Chapter 1
Electric Charges and Fields

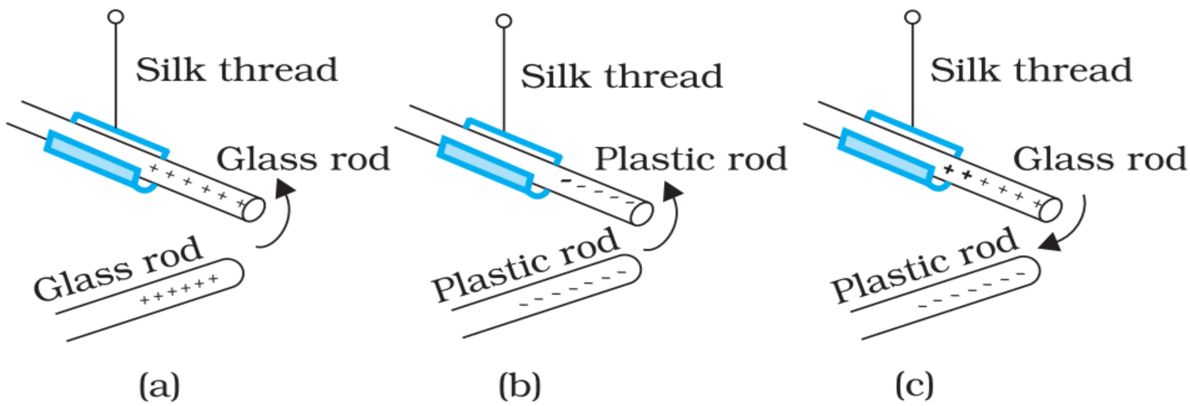
Introduction

A common example of electric discharge is the lightning that we see in the sky during thunderstorms. We also experience a sensation of an electric shock either while opening the door of a car or holding the iron bar of a bus after sliding from our seat. The reason for these experiences is discharge of electric charges through our body, which were accumulated due to rubbing of insulating surfaces. This is due to generation of static electricity. Static means anything that does not move or change with time.

Electrostatics deals with the study of forces, fields and potentials arising from static charges.

Electric charge

Historically the credit of discovery of the fact that amber rubbed with wool or silk cloth attracts light objects goes to Thales of Miletus, Greece, around 600 BC. The name electricity is coined from the Greek word **elektron meaning amber.**



If Two glass rods rubbed with wool or silk cloth are brought close to each other, they repel each other [Fig. 1.1(a)]. Similarly, two plastic rods rubbed with cat's fur repelled each other [Fig. 1.1(b)] . On the other hand, the plastic rod attracts the glass rod [Fig. 1.1(c)] .

There are two kinds of electrification and we find that

(i) like charges repel and

(ii) unlike charges attract each other.

The property which differentiates the two kinds of charges is called the polarity of charge.

The charges were named as positive and negative by the American scientist Benjamin Franklin.

On rubbing electrons are transferred from one body to the other. The body, which loses electrons, will become positively charged and which gains electrons becomes negatively charged.

- When a glass rod is rubbed with silk, glass rod becomes positively charged and silk negative.
- When a plastic rod is rubbed with fur, plastic rod becomes negatively charged and fur positive.

Conductors and Insulators

Conductors

Conductors are those substances which allow passage of electricity through them. Eg. Metals, human and animal bodies and earth are conductors.

- They have electric charges (electrons) that are comparatively free to move inside the material.
- When some charge is transferred to a conductor, it readily gets distributed over the entire surface of the conductor.
- Metals cannot be charged by friction,because the charges transferred to the metal leak through our body to the ground as both are conductors of electricity.

