Capacitance

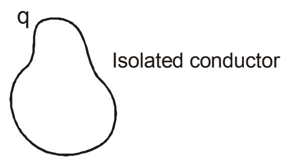
Section A - Capacitance Calculation (Sphere, Cylinderical, Parallel plate), Sharing of Charge, Combination of Capacitor

1. INTRODUCTION

A capacitor can store energy in the form of potential energy in an electric field. In this chapter we'll discuss the capacity of conductors to hold charge and energy.

1.1 Capacitance of an isolated conductor

When a conductor is charged its potential increases. It is found that for an isolated conductor (conductor should be of finite dimension, so that potential of infinity can be assumed to be zero) potential of the conductor is proportional to charge given to it.



q = charge on conductor

V = potential of conductor

$q \propto V \quad \Rightarrow \quad q = CV$

Where C is proportionally constant called capacitance of the conductor.

1.2 Definition of capacitance :

Capacitance of conductor is defined as charge required to increase the potential of conductor by one unit.

1.3 Important point about the capacitance of an isolated conductor:

- It is a scalar quantity.
- Unit of capacitance is farad in SI unis and its dimensional formula is $M^{-1}L^{-2}I^{2}T^{4}$
- **1 Farad :** 1 Farad is the capacitance of a conductor for which 1 coulomb charge increases potential by 1 volt.

$$1 \text{ Farad} = \frac{1\text{Coulomb}}{1 \text{ Volt}}$$

$1\ \mu F = 10^{-6}$ F, 1nF = 10^{-9} F or 1 pF = 10^{-12} F

- **Capacitance of an isolated conductor depends on following factors :**
  - **(a) Shape and size of the conductor:**
    On increasing the size, capacitance increase.
  - **(b) On surrounding medium :**
    With increase in dielectric constant K, capacitance increases.
  - **(c) Presence of other conductors:**
    When a neutral conductor is placed near a charged conductor capacitance of conductors increases.
- Capacitance of a conductor does not depend on
  - (a) Charge on the conductor
  - (b) Potential of the conductor
  - (c) Potential energy of the conductor.

1.4 Capacitance of an isolated Spheical Conductor.

EXAMPLE 01

Find out the capacitance of an isolated spherical conductor of radius R.

Sol. Let there is charge Q on sphere.

$\therefore$ Potential $V = \dfrac{KQ}{R}$

Hence by formula : Q = CV

$$Q = \frac{CKQ}{R}$$

$$C = 4\pi\varepsilon_0 R$$

(i) If the medium around the conductor is vacuum or air :

$$C_{vacuum} = 4\pi\varepsilon_0 R$$

R = Radius of spherical conductor. (may be solid or hollow)

(ii) If the medium around the conductor is a dielectric of constant K from surface of sphere to infinity then

$$C_{medium} = 4\pi\varepsilon_0 KR$$

(iii) $\frac{C_{medium}}{C_{air / vaccum}} =$ K = dielectric constant.

2. CAPACITOR :

A capacitor or condenser consists of two conductors separated by an insulator or dielectric.

(i) When uncharged conductor is brought near to a charged conductor, the charge on conductors remains same but its potential dcreases resulting in the increase of capacitance.

(ii) In capacitor two conductors have equal but opposite charges.

(iii) The conductors are called the plates of the capacitor. The name of the capacitor depends on the shape of the capacitor.

(iv) Formulae related with capacitors:

(a) Q = CV

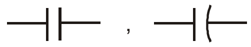
$$\Rightarrow \quad C = \frac{Q}{V} = \frac{Q_A}{V_A - V_B} = \frac{Q_B}{V_B - V_A}$$

Q = Charge of positive plate of capacitor.

V = Potential difference between positive and negative plates of capacitor

C = Capacitance of capacitor.

(v) The capacitor is represented as following :



(vi) Based on shape and arrangement of capacitor plates there are various types of capacitors:

(a) Parallel plate capacitor

(b) Spherical capacitor.

(c) Cylindrical capacitor

(v) Capacitance of a capacitor depends on

(a) Area of plates.

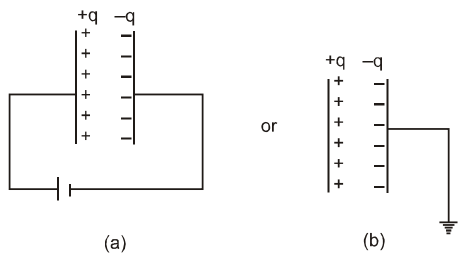
(b) Distance between the plates.

(c) Dielectric medium between the plates.

2.1 Parallel Plate Capacitor

Two metallic parallel plates of any shape but of same size and separated by small distance constitute parallel plate capacitor. Suppose the area of each plate is A and the separation between the two plates is d. Also assume that the space between the plates contains vacuum.

We put a charge q on one plate and a charge –q on the other. This can be done either by connecting one plate with the positive terminal and the other with negative plate of a battery (as shown in figure a) or by connecting one plate to the earth and by giving a charge +q to the other plate only. This charge will induce a charge – q on the earthed plate. The charges will appear on the facing surfaces. The charges density on each of these surfaces has a magnitude $\sigma = q/A$.



If the plates are large as compard to the separation between them, then the electric field between the plates (at point B) is uniform and perpendicular to the plates except for a small region near the edge. The magnitude of this uniform field E may be calculated by using the fact that both positive and negative plates produce the electric field in the same direction (from positive plate towards negative plate) of magnitude $\sigma/2\varepsilon_0$ and therefore, the net electric field between the plates will be,

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

Outside the plates (at point A and C) the field due to positive sheet of charge and negative sheet of charge are in opposite directions. Therefore, net field at these points is zero.

The potential difference between the plates is,

$$\therefore \quad V = E.d = \left(\frac{\sigma}{\varepsilon_0}\right) d = \frac{qd}{A\varepsilon_0}$$

$\therefore$ The capacitance of the parallel plate capacitor is,

$$C = \frac{q}{V} = \frac{A\varepsilon_0}{d} \quad \text{or} \quad C = \frac{\varepsilon_0 A}{d}$$

2.2 Cylindrical Capacitor

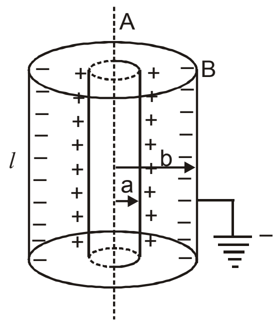
Cylindrical capacitor consists of two co-axial cylinders of radii a and b and length l. If a charge q is given to the inner cylinder, induced change –q will reach the inner surface of the outer cylinder. By symmetry, the electric field in region between the cylinders is radially outwards.

By Gauss's theorem, the electric field at a distance r from the axis of the cylinder is given by

$$E = \frac{1}{2\pi\varepsilon_0 l}\frac{q}{r}$$

The potential difference between the cylinders is given by

$$V = -\int_b^a \vec{E}\,\vec{dr} = -\frac{1}{2\pi\varepsilon_0 l} q \int_b^a \frac{dr}{r} = \frac{-q}{2\pi\varepsilon_0 l}\left(\ln\frac{a}{b}\right)$$



$$\text{or,}\quad C = \frac{q}{V} = \frac{2\pi\varepsilon_0 l}{\left(\ln\frac{a}{b}\right)}$$

2.3 Spherical Capacitor

A spherical capacitor consists of two concentric spheres of radii a and b as shown. The inner sphere is positively charged to potential V and outer sphere is at zero potential.

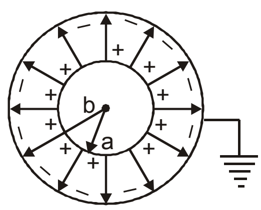
The inner surface of the outer sphere has an equal negative charge.

The potential difference between the spheres is

$$V = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}$$

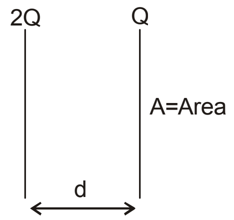
Hence, capacitance

$$C = \frac{Q}{V}\frac{4\pi\varepsilon_0 ab}{(b-a)}$$



EXAMPLE 02

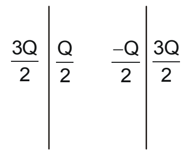
Find capacitance of the given system.



