02

Motion in a Straight Line

TOPIC 1
Terms Related to Motion

01 If the velocity of a body related to displacement x is given by $v=\sqrt{5000+24x}$ m/s, then the acceleration of the body is ...... m/s^2. **[2021, 27 Aug Shift-I]**

Ans. *(12)*

Velocity of body is

$$v=\sqrt{5000+24x} \text{ m/s}$$

Acceleration of body can be calculated as

$$a=\frac{dv}{dt}=\frac{dv}{dx}\times\frac{dx}{dt}$$

$$\Rightarrow \quad a=v\frac{dv}{dx} \quad \text{...(i)}$$

$$\frac{dv}{dx}=\frac{d}{dx}(\sqrt{5000+24x})$$

$$\Rightarrow \quad \frac{dv}{dx}=\frac{1}{2(\sqrt{5000+24x})}\times 24$$

$$=\frac{12}{\sqrt{5000+24x}}$$

Substituting the value of $\frac{dv}{dx}$ in Eq. (i), we get

$$a=\sqrt{5000+24x}\times\frac{12}{\sqrt{5000+24x}}$$

$$=12 \text{ m/s}^2$$

Thus, the acceleration of body is 12 m/s^2.

02 The instantaneous velocity of a particle moving in a straight line is given as $v=\alpha t+\beta t^2$, where α and β are constants. The distance travelled by the particle between 1s and 2s is **[2021, 25 July Shift-II]**

(a) $3\alpha+7\beta$ (b) $\frac{3}{2}\alpha+\frac{7}{3}\beta$

(c) $\frac{\alpha}{2}+\frac{\beta}{3}$ (d) $\frac{3}{2}\alpha+\frac{7}{2}\beta$

Ans. *(b)*

Given, instantaneous velocity

$$(v)=\alpha t+\beta t^2$$

Let distance travelled by particle be s and t_1 and t_2 be the initial and final time i.e. $t_1=1$s and $t_2=2$s

As, $\quad v=\frac{ds}{dt}$

$$\Rightarrow \quad ds=vdt \Rightarrow ds=(\alpha t+\beta t^2)$$

Integrating both sides, we get

$$s=\left(\alpha\frac{t^2}{2}+\frac{\beta t^3}{3}\right)\Bigg|_1^2$$

$$=\frac{\alpha}{2}[2^2-1^2]+\frac{\beta}{3}[2^3-1^3]$$

$$=\frac{\alpha}{2}[4-1]+\frac{\beta}{3}[8-1]$$

$$=\frac{3\alpha}{2}+\frac{7\beta}{3}$$

03 The relation between time t and distance x for a moving body is given as $t=mx^2+nx$, where m and n are constants. The retardation of the motion is (when v stands for velocity) **[2021, 25 July Shift-II]**

(a) $2mv^3$ (b) $2mnv^3$

(c) $2nv^3$ (d) $2n^2v^3$

Ans. *(a)*

Given, distance-time relation

$$t=mx^2+nx \quad \text{...(i)}$$

Let a be the acceleration.

$$\Rightarrow \quad a=\frac{dv}{dt}=\frac{d}{dt}\left(\frac{dx}{dt}\right)$$

where, v is speed and x is distance.

On differentiating both sides of Eq. (i) with respect to time t, we get

$$\frac{d}{dt}(t)=\frac{d}{dt}(mx^2+nx)$$

$$\Rightarrow \quad 1=m\left[2x\frac{dx}{dt}\right]+n\frac{dx}{dt}\Rightarrow 1=\frac{dx}{dt}(2mx+n)$$

$$\Rightarrow \quad v=\frac{1}{2mx+n} \quad \text{...(ii)}$$

Now, on differentiating Eq. (ii) with respect to time t, we get

$$\frac{dv}{dt}=\frac{-1\left(2m\frac{dx}{dt}\right)}{(2mx+n)^2}$$

$$\Rightarrow \quad a=\frac{-2mv}{(2mx+n)^2} \quad \text{...(iii)}$$

On squaring Eq. (ii) both sides, we get

$$v^2=\frac{1}{(2mx+n)^2}$$

Substituting the value of v^2 in Eq. (iii), we get

$$a=-2mv\cdot v^2=-2mv^3$$

$\therefore$ Retardation $=-a=2mv^3$

04 A boy reaches the airport and finds that the escalator is not working. He walks up the stationary escalator in time t_1. If he remains stationary on a moving escalator, then the escalator takes him up in time t_2. The time taken by him to walk up on the moving escalator will be **[2021, 20 July Shift-II]**

(a) $\frac{t_1t_2}{t_2-t_1}$ (b) $\frac{t_1+t_2}{2}$

(c) $\frac{t_1t_2}{t_2+t_1}$ (d) t_2-t_1

Ans. *(c)*

Let the length of escalator be L.

Also, suppose that the velocity of boy with respect to escalator be v_1 and the velocity of escalator be v_2.

$\therefore$ Velocity of boy with respect to stationary escalator can be given as

$$v_1 = \frac{L}{t_1} \quad \text{...(i)}$$

When escalator is moving and boy is at rest velocity of escalator can be given as

$$v_2 = \frac{L}{t_2} \quad \text{...(ii)}$$

So, the resultant velocity can be given as when both escalator and boy are moving,

$$v = v_1 + v_2 \quad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$v = \frac{L}{t_1} + \frac{L}{t_2} \Rightarrow \frac{v}{L} = \frac{1}{t_1} + \frac{1}{t_2}$$

$$\Rightarrow \quad \frac{v}{L} = \frac{t_1 + t_2}{t_1 t_2} \Rightarrow \frac{L}{v} = \frac{t_1 t_2}{t_1 + t_2}$$

$$\Rightarrow \quad t = \frac{t_1 t_2}{t_1 + t_2} \quad [\because t = L/v]$$

05 The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is $x = 0$ at $t = 0$, then its displacement after time $(t = 1)$ is **[2021, 17 March Shift-II]**

(a) $v_0 + g + F$ (b) $v_0 + \frac{g}{2} + \frac{F}{3}$

(c) $v_0 + \frac{g}{2} + F$ (d) $v_0 + 2g + 3F$

Ans. *(b)*

Given, equation of velocity,

$$v = v_0 + gt + Ft^2$$

At $t = 0$, $x = 0$

At $t = 1$ s $x = ?$

We know that, $v = \frac{dx}{dt} \Rightarrow dx = vdt$

Integrating the above equation with proper limits, we get

$$\int_0^x dx = \int_0^{t=1} vdt$$

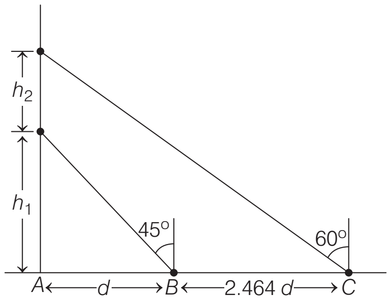
$$x - 0 = \int_0^{t=1} (v_0 + gt + Ft^2)\, dt$$

$$\Rightarrow \quad x = \left[v_0 t + \frac{gt^2}{2} + \frac{Ft^3}{3}\right]_0^1$$

$$\Rightarrow x = \left[v_0(1) + \frac{g(1)^2}{2} + \frac{F(1)^3}{3}\right] - 0$$

$$\Rightarrow \quad x = \left[v_0 + \frac{g}{2} + \frac{F}{3}\right]$$

06 A balloon is moving up in air vertically above a point A on the groun(d) When it is at a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance 2.464 d (point C). Then, the height h_2 is (Given, $\tan 30° = 0.5774$) **[2020, 5 Sep Shift-I]**

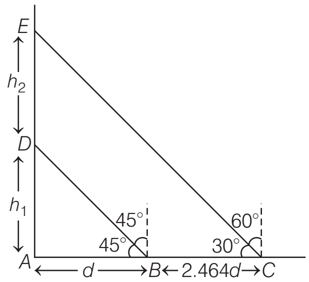


(a) 1.464 d (b) d

(c) 0.464 d (d) 0.732 d

Ans. *(b)*

The given figure is shown below,



In ΔABD, $\tan 45° = \frac{h_1}{d}$

$\because \quad \tan 45° = 1 \Rightarrow 1 = \frac{h_1}{d} \Rightarrow h_1 = d \quad$ ...(i)

Now, in ΔACE,

$$\tan 30° = \frac{AD + DE}{AB + BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h_1 + h_2}{d + 2.464d}$$

$$0.5774 = \frac{h_1 + h_2}{3.464d}$$

$$\Rightarrow \quad h_1 + h_2 = (0.5774) \times (3.464d)$$

$$h_1 + h_2 = 2d$$

$$d + h_2 = 2d \quad [\text{Using eq. (i)}]$$

$$h_2 = 2d - d$$

$$\Rightarrow \quad h_2 = d$$

Hence, correct option is (b).