Insulators

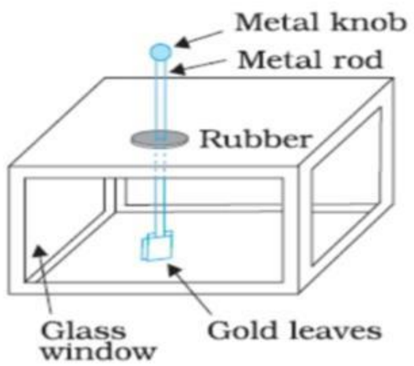
The substances which offer high resistance to the passage of electricity through them are called Insulators . Eg. glass, porcelain, plastic, nylon, wood

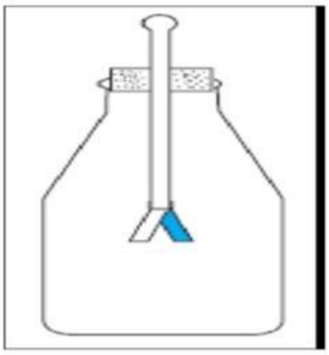
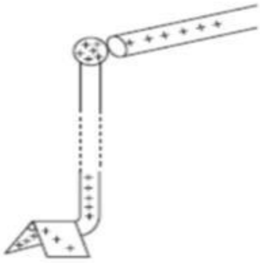
- If some charge is put on an insulator, it stays at the same place. So insulators gets electrified on combing dry hair or on rubbing.

Gold Leaf Electroscope

A simple apparatus to detect charge on a body is called a gold-leaf electroscope.

Apparatus It consists of a vertical metal rod placed in a box. Two thin gold leaves are attached to its bottom end as shown in figure.



Working

When a charged object touches the metal knob at the top of the rod, charge flows on to the leaves and they diverge. The degree of divergence is an indicator of the amount of charge.

Basic properties of electric charges

1. Quantization of charge : According to quantisation of electric charge, charge of a body is an integral multiple of a basic charge, which is the electronic charge.

Charge on a body, $q = \pm ne$; where, $n = 1,2,3.........$

e is the electronic charge. $e = 1.602 \times 10^{-19}$ C

2. Charge is conserved: It means that total charge of an isolated system remains constant. It also implies that electric charges can neither be created nor destroyed. If an object loses some charge, an equal amount of charge appears somewhere else.

3. Additivity of charge: The total charge on a surface is the algebraic sum of individual charges present on that surface.

If q_1, q_2, q_3......................, q_n are the charges on a surface, then total or net charge,

$$q = q_1 + q_2 + q_3 + \dots\dots + q_n$$

Example 1

How many electronic charges form 1 C of charge?

$$q = \pm ne,$$

$$n = \frac{q}{e}$$

$$n = \frac{1}{1.602 \times 10^{-19}}$$

$$= 6.25 \times 10^{18}$$

Example 2

A comb drawn through person's hair causes 10^{22} electrons to leave the person's hair and stick to the comb. Calculate the charge carried by the comb.

$$q = ne,$$

$$q = 10^{22} \text{ x } 1.602 \text{ x } 10^{-19}\text{C} \quad = -1.602 \text{ x } 10^{3} \text{ C}$$

As the comb gains electrons it gets negatively charged.

Coulomb's Law

The force of attraction or repulsion between two stationery electric charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

If charges are placed in free space, $\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

If charges are placed in a medium, $\mathbf{F} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1 q_2}{r^2}$

- Where ε_0 - permittivity of free space, $\varepsilon_0 = 8.854\text{x}10^{-12}\text{C}^2\text{N}^{-1}\text{m}^{-2}$

  ε_r - relative permittivity.

  $\varepsilon = \varepsilon_0 \varepsilon_r \qquad \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$

  ε - Permittivity of the medium.

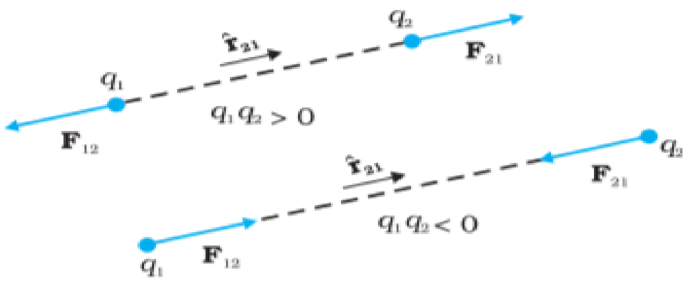
Thus $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}\text{ C}^{-2}\text{N}^{1}\text{m}^{2}$

Definition of coulomb

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

- When $q_1 = q_2 = 1$ C, r = 1 m , $F = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}$ N
- 1 C is the charge that when placed at a distance of 1 m from another charge of the same magnitude in vacuum experiences an electrical force of repulsion of magnitude 9×10^{9} N.

