

# PHYSICS

## Day-2 : Assignment

### Chapter(s): Vectors, Motion In a Plane, Laws Of Motion

1. Vector  $\vec{A}$  makes equal angles with x, y and z axis. Value of its components (in terms of magnitude of  $\vec{A}$ ) will be

- 1)  $\frac{A}{\sqrt{3}}$                       2)  $\frac{A}{\sqrt{2}}$   
 3)  $\sqrt{3} A$                       4)  $\frac{\sqrt{3}}{A}$

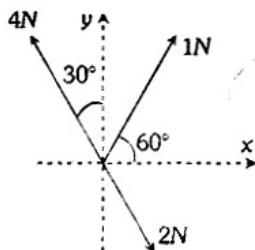
2. A particle starting from the origin (0, 0) moves in a straight line in the (x, y) plane. Its coordinates at a later time are  $(\sqrt{3}, 3)$ . The path of the particle makes with the x-axis an angle of

- 1)  $30^\circ$                       2)  $45^\circ$   
 3)  $60^\circ$                       4)  $0^\circ$

3. If  $\vec{P} = \vec{Q}$  then which of the following is NOT correct

- 1)  $\hat{P} = \hat{Q}$                       2)  $|\vec{P}| = |\vec{Q}|$   
 3)  $P\hat{Q} = Q\hat{P}$                       4)  $\vec{P} + \vec{Q} = \hat{P} + \hat{Q}$

4. Three forces acting on a body are shown in the figure. To have the resultant force only along the y-direction, the magnitude of the minimum additional force needed is



- 1)  $\frac{\sqrt{3}}{4} N$                       2)  $\sqrt{3} N$   
 3) 0.5 N                      4) 1.5 N

5. The magnitudes of vectors  $\vec{A}$ ,  $\vec{B}$  and  $\vec{C}$  are 3, 4 and 5 units respectively. If  $\vec{A} + \vec{B} = \vec{C}$ , the angle between  $\vec{A}$  and  $\vec{B}$  is

- 1)  $\frac{\pi}{2}$   
 2)  $\cos^{-1}(0.6)$   
 3)  $\tan^{-1}\left(\frac{7}{5}\right)$

- 4)  $\frac{\pi}{4}$

6. The maximum and minimum magnitudes of the resultant of two given vectors are 17 units and 7 unit respectively. If these two vectors are at right angles to each other, the magnitude of their resultant is

- 1) 14                      2) 16  
 3) 18                      4) 13

7. Five equal forces of 10N each are applied at one point and all are lying in one plane. If the angles between them are equal, the resultant force will be

- 1) Zero  
 2) 10 N  
 3) 20 N  
 4)  $10\sqrt{2} N$

8. A body is moving with velocity 30 m/s towards east. After 10 seconds its velocity becomes 40 m/s towards north. The average acceleration of the body is

- 1)  $5 m/s^2$                       2)  $1 m/s^2$   
 3)  $7 m/s^2$                       4)  $\sqrt{7} m/s^2$

9. A vector  $\vec{A}$  points vertically upward and  $\vec{B}$  points towards north. The vector product  $\vec{A} \times \vec{B}$  is

- (1) Zero
- (2) Along west
- (3) Along east
- (4) Vertically downward

10. The position vector of a particle is  $\vec{r} = a \cos \omega t \hat{i} + a \sin \omega t \hat{j}$ ; the velocity of the particle is

- (1) parallel to the position vector
- (2) perpendicular to the position vector
- (3) directed towards the origin
- (4) directed away from the origin

11. From a light-house an observer observes two ships A and B. Ship A proceeding towards north at a speed  $20\sqrt{2} \text{ km/h}$  and ship B proceeding towards north-east at a speed of  $20 \text{ km/h}$ .

Find in which direction and at what speed the ship B would appear to move to an observer standing on the deck of the ship A.

- (1)  $20 \text{ km/hr S-E}$
- (2)  $10 \text{ km/hr N-E}$
- (3)  $5 \text{ km/hr N-W}$
- (4)  $30 \text{ km/hr S-W}$

12. To the man walking towards east with a speed of  $4 \text{ km/h}$ , the wind appears to blow from the north. It appears to blow from north-east when he doubles his speed. Find the absolute velocity and direction of the wind.

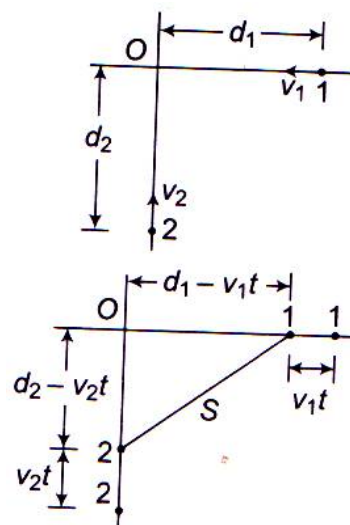
- (1)  $4\sqrt{2} \text{ km/hr S-E direction}$

- (2)  $4 \text{ km/hr S-E direction}$
- (3)  $4\sqrt{2} \text{ km/hr N-E direction}$
- (4)  $4 \text{ km/hr N-W direction}$

13. A man running on a horizontal road at  $6 \text{ km/h}$  finds the rain falling vertically. He doubles his speed and finds that raindrops make an angle  $37^\circ$  with the vertical. Find the velocity of rain with respect to the ground.

- (1)  $20 \text{ km/hr}$
- (2)  $5 \text{ km/hr}$
- (3)  $10 \text{ km/hr}$
- (4)  $25 \text{ km/hr}$

14. Two particles, 1 and 2, move with constant velocities  $v_1$  and  $v_2$  along two mutually perpendicular straight lines toward the intersection point. At  $t=0$ , particles were located at the distances  $d_1$  and  $d_2$  from the point. After how much time the distance between the particles is minimum?



- (1)  $t = \frac{d_1 v_1 + d_2 v_2}{v_1^2 + v_2^2}$
- (2)  $t = \frac{d_1 v_2 - d_2 v_1}{v_1^2 + v_2^2}$
- (3)  $t = \frac{d_1 v_1 - d_2 v_2}{v_1^2 + v_2^2}$
- (4)  $t = \frac{d_1 v_1 + d_2 v_2}{v_1^2 - v_2^2}$

15. A motorboat covers a distance between two points in 4 h along the flow and in 8 h opposite to the flow. In how much time, this distance can be covered in still water?

- (1)  $\frac{16}{3}h$  (2)  $\frac{8}{3}h$   
 (3)  $\frac{10}{3}h$  (4)  $\frac{7}{3}h$

16. A motorboat going downstream passes an object moving with water at a point A: 60 min later it turned back and after some time passed the object at a distance 6 km from the point A. Find the river flow velocity, assuming that the speed of motorboat is constant.

- (1)  $2km/hr$  (2)  $5km/hr$   
 (3)  $3km/hr$  (4)  $10km/hr$

17. Two boats, A and B, move away from a point P at the middle of a river along the mutually perpendicular straight lines. Boat A moves along the river and boat B cross the river. Having moved off equal distance from point P the boats returned. Find the ratio of times of motion of boats  $\frac{t_A}{t_B}$ , if

$v = \eta u$  ( $\eta > 1$ ). ( $v$ : velocity of boat with respect to water,  $u$ : stream velocity)

- (1)  $\frac{\eta}{\sqrt{\eta^2 + 1}}$  (2)  $\frac{\eta}{\sqrt{\eta^2 - 1}}$   
 (3)  $\frac{\eta}{\eta + 1}$  (4)  $\frac{\eta}{\eta - 1}$

18. An aeroplane has to go from a point O to another point A, at distance  $d$  due  $37^\circ$  east or north. A wind is blowing due north at a speed of 20

m/s. The air speed of the plane is  $v$ . (a) Find the direction in which the pilot should head the plane to reach the point A. (b) Find the time taken by the plane to go from O to A.

- (1)  $\theta = \sin^{-1}\left(\frac{12}{v}\right)$  (2)  $\theta = \sin^{-1}\left(\frac{6}{v}\right)$   
 (3)  $\theta = \sin^{-1}\left(\frac{8}{v}\right)$  (4)  $\theta = \sin^{-1}\left(\frac{10}{v}\right)$

19. Two cars are moving in the same direction with the same speed  $30km/h$ . They are separated by a distance of 5 km, the speed of a car moving in the opposite direction if it meets these two cars at an interval of 4 min, will be

- (1)  $40km/h$  (2)  $45km/h$   
 (3)  $30km/h$  (4)  $15km/h$

20. An express train is moving with a velocity with a velocity  $v_1$ . Its driver finds another train is moving on the same track in the same direction with velocity  $v_2$ . To escape collision, the driver applies a retardation  $a$  on the train. The minimum time of escaping collision will be

- (1)  $t = \frac{v_1 - v_2}{a}$  (2)  $t = \frac{v_1^2 - v_2^2}{2a}$   
 (3) None (4) Both (1) and (2)

21. A ball A is thrown up vertically with a speed  $u$  and at the same instant another ball B is released from a height  $h$ . At time  $t$ , the speed of A relative to B is

- (1)  $u$  (2)  $2u$   
 (3)  $u - gt$  (4)  $\sqrt{u^2 - gt}$

22. A car A is going north-east at  $80\text{km/h}$  and another car B is going south-east at  $60\text{km/h}$ .

The direction of the velocity of A relative to B makes an angle with the north equal to:

- (1)  $\tan^{-1}(2/7)$       (2)  $\tan^{-1}(7/2)$   
(3)  $\tan^{-1}(7)$       (4)  $\tan^{-1}(1/7)$

23. A ship is travelling due east at a speed of  $15\text{km/h}$ . Find the speed of a boat heading  $30^\circ$  east of north if it appears always due north from the ship.

- (1)  $30\text{km/h}$       (2)  $\frac{15\sqrt{3}}{2}\text{km/h}$   
(3)  $10\sqrt{3}\text{km/h}$       (4)  $20\text{km/h}$

24. Three particles A, B and C are situated at the vertices of an equilateral triangle ABC of side  $d$  at  $t=0$ . Each of the particles moves with constant speed  $v$ . A always has its velocity along AB, B along BC and C along CA. At what time will the particles meet each other?

- (1)  $2d/3v$       (2)  $d/3v$   
(3)  $3d/2v$       (4)  $4d/3v$

25. Four particles A, B, C and D are situated at the vertices of a rectangle of side  $d$  at  $t=0$ . Each of the particles moves with constant speed  $v$ . A always has its velocity along AB, B along BC, C along CD and D along DA. At what time will the particles meet each other?

- (1)  $2d/3v$       (2)  $d/v$   
(3)  $3d/2v$       (4)  $4d/3v$

26. Six particles situated at the corners of a regular hexagon of side  $a$  move at a constant speed  $v$ . Each particle maintains a direction towards the particle at the next corner. The time the particles will take to meet each other.

- (1)  $2d/3v$       (2)  $2d/v$   
(3)  $3d/2v$       (4)  $4d/3v$

27. A particle A is at origin and particle B is at distance  $y = -d$  at  $t = 0$ . They move with constant velocity  $v$ , A towards positive  $x$ -axis and B towards origin. The time at which distance between them is minimum and minimum distance will be

- (1)  $\frac{d}{2v}, d$       (2)  $\frac{d}{v}, \frac{d}{\sqrt{2}}$   
(3)  $\frac{d}{2v}, \frac{d}{\sqrt{2}}$       (4)  $\frac{d}{v}, d$

28. A police jeep is chasing with a velocity of  $45\text{km/h}$  a thief in another jeep moving with a velocity of  $153\text{km/h}$ . Police fires a bullet with muzzle velocity of  $180\text{ m/s}$ . The velocity with which it will strike the jeep of the thief is

- (1)  $150\text{ m/s}$       (2)  $27\text{ m/s}$   
(3)  $450\text{ m/s}$       (4)  $250\text{ m/s}$

29. A boat is moving with a velocity  $3i + 4j$  with respect to the ground. The water in the river is moving with a velocity  $-3i - 4j$  with respect to the ground. The relative velocity of the boat with respect to water is

- (1)  $8j$       (2)  $-6i - 8j$   
(3)  $6i + 8j$       (4)  $5\sqrt{2}$

30. The speed of a boat is  $5\text{ km/h}$  in still water. It crosses a river of width  $1\text{ km}$  along the shortest possible path in  $15\text{ min}$ . The velocity of the river water is

- (1)  $1\text{ km/h}$  (2)  $3\text{ km/h}$   
(3)  $4\text{ km/h}$  (4)  $5\text{ km/h}$

31. A man can swim at a speed of  $3\text{ km/h}$  in still water. He wants to cross a  $500\text{ m}$  wide river flowing at  $2\text{ km/h}$ . He keeps himself always at an angle of  $120^\circ$  with the river flow while swimming. The time he takes to cross the river is

- (1)  $10/\sqrt{3}\text{ min}$  (2)  $20/\sqrt{3}\text{ min}$   
(3)  $30/\sqrt{3}\text{ min}$  (4)  $40/\sqrt{3}\text{ min}$

32. A man can swim with velocity  $v$  relative to water. He has to cross a river of width  $d$  flowing with a velocity  $u$  ( $u > v$ ). The distance travelled by man along the flow is  $x$ , when he reaches to opposite bank. For  $x$  to be minimum the person should swim at an angle  $\alpha$  with the direction of the flow of water, where  $\alpha$  is

- (1)  $\sin^{-1}(v/u)$  (2)  $\sin^{-1}(u/v)$   
(3)  $\frac{\pi}{2} + \sin^{-1}(v/u)$  (4)  $\frac{\pi}{2} + \sin^{-1}(u/v)$

33. To cross the river in shortest distance, a swimmer should swim making an angle  $\theta$  with the upstream. What is the ratio of the time taken to swim across in the shortest time to that in swimming across over shortest distance.

- (1)  $\cos \theta$  (2)  $\sin \theta$   
(3)  $\tan \theta$  (4)  $\cot \theta$

34. A person walks up a stationary escalator in time  $t_1$ . If he remains stationary on the escalator, then it can take him up in time  $t_2$ . How much time would it take him to walk up the moving escalator?

- (1)  $\frac{t_1 + t_2}{2}$  (2)  $\sqrt{t_1 t_2}$   
(3)  $\frac{t_1 t_2}{t_1 + t_2}$  (4)  $t_1 + t_2$

25. A swimmer crosses a flowing stream of width  $\omega$  to and from in time  $t_1$ . The time taken to cover the same distance up and down the stream is  $t_2$ . If  $t_3$  is the time the swimmer would take to swim a distance  $2\omega$  in still water, then

- (1)  $t_1^2 = t_2 t_3$  (2)  $t_2^2 = t_1 t_3$   
(3)  $t_3^2 = t_1 t_2$  (4)  $t_3 = t_1 + t_2$

36. River is flowing with a velocity  $\vec{v}_{BR} = 4\hat{i}\text{ m/s}$ . A boat is moving with a velocity of  $\vec{v}_{Br} = (-2\hat{i} + 4\hat{j})\text{ m/s}$  relative to river. The width of the river is  $100\text{ m}$  along  $y$ -direction. Choose the correct alternative(s).

- (1) The boatman will cross the river in  $25\text{ s}$   
(2) Absolute velocity of boatman is  $2\sqrt{5}\text{ m/s}$   
(3) Drift of the boatman along the river current is  $50\text{ m}$   
(4) The boatman can never cross the river

37. The rain is falling vertically downward with velocity 6 m/s and a man is moving horizontally with velocity 8 m/s. Find the velocity of rain with respect to the man.

- (1) 10 m/s                      (2) 5 m/s  
(3) 15 m/s                      (4) 20 m/s

38. Two trains A and B of length 100 m and 200 m, respectively, are approaching each other on parallel tracks. If they take 15 s to pass each other and velocity of A is three times that of B, find their velocities.

- (1) 2 m/s                      (2) 6 m/s  
(3) 5 m/s                      (4) 4 m/s

39. A person aiming to reach the exactly opposite point on the bank of a stream is swimming with a speed of 0.5 m/s at an angle of  $120^\circ$  with the direction of flow of water. The speed of water in the stream is

- (1) 1 m/s                      (2) 0.5 m/s  
(3) 0.25 m/s                      (4) 0.433 m/s

40. A river is flowing from west to east with a speed of 5 m/min. A man can swim in still water with a velocity 10 m/min. In which direction should the man swim so as to take the shortest possible path to go to the south.