07 The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is .... **[2020, 9 Jan Shift-I]**

Ans. *(3)*

Given, displacement (x) and time (t) relation,

$$x^2 = at^2 + 2bt + c \quad \text{...(i)}$$

On differentiating Eq. (i) w.r.t. t, we get

$$\Rightarrow \quad 2x\frac{dx}{dt} = 2at + 2b$$

or $\quad xv = at + b \quad$ ...(ii)

Differentiating again, we have

$$x \cdot \frac{dv}{dt} + v \cdot \frac{dx}{dt} = a \left(\because \frac{dx}{dt} = v \text{ and } \frac{dv}{dt} = A\right)$$

$$\Rightarrow \quad xA + v^2 = a \Rightarrow xA = a - v^2$$

$$\Rightarrow \quad xA = a - \left(\frac{at + b}{x}\right)^2$$

$$[\text{from Eq. (ii)}, v = at + b/x]$$

$$\Rightarrow \quad A = \frac{ac - b^2}{x^3} \Rightarrow A \propto x^{-3}$$

Thus, $\quad n = 3$

08 A particle is moving with speed $v = b\sqrt{x}$ along positive X-axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at $t = 0$). **[2019, 12 April Shift-II]**

(a) $\frac{b^2\tau}{4}$ (b) $\frac{b^2\tau}{2}$ (c) $b^2\tau$ (d) $\frac{b^2\tau}{\sqrt{2}}$

Ans. *(b)*

Given, speed, $v = b\sqrt{x}$

Now, differentiating it with respect to time, we get $\frac{dv}{dt} = \frac{d}{dt} b\sqrt{x}$

Now, acceleration

$$\Rightarrow \quad a = \frac{b}{2\sqrt{x}} \cdot \frac{dx}{dt} \quad \left[\because \frac{dv}{dt} = a\right]$$

$$\Rightarrow a = \frac{b}{2\sqrt{x}} \cdot v = \frac{b}{2\sqrt{x}} \cdot b\sqrt{x} = \frac{b^2}{2}$$

As acceleration is constant, we use

$$v = u + at \quad \text{...(i)}$$

Now, it is given that $x = 0$ at $t = 0$.

So, initial speed of particle is

$$u = b\sqrt{x}\Big|_{x=0} = b \times 0 = 0$$

Hence, when time $t = \tau$, speed of the particle using Eq. (i) is

$$v = u + at = 0 + \frac{b^2}{2} \cdot \tau = \frac{b^2}{2} \cdot \tau$$

09 The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$ where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be **[2019, 9 April Shift-II]**

(a) $a + \frac{b^2}{2c}$ (b) $a + \frac{b^2}{4c}$

(c) $a + \frac{b^2}{3c}$ (d) $a + \frac{b^2}{c}$

Ans. ***(c)***

Position of particle is,

$$x(t) = at + bt^2 - ct^3$$

So, its velocity is,

$$v = \frac{dx}{dt} = a + 2bt - 3ct^2$$

and acceleration is,

$$a = \frac{dv}{dt} = 2b - 6ct$$

Acceleration is zero, then $2b - 6ct = 0$

$$\Rightarrow \quad t = \frac{2b}{6c} = \frac{b}{3c}$$

Substituting this 't' in expression of velocity, we get

$$v = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)^2$$

$$= a + \frac{2b^2}{3c} - \frac{b^2}{3c} = a + \frac{b^2}{3c}$$

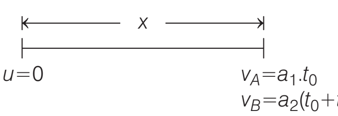
10 In a car race on a straight path, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to **[2019, 9 Jan Shift-II]**

(a) $\frac{2a_1a_2}{a_1 + a_2}t$ (b) $\sqrt{2a_1a_2}\ t$

(c) $\sqrt{a_1a_2}\ t$ (d) $\frac{a_1 + a_2}{2}t$

Ans. ***(c)***

Let car B takes time $(t_0 + t)$ and car A takes time t_0 to finish the race.



Then,

Given, $v_A - v_B = v = (a_1 - a_2)t_0 - a_2 t$ ...(i)

$$s_B = s_A = \frac{1}{2}a_1t_0^2 = \frac{1}{2}a_2(t_0 + t)^2$$

or $\sqrt{a_1}\,t_0 = \sqrt{a_2}\,(t_0 + t)$

or $\sqrt{a_1}t_0 = \sqrt{a_2}t_0 + \sqrt{a_2}t$

or $(\sqrt{a_1} - \sqrt{a_2})\,t_0 = \sqrt{a_2}\ t$

or $t_0 = \frac{\sqrt{a_2}\,.\,t}{(\sqrt{a_1} - \sqrt{a_2})}$ ...(ii)

Substituting the value of t_0 from Eq. (ii) into
Eq. (i), we get

$$v = (a_1 - a_2)\frac{\sqrt{a_2}\ t}{\sqrt{a_1} - \sqrt{a_2}} - a_2t$$

$$= (\sqrt{a_1} - \sqrt{a_2})(\sqrt{a_1} + \sqrt{a_2})\cdot\frac{\sqrt{a_2}t}{(\sqrt{a_1} - \sqrt{a_2})} - a_2t$$

or $v = (\sqrt{a_1} + \sqrt{a_2})\cdot\sqrt{a_2}\ t - a_2t$

$= \sqrt{a_1a_2}\cdot t + a_2t - a_2t$

or $v = \sqrt{a_1\cdot a_2}\ t$

11 An object moving with a speed of 6.25 m/s, is decelerated at a rate given by $dv/dt = -2.5\sqrt{v}$, where v is the instantaneous spee(d) The time taken by the object to come to rest, would be **[AIEEE 2011]**

(a) 2 s (b) 4 s
(c) 8 s (d) 1 s

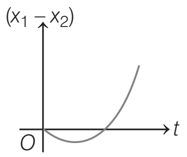
Ans. ***(a)***

Given, $\frac{dv}{dt} = -2.5\sqrt{v} \Rightarrow \frac{dv}{\sqrt{v}} = -2.5dt$

$$\Rightarrow \int_{6.25}^{0} v^{-1/2}\,dv = -2.5\int_0^t dt$$

$$\Rightarrow -2.5\,[t]_0^t = [2v^{1/2}]_{6.25}^{0} \Rightarrow t = 2\text{ s}$$

12 The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time $(t = 1)$ is **[AIEEE 2007]**



(a) $v_0 + 2g + 3f$ (b) $v_0 + \frac{g}{2} + \frac{f}{3}$

(c) $v_0 + g + f$ (d) $v_0 + \frac{g}{2} + f$

Ans. ***(b)***

Given, $v = v_0 + gt + ft^2$

After differentiating with respect to time, we get

$$\frac{dx}{dt} = v_0 + gt + ft^2$$

$$\Rightarrow \quad dx = (v_0 + gt + ft^2)\,dt$$

So, $\int_0^x dx = \int_0^1 (v_0 + gt + ft^2)\,dt$

$$\Rightarrow \quad x = v_0 + \frac{g}{2} + \frac{f}{3}$$

Since, vertical component of velocity is zero.