Coulomb's Law in vector form



$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

$$\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{21}^2}\hat{\mathbf{r}}_{21}$$

Thus $F_{12} = -F_{21}$, Coulomb's law agrees with Newton's third law.

Super position principle

Force on a charge due to a number of charges is the vector sum of forces due to individual charges.For a system of n charges.

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \ldots + \mathbf{F}_{1n} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13} + \ldots + \frac{q_1 q_n}{r_{1n}^2}\hat{\mathbf{r}}_{1n}\right]$$

Electric Field

Electric field is the region around a charge where its effect can be felt.

Intensity of electric field at a point is the force per unit charge.

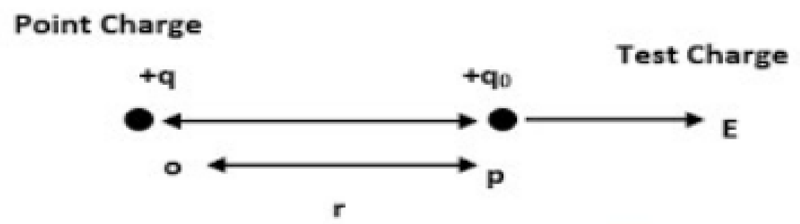
$$\mathbf{E} = \frac{\mathbf{F}}{\mathbf{q}}$$

F= qE

Unit of electric field is N/C or V/m.

It is a vector quantity.

Electric field due to a point charge



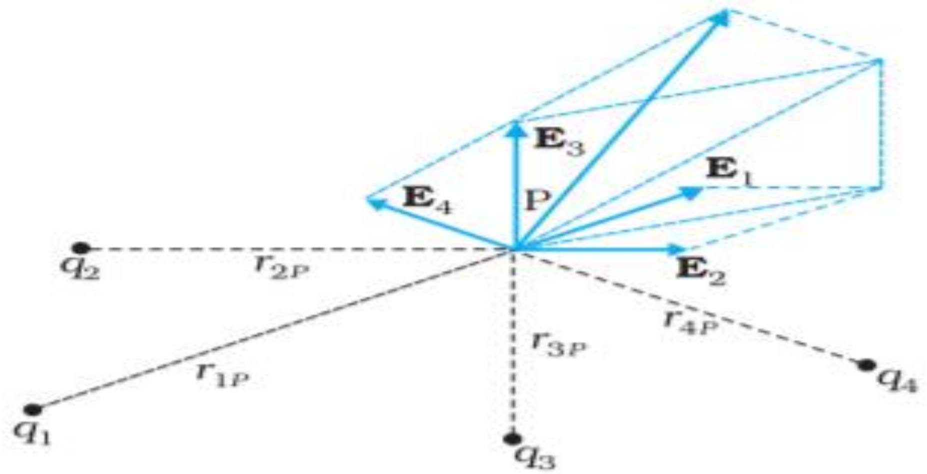
By Coulomb's law, $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

$$E = \frac{F}{q_0}$$

$$\mathbf{E} = \frac{\mathbf{1}}{\mathbf{4\pi\varepsilon_0}}\frac{\mathbf{q}}{\mathbf{r^2}}$$

Electric field due to a system of charges

Electric field at a point due to a system of charges is the vector sum of the electric fields at the point due to individual charges.



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots\ldots\ldots\ldots + \vec{E}_n$$

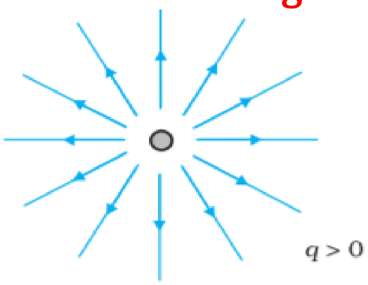
$$= \frac{1}{4\Pi\,\epsilon_0}\frac{q_1}{r_{1p}^2}\,\hat{r_{1p}} + \frac{1}{4\Pi\,\epsilon_0}\frac{q_2}{r_{2p}^2}\,\hat{r_{2p}} + \ldots\ldots + \frac{1}{4\Pi\,\epsilon_0}\frac{q_n}{r_{np}^2}\,\hat{r_{np}}$$

Electric Field Lines

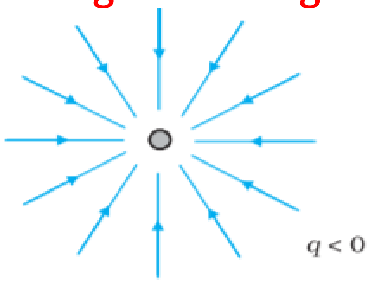
An electric field line is a curve drawn in such a way that the tangent to it at each point is in the direction of the net field at that point.

