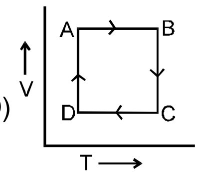


Which of the following curves represents the same process?

(A) 

(B) 

(C) 

(D) 

12. The following sets of values for C_v and C_p of an ideal gas have been reported by different students. The units are cal mole^{-1} K^{-1}. Which of these sets is most reliable ?

(A) $C_v = 3$, $C_p = 5$ (B) $C_v = 4$, $C_p = 6$ (C) $C_v = 3$, $C_p = 2$ (D) $C_v = 3$, $C_p = 4.2$

13. A gas kept in a container, if the container is of finite conductivity, then the process

(A) must be very nearly adiabatic (B) must be very nearly isothermal

(C) may be very nearly adiabatic (D) may be very nearly isothermal

14. Oxygen, nitrogen and helium gas are kept in three identical adiabatic containers P, Q and R respectively at equal pressure. When the gases are pushed to half their original volumes. (initial temperature is same)

(A) The final temperature in the three containers will be the same.

(B) The final pressures in the three containers will be the same.

(C) The pressure of oxygen and nitrogen will be the same but that of helium will be different.

(D) The temperature of oxygen and nitrogen will be the same but that of helium will be different

15. The pressure P and volume V of an ideal gas both decreases in a process.

(A) The work done by the gas is negative

(B) The work done by the gas is positive

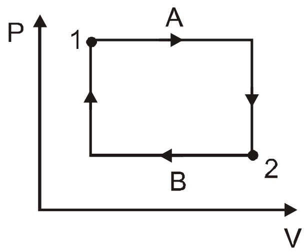
(C) The temperature of the gas must decrease

(D) Heat supplied to the gas is equal to the change in internal energy.

16. An ideal gas can be taken from initial state 1 to final state 2 by two different process. Let ΔQ and W represent the heat given and work done by the system. Then which quantities is/are same in both process (where ΔU = internal energy of gas)

(A) ΔQ (B) W (C) ΔU (D) ΔQ - W

17. In given figure, let ΔU_1 and ΔU_2 be change in internal energy in process A and B respectively. ΔQ and W be the net heat given and net work done by the system in the process A + B, then



(A) $\Delta U_1 + \Delta U_2 = 0$ (B) $\Delta U_1 - \Delta U_2 = 0$

(C) $\Delta Q - W = 0$ (D) $\Delta Q + W = 0$

18. For an ideal gas of fixed amount, the initial pressure and volume are equal to the final pressure and volume.

(A) The initial temperature must be equal to the final temperature

(B) The initial internal energy must be equal to the final internal energy.

(C) The net heat given to an ideal gas in the process must be zero

(D) The net work done by an ideal gas in the process may be zero

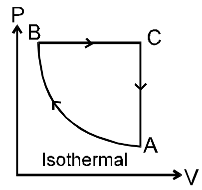
19. A cyclic process of an ideal monoatomic gas is shown in figure. The correct statement is (are) :

(A) Work done by gas in process AB is more than that of the process BC.

(B) net heat energy has been supplied to the system.

(C) temperature of the gas is maximum in state B.

(D) in process CA, heat energy is rejected out by system.



20. Let n_1 and n_2 moles of two different ideal gases be mixed. If adiabatic coefficient of the two gases are γ_1 and γ_2 respectively, then adiabatic coefficient γ of the mixture is given through the relation [Olympiad 2011-2012]

(A) $(n_1 + n_2)\gamma = n_1 + n_1\gamma_2$

(B) $\dfrac{(n_1 + n_2)}{\gamma - 1} = \dfrac{n_1}{\gamma_1 - 1} + \dfrac{n_2}{\gamma_2 - 1}$

(C) $(n_1 + n_2)\dfrac{\gamma}{\gamma - 1} = n_1\dfrac{\gamma_1}{\gamma_1 - 1} + n_2\dfrac{\gamma_2}{\gamma_2 - 1}$

(D) $(n_1 + n_2)(\gamma - 1) = n_1(\gamma_1 - 1) + n_2(\gamma_2 - 1)$

21. Which of the following statement/s in case of a thermodynamic process is /are correct?

(A) ΔE_{int} = W indicates an adiabatic process **[Olympiad 2015 (stage-1)]**

(B) ΔE_{int} = Q suggests an isochoric process

(C) ΔE_{int} = 0 is true for a cyclic process

(D) ΔE_{int} = –W indicates an adiabatic

22. If a system is made to undergo a change from an initial state to a final state by adiabatic process only, then

(A) the work done is different for different paths connecting the two states **[Olympiad (Stage-1) 2017]**

(B) there is no work done since there is no transfer of heat

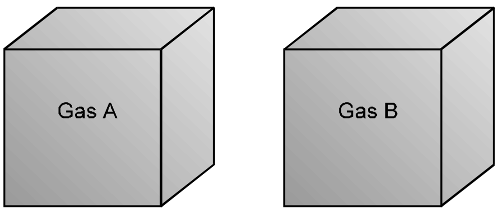
(C) the internal energy of the system will change

(D) the work done is the same for all adiabatic paths.

PART - IV : COMPREHENSION

Comprehension # 1

Two closed identical conducting containers are found in the laboratory of an old scientist. For the verification of the gas some experiments are performed on the two boxes and the results are noted.



Experiment 1.

When the two containers are weighed W_A = 225 g , W_B = 160 g and mass of evacuated container W_C = 100 g.

Experiment 2.

When the two containers are given same amount of heat same temperature rise is recorded. The pressure changes found are

ΔP_A = 2.5 atm. ΔP_B = 1.5 atm.

Required data for unknown gas :

Mono (molar mass)	He 4g	Ne 20g	Ar 40 g	Kr 84 g	Xe 131 g	Rd 222 g
Dia (molar mass)	H_2 2g	F_2 19 g	N_2 28g	O_2 32g	Cl_2 71 g	

1. Identify the type of gas filled in container A and B respectively.

(A) Mono, Mono (B) Dia, Dia (C) Mono, Dia (D) Dia, Mono.

2. Identify the gas filled in the container A and B.

(A) N_2, Ne (B) He, H_2 (C) O_2 , Ar (D) Ar, O_2

3. Total number of molecules in 'A' (here N_A = Avagadro number)

(A) $\frac{125}{64}N_A$ (B) 3.125 N_A (C) $\frac{125}{28}N_A$ (D) 31.25 N_A

4. The initial internal energy of the gas in container 'A', If the container were at room temperature 300K initially

(A) 1406.25 cal (B) 1000 cal (C) 2812.5 cal (D) none of these

Comprehension # 2

A mono atomic ideal gas is filled in a non conducting container. The gas can be compressed by a movable non conducting piston. The gas is compressed slowly to 12.5% of its initial volume.

5. The percentage increase in the temperature of the gas is

(A) 400% (B) 300% (C) – 87.5% (D) 0%

6. The ratio of initial adiabatic bulk modulus of the gas to the final value of adiabatic bulk modulus of the gas is

(A) 32 (B) 1 (C) 1/32 (D) 4

7. The ratio of work done by the gas to the change in internal energy of the gas is

(A) 1 (B) –1 (C) ∞ (D) 0

Comprehension # 3

An ideal gas initially at pressure p_0 undergoes a free expansion (expansion against vacuum under adiabatic conditions) until its volume is 3 times its initial volume. The gas is next adiabatically compressed back to its original volume. The pressure after compression is $3^{2/3}$ p_0.

8. The pressure of the gas after the free expansion is :

(A) $\frac{p_0}{3}$ (B) $p_0^{1/3}$ (C) p_0 (D) $3p_0$

9. The gas

(A) is monoatomic. (B) is diatomic.

(C) is polyatomic. (D) type is not possible to decide from the given information.