Sol. Arranging charges

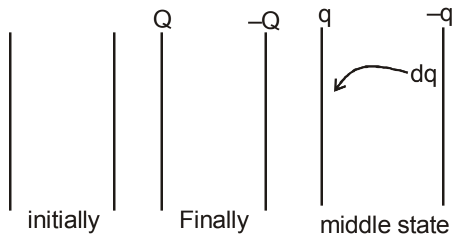
$$\Rightarrow \quad E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

$$\text{Now, } V = Ed = \frac{Qd}{2A\varepsilon_0}$$



$$\Rightarrow \quad C = \frac{Q/2}{V} = \frac{A\varepsilon_0}{d}$$

3. ENERGY STORED IN A CHARGED CAPACITOR



Work has to be done in charging a conductor against the force of repulsion by the already existing charges on it. The work is stored as a potential energy in the electric field of the conductor. Suppose a conductor of capacity C is charged to a potential V_0 and let q_0 be the charge on the conductor at this instant. The potential of the conductor when (during charging) the charge on it was q (< q_0) is,

$$V = \frac{q}{C}$$

Now, work done in bringing a small charge dq at this potential is,

$$dW = Vdq = \left(\frac{q}{C}\right)dq$$

$\therefore$ total work done in charging it from 0 to q_0 is,

$$W = \int_0^{q_0} dW = \int_0^{q_0} \frac{q}{C} dq = \frac{1}{2}\frac{q_0^2}{C}$$

This work is stored as the potential energy,

$$\therefore \quad U = \frac{1}{2}\frac{q_0^2}{C}$$

Further by using $q_0 = CV_0$ we can write this expression also as,

$$U = \frac{1}{2}CV_0^2 = \frac{1}{2}q_0V_0$$

In general if a conductor of capacity C is charged to a potential V by giving it a charge q, then

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV$$

3.1 Energy Density of a Charged Capacitor

This energy is localized on the charges or the plates but is distributed in the field. Since in case of a parallel plate capacitor, the electric field is only between the plates, i.e., in a volume (A × d), the energy density

$$U_E = \frac{U}{\text{volume}} = \frac{\frac{1}{2}CV^2}{A \times d} = \frac{1}{2}\left[\frac{\varepsilon_0 A}{d}\right]\frac{V^2}{Ad}$$

or $$U_E = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2 \quad \left[\because \frac{v}{d} = E\right]$$

3.2 Calculation of Capacitance

The method for the calculation of capacitance involves integration of the electric field between two conductors or the plates which are just equipotential surfaces to obtain the potential difference V_{ab}. Thus,

$$V_{ab} = -\int_b^a \vec{E}.\vec{dr}$$

$$\therefore \quad C = \frac{q}{V_{ab}} = \frac{q}{-\int_b^a \vec{E}.\vec{dr}}$$

3.3 Heat Generated :

(1) Work done by battery

$W = QV$

Q = charge flow in the battery

V = EMF of battery

(2) W = +Ve (When Battery discharging)

W = –Ve (When Battery charging)

(3) $\because$ Q = CV (C = equivalent capacitance)

so $W = CV \times V = CV^2$

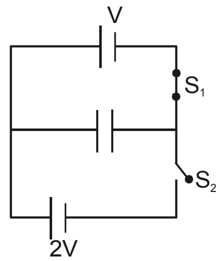
Now energy on the capacitor $= \frac{1}{2}CV^2$

$\therefore$ Energy dissipated in form of heat (due to resistance)

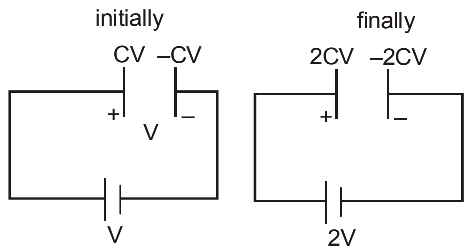
H = Work done by battery – {final energy of capacitor - initial energy of capacitor}

EXAMPLE 03

At any time S_1 switch is opened and S_2 is closed then find out heat generated in circuit.



Sol.



Charge flow through battery = $Q_f - Q_i$

$= 2CV - CV = CV$

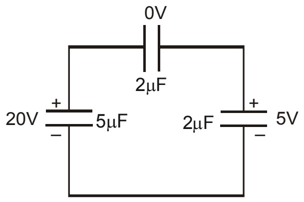
$$H = (CV \times 2V) - \left\{\frac{1}{2}C(2V)^2 - \frac{1}{2}CV^2\right\}$$

$$= 2CV^2 - \left\{2CV^2 - \frac{1}{2}CV^2\right\}$$

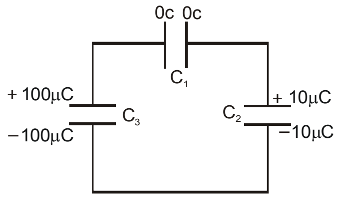
$$H = \frac{1}{2}CV^2$$

EXAMPLE 04

(a) Find the final charge on each capacitor if they are connected as shown in the figure.



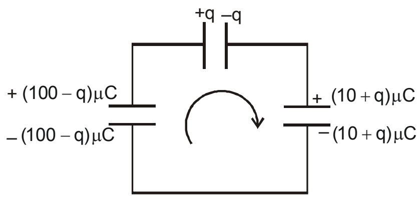
Sol. Initially



Finally let q charge flows clockwise then

Now applying KVL

$$\frac{+q}{C_1}+\frac{(10+q)}{C_2}-\frac{(100-q)}{C_3}=0$$



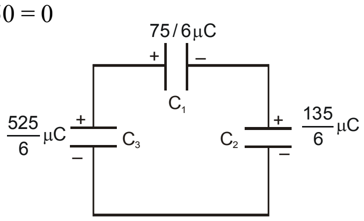
$$\Rightarrow \quad \frac{q}{2}+\frac{10+q}{2}-\frac{100-q}{5}=0$$

$$5q + 50 + 5q - 200 + 2q = 0$$

$$12\,q - 150 = 0$$

$$\Rightarrow \quad q=\frac{75}{6}\mu C$$

so finally



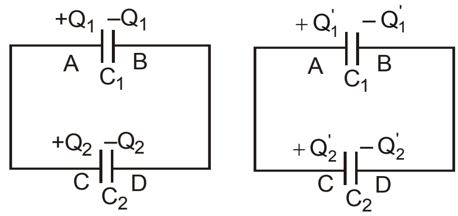
(b) Find heat loss in the above circuit.

$\Delta H =$ Energy [initially – finally] on capacitor

$$=\left[\left\{\frac{1}{2}\times5\times(20)^2+\frac{1}{2}\times2\times(5)^2\right\}-\left\{\frac{1}{2}\times\left(\frac{525}{6}\right)^2\times\frac{1}{5}+\frac{1}{2}\times\left(\frac{75}{6}\right)^2\times\frac{1}{2}+\frac{1}{2}\left(\frac{135}{6}\right)^2\times\frac{1}{2}\right\}\right]\times10^{-6}\,J$$

4. DISTRIBUTION OF CHARGES ON CONNECTING TWO CHARGED CAPACITORS :

When two capacitors C_1 and C_2 are connected as shown in figure



Initially Finally

Before connecting the capacitors		
Parameter	**Ist Capacitor**	**IInd Capacitor**
Capacitance	C_1	C_2
Charge	Q_1	Q_2
Potential	V_1	V_2

After connecting the capacitors		
Parameter	**Ist Capacitor**	**IInd Capacitor**
Capacitance	C_1	C_2
Charge	Q_1'	Q_2'
Potential	V_1	V_2

(a) Common potential :

By charge conservation on plates A and C before and after connection.

$Q_1 + Q_2 = C_1V + C_2V$

$$\Rightarrow \quad V=\frac{Q_1+Q_2}{C_1+C_2}=\frac{C_1V_1+C_2V_2}{C_1+C_2}=\frac{\text{Total charge}}{\text{Total capacitance}}$$

(b) $$Q_1'=C_1V=\frac{C_1}{C_1+C_2}(Q_1+Q_2)$$

$$Q_2'=C_2V=\frac{C_2}{C_1+C_2}(Q_1+Q_2)$$

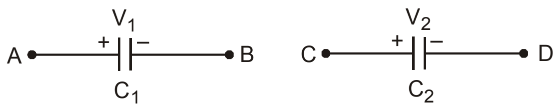
(c) Heat loss during redistribution :

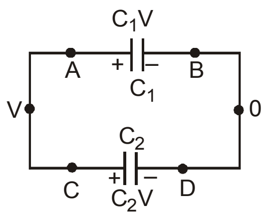
$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$$

The loss of energy is in the form of Joule heating in the wire.

- When plates of similar charges are connected with each other (+with + and – with –) then put all values (Q_1, Q_2, V_1, V_2) with positive sign.
- When plates of opposite polarity are connected with each other (+ with –) then take charge and potential of one of the plate to be negative.

Derivation of above formulae :





Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V.

$$C_1V + C_2V = C_1V_1 + C_2V_2$$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

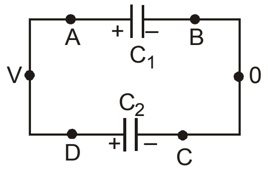
$$H = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1 + C_2)V^2$$

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}$$

$$= \frac{1}{2}\left[\frac{C_1^2V_1^2 + C_1C_2V_1^2 + C_2C_1V_2^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2V_2^2 - 2C_1C_2V_1V_2}{C_1 + C_2}\right]$$

$$= \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$$

$$H = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$$



when oppositely charged terminals are connected then

$\therefore$ $C_1V + C_2V = C_1V_1 - C_2V_2$

$$V = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$$

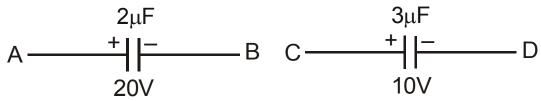
$$H = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 + V_2)^2$$

EXAMPLE 05

Find out the following if A is connected with C and B is connected with D.

(i) How much charge flows in the circuit.

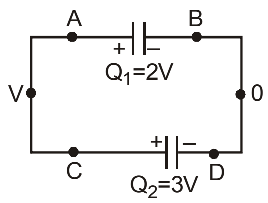
(ii) How much heat is produced in the circuit.



Sol. Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V.