- Electric Field lines tart from positive charge, end at negative charge.
- Electric field lines of a positive charge are radially outwards and that of a negative charge is radially inwards
- Electric field lines do not form closed loops.
- In a charge free region field lines are continuous.
- Two field lines never intersect.(Two directions for electric field is not possible at a point)
- Field lines are parallel ,equidistant and in same direction in uniform electric field.

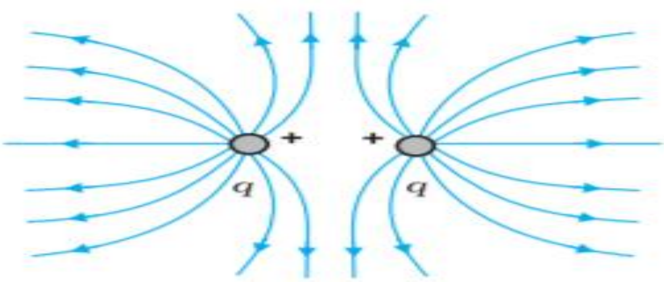
Positive Charge



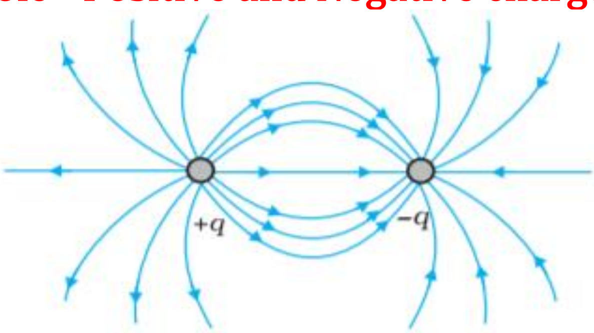
Negative Charge



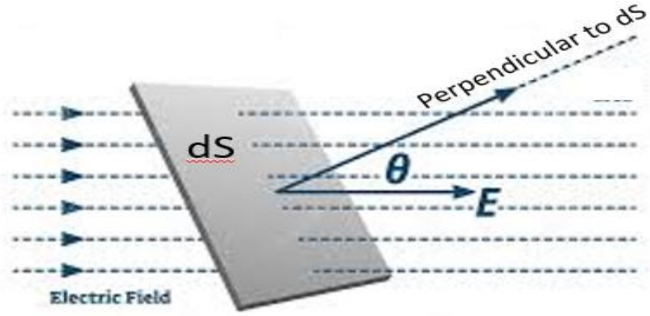
Two positive Charges



Dipole - Positive and Negative charge



Electric Flux



The electric flux associated with a surface is the number of electric field lines passing normal through a surface.

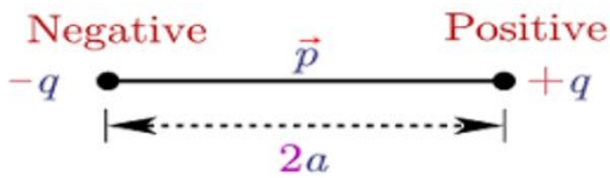
$$\phi = \int \mathbf{E} \cdot \mathbf{dS}$$

$$\phi = \int E\, dS \cos\theta$$ (θ is the angle between E and normal to dS)

- Unit – Nm^2 / C
- It is a scalar quantity

Electric Dipole

An electri dipole is a pair of equal and opposite charges separated by a distance



The total charge of the system is +q + -q =0

Electric Dipole moment ($\vec{p}$)

Electric Dipole moment(p) is the product of magnitude of one of the charges and the distance between charges.

$$p = q \times 2a$$

q- magnitude of charge

2a- distance between the charges or dipole length

Unit of dipole moment is Cm

Dipole moment is a vector quantity.