13 A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as **[AIEEE 2006]**

(a) t^2 (b) t (c) $t^{1/2}$ (d) t^3

Ans. ***(a)***

Given, $v = \alpha\sqrt{x}$ or $\frac{dx}{dt} = \alpha\sqrt{x}$ $\left[\because v = \frac{dx}{dt}\right]$

or $\frac{dx}{\sqrt{x}} = \alpha\,dt$

On integrating, we get

$$\int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\,dt$$

[$\because$ at $t = 0$, $x = 0$ and let at any time t, particle be at x]

$$\Rightarrow \left[\frac{x^{1/2}}{1/2}\right]_0^x = \alpha t \text{ or } x^{1/2} = \frac{\alpha}{2}t$$

or $x = \frac{\alpha^2}{4}\times t^2$ or $x \propto t^2$

14 The relation between time t and distance x is $t = ax^2 + bx$, where a and b are constants. The acceleration is **[AIEEE 2005]**

(a) $-2abv^2$ (b) $2bv^3$
(c) $-2av^3$ (d) $2av^2$

Ans. ***(c)***

Given, $t = ax^2 + bx$

Differentiating it w.r.t. t, we get

$$\frac{dt}{dt} = 2ax\frac{dx}{dt} + b\frac{dx}{dt}$$

$$v = \frac{dx}{dt} = \frac{1}{(2ax + b)}$$

Again, differentiating w.r.t. t, we get

$$\frac{d^2x}{dt^2} = \frac{-2a}{(2ax + b)^2}\cdot\frac{dx}{dt}$$

$$\therefore \quad f = \frac{d^2x}{dt^2}$$

$$= \frac{-1}{(2ax + b)^2}\cdot\frac{2a}{(2ax + b)}$$

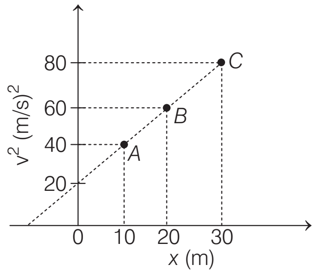
or $f = \frac{-2a}{(2ax + b)^3}$

$$\Rightarrow \quad f = -2av^3$$

TOPIC 2

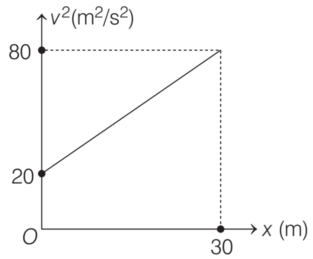
Kinematics Equations of Uniformly Accelerated Motion

15 A particle is moving with constant acceleration a. Following graph shows v^2 *versus* x (displacement) plot. The acceleration of the particle is .... m/s^2 **[2021, 31 Aug Shift-II]**



Ans. ***(1)***

A v^2 *versus* x graph is shown below.



Using equation of motion,

$$v^2 = u^2 + 2ax \quad \text{...(i)}$$

We know that, the equation of straight line.

$$y = mx + c \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii),

Slope, $m = 2a = \dfrac{80-20}{30-0} = \dfrac{60}{30} = 2$

Now, the acceleration, $a = \dfrac{2}{2} = 1\,\text{m/s}^2$

16 Water drops are falling from a nozzle of a shower onto the floor, from a height of 9.8 m. The drops fall at a regular interval of time. When the first drop strikes the floor, at that instant, the third drop begins to fall. Locate the position of second drop from the floor when the first drop strikes the floor.

[2021, 27 Aug Shift-II]

(a) 4.18 m (b) 2.94 m
(c) 2.45 m (d) 7.35 m

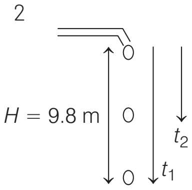
Ans. ***(d)***

Given, height of nozzle from floor, $H = 9.8\,\text{m}$

Let, t_1 = time taken by first drop to reach the ground

and t_2 = time taken by 2nd drop.

$$t_2 = \frac{t_1}{2}$$



g = acceleration due to gravity (9.8 ms^{-2})

u = initial speed = 0

Since, $H = ut + \frac{1}{2}gt_1^2 \quad \text{...(i)}$

$\therefore \quad t_1 = \sqrt{\dfrac{2H}{g}}$

$\Rightarrow \quad t_1 = \sqrt{\dfrac{2\times 9.8}{9.8}} = \sqrt{2}\,\text{s}$

$\therefore \quad t_2 = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}\,\text{s}$

Again from Eq. (i)

Distance covered by 2nd drop, $x = \dfrac{1}{2}gt_2^2$

$\therefore \quad x = \dfrac{1}{2}\times 9.8 \times \left(\dfrac{1}{\sqrt{2}}\right)^2$

$= \dfrac{1}{2}\times 9.8\times\dfrac{1}{2} = \dfrac{9.8}{4} = 2.45\,\text{m}$

and position of second drop from ground,

$H_2 = H_1 - x = 9.8 - 2.45 = 7.35\,\text{m}$

17 Two spherical balls having equal masses with radius of 5 cm each are thrown upwards along the same vertical direction at an interval of 3s with the same initial velocity of 35 m/s, then these balls collide at a height of .... m. (Take, $g = 10\,\text{m/s}^2$)

[2021, 26 Aug Shift-I]

Ans. ***(50)***

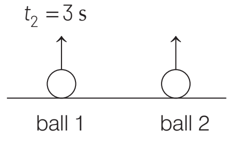
Given, $m_1 = m_2 = m$

$r_1 = r_2 = r = 5\,\text{cm} = 5\times 10^{-2}\,\text{m}$

$u_1 = u_2 = 35\,\text{m/s}$

$t_1 = 0$

and $t_2 = 3\,\text{s}$



When both ball will collide,

$$(S_1)_{\text{vertical}} = (S_2)_{\text{vertical}} \quad \text{...(i)}$$

Let after time t both ball will collide.

$$(S_1)_{\text{vertical}} = u_1 t - \frac{1}{2}gt^2$$

$$(S_2)_{\text{vertical}} = u_2(t-3) - \frac{1}{2}g(t-3)^2$$

From Eq. (i), we have

$\Rightarrow u_1 t - \dfrac{1}{2}gt^2 = u_2(t-3) - \dfrac{1}{2}g(t-3)^2$

$\Rightarrow 35t - \dfrac{1}{2}\times 10\times t^2$

$= 35(t-3) - \dfrac{1}{2}\times 10(t-3)^2$

$\Rightarrow 35t - 35(t-3) = 5t^2 - 5(t-3)^2$

$\Rightarrow 35(t-t+3) = 5[t^2-(t-3)^2]$

$\Rightarrow 35\times 3 = 5(t+t-3)(t-t+3)$

$\Rightarrow \dfrac{105}{5} = (2t-3)(3)$

$\Rightarrow \dfrac{21}{3} = 2t-3 \Rightarrow 7 = 2t-3$

$\Rightarrow 2t = 10 \Rightarrow t = 5\,\text{s}$

So, the height at which ball will collide is

$(S_1)_{\text{vertical}} = u_1 t - \dfrac{1}{2}gt^2$

$= 35\times 5 - \dfrac{1}{2}\times 10\times(5)^2$

$= 175 - 125$

$= 50\,\text{m}$

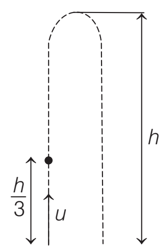
18 A ball is thrown up with a certain velocity, so that it reaches a height h. Find the ratio of the two different times of the ball reaching $\dfrac{h}{3}$ in both the directions. **[2021, 27 July Shift-I]**

(a) $\dfrac{\sqrt{2}-1}{\sqrt{2}+1}$ (b) $\dfrac{1}{3}$

(c) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (d) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

Ans. ***(c)***

The given scenario can be represented as follows.



$\because$ From Newton's third equation of motion,

When the ball is thrown up, we have

$$v^2 = u^2 + 2as$$

$\Rightarrow \quad v^2 = u^2 + 2(-g)h$

$\{\because a = -g$ (against gravity) and $s = h\}$

$$\Rightarrow \quad 0 = u^2 + 2(-g)h \quad \{\because v = 0\}$$
$$\Rightarrow \quad u^2 = 2gh \quad \text{...(i)}$$
From Newton's second equation of motion, we have
$$s = ut + \frac{1}{2}at^2$$
When the ball reaches $\frac{h}{3}$ height which coming downward
$$\Rightarrow \quad \frac{h}{3} = ut - \frac{1}{2}gt^2$$
$$\text{...(ii)} \left\{\because a = -g \text{ and } s = \frac{h}{3}\right\}$$
From Eqs. (i) and (ii), we get
$$\frac{h}{3} = \sqrt{2gh}\,t - \frac{1}{2}gt^2$$
$$\Rightarrow \quad \frac{1}{2}gt^2 - \sqrt{2gh}\,t + \frac{h}{3} = 0 \quad \text{...(iii)}$$
$\because$ Eq. (iii) is a quadratic equation, so we have to calculate its roots using formula,
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$\therefore$ On solving, we will get
$$t_1, t_2 = \frac{\sqrt{2gh} \pm \sqrt{2gh - \frac{4g}{2}\frac{h}{3}}}{g}$$
$$\therefore \frac{t_1}{t_2} = \frac{\sqrt{2gh} - \sqrt{\frac{4gh}{3}}}{\sqrt{2gh} + \sqrt{\frac{4gh}{3}}} \Rightarrow \frac{t_1}{t_2} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

19 A balloon was moving upwards with a uniform velocity of 10 m/s. An object of finite mass is dropped from the balloon when it was at a height of 75 m from the ground level. The height of the balloon from the ground when object strikes the ground was around, is (Take, the value of $g = 10$ m/s^2)
[2021, 25 July Shift-II]

(a) 300 m (b) 200 m
(c) 125 m (d) 250 m

Ans. *(c)*

Given, initial speed of balloon,
$$u_b = 10 \text{ ms}^{-1}$$
Initial height of balloon from ground, $h = 75$ m

Initial speed of object, $u_a = -10 \text{ ms}^{-1}$

Let the time taken by object to reach ground be t.