By charge conservation,

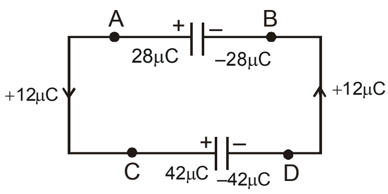
$$3V + 2V = 40 + 30$$



$5V = 70 \quad \Rightarrow \quad V = 14$ volt

Charge flow = 40 – 28 = 12 μC

Now final charges on each plate is shown in the figure.



(ii) Heat produced $= \frac{1}{2} \times 2 \times (20)^2 + \frac{1}{2} \times 3 \times (10)^2$

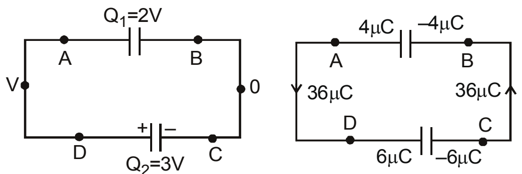
$- \frac{1}{2} \times 5 \times (14)^2 = 400 + 150 - 490$

$= 550 - 490 = 60$ μJ

- When capacitor plates are joined then the charge remains conserved.
- We can also use direct formula of redistribution as given above.

EXAMPLE 06

Repeat above question if A is connected with D and B is connected with C.



Initial Final

Sol. Let potential of B and C is zero and common potential on capacitors is V, then at A and D it will be V

$2V + 3V = 10 \quad \Rightarrow \quad V = 2$ volt

Now charge on each plate is shown in the figure.

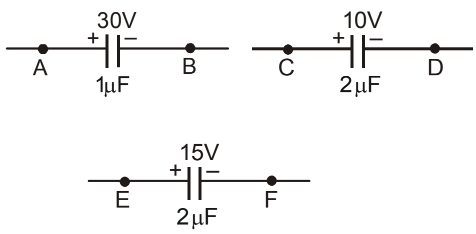
Heat produced $= 400 + 150 - \frac{1}{2} \times 5 \times 4$

$= 550 - 10 = 540\ \mu J$

- Here heat produced is more. Think why ?

EXAMPLE 07

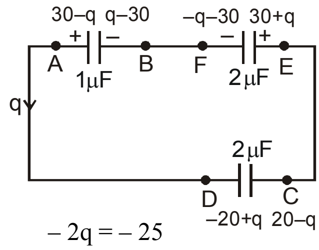
Three capacitors as shown of capacitance 1μF, 2μF and 2μF are charged upto potential difference 30 V, 10 V and 15V respectively. If terminal A is connected with D, C is connected with E and F is connected with B. Then find out charge flow in the circuit and find the final charges on capacitors.



Sol. Let charge flow is q.

Now applying Kirchhoff's voltage low

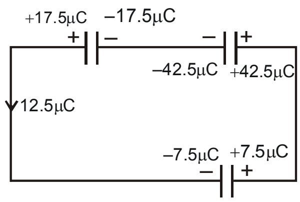
$$-\frac{(q-20)}{2} - \frac{(30+q)}{2} + \frac{30-q}{1} = 0$$



$-2q = -25$

$q = 12.5\ \mu C$

Final charges on plates



Note

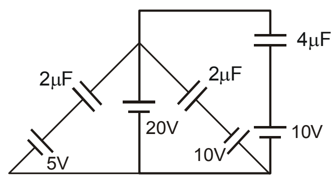
The student can now attempt section A from exercise.

Section B - Circuit Problems (Switch, Energy etc.)

5. CAPACITOR CIRCUITS

EXAMPLE 08

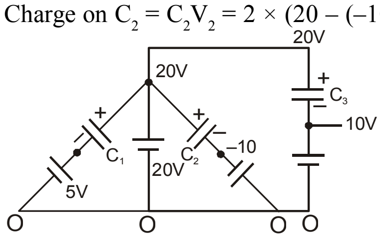
Find charge on each capacitor.



Sol. Charge on $C_1 = C_1V_1 = 2 \times (20 - 5)\mu C$

$= 30\ \mu C$

Charge on $C_2 = C_2V_2 = 2 \times (20 - (-10))\mu C$



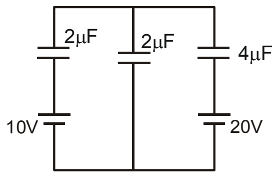
$= 60\ \mu C$

Charge on $C_3 = C_3V_3 = 4 \times (20 - 10)\mu C$

$= 40\ \mu C$

EXAMPLE 09

Find charge on each capacitor.

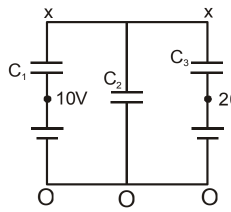


Sol. Charge on $C_1 = (x - 10)\, C_1$

Charge on $C_2 = (x - 0)\, C_2$

Charge on $C_3 = (x - 20)\, C_3$

Now from charge conservation at node x



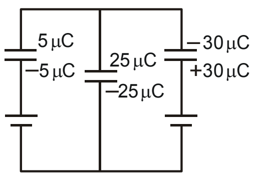
$(x - 10)C_1 + (x - 0)C_2 + (x - 20)C_3 = 0$

$\Rightarrow$ $2x - 20 + 2x + 4x - 80 = 0 \Rightarrow x = 25/2$

so $Q_{C_1} = \left(\frac{25}{2} - 10\right) 2\mu C = 5\mu C$

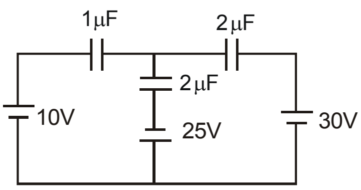
$Q_{C_2} = \frac{25}{2} \times 2\mu C = 25\mu C$

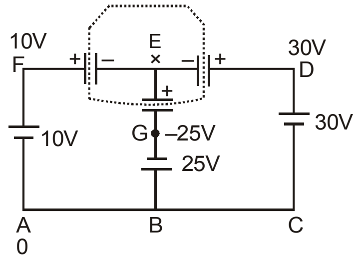
$Q_{C_3} = (25/2 - 40) \times 4\mu C = -30\mu C$



EXAMPLE 10

In the given circuit find out the charge on each capacitor. (Initially they are uncharged)





Sol. Let potential at A is 0, so at D it is 30 V, at F it is 10 V and at point G potential is –25V. Now apply Kirchhoff's I[st] law at point E. (total charge of all the plates connected to 'E' must be same as before i.e. 0)

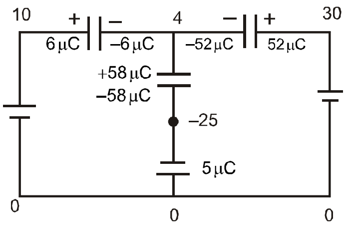
$\therefore$ $(x - 10) + (x - 30)\,2 + (x + 25)\,2 = 0$

$5x = 20$

$x = 4$

Final charges :

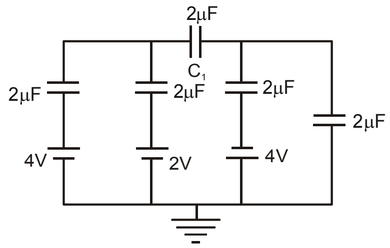
$Q_{2\mu F} = (30 - 4)\,2 = 52\ \mu C$



$Q_{1\mu F} = (10 - 4) = 6\ \mu C$

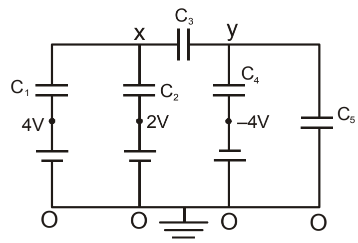
$Q_{2\mu F} = (4 - (-25))\,2 = 58\ \mu C$

EXAMPLE 11



Find voltage across capacitor C_1.

Sol.



Now from charge conservation at node x and y

for x

$(x-4)C_1 + (x-2)C_2 + (x-y)C_3 = 0$

$\Rightarrow$ $2(x-4) + 2(x-2) + (x-y)\,2 = 0$

$6x - 2y - 12 = 0$ .....(1)

For y

$(y-x)C_3 + [y-(-4)]C_4 + (y-0)C_5 = 0$

$\Rightarrow$ $(y-x)2 + (y+4)\,2 + y\,2 = 0$

$= 6y - 2x + 8 = 0$ .....(2)

eq. (1) & (2)

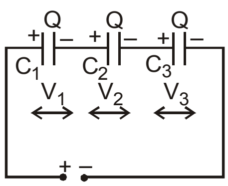
$y = -3/4$ $x = 7/4$

So potential difference $= x - y = \frac{7}{4} - \left(-\frac{3}{4}\right) = \frac{5}{3}$ volt

6. COMBINATION OF CAPACITORS :

6.1 Series Combination :

(i) When initially uncharged capacitors are connected as shown, then the combination is called series combination



(ii) All capacitors will have same charge but different potential difference across then.