10. What is the ratio of the average kinetic energy per molecule in the final state to that in the initial state ?

(A) 1 (B) $3^{2/3}$ (C) $3^{1/3}$ (D) $3^{1/6}$

Exercise # 3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

1. A real gas behaves like an ideal gas if its **[JEE- 2010; 3/163, –1]**

(A) pressure and temperature are both high

(B) pressure and temperature are both low

(C) pressure is high and temperature is low

(D) pressure is low and temperature is high

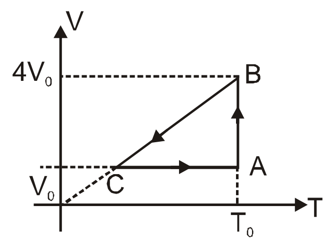
2. One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following : **[JEE- 2010; 3/163]**

(A) Internal energies at A and B are the same

(B) Work done by the gas in process AB is $P_0V_0\ \ell n\ 4$

(C) Pressure at C is $\frac{P_0}{4}$

(D) Temperature at C is $\frac{T_0}{4}$

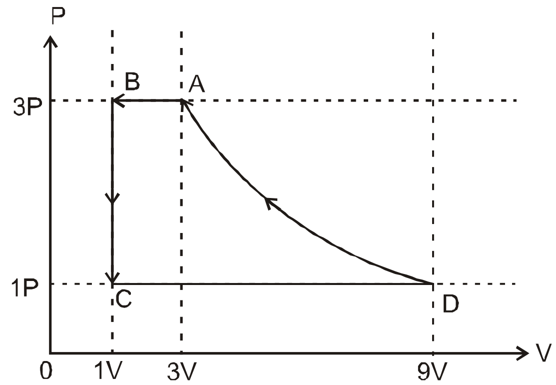


3. A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is : **[JEE, 2010, 3/163]**

4. 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is : **[JEE, 2011, 3/160, –1]**

(A) $\frac{9}{8}RT_1$ (B) $\frac{3}{2}RT_1$ (C) $\frac{15}{8}RT_1$ (D) $\frac{9}{2}RT_1$

5. One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in the P-V diagram. Column II gives the characteristics involved in the cycle. Match them with each of the processes given in Column I. **[JEE, 2011, 8/160]**



Column I	Column II
(A) Process A $\rightarrow$ B	(p) Internal energy decreases
(B) Process B $\rightarrow$ C	(q) Internal energy increases
(C) Process C $\rightarrow$ D	(r) Heat is lost
(D) Process D $\rightarrow$ A	(s) Heat is gained
	(t) Work is done on the gas.

6. A mixture of 2 moles of helium gas (atomic mass = 4 amu), and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{rms}(\text{helium})}{v_{rms}(\text{argon})}\right)$ is :

[IIT-JEE-2012, Paper-1 : 3/70, –1]

(A) 0.32 (B) 0.45 (C) 2.24 (D) 3.16

7. Two moles of ideal helium gas are in a rubber balloon at 30° C .The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C.The amount of heat required in raising the temperature is nearly (take R = 8.31 J/mol.K)

[IIT-JEE-2012, Paper-2 : 3/66, –1]

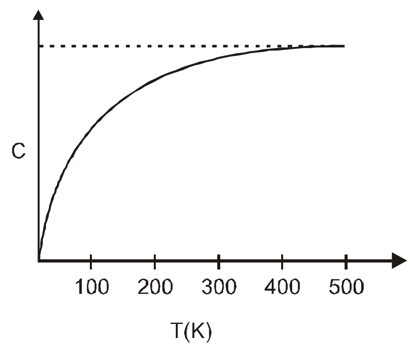
(A) 62J (B) 104 J (C) 124 J (D) 208 J

8. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is:

[JEE-2013]

(A) 1 : 4 (B) 1 : 2 (C) 6 : 9 (D) 8 : 9

9.* The figure below shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation. **[JEE ADVANCED 2013]**



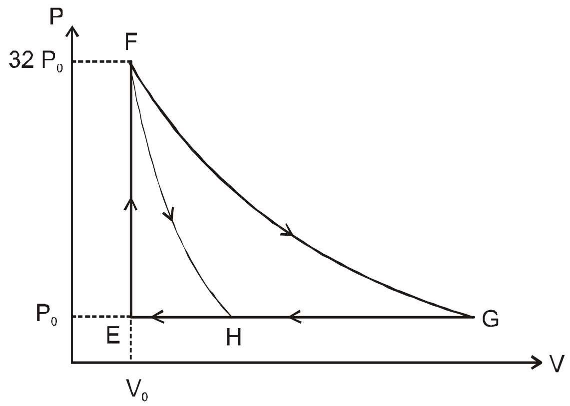
(A) the rate at which heat is absorbed in the range 0–100 K varies linearly with temperature T.

(B) heat absorbed in increasing the temperature from 0–100 K is less than the heat required for increasing the temperature from 400–500 K.

(C) there is no change in the rate of heat absorbtion in the range 400–500 K.

(D) the rate of heat absorption increases in the range 200–300 K.

10. One mole of a monatomic ideal gas is taken along two cyclic processes $E \rightarrow F \rightarrow G \rightarrow E$ and $E \rightarrow F \rightarrow H \rightarrow E$ as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic.



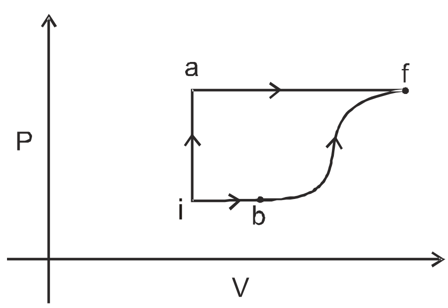
Match the paths in List I with the magnitudes of the work done in List II and select the correct answer using the codes given below the lists. **[JEE ADVANCED 2013]**

	List I		**List II**
P.	$G \rightarrow E$	1.	$160\ P_0V_0\ \ln 2$
Q.	$G \rightarrow H$	2.	$36\ P_0V_0$
R.	$F \rightarrow H$	3.	$24\ P_0V_0$
S.	$F \rightarrow G$	4.	$31\ P_0V_0$

Codes :

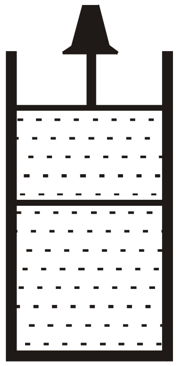
	P	Q	R	S
(A)	4	3	2	1
(B)	4	3	1	2
(C)	3	1	2	4
(D)	1	3	2	4

11. A thermodynamic system is taken fr0m an initial state i with internal energy $U_i = 100$ J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are $W_{af} = 200$ J, $W_{ib} = 50$ J and $W_{bf} = 100$ J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{bf} and Q_{ib} respectively. If the internal energy of the sytem in the state b is $U_b = 200$ J and $Q_{iaf} = 500$ J, the ratio Q_{bf}/Q_{ib} is: **[JEE (Advanced)-2014,P-1, 3/60]**



Paragraph For Questions 12 to 13

In the figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_V = \frac{3}{2}R$, $C_P = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \frac{5}{2}R$, $C_P = \frac{7}{2}R$. **[JEE (Advanced)-2014, 3/60, –1]**



12. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be : **[JEE (Advanced)-2014, 3/60, –1]**

(A) 550 K (B) 525 K (C) 513 K (D) 490 K

13. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be :

[JEE (Advanced)-2014, 3/60, –1]

(A) 250 R (B) 200 R (C) 100 R (D) –100 R

14.* A container of fixed volume has a mixutre of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is (are) : **[JEE-Advance-2015]**

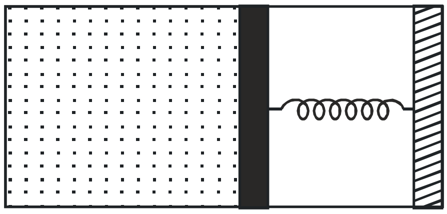
(A) The average energy per mole of the gas mixture is 2RT.

(B) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$.

(C) The ratio of the rms speed of helium atoms to that of hydrogen molecules is 1/2.

(D) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$.

15.* An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are) : **[JEE-Advance-2015]**



(A) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\frac{1}{4}P_1V_1$

(B) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

(C) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3}P_1V_1$

(D) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6}P_1V_1$

16. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^5$ Pa and volume $V_i = 10^{-3}$ m^3 changes to a final state at $P_f = \left(\frac{1}{32}\right) \times 10^5 \text{Pa}$ and $V_f = 8 \times 10^{-3}$ m^3 in an adiabatic quasi-static process, such that P^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volumes V_f. The amount of heat supplied to the system in the two step process is approximately **[JEE-Advance 2016]**

(A) 112 J (B) 294 J (C) 588 J (D) 813 J

17.* A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true ? **[JEE-Advance 2017]**

(A) The resistive force experienced by the plate is proportional to v

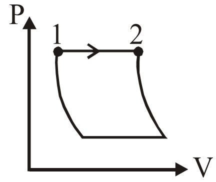
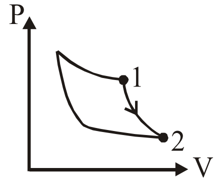
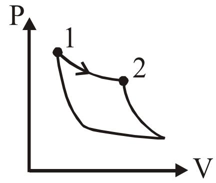
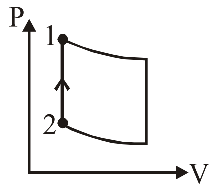
(B) The pressure difference between the leading and trailing faces of the plate is proportional to uv.

(C) The plate will continue to move with constant non-zero acceleration, at all times

(D) At a later time the external force F balances the resistive force.

Answer Q.18, Q.19 and Q.20 by appropriately matching the information given in the three columns of the following table.