- (1)  $30^\circ$  with downstream  
(2)  $60^\circ$  with downstream  
(3)  $120^\circ$  with downstream  
(4) South

41. A stone projected with a velocity  $u$  at an angle  $\theta$  with the horizontal reaches maximum height  $H_1$ . When it is projected with velocity  $u$  at an angle  $\left(\frac{\pi}{2} - \theta\right)$  with the horizontal, it reaches maximum height  $H_2$ . The relation between the horizontal range  $R$  of the projectile,  $H_1$  and  $H_2$  is

- (1)  $R = 4\sqrt{H_1 H_2}$                       (2)  $R = 4(H_1 - H_2)$   
(3)  $R = 4(H_1 + H_2)$                       (4)  $R = \frac{H_1^2}{H_2^2}$

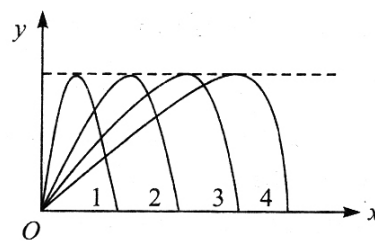
42. An object is projected with a velocity of 20 m/s making an angle of  $45^\circ$  with horizontal. The equation for the trajectory is  $h = Ax - Bx^2$  where  $h$  is height,  $x$  is horizontal distance,  $A$  and  $B$  are constants. The ratio  $A:B$  is ( $g = 10 \text{ ms}^{-2}$ )

- (1) 1 : 5                      (2) 5 : 1  
(3) 1 : 40                      (4) 40 : 1

43. For a given velocity, a projectile has the same range  $R$  for two angles of projection if  $t_1$  and  $t_2$  are the times of flight in the two cases then

- (1)  $t_1 t_2 \propto R^2$                       (2)  $t_1 t_2 \propto R$   
(3)  $t_1 t_2 \propto \frac{1}{R}$                       (4)  $t_1 t_2 \propto \frac{1}{R^2}$

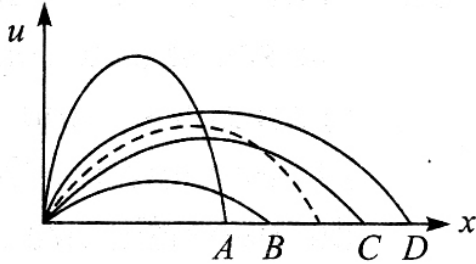
43. Figure shows four paths for a kicked football. Ignoring the effects of air on the flight, rank the paths according to initial horizontal velocity component, highest first



- (1) 1, 2, 3, 4                      (2) 2, 3, 4, 1  
(3) 3, 4, 1, 2                      (4) 4, 3, 2, 1



45. The path of a projectile in the absence of air drag is shown in the figure by dotted line. If the air resistance is not ignored then which one of the path shown in the figure is appropriate for the projectile



- (1) B (2) A (3) D (4) C
46. A man standing on the roof of a house of height  $h$  throws one particle vertically downwards and another particle horizontally with the same velocity  $u$ . The ratio of their velocities when they reach the earth's surface will be

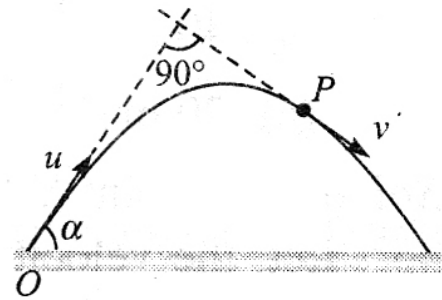
- (1)  $\sqrt{2gh+u^2} : u$  (2) 1 : 2  
(3) 1 : 1 (4)  $\sqrt{2gh+u^2} : \sqrt{2gh}$

47. A cannon on a level plane is aimed at an angle  $\theta$  above the horizontal and a shell is fired with a muzzle velocity  $v_0$  towards a vertical cliff a distance  $S$  away. Then the height from the bottom at which the shell strikes the side walls of the cliff is

- (1)  $S \sin \theta - \frac{gS^2}{2v_0^2 \sin^2 \theta}$   
(2)  $S \sin \theta - \frac{gS^2}{2v_0^2 \cos^2 \theta}$   
(3)  $S \tan \theta - \frac{gS^2}{2v_0^2 \cos^2 \theta}$   
(4)  $S \tan \theta - \frac{gS^2}{2v_0^2 \sin^2 \theta}$

48. A particle is projected from point  $O$  with velocity  $u$  in a direction making an angle  $\alpha$  with the horizontal. At any instant its position is at point  $P$  at

right angles to the initial direction of projection. Its velocity at point  $P$  is

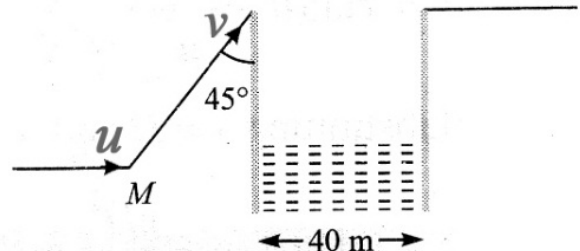


- (1)  $u \tan \alpha$  (2)  $u \cot \alpha$   
(3)  $u \cos \alpha$  (4)  $u \sec \alpha$

49. Two seconds after projection a projectile is travelling in a direction inclined at  $30^\circ$  to the horizontal after one more sec, it is travelling horizontally, the magnitude and direction of its velocity are

- (1)  $2\sqrt{20} \text{ m/s}, 60^\circ$   
(2)  $20\sqrt{3} \text{ m/s}, 60^\circ$   
(3)  $6\sqrt{40} \text{ m/s}, 30^\circ$   
(4)  $40\sqrt{6} \text{ m/s}, 30^\circ$

50. A body is projected up a smooth inclined plane (length =  $20\sqrt{2} \text{ m}$ ) with velocity  $u$  from the point  $M$  as shown in the figure. The angle of inclination is  $45^\circ$  and the top is connected to a well of diameter  $40 \text{ m}$ . If the body just manages to cross the well, what is the value of  $v$ ?



- (1)  $40 \text{ ms}^{-1}$  (2)  $40\sqrt{2} \text{ ms}^{-1}$   
(3)  $20 \text{ ms}^{-1}$  (4)  $20\sqrt{2} \text{ ms}^{-1}$

51. A particle is projected from a point  $O$  with a velocity  $u$  in a direction making an angle  $\theta$  upward with the horizontal. After some time at point  $P$ , it is moving at right angle with its initial direction of projection. The time of flight from  $O$  and  $P$  is

- (1)  $\frac{u \sin \theta}{g}$  (2)  $\frac{u \operatorname{cosec} \theta}{g}$   
 (3)  $\frac{u \tan \theta}{g}$  (4)  $\frac{u \sec \theta}{g}$

52. Raju and Kiran are playing with two different balls of masses  $m$  and  $2m$ , respectively. If Raju throws his ball vertically up and Kiran at an angle  $\theta$ , both of them stay in our view for the same period. The height attained by the two balls are in the ratio

- (1) 2 : 1 (2) 1 : 1  
 (3) 1 :  $\cos \theta$  (4) 1 :  $\sec \theta$

53. A helicopter is flying horizontally at 8 m/s at an altitude 180 m when a package of emergency medical supplies is ejected horizontally backward with a speed of 12 m/s relative to the helicopter. Ignoring air resistance, what is the horizontal distance between the package and the helicopter when the package hits the ground ( $g = 10 \text{ m/sec}^2$ )?

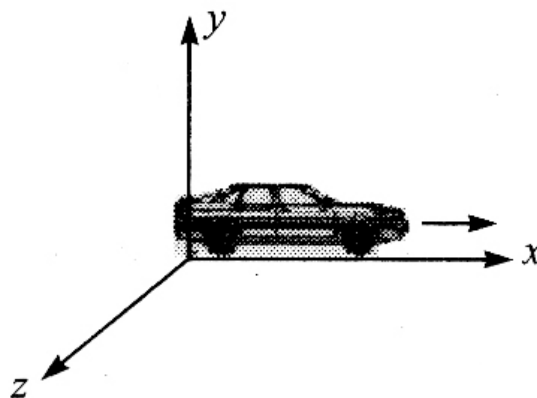
- (1) 120 m (2) 24 m  
 (3) 36 m (4) 72 m

54. In horizontal level, ground to ground projectile if at any instant velocity becomes perpendicular to initial velocity, then what can you say about projection angle with horizontal?

- (1)  $\theta = 45^\circ$   
 (2)  $\theta \leq 45^\circ$   
 (3)  $\theta \leq 45^\circ$   
 (4) for any value of  $\theta$  it is possible

55. A car is travelling on a highway at a speed of 25 m/s along  $x$ -axis. A passenger in a car throws a ball at an

angle  $37^\circ$  with horizontal in a plane perpendicular the motion of the car. The ball is projected with a speed of 10 m/s relative to car. What a speed of 10 m/s relative to car. What may be the initial velocity of ball in unit vector notation?



- (1)  $25i + 8j + 6k$   
 (2)  $10i + 8j + 6k$   
 (3)  $10i + 25j + 6k$   
 (4)  $25i + 6j + 8k$

56. Velocity of a particle at time  $t = 0$  is 2 m/s. A constant acceleration of  $2 \text{ m/s}^2$  act on the particle of 2 s at an angle  $60^\circ$  with the initial velocity. The magnitude of velocity of particle at the end of 2 s will be

- (1)  $2\sqrt{5} \text{ m/s}$  (2)  $2\sqrt{7} \text{ m/s}$   
 (3)  $2\sqrt{6} \text{ m/s}$  (4)  $2\sqrt{3} \text{ m/s}$

57. A particle is projected with a certain velocity at an angle  $\alpha$  above the horizontal from the foot of an inclined plane of inclination  $30^\circ$ . If the particle strikes the plane normally, then  $\alpha$  is equal to:

- (1)  $30^\circ + \tan^{-1} \left( \frac{\sqrt{3}}{2} \right)$   
 (2)  $45^\circ$   
 (3)  $60^\circ$   
 (4)  $30^\circ + \tan^{-1} (2\sqrt{3})$



58. A particle is projected with a velocity  $v$  so that its range on a horizontal plane is twice the greatest height attained. If  $g$  is acceleration due to gravity, then its range is

- (1)  $\frac{4v^2}{5g}$  (2)  $\frac{4g}{5v^2}$   
 (3)  $\frac{4v^3}{5g^2}$  (4)  $\frac{4v}{5g^2}$

59. A particle is projected from ground at some angle with the horizontal. Let  $P$  be the point at maximum height  $H$ . At what height above the point  $P$  should the particle be aimed to have range equal to maximum height?

- (1)  $H$  (2)  $2H$  (3)  $H/2$  (4)  $3H$

60. A shot is fired from a point at a distance of 200 m from the foot of a tower 100 m high so that it just passes over it horizontally. The direction of shot with horizontal is

- (1)  $30^\circ$  (2)  $45^\circ$   
 (3)  $60^\circ$  (4)  $70^\circ$

61. A ball is thrown from a point with a speed  $v_o$  at an angle of projection  $\theta$ . From the same point and at the same instant, a person starts running with a constant speed  $v_o/2$  to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?

- (1) Yes,  $60^\circ$  (2) Yes,  $30^\circ$   
 (3) No (4) Yes,  $45^\circ$

62. A body has an initial velocity of  $3 \text{ ms}^{-1}$  and has an acceleration of  $1 \text{ ms}^{-2}$  normal to the direction of the initial velocity. Then its velocity 4 s after the start is

- (1)  $7 \text{ ms}^{-1}$  along the direction of initial velocity  
 (2)  $7 \text{ ms}^{-1}$  along the normal to the direction of initial velocity.

(3)  $7 \text{ ms}^{-1}$  midway between the two directions

(4)  $5 \text{ ms}^{-1}$  at an angle  $\tan^{-1}(4/3)$  with the direction of initial velocity.

23. Two paper screens  $A$  and  $B$  are separated by 150 m. A bullet pierces  $A$  and  $B$ . The hole in  $B$  is 15 cm below the hole in  $A$ . If the bullet is travelling horizontally at the time of hitting  $A$ , then the velocity of the bullet at  $A$  is ( $g = 10 \text{ ms}^{-2}$ )

- (1)  $100\sqrt{3} \text{ ms}^{-1}$  (2)  $200\sqrt{3} \text{ ms}^{-1}$   
 (3)  $300\sqrt{3} \text{ ms}^{-1}$  (4)  $500\sqrt{3} \text{ ms}^{-1}$

64. The equation of motion of a projectile is  $y = 12x - \frac{3}{4}x^2$ . The horizontal component of velocity is  $3 \text{ ms}^{-1}$ . What is the range of the projectile?

- (1) 18 m (2) 16 m  
 (3) 12 m (4) 21.6 m

65. At what angle with the horizontal should a ball be thrown so that the range  $R$  is related to the time of flight as  $R = 5T^2$ ? (Take  $g = 10 \text{ ms}^{-2}$ )

- (1)  $30^\circ$  (2)  $45^\circ$   
 (3)  $60^\circ$  (4)  $90^\circ$

66. A ball rolls off the top of a staircase with a horizontal velocity  $u \text{ ms}^{-1}$ . If the steps are  $h$  metre high and  $b$  metre wide, the ball hit the edge of the  $n^{\text{th}}$  step, if

- (1)  $n = \frac{2hu}{gb^2}$  (2)  $n = \frac{2hu^2}{gb}$   
 (3)  $n = \frac{2hu^2}{gb^2}$  (4)  $n = \frac{hu^2}{gb^2}$

67. At a height 0.4 m from the ground, the velocity of a projectile in vector form is  $\vec{v} = (6\hat{i} + 2\hat{j}) \text{ ms}^{-1}$ . The angle of projection is

- (1)  $45^\circ$  (2)  $60^\circ$   
 (3)  $30^\circ$  (4)  $\tan^{-1}(3/4)$

68. A projectile is thrown in the upward direction making an angle of  $60^\circ$  with the horizontal direction with a velocity of  $150\text{ms}^{-1}$ . Then the time after which its inclination with the horizontal is  $45^\circ$  is

- (1)  $15(\sqrt{3}-1)s$  (2)  $15(\sqrt{3}+1)s$   
 (3)  $7.5(\sqrt{3}-1)s$  (4)  $7.5(\sqrt{3}+1)s$

69. A person sitting in the rear end of a compartment throws a ball towards the front end. The ball follows a parabolic path. The train is moving with the uniform velocity of  $20\text{ms}^{-1}$ . A person standing outside on the ground also observes the ball. How will the maximum heights ( $h_m$ ) attained and the ranges ( $R$ ) seen by the thrower and the outside observer compare each other?

- (1) same  $h_m$ , different  $R$   
 (2) same  $h_m$ , and  $R$   
 (3) different  $h_m$ , same  $R$   
 (4) different  $h_m$ , and  $R$

70. A body is projected horizontally from the top of a tower with initial velocity  $18\text{ms}^{-1}$ . It hits the ground at angle  $45^\circ$ . What is the vertical component of velocity when strikes the ground?

- (1)  $9\text{ms}^{-1}$  (2)  $9\sqrt{2}\text{ms}^{-1}$   
 (3)  $18\text{ms}^{-1}$  (4)  $18\sqrt{2}\text{ms}^{-1}$

71. A plane flying horizontally at  $100\text{ms}^{-1}$  releases an object which reaches the ground in 10 s. At what angle with horizontal it hits the ground?

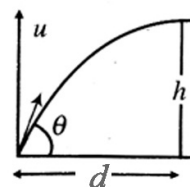
- (1)  $55^\circ$  (2)  $45^\circ$   
 (3)  $60^\circ$  (4)  $75^\circ$

72. A hose lying on the ground shoots a stream of water upward at an angle of  $60^\circ$  to the horizontal with the velocity

of  $16\text{ms}^{-1}$ . The height at which the water strikes the wall 8 m away is

- (1) 8.9 m (2) 10.9 m  
 (3) 12.9 m (4) 6.9 m

73. If a stone is to hit at a point which is at a distance  $d$  away and at a height  $h$ , above the point from where the stone starts, then what is the value of initial speed  $u$  is the stone is launched at an angle  $\theta$ ?