(iii) We can say that

$$V_1 = \frac{Q}{C_1}$$

V_1 = potential across C_1

Q = charge on positive plate of C_1

C_1 = capacitance of capacitor similarly

$$V_2 = \frac{Q}{C_2},\ V_3 = \frac{Q}{C_3}.........$$

(iv) $V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$

We can say that potential difference across capacitor is inversely proportional to its capacitance in series combination.

$$V \propto \frac{1}{C}$$

Note

In series combination the smallest capacitor gets maximum potential

(v) $$V_1 = \frac{\frac{1}{C_1}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+......}V,$$

$$V_2 = \frac{\frac{1}{C_2}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+......}V,$$

$$V_3 = \frac{\frac{1}{C_3}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+......}V$$

Where $V = V_1 + V_2 + V_3$

(vi) Equivalent Capacitance :

Equivalent capacitance of any combination is that capacitance which when connected in place of the combination stores same charge and energy as that of the combination

In series :

$$\frac{1}{C_{eq}} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\ ....$$

- In series combination equivalent is always less then smallest capacitor of combination.

(vii) Energy stored in the combination

$$U_{combination} = \frac{Q^2}{2C_1}+\frac{Q^2}{2C_2}+\frac{Q^2}{2C_3}$$

$$U_{combination} = \frac{Q^2}{2C_{eq}}$$

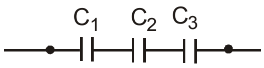
Energy supplied by the battery in charging the combination

$$U_{battery} = Q \times V = Q \cdot \frac{Q}{C_{eq}} = \frac{Q^2}{C_{eq}}$$

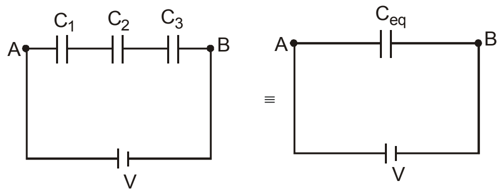
$$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$$

- Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance.

Derivation of Formulae :



Meaning of equivalent capacitor



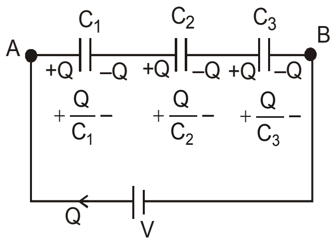
$$C_{eq} = \frac{Q}{V}$$

Now,

Initially, the capacitor has no charge. Applying kirchhoff's voltage law

$$\frac{-Q}{C_1} + \frac{-Q}{C_2} + \frac{-Q}{C_3} + V = 0$$

$$V = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$



$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

in general

$$\frac{1}{C_{eq}} = \sum_{n=1}^{n} \frac{1}{C_n}$$

EXAMPLE 12

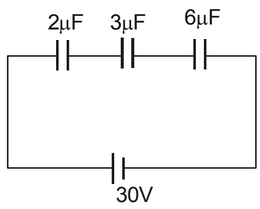
Three initially uncharged capacitors are connected in series as shown in circuit with a battery of emf 30V. Find out following :

(i) charge flow through the battery,

(ii) potential energy in 3 μF capacitor.

(iii) U_{total} in capacitors

(iv) heat produced in the circuit



Sol. $$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = 1$$

$C_{eq} = 1$ μF.

(i) $Q = C_{eq}\,V = 30$ μC

(ii) charge on 3μF capacitor = 30 μC

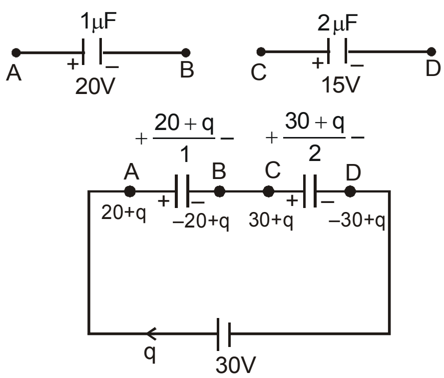
$$\text{energy} = \frac{Q^2}{2C} = \frac{30\times 30}{2\times 3} = 150\ \mu J$$

(iii) $U_{total} = \frac{30\times 30}{2}\mu J = 450\ \mu J$

(iv) Heat produced = (30 μC) (30) – 450 μJ = 450 μJ

EXAMPLE 13

Two capacitors of capacitance 1 μF and 2μF are charged to potential difference 20 V and 15 V as shown in figure. If now terminal B and C are connected together terminal A with positive of battery and D with negative terminal of battery then find out final charges on both the capacitor.



Now applying kirchoff voltage law

$$\frac{-(20+q)}{1} - \frac{30+q}{2} + 30 = 0$$

$$-40 - 2q - 30 - q = -60$$

$$3q = -10$$

Charge flow $= -\frac{10}{3}$ μC.

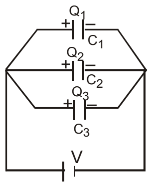
Charge on capacitor of capacitance $1\mu F = 20 + q = \frac{50}{3}$

Charge on capacitor of capacitance $2\mu F = 30 + q = \frac{80}{3}$

6.2 Parallel Combination :

(i) When one plate of one capacitor is connected with one plate of the other capacitor, such combination is called parallel combination.

(ii) All capacitors have same potential difference but different charges.



(iii) We can say that :

$Q_1 = C_1V$

Q_1 = Charge on capacitor C_1

C_1 = Capacitance of capacitor C_1

V = Potential across capacitor C_1

(iv) $Q_1 : Q_2 : Q_3 : C_1 : C_2 : C_3$

The charge on the capacitor is proportional to its capacitane $Q \propto C$

(v) $$Q_1 = \frac{C_1}{C_1 + C_2 + C_3}Q$$

$$Q_2 = \frac{C_2}{C_1 + C_2 + C_3}Q$$

$$Q_3 = \frac{C_3}{C_1 + C_2 + C_3}Q$$

Where $Q = Q_1 + Q_2 + Q_3$ ....

☛

- Maximum charge will flow thorugh the capacitor of largest value.

(vi) Equivalent capacitance of parallel combination

$$C_{eq} = C_1 + C_2 + C_3$$

- Equivalent capacitance is always greater then the largest capacitor of combination.

(vii) Energy stored in the combination :

$$V_{combination} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + ...$$

$$= \frac{1}{2}(C_1 + C_2 + C_3........)V^2 = \frac{1}{2}C_{eq}V^2$$

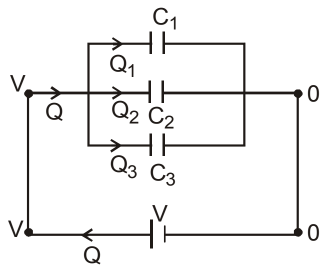
$$U_{battery} = QV = CV^2$$

$$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$$

- Half of the energy supplied by the battery is stored in the form of electrostatic energy and half of the energy is converted into heat through resistance.

Formulae Derivation for parallel combination :

$$Q = Q_1 + Q_2 + Q_3$$
$$= C_1V + C_2V + C_3V$$
$$= V(C_1 + C_2 + C_3)$$



$$\frac{Q}{V} = C_1 + C_2 + C_3$$

$$C_{eq} = C_1 + C_2 + C_3$$

In general

$$C_{eq} = \sum_{n=1}^{n} C_n$$

EXAMPLE 14

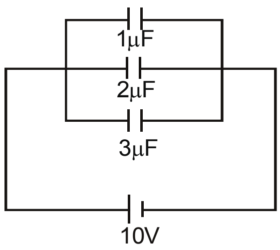
Three initially uncharged capacitors are connected to a battery of 10 V in parallel combination. Find out the following ?

(i) charge flow from the battery

(ii) total energy stored in the capacitors

(iii) heat produced in the circuit

(iv) potential energy in the 3μF capacitor.



Sol. (i) $Q = (30 + 20 + 10)\ \mu C = 60\ \mu C$

(ii) $U_{total} = \frac{1}{2} \times 6 \times 10 \times 10 = 300\ \mu J$

(iii) Heat produced $= 60 \times 10 - 300 = 300\ \mu J$

(iv) $U_{3\mu F} = \frac{1}{2} \times 3 \times 10 \times 10 = 150\ \mu J$

6.3 Mixed Combination :

The combination which contains mixing of series combinations or other complex combinations fall in mixed category.

There are two types of mixed combinations

(i) Simple (ii) Complex.

EXAMPLE 15

Two capacitors of capacitances $C_1 = 6\mu F$ and $C_2 = 3\mu F$ are connected in series across a cell of emf 18 V. Calculate

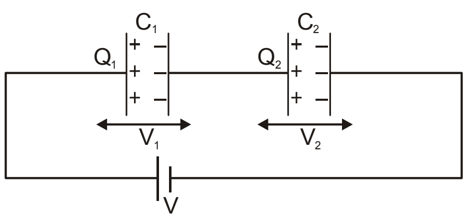
(i) the equivalent capacitance,

(ii) the potential difference across each capacitor,

(iii) the charge on each capacitor.