An ideal gas is undergoing a cyclic thermodynamics process in different ways as shown in the corresponding P–V diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamics processes. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n. **[JEE-Advance 2017]**

Column-1	Column-2		Column-3
(I) $W_{1\to2} = \dfrac{1}{\gamma-1}(P_2V_2 - P_1V_1)$	(i) Isothermal	(P)	
(II) $W_{1\to2} = -PV_2 + PV_1$	(ii) Isochoric	(Q)	
(III) $W_{1\to2} = 0$	(iii) Isobaric	(R)	
(IV) $W_{1\to2} = -nRT \ln \dfrac{V_2}{V_1}$	(iv) Adiabatic	(S)	

18. Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$?

(A) (II) (iv) (R) (B) (II) (iii) (P) (C) (II) (iii) (S) (D) (III) (iii) (P)

19. Which one of the following options is the correct combination ?

(A) (III) (ii) (S) (B) (II) (iv) (R) (C) (II) (iv) (P) (D) (IV) (ii) (S)

20. Which one of the following options correctly represents a thermodynamics process that is used as a correction in the determination of the speed of sound in an ideal gas ?

(A) (III) (iv) (R) (B) (I) (ii) (Q) (C) (IV) (ii) (R) (D) (I) (iv) (Q)

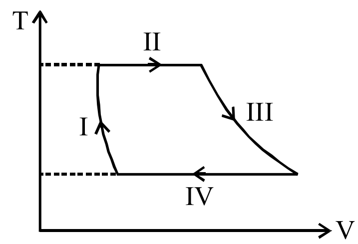
21.* One mole of a monatomic ideal gas undergoes a cyclic process as shown in the figure (where V is the volume and T is the temperature). Which of the statements below is (are) true? **[JEE-Advance 2018]**

(A) Process I is an isochoric process

(B) In process II, gas absorbs heat

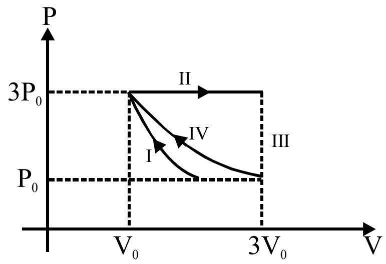
(C) In process IV, gas releases heat

(D) Processes I and II are not isobaric



22. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant R = 8.0 J mol^{-1} K^{-1}, the decrease in its internal energy, in Joule, is............ . **[JEE-Advance 2018]**

23. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List–II. **[JEE-Advance 2018]**



	List–I		List–II
P.	In process I	1.	Work done by the gas is zero
Q.	In process II	2.	Temperature of the gas remains unchanged
R.	In process III	3.	No heat is exchanged between the gas and its surroundings
S.	In process IV	4.	Work done by the gas is 6 P_0V_0

(A) $P \rightarrow 4$; $Q \rightarrow 3$; $R \rightarrow 1$; $S \rightarrow 2$

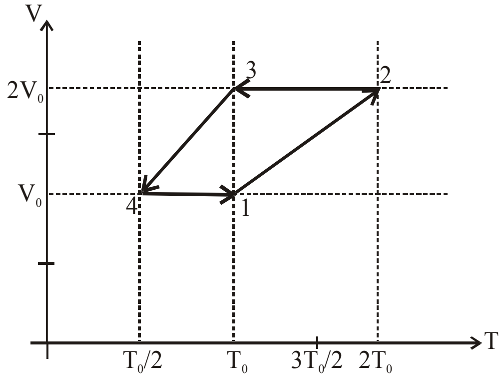
(B) $P \rightarrow 1$; $Q \rightarrow 3$; $R \rightarrow 2$; $S \rightarrow 4$

(C) $P \rightarrow 3$; $Q \rightarrow 4$; $R \rightarrow 1$; $S \rightarrow 2$

(D) $P \rightarrow 3$; $Q \rightarrow 4$; $R \rightarrow 2$; $S \rightarrow 1$

24.* One mole of a monoatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V-T) diagram. The correct statement(s) is/are :**[JEE-Advanced-2019, Paper-1; 4/62, –1]**

[R is the gas constant]



(A) Work done in this thermodynamic cycle (1→2→3→4→1) is $|W| = \frac{1}{2}RT_0$

(B) The ratio of heat transfer during processes 1→2 and 2→3 is $\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \frac{5}{3}$

(C) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(D) The ratio of heat transfer during processes 1→2 and 3→4 is $\left|\frac{Q_{1\to2}}{Q_{3\to4}}\right| = \frac{1}{2}$

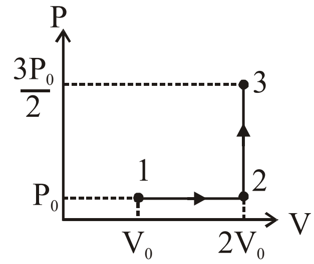
25. Answer the following by appropriately matching the lists based on the information given in the paragraph.

In a thermodynamics process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monoatomic ideal gas $X = \frac{3}{2}R \ln\left(\frac{T}{T_A}\right) + R \ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities. **[JEE-Advanced-2019, Paper-2; 3/62, –1]**

List-I		**List-II**
(I) Work done by the system in process $1 \to 2 \to 3$	(P)	$\frac{1}{3}RT_0 \ln 2$
(II) Change in internal energy in process $1 \to 2 \to 3$	(Q)	$\frac{1}{3}RT_0$
(III) Heat absorbed by the system in process $1 \to 2 \to 3$	(R)	RT_0
(IV) Heat absorbed by the system in process $1 \to 2$	(S)	$\frac{4}{3}RT_0$
	(T)	$\frac{1}{3}RT_0(3+\ln 2)$
	(U)	$\frac{5}{6}RT_0$

If the process carried out on one mole of monatomic ideal gas is as shown in figure in the PV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is,



(A) I $\rightarrow$ Q, II $\rightarrow$ R, III $\rightarrow$ P, IV $\rightarrow$ U

(B) I $\rightarrow$ S, II $\rightarrow$ R, III $\rightarrow$ Q, IV $\rightarrow$ T

(C) I $\rightarrow$ Q, II $\rightarrow$ R, III $\rightarrow$ S, IV $\rightarrow$ U

(D) I $\rightarrow$ Q, II $\rightarrow$ S, III $\rightarrow$ R, IV $\rightarrow$ U

26. Consider one mole of helium gas enclosed in a container at initial pressure P_1 and volume V_1. It expands isothermally to volume $4V_1$. After this, the gas expands adiabatically and its volume becomes $32V_1$. The work done by the gas during isothermal and adiabatic expansion processes are W_{iso} and W_{adia}, respectively. If the ratio $\frac{W_{iso}}{W_{adia}} = f\ln 2$, then f is _________. **[JEE-Advanced-2020, Paper-1; 3/62, –1]**

27. A spherical bubble inside water has radius R. Take the pressure inside the bubble and the water pressure to be p_0. The bubble now gets compressed radially in an adiabatic manner so that its radius becomes (R – a). For a << R the magnitude of the work done in the process is given by $(4\pi p_0 Ra^2)X$, where X is a constant and $\gamma = C_pV_v$ = 4/130. The value of X is________.

[JEE-Advanced-2020, Paper-2; 3/62, –1]

PART - II : JEE(MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32 V, the efficiency of the engine is :

[AIEEE - 2010; 4/144, –1]

(1) 0.5 (2) 0.75 (3) 0.99 (4) 0.25

2. 100g of water is heated from 30ºC to 50ºC ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/Kg/K) : **[AIEEE - 2011; 4/120, –1]**

(1) 4.2 kJ (2) 8.4 kJ (3) 84 kJ (4) 2.1 kJ

3. A Carnot engine operating between temperatures T_1 and T_2 has effeiciency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively : **[AIEEE - 2011, 4/120, –1]**

(1) 372 K and 310 K (2) 372 K and 330 K (3) 330 K and 268 K (4) 310 K and 248 K

4. Three perfect gases at absolute temperature T_1,T_2 and T_3 are mixed. The masses of molecules are m_1,m_2 and m_3 and the number of molecules are n_1,n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is : **[AIEEE - 2011, 4/120, –1]**

(1) $\frac{(T_1+T_2+T_3)}{3}$ (2) $\frac{n_1T_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}$ (3) $\frac{n_1T_1^2+n_2T_2^2+n_3T_3^2}{n_1T_1+n_2T_2+n_3T_3}$ (4) $\frac{n_1^2T_1^2+n_2^2T_2^2+n_3^2T_3^2}{n_1T_1+n_2T_2+n_3T_3}$

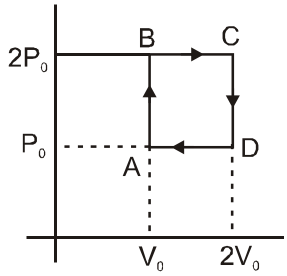
5. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by : **[AIEEE - 2011, 4/120, –1]**