- (1)  $\frac{g}{\cos \theta} \sqrt{\frac{d}{2(d \tan \theta - h)}}$   
 (2)  $\frac{d}{\cos \theta} \sqrt{\frac{g}{2(d \tan \theta - h)}}$   
 (3)  $\sqrt{\frac{gd^2}{h \cos^2 \theta}}$  (4)  $\sqrt{\frac{gd^2}{(d-h)}}$

74. The speed of a projectile at its maximum height is  $\sqrt{3}/2$  times its initial speed. If the range of the projectile is  $P$  times the maximum height attained by it,  $P$  is equal to

- (1)  $4/3$  (2)  $2\sqrt{3}$   
 (3)  $4\sqrt{3}$  (4)  $3/4$

75. Two balls A and B are thrown with speeds  $u$  and  $u/2$ , respectively. Both the balls cover the same horizontal distance before returning to the plane of projection. If the angle of projection of ball B is  $15^\circ$  with the horizontal, then the angle of projection of A is

- (1)  $\sin^{-1}\left(\frac{1}{8}\right)$  (2)  $\frac{1}{2}\sin^{-1}\left(\frac{1}{8}\right)$   
 (3)  $\frac{1}{3}\sin^{-1}\left(\frac{1}{8}\right)$  (4)  $\frac{1}{4}\sin^{-1}\left(\frac{1}{8}\right)$

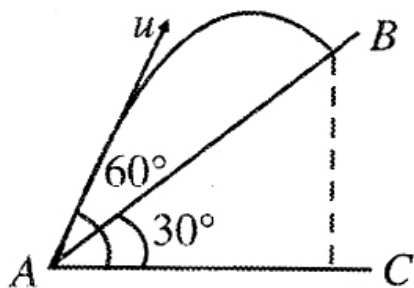
76. A car is moving horizontally along a straight line with a uniform velocity of  $25\text{ms}^{-1}$ . A projectile is to be fired from this car in such a way that it will return to it after it has moved 100 m. The speed of the projection must be

- (1)  $10\text{ms}^{-1}$  (2)  $20\text{ms}^{-1}$   
(3)  $15\text{ms}^{-1}$  (4)  $25\text{ms}^{-1}$

77. The horizontal range and maximum height attained by a projectile are R and H, respectively. If a constant horizontal acceleration  $a = g/4$  is imparted to the projectile due to wind, then its horizontal range and maximum height will be

- (1)  $(R+H), \frac{H}{2}$  (2)  $\left(R+\frac{H}{2}\right), 2H$   
(3)  $(R+2H), H$  (4)  $(R+H), H$

78. As shown in the below figure, time taken by the projectile to reach from A and B is t. Then the distance AB is equal to



- (1)  $\frac{ut}{\sqrt{3}}$  (2)  $\frac{\sqrt{3}ut}{2}$   
(3)  $\sqrt{3}ut$  (4)  $2ut$

79. The coordinates of a moving particle at any time t are given by  $x = \alpha t^3$  and  $y = \beta t^3$ . The speed of the particle at time t is given by

- (1)  $\sqrt{\alpha^2 + \beta^2}$  (2)  $3t\sqrt{\alpha^2 + \beta^2}$   
(3)  $3t^2\sqrt{\alpha^2 + \beta^2}$  (4)  $t^2\sqrt{\alpha^2 + \beta^2}$

80. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of  $30^\circ$

with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground?

( $g = 10\text{m/s}^2$ )

- (1) 8.66 m (2) 5.20 m  
(3) 4.33 m (4) 2.60 m

81. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be

- (1)  $20\sqrt{2}\text{m}$  (2) 10m  
(3)  $10\sqrt{2}\text{m}$  (4) 20m

82. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is

- (1)  $2gH = nu^2(n-2)$  (2)  $gH = (n-2)u^2$   
(3)  $2gH = n^2u^2$  (4)  $gH = (n-2)2u^2$

83. The speed of a projectile when it is at

its greatest height is  $\sqrt{\frac{2}{3}}$  times its

speed at half the maximum height.

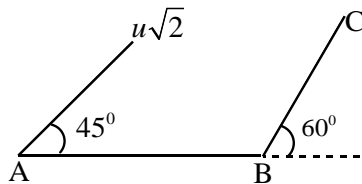
The angle of projection is

- (1)  $30^\circ$  (2)  $60^\circ$   
(3)  $45^\circ$  (4)  $\tan^{-1}\left(\frac{3}{4}\right)$

84. With what minimum speed must a particle be projected from origin so that it is able to pass through a given point (30m, 40m)? Take  $g = 10\text{m/s}^2$ .

- (1) 60m/s (2) 30m/s  
(3) 50m/s (4) 40m/s

85. A particle is projected from a point A with velocity  $u\sqrt{2}$  at an angle of  $45^\circ$  with horizontal as shown in figure. It strikes the plane BC at right angles. The velocity of the particle at the time of collision is



(1)  $\frac{\sqrt{3}}{2}u$

(2)  $\frac{u}{2}$

(3)  $\frac{2u}{\sqrt{3}}$

(4)  $u$

86. A projectile is given an initial velocity of  $\hat{i} + 2\hat{j}$ . The Cartesian equation of its path is ( $g = 10 \text{ ms}^{-2}$ )

1)  $y = 2x - 5x^2$

2)  $y = x - 5x^2$

3)  $4y = 2x - 5x^2$

4)  $y = 2x - 25x^2$

87. A particle is projected from the ground with an initial speed of  $v$  at an angle  $\theta$  with horizontal. The average velocity of the particle between its point of projection and highest point of trajectory is

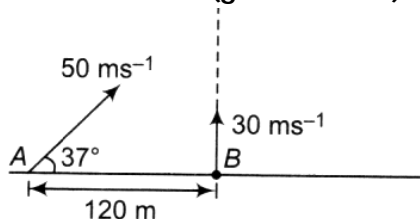
1)  $\frac{v}{2}\sqrt{1+2\cos^2\theta}$

2)  $\frac{v}{2}\sqrt{1+4\cos^2\theta}$

3)  $\frac{v}{2}\sqrt{1+3\cos^2\theta}$

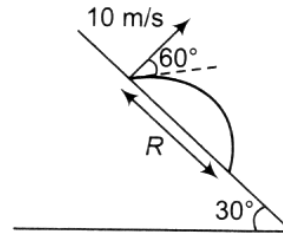
4)  $v \cos \theta$

88. Balls A and B are thrown from two points lying on the same horizontal plane separated by a distance, 120 m. Which of the following statements (s) is/are correct ( $g = 10 \text{ m/s}^2$ ).



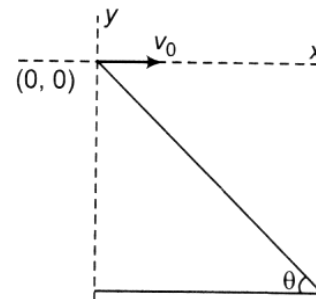
- 1) The two balls can never meet
- 2) The balls can meet if the ball B is thrown 1 s later
- 3) The two balls meet at a height of 45
- 4) None of the above

89. A projectile is launched with speed of 10 m/s at an angle  $60^\circ$  with the horizontal from a sloping surface of inclination  $30^\circ$ . The range  $R$  is (Take  $g = 10 \text{ m/s}^2$ )



- 1) 4.9 m
- 2) 13.3 m
- 3) 9.1 m
- 4) 12.6 m

90. A man standing on a hill top projects a stone horizontally with speed  $v_0$  as shown in figure. Taking the coordinates system as given in the figure. The coordinates of the point where the stone will hit the hill surface are



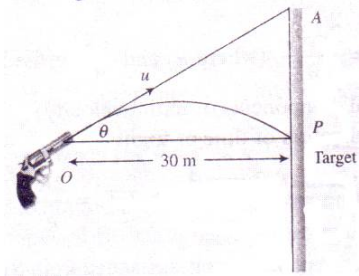
1)  $\left( \frac{2v_0^2 \tan \theta}{g}, -\frac{2v_0^2 \tan^2 \theta}{g} \right)$

2)  $\left( \frac{2v_0^2}{g}, -\frac{2v_0^2 \tan^2 \theta}{g} \right)$

3)  $\left( \frac{2v_0^2 \tan \theta}{g}, -\frac{2v_0^2}{g} \right)$

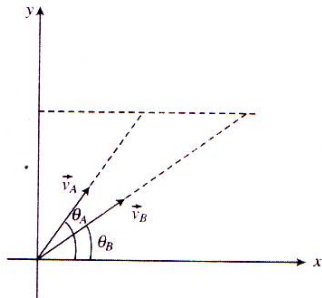
4)  $\left( \frac{2v_0^2 \tan \theta}{g}, -\frac{2v_0^2 \tan \theta}{g} \right)$

91. A bullet with muzzle velocity  $100 \text{ ms}^{-1}$  is to be shot at a target 30 m away in the same horizontal line. How high above the target must the rifle be aimed so that the bullet will hit the target?



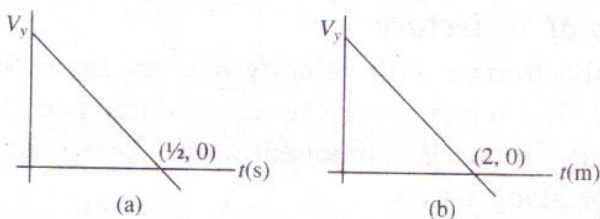
- (1) 45 cm (2) 30 cm  
(3) 20 cm (4) 40 cm

92. Two particles  $A$  and  $B$  are projected from the same point in different directions in such a manner that vertical components of their initial velocities are same. Find the ratio of ranges.



- (1)  $\frac{\tan \theta_A}{\tan \theta_B}$  (2)  $\frac{\sin \theta_A}{\sin \theta_B}$   
(3)  $\frac{\cos \theta_A}{\cos \theta_B}$  (4)  $\frac{\tan \theta_B}{\tan \theta_A}$

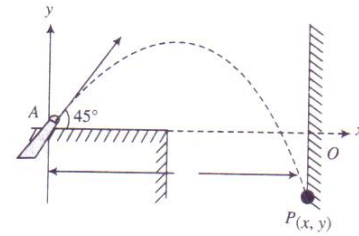
93. Two graphs of the same projectile motion (in the  $x - y$  - plane) projected from origin are shown in figure below.  $X$  - axis is along horizontal direction and  $Y$  - axis is vertically upwards. Take  $g = 10 \text{ ms}^{-2}$ .



- (1) 2 m/s (2) 2m/s  
(3) 4m/s (1) 1 m/s

94. A jet of water is projected at an angle  $\theta = 45^\circ$  with horizontal from point  $A$  which is situated at a distance  $x = OA = (a) 1/2m, (b) 2m$  from a

vertical wall. If the speed of projection is  $v_0 = \sqrt{10} \text{ ms}^{-1}$ , find point  $P$  of striking of the water jet with the vertical wall.



- (1) (2m, 1m) (2) (1m, 2m)  
(3) (3m, 4m) (4) (1m, 4m)

95. From a point on the ground at a distance  $a$  from the foot of a pole, a ball is thrown at an angle of  $45^\circ$  which just touches the top of the pole and strikes the ground at a distance of  $b$ , on the other side of it. Find the height of the pole.

- (1)  $\frac{ab}{a+b}$  (2)  $\frac{a}{b}$   
(3)  $\frac{b}{a}$  (4)  $\frac{a+b}{ab}$

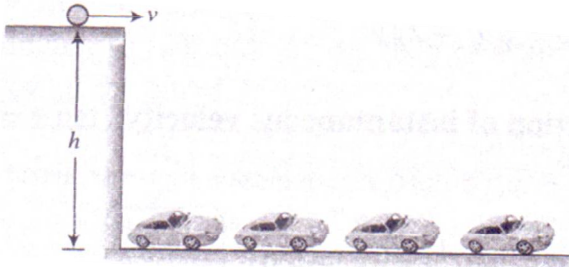
96. A projectile thrown with an initial speed  $u$  and the angle of projection  $15^\circ$  to the horizontal has a range  $R$ . If the same projectile is thrown at an angle of  $45^\circ$  to the horizontal with speed  $2u$ , what will be its range?

- (1)  $R' = 8R_1$  (2)  $R' = 4R_1$   
(3)  $R' = 16R_1$  (4)  $R' = 2R_1$

97. A grasshopper can jump a maximum distance of 1.6 m. It spends negligible time on the ground. How far can it go in 10 s? ( $g = 10 \text{ m/s}^2$ )

- (1)  $10\sqrt{2}$  (2)  $15\sqrt{2}$   
(3)  $5\sqrt{2}$  (4)  $20\sqrt{2}$

98. A rubber ball escapes from the horizontal roof with a velocity  $v = 5 \text{ ms}^{-1}$ . The roof is situated at a height,  $h = 20 \text{ m}$ . If the length of each car is equal to  $x_0 = 4 \text{ m}$ , with which car will the ball hit?



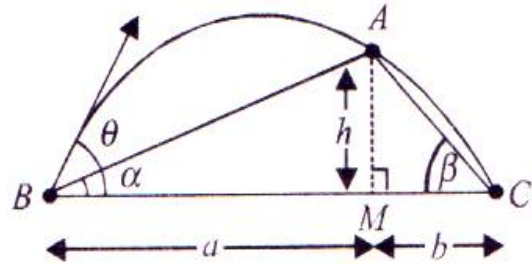
- (1) 1<sup>st</sup> car                      (2) 2<sup>nd</sup> car  
(3) 3<sup>rd</sup> car                    (4) 4<sup>th</sup> car

99. A boy of height  $1.5\text{m}$ , making move on a skateboard due east with velocity  $4\text{ms}^{-1}$ , throws a coin vertically up with a velocity of  $3\text{ms}^{-1}$  relative to himself. Find the total displacement of the coin relative to ground till it comes to the hand of the boy.

- (1)  $\frac{12}{5}\text{m}$                       (2)  $\frac{7}{5}\text{m}$   
(3)  $\frac{8}{5}\text{m}$                       (4)  $\frac{2}{5}\text{m}$

100. A particle is projected over a triangle from one extremity of its horizontal base. Grazing over the vertex, it falls on the other extremity of the base. If  $\alpha$  and  $\beta$  are the base angles of the triangle and  $\theta$

is the angle of projection then  $\tan \theta$  =?



- (1)  $\tan \theta = \tan \alpha + \tan \beta$   
(2)  $\tan \theta = \tan \alpha \cdot \tan \beta$   
(3)  $\tan \theta = \tan \alpha - \tan \beta$   
(4)  $\tan \theta = \frac{\tan \alpha}{\tan \beta}$



## ANSWERS

|       |       |       |       |       |       |       |       |       |        |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| 1) 1  | 2) 3  | 3) 4  | 4) 3  | 5) 1  | 6) 4  | 7) 1  | 8) 1  | 9) 2  | 10) 2  |
| 11) 1 | 12) 1 | 13) 3 | 14) 1 | 15) 1 | 16) 3 | 17) 2 | 18) 1 | 19) 2 | 20) 3  |
| 21) 1 | 22) 4 | 23) 1 | 24) 1 | 25) 2 | 26) 2 | 27) 3 | 28) 1 | 29) 3 | 30) 2  |
| 31) 2 | 32) 3 | 33) 2 | 34) 3 | 35) 1 | 36) 2 | 37) 1 | 38) 3 | 39) 3 | 40) 3  |
| 41) 1 | 42) 4 | 43) 2 | 44) 4 | 45) 1 | 46) 3 | 47) 3 | 48) 2 | 49) 2 | 50) 4  |
| 51) 2 | 52) 2 | 53) 4 | 54) 2 | 55) 4 | 56) 2 | 57) 1 | 58) 1 | 59) 1 | 60) 2  |
| 61) 1 | 62) 4 | 63) 4 | 64) 2 | 65) 2 | 66) 3 | 67) 3 | 68) 3 | 69) 1 | 70) 3  |
| 71) 2 | 72) 1 | 73) 2 | 74) 3 | 75) 2 | 76) 2 | 77) 4 | 78) 1 | 79) 3 | 80) 1  |
| 81) 4 | 82) 1 | 83) 3 | 84) 2 | 85) 3 | 86) 1 | 87) 3 | 88) 3 | 89) 2 | 90) 1  |
| 91) 1 | 92) 4 | 93) 3 | 94) 1 | 95) 1 | 96) 1 | 97) 4 | 98) 3 | 99) 1 | 100) 1 |

## HINTS AND SOLUTIONS

1. Let the components of  $\vec{A}$  makes angles  $\alpha, \beta$  and  $\gamma$  with x, y and z axis respectively then  $\alpha = \beta = \gamma$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$3 \cos^2 \alpha = 1 \Rightarrow \cos \alpha = \frac{1}{\sqrt{3}}$$

$$\therefore A_x = A_y = A_z = A \cos \alpha = \frac{A}{\sqrt{3}}$$

2. Final position vector of a particle,  
 $\vec{r} = \sqrt{3}\hat{i} + 3\hat{j}$

Angle of path from x-axis,

$$\theta = \tan^{-1} \left( \frac{y}{x} \right) = \tan^{-1} \left( \frac{3}{\sqrt{3}} \right)$$

$$= \tan^{-1}(\sqrt{3}) = 60^\circ$$

3.  $\vec{P} + \vec{Q} = P\hat{P} + Q\hat{Q}$

4. According to problem there should be  
 $\sum F_x = 0$

In the given figure net force along x-axis

$$\vec{F} = 1 \cos 60^\circ \hat{i} + 2 \sin 30^\circ \hat{i} - 4 \sin 30^\circ \hat{i}$$

$$= \frac{1}{2} \hat{i} + \hat{i} - 2 \hat{i} \Rightarrow \vec{F} = -\frac{1}{2} \hat{i} = 0.5 \hat{i}$$

To cancel this force minimum additional force needed is 0.5N along the positive direction of x-axis.