Sol. (i) $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

$\Rightarrow \quad C = \frac{C_1C_2}{C_1 + C_2} = \frac{6 \times 3}{6+3} = 2\mu F$



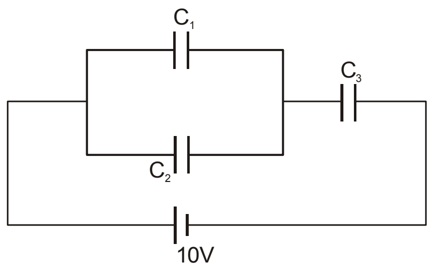
(ii) $V_1 = \left(\frac{C_2}{C_1 + C_2}\right)V = \left(\frac{6}{6+3}\right) \times 18 = 12\ V$

(iii) $v_2 = 18 - v_1 = 6v$

(iv) $Q_1 = Q_2 = C_1V_1 = C_2V_2 = CV$

EXAMPLE 16

In the circuit shown the capacitors are $C_1 = 15\mu F$; $C_2 = 10\mu F$ and $C_3 = 25\mu F$. Find

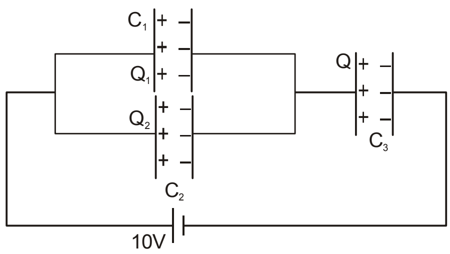


(i) the equivalent capacitance of the circuit,

(ii) the charge on each capacitor and

(iii) the potential difference across each capacitor.

Sol. (i) $\frac{(C_1 + C_2)C_3}{(C_1 + C_2) + C_3} = \left(\frac{25 \times 25}{25 + 25}\right)\mu F = 12.5\mu F$



(ii) Q = Total charge supplied by the cell = CV = $(12.5 \times 10)\ \mu C = 125\ \mu C$

Charge on

$C_1 = Q_1 = \left(\frac{C_1}{C_1 + C_2}\right)Q = \left(\frac{15}{15 + 10}\right) \times 125 = 75\mu C$

Charge on

$C_2 = Q_2 = \left(\frac{C_2}{C_1 + C_2}\right)Q = \left(\frac{10}{15 + 10}\right) \times 125 = 50\mu C$

Charge on $C_3 = Q = 125\ \mu C$

(iii) p.d. across $C_1 = V_1 = \frac{Q_1}{C_1} = \frac{75}{15} = 5V$

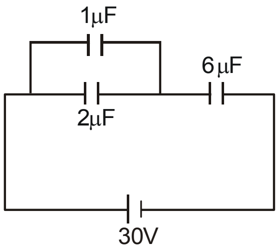
p.d. across $C_2 = V_2 = V_1 = 5$ V

p.d. across $C_3 = V_3 = \frac{Q_3}{C_3} = \frac{125}{25} = 5V$

EXAMPLE 17

In the given circuit find out charge across 6μF and 1μF capacitor.

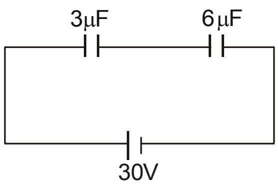
Sol.



It can be simplified as

$$C_{eq} = \frac{18}{9} = 2\mu F$$

Charge flow through the cell = 30 × 2 μC



Q = 60 μC

Now charge on 3μF = Charge on 6μF = 60 μC

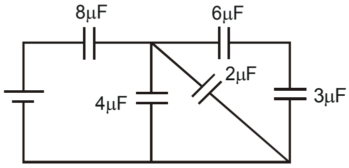
Potential difference across 3μF

$$= \frac{60}{3} = 20V$$

∴ Charge on 1μF = 20 μC

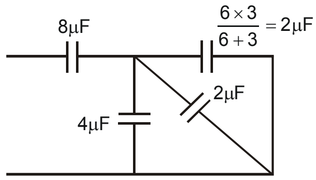
6.4 Series-Parallel

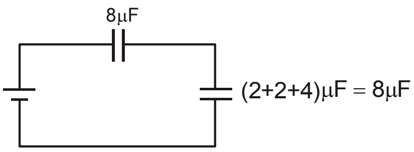
EXAMPLE 18

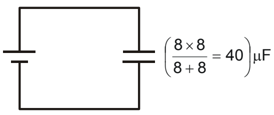


Find equivalent capacitance of the circuit

Sol.



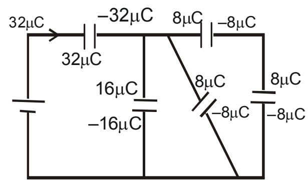




EXAMPLE 18

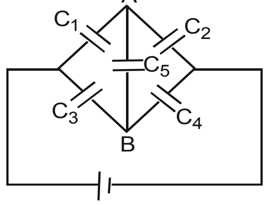
Find charge in each capacitor :

Sol. In series charge will be same on all capacitors and in parallel charge will be propotional to capacitane.



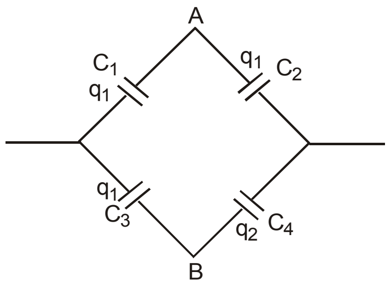
EXAMPLE 19

Find out the Relation between C_1, C_2, C_3 and C_4 such that point A and B are equipotential. [Balanced wheat stone bridge]



Sol. When A and B are equipotential then there will be no charge on capacitor C_5. So remove it. Now C_1, C_2 are in series and C_3, C_4 are in series so they will have same charges respectively.

$$\Rightarrow \quad \frac{q_1}{C_1} = \frac{q_2}{C_3} \qquad ...(1)$$

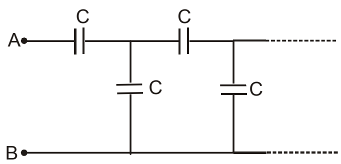


and $\dfrac{q_1}{C_2} = \dfrac{q_2}{C_4}$ ...(2)

$$\frac{C_2}{C_1} = \frac{C_4}{C_3} \quad \Rightarrow \quad C_2C_3 = C_1C_4$$

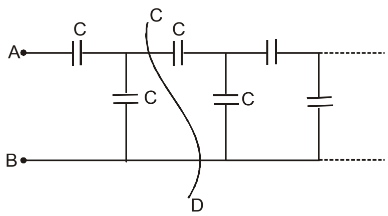
6.5 Ladder Problems

EXAMPLE 20

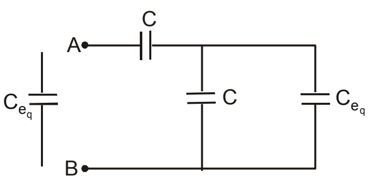


Find equivalent capacitance between point A and B.

Sol.



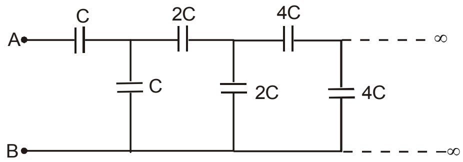
Let Capacitance is C_{eq} then after line CD capacitance again will be equal to C_{e_q}, because circuit is infinite. Then



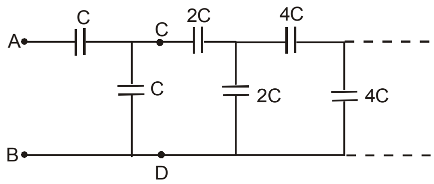
$$\Rightarrow \quad C_{eq} = \frac{(C_{eq} + C) \times C}{C + C + C_{eq}} \quad \Rightarrow C_{eq} = \frac{\sqrt{5} - 1}{2} C$$

EXAMPLE 21

Find equivalent capacitance between points A and B.

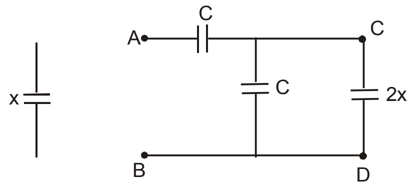


Sol.



Let equivalent capacitance x then capacitance after C,D point will be 2x because every capacitance becomes 2 times as compared to A, B.

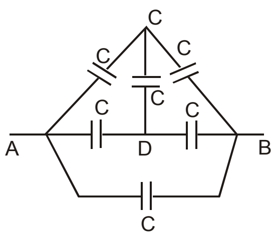
So



$$\Rightarrow \quad x = \frac{(2x + C)C}{(2x + C + C)} \quad \Rightarrow \quad x = \frac{C}{\sqrt{2}}$$

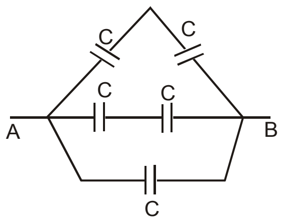
6.6 Symmetric Circuits :

EXAMPLE 22



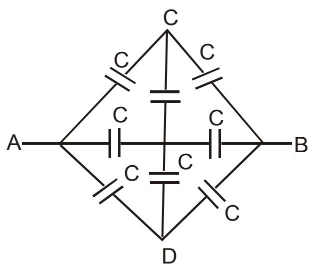
Find equivalent Capacitance between A and B.

Sol. Because line CD is symmetric about A and B so we remove it then



$\Rightarrow \quad C_{e_q} = 2C$

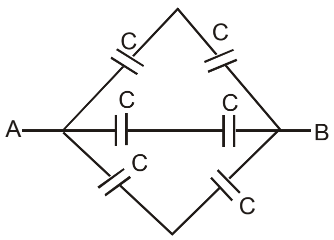
EXAMPLE 23



Find equivalent Capacitance between A and B.