(1) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2K$ (2) $\frac{(\gamma-1)}{2\gamma R}Mv^2K$ (3) $\frac{\gamma Mv^2}{2R}K$ (4) $\frac{(\gamma-1)}{2R}Mv^2K$

6. A container with insulating walls is divided into equal parts by a partition fitted with a valve. One part is filled with an ideal gas at a pressure P and temperature T, whereas the other part is completely evacuated. If the valve is suddenly opened, the pressure and temperature of the gas will be: **[AIEEE 2011, 11 May; 4/120, –1]**

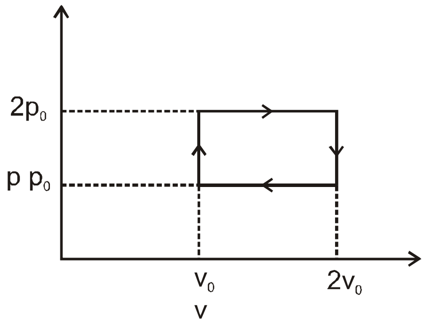
(1) $\frac{P}{2},\frac{T}{2}$ (2) P, T (3) P, $\frac{T}{2}$ (4) $\frac{P}{2}$, T

7. Helium gas goes through a cycle ABCDA (consisting of two isochoric and isobaric lines) as shown in figure. Efficiency of this cycle is nearly : (Assume the gas to be close to ideal gas) **[AIEEE 2012 ; 4/120, –1]**



(1) 15.4% (2) 9.1% (3) 10.5% (4) 12.5%

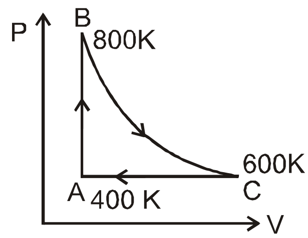
8.



The above p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is : **[JEE-Mains 2013, 4/120, –1]**

(1) p_0v_0 (2) $\left(\frac{13}{2}\right)p_0v_0$ (3) $\left(\frac{11}{2}\right)p_0v_0$ (4) $4p_0v_0$

9. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400K, 800K and 600 K respectively. Choose the correct statement : **[JEE-Mains 2014; 4/120, –1]**



(1) The change in internal energy in whole cyclic process is 250 R.

(2) The change in internal energy in the process CA is 700 R

(3) The change in internal energy in the process AB is – 350 R

(4) The change in internal energy in the process BC is – 500 R

10. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now ?

(Atmospheric pressure = 76 cm of Hg) **[JEE-Main 2014; 4/120, –1]**

(1) 16 cm (2) 22 cm (3) 38 cm (4) 6 cm

11. A solid body of constant heat capacity 1 J°/C is being heated by keeping it in contact with reservoirs in two ways - **[JEE-Main-2015; 4/120, –1]**

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is -

12. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume u = $\frac{U}{V} \propto T^4$ and pressure p = $\frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is -

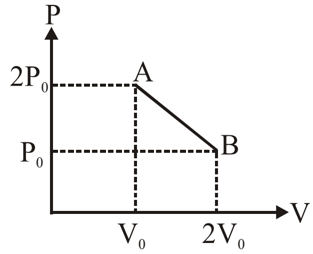
[JEE-Main-2015; 4/120, –1]

(1) $T \propto \frac{1}{R}$ (2) $T \propto \frac{1}{R^3}$ (3) $T \propto e^{-R}$ (4) $T \propto e^{-3R}$

13. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is : $\left(\gamma = \frac{C_p}{C_v}\right)$ **[JEE-Main-2015; 4/120, –1]**

(1) $\frac{\gamma+1}{2}$ (2) $\frac{\gamma-1}{2}$ (3) $\frac{3\gamma+5}{6}$ (4) $\frac{3\gamma-5}{6}$

14. 'n' moles of an ideal gas undergoes a process A $\rightarrow$ B as shown in the figure. The maximum temperature of the gas during the process will be : **[JEE-Main-2016; 4/120, –1]**



(1) $\frac{9P_0V_0}{nR}$ (2) $\frac{9P_0V_0}{4nR}$ (3) $\frac{3P_0V_0}{2nR}$ (4) $\frac{9P_0V_0}{2nR}$

15. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively) :

[JEE-Main-2016; 4/120, –1]

(1) $n=\frac{C-C_V}{C-C_P}$ (2) $n=\frac{C_P}{C_V}$ (3) $n=\frac{C-C_P}{C-C_V}$ (4) $n=\frac{C_P-C}{C-C_V}$

16. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be : **[JEE-Main 2017; 4/120, –1]**

(1) 2.5×10^{25} (2) -2.5×10^{25} (3) -1.61×10^{23} (4) 1.38×10^{23}

17. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that

$C_p - C_v$ = a for hydrogen gas

$C_p - C_v$ = b for nitrogen gas

The correct relation between a and b is : **[JEE-Main 2017; 4/120, –1]**

(1) a = 14 b (2) a = 28 b (3) $a = \frac{1}{14}b$ (4) a = b

18. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

[JEE-Main 2018; 4/120, –1]

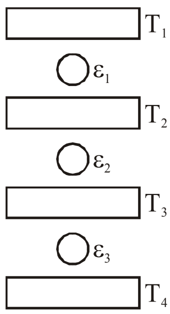
(1) (a) 195 K (b) –2.7 kJ (2) (a) 189 K (b) –2.7 kJ

(3) (a) 195 K (b) 2.7 kJ (4) (a) 189 K (b) 2.7 kJ

19. Two Carrnot engines A and B are operated in series. The first one, A, receives heat at T_1(= 600 K) and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and, in turn, rejects to a heat reservoir at T_3(= 400 K). Calculate the temperature T_2 if the work outputs of the two engines are equal : **[JEE (Main) 2019, Jan.; 4/120, –1]**

(1) 400 K (2) 600 K (3) 500 K (4) 300 K

20. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2, and T_3, as shown, with $T_1 > T_2 > T_3 > T_4$. The three engines are equally efficient if: **JEE (Main) 2019, Jan.; 4/120, –1]**



(1) $T_2=\left(T_1^2T_4\right)^{1/3};T_3=\left(T_1T_4^2\right)^{1/3}$ (2) $T_2=\left(T_1T_4^2\right)^{1/3};T_3=\left(T_1^2T_4\right)^{1/3}$

(3) $T_2=\left(T_1^3T_4\right)^{1/4};T_3=\left(T_1T_4^3\right)^{1/4}$ (4) $T_2=(T_1T_4)^{1/2};T_3=(T_1^2T_4)^{1/3}$

21. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to : **[JEE(Main) 2019, April; 4/120, –1]**

[Boltzmann Constant $k_B = 1.38 \times 10^{-23}$ J/K

Avogadro Number $N_A = 6.02 \times 10^{26}$ /kg

Radius of Earth : 6.4×10^6 m

Gravitational acceleration on Earth = $10 ms^{-2}$]

(1) 650 K (2) 3×10^5 K (3) 10^4 K (4) 800 K

22. Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures, T_1 and T_2. The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2. T is temperature of the sink of first engine which is also the source for the second engine. How is T related to T_1 and T_2, if both the engines perform equal amount of work?

(1) $T = \frac{2T_1T_2}{T_1+T_2}$ (2) $T = \sqrt{T_1T_2}$ (3) $T = \frac{T_1+T_2}{2}$ (4) T = 0

23. Starting at temperature 300 K, one mole of an ideal diatomic gas (γ = 1.4) is first compressed adiabatically from volume V_1 to $V_2 = \frac{V_1}{16}$. It is then allowed to expand isobarically to volume $2V_2$. If all the processes are the quasi-static then the final temperature of the gas (in °K) is (to the nearest integer) ________.

[JEE(Main) 2020, Jan.; 4/100, –1]

24. An engine takes in 5 moles of air at 20°C and 1 atm, and compresses it adiabaticaly to l/l0th of the original volume. Assuming air to be a diatomic ideal gas made up of rigid molecules, the change in its internal energy during this process comes out to be X kJ. The value of X to the nearest integer is ________.