Dipole moment is directed from the negative charge to the positive charge along the dipole axis.

Physical significance of electric dipole

Non Polar molecules

In non polar molecules the centres of positive charges and negative charges lie at the same place. Dipole moment is zero for a non polar molecule in the absence of an external field.

They develop a dipole moment when an electric field is applied.

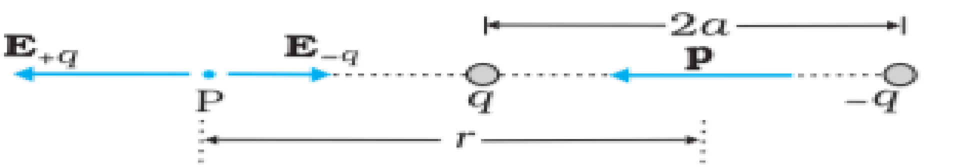
- Eg: CO_2 , CH_4

Polar molecules

The molecules in which the centres of negative charges and of positive charges do not coincide. . Therefore they have a permanent electric dipole moment, even in the absence of an electric field.

- Eg: water (H_2O)

Electric Field due to a Dipole along the Axial Line



The electric field at P due to +q

$$E_{+q} = \frac{1}{4\pi\varepsilon_0}\ \frac{q}{(r-a)^2} \qquad \text{(in the direction of dipole moment } \vec{p}\text{)}$$

The electric field at Pdue to -q

$$E_{-q} = \frac{1}{4\pi\varepsilon_0}\ \frac{q}{(r+a)^2} \qquad \text{(opposite to the direction of dipole moment } \vec{p}\text{)}$$

Total field, $\quad \mathrm{E} = E_{+q} - E_{-q}$

$$E = \frac{1}{4\pi\varepsilon_0}\ \frac{q}{(r-a)^2} - \frac{1}{4\pi\varepsilon_0}\ \frac{q}{(r+a)^2}$$

Thus the total electric field at P is

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

Simplifying

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ar}{(r^2-a^2)^2}\right]$$

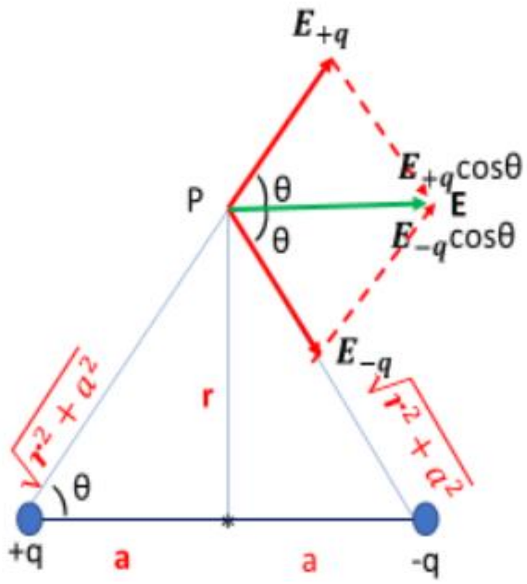
For r≫ a ,we get

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{4qa}{r^3}\right]$$

2qa=$\vec{p}$ (dipole moment)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2\vec{p}}{r^3}\right]$$

Electric Field due to a Dipole along the Equatorial Line



The magnitude of electric field at P due to +q

$$E_{+q} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2 + a^2} \text{ ----------------(1)}$$

The magnitude of electric field at P due to -q

$$E_{-q} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2 + a^2} \text{ ------------------(2)}$$

The vertical componennts cancel each other and horizontal components add up

Total electric field at P,

$$\text{E} = E_{+q}cos\theta + E_{-q}cos\theta$$

But , $E_{+q} = E_{-q}$

$$\text{E} = 2E_{+q}cos\theta \text{ --------------(3)}$$

$$\text{Cos}\theta = \frac{a}{\sqrt{r^2+a^2}} = \frac{a}{(r^2+a^2)^{1/2}} \text{ ---------------(4)}$$

Substituting eq(1) and (4) in eq(3)

$$\text{E} = 2\ x \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2 + a^2}\ x \frac{a}{(r^2+a^2)^{1/2}}$$

p=2qa (dipole moment)

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{p}{(r^2+a^2)^{3/2}}\right]$$