As, $$h = ut + \frac{1}{2}gt^2$$

For object a,
$$\Rightarrow 75 = -10t + \frac{1}{2} \times 10 \times t^2 = -10t + 5t^2$$
$$\Rightarrow \quad t^2 - 2t - 15 = 0$$
$$\Rightarrow \quad t^2 - 5t + 3t - 15 = 0$$
$$\Rightarrow \quad t(t-5) + 3(t-5) = 0 \Rightarrow t = 5, -3$$
$$\therefore \quad t = 5\text{ s} \quad (\because t \neq -\text{ve})$$
Also, $$u_b = \frac{s}{t}$$
where, s is distance covered by balloon and t is time taken by object to reach ground

$\therefore$ Final height, $h_f = h + s$
$$= h + u_b \times t$$
$$= 75 + 10 \times 5$$
$$= 75 + 50 = 125 \text{ m}$$

20 Water droplets are coming from an open tap at a particular rate. The spacing between a droplet observed at 4th second after its fall to the next droplet is 34.3 m. At what rate, the droplets are coming from the tap ? (Take, $g = 9.8$ m/s^2)
[2021, 25 July Shift-I]

(a) 3 drops / 2 s (b) 2 drops / s
(c) 1 drop / s (d) 1 drop / 7 s

Ans. *(c)*

Given, distance travelled in 4th second,
$$s_2 = 34.3 \text{ m},$$
Acceleration due to gravity, $g = 9.8 \text{ ms}^{-2}$

Let distance covered by 1st drop in time $t (= 4\text{ s})$ be s_1.

As we know that,
$$s = ut + \frac{1}{2}gt^2 \quad \text{...(i)}$$
where, u = initial speed of drop = 0 ms^{-1}
$$\Rightarrow \quad s_1 = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8 \times 4^2$$
$$= \frac{1}{2} \times 9.8 \times 16 = 9.8 \times 8 = 78.4 \text{ m}$$
Now, distance covered by 2nd drop will be s.
$$s = s_1 - s_2$$
$$= 78.4 - 34.3 = 44.1 \text{ m}$$
Again, By using Eq. (i)
$$s = \frac{1}{2} 9.8\, t'^2$$
where, t' is the time taken by 2nd drop to cover 44.1 m.
$$\therefore \quad 44.1 = 4.9t'^2 \Rightarrow t'^2 = \frac{44.1}{4.9}$$
$$\Rightarrow \quad t' = \sqrt{\frac{441}{49}} = \sqrt{9} = 3 \text{ s} \quad \text{...(ii)}$$

Since, time taken by 1st drop is 4 s, and time taken by 2nd drop is 3 s, so one drop is falling per second.

21 A scooter accelerates from rest for time t_1 at constant rate a_1 and then retards at constant rate a_2 for time t_2 and comes to rest. The correct value of $\frac{t_1}{t_2}$ will be
[2021, 26 Feb Shift-II]

(a) $\frac{a_1 + a_2}{a_2}$ (b) $\frac{a_2}{a_1}$
(c) $\frac{a_1}{a_2}$ (d) $\frac{a_1 + a_2}{a_1}$

Ans. *(b)*

According to given data;

Initial speed of scooter, $u_1 = 0 \text{ ms}^{-1}$

and final speed, $v_2 = 0 \text{ ms}^{-1}$

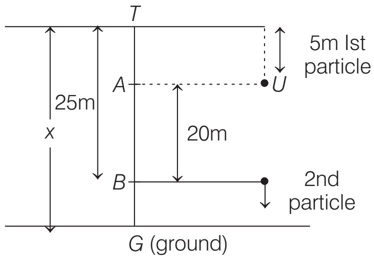
By using first equation of motion,
$$v_1 = u + a_1 t_1$$
$$\Rightarrow \quad v_1 = a_1 t_1 \quad \text{...(i)}$$
and for second case,
$$v_2 = v_1 - a_2 t_2$$
$$\Rightarrow \quad 0 = v_1 - a_2 t_2$$
$$\Rightarrow \quad v_1 = a_2 t_2 \quad \text{...(ii)}$$
On dividing Eq. (i) by Eq. (ii), we get
$$v_1 / v_1 = 1 = \frac{a_1 t_1}{a_2 t_2}$$
$$\Rightarrow \quad a_2 t_2 = a_1 t_1$$
$$\Rightarrow \quad t_1 / t_2 = a_2 / a_1$$

22 A stone is dropped from the top of a building. When it crosses a point 5 m below the top, another stone starts to fall from a point 25 m below the top. Both stones reach the bottom of building simultaneously. The height of the building is **[2021, 25 Feb Shift-II]**

(a) 45 m (b) 25 m (c) 35 m (d) 50 m

Ans. *(a)*

Let the total height of building be x.



$$TA = 5\,\text{m}$$
$$TB = 25\,\text{m}$$
$\therefore$ $AG = x - 5$ and $BG = x - 25$

For initial conditions, from second equation of motion under gravity,

$$s = ut + 1/2gt^2$$

where, $g = 10\,\text{ms}^{-2}$

$\therefore$ $5 = 0 + 1/2 \times 10t^2 \Rightarrow t = 1\text{s}$

Now, by first equation of motion under gravity,

$$v_A = u + gt$$
$$= 0 + 10 = 10\,\text{ms}^{-1}$$

From second equation of motion,

$$x - 5 = v_A t + 1/2gt^2 \quad \ldots \text{(i)}$$

Similarly, $x - 25 = 1/2gt^2$

Put the above value in Eq. (i), we get

$$x - 5 = 10t + x - 25$$
$$20 = 10t$$
$$\Rightarrow \quad t = 2\text{s}$$

Put the value of t in Eq. (i), we get

$$x - 5 = 10 \times 2 + 1/2 \times 10 \times 4$$
$$\Rightarrow \quad x - 5 = 20 + 20$$
$$\Rightarrow \quad x = 45\,\text{m}$$

23 An engine of a train moving with uniform acceleration, passes the signal-post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is **[2021, 25 Feb Shift-I]**

(a) $\sqrt{\dfrac{v^2 + u^2}{2}}$ (b) $\dfrac{v - u}{2}$

(c) $\dfrac{u + v}{2}$ (d) $\sqrt{\dfrac{v^2 - u^2}{2}}$

Ans. ***(a)***

Given, initial speed of engine $= u$

Speed of engine last compartment $= v$

Let the length of train be l

and the speed of mid-point of train be v'.

Using third equation of motion,

$$v^2 = u^2 + 2as$$
$$\Rightarrow \quad v'^2 = u^2 + 2al/2$$
$$\Rightarrow \quad v'^2 - u^2 = al \quad \ldots \text{(i)}$$

Also, $v^2 - u^2 = 2al \Rightarrow a = \dfrac{v^2 - u^2}{2l}$

Substituting the value of a in Eq. (i), we get

$$v'^2 - u^2 = \frac{v^2 - u^2}{2l} \cdot l = \frac{v^2 - u^2}{2}$$
$$\Rightarrow \quad v' = \sqrt{\frac{v^2 - u^2}{2} + u^2} = \sqrt{\frac{v^2 + u^2}{2}}$$

24 A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is at a height h. The time taken by the packet to reach the ground is close to (Here, g is the acceleration due to gravity). **[2020, 5 Sep Shift-I]**

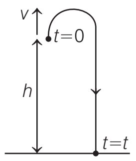
(a) $t = \dfrac{2}{3}\sqrt{\dfrac{h}{g}}$ (b) $t = 1.8\sqrt{\dfrac{h}{g}}$

(c) $t = \sqrt{\dfrac{2h}{3g}}$ (d) $t = 3.4\sqrt{\dfrac{h}{g}}$

Ans. ***(d)***

Let t be the time taken by the packet to reach the groun(d) As, the helicopter rises from rest in upward direction, its final velocity is

$$v = \sqrt{0^2 + 2gh} \Rightarrow v = \sqrt{2gh}$$



From second equation of motion,

$$s = ut + \frac{1}{2}at^2$$

Here, $s = -h$

u or $v = \sqrt{2gh} \Rightarrow a = g$

Substituting all these values in above equation, we get

$$-h = \sqrt{2gh}\,t + \frac{1}{2}(-g)t^2$$
$$\Rightarrow \quad \frac{1}{2}gt^2 - \sqrt{2gh}\,t - h = 0$$

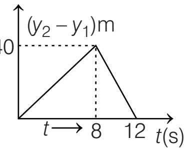
This is a quadratic equation in t.

$$\therefore \quad t = \frac{\sqrt{2gh} \pm \sqrt{(\sqrt{2gh})^2 - 4 \times \frac{g}{2}(-h)}}{2 \times \frac{g}{2}}$$
$$= \frac{\sqrt{2gh} \pm \sqrt{2gh + 2gh}}{g} = \frac{\sqrt{2gh}}{g}(1 + \sqrt{2})$$
$$= \sqrt{\frac{2h}{g}}(1 + \sqrt{2}) = (2 + \sqrt{2})\sqrt{\frac{h}{g}} = 3.4\sqrt{\frac{h}{g}}$$

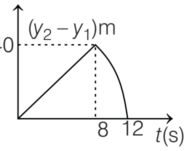
Hence, correct option is (d).