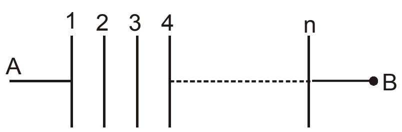
Sol. Because line CD is symmetric about points A and B so we remove it.

Now



So $\quad C_{e_q} = \dfrac{3C}{2}$

EXAMPLE 24



Find equivalent Capacitance between point A and B if Capacitance between any two plates is C.

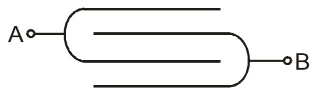
Sol. There are total (n – 1) capacitors which are in series.

so $\dfrac{1}{C_{eq}} = \dfrac{1}{C} + \dfrac{1}{C} + ...(n-1)$ times

$\dfrac{1}{C_{eq}} = \dfrac{(n-1)}{C} \quad \Rightarrow \quad C_{eq} = \dfrac{C}{n-1}$

EXAMPLE 25

Find out equivalent capacitance between A and B.

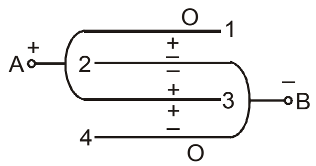


Sol. Put numbers on the plates The charges will be as shown in the figure.

$V_{12} = V_{32} = V_{34}$

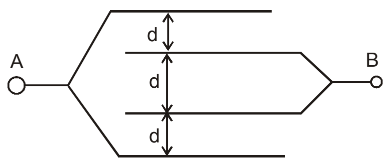
So all the capacitors are in parallel combination.

$C_{eq} = C_1 + C_2 + C_3$

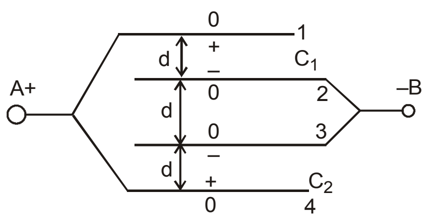


EXAMPLE 26

Find out equivalent capacitance between A and B.



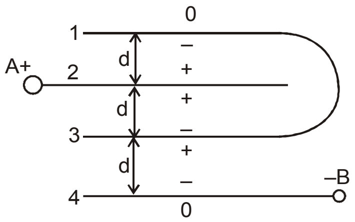
Sol.



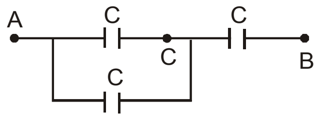
There are only two capacitors. $C_{eq} = C_1 + C_2$

EXAMPLE 27

Find out equivalent capacitance between A and B.

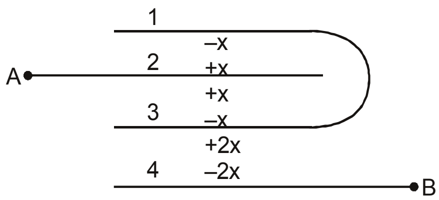


The modified circuit is



$$C_{eq} = \frac{2C}{3}$$

Other method :



$$C_{eq} = \frac{Q}{V} = \frac{2xA}{V}$$

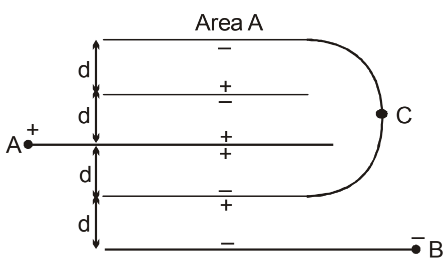
$$V = V_2 - V_4 = (V_2 - V_3) + (V_3 - V_4)$$

$$= \frac{xd}{\varepsilon_0} + \frac{2xd}{\varepsilon_0} = \frac{3xd}{\varepsilon_0}$$

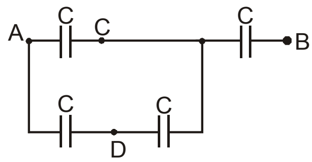
$$\therefore \quad C_{eq} = \frac{2Ax\varepsilon_0}{3xd} = \frac{2A\varepsilon_0}{3d} = \frac{2C}{3}$$

EXAMPLE 28

Find out equivalent capacitance between A and B.



Sol. $C = \dfrac{A\varepsilon_0}{d}$



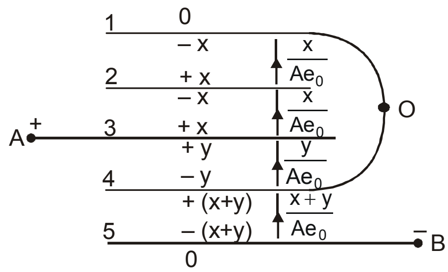
$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{2}{3C} = \frac{5}{3C}$$

$$\Rightarrow \quad C_{eq} = \frac{3C}{5} = \frac{3A\varepsilon_0}{5d}$$

Alternative method :

$$C = \frac{Q}{V} = \frac{x+y}{V_{AB}}$$

$$C = \frac{Q}{V} = \frac{x+y}{V_{AB}}$$



Potential of 1 and 4 is same

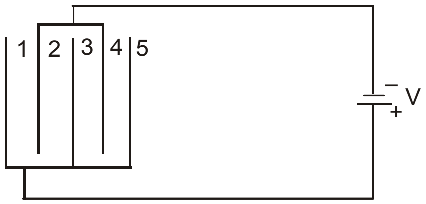
$$\frac{y}{A\varepsilon_0} = \frac{2x}{A\varepsilon_0} \qquad y = 2x$$

$$V = \left(\frac{2y + x}{A\varepsilon_0}\right)d$$

$$C = \frac{(x+2x)A\varepsilon_0}{(5x)d} = \frac{3A\varepsilon_0}{5d}$$

EXAMPLE 29

Five similar condenser plates, each of area A, are placed at equal distance d apart and are connected to a source of e.m.f. E as shown in the following diagram. The charge on the plates 1 and 4 will be



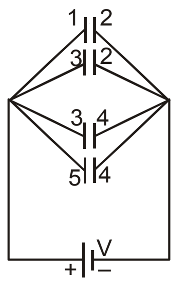
(A) $\frac{\varepsilon_0 A}{d}, \frac{-2\varepsilon_0 A}{d}$ (B) $\frac{\varepsilon_0 AV}{d}, \frac{-2\varepsilon_0 AV}{d}$

(C) $\frac{-\varepsilon_0 AV}{d}, \frac{-3\varepsilon_0 AV}{d}$ (D) $\frac{\varepsilon_0 AV}{d}, \frac{-4\varepsilon_0 AV}{d}$

Sol. From equivalent circuit diagram Charge on first plate

$Q = CV$

$\Rightarrow \quad Q = \frac{\varepsilon_0 AV}{d}$



Charge on fourth plate

$Q' = C(-V)$

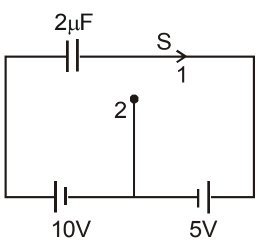
$Q' = \frac{-\varepsilon_0 AV}{d}$

As plate 4 is repeated twice, hence charge on 4 will be $Q'' = 2Q'$

$Q'' = -\frac{2\varepsilon_0 AV}{d}$

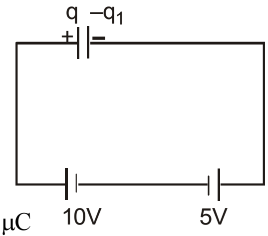
Hence the correct answer will be (B).

EXAMPLE 30



When switch is shifted from 1 to 2 then find charge flown in circuit and heat generated.

Sol. Initially



$\frac{q}{C}1 - 5 + 10 = 0$

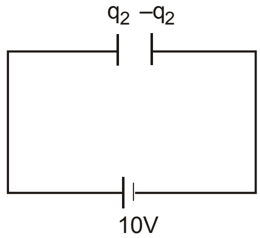
$\Rightarrow \quad q_1 = 10\ \mu F$

Finally

$\Rightarrow \quad q_2 = 10 \times 2\ \mu C = 20\ \mu C$

So charge Flown (Q) = $(q_2 - q_1) = 10\ \mu C$

Work done by battery = QV = 10 × 10 = 100 μJ



So

Heat

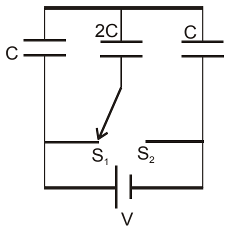
=[W.D$_B$ – {final initial}energy of capacitor]

$$= \left[100 - \left\{\frac{1}{2} \times 2 \times (0)^2 - \frac{1}{2} \times 2 \times 5\right\}\right] \mu J$$

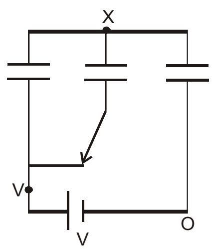
$$= \left[100 - \{100 - 25\}\right] \mu J = 25\ \mu J$$

EXAMPLE 31

Find charge flown through battery and heat generated in the circuit after shifting switch from S_1 to S_2.