For r≫ a ,we get

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{\vec{p}}{r^3}\right]$$

Relation connecting Axial field and Equatorial field of a Dipole

Axial field, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2\vec{p}}{r^3}\right]$

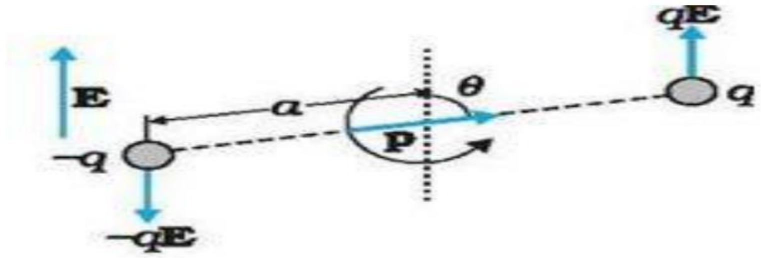
Equatorial field , $\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{\vec{p}}{r^3}\right]$

Axial field = 2 x Equatorial field

Dipole in a Uniform External field

In a uniform electric field there will be a net torque on the dipole, but the net force will be zero. Due to the torque ,the dipole rotates. There will be no translatory motion as the net force is zero.

Torque on a Dipole in a Uniform External field



Torque, τ = one of the forces x perpendicular distance between them.

$\tau = qE \times 2a \sin\theta$

$\tau = pE \sin\theta$

$$\vec{\tau} = \vec{P} \times \vec{E}$$

• When p and E are in the same direction or opposite direction(θ=0 or180)

$$\tau = pE \sin 0 = 0$$

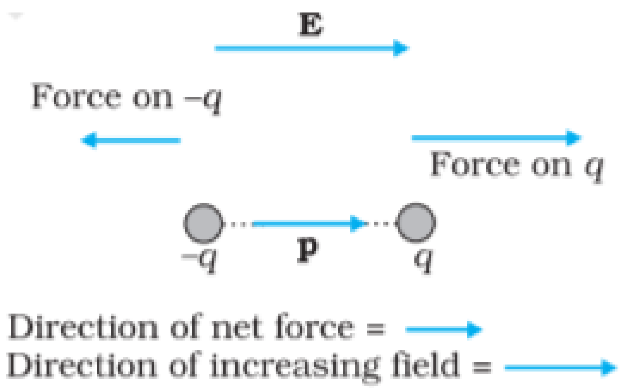
• Torque is maximum , when p and E are perpendicular. (θ=90)

$$\tau = pE \sin 90 = pE$$

Dipole in a non uniform electric field

In a non uniform electric field the dipole experiences a net force as well as a net torque in general .

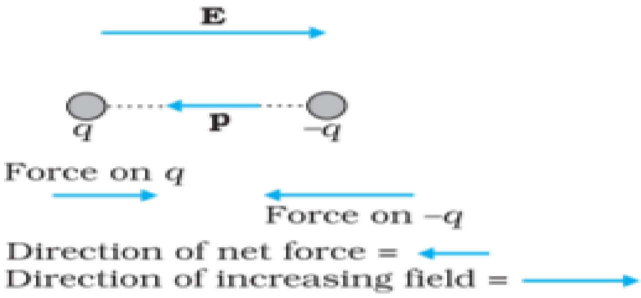
Case 1 -when p is parallel to E



when p is parallel to E, the dipole has a net force in the direction of increasing field.

But the net torque will be zero $\tau = pE \sin 0 = 0$

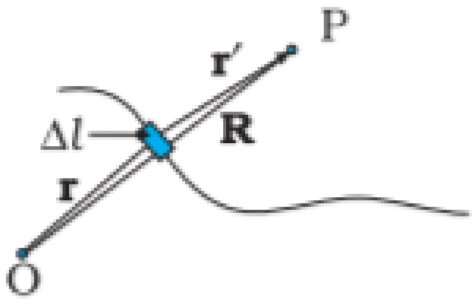
Case 2-When p is antiparallel to E.