5.  $C = \sqrt{A^2 + B^2} = \sqrt{3^2 + 4^2} = 5$

$\therefore$  Angle between  $\vec{A}$  and  $\vec{B}$  is  $\frac{\pi}{2}$

6.  $R_{\max} = A + B = 17$  when  $\theta = 0^\circ$

$$R_{\min} = A - B = 7 \text{ when } \theta = 180^\circ$$

by solving we get  $A = 12$  and  $B = 5$

Now when  $\theta = 90^\circ$  then  $R = \sqrt{A^2 + B^2}$

$$\Rightarrow R = \sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13$$

7. If the angle between all forces which are equal and lying in one plane are equal then resultant force will be zero.

$$8. a = \frac{|\vec{v}_f - \vec{v}_i|}{t} = \frac{\sqrt{30^2 + 40^2}}{10} = 5 \text{ m/s}^2$$

$$9. \vec{A} \times \vec{B} = (A\hat{k}) \times (B\hat{j}) = -AB\hat{i}$$

- 10.

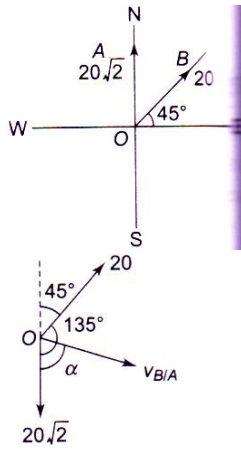
$$\vec{r} = a \cos \omega t \hat{i} + \alpha \sin \omega t \hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = -a \sin \omega t \cdot \omega \hat{i} + \alpha \cos \omega t \cdot \omega \hat{j}$$

$$\vec{r} \cdot \vec{v} = 0, \vec{r} \text{ and } \vec{v} \text{ are } \perp^{ar}.$$

11. In this problem, the velocity of B with respect to A is required.

$v_{B/A}$  = resultant of (velocity of B and velocity of A in opposite direction)



$$v_{B/A} = \sqrt{(20)^2 + (20\sqrt{2})^2 + 2 \times 20 \times 20\sqrt{2} \cos 135^\circ}$$

$$= \sqrt{400 + 800 - 800} = 20 \text{ km/h}$$

$$\tan \alpha = \frac{20 \sin 135^\circ}{20\sqrt{2} + 20 \cos 135^\circ}$$

$$= \frac{\frac{20}{\sqrt{2}}}{20\sqrt{2} - \frac{20}{\sqrt{2}}} = 1 \Rightarrow \alpha = 45^\circ$$

For an observer on the deck of A, ship B is moving with a speed  $20 \text{ km/h}$  along the south-east.

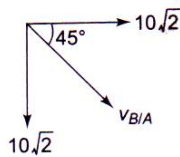
**OR**

$$\vec{v}_{A/g} = 20\sqrt{2}\hat{j}, \vec{v}_{B/g} = 20\cos 45^\circ\hat{i} + 20\sin 45^\circ\hat{j}$$

$$= 10\sqrt{2}\hat{i} + 10\sqrt{2}\hat{j}$$

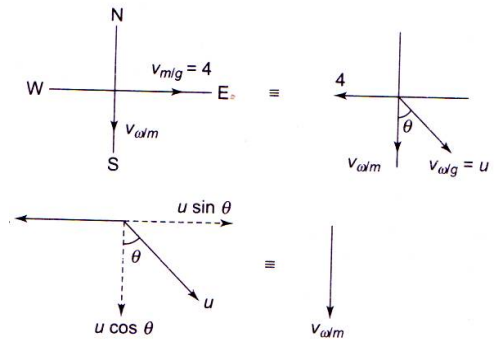
$$\vec{v}_{B/A} = \vec{v}_{B/g} - \vec{v}_{A/g} = 10\sqrt{2}\hat{i} - 10\sqrt{2}\hat{j}$$

$$|\vec{v}_{B/A}| = v_{B/A} = 20 \text{ km/h}$$



12. Absolute velocity, i.e. velocity with respect to the ground. Let the velocity of wind w.r.t. the ground is  $u$ .

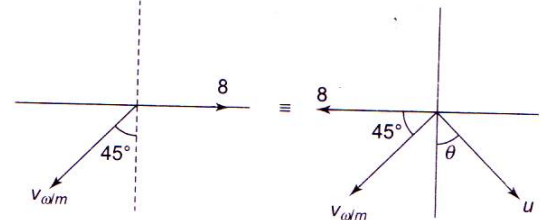
**Case I:**



$$u \sin \theta - 4 = 0$$

$$u \sin \theta = 4$$

**Case II:**



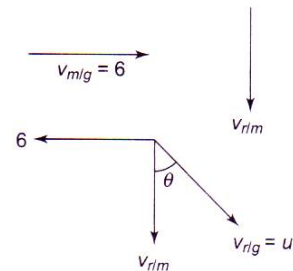
$$\tan 45^\circ = \frac{u \sin(90 + \theta)}{8 + u \cos(90 + \theta)} = \frac{u \cos \theta}{8 - u \sin \theta}$$

$$1 = \frac{u \cos \theta}{8 - 4} \Rightarrow u \cos \theta = 4$$

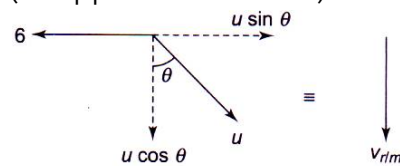
$$u = 4\sqrt{2} \text{ km/h}, \theta = 45^\circ$$

Wind is blowing with a speed of  $4\sqrt{2} \text{ km/h}$  and blowing in south-east direction.

13. Let the velocity of rain w.r.t. the ground is  $u$ .



**Recall:**  $v_{r/m}$  is resultant of  $v_{r/g}$  and  $v_{m/g}$  (in opposite direction).

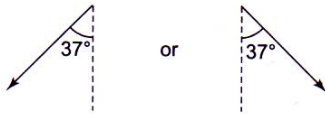


$$u \sin \theta - 6 = 0$$

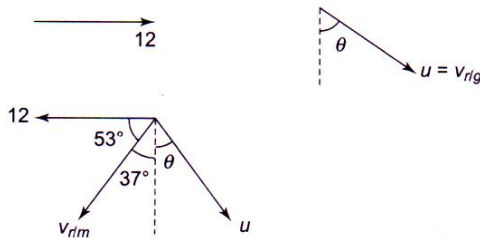
$$u \sin \theta = 6$$

Now the man doubles his speed and

$v_{r/m}$  makes  $37^\circ$  with vertical, it may be



**Recall:** Resultant is towards larger vector.



$$\tan 53^\circ = \frac{u \sin(90 + \theta)}{12 + u \cos(90 + \theta)}$$

$$\frac{4}{3} = \frac{u \cos \theta}{12 - u \sin \theta} = \frac{u \cos \theta}{12 - 6}$$

$$u \cos \theta = 8$$

$$u = \sqrt{(u \sin \theta)^2 + (u \cos \theta)^2}$$

$$= 8\sqrt{(6)^2 + (8)^2} = 10 \text{ km/h}$$

$$\tan \theta = \frac{u \sin \theta}{u \cos \theta} = \frac{6}{8} = \frac{3}{4} \Rightarrow \theta = 37^\circ$$

$v_{r/g} = 10 \text{ km/h}$  at an angle  $37^\circ$  with vertical direction

14. At time  $t$ ,

$$S^2 = (d_1 - v_1 t)^2 + (d_2 - v_2 t)^2$$

For  $S$  to be minimum

$$\frac{d(S^2)}{dt} = 2(d_1 - v_1 t)(-v_1) + 2(d_2 - v_2 t)(-v_2) = 0$$

$$t = \frac{d_1 v_1 + d_2 v_2}{v_1^2 + v_2^2}$$

15. Let  $d$ : distance between two points  
 $v$ : velocity of motorboat in still water

$u$ : velocity of river flow

Along the flow :  $v_{b/g} = v + u$

$$\frac{d}{v + u} = 4 \quad (i)$$

$$\Rightarrow v + u = \frac{d}{4}$$

(ii)

Opposite to the flow :  $v_{b/g} = v - u$

$$\frac{d}{v - u} = 8 \quad (iii) \quad (iv)$$

$$\Rightarrow v - u = \frac{d}{8}$$

Time taken boat to cover distance  $d$  in still water  $t_o = \frac{d}{v}$ . Adding (ii) and

(iv), we get

$$2v = \frac{d}{4} + \frac{d}{8} = \frac{3d}{8}$$

$$\frac{d}{v} = \frac{16}{3} h$$

The motorboat will cover the distance in  $\frac{16}{3} h$ .

16.  $v$ : Speed of boat with respect to river flow

$u$ : river flow speed

$v_{b/g} = v + u$  (along the flow)

$v_{b/g} = v - u$  (opposite to the flow)

Boat: AC-CB = 6

$$(v + u) \times 1 - (v - u)t = 6$$

$t$ : time taken by the boat from C to B

Object : AB = 6

$$u(t + 1) = 6$$

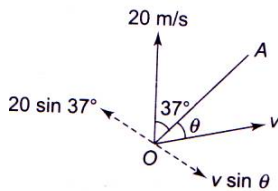
Solving  $u = 3 \text{ km/h}$

$$17. \text{ Boat A: } t_A = \frac{d}{v - u} + \frac{d}{v + u} = \frac{2dv}{v^2 - u^2}$$

$$\text{Boat B: } t_B = \frac{2d}{\sqrt{v^2 - u^2}}$$

$$\frac{t_A}{t_B} = \frac{v}{\sqrt{v^2 - u^2}} = \frac{\eta u}{\sqrt{(\eta u)^2 - u^2}} = \frac{\eta}{\sqrt{\eta^2 - 1}}$$

18. (a) If a particle moves in a straight line, the velocity of particle, perpendicular to straight line should be zero. If an aeroplane moves along OA, the pilot should head the plane towards  $\perp^{ar}$  to line OA:



$$v \sin \theta = 20 \sin 37^\circ = 12$$

$$\sin \theta = \frac{12}{v}$$

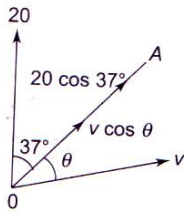
$$\theta = \sin^{-1}\left(\frac{12}{v}\right)$$

The pilot should head the plane at angle

$$\sin^{-1}\left(\frac{12}{v}\right) \text{ east of line OA.}$$

(b) Along line OA:

Time taken by plane to move from O to A:



$$t = \frac{d}{20^\circ \cos 37^\circ + v \cos \theta} = \frac{d}{16 + v \cos \theta}$$

19. Speed of car w.r.t. cars in opposite direction

$$v_r = (v + 30)$$

$$t = \frac{d}{v_r} \Rightarrow \frac{4}{60} = \frac{5}{v + 30}$$

$$4v + 120 = 300 \Rightarrow v = 45 \text{ km/h}$$

20.  $v_{A/B} = (v_1 - v_2)$

Let car A stop after travelling a distance s.

$$0 = (v_{A/B})^2 - 2as$$

$$s = \frac{(v_1 - v_2)^2}{2a}$$

For no collision,  $s < d$

$$\frac{(v_1 - v_2)^2}{2a} < d \quad \text{or} \quad d > \frac{(v_1 - v_2)^2}{2a}$$

21. After time t,

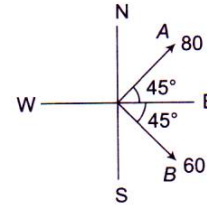
$$\vec{v}_A = (u - gt)\hat{j}$$

$$\vec{v}_B = -gt\hat{j}$$

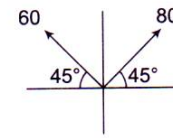
$$\vec{v}_A - \vec{v}_B = u\hat{j}$$

$$|\vec{v}_{A/B}| = u$$

22.

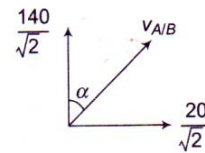


$$\vec{v}_{A/B} = \vec{v}_{A/g} - \vec{v}_{B/g} = \vec{v}_{A/g} + (-\vec{v}_{B/g})$$



$$v_x = 80 \cos 45^\circ - 60 \cos 45^\circ = \frac{20}{\sqrt{2}}$$

$$v_y = 80 \sin 45^\circ + 60 \sin 45^\circ = \frac{140}{\sqrt{2}}$$



$$\tan \alpha = \frac{20/\sqrt{2}}{140/\sqrt{2}} = \frac{1}{7}$$

$$\alpha = \tan^{-1}(1/7)$$

23.  $v_{b/s}$  is along north

$v_{b/s}$  is resultant of  $v_{b/g}$  and  $v_{s/g}$  in opposite direction

$$v \cos 30^\circ = v_{b/s}$$

$$v \sin 30^\circ = 15 \Rightarrow v = 30 \text{ km/h}$$

24.  $(v_{A/B})$  along x-axis  $= v + v \cos 60^\circ = \frac{3v}{2}$

$$t = \frac{d}{3v/2} = \frac{2d}{3v}$$

25.  $(v_{A/B})_x = v - 0 = v; t = \frac{d}{v}$

$$(v_{A/B})_x = v - v \cos 60^\circ = \frac{v}{2}$$

- 26.

$$t = \frac{d}{v/2} = \frac{2d}{v}$$

27. After time  $t$ ,

$$s^2 = (vt)^2 + (d - vt)^2$$

For  $s$  to be minimum,

$$\frac{ds}{dt} \quad \text{or}$$

$$\frac{d(s^2)}{dt} = v^2 \cdot 2t + s(d - vt)(-v) = 0$$

$$vt = (d - vt) \Rightarrow t = \frac{d}{2v}$$

$$s_{\min} = \sqrt{\left(\frac{d}{2}\right)^2 + \left(\frac{d}{2}\right)^2} = \frac{d}{\sqrt{2}}$$

28. Velocity of police jeep =  $45 \text{ km/h}$

$$= 45 \times \frac{5}{18} = \frac{25}{2} \text{ m/s}$$

Velocity of thief jeep =  $153 \text{ km/h}$

$$= 153 \times \frac{5}{18} = \frac{85}{2} \text{ m/s}$$

Velocity of bullet w.r.t. police jeep =  $180 \text{ m/s}$

Velocity of bullet w.r.t. police car =  $180 + \frac{25}{2}$

Velocity of bullet w.r.t thief jeep =  $180 + \frac{25}{2} - \frac{85}{2} = 150 \text{ m/s}$

- 29.

$$\vec{v}_{b/g} = 3\mathbf{i} + 4\mathbf{j}$$

$$\vec{v}_{w/g} = -3\mathbf{i} - 4\mathbf{j}$$

$$\vec{v}_{b/w} = \vec{v}_{b/g} - \vec{v}_{w/g} = 6\mathbf{i} + 8\mathbf{j}$$

30.  $(v_{m/r})_x = (v \sin \theta - u)$

To solve problems, convert all velocities with respect to the ground.

$$x : (v \sin \theta - u)t = 0 \Rightarrow \sin \theta = \frac{u}{v}$$

$$y : d = v \cos \theta t \Rightarrow t = \frac{d}{v \cos \theta}$$

$$= \frac{d}{v \sqrt{1 - \sin^2 \theta}} = \frac{d}{v \sqrt{1 - \frac{u^2}{v^2}}} = \frac{d}{\sqrt{v^2 - u^2}}$$

$$t = 15 \text{ min} = \frac{15}{60} = \frac{1}{4} \text{ h}$$

$$t = \frac{d}{\sqrt{v^2 - u^2}} \Rightarrow \frac{1}{4} = \frac{1}{\sqrt{5^2 - u^2}}$$

$$u = 3 \text{ km/h}$$

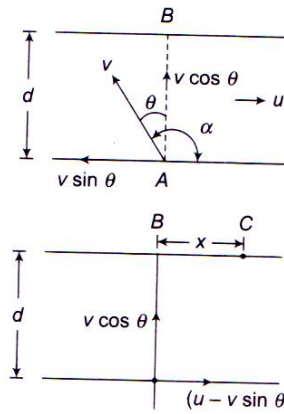
$$31. \quad y = \frac{1}{2} = \frac{3\sqrt{3}}{2} t \Rightarrow t = \frac{1}{3\sqrt{3}} \text{ h} = \frac{60}{3\sqrt{3}} = \frac{20}{\sqrt{3}} \text{ min}$$