25 Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first? Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10\,\text{m/s}^2$) **[JEE Main 2015]**

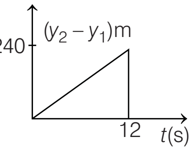
(a)



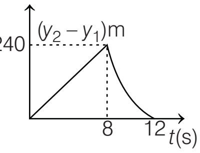
(b)



(c)



(d)



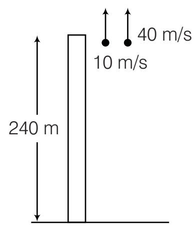
Ans. ***(b)***

Key Idea Concept of relative motion can be applied to predict the nature of motion of one particle with respect to the other.

Consider the stones thrown up simultaneously as shown in the diagram below.

Considering motion of the second particle with respect to the first we have relative acceleration

$|a_{21}| = |a_2 - a_1| = g - g = 0$



Thus, motion of first particle is straight line with respect to second particle till the first particle strikes ground at a time given by

$$-240 = 10t - \frac{1}{2} \times 10 \times t^2$$

or $t^2 - 2t - 48 = 0$

or $t^2 - 8t + 6t - 48 = 0$

or $t = 8, -6$ (not possible)

Thus, distance covered by second particle with respect to first particle in 8 s is

$$s_{12} = (v_{21})\,t = (40 - 10)\,(8\text{s})$$
$$= 30 \times 8 = 240\,\text{m}$$

Similarly, time taken by second particle to strike the ground is given by

$$-240 = 40t - \frac{1}{2} \times 10 \times t^2$$

or $\quad -240 = 40t - 5t^2$

or $\quad 5t^2 - 40t - 240 = 0$

or $\quad t^2 - 8t - 48 = 0$

$\quad t^2 - 12t + 4t - 48 = 0$

or $\quad t(t-12) + 4(t-12) = 0$

or $\quad t = 12, -4 \quad$ (not possible)

Thus, after 8 s, magnitude of relative velocity will increase upto 12 s when second particle strikes the ground.

26 From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle to hit the ground is n times that taken by it to reach the highest point of its path. The relation between H, u and n is **[JEE Main 2014]**

(a) $2gH = n^2 u^2$

(b) $gH = (n-2)^2 u^2$

(c) $2gH = nu^2 (n-2)$

(d) $gH = (n-2)^2 u^2$

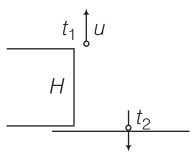
Ans. ***(c)***

Time taken to reach the maximum height

$$t_1 = \frac{u}{g}$$

If t_2 is the time taken to hit ground

i.e., $\quad -H = ut_2 - \frac{1}{2} gt^2$



But $\quad t_2 = nt_1 \quad$ [given]

So, $\quad -H = u\frac{nu}{g} - \frac{1}{2} g \frac{n^2u^2}{g^2}$

$$-H = \frac{nu^2}{g} - \frac{1}{2}\frac{n^2u^2}{g}$$

$$2gH = nu^2 (n-2)$$

27 A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 ms^{-2}. He reaches the ground with a speed of 3 ms^{-1}. At what height, did he bail out? **[AIEEE 2005]**

(a) 91 m (b) 182 m

(c) 293 m (d) 111 m

Ans. ***(c)***

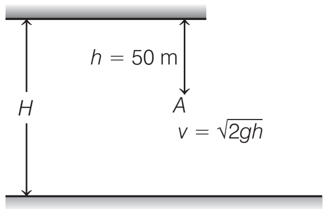
Parachute bails out at height H from groun(d) Velocity at A

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 50}$$

$$= \sqrt{980}\ \text{ms}^{-1}$$

Velocity at ground, $v_1 = 3\ \text{ms}^{-1}$ [given]

Acceleration $= -2\ \text{ms}^{-2}$ [given]



Ground

$\therefore \quad H - h = \frac{v^2 - v_1^2}{2 \times 2} = \frac{980 - 9}{4}$

$$= \frac{971}{4} = 242.75$$

$\Rightarrow \quad H = 242.75 + h$

$$= 242.75 + 50 \approx 293 \text{ m}$$

28 A ball is released from the top of a tower of height h metre. It takes T second to reach the groun(d) What is the position of the ball in $T/3$ second? **[AIEEE 2004]**

(a) $h/9$ m from the ground

(b) $7h/9$ m from the ground

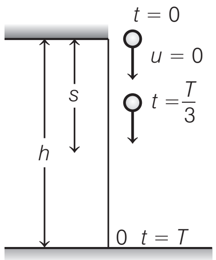
(c) $8h/9$ m from the ground

(d) $17h/18$ m from the ground

Ans. ***(c)***

Second law of motion gives

$$s = ut + \frac{1}{2} gT^2$$



Ground

or $\quad h = 0 + \frac{1}{2} gT^2$

$\Rightarrow \quad T = \sqrt{\left(\frac{2h}{g}\right)} \quad [\because u = 0]$

At $\quad t = \frac{T}{3}$ s, $s = 0 + \frac{1}{2} g\left(\frac{T}{3}\right)^2$

or $\quad s = \frac{1}{2} g \cdot \frac{T^2}{9}$

$\Rightarrow \quad s = \frac{g}{18} \times \frac{2h}{g}$

or $\quad s = \frac{h}{9}$ m $\quad \left[\because T = \sqrt{\frac{2h}{g}}\right]$

Hence, the position of ball from the ground $= h - \frac{h}{9} = \frac{8h}{9}$ m

29 An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20 m. If the car is going twice as fast, *i.e*, 120 km/h, the stopping distance will be **[AIEEE 2004]**

(a) 20 m

(b) 40 m

(c) 60 m

(d) 80 m

Ans. ***(d)***

The braking retardation will remain same and assumed to be constant, let it be a.

From third equation of motion,

$v^2 = u^2 + 2as$

Case I $\quad 0 = \left(60 \times \frac{5}{18}\right)^2 - 2a \times s_1$

$\Rightarrow \quad s_1 = \frac{(60 \times 5/18)^2}{2a}$

Case II $\quad 0 = \left(120 \times \frac{5}{18}\right)^2 - 2a \times s_2$

$\Rightarrow \quad s_2 = \frac{(120 \times 5/18)^2}{2a}$

$\therefore \quad \frac{s_1}{s_2} = \frac{1}{4} \Rightarrow s_2 = 4s_1 = 4 \times 20 = 80$ m

30 A car moving with a speed of 50 km/h, can be stopped by brakes after atleast 6 m. If the same car is moving at a speed of 100 km/h, the minimum stopping distance is **[AIEEE 2003]**

(a) 12 m (b) 18 m

(c) 24 m (d) 6 m

Ans. ***(c)***

As the first equation of motion

$v^2 = u^2 + 2as$

$\Rightarrow \quad 0 = \left(50 \times \frac{5}{18}\right)^2 + 2a \times 6$

or $\quad a = -16\ \text{m/s}^2 \quad [\because a$ is retardation$]$

Again $\quad v^2 = u^2 + 2as$

$\Rightarrow \quad 0 = \left(100 \times \frac{5}{18}\right)^2 - 16 \times 2 \times s$

or $\quad s = \frac{(100 \times 5)^2}{18 \times 18 \times 32}$ or $s = 24.1 \approx 24$ m

31 Speeds of two identical cars are u and $4u$ at a specific instant. The ratio of the respective distances at which the two cars are stopped from that instant is **[AIEEE 2002]**

(a) 1:1 (b) 1:4

(c) 1:8 (d) 1:16

Ans. ***(d)***

In this question, the cars are identical means coefficient of friction between the tyre and the ground is same for both the cars, as a result retardation is same for both the cars equal to μg.

