Derivatives of Implicit Function

Q.1. If y = x log y, show that x dy/dx = y^2/(y – x) .

Solution : 1

We have y = x log y,

Differentiating both sides with respect to x we get , dy/dx = x . (1/y) .

dy/dx + log y . 1

Or, (1 – x /y) dy/dx = log y

Or, [(y – x)/y] dy/dx = log y

Or, dy/dx = y log y /(y – x)

Or, x dy/dx = x y log y/(y – x)

= y(x log y)/(y – x) [As, x log y = y]

= y2/(y – x) .

Q.2. If y log x = x – y , prove that dy/dx = log x/$(1 + \log x)^2$.

Solution : 2

We have , y log x = x – y ,

Differentiating with respect to x we get ,

y × 1/x + log x × dy/dx = 1 – dy/dx

Or, (1 + log x)dy/dx = 1 – y/x --- --- --- (i)

We are given, y log x = x – y

Or, y log x + y = x

Or, y (1 + log x) = x

Or, y/x = 1/(1 + log x) --- --- --- (ii)

Hence, putting y/x from (ii) to (i) we get,

$(1 + \log x)\, dy/dx = 1 - 1/(1 + \log x)$

$= (1 + \log x - 1)/(1 + \log x) = \log x/(1 + \log x)$

Therefore , $dy/dx = \log x/(1 + \log x)^2$. **[Proved.]**

Q.3. If $\log (x^2 + y^2) = 2 \tan^{-1} (y/x)$, show that $dy/dx = (x + y)/(x - y)$.

Solution : 3

We have , $\log (x^2 + y^2) = 2 \tan -1 (y/x)$,

Differentiating with respect to x we get ,

$1/(x^2 + y^2) . d/dx (x^2 + y^2) = 2 . 1/\{1 + (y/x)^2\}. d/dx (y/x)$

Or, $1/(x^2 + y^2) \{d/dx (x^2) + d/dx (y^2)\} = 2x^2/(x^2 + y^2) . [\{x .dy/dx - y .1\}/x^2]$

Or, $1/(x^2 + y^2) (2x + 2y\, dy/dx) = 2/(x^2 + y^2) .\{x\, dy/dx - y\}$

Or, $2 (x + y\, dy/dx) = 2 (x\, dy/dx - y)$

Or, $x + y\, dy/dx = x\, dy/dx - y$

Or, $(y - x) dy/dx = -(x + y)$

Or, $dy/dx = (x + y)/(x - y)$. **[Proved.]**

Q.4. If $x \sqrt{(1 + y)} + y \sqrt{(1 + x)} = 0$, prove that $dy/dx = -1/(x + 1)^2$.

Solution : 4

We have , $x \sqrt{(1 + y)} + y \sqrt{(1 + x)} = 0$

$x \sqrt{(1 + y)} = -y \sqrt{(1 + x)}$

Squaring both sides we get ,

$x^2 (1 + y) = y^2 (1 + x)$

Or, $x^2 - y^2 = y^2 x - x^2 y$

Or, $(x + y)(x - y) = - xy (x - y)$

Or, $x + y = - xy$ [As $x = y$ does not satisfy the given equation, $x - y \neq 0$]

Or, $x = - y - xy$ = > $y (1 + x) = - x$ Or, $y = - x /(1 + x)$

Therefore , $dy/dx = - [\{(1 + x) . 1 - x (0 + 1)\}/\{(1 + x)^{2}\}] = - 1 /(1 + x)^{2}$. **[Proved.]**

Q.5. If $y\sqrt{(1 - x2)} + x\sqrt{(1 - y2)} = 1$, prove that $dy/dx = - \sqrt{\{(1 - y^2)/(1 - x^2)\}}$.

Solution : 5

Let $x = \sin \theta$ and $y = \cos \varphi$, then

$y \sqrt{(1 - x^{2})} + x \sqrt{(1 - y^{2})} = 1$ reduces to

$\cos \varphi \sqrt{(1 - \sin^{2}\theta)} + \sin \theta \sqrt{(1 - \cos^{2}\varphi)} = 1$

Or, $\cos \varphi \cos \theta + \sin \theta \sin \varphi = 1$

Or, $\cos (\theta - \varphi) = 1$

Or, $\theta - \varphi = \cos -1\ 1$

Or, $\sin -1\ x - \cos -1\ y = \cos -1\ 1$

Or, $\sin -1\ x - \cos -1\ y = 0$

[As, $\cos -1\ 1 = 0$]

Differentiating we get , $1/\sqrt{(1 - x^{2})} - (-1)/\sqrt{(1 - y^{2})}\ dy/dx = 0$

Or, $1/\sqrt{(1 - y^{2})}\ dy/dx = -1/\sqrt{(1 - x^{2})}$

Or, $dy/dx = - \sqrt{\{(1 - y^{2})/(1 - x^{2})\}}$. **[Proved.]**

Q.6. If $\sin y = x \sin (a + y)$, prove that $dy/dx = \sin^2 (a + y)/\sin a$.

Solution : 6

We have $\sin y = x \sin (a + y)$

Differentiating both sides with respect to y we get ,

dx/dy = [sin (a + y) . cos y – sin y . cos (a + y)] / $\sin^2$ (a + y) .

= sin (a + y – y)/$\sin^2$ (a + y)

= sin a / $\sin^2$ (a + y)

Hence , dy/dx = $\sin^2$ (a + y) / sin a . **[Proved.]**

Q.7. If y = x sin y , prove that x. dy/dx = y/(1 – x cos y) .

Solution : 7

We have y = x sin y

Differentiating both sides with respect to x we get

dy/dx = x . cos y dy/dx + 1. sin y

Or, dy/dx (1 – x cos y) = sin y

Or, dy/dx = sin y / (1 – x cos y)

Or, x . dy/dx = (x . sin y)/(1 – x cos y) [As, y = x sin y]

= y/(1- x cos y) **[Proved.]**