- 32.

$$y : d = v \cos \theta t \Rightarrow t = \frac{d}{v \cos \theta}$$

$$x : x = (u - v \sin \theta)t$$

$$= (u - v \sin \theta) \frac{d}{v \cos \theta} = \left( \frac{u}{v} \sec \theta - \tan \theta \right) d$$



For  $x$  to be minimum

$$\frac{dx}{d\theta} = d \left( \frac{u}{v} \sec \theta \tan \theta - \sec^2 \theta \right) = 0$$

$$\frac{u}{v} \tan \theta = \sec \theta$$

$$\sin \theta = \frac{v}{u}$$

$$\alpha = 90^\circ + \theta = \frac{\pi}{2} + \sin^{-1} \left( \frac{v}{u} \right)$$

$$33. \quad t_1 = \frac{d}{v_{br}}, t_2 = \frac{d}{v_{br} \sin \theta} \therefore \frac{t_1}{t_2} = \sin \theta$$

34. Let  $L$  be the length of escalator. Speed of man w.r.t. escalator is

$$v_{mc} = \frac{L}{t_1}$$

$$\text{Speed of escalator is } v_c = \frac{L}{t_2}$$

$\therefore$  Speed of man w.r.t. ground would be

$$v_m = v_{mc} + v_c = L \left( \frac{1}{t_1} + \frac{1}{t_2} \right)$$

$$\therefore \text{The Desired time is } t = \frac{L}{v_m} = \frac{t_1 t_2}{t_1 + t_2}$$

35. Let  $v$  be the river velocity and  $u$  the velocity of swimmer in still water. Then

$$t_1 = 2 \left( \frac{\omega}{\sqrt{u^2 - v^2}} \right)$$

$$t_2 = \frac{\omega}{u+v} + \frac{\omega}{u-v} = \frac{2u\omega}{u^2 - v^2}$$

$$\text{And } t_3 = \frac{2\omega}{u}$$

$$\text{Now, we can see that } t_1^2 = t_2 t_3$$

36. Absolute velocity of boatman is

$$\vec{v}_B = \vec{v}_{BR} + \vec{v}_R$$

$$= (-2\hat{i} + 4\hat{j}) + (4\hat{i}) = 2\hat{i} + 4\hat{j}$$

$$\text{Time } t = \frac{100}{4} = 25s$$

$$\text{Drift } x = (2)(25) = 50m$$

$$|\vec{v}_B| = \sqrt{(4)^2 + (2)^2} = 2\sqrt{5}m/s$$

- 37.

$$v_{r/g} = 6m/s, v_{m/g} = 8m/s$$

$$\vec{v}_{r/m} = \vec{v}_{r/g} - \vec{v}_{m/g} = \vec{v}_{r/g} + (-\vec{v}_{m/g})$$

$$v_{m/g} = 8m/s$$

$$v_{r/g} = 6m/s$$

$$v_{r/m} = \sqrt{(6)^2 + (8)^2} = 10m/s$$

$$\tan \alpha = \frac{6}{8} = \frac{3}{4} \Rightarrow \alpha = 37^\circ$$

$$\tan \beta = \frac{8}{6} = \frac{4}{3} \Rightarrow \beta = 53^\circ$$

Hence, the velocity of rain with respect to the man is 10 m/s, at an angle  $37^\circ$  with horizontal or at an angle  $53^\circ$  with vertical.

OR

$$\vec{v}_{r/g} = -6\hat{k}, \vec{v}_{m/g} = 8\hat{i}$$

$$\vec{v}_{r/m} = \vec{v}_{r/g} - \vec{v}_{m/g} = 6\hat{k} - 8\hat{i}$$

$$|\vec{v}_{r/m}| = v_{r/m} = \sqrt{(-6)^2 + (-8)^2} = 10m/s$$

38.  $v_{A/B} = 3v + v = 4v$ , now train B is at rest and train B is moving with velocity  $4v$ .

Time in which trains cross each other

$$t = \frac{l_A + l_B}{v_{A/B}} = \frac{100 + 200}{4v} = \frac{300}{4v}$$

$$15 = \frac{300}{4v} \Rightarrow v = 5m/s$$

$$v_A = 3v = 15m/s$$

$$v_B = v = 5m/s$$

39. To reach directly opposite, the velocity perpendicular to the line AB should be zero.

$$(v_{m/g})_x = 0.5 \sin 30^\circ - u = 0$$

$$u = 0.25m/s$$

40.  $\theta = \sin^{-1}(u/v) = \sin^{-1}(5/10) = 30^\circ$

$$41. H_1 = \frac{u^2 \sin^2 \theta}{2g}$$

$$\text{and } H_1 = \frac{u^2 \sin^2 (90 - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$$

$$H_1 H_2 = \frac{u^2 \sin^2 \theta}{2g} \times \frac{u^2 \cos^2 \theta}{2g} = \frac{(u^2 \sin 2\theta)^2}{16g^2} = \frac{R^2}{16}$$

$$\therefore R = 4\sqrt{H_1 H_2}$$

42. Standard equation of projectile motion

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

Comparing with given equation

$$A = \tan \theta \text{ and } B = \frac{g}{2u^2 \cos^2 \theta}$$

$$\text{So } \frac{A}{B} = \frac{\tan \theta \times 2u^2 \cos^2 \theta}{g} = 40$$

$$(As \theta = 45^\circ, u = 20m/s, g = 10m/s^2)$$

43. For same range angle of projection should be  $\theta$  and  $90 - \theta$

$$\text{So, time of flights } t_1 = \frac{2u \sin \theta}{g} \text{ and}$$

$$t_2 = \frac{2u \sin(90 - \theta)}{g} = \frac{2u \cos \theta}{g}$$

$$\text{By multiplying } t_1 t_2 = \frac{4u^2 \sin \theta \cos \theta}{g^2}$$



$$t_1 t_2 = \frac{2(u^2 \sin 2\theta)}{g} = \frac{2R}{g} \Rightarrow t_1 t_2 \propto R$$

$$44. \quad R = \frac{u^2 \sin 2\theta}{g} = \frac{2u_x u_y}{g}$$

$\therefore$  Range  $\propto$  horizontal initial velocity ( $u_x$ )

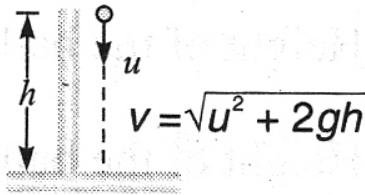
In path 4 range is maximum so football possess maximum horizontal velocity in this path.

45. If air resistance is taken into consideration then range and maximum height, both will decrease.

46. When particle is thrown in vertical downward direction with velocity  $u$ , then the final velocity of the ground level is

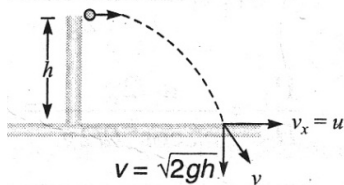
$$v^2 = u^2 + 2gh$$

$$\therefore v = \sqrt{u^2 + 2gh}$$



Another particle is thrown horizontally same velocity then at the surface of earth.

Horizontal component of velocity  $v_x = u$



Therefore, resultant velocity,

$$v = \sqrt{u^2 + 2gh}$$

Both the particles, final velocities when they reach the earth's surface are equal.

47. Equation of trajectory for oblique projectile motion

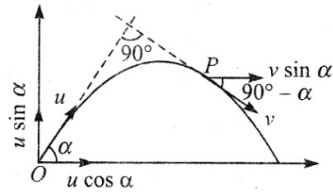
$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

Substituting  $x = S$  and  $u = v_0$

$$h = S \tan \theta - \frac{gS^2}{2u_0^2 \cos^2 \theta}$$

48. Horizontal velocity at point  $O = u \cos \alpha$

Horizontal velocity at point  $P = v \sin \alpha$



In projectile motion horizontal component of velocity remains constant throughout the motion

$$\therefore v \sin \alpha = u \cos \alpha$$

$$\Rightarrow v = u \cot \alpha$$

49. Let in 2 seconds, the body reaches upto point A and after one more second upto point B. Total time of ascent for a body

$$\text{is given 3 sec, i.e., } t = \frac{u \sin \theta}{g} = 3$$

$$\therefore u \sin \theta = 10 \times 3 = 30 \dots (i)$$

Horizontal component of velocity remains always constant

$$u \cos \theta = v \cos 30^\circ \dots (ii)$$

For vertical upward motion between point O and A

$$v \sin 30^\circ = u \sin \theta - g \times 2$$

$$v \sin 30^\circ = 30 - 20$$

$$\therefore v = 20 \text{ m/s.}$$

Substituting this value in equation (ii),

$$u \cos \theta = 20 \cos 30^\circ = 10\sqrt{3} \dots (iii)$$

From equation (i) and (iii),

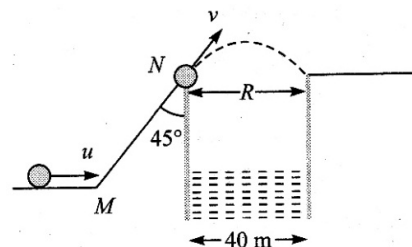
$$u = 20\sqrt{3} \text{ and } \theta = 60^\circ$$

50. At point N, the angle of projection of the body will be  $45^\circ$ . Let, the velocity of projection at this point is  $v$ . If the body just manages to cross the well, then Range = Diameter of well

$$\frac{v^2 \sin 2\theta}{g} = 40$$

$$[As \theta = 45^\circ]$$

$$v^2 = 400 \Rightarrow v = 20 \text{ ms}^{-1}$$



But we have to calculate the velocity ( $u$ ) of the body at point M

For motion along the inclined plane (from M to N)

Final velocity ( $v$ ) = 20 ms,

Acceleration ( $a$ ) =  $-g \sin \alpha = -g \sin 45^\circ$ ,

distance of inclined plane ( $s$ ) =  $20\sqrt{2}m$

$$(20) = u^2 - 2 \frac{g}{\sqrt{2}} \cdot 20\sqrt{2} \quad [\text{Using } v^2 = u^2 + 2as]$$

$$u^2 = 20^2 + 400 \Rightarrow 20\sqrt{2}m/s.$$

51.  $\vec{u} = u \cos \theta \vec{i} + u \sin \theta \vec{j}$  after time  $t$

$$\vec{v} = u \cos \theta \vec{i} + (u \sin \theta - gt) \vec{j}$$

Here  $\vec{u} \cdot \vec{v} = 0$

$$(u \cos \theta \vec{i} + u \sin \theta \vec{j}) \cdot (u \cos \theta \vec{i} + (u \sin \theta - gt) \vec{j}) = 0$$

$$u^2 \cos^2 \theta + u \sin^2 \theta - u \sin \theta gt = 0$$

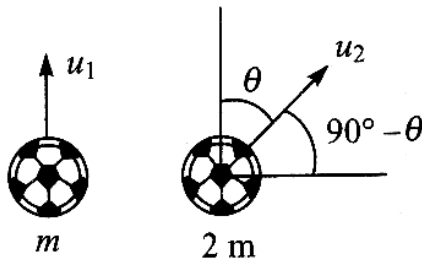
$$u^2 = u \sin \theta gt$$

$$t = \frac{u}{g \sin \theta} = \frac{u}{g} \operatorname{cosec} \theta$$

52. Time of flight for the ball thrown by Raju,  $T_1 = \frac{2u_1}{g}$  Time of flight for the ball thrown by Kiran

$$T_2 = \frac{2u^2 \sin(90^\circ - \theta)}{g} = \frac{2u^2 \cos \theta}{g}$$

According to problem  $T_1 = T_2$



$$\Rightarrow \frac{2u_1}{g} = \frac{2u^2 \cos \theta}{g}$$

$$\Rightarrow u_1 = u_2 \cos \theta$$

Height of the ball thrown by Pankaj

$$H_1 = \frac{u_1^2}{2g}$$

Height of the ball thrown by Sudhir

$$H_2 = \frac{u_2^2 \sin^2(90^\circ - \theta)}{2g} = \frac{u_2^2 \cos^2 \theta}{2g}$$

$$\therefore \frac{H_1}{H_2} = \frac{u_1^2 / 2g}{u_2^2 \cos^2 \theta / 2g} = 1 \quad [\text{As}]$$

$$u_1 = u_2 \cos \theta]$$

53. Time taken by packet to reach the

$$\text{ground: } T = \sqrt{\frac{2 \times 180}{10}} = 6s$$

Horizontal distance travelled by helicopter in this time =  $8 \times 6 = 48m$ .

Velocity of package w.r.t. ground =  $12 - 8 = 4m/s$  in backward direction.

Horizontal distance travelled by package in time,  $T = 4 \times 6 = 24m$ .

So, horizontal distance between them =  $48 + 24 = 72m$

54. Velocity at any time,

$$\vec{v} = \vec{u} + \vec{g}t \Rightarrow \vec{v} = u \cos \theta \vec{i} + (u \sin \theta - gt) \vec{j}$$

Let at any time, this velocity becomes perpendicular to initial velocity. Then

$$\vec{v} \cdot \vec{u} = 0$$

$$\text{Solve to get } t = \frac{u}{\sin \theta}$$

Now  $t$  should be less than/equal to time of flight. So  $t \leq T$ .

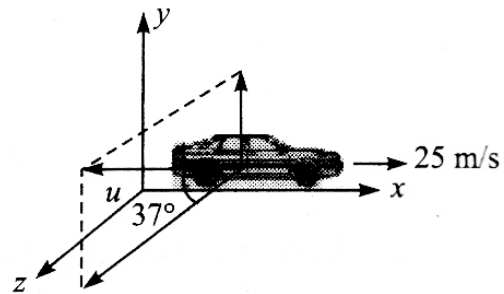
$$\frac{u}{g \sin \theta} \leq \frac{2u \sin \theta}{g} \Rightarrow \sin^2 \theta \geq \frac{1}{2} \Rightarrow \theta \geq 45^\circ$$

55. Velocity of the car,  $\vec{v}_c = 25\vec{i}$  Velocity of ball with respect to car

$$\vec{v}_{b/c} = u \sin 37^\circ \vec{j} + u \cos 37^\circ \vec{k} = 10 \times \frac{3}{5} \vec{j} + 10 \times \frac{4}{5} \vec{k} = 6\vec{j} + 8\vec{k}$$

Now : using  $\vec{v}_{b/c} = \vec{v}_b - \vec{v}_c$

$$\Rightarrow \vec{v}_b = \vec{v}_c + \vec{v}_{b/c} = 25\vec{i} + 6\vec{j} + 8\vec{k}$$



56. Let us make the components of initial velocity in the direction and perpendicular to the direction of acceleration as shown in the figure.