Let first car travel distance s_1, before stopping while second car travel distance s_2, then from

$$v^2 = u^2 - 2as$$

$$\Rightarrow \quad 0 = u^2 - 2\mu g \times s_1$$

$$\Rightarrow \quad s_1 = \frac{u^2}{2\mu g}$$

and $\quad 0 = (4u)^2 - 2\mu g \times s_2$

$$\Rightarrow \quad s_2 = \frac{16u^2}{2\mu g} = 16 s_1$$

$$\therefore \quad \frac{s_1}{s_2} = \frac{1}{16}$$

32 From a building, two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then **[AIEEE 2002]**

(a) $v_B > v_A$

(b) $v_A = v_B$

(c) $v_A > v_B$

(d) their velocities depend on their masses

Ans. ***(b)***

From conservation of energy,

Potential energy at height h = Kinetic energy at ground

Therefore, at height h, PE of ball A

$$= m_A gh$$

$$\text{KE at ground} = \frac{1}{2} m_A v_A^2$$

So, $\quad m_A gh = \frac{1}{2} m_A v_A^2$

or $\quad v_A = \sqrt{2gh}$

Similarly, $\quad v_B = \sqrt{2gh}$

Therefore, $v_A = v_B$

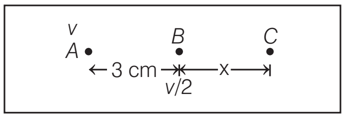
Note In this question, it is not mentioned that magnitude of thrown velocity of both balls are same which is assumed in solution.

33 If a body losses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest? **[AIEEE 2002]**

(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm

Ans. ***(a)***

Let initial velocity of body at point A be v, AB is 3 cm.



From $\quad v^2 = u^2 - 2as$

$$\Rightarrow \quad \left(\frac{v}{2}\right)^2 = v^2 - 2a \times 3 \text{ or } a = \frac{v^2}{8}$$

Let on penetrating 3 cm in a wooden block, the body moves x distance from B to C.

So, for B to C, $\quad u = v/2, \quad v = 0,$

$$s = x, a = \frac{v^2}{8} \qquad \text{[deceleration]}$$

$$\therefore \quad (0)^2 = \left(\frac{v}{2}\right)^2 - 2 \cdot \frac{v^2}{8} \cdot x$$

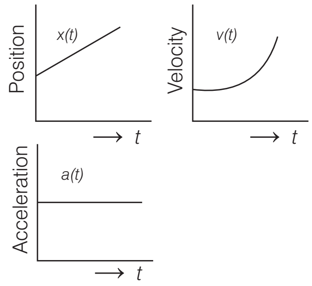
or $\quad x = 1\text{cm}$

Note Here, it is assumed that retardation is uniform.

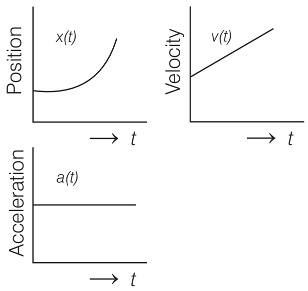
TOPIC 3
Graphs in Motion

34 The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by **[2021, 18 March Shift-I]**

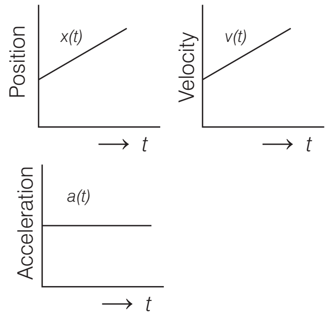
(a)



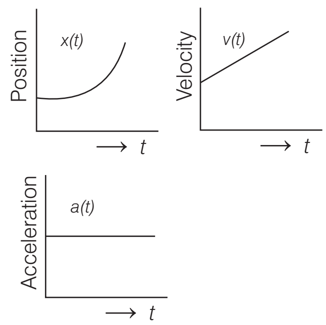
(b)



(c)



(d)

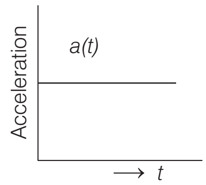


Ans. ***(b)***

Since, the particle is moving with constant acceleration.

Acceleration = Constant, (the acceleration-time graph is parallel to the time axis, and acceleration is positive)

$$a = \frac{dv}{dt}$$



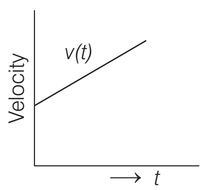
As we know,

$$\frac{dv}{dt} = C \Rightarrow \int dv = C \int dt$$

$$\Rightarrow \quad v = Ct + C_0$$

The velocity-time graph is straight line with positive slope

$$v = \frac{dx}{dt}$$



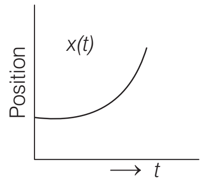
As we know,

$$\frac{dx}{dt} = Ct + C_0$$

$$\Rightarrow \quad \int dx = \int (Ct + C_0)\, dt$$

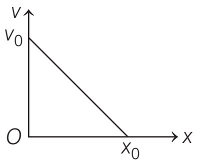
$$\Rightarrow \quad x = \frac{Ct^2}{2} + C_0 t + C_1$$

This equation shows that the distance-time graph is upward concave parabola.



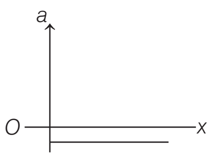
Hence, the correct option is (b).

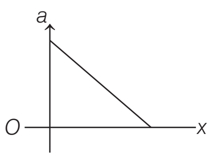
35 The velocity- displacement graph of a particle is shown in the figure.

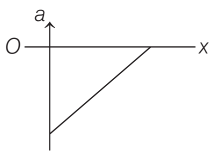


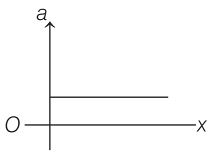
The acceleration-displacement graph of the same particle is represented by

[2021, 18 March Shift-II]

(a)


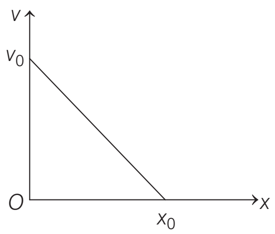
(b)


(c)


(d)


Ans. ***(c)***

For the given velocity-displacement graph,



Intercept, $c = v_0$

Slope, $m = -\frac{v_0}{x_0}$

$\therefore$ Slope, $m = \frac{dv}{dx} = -\frac{v_0}{x_0}$ ...(i)

For given diagram, the equation of line,

$$y = mx + c$$

$$v = -\frac{v_0}{x_0}x + v_0 \quad \text{...(ii)}$$

Acceleration, $a = \frac{dv}{dx} \times \frac{dx}{dt}$

$\Rightarrow \quad a = v\frac{dv}{dx}$

$\Rightarrow \quad a = \left(-\frac{v_0}{x_0}x + v_0\right)\left(-\frac{v_0}{x_0}\right)$

[From Eqs. (i) and (ii)]

$\Rightarrow \quad a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$

Hence, the intercept is negative and the slope is positive, so the correct graph is given in option (c).

36 A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t seconds, the total distance travelled is

[2021, 17 March Shift-I]

(a) $\frac{4\alpha\beta}{(\alpha+\beta)}t^2$ (b) $\frac{2\alpha\beta}{(\alpha+\beta)}t^2$

(c) $\frac{\alpha\beta}{2(\alpha+\beta)}t^2$ (d) $\frac{\alpha\beta}{4(\alpha+\beta)}t^2$

Ans. ***(c)***

Consider 1st case,

A car starts from rest at a constant acceleration α. It means

Acceleration, $a_1 = \alpha$

initial velocity, $u = 0$

$$v = u + a_1 t \Rightarrow v_0 = 0 + \alpha t_1$$

$\Rightarrow \quad v_0 = \alpha t_1$ ...(i)

Consider 2nd case,

The same car decelerates after some time at a constant acceleration β and comes to rest. It means

Acceleration, $a_2 = \beta$

Final velocity $= 0$

$\Rightarrow \quad v_0 = \beta t_2$ ...(ii)

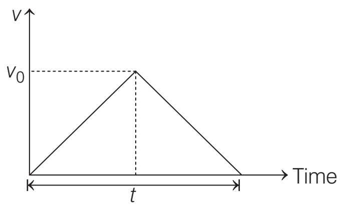
According to question,

$$t_1 + t_2 = t$$

$\Rightarrow \quad v_0\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = t$ [from Eqs. (i) & (ii)]

$\Rightarrow \quad v_0 = \frac{\alpha\beta t}{\alpha+\beta}$ ...(iii)