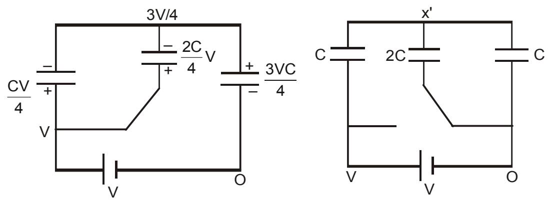
Sol. Initially



For node x

$$(x - v)\,2c + (x - v)\,c + (x + 0)\,c = 0$$

$$\Rightarrow \quad 4x = 3v \quad \Rightarrow \quad x = \frac{3v}{4}$$

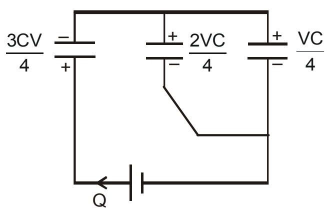


$$(x' - 0)C + (x' - 0)2C + (x' - V)C + = 0$$

$$\Rightarrow \quad x' = \frac{V}{4}$$

So now charge on capacitor

So charge flow from battery



$$Q = \left(\frac{3CV}{4} - \frac{CV}{4}\right) = \frac{CV}{2}$$

So heat generated

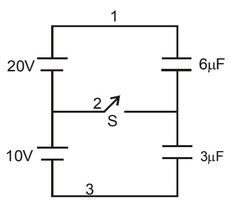
$$\Delta H = QV - \left[\left\{\frac{1}{2}C\left(\frac{3V}{4}\right)^2 + \frac{1}{2}(2C)\left(\frac{V}{4}\right)^2 + \frac{1}{2}\times C\times\left(\frac{V}{4}\right)^2\right\}\right.$$

$$\left. - \left\{\frac{1}{2}C\left(\frac{V}{4}\right)^2 + \frac{1}{2}(2C)\left(\frac{V}{4}\right)^2 + \frac{1}{2}\times C\times\left(\frac{3V}{4}\right)^2\right\}\right]$$

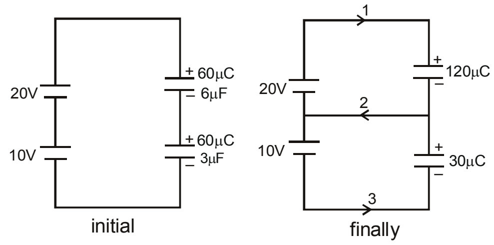
$$\Delta H = QV = \frac{CV^2}{2}$$

EXAMPLE 32

Find the charge flown through the path 1, 2, 3 as shown in figure after closing switch S and heat generated in the circuit.



Sol.



So charge flow in path 1 = 120 – 60 = 60μC

So charge flow in path 2 = 60 + (60 – 30) = 90μC

So charge flow in path 3 = (60 – 30) = 30μC

So work done by battery of 20 V = 60 × 20μJ

= 1200μJ

Wwork done by battery of 10 V = – 30 × 10μJ

= –300μJ

$$\text{Initial energy of capacitors} = \left[\frac{1}{2}\frac{(60)^2}{6} + \frac{1}{2}\frac{(60)^2}{3}\right]\mu J$$

= 900 μJ

Final energy of capacitor

$$= \left[\frac{1}{2}6\times(20)^2 + \frac{1}{2}\times 3\times(10)^2\right]\mu J = 1350\ \mu J$$

$$\text{So heat loss} = (1200 - 300) - (1350 - 900)\ \mu J = 450\ \mu J$$

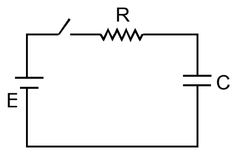
Note

The student can now attempt section B from exercise.

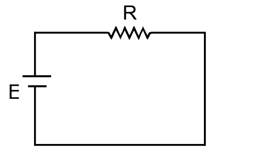
Section C,D - Dielectric, R-C Circuit

7. CIRCUIT SOLUTION FOR R – C CIRCUIT AT T = 0 (INITIAL STATE) AND T = ∞ (FINAL STATE)

- Charge on the capacitor does not change instantaneously or suddenly if there is a resistance in the path (series) of the capacitor.
- When an uncharged capacitor is connected with battery then its charge is zero initially hence potential difference across it is zero initially. At this time the capacitor can be treated as a conducting wire

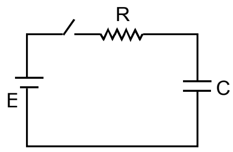


Before connection

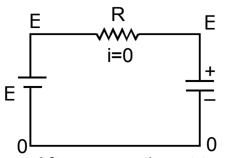


Just after connection at t = 0

- The current will become zero finally (that means in steady state) in the branch which contains capacitor.



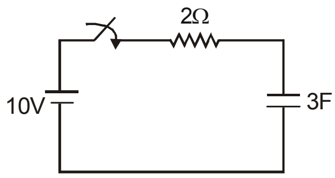
Before connection



After connection at $t = \infty$

EXAMPLE 33

Find out current in the circuit and charge on capacitor which is initially uncharged in the following situations.



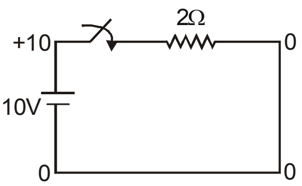
(a) Just after the switch is closed

(b) After a long time when switch was closed.

Sol. **(a) For just after closing the switch :**

Potential difference across capacitor = 0

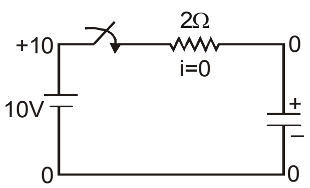
$\therefore \quad Q_C = 0 \quad \therefore\ i = \dfrac{10}{2} = 5A$



(b) After a long time

At steady state current i = 0

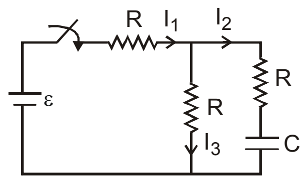
and potential difference across capacitor = 10 V



$\therefore \quad Q_C = 3 \times 10 = 30\ C$

EXAMPLE 34

Find out current I_1, I_2, I_3, charge on capacitor and $\dfrac{dQ}{dt}$ of capacitor in the circuit when it is initially uncharged in the following situations



(a) Just after the switch is closed

(b) After a long time when switch is closed.

Sol. (a) Initially the capacitor is uncharged so its behaviour is like a conductor

Let potential at A is zero so at B and C also zero and at F it is ε. Let potential at E is x so at D also x.

Apply Kirchoff's Ist law at point E :

$$\frac{x-\varepsilon}{R} + \frac{x-0}{R} + \frac{x-0}{R} = 0$$

$$\frac{3x}{R} = \frac{\varepsilon}{R}$$

$$x = \frac{\varepsilon}{3} \qquad Q_c = 0$$

$$\therefore \quad I_1 = \frac{-\varepsilon/3+\varepsilon}{R} = \frac{2\varepsilon}{3R}$$

$$I_2 = \frac{dQ}{dt} = \frac{\varepsilon}{3R}, \qquad I_3 = \frac{\varepsilon}{3R}$$

Alternatively

$$i_1 = \frac{\varepsilon}{R_{eq}} = \frac{\varepsilon}{R + \dfrac{R}{2}} = \frac{2\varepsilon}{3R}$$

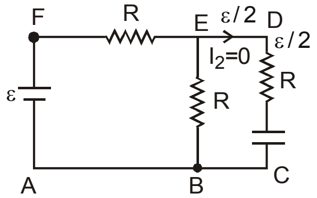
$$i_2 = i_3 = \frac{i_1}{2} = \frac{\varepsilon}{3R}$$

(b) at $t = \infty$ (finally)

Capacitor is completely charged. So their will be no current through it.

$$I_2 = 0,\ I_1 = I_3 = \frac{\varepsilon}{2R}$$

$$V_E - V_B = V_D - V_C = \left(\frac{\varepsilon}{2R}\right)R = \frac{\varepsilon}{2}$$

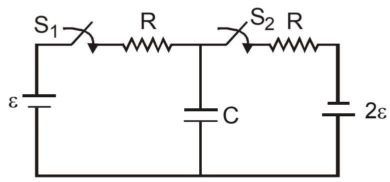


$\Rightarrow \quad Q_C = \frac{\varepsilon C}{2}, \frac{dQ}{dt} = I_2 = 0$

Time	I_1	I_2	I_3	Q	dQ/dt
t = 0	$\frac{2\varepsilon}{3R}$	$\frac{\varepsilon}{3R}$	$\frac{\varepsilon}{3R}$	0	$\frac{\varepsilon}{3R}$
Finally t =	$\frac{\varepsilon}{2R}$	0	$\frac{\varepsilon}{2R}$	$\frac{\varepsilon C}{2}$	0

EXAMPLE 35

At t = 0 switch S_1 is closed and remains closed for a long time and S_2 remains open. Now S_1 is opened and S_2 is closed. Find out



(i) The current through the capacitor immediately after that moment

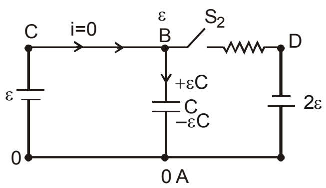
(ii) Charge on the capacitor long after that moment.

(iii) Total charge flown through the cell of emf 2ε after S_2 is closed.

Sol. (i) Let Potential at point A is zero. The potential at point B and C will be ε (because current through the circuit is zero.)

$V_B - V_A = (\varepsilon - 0)$

∴ Charge on capacitor = C (ε – 0) = Cε



Now S_2 is closed and S_1 is open.