In the direction of acceleration:

$$\text{Using } v = u + at, \text{ we get } v_1 = u \cos 60^\circ + at$$

$$= 2 \times \frac{1}{2} + 2 \times 2 = 5m/s$$

Velocity perpendicular to direction of acceleration remains same, which is

$$V_2 = u \sin 60^\circ = \sqrt{3}m/s$$

So net velocity at the end of 2s,

$$V = \sqrt{V_1^2 + V_2^2}$$

$$v = \sqrt{v_1^2 + (\sqrt{3})^2} = \sqrt{5^2 + 3} = \sqrt{28} = 2\sqrt{7} \text{ m/s}$$

57.  $t_{AB}$  = time of flight of projectile

$$= \frac{2u \sin(\alpha - 30^\circ)}{g \cos 30^\circ}$$

Now the component of velocity along the plane becomes zero at point B.

$$V = u \cos(\alpha - 30^\circ) - g \sin 30^\circ \times T_{AB} \text{ or}$$

$$u \cos(\alpha - 30^\circ) = g \sin 30^\circ \times \frac{2u \sin(\alpha - 30^\circ)}{g \cos 30^\circ} \text{ or}$$

$$\tan(\alpha - 30^\circ) = \frac{\cot 30^\circ}{2} = \frac{\sqrt{3}}{2}$$

$$\alpha = 30^\circ + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

58. Since  $R = 2H$

$$\text{Or } \frac{v^2 \sin 2\theta}{g} = 2 \times \frac{v^2 \sin^2 \theta}{2g}$$

$$\text{Or } 2 \sin \theta \cos \theta = \sin^2 \theta \text{ Or } \tan \theta = 2$$

$$R = \frac{v^2 \sin 2\theta}{g}$$

$$= \frac{v^2 2 \sin \theta \cos \theta}{g} = \frac{2v^2}{g} \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{4v^2}{5g}$$

59.  $AC = R/2$ ,  $PC = H$

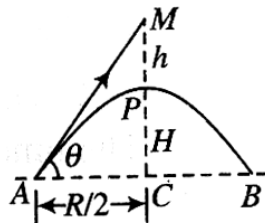
We have to find  $h = MP$

From the figure in previous question, we know that if  $H = R$ , then  $\tan \theta = 4$

$$\text{Now } \tan \theta = \frac{MC}{AC} = \frac{MP + PC}{AC}$$

$$4 = \frac{h + H}{R/2} \Rightarrow 4 = \frac{(h + H)2}{H}$$

$$\Rightarrow h = H$$



60.  $H = 100\text{m}$ ,  $R = 2 \times 200 = 400\text{m}$

$$\tan \theta = \frac{4H}{R} \Rightarrow \tan \theta = \frac{4 \times 100}{400} = 1$$

$$\Rightarrow \theta = 45^\circ$$

$$\left[ \therefore \frac{H}{R} = \frac{\tan \theta}{4} \right]$$

61. For the person to be able to catch the ball, the horizontal component of velocity of the ball should be same as the speed of the person, i.e.,

$$v_0 \cos \theta = \frac{v_0}{2} \text{ or } \cos \theta = \frac{1}{2} \text{ or } \theta = 60^\circ$$

62. Let  $u_x = 3\text{ms}^{-1}$ ,  $a_x = 0$

$$v_y = u_y + a_y t = 0 + 1 \times 4 = 4\text{ms}^{-1}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5\text{m/s}$$

Angle made by the resultant velocity w.r.t. direction of initial velocity, i.e., x-axis, is

$$\beta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \frac{4}{3}$$

63. Range = 150 =  $ut$  and  $h = \frac{15}{100} = \frac{1}{2} \times gt^2$

$$\text{Or } r^2 = \frac{2 \times 15}{100 \times g} = \frac{30}{100} \text{ or } t = \frac{\sqrt{3}}{10}$$

$$u = \frac{150}{t} = \frac{150 \times 10}{\sqrt{3}} = 500\sqrt{3}\text{ms}^{-1}$$

64. Given  $y = 12x - \frac{3}{4}x^2$ ,  $u_x = 3\text{ms}^{-1}$

$$v_y = \frac{dy}{dt} = 12 \frac{dx}{dt} - \frac{3}{2}x \frac{dx}{dt}$$

At

$$x = 0, v_y = u_y = 12 \frac{dx}{dt} = 12u_x = 12 \times 3 = 36\text{ms}^{-1}$$

$$a_y = \frac{d}{dt} \left( \frac{dy}{dt} \right) = 12 \frac{dx}{dt} - \frac{3}{2} \left[ \left( \frac{dx}{dt} \right)^2 + x \frac{d^2x}{dt^2} \right]$$

But  $\frac{d^2x}{dt^2} = a_x = 0$ . Hence

$$a_y = -\frac{3}{2} \left( \frac{dx}{dt} \right)^2 = -\frac{3}{2} u_x^2 = -\frac{3}{2} \times (3)^2 = -\frac{27}{2} \text{ms}^{-2}$$

$$\text{Range, } R = \frac{2u_x u_y}{a_y} = \frac{2 \times 3 \times 36}{27/2} = 16\text{m}$$

Alternatively: We have  $y = 12x - \frac{3}{4}x^2$ .

When projectile again comes to ground,  $y = 0$  and  $x = R$

$$0 = 12R - \frac{3}{4}R^2 \Rightarrow R = 16\text{m}$$

$$y = Ax - Bx^2$$

Or

$$12x = Ax; \quad A = 12$$

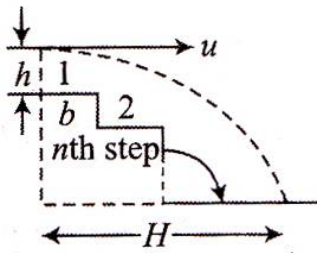
$$\frac{3}{4}x^2 = Bx^2; \quad B = \frac{3}{4}$$

$$\text{Range } R = \frac{A}{B} = 12 \times \frac{4}{3} = 16\text{m}$$

$$65. \frac{R}{T^2} = g \frac{\sin 2\theta}{4 \sin^2 \theta} = \frac{g}{2} \cot \theta = 5 \cot \theta$$

$$\text{Given } \frac{R}{T^2} = 5; \quad \text{Hence } 5 = 5 \cot \theta \quad \text{or}$$

$$\theta = 45^\circ$$



66.

If the ball hits the  $n^{\text{th}}$  step, then horizontal distance traversed  $= nh$ . Here, velocity along horizontal direction  $= u, n = ut$  (i)

Initial velocity along vertical direction is 0.

$$nh = 0 + \frac{1}{2}gt^2 \quad \text{(ii)}$$

From  $t = \frac{nb}{u}$ , putting  $t$  in (ii), we get

$$nh = \frac{1}{2}g \times \left(\frac{nb}{u}\right)^2 \text{ or } n = \frac{2hu^2}{gb^2}$$

$$67. v^2 = u^2 + 2gh \text{ or } u^2 = v^2 + 2gh$$

$$\text{Or } u_x^2 + u_y^2 = v_x^2 + v_y^2 + 2gh, u_x = v_x$$

$$\text{So, } u_y^2 = v_y^2 + 2gh \quad \text{or}$$

$$u_y^2 = (2)^2 + 2 \times 10 \times 0.4 = 12$$

$$u_y = \sqrt{12} = 2\sqrt{3}\text{ms}^{-1}, u_x = v_x = 6\text{ms}^{-1}$$

$$\tan \theta = \frac{u_y}{u_x} = \frac{2\sqrt{3}}{6} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

68. At the two points of the trajectory during projectile motion, the horizontal component of the velocity is same. Then

$$150 \times \frac{1}{2} = v \times \frac{1}{\sqrt{2}} \text{ or } v = \frac{150}{\sqrt{2}}\text{ms}^{-1}$$

$$\text{Initially: } u_y = u \sin 60^\circ = \frac{150\sqrt{3}}{2}\text{ms}^{-1}$$

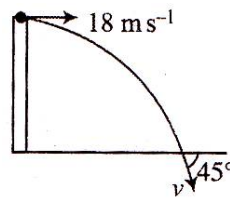
$$\text{Finally: } v_y = v \sin 45^\circ = \frac{150}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{150}{2}\text{ms}^{-1}$$

$$\text{But: } v_y = u_y + a_y t \quad \text{or} \quad \frac{150}{2} = \frac{150\sqrt{3}}{2} - 10t$$

$$10t = \frac{150}{2}(\sqrt{3} - 1) \text{ or } t = 7.5(\sqrt{3} - 1)$$

69. The motion of the train will affect only the horizontal component of the velocity of the ball. Since, vertical component is same for both observers,  $h_m$  will be same, but  $R$  will be different.

70.



$$v \cos 45^\circ = u = 18\text{ms}^{-1}$$

$$\Rightarrow v = 18\sqrt{2}\text{ms}^{-1}$$

Vertical component

$$v \sin 45^\circ = 18\sqrt{2} \times \frac{1}{\sqrt{2}} = 18\text{ms}^{-1}$$

$$71. v_x = u_x = 100\text{ms}^{-1}, v_y = u_y + a_y t = 0 + 10 \times 10$$

$$\tan \theta = \frac{v_y}{v_x} = \frac{100}{100} = 1 \Rightarrow \theta = 45^\circ$$

$$72. u_x = 16 \cos 60^\circ = 8\text{ms}^{-1}$$

Time taken to reach the wall  $= 8/8 = 1\text{ s}$

$$\text{Now } u_y = 16 \sin 60^\circ = 8\text{ms}^{-1}$$

$$h = 8\sqrt{3} \times 1 - \frac{1}{2} \times 10 \times 1 = 13.86 - 5 = 8.9\text{m}$$

$$73. h = (u \sin \theta)t - \frac{1}{2}gt^2$$

$$d = (u \cos \theta)t \text{ or } t = \frac{d}{u \cos \theta}$$

$$h = u \sin \theta \frac{d}{u \cos \theta} - \frac{1}{2}g \frac{d^2}{u^2 \cos^2 \theta}$$

$$u = \frac{d}{\cos \theta} \sqrt{\frac{g}{2(d \tan \theta - h)}}$$

74. Given  $\frac{\sqrt{3}u}{2} = u \cos \theta = \text{speed at maximum height}$

$$\text{Or } \cos \theta = \frac{\sqrt{3}}{2} \text{ or } \theta = 30^\circ$$

Given that  $PH_{\max} = R$

We know  $H_{\max} = \frac{R \tan \theta}{4}$

$$P = \frac{R}{H_{\max}} = \frac{4}{\tan 30^\circ} = 4\sqrt{3}$$

$$75. \frac{u^2 \sin 2\theta}{g} = \frac{(u/2)^2 \sin 30^\circ}{g} = \frac{u^2}{8g}$$

$$\therefore \sin 2\theta = \frac{1}{8} \text{ or } \theta = \frac{1}{2} \sin^{-1} \left( \frac{1}{8} \right)$$

76.

$$T = \frac{100}{25} = 4s \Rightarrow \frac{2u \sin \theta}{g} = 4 \Rightarrow u \sin \theta = 20ms^{-1}$$

$$77. H = \frac{u^2 \sin^2 \theta}{2g}, R = \frac{u^2 \sin 2\theta}{g}$$

(i) Maximum height will be same because acceleration  $a = g/4$  is in horizontal direction

$$R' = u \cos \theta T + \frac{1}{2} a T^2$$

$$= R + \frac{1}{2} \frac{g}{4} \left( \frac{2u \sin \theta}{g} \right)^2 = R + H$$

78. Horizontal component of velocity,

$$u_H = u \cos 60^\circ = \frac{u}{2}$$

$$AC = u_H \times t = \frac{ut}{2}$$

$$\text{And } AB = AC \sec 30^\circ = \left( \frac{ut}{2} \right) \left( \frac{2}{\sqrt{3}} \right) = \frac{ut}{\sqrt{3}}$$

$$79. \text{ Here, } v_x = \frac{dx}{dt} = 3\alpha t^2, v_y = 3\beta t^2$$

$$\therefore |v| = \sqrt{v_x^2 + v_y^2} = \sqrt{(3\alpha t^2)^2 + (3\beta t^2)^2} = 3t^2 \sqrt{\alpha^2 + \beta^2}$$

$$80. \text{ Here } R = \frac{u \sin 2\theta}{g} = \frac{(10)^2 \sin 60^\circ}{10} = 5\sqrt{3} = 8.66m$$

$$81. h_{\max} = \frac{u^2}{2g} = 10; u^2 = 200; R_{\max} = \frac{u^2}{g} = 20m$$

82. Time to reach the maximum height,

$$t_1 = \frac{u}{g}$$

If  $t_2$  be the time taken to hit the ground, then

$$-H = ut_2 - \frac{1}{2} g t_2^2$$

$$\text{But } t_2 = nt_1 (\text{given}) \Rightarrow -H = u \frac{nu}{g} - \frac{1}{2} g \frac{n^2 u^2}{g}$$

$$\Rightarrow 2gH = nu^2(n-2)$$

$$83. u \cos \theta = \sqrt{\frac{2}{3}} u \sqrt{\frac{1 + \cos^2 \theta}{2}}$$

$$\cos^2 \theta = \frac{2}{3} \left( \frac{1 + \cos^2 \theta}{2} \right)$$

$$3 \cos^2 \theta = 1 + \cos^2 \theta \Rightarrow \cos^2 \theta = \frac{1}{2}$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

84. For the projectile to pass through (30m, 40m)

$$40 = 30 \tan \alpha - \frac{g(30)^2}{2u^2} (1 + \tan^2 \alpha)$$

$$900 \tan^2 \alpha - (6u^2 \tan \alpha) + (900 + 8u^2) = 0$$

For real value of  $\alpha$

$$(6u^2)^2 \geq 3600(900 + 8u^2)$$

$$(u^4 - 800u^2) \geq 900000$$

$$(u^2 - 400)^2 \geq 250000$$

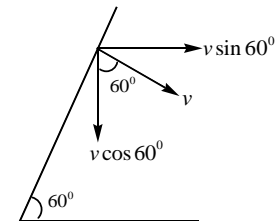
$$(u^2 - 400) \geq 500$$

$$u^2 \geq 900 \Rightarrow u \geq 30m/s$$

85. Let  $v$  be the velocity at the time of collision

$$\text{Then } u\sqrt{2} \cos 45^\circ = v \sin 60^\circ$$

$$u\sqrt{2} \left( \frac{1}{\sqrt{2}} \right) = v \left( \frac{\sqrt{3}}{2} \right) \Rightarrow v = \frac{2}{\sqrt{3}} u$$



$$86. \tan \theta = \frac{u \sin \theta}{u \cos \theta} = 2$$

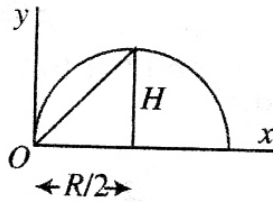
The desired equation is

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta} = 2x - \frac{10x^2}{2}$$

$$y = 2x - 5x^2$$

87.

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{\sqrt{H^2 + R^2}}{T/2}$$



$$H = \frac{u^2 \sin^2 \theta}{2g}, R = \frac{u^2 \sin 2\theta}{g}, T = \frac{2u \sin \theta}{g}$$

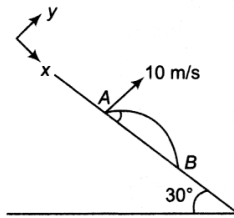
$$v_{av} = \frac{v}{2} \sqrt{1 + 3 \cos^2 \theta}$$

88. Two balls will meet if,  
 $(50 \cos 37^\circ) t_A = 120$  or  $t_A = 3$  s  
 Vertical component of A is also  $50 \sin 37^\circ$  or  $30$  m/s, so they will meet if thrown simultaneously.

$$h_A = h_B = ut - \frac{1}{2}gt^2 = 30 \times 3 - \frac{1}{2} \times 10 \times (3)^2 = 45 \text{ m}$$

89.

At B,  $S_y = 0$



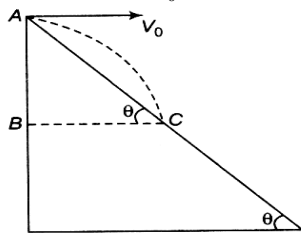
$$\therefore u_y t + \frac{1}{2} a_y t^2 = 0$$

$$\text{or } t = -\frac{2u_y}{a_y} = -\frac{2(10)}{-10 \times \sqrt{3}/2} = \frac{4}{\sqrt{3}} \text{ s}$$

$$\text{Now, } AB = R = \frac{1}{2} a_x t^2 = \frac{1}{2} \left( 10 \times \frac{1}{2} \right) \left( \frac{16}{3} \right) = 13.33 \text{ m}$$

90.

$$\frac{AB}{BC} = \frac{\frac{1}{2}gt^2}{v_0 t} = \tan \theta$$



$$\therefore t = \frac{2v_0 \tan \theta}{g}$$

$$\text{Now } x\text{-coordinate} = v_0 t = \frac{2v_0^2 \tan \theta}{g}$$

$$\text{and } y\text{-coordinate} = -\frac{1}{2}gt^2 = -\frac{2v_0^2 \tan^2 \theta}{g}$$

91. Horizontal range of bullet is 30 m.

Using range formula,

$$R = \frac{u^2 \sin 2\theta}{g} = 30$$

$$\text{Or } \sin 2\theta = \frac{30 \times 10}{(100)^2} \text{ Or } \sin 2\theta = 0.03$$

For small  $\theta$ ,

$$\sin \theta = \theta = \tan \theta, \text{ i.e., } 2\theta = 0.03$$

Therefore,  $\theta = 0.015$  and

$$\text{In } \triangle OAP, \tan \theta = \frac{AP}{OP} \Rightarrow AP = OP \tan \theta$$

The rifle must be aimed at an angle  $\theta = 0.015$  above horizontal.