If we draw the velocity-time graph for the given situation, it will be as follows

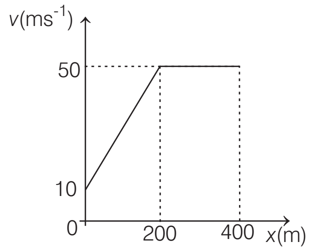


$\therefore$ Distance travelled = Area of v - t graph

$= \frac{1}{2} \times t \times v_0 = \frac{1}{2} \times t \times \frac{\alpha\beta t}{(\alpha+\beta)}$ [from Eq. (iii)]

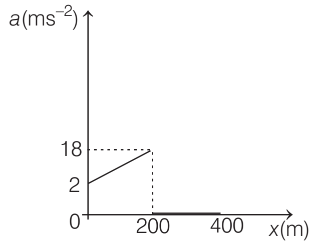
$= \frac{1}{2}\frac{\alpha\beta t^2}{(\alpha+\beta)} = \frac{\alpha\beta t^2}{2(\alpha+\beta)}$

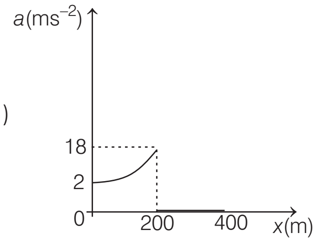
37 The velocity-displacement graph describing the motion of a bicycle is shown in the following figure.

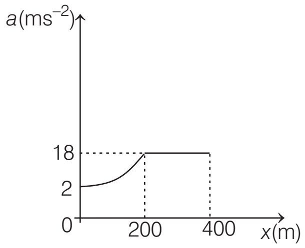


The acceleration-displacement graph of the bicycle's motion is best described by

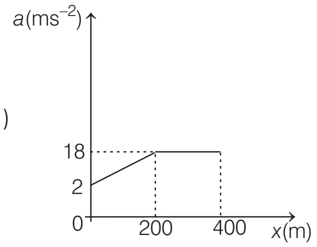
[2021, 16 March Shift-I]

(a)


(b)


(c)


(d)



Ans. *(a)*

From the given velocity-displacement graph, for $0 \le x \le 200$, the graph is a straight line i.e., varying linearly.

As, the equation of straight line is

$$y = mx + c$$

Here, it can be given as $v = mx + c$

where, $m = \text{slope} = \dfrac{(50-10)}{200-0} = \dfrac{1}{5}$

and $c = 10$

$$\Rightarrow \quad v = \frac{x}{5} + 10 \quad \text{...(i)}$$

As, acceleration, $a = dv/dt = vdv/dx$

$$a = \left(\frac{x}{5} + 10\right)\frac{d}{dx}\left(\frac{x}{5} + 10\right) \quad \text{[using Eq. (i)]}$$

$$= \left(\frac{x}{5} + 10\right)\left(\frac{1}{5}\right) = 2 + \frac{x}{25}$$

For $x = 0, a = 2 + \dfrac{0}{25} = 2\,\text{ms}^{-2}$

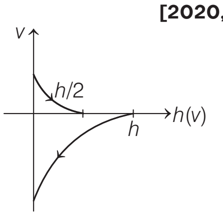
and for $x = 200, a = 2 + \dfrac{200}{25} = 10\,\text{ms}^{-2}$

Therefore, the graph will have straight line till $x = 200$ and for $x > 200$;
v = constant and $a = 0$

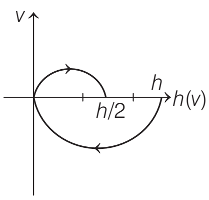
So, from the graphs given in the options, the above conditions are satisfied in option (a) only.

38 A tennis ball is released from a height h and after freely falling on a wooden floor, it rebounds and reaches height $h/2$. The velocity *versus* height of the ball during its motion may be represented graphically by (Graphs are drawn schematically and on not to scale) **[2020, 4 Sep Shift-I]**

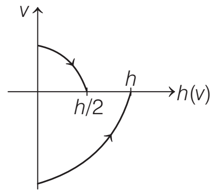
(a)



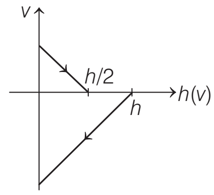
(b)



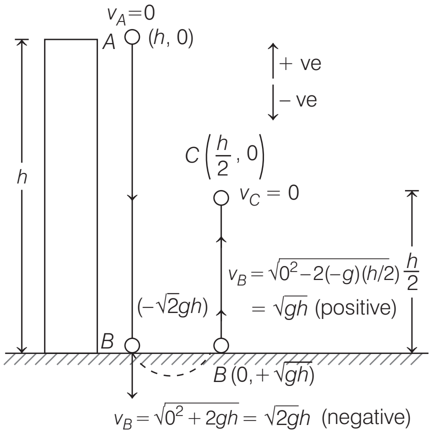
(c)



(d)



Ans. *(c)*

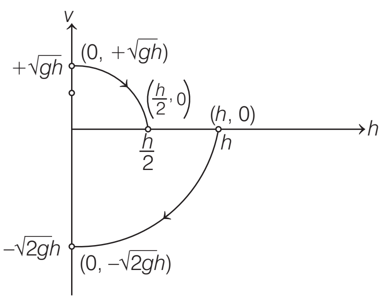


Using third equation of motion, the relation between v and h is given by,

$$v^2 = u^2 + 2gh$$
$$= 0^2 + 2gh = 2gh$$
$$\Rightarrow \quad v^2 \propto h$$

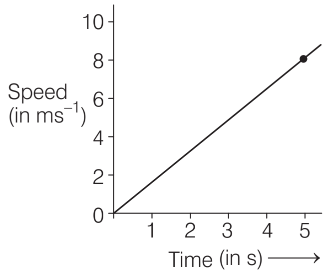
It is the equation of a parabol(a) So, the shape of v *versus* h graph will be paraboli(c).

Now, as we have some co-ordinate points from diagram. On plotting them and tracing with a parabolic locus, we will get the following graph :



Hence, correct option is (c).

39 The speed *versus* time graph for a particle is shown in the figure. The distance travelled (in metre) by the particle during the time interval $t = 0$ s to $t = 5$ s will be .... **[2020, 4 Sep Shift-II]**



Ans. *(20)*

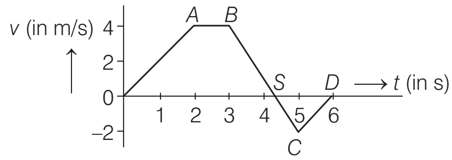
Distance travelled = Area under speed-time graph

Distance travelled (from $t = 0$ s to $t = 5$ s)

$= \text{Area of } \Delta\, OAB = \dfrac{1}{2} \times \text{Base} \times \text{Height}$

$= \dfrac{1}{2} \times (5\text{ s}) \times (8\text{ m/s}) = 20\text{ m}$

40 The v-t graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 s. The total distance covered by the body in 6 s is **[2020, 5 Sep Shift-II]**

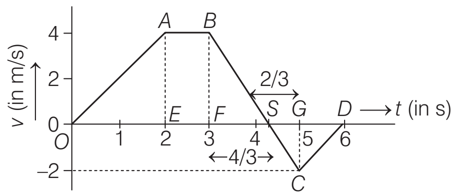


(a) $\dfrac{37}{3}$ m (b) 12 m

(c) 11 m (d) $\dfrac{49}{4}$ m

Ans. *(a)*

The given v-t graph is shown below,



Distance covered by the body in 6s = Sum of magnitudes of area of each part in v-t curve

s = Area of ΔAOE + Area of rectangle $ABFE$ + Area of ΔBSF + Area of ΔSCG + Area of ΔGCD

$$= \frac{1}{2}(OE \times AE) + (EF \times AE) + \frac{1}{2}(SF \times BF) + \frac{1}{2}(SG \times GC) + \frac{1}{2}(GD \times GC)$$

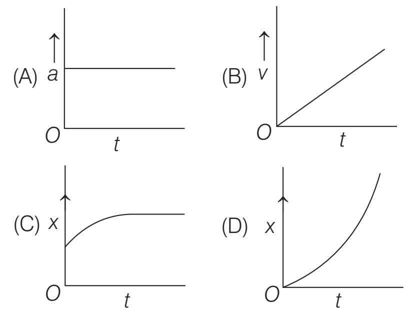
$$= \frac{1}{2}(2 \times 4) + 1 \times 4 + \frac{1}{2} \times 4 \times \left(\frac{4}{3}\right) + \frac{1}{2} \times 2 \times \left(\frac{2}{3}\right) + \frac{1}{2} \times 2 \times 1$$

$$= 4 + 4 + \frac{8}{3} + \frac{2}{3} + 1 = \frac{37}{3} \text{ m}$$

Hence, correct option is (a).