[JEE(Main) 2020, Sep.; 4/100, –1]

25. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T. The total internal energy, U of a mole of this gas, and the value of $\gamma\left(=\frac{C_P}{C_v}\right)$ given, respectively, by: **[JEE(Main) 2020, Sep.; 4/100, –1]**

(1) $U = \frac{5}{2}RT$ and $\gamma = \frac{6}{5}$ (2) U = 5RT and $\gamma = \frac{7}{5}$

(3) U = 5RT and $\gamma = \frac{6}{5}$ (4) $U = \frac{5}{2}RT$ and $\gamma = \frac{7}{5}$

Answers

Exercise # 1

PART - I

SECTION (A) :

A-1. $\Delta T = \frac{m v_0^2}{3R}$ **A-2.** $\left(\frac{m v_0^2}{\ell}\right)$N

SECTION (B) :

B-1. 3600 R **B-2.** H_2, 1200 K

B-3. $\sqrt{\frac{50}{3}}$ V

B-4. $\frac{6P_0}{5}$

B-5. $\frac{240}{7}$ cm

B–6. **(i)** $\frac{1492}{293}$ cm ; **(ii)** $\frac{320\pi}{293}\left(1+\frac{13}{4\pi}\right)$ kg

SECTION (C) :

C-1. 1 : 2 **C-2.** 1 : $\sqrt{2}$

C-3. $\frac{1452\pi}{25}\times 10^3$ K **C-4.** $\frac{83}{3\sqrt{10}}\times 10^{-23}$ kg-m/s

SECTION (D) :

D-1. 196ºC **D-2.** 32ºC

D-3. $U = \frac{fnRT}{2} = \frac{f}{2}PV = \frac{f}{2}P_{atm} \cdot V_{room}$ = constant.

SECTION (E) :

E-1. 450 J **E-2.** -100πJ

E-3. 1500 J **E-4.** 750 J

SECTION (F) :

F-1. 16.9 J **F-2.** 60 cal

F-3. 12 K **F-4.** 0.0091 J

F-5. **(i)** 765 J **(ii)** $\frac{208}{1921}$ **F-6.** 110 J

F-7. (33600 + 0.02) J **F-8.** $\frac{25}{6}$ J/cal

F-9. (a) 120 K, 240 K, 480 K, 240 K,

(b) 3500 J, 5000 J, 7000 J, 2500 J

(c) –1000 J

SECTION (G) :

G-1. 40 **G-2.** $\frac{2}{\gamma-1}$

G-3. 2.5 R **G-4.** 3R

G-5. (a) 40 J (b) $\frac{9}{500}$ moles (c) $\frac{125}{9}$ J/mol–K

(d) $\frac{50}{9}$ J/mol–K

G-6. 0.8 JK^{-1} **G-7.** 7

SECTION (H) :

H-1. (a) 0.5 atm; (b) zero; (c) zero (d) no

H-2. Q – W **H-3.** $\frac{3}{2}$

H-4. 128 **H-5.** 32P

H-6. 4/9 %

H-7. (a) $\sqrt{2} \times 10^5$ Pa (b) 200 $\sqrt{2}$ K

(c) $40(2-\sqrt{2})$ J

H-8. (a) 800 kPa, 100 K (b) 1600 kPa, 200 K

SECTION (I) :

I-1. 7.5 R **I-3.** $\frac{7}{2}$R

SECTION (J) :

J-1. 66.6%

J-2. 2.8×10^6Joule

J-3. 373.3 K

J-4. 879 kcal

J-5. $= \frac{800\times10^3}{(879-800)\times10^3} = 10.13$

J-6. 900 Calories

PART - II

SECTION (A) :

A-1.	(C)	**A-2.**	(C)	**A-3.**	(D)

SECTION (B) :

B-1.	(A)	**B-2.**	(A)	**B-3.**	(C)
B-4.	(A)	**B-5.**	(C)	**B-6.**	(B)

SECTION (C) :

C-1.	(A)	**C-2.**	(B)

SECTION (D) :

D-1.	(A)	**D-2.**	(C)	**D-3.**	(D)

SECTION (E) :

E-1.	(C)	**E-2.**	(D)	**E-3.**	(C)
E-4.	(C)	**E-5.**	(B)	**E-6.**	(B)
E-7.	(A)	**E-8.**	(C)		

SECTION (F) :

F-1.	(D)	**F-2.**	(D)	**F-3.**	(B)
F-4.	(B)	**F-5.**	(B)	**F-6.**	(C)
F-7.	(D)				

SECTION (G) :

G-1.	(D)	**G-2.**	(D)	**G-3.**	(B)
G-4.	(C)	**G-5.**	(C)	**G-6.**	(C)

SECTION (H) :

H-1.	(A)	**H-2.**	(B)	**H-3.**	(A)
H-4.	(A)	**H-5.**	(C)	**H-6.**	(B)
H-7.	(B)	**H-8.**	(B)	**H-9.**	(D)
H-10.	(A)				

SECTION (I) :

I-1.	(C)	**I-2.**	(D)	**I-3.**	(C)
I-4.	(A)	**I-5.**	(D)	**I-6.**	(B)
I-7.	(C)				

SECTION (J) :

J-1.	(3)	**J-2.**	(2)	**J-3.**	(4)
J-4.	(1)	**J-5.**	(2)		

PART - III

1. (A) p,r,s (B) q (C) p,r,s (D) q,r

2. (A) p, s (B) s (C) p, s (D) q, r

Exercise # 2

PART - I

1.	(D)	**2.**	(A)	**3.**	(D)
4.	(A)	**5.**	(D)	**6.**	(A)
7.	(A)	**8.**	(D)	**9.**	(D)
10.	(C)	**11.**	(C)	**12.**	(D)
13.	(D)	**14.**	(B)	**15.**	(B)
16.	(C)				

PART - II

1.	500	**2.**	200	**3.**	1.5
4.	75	**5.**	C = 2R	**6.**	41.25
7.	53.66	**8.**	3.33	**9.**	1200
10.	–9000	**11.**	3	**12.**	4

PART - III

1.	(B,C)	**2.**	(A,B)	**3.**	(A,D)
4.	(A,B,C,D)	**5.**	(B,D)	**6.**	(B,D)
7.	(A,D)	**8.**	(A,B,C,D)	**9.**	(B,D)
10.	(A,B,D)	**11.**	(A,B)	**12.**	(A,B)
13.	(C,D)	**14.**	(C,D)	**15.**	(A,C)
16.	(C,D)	**17.**	(A,C)	**18.**	(A,B,D)
19.	(B,D)	**20.**	(B,C)	**21.**	(B,C,D)
22.	(C,D)				

PART - IV

1.	(C)	**2.**	(D)	**3.**	(B)
4.	(C)	**5.**	(B)	**6.**	(C)
7.	(B)	**8.**	(A)	**9.**	(A)
10.	(B)				

Exercise # 3

PART - I

1.	(D)	**2.**	(A,B,C,D)	**3.**	a = 4
4.	(A)				

5. (A) – p,r,t , (B) – p,r (C) – q,s, (D) – r, t

6.	(D)	**7.**	(D)	**8.**	(D)
9.	(A,B,C,D)	**10.**	(A)	**11.**	2
12.	(D)	**13.**	(D)	**14.**	(A,B,D)
15.	(A,B,C)	**16.**	(C)	**17.**	(A,B,D)
18.	(B)	**19.**	(A)	**20.**	(D)
21.	(B,C,D)	**22.**	900 [899.95, 900.05]		
23.	(C)	**24.**	(A,B)	**25.**	(C)
26.	1.77	**27.**	4.10 or 2.05		

PART - II

1.	(2)	**2.**	(2)	**3.**	(1)
4.	(2)	**5.**	(4)	**6.**	(4)
7.	(1)	**8.**	(2)	**9.**	(4)
10.	(1)	**11.**	(Bonus)	**12.**	(1)
13.	(1)	**14.**	(2)	**15.**	(3)
16.	(2)	**17.**	(1)	**18.**	(2)
19.	(3)	**20.**	(1)	**21.**	(3)
22.	(3)	**23.**	14	**24.**	46
25.	(4)				

RANKER PROBLEMS

SUBJECTIVE QUESTIONS

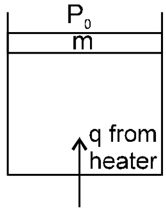
1. A vessel of volume V is evacuated by means of a piston air pump. In one stroke the piston is pulled out to make the volume of gas $V + \Delta V$ then ΔV volume from this is taken out leaving volume V in the cylinder. How many strokes are needed to reduce the pressure in the vessel to $1/\eta$ times the initial pressure? The process is assumed to be isothermal, and the gas is an ideal.

2. Find the pressure of air in a vessel being evacuated as a function of evacuation time t. The vessel volume is V, the initial pressure is p_0. The process is assumed to be isothermal, and the evacuation rate equal to C and independent of pressure.

Note: The evacuation rate is the gas volume being evacuated per unit time, with that volume being measured under the gas pressure attained by that moment.

3. Find the maximum attainable temperature of an ideal gas in the following process : where (a) $p = p_0 - \alpha V^2$; (b) $p = p_0 e^{-\beta V}$, where p_0, α and β are positive constants and V is the volume of one mole of gas.