Height to be aimed =

$$30 \tan \theta = 30(\theta) = 30 \times 0.015 = 45 \text{ cm}$$

92. We know that the range of projectile,

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(2u \sin \theta)}{g} \times u \cos \theta = \frac{2u_x u_y}{g}$$

$$As (u_y)_1 = (u_y)_2 \Rightarrow u_A \sin \theta_A = u_B \sin \theta_B$$

$$\text{Or } \begin{aligned} &= \left[ \frac{u_A \cos \theta_A}{u_B \cos \theta_B} \right] \\ &= \frac{\sin \theta_B \cos \theta_A}{\sin \theta_A \cos \theta_B} = \frac{\tan \theta_B}{\tan \theta_A} \end{aligned}$$

93. From graph (i):  $v_y = 0$  at  $t = \frac{1}{2} \text{ s}$ , i.e.,  
 time taken to reach maximum height H is

$$t = \frac{u_y}{g} = \frac{1}{2} \Rightarrow u_y = 5 \text{ ms}^{-1}$$

From graph (2):  $v_y = 0$  at  $x = 2 \text{ m}$ , i.e.,  
 when the particle is at maximum height, its displacement along horizontal,  $x = 2 \text{ m}$ .

$$x = u_x \times t \Rightarrow 2 = u_x \times \frac{1}{2} \Rightarrow u_x = 4 \text{ ms}^{-1}$$

94. Point P lies at the trajectory of jet of water. Hence, the coordinate of point P(x,y) should satisfy the trajectory equation.

$$y = x \tan \theta \left( 1 - \frac{x}{R} \right)$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(\sqrt{10})^2 \cdot \sin 2 \times 45^\circ}{10} = 1 \text{ m} \quad \text{--(i)}$$

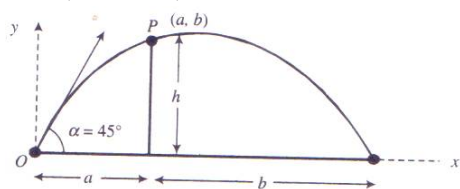
a. If  $x = 1/2 \text{ m}$  from (i)

$$y = \frac{1}{2} 45^\circ \left( 1 - \frac{1/2}{1} \right) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \text{ m}$$



Hence, the coordinate of

$$P = \left( \frac{1}{2}m, \frac{1}{4}m \right)$$

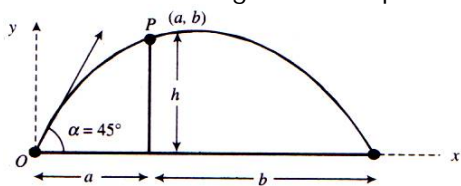


b. If  $x = 2m$ , from (i),

$$y = 2 \cdot \tan 45^\circ \left( 1 - \frac{2}{1} \right) = -1m$$

Hence, water jet will strike below the horizontal dotted line ( $x$ -axis) at coordinate  $(2m, 1m)$ .

95. Let  $h$  be the height of the pole.



$$\text{We have } y = x \tan \alpha \left( 1 - \frac{x}{R} \right)$$

Since the top of pole lies on curve (1),

$$h = a \tan 45^\circ \left( 1 - \frac{a}{a+b} \right) = a \left[ \frac{a+b-a}{a+b} \right] = \frac{ab}{a+b}$$

98. The initial velocity of the ball is in horizontal direction. The initial velocity in vertical direction is zero.

For vertical motion,  $u_x = 5 \text{ ms}^{-1}$ ,  $u_y = 0$ .

Displacement in vertical direction,  $y = -h = -20 \text{ m}$

$$-h = \frac{1}{2}gt^2 \Rightarrow \frac{2 \times 20}{10} = t^2 \Rightarrow t = 2 \text{ s}$$

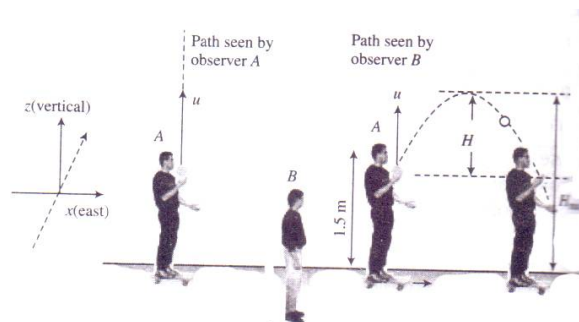
Horizontal displacement: Distance covered in 2s,  $x = d = u_x \times t = 5 \times 2 = 10 \text{ m}$

Since the length of one car is  $4 \text{ m}$ , it will hit the third car for which  $8 \text{ m} < d < 12 \text{ m}$ .

99. The path of the coin as seen from boy A is a straight line (up and down), but that seen from ground observer B is parabola. Velocity of coin,

$$\vec{v}_{\text{coin}} = \vec{v}_{\text{coin, boy}} + \vec{v}_{\text{boy}} \vec{v}_{\text{coin}} = 4\hat{i}3\hat{k}(\text{ms}^{-1})$$

Acceleration of coin,  $\vec{a}_{\text{coin}} = -10\hat{k}(\text{ms}^{-2})$



The displacement of coin in  $z$ -direction will be zero.

Using  $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$  in vertical direction ( $z$ -direction),

$$0 = (3T) - \frac{1}{2} \times 10 \times (T)^2$$

( $T$  = Time of flight of coin)

$$\text{Or } 0 = T(3 - 5T) \Rightarrow T = \frac{3}{5} \text{ s}$$

The displacement of the coin will be only  $x$ -direction. Let the displacement in  $x$ -direction be  $x$ .

$$\text{Then } x = v_x \times T = 4 \times \frac{3}{5} = \frac{12}{5} \text{ m}$$

100. ABC be the triangle with base BC. Let  $h$  be the height of the vertex A above BC. If AM is the perpendicular drawn on base BC from vertex A,

Then  $\tan \alpha = h/a$  and  $\tan \beta = h/b$

, where  $BM = a$  and  $CM = b$

Since  $A(a, h)$  lies on the trajectory of the projectile,  $y = x \tan \theta \left( 1 - \frac{x}{R} \right)$

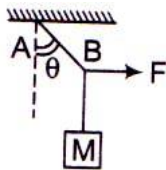
$$h = a \tan \theta \left( 1 - \frac{a}{a+b} \right)$$

$$\therefore \frac{h}{a} = \tan \theta \left[ \frac{b}{a+b} \right] \Rightarrow \tan \theta = \frac{h}{a} + \frac{h}{b} = \tan \alpha + \tan \beta$$

# Neet Medical Academy

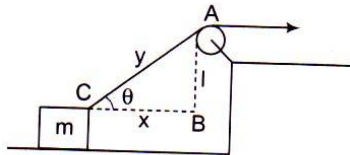
## Day-2 : Laws Of Motion

1. A mass  $M$  is suspended by a rope from a rigid support at  $B$  as shown in figure. Another rope is tied at the end  $B$ , and it is pulled horizontally with a force  $F$ . If the rope  $AB$  makes an angle  $\theta$  with the vertical in equilibrium, then the tension in the string  $AB$  is



- (1)  $F \sin \theta$  (2)  $F / \sin \theta$   
(3)  $F \cos \theta$  (4)  $F / \cos \theta$

2. A block is dragged on smooth plane with the help of a rope which moves with velocity  $v$ . The horizontal velocity of the block is



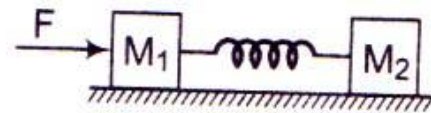
In  $\Delta ABC$ :

- (1)  $v$  (2)  $\frac{v}{\sin \theta}$   
(3)  $v \sin \theta$  (4)  $\frac{v}{\cos \theta}$

3. A force produces an acceleration of  $4ms^{-2}$  in a body of mass  $m_1$  and the same force produces an acceleration of  $6ms^{-2}$  in another body of mass  $m_2$ . If the same force is applied to  $(m_1 + m_2)$ , then the acceleration will be

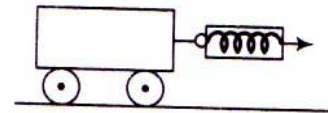
- (1)  $1.6ms^{-2}$  (2)  $2ms^{-2}$   
(3)  $2.4ms^{-2}$  (4)  $3.2ms^{-2}$

4. Two blocks of masses  $M_1$  and  $M_2$  are connected to each other through a light spring as shown in figure. If we push mass  $M_1$  with force  $F$  and cause acceleration  $a_1$  in mass  $M_1$ , what will be the magnitude of acceleration in  $M_2$ ?



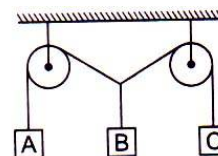
- (1)  $F / M_2$  (2)  $F / (M_1 + M_2)$   
(3)  $a_1$  (4)  $(F - M_1 a_1) / M_2$

5. A mass less spring balance is attached to 2kg trolley and is used to pull the trolley along a flat smooth surface as shown in the figure. The reading on the spring balance remains at 10kg during the motion. The acceleration of the trolley is (Use  $g = 9.8ms^{-2}$ )



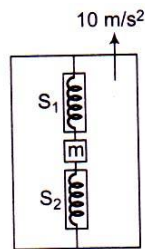
- (1)  $4.9ms^{-2}$  (2)  $9.8ms^{-2}$   
(3)  $49ms^{-2}$  (4)  $98ms^{-2}$

6. Three blocks A, B and C are suspended as shown in the figure. Mass of each block A and C is  $m$ . If the system is in equilibrium and mass of B is  $M$ , then



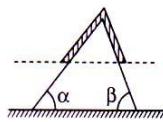
- (1)  $M = 2m$  (2)  $M < 2m$   
(3)  $M > 2m$  (4)  $M = m$

7. Reading shown in two spring balances  $S_1$  and  $S_2$  is 90kg and 30kg respectively and lift is accelerating upwards with acceleration  $10\text{ m/s}^2$ .  $S_1$  is elongated and  $S_2$  is compressed. The mass is stationary with respect to lift. Then the mass  $m$  of the block will be



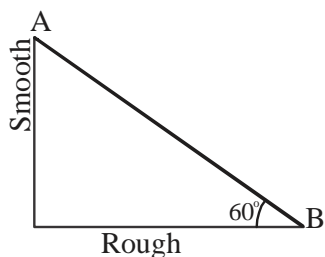
- (1) 60kg (2) 30kg  
(3) 120kg (4) None of these

8. A uniform rope of length  $L$  and



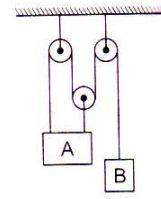
- (1) zero (2)  $(\cos \alpha - \cos \beta)g$   
(3)  $(\tan \alpha - \tan \beta)g$  (4)  $(\sin \alpha - \sin \beta)g$

9. A ladder AB of mass 10kg is held at rest against a smooth wall on rough ground as shown in figure. The normal reaction between the wall and the ladder is



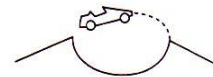
- (1) 50N (2)  $50\sqrt{3}\text{N}$   
(3)  $\frac{50}{\sqrt{3}}\text{N}$  (4) 100N

10. At a given instant, A is moving with velocity of  $5\text{ m/s}$  upwards. What is velocity of B at that time



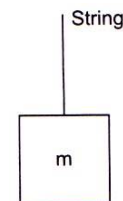
- (1)  $15\text{ m/s}$  ↓ (2)  $15\text{ m/s}$  ↑  
(3)  $5\text{ m/s}$  ↓ (4)  $10\text{ m/s}$  ↓

11. A stunt man jumps his car over a crater as shown



- (1) during the whole flight the driver experiences weightlessness  
(2) during the whole flight the driver never experiences weightlessness  
(3) during the whole flight the driver experiences weightlessness only at the highest point  
(4) The apparent weight increases during upwards journey

12. A block of mass  $m$  is attached with a mass less string. Breaking strength is  $4mg$ . Block is moving up. The maximum acceleration and maximum retardation of the block can be

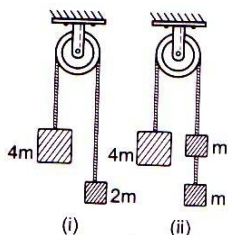


- (1)  $4g, 3g$  (2)  $4g, g$   
(3)  $3g, g$  (4)  $3g, 4g$

13. In order to raise a mass of 100kg a man of mass 60kg fastens a rope to it and passes the rope over a smooth pulley. He climbs the rope with an acceleration  $5g/4$  relative to rope. The tension in the rope is

(1) 1432 N                      (2) 928 N  
(3) 1218 N                      (4) 642 N

14. In the arrangement shown in figure, pulley is smooth and mass less and all the strings are light. Let  $F_1$  be the force exerted on the pulley in case (i) and  $F_2$  the force is case (ii). Then,



(1)  $F_1 > F_2$                       (2)  $F_1 < F_2$   
(3)  $F_1 = F_2$                       (4)  $F_1 = 2F_2$

15. An elastic spring has a length  $l_1$  when tension in it is 4 N. Its length is  $l_2$  when tension in it is 5 N. What will be its length when tension in it is 9 N ?

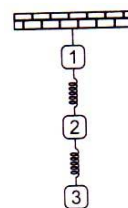
(1)  $5l_1 - 4l_2$                       (2)  $5l_2 - 4l_1$   
(3)  $4l_1 + 5l_2$                       (4)  $4l_2 + 5l_1$

16. The pulley shown in the diagram is frictionless. A cat of mass 1kg moves up on the mass less string so as to just lift a block of mass 2kg. After some time, the cat stops moving with respect to the string. The magnitude of the change in the cat's acceleration is



(1)  $g$                                       (2)  $\frac{g}{3}$   
(3)  $\frac{2g}{3}$                                       (4)  $\frac{4g}{3}$

17. Three identical blocks are suspended on two identical springs one below the other as shown in figure. If thread is cut that supports block 1, then initially

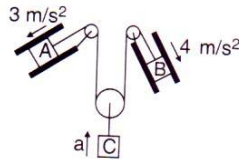


(1) the second ball falls with zero acceleration  
(2) the first falls with maximum acceleration  
(3) Both (1) and (2) are wrong  
(4) Both (1) and (2) are correct

18. The acceleration of a particle as seen from two frames  $S_1$  and  $S_2$  have equal magnitudes  $4m/s^2$

(1) the frames must be at rest with respect to each other  
(2) the frames may be moving with respect to each other but neither should be accelerated with respect to the other  
(3) the acceleration of  $S_2$  with respect to  $S_1$  may be either zero or  $8m/s$   
(4) the acceleration of  $S_2$  with respect to  $S_1$  may have any value between zero and  $8m/s$

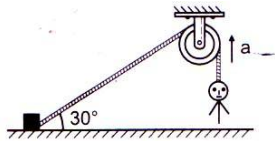
19. Assuming all the surfaces to be frictionless, acceleration of the block C shown in the figure is



- (1)  $5m/s^2$  (2)  $7m/s^2$   
 (3)  $3.5m/s^2$  (4)  $4m/s^2$
20. A lift of total mass  $M$  is raised by cables from rest to rest through a height  $h$ . The greatest tension which the cables can safely bear is  $n Mg$ . The maximum speed of lift during its journey if the ascent is to be made in shortest time is

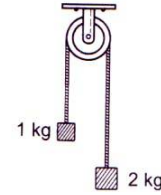
- (1)  $\sqrt{2gh\left(\frac{n+1}{n}\right)}$  (2)  $\sqrt{2ghn}$   
 (3)  $\sqrt{2gh\left(\frac{n}{n+1}\right)}$  (4)  $\sqrt{2gh\left(\frac{n-1}{n}\right)}$

21. A light string fixed at one end to a clamp on ground passes over a fixed pulley and hangs at the other side. It makes an angle of  $30^\circ$  with the ground. A monkey of mass  $5\text{ kg}$  climbs up the rope. The clamp can tolerate a vertical force of  $40\text{ N}$  only. The maximum acceleration in upward direction with which the monkey can climb safely is (Neglect friction and take  $g=10\text{ m/s}^2$ )

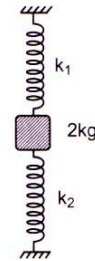


- (1)  $2m/s^2$  (2)  $4m/s^2$   
 (3)  $6m/s^2$  (4)  $8m/s^2$
22. Two unequal masses are connected on two sides of a light string passing over a light and smooth pulley as shown in figure. The system is

released from rest. The larger mass is stopped for a moment,  $1.0\text{ s}$  after the system is set into motion. The time elapsed before the string is tight again is ( $g=10\text{ m/s}^2$ )



- (1)  $1/4\text{ s}$  (2)  $1/2\text{ s}$   
 (3)  $2/3\text{ s}$  (4)  $1/3\text{ s}$
23. A  $2\text{ kg}$  block is connected with two springs of force constants  $k_1=100\text{ N/m}$  and  $k_2=300\text{ N/m}$  as shown in figure. The block is released from rest with the springs unstretched. The acceleration of the block in its lowest position is ( $g=10\text{ m/s}^2$ )



- (1) zero  
 (2)  $10\text{ m/s}^2$  upwards  
 (3)  $10\text{ m/s}^2$  downwards  
 (4)  $5\text{ m/s}^2$  upwards
24. A plumb bob is hung from the ceiling of a train compartment. The train moves on an inclined track of inclination  $30^\circ$  with horizontal. Acceleration of train up the plane is  $a = g/2$ . The angle which the string supporting the bob makes with normal to the ceiling in equilibrium is

(1)  $30^\circ$  (2)  $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(3)  $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$  (4)  $\tan^{-1}(2)$

25. A balloon carrying some sand is moving down with a constant acceleration  $a_0$ . How much mass of sand should be removed, so that the balloon move up with same acceleration?