When p is antiparallel to E, the net force on the dipole is in the direction of decreasing field.
But the net torque will be zero, τ =pE sin 180 =0

Continuous Charge Distribution

Linear charge density



The linear charge density λ of a wire is defined as

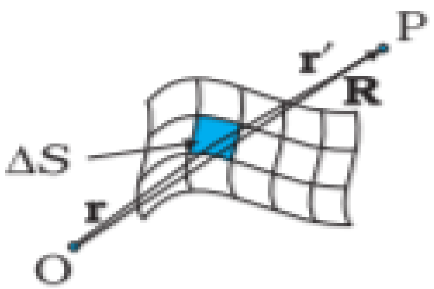
$$\lambda = \frac{\Delta q}{\Delta l}$$

$$\boldsymbol{\lambda = \frac{q}{l}}$$

The unit of λ is C/m

Line charge **q = λl**

Surface charge density



The surface charge density σ of a area element is defined as

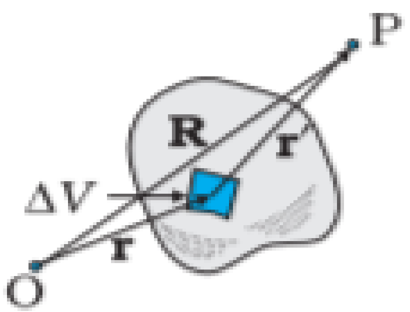
$$\sigma = \frac{\Delta q}{\Delta S}$$

$$\boldsymbol{\sigma = \frac{q}{S}}$$

The units for σ is C/m^2

Surface charge, **q= σS**

Volume charge density



The volume charge density ρ of a volume element is defined as

$$\rho = \frac{\Delta q}{\Delta V}$$

$$\boldsymbol{\rho = \frac{q}{V}}$$

The units for ρ is C/m^3

Volume charge , **q= ρV**

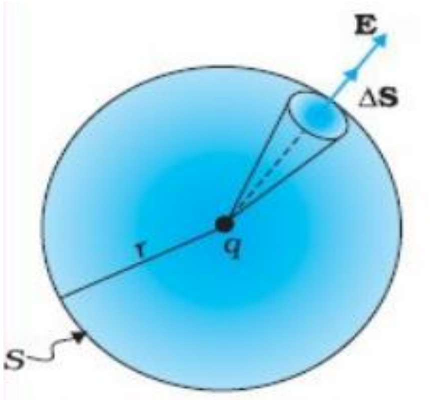
Gauss's Theorem

Gauss's theorem states that the total electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed by the surface.

$$\phi = \oint E \cdot dS = \frac{q}{\varepsilon_0}$$

The surface over which we calculate the flux is called Gaussian surface.

Proof



Consider a sphere of radius r enclosing a point charge q. the electric flux through the surface dS

$$\phi = \int E \cdot dS$$

$$\phi = \int E\, dS \cos 0 = \int E\, dS = E\int dS$$

$$\phi = ES$$

$$\phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \text{ x } 4\pi r^2$$

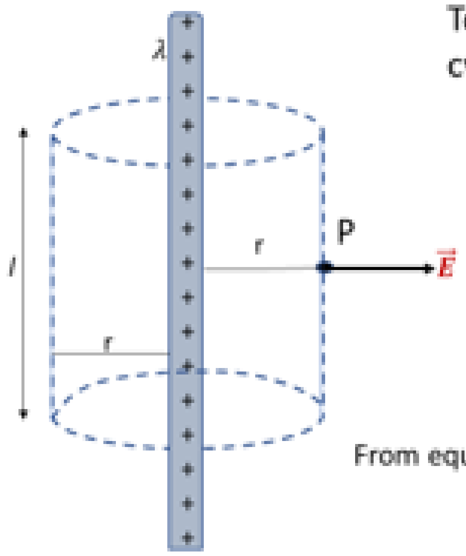
$$\phi = \frac{q}{\varepsilon_0}$$

Features of Gauss's Law

- Gauss's law is true for any surface irrespective of the size and shape.
- The charge includes the sum of all charges enclosed by the surface.
- The surface that we choose for the application of Gauss's law is called the Gaussian Surface.
- Gauss's law is applicable to both symmetric and asymmetric system, but it will be much easier if the system has some symmetry.
- Gauss's law is based on inverse square dependence on distance contained in the Coulomb's law.