4. Two moles of an ideal monoatomic gas are contained in a vertical cylinder of cross sectional area A as shown in the figure. The piston is frictionless and has a mass m. At a certain instant a heater starts supplying heat to the gas at a constant rate q J/s. Find the steady velocity of the piston under isobaric condition. All the boundaries are thermally insulated.



5. A piston can freely move inside a horizontal cylinder closed from both ends. Initially, the piston separates the inside space of the cylinder into two equal parts each of volume V_0, in which an ideal gas is contained under the same pressure p_0 and at the same temperature. What work has to be performed in order to increase isothermally the volume of one part of gas η times compared to that of the other by slowly moving the piston?

6. At 27° C two moles of an ideal monoatomic gas occupy a volume V. The gas expands adiabatically to a volume 2V. Calculate: **[JEE '96, 5/100]**

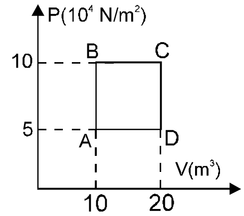
(i) the final temperature of the gas,

(ii) change in its internal energy and

(iii) the work done by the gas during the process.

7. A vertical hollow cylinder contains an ideal gas. The gas is enclosed by a 5kg movable piston with an area of cross-section 5×10^{-3} m^2. Now, the gas is heated slowly from 300 K to 350 K and the piston rises by 0.1 m. The piston is now clamped at this position and the gas is cooled back to 300 K. Find the difference between the heat energy added during heating process and energy lost during the cooling process. [1 atm pressure = 10^5 N m^{-2}] **[REE 1996, 5]**

8. A sample of 2 kg of monoatomic Helium (assumed ideal) is taken through the process ABC and another sample of 2 kg of the same gas is taken through the process ADC as in figure. Given, molecular mass of Helium = 4. **[JEE '97, 5/100]**



(i) What is the temperature of Helium in each of the states A, B, C and D ?

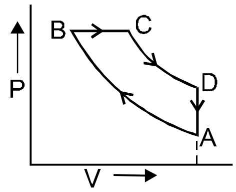
(ii) Is there any way of telling afterwards which sample of Helium went through the process ABC and which went through the process ADC? Write Yes or No.

(iii) How much is the heat involved in each of the processes ABC and ADC.

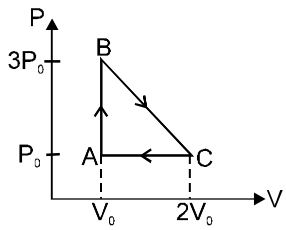
9. Two moles of an ideal monoatomic gas are confined within a cylinder by a massless and frictionless spring loaded piston of cross-sectional area 4×10^{-3} m^2. The spring is, initially in its relaxed state. Now the gas is heated by an electric heater, placed inside the cylinder, for some time. During this time, the gas expands and does 50 J of work in moving the piston through a distance 0.10 m. The temperature of the gas increases by 50 K. Calculate the spring constant and the heat supplied by the heater. $P_{atm} = 1 \times 10^5$ N/m^2. **[REE 1997, 5]**

10. Two vessels A and B, thermally insulated, contain an ideal monoatomic gas. A small tube fitted with a valve connects these vessels. Initially the vessel A has 2 liters of gas at 300 K and 2×10^5 N m^{-2} pressure while vessel B has 4 liters of gas at 350 K and 4×10^5 Nm^{-2} pressure. The valve is now opened and the system reaches equilibrium in pressure and temperature. Calculate the new pressure and temperature. ($R = \frac{25}{3}$ J/mol-K) **[REE 1997, 5]**

11. One mole of a diatomic ideal gas (γ = 1.4) is taken through a cyclic process starting from point A. The process A $\to$ B is an adiabatic compression. B$\to$C is isobaric expansion. C $\to$ D an adiabatic expansion and D $\to$ A is isochoric as shown in P-V diagram. The volume ratios are $\frac{V_A}{V_B} = 16$ & $\frac{V_C}{V_B} = 2$ and the temperature at A is T_A = 300 K. Calculate the temperature of the gas at the points B and D and find the efficiency of the cycle. **[JEE 1997, 5/100]**



12. One mole of an ideal monoatomic gas is taken around the cyclic process ABCA as shown in the figure. Calculate, **[JEE 1998, 8/200]**



(a) the work done by the gas ;

(b) the heat rejected by the gas in the path CA and the heat absorbed by the gas in the path AB ;

(c) the net heat absorbed by the gas in the path BC ;

(d) the maximum temperature attained by the gas during the cycle.

13. Two moles of a monatomic gas, initially at pressure P_1 and volume V_1, undergo an adiabatic compression until its volume is V_2. Then the gas is given heat Q at constant volume V_2. **[JEE 1999, 2 + 8 / 200]**

(a) Sketch the complete process on a P-V diagram.

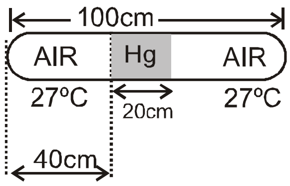
(b) Find the total work done by the gas, the total change in its internal energy and the final temperature of the gas. [Give your answers in terms of P_1, V_1, V_2 and Q and R.]

14. A weightless piston divides a thermally insulated cylinder into two parts of volumes V and 3V. 2 moles of an ideal gas at pressure P = 2 atmosphere are confined to the part with volume V = 1 liter. The remainder of the cylinder is evacuated. The piston is now released and the gas expands to fill the entire space of the cylinder. The piston is then pressed back to the initial position. Find the increase of internal energy in the process and final temperature of the gas. The ratio of the specific heats of the gas, $\gamma = 1.5$. **[REE -1999]**

15. In given figure, an adiabatic cylindrical tube of volume $2V_0$ is divided in two equal parts by a frictionless adiabatic separator. An ideal gas in left side of a tube having pressure P_1 and temperature T_1 where as in the right side having pressure P_2 and temperature T_2. $C_p/C_v = \gamma$ is the same for both the gases. The separator is slid slowly and is released at a position where it can stay in equilibrium. Find (a) the final volumes of the two parts, (b) the heat given to the gas in the left part and (c) the final common pressure of the gases.

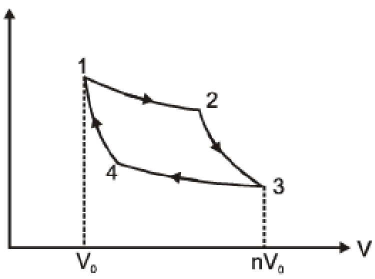
16. Two containers A and B of equal volume $V_0 / 2$ each are connected by a narrow tube which can be closed by a valve. The containers are fitted with pistons which can be moved to change the volumes. Initially, the valve is open and the containers contain an ideal gas ($C_p/C_v = \gamma$) at atmospheric pressure P_0 and atmospheric temperature $2T_0$. The walls of the containers A are highly conducting and of B are non-conducting. The valve is now closed and the pistons are slowly pulled out to increase the volumes of the containers to double the original value. (a) Calculate the temperatures and pressures in the two containers. (b) The valve is now opened for sufficient time so that the gases acquire a common temperature and pressure. Find the new values of the temperature and the pressure.

17. In the given figure a glass tube lies horizontally with the middle 20 cm containing mercury. The two ends of the tube contains air at 27ºC and at a pressure 76 cm of mercury. Now the air column on one side is maintained at 0ºC and the other side is maintained at 127ºC. Find the new length of the air column on the cooler side. Neglect the changes in the volume of mercury and of the glass.



18. An ideal gas ($C_p/C_v = \gamma$) having initial pressure P_0 and volume V_0. (a) The gas is taken isothermally to a pressure $2P_0$ and then adiabatically to a pressure $4P_0$. Find the final volume. (b) The gas is brought back to its initial state. It is adiabatically taken to a pressure $2P_0$ and then isothermally to a pressure $4P_0$. Find the final volume.

19. A Carnot engine cycle is shown in the Fig. (2). The cycle runs between temperatures $T_H = \alpha T_0$ and $T_L = T_0$ ($\alpha > 1$). Minimum and maximum volume at state 1 and state 3 are V_0 and nV_0 respectively. The cycle uses one mole of an ideal gas with $C_P / C_V = \gamma$. Here C_P and C_V are the specific heats at constant pressure and volume respectively. You must express all answers in terms of the given parameters {α, n, T_0, V_0, ?} and universal gas constant R.



(a) Find P, V, T for all the states

(b) Calculate the work done by the engine in each process: W_{12}, W_{23}, W_{34}, W_{41}.

(c) Calculate Q, the heat absorbed in the cycle.

20. A thermally insulated piston divides a nonconducting container in two compartments, right compartment of 2V, T and 2P, while in the left compartment the respective values are V, T and P. Total moles in total system of both compartments is 5 moles (same molar mass). If the piston can slide freely, and in the final equilibrium position, volume of right compartment is $\frac{xV}{5}$ then find the value of x.