(1)  $\frac{Ma_0}{a_0 + g}$  (2)  $\frac{3Ma_0}{a_0 + g}$

(3)  $\frac{4Ma_0}{a_0 + g}$  (4)  $\frac{2Ma_0}{a_0 + g}$

26. A boy sitting on the topmost berth in the compartment of a train, which is just going to stop on a railway station, drops an apple aiming at the open hand of his brother sitting vertically below his hands at a distance of about 2 meter. The apple will fall

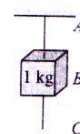
- (1) precisely on the hand of his brother  
 (2) slightly away from the hand of his brother in the direction of motion of the train  
 (3) slightly away from the hand of his brother in the direction opposite to the direction of motion of the train  
 (4) none of the above

27. A person sitting in an open car moving at constant velocity throws a ball vertically up into air. The ball falls.

- (1) outside the car

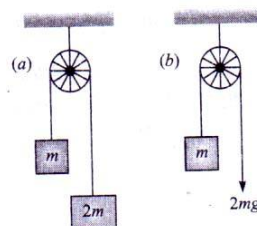
- (2) in the car ahead of the person  
 (3) in the car to the side of the person  
 (4) exactly in the hand which threw it up

28. A mass of 1kg is suspended by a string A. Another string C is connected to its lower end as shown in the figure. If a sudden jerk is given to C, then



- (1) the portion AB of the string will break  
 (2) the portion BC of the string will break  
 (3) none of the strings will break  
 (4) the mass will start rotating

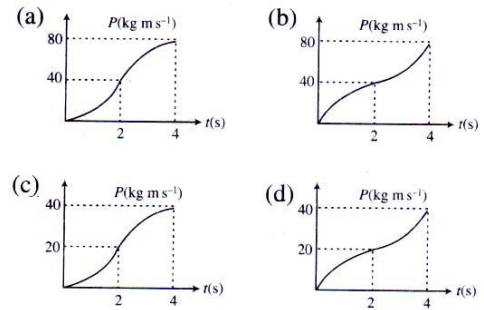
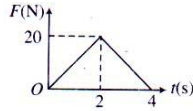
29. The two pulley arrangements as shown in figure are identical. The mass of the rope is negligible. In (a) the mass  $m$  is lifted up by attaching a mass  $2m$  to the other end of the rope. In (b)  $m$  is lifted up by pulling the other end of the rope with a constant downward force of  $2mg$ . The ratio of accelerations in two cases will be



- (1) 1 : 1 (2) 1 : 2  
 (3) 1 : 3 (4) 1 : 4



30. As shown in figure the variation of force acting on a body with time. Assuming the body to start from rest, the variation of its momentum with time is best represented by which plot?



## ANSWERS

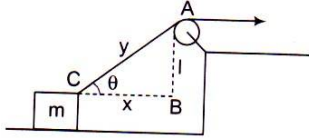
|       |       |       |       |       |       |       |       |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1) 2  | 2) 4  | 3) 3  | 4) 4  | 5) 3  | 6) 2  | 7) 1  | 8) 1  | 9) 3  | 10) 1 |
| 11) 1 | 12) 3 | 13) 3 | 14) 3 | 15) 2 | 16) 3 | 17) 4 | 18) 4 | 19) 3 | 20) 4 |
| 21) 3 | 22) 4 | 23) 2 | 24) 2 | 25) 4 | 26) 2 | 27) 4 | 28) 2 | 29) 3 | 30) 3 |

## Solutions

1.  $T \cos \theta = Mg$  and  
 $T \sin \theta = F$

$$T = \frac{F}{\sin \theta}$$

2.  $AB = \text{constant} = l$  (say)



$$\text{In } \Delta ABC: \\ y^2 = x^2 + l^2$$

Differentiating w.r.t time, we have

$$2y \left( \frac{dy}{dt} \right) = 2x \left( \frac{dx}{dt} \right) + 0$$

$$\therefore \left( \frac{dx}{dt} \right) = \left( \frac{y}{x} \right) \left( \frac{dy}{dt} \right)$$

$$\text{Or } v_m = \frac{v}{\cos \theta}$$

3.  $m_1 = \frac{F}{4}$  and  $m_2 = \frac{F}{6}$

With both

$$a = \frac{F}{m_1 + m_2} = \frac{F}{(F/4) + (F/6)} = 2.4 \frac{m}{s^2}$$

4.  $F - kx = M_1 a_1$

$$\therefore kx = F - M_1 a_1$$

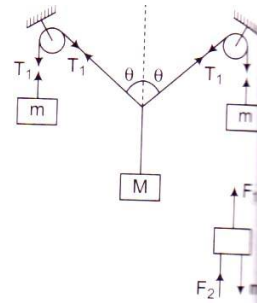
Only force  $M_2$  is  $kx$ .

$$\therefore a_2 = \frac{kx}{M_2} = \frac{F - M_1 a_1}{M_2}$$

5. Reading of spring balance means tension (T) or spring force (kx).

$$\therefore a = \frac{T}{m} = \frac{10g}{2} = 5g = 5 \times 9.8 = 49 \frac{m}{s^2}$$

- 6.



$$T_1 = mg$$

$$2T_1 \cos \theta = Mg \text{ or } 2(mg) \cos \theta = Mg$$

$$\text{or } M = 2m \cos \theta$$

$$\therefore M < 2m \text{ as } \cos \theta < 1$$

7.  $F_1 + F_2 - mg = ma$

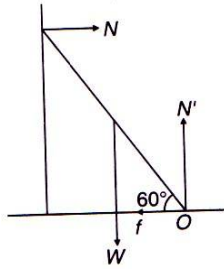
$$\therefore m = \frac{F_1 + F_2}{(g + a)} = \frac{(90 + 30)g}{(g + a)} = \frac{120 \times 10}{10 + 10} = 60 \text{ kg}$$

8. Vertical height on both sides is same.

9.  $\sum M_0 = 0$ . Moments of  $N'$  and  $f$  are already zero.

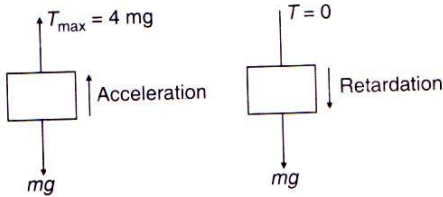
$$\therefore N \times l \sin 60^\circ = W \left( \frac{l}{2} \cos 60^\circ \right)$$

$$\text{or } N = 100 \times \frac{1}{2} \times \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{50}{\sqrt{3}} N$$



10.  $T_A = 3T$  and  $T_B = T$  In such problems,  $v \propto \frac{1}{T}$   
 $T_B$  is one third. Therefore  $v_B$  will be three times.

12. Maximum acceleration  $= \frac{4mg - mg}{m} = 3g$



$$\text{Maximum retardation} = \frac{mg}{m} = g$$

13. Let  $T$  be the tension in the rope and  $a$  the acceleration of rope. The absolute acceleration of man is therefore  $\left( \frac{5g}{4} - a \right)$ .

Equations of motion for mass and man gives

$$T - 100g = 100a$$

$$T - 60g = 60 \left( \frac{5g}{4} - a \right)$$

$$T = 1218N$$

14. Acceleration of the system and hence tension ( $T$ ) on the string attached with the pulley is same in both the cases. Hence  $F_1 = F_2 = 2T$

15. Let  $l$  be the natural length of spring. Then,  
 $4 = k(l_1 - l)$   
 ( $k$  = spring constant) and  $5 = k(l_2 - l)$   
 Solving these two equations we get,  
 $l = 5l_1 - 4l_2$  and  $k = \frac{1}{l_2 - l_1}$

Now let  $l_3$  be the length of spring when tension in the spring is 9N. Then,

$$9 = k(l_3 - l)$$

Substituting the values

$$9 = \frac{1}{l_2 - l_1} (l_3 - 5l_1 + 4l_2)$$

$$\text{or } 9l_2 - 9l_1 = l_3 - 5l_1 + 4l_2$$

$$\text{or } l_3 = 5l_2 - 4l_1$$

16. In first case,  $T_1 = 2g$

$$\therefore a_1 = \frac{T_1 - 1 \times g}{1}$$

$$= \frac{2g - g}{1} = g$$

In second case,

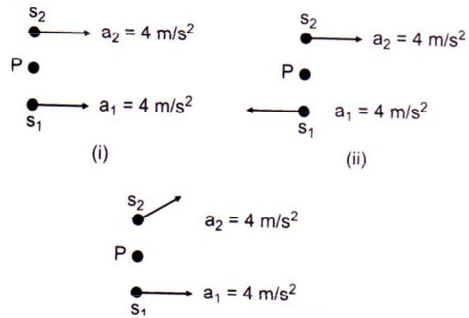
$$a_2 = \frac{2g - 1 \times g}{1 + 2} = \frac{g}{3}$$

$$\therefore |\Delta a| = a_1 - a_2 = \frac{2g}{3}$$

17. In equilibrium, free body diagrams of all the three blocks are as shown in the given figure.

$kx = mg$  when the thread is cut,  $T=0$   $a_1$  is maximum while  $a_2$  and  $a_3$  is zero.

- 18.



(iii) In all the three cases particle  $P$  is at rest while the frames  $S_1$  and  $S_2$  are moving with accelerations  $a_1$  and  $a_2$  in the directions shown in figure. In all three cases acceleration of particle as seen from frames  $S_1$  and  $S_2$  is  $4m/s^2$ , while relative acceleration between  $S_1$  and  $S_2$  is zero in case (i),  $8m/s^2$  in case (ii) and between 0 and  $8m/s^2$  in case (iii).

19.  $a_c = \frac{a_A + a_B}{2}$

Maximum tension =  $nMg$   
 $\therefore$  Maximum acceleration =  

$$\frac{nMg - Mg}{M} = (n-1)g \text{ and}$$

Maximum retardation =  $g$   
 Corresponding velocity- time graph for shortest time will be as shown.

$$\text{Here } (n-1)g = \frac{v_m}{t_1} \text{ or } t_1 = \frac{v_m}{(n-1)g} \text{ -----(1)}$$

$$\text{And } g = \frac{v_m}{t_2} \text{ or } t_2 = \frac{v_m}{g} \text{ -----(2)}$$

Area under v-t graph is total displacement  $h$ .

$$\text{Hence, } h = \frac{1}{2}(t_1 + t_2)v_m \text{ -----(3)}$$

From eqs.(1) (2) and (3) we get,

$$v_m = \sqrt{2gh \left( \frac{n-1}{n} \right)}$$

21. Let  $T$  be the tension in the string. The upward force exerted on the clamp =

$$T \sin 30^\circ = \frac{T}{2}$$

$$\text{Given } \frac{T}{2} = 40N$$

$$T = 80N$$

If  $a$  is the acceleration of monkey in upward direction. Equation of monkey in upward direction is

$$a = \frac{T - mg}{m} = \frac{80 - (5)(10)}{5} = 6m/s^2$$

22. Net pulling force =  $2g - 1g = 10N$

Mass being pulled =  $2 + 1 = 3kg$

$\therefore$  Acceleration of the system is

$$a = \frac{10}{3} m/s^2$$

$\therefore$  Velocity of both the blocks at  $t=1s$  will be

$$v_0 = at = \left( \frac{10}{3} \right)(1) = \frac{10}{3} m/s.$$

Now, at this moment velocity of 2kg block becomes zero, while that of 1kg block is

$\frac{10}{3} m/s$  upwards. Hence string becomes

tight again when, displacement of 1kg block = displacement of 2kg block.

$$v_0 t - \frac{1}{2} g t^2 = \frac{1}{2} g t^2 \Rightarrow t = \frac{v_0}{g} = \frac{10/3}{10} = \frac{1}{3} s$$

23. Let  $x$  be the maximum displacement of block downwards. Then from conservation of mechanical energy in decrease potential energy of 2kg block = increase in elastic potential energy of both the springs

$$\therefore mgx = \frac{1}{2}(k_1 + k_2)x^2$$

$$x = \frac{2mg}{k_1 + k_2} = \frac{(2)(2)(10)}{100 + 300} = 0.1m$$

Acceleration of block in this position is

$$T_1 = 2g$$

$$a = \frac{(k_1 + k_2)x - mg}{m}$$

$$= \frac{(400)(0.1) - (2)(10)}{2} = 10m/s^2 \text{ (upwards)}$$

24. Drawing free body diagram bob Let  $m$  be the mass of the bob. Then,

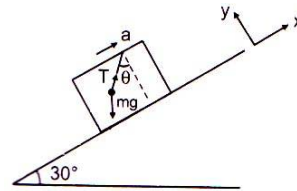
$$\sum F_y = 0$$

$$\therefore T \cos \theta = mg \cos 30^\circ$$

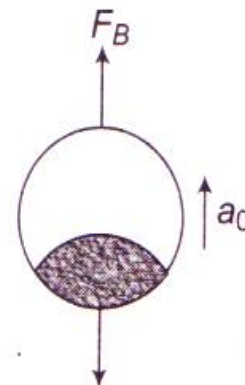
$$\sum F_x = ma$$

$$T \sin \theta - mg \sin 30^\circ = m \frac{g}{2}$$

Solving (1) and (2), we get  $\theta = \tan^{-1}(2\sqrt{3})$

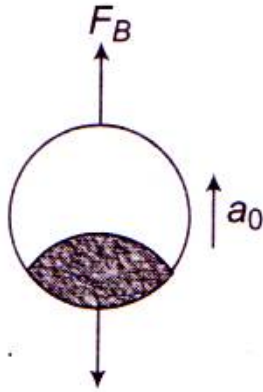


25.  $Mg - F_B = Ma_0$  ----- (i)



$F_B$ : upthrust

Let the mass removed be  $m$ .



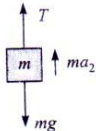
$$F_B - (M - m)g = (M - m)a_0 \text{-----(ii)}$$

Adding (i) and (ii), we get

$$m = \frac{2Ma_0}{a_0 + g}$$

26. The horizontal velocity of apple will remain same but due to the retardation of train, the velocity of train and hence the velocity of boy w.r.t. ground decreases. So apple falls away from the hand of boy in the direction of motion of the train.
27. The horizontal velocities of ball and person are same. So both will cover equal horizontal distance in a given interval of time, and after following the parabolic path, the ball falls exactly in the hand which threw it up.
28. When a sudden jerk is given to C, an impulsive tension exceeding the breaking tension develops in C first, which breaks before this impulse can reach A as a wave through block.
29. 
$$a_1 = \frac{m_2 - m_1}{m_1 + m_2} g = \left( \frac{2m - m}{m + 2m} \right) g = \frac{g}{3}$$

For second case,



From free body diagram of m,

$$ma_2 = T - mg$$

$$ma_2 = 2mg - mg$$

$$\therefore a_2 = g$$

$$\therefore \frac{a_1}{a_2} = \frac{g/3}{g} = 1/3$$

30. From 0 to 2s: at any time t,  $F = 10t$   
 $\Rightarrow a = F/m = 10t/m$

$$\Rightarrow \int_0^v dv = \int_0^t \frac{10t}{m} dt \Rightarrow v = \frac{5t^2}{m}$$

$$\text{Momentum : } P = mv = 5t^2$$

$$\text{At } t = 2s, P = 5(2)^2 = 20 \text{ kg ms}^{-1}, v = 20/m$$

$$\text{From 2 to 4s : } F = 40 - 10t$$

$$\int_{20/m}^v dv = \int_2^t \frac{40 - 10t}{m} dt \Rightarrow v = \frac{1}{m} [40t - 40 - 5t^2]$$

$$P = mv = 40t - 40 - 5t^2$$