Applications of Gauss's law

1) Electric field due to an infinitely long straight uniformly charged wire



To find the electric field at point P at distance r consider cylindrical Gaussian surface of radius r

$$\phi = E\,S$$

$$\phi = E \times 2\pi r l \longrightarrow (1)$$

By Gauss's law $\phi = \frac{q}{\varepsilon_0}$

$$\lambda = \frac{q}{l}$$
$$q = \lambda l$$

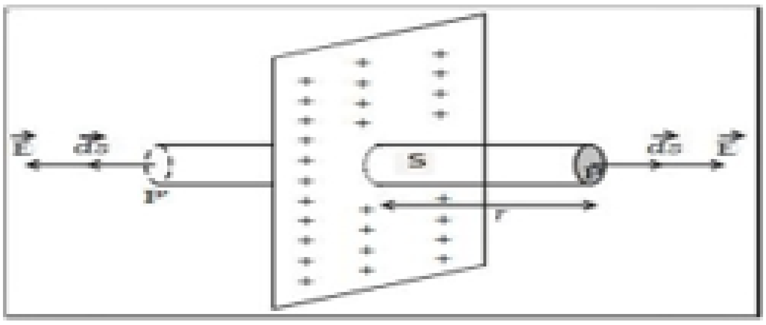
$$\phi = \frac{\lambda l}{\varepsilon_0} \longrightarrow (2)$$

From equations (1) and (2) $E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0}$

$$\boxed{E = \frac{\lambda}{2\pi r \varepsilon_0}}$$

$$E \alpha \frac{1}{r}$$

2. Electric field due to a uniformly charged infinite planesheet



The field lines crosses only the 2 end faces of the Gaussian surface.
So the flux through the Gaussian surface

$$\phi = 2\,E\,S \text{ ---------------- } (1)$$

By Gauss law

$$\phi = \frac{q}{\varepsilon_0} \qquad (\sigma = q/S)$$

$$\phi = \frac{\sigma S}{\varepsilon_0} \text{ -------------------- } (2)$$

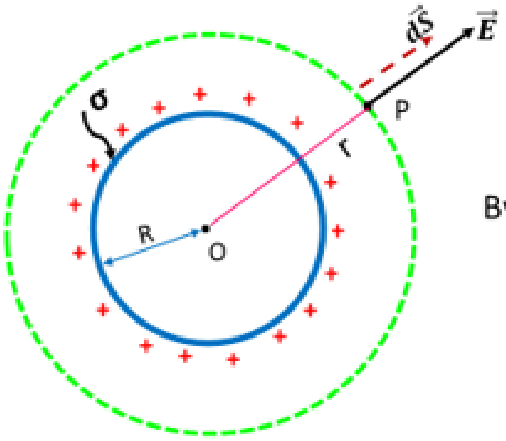
From equations (1) and (2)

$$2\,E\,S = \frac{\sigma S}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Electric field is independent of distance r

3) Electric field due to a uniformly charged thin spherical shell



a) Field outside the shell

$$\phi = ES$$

$$\phi = E \times 4\pi r^2 \longrightarrow \text{(1)}$$

By Gauss's law $\phi = \frac{q}{\varepsilon_0}$

$$\sigma = \frac{q}{A}$$

$$q = \sigma A$$

$$\phi = \frac{\sigma A}{\varepsilon_0}$$

$$\phi = \frac{\sigma \times 4\pi R^2}{\varepsilon_0} \longrightarrow \text{(2)}$$

From equations (1) and (2) $E \times 4\pi r^2 = \frac{\sigma \times 4\pi R^2}{\varepsilon_0}$

$$\boxed{E = \frac{\sigma R^2}{\varepsilon_0 r^2}}$$

$$E \propto \frac{1}{r^2}$$

b) field on the surface of the shell

On the surface of shell $r = R$

$$E = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

E is maximum at the surface of the shell

c) field inside the shell

$$\phi = ES \longrightarrow (1)$$

Inside the shell q=0

By Gauss's law $\phi = \frac{q}{\varepsilon_0} = 0 \longrightarrow$ **(2)**

From equations (1) and (2)

$$ES = 0 \qquad (S \neq 0)$$

$$\boxed{E = 0}$$

Example

Find the electric field due two plane sheets of charge in regions I ,II and III