21. Cloud formation condition

Consider a simplified model of cloud formation. Hot air in contact with the earth's surface contains water vapor. This air rises convectively till the water vapor content reaches its saturation pressure. When this happens, the water vapor starts condensing and droplets are formed. We shall estimate the height at which this happens. We assume that the atmosphere consists of the diatomic gases oxygen and nitrogen in the mass proportion 21:79 respectively. We further assume that the atmosphere is an ideal gas, g the acceleration due to gravity is constant and air processes are adiabatic. Under these assumptions one can show that the pressure is given by

$$p = p_0\left(\frac{T_0 - \Gamma z}{T_0}\right)^{\alpha}$$

Here p_0 and T_0 is the pressure and temperature respectively at sea level (z = 0), Γ is the lapse rate (magnitude of the change in temperature T with height z above the earth's surface, i.e. $\Gamma > 0$).

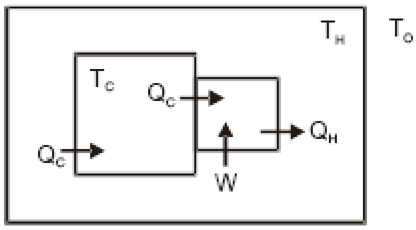
(a) Obtain an expression for the lapse rate Γ in terms of γ, R, g and m_a. Here γ is the ratio of specific heat at constant pressure to specific heat at constant volume; R, the gas constant; and m_a, the relevant molar mass.

(b) Estimate the change in temperature when we ascend a height of one kilometer ?

(c) Show that pressure will depend on height as given by Eq. (1). Find an explicit expression for exponent α in terms of γ.

(d) According to this model what is the height to which the atmosphere extends? Take T_0 = 300 K and p_0 = 1 atm.

22. It is well known that the temperature of a closed room goes up if the refrigerator is switched on inside it. A refrigerator compartment set to temperature T_c is turned on inside a hut in Leh (Ladakh). The atmosphere (outside the hut) can be considered to be a vast reservoir at constant temperature T_o. Walls of hut and refrigerator compartment are conducting. The temperature of the refrigerator compartment is maintained at T_c with the help of a compressor engine. We explain the working of the refrigerator engine and the heat flow with the help of the associated figure.



The larger square is the refrigerator compartment with heat leak per unit time Q_c into it from the room. The same heat per unit time Q_c is pumped out of it by the engine (also called compressor and indicated by the smaller square in thick). The compressor does work W and rejects heat per unit time Q_H into the hut. The thermal conductance (in units of watt per kelvin) of the walls of the compartment and hut respectively are K_c and K_H. After a long time it is found that temperature of the hut is T_H. The compressor works as a reverse Carnot engine and it does not participate in heat conduction process.

(a) State the law of heat conduction for the walls of the hut and the refrigerator compartment.

(b) We define the dimensionless quantities $k = K_H/K_c$, $h = T_H/T_o$ and $c = T_c/T_o$. Express h in terms of c and k.

(c) Calculate stable temperature T_H given T_0 = 280.0 K, T_c = 252.0 K and k = 0.90.

(d) Now another identical refrigerator is put inside the hut. T_c and T_0 do not change but T_H, the hut temperature will change to T_H. State laws of heat conduction for hut and one of the two identical refrigerator compartments.

(e) Assume that dimensionless quantities k and c do not change. Let $h' = T'_H / T_0$. Obtain an expression for h'.

Answers

1. $n = \dfrac{\ln \eta}{\ln(1+\Delta V / V)}$

2. $p = p_0 e^{-Ct/V}$

3. (a) $T_{max} = \dfrac{2}{3}(p_0 / R)\sqrt{p_0 / 3\alpha}$] (b) $T_{max} = p_0 / e\beta R$.

4. $\dfrac{2q}{5(mg+P_0A)}$

5. $W = p_0V_0 \ln [(\eta +1)^2 /4\eta$

6. **(i)** $300\left(\dfrac{1}{2}\right)^{2/3}$ K **(ii)** $7500\left(2^{-2/3}-1\right)$ J **(iii)** $-7500\left(2^{-2/3}-1\right)$ J

7. 55 J

8. **(i)** T_A = 120 K, T_B = 241 K, T_C = 481 K, T_D = 241 K **(ii)** No

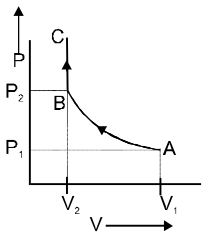
   (iii) $\Delta Q_{ABC} = \dfrac{13}{4} \times 10^6$ J ; $\Delta Q_{ADC} = \dfrac{11}{4} \times 10^6$ J

9. K = 2000 N/m, Q = 50 + 150 R Joules

10. $P = \dfrac{10}{3} \times 10^5$ N/m², $T = \dfrac{10500}{31}$ K

11. $T_B = 600 \times 2^{3/5}$ K, $T_D = 1200 \times 2^{-3/5}$ K, $\eta = 61.37\%$

12. (a) P_0V_0 (b) $5/2P_0V_0$, $3P_0V_0$ (c) $1/2P_0V_0$ (d) $25P_0V_0/8R$

13. (a) 

, (b) $W = \dfrac{3}{2}P_1V_1\left[1-\left(\dfrac{V_1}{V_2}\right)^{2/3}\right]$; $\Delta U = Q - W$, $T_F = \dfrac{\Delta U}{3R} + T_i = \dfrac{\Delta U}{3R} + \dfrac{P_1V_1}{2R}$

$$= \left[\frac{P_1V_1^{5/3} \cdot V_2^{-2/3}}{2R} + \frac{Q}{3R}\right]$$

14. 400 J, 24 K

15. (a) $\dfrac{2p_1^{1/\gamma}V_0}{A}, \dfrac{2p_2^{1/\gamma}V_0}{A}$, (b) zero, (c) $(A/2)^\gamma$ where $A = p_1^{1/\gamma} + p_2^{1/\gamma}$

16. (a) $2T_0$, $\dfrac{p_0}{2}$ in the vessel A and $\dfrac{T_0}{2^{\gamma-2}}$, $p_0/2^\gamma$ in vessel B, (b) $2T_0$, $P_0/2$

17. $\ell = \dfrac{21840}{673}$ cm

18. $\dfrac{V_0}{2^{\frac{\gamma+1}{\gamma}}}$ **in each cases**

19. (a) $P_1 = \dfrac{R\alpha T_0}{V_0}$, $P_2 = \dfrac{\alpha^{\frac{\gamma}{\gamma-1}} RT_0}{nV_0}$, $P_3 = \dfrac{RT_0}{nV_0}$, $P_4 = \dfrac{RT_0}{\alpha^{\frac{1}{\gamma-1}} V_0}$

$V_1 = V_0$, $V_2 = \dfrac{nV_0}{\alpha^{\frac{1}{\gamma-1}}}$, $V_3 = nV_0$, $V_4 = \alpha^{\frac{1}{\gamma-1}}.V_0$

$T_1 = \alpha T_0$, $T_2 = \alpha T_0$, $T_3 = T_0$, $T_4 = T_0$

(b) $W_{12} = R\,\alpha\,T_0 \ln\left(\dfrac{V_2}{V_1}\right) = \alpha RT_0 \ln\left(\dfrac{n}{\alpha^{\frac{1}{\gamma-1}}}\right)$

$$W_{23} = -\frac{R}{\gamma-1}(T_0 - \alpha T_0)$$

$$W_{34} = RT_0\left(\frac{\alpha^{\frac{1}{\gamma-1}}}{n}\right)$$

$$W_{41} = -\frac{R}{\gamma-1}(\alpha T_0 - T_0)$$

(c) $Q = RT_0(\alpha - 1)\ \ell n\left(\dfrac{n}{\alpha^{\left(\frac{1}{\gamma-1}\right)}}\right)$

20. 12

21. (a) $\Gamma = \dfrac{dT}{dz} = \left(\dfrac{\rho T}{P}\right) g\dfrac{\gamma-1}{\gamma} = \dfrac{m_a}{R} g\dfrac{(\gamma-1)}{\gamma}$

(b) Change in temperature = 9.9 Kelvin

(c) Comparing $\alpha = \dfrac{\gamma}{\gamma-1}$

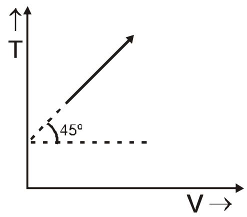
(d) $z = \dfrac{T_0}{\Gamma} = \dfrac{300}{9.9} = 30.3\,\text{km}$

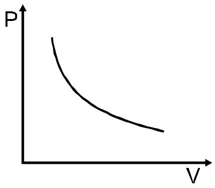
SELF ASSESSMENT PAPER

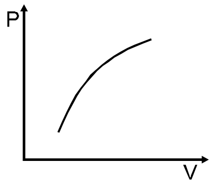
JEE (ADVANCED) PAPER-1

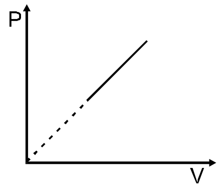
SECTION-1 : ONE OPTION CORRECT TYPE (Maximum Marks - 12)

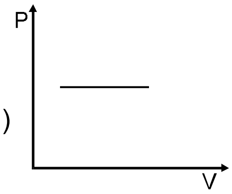
1. The given curve represents the variation of temperature as a function of volume for one mole of an ideal gas. Which of the following curves best represents the variation of pressure as a function of volume?