41 A particle starts from origin O from rest and moves with a uniform acceleration along the positive X-axis. Identify all figures that correctly represent the motion qualitatively. (a = acceleration, v = velocity, x = displacement, t = time)

[2019, 8 April Shift-II]



(a) (A) (b) (A), (B), (C)
(c) (B), (C) (d) (A), (B), (D)

Ans. *(d)*

Since, the particle starts from rest, this means, initial velocity, $u = 0$

Also, it moves with uniform acceleration along positive X-axis. This means, its acceleration (a) is constant.

$\therefore$ Given, a - t graph in (A) is correct.

As we know, for velocity-time graph, slope = acceleration.

Since, the given v-t graph in (B) represents that its slope is constant and non-zero.

$\therefore$ Graph in (B) is also correct.

Also, the displacement of such a particle w.r.t. time is given by

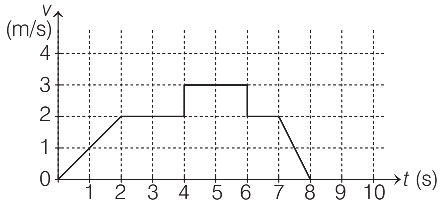
$$x = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2}at^2 \Rightarrow x \propto t^2$$

So, x *versus* t graph would be a parabola with starting from origin. This is correctly represented in displacement-time graph given in (D).

42 A particle starts from the origin at time $t = 0$ and moves along the positive X-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time $t = 5$ s?

[2019, 10 Jan Shift-II]

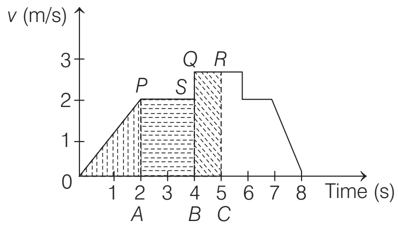


(a) 6 m (b) 3 m (c) 10 m (d) 9 m

Ans. *(d)*

Key Idea Area under the velocity-time curve represents displacement.

To get exact position at $t = 5$ s, we need to calculate area of the shaded part in the curve as shown below



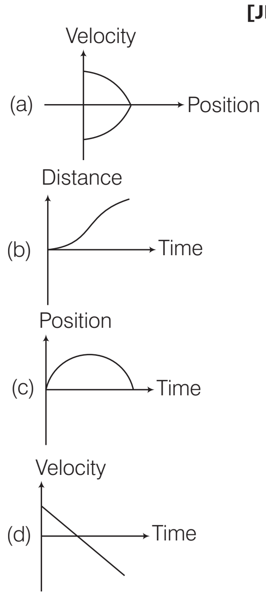
$\therefore$ Displacement of particle = Area of OPA + Area of $PABSP$ + Area of $QBCRQ$

$= (1/2 \times 2 \times 2) + (2 \times 2) + (3 \times 1)$

$= 2 + 4 + 3 = 9$ m

43 All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

[JEE Main 2018]



Ans. *(b)*

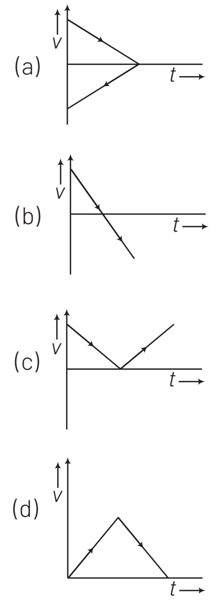
If velocity versus time graph is a straight line with negative slope, then acceleration is constant and negative.

With a negative slope distance-time graph will be parabolic $\left(s = ut - \frac{1}{2}at^2\right)$.

So, option (b) will be incorrect.

44 A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity *vs* time?

[JEE Main 2017]

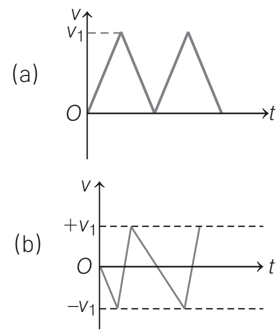


Ans. *(b)*

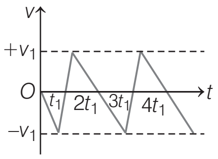
Initially velocity keeps on decreasing at a constant rate, then it increases in negative direction with same rate.

45 Consider a rubber ball freely falling from a height $h = 4.9$ m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then, the velocity as a function of time the height as function of time will be

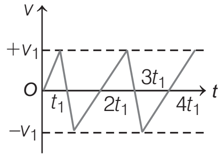
[AIEEE 2009]



(c)



(d)



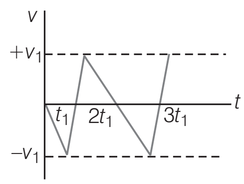
Ans. ***(c)***

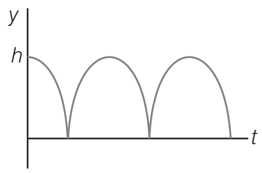
As we know that for vertical motion,

$$h = \frac{1}{2} gt^2 \qquad [\text{parabolic}]$$

$v = -gt$ and after collision, $v = gt$ (straight line).

Collision is perfectly elastic, then ball reaches to same height again and again with same velocity.

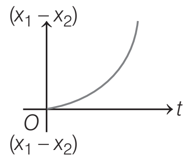




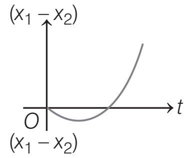
Hence, option (c) is true.

46 A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant, another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time? **[AIEEE 2008]**

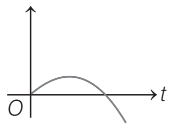
(a)



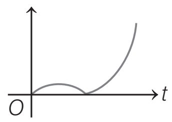
(b)



(c)



(d)



Ans. ***(b)***

Here, $x_2 = vt$ and $x_1 = \dfrac{at^2}{2}$

$\therefore \quad x_1 - x_2 = -\left(vt - \dfrac{at^2}{2}\right)$

From the above expression, it is clear that at $t = 0$, $x_1 - x_2 = 0$ further for increasing values, the graph is as follows.

Hence, option (c) is true.

47 A car starting from rest, accelerates at the rate f through a distance s, then continues at constant speed for time t and then decelerates at the rate $f/2$ to come to rest. If the total distance travelled is 15 s, then **[AIEEE 2005]**

(a) $s = ft$

(b) $s = \dfrac{1}{6} ft^2$

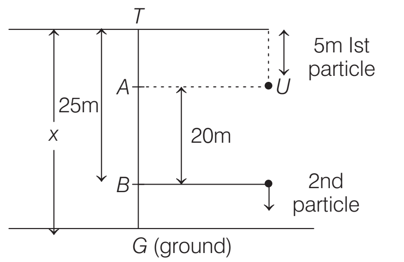
(c) $s = \dfrac{1}{2} ft^2$

(d) None of the above

Ans. ***(d)***

The velocity-time graph for the given situation can be drawn as below.

Magnitudes of slope of $OA = f$



and slope of $BC = \dfrac{f}{2}$

$\Rightarrow \qquad v = ft_1 = \dfrac{f}{2} t_2$

$\therefore \qquad t_2 = 2t_1$

In graph, area of ΔOAD gives distance,

$$s = \frac{1}{2} ft_1^2 \qquad \text{...(i)}$$

Area of rectangle $ABED$ gives distance travelled in time t.

$$s_2 = (ft_1)t$$

Distance travelled in time t_2,

$$s_3 = \frac{1}{2}\frac{f}{2}(2t_1)^2$$

Thus, $s_1 + s_2 + s_3 = 15s$

$\Rightarrow \quad s + (ft_1)t + ft_1^2 = 15s$

or $\quad s + (ft_1)t + 2s = 15s \quad \left[\because s = \dfrac{1}{2} ft_1^2\right]$

or $\qquad (ft_1)t = 12s \qquad$ ...(ii)

From Eqs. (i) and (ii), we have

$$\frac{12s}{s} = \frac{(ft_1)\,t}{\frac{1}{2}(ft_1)\,t_1}$$

or $\qquad t_1 = \dfrac{t}{6}$

From Eqs. (i) and (ii), we get

$$s = \frac{1}{2} f(t_1)^2$$

$\Rightarrow \qquad s = \dfrac{1}{2} f\left(\dfrac{t}{6}\right)^2 = \dfrac{1}{72} ft^2$

Hence, none of the given options is correct.