(A)


(B)


(C)


(D)


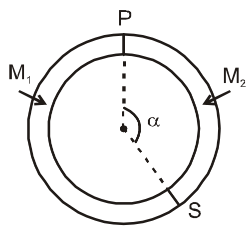
2. A one dimensional gas is a hypothetical gas with molecules that can move along only a single axis. The following table gives four situations, the velocities in meter per second of such a gas having four molecules. The plus and minus sign refer to the direction of the velocity along the axis.

Situation	Velocities			
a	–2	+3	–4	+5
b	+1	–3	+4	–6
c	+2	+3	+4	+5
d	+3	+3	–4	–5

In which situation root-mean-square speed of the molecules is greatest

(A) a (B) b (C) c (D) d

3. A ring shaped tube contains two ideal gases with equal masses and atomic mass numbers $M_1 = 32$ and $M_2 = 28$. The gases are separated by one fixed partition P and another movable conducting partition S which can move freely without friction inside the ring. The angle α as shown in the figure (in degrees) in equilibrium is:



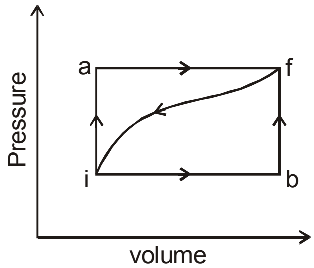
(A) $\frac{7\pi}{8}$ (B) $\frac{8\pi}{7}$ (C) $\frac{15\pi}{16}$ (D) $\frac{16\pi}{15}$

4. In a process the pressure of a gas is inversely proportional to the square of the volume. If temperature of the gas increases, then work done by the gas:

(A) is positive (B) is negative (C) is zero (D) may be positive

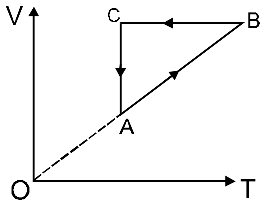
SECTION-2 : ONE OR MORE THAN ONE CORRECT TYPE (Maximum Marks - 32)

5. When a sample of a gas is taken from state i to state f along path ' iaf', heat supplied to the gas is 50 cal and work done by the gas is 20 cal.
If it is taken by path ' ibf ', then heat supplied is 36 cal.

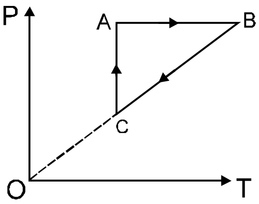


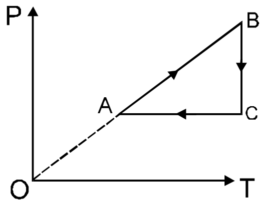
(A) Work done by the gas along path ibf is 6 cal
(B) If work done on the gas is 13 cal for the return path ' fi', then heat rejected by the gas along path ' fi ' is 43 cal.
(C) If internal energy of the gas at state i is 10 cal, then internal energy at state 'f' is 40 cal.
(D) If internal energy at state 'b' is 22 cal and at 'i' is 10 cal then heat supplied to the gas along path ' ib' is 18 cal.

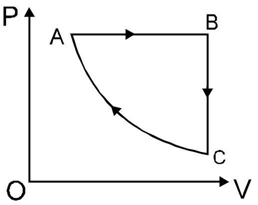
6. An ideal gas undergoes a thermodynamic cycle as shown in figure.:

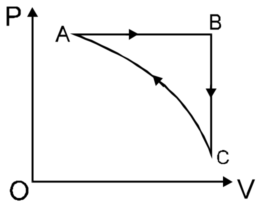


Which of the following graphs represents the same cycle ?

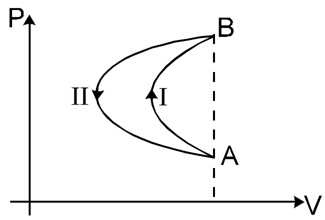
(A)


(B)


(C)


(D)


7. In a cyclic process, a gas is taken from state A to B via path-I as shown in the indicator diagram and taken back to state A from state B via path-II. In the complete cycle :



(A) work is done on the gas.
(B) heat is given to the gas
(C) no work is done by the gas.
(D) nothing can be said about work as data is insufficient

8. During the melting of a slab of ice at 273 K at atmospheric pressure

(A) positive work is done by the ice water system on the atmosphere.

(B) positive work is done on the ice water system by the atmosphere.

(C) the internal energy of the ice water system increases.

(D) the internal energy of the ice water system decreases.

9. Consider a collision between an argon molecule and a nitrogen molecule in a mixture of argon and nitrogen kept at room temperature. Which of the following are possible?

(A) The kinetic energies of both the molecules decrease.

(B) The kinetic energies of both the molecules increase

(C) The kinetic energy of the argon molecule increases and that of the nitrogen molecules decrease.

(D) The kinetic energy of the nitrogen molecules increases and that of the argon molecule decrease.

10. A thermally insulated piston divides a nonconducting container in two compartments, right compartment of 2V, T and 2P, while in the left compartment the respective values are V ,T and P. Total moles in total system of both compartments is 5 moles. If the piston can slide freely, then in the final equilibrium position.

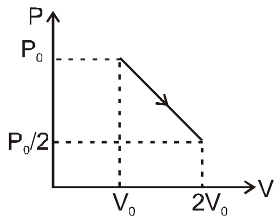
(A) Right compartment has 4 mole

(B) Left compartment has 1 mole

(C) Volume of right compartment $\frac{12V}{5}$

(D) Volume of left compartment $\frac{4V}{5}$

11. An ideal monatomic gas is at P_0, V_0. It is taken to final volume $2V_0$ when pressure is $P_0/2$ in a process which is straight line on P-V diagram. Then



(A) The final temperature is equal to initial temperature

(B) There is no change in internal energy

(C) The work done by the gas is $+\frac{P_0V_0}{4}$

(D) The heat is absorbed in the process.

12. For an ideal gas molecules the following quantities having zero average values are.

(A) velocity (B) momentum (C) kinetic energy (D) density

SECTION-3 : NUMERICAL VALUE TYPE (Maximum Marks - 18)

13. Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then find the rise in temperature (in K) of the gas in B.

14. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes the total internal energy of the system is xRT. Find x.

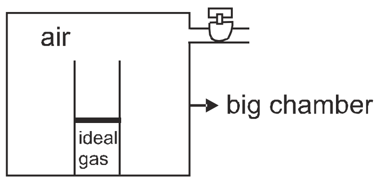
15. A gas has molar heat capacity C = 37.35 $Jmol^{-1}$ K^{-1} in the process PT = constant. Find the number of degrees of freedom of the molecules in the gas.

16. Initially volume and temperature of three samples A, B and C of an ideal gas ($\gamma = 3/2$) are equal. The volume of each sample is doubled, the process being isobaric for A, adiabatic for B and isothermal for C. If the final pressure are equal for the three samples, then the ratio of the initial pressures is found to be m : n$\sqrt{2}$: q. Find the value of mnq.

17. A vessel contains a mixture consisting of m_1 = 7 g of nitrogen (M_1 = 28) and m_2 = 11 g of carbon-dioxide (M_2 = 44) at temperature T = 300 K and pressure P_0 = 1 atm. Find the density (in $\frac{kg}{m^3}$) of the mixture.

18. A big chamber containing air at pressure 400 kPa in which an ideal gas is kept in a long cylindrical vessel fitted with a frictionless piston of cross-sectional area 5 cm^2 and mass 1 kg as shown in figure. The length of the gas column is 10 cm. If the chamber is now completely evacuated by an exhaust pump, what will be the length (in cm) of the gas column? Assume temperature remains constant throughout the process.

(g = 10 m/s^2)



Answers

1.	(A)	**2.**	(B)	**3.**	(D)	**4.**	(B)	**5.**	(A,B,C,D)	**6.**	(A,C)
7.	(A,C)	**8.**	(B,C)	**9.**	(C,D)	**10.**	(A,B,C)	**11.**	(A,B,D)	**12.**	(A,B)
13.	42	**14.**	11	**15.**	5	**16.**	4	**17.**	1.44	**18.**	210