(p.d. across capacitor and charge on it will not change suddenly)

Potential at A is zeo so at D it is –2ε

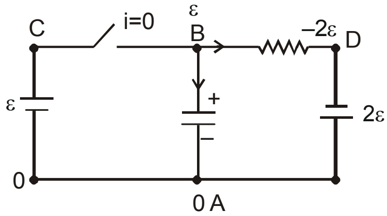
∴ current through the capacitor

$$= \frac{\varepsilon - (-2\varepsilon)}{R} = \frac{3\varepsilon}{R} \text{(B to D)}$$

(ii) after a long time, i = 0

$V_B - V_A = V_D - V_A = -2\varepsilon$

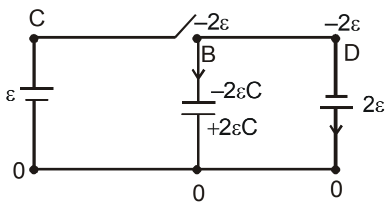
∴ Q = C(–2ε – 0) = – 2εC



(iii) The charge on the lower plate (which is connected to the battery) changes from –εC to 2εC.

∴ this charge will come from the battery,

∴ charge flown from that cell is 2εC downward.

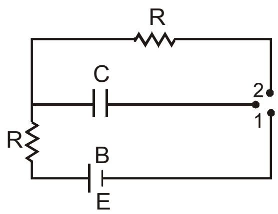


8. CHARGING AND DISCHARGING OF A CAPACITOR

8.1 Charging of a condenser :

- In the following circuit. If key 1 is closed then the condenser gets charged. Finite time is taken in the charging process. The quantity of charge at any instant of time t is given by

$$q = q_0[1 - e^{-(t/RC)}]$$



Where q_0 = maximum final value of charge at $t = \infty$.

According to this equation the quantity of charge on the condenser increases exponentially with increase of time.

- If $t = RC = \tau$ then

$$q = q_0 [1 - e^{-(RC/RC)}] = q_0\left[1-\frac{1}{e}\right]$$

or $q = q_0 (1 - 0.37) = 0.63\, q_0$

$= 63\%$ of q_0

- Time t = RC is known as Time Constant.

i.e. the time constant is that time during which the charge rises on the condenser plates to 63% of its maximum value.

- The potential difference across the condenser plates at any instant of time is given by

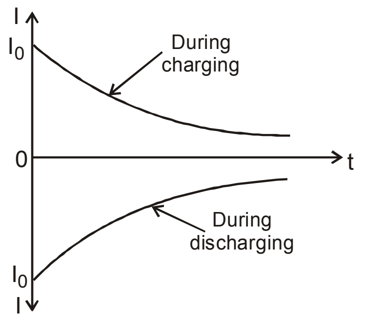
$$V = V_0[1 - e^{-(t/RC)}]\,\text{volt}$$

- The potential curve is also similar to that of charge. During charging process an electric current flows in the circuit for a small interval of time which is known as the transient current. The value of this current at any instant of time is given by

$$I = I_0[e^{-(t/RC)}] \text{ ampere}$$

According to this equation the current falls in the circuit exponentially (Fig.)

- If $t = RC = \tau$ = Time constant



$$I = I_0[e^{-(RC/RC)} = \frac{I_0}{e} = 0.37 I_0 = 37\% \text{ of } I_0$$

i.e. time constant is that time during which current in the circuit falls to 37% of its maximum value.

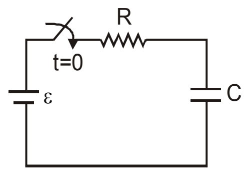
Derivation of formulae for charging of capacitor:

It is given that initially capacitor is uncharged. Let at any time t

Applying Kirchoff voltage law

$$\varepsilon - iR - \frac{q}{C} = 0 \quad \Rightarrow \quad iR = \frac{\varepsilon C - q}{C}$$

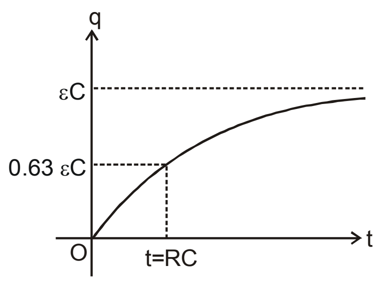
$$i = \frac{\varepsilon C - q}{CR} \quad \Rightarrow \quad \frac{dq}{dt} = \frac{\varepsilon C - q}{CR}$$



$$\frac{dq}{dt} = \frac{\varepsilon C - q}{CR} \quad \Rightarrow \quad \frac{CR}{\varepsilon C - q}.dq = dt$$

$$\int_0^q \frac{dq}{\varepsilon C - q} = \int_0^t \frac{dt}{RC} \Rightarrow -\ln(\varepsilon C - q) + \ln \varepsilon C = \frac{t}{RC}$$

$$\ln \frac{\varepsilon C}{\varepsilon C - q} = \frac{t}{RC}$$



$$\varepsilon C - q = \varepsilon C.\ e^{-t/RC} \quad \Rightarrow q = \varepsilon C(1 - e^{-t/RC})$$

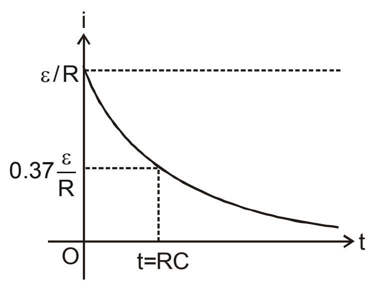
RC = time constant of the RC series circuit.

After one time constant

$$q = \varepsilon C\left(1 - \frac{1}{e}\right) = \varepsilon C\ (1 - 0.37) = 0.63\ \varepsilon C$$

Current at any time t

$$i = \frac{dq}{dt} = \varepsilon C\left(-e^{-t/RC}\left(-\frac{1}{RC}\right)\right)$$

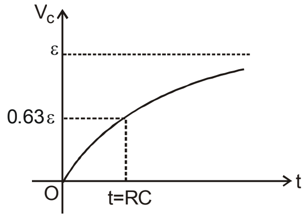


$$= \frac{\varepsilon}{R} e^{-t/RC}$$

Voltage across capacitor after one time constant V = 0.63 ε

$Q = CV$

$V_C = \varepsilon(1-e^{-t/RC})$



Voltage across the resistor

$V_R = iR$

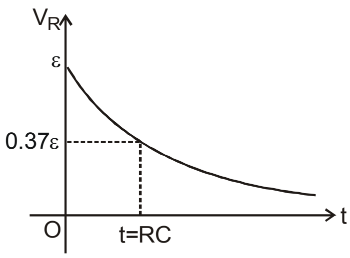
$= \varepsilon e^{-t/RC}$

By energy conservation,

Heat dissipated = work done by battery – ΔU capacitor

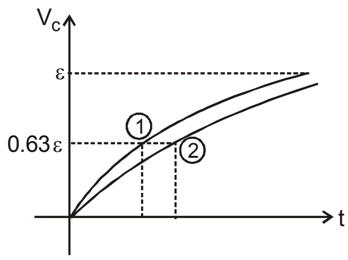
$$= C\varepsilon(\varepsilon) - (\frac{1}{2}C\varepsilon^2 - 0) = \frac{1}{2}C\varepsilon^2$$

Alternatively :



Heat $= H = \int_0^{\infty} i^2 R dt$

$$= \int_0^{\infty} \frac{\varepsilon^2}{R^2} e^{\frac{-2t}{RC}} R dt = \frac{\varepsilon^2}{R}\int_0^{\infty} e^{-2t/RC} dt$$



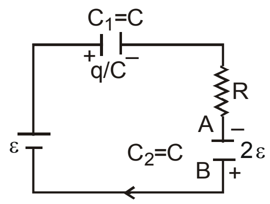
In the figure time constant of (2) is more than (1)

$$= \frac{\varepsilon^2}{R}\left[\frac{e^{-\frac{2t}{RC}}}{-2/RC}\right]_0^{\infty}$$

$$= -\frac{\varepsilon^2 RC}{2R}\left[e^{-\frac{2t}{RC}}\right]_0^{\infty} = \frac{\varepsilon^2 C}{2}$$

EXAMPLE 36

In the figure shown below, find out the current as function of time and charge on capacitor C_1 and also plot the graph of charge on plate A and B of capacitor C_2 as a function of time.

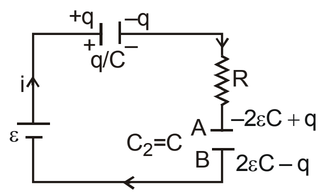


Sol. Let at any time charge on $C_1 = q$

Now apply, K.V.L

$$-iR - \frac{q}{c} - \frac{(q-2\varepsilon C)}{C} + \varepsilon = 0$$

$$3\varepsilon = \frac{2q}{c} + iR \quad \Rightarrow \quad 3\varepsilon C = 2q + iRC$$



$$3\varepsilon C = 2q + iRC \Rightarrow \int_0^q \frac{dq}{3\varepsilon C - 2q} = \int_0^t \frac{dt}{RC}$$

$$\left.\frac{-\ell n(3\varepsilon C - 2q)}{2}\right|_0^q = \frac{t}{RC}$$

$$\Rightarrow \quad \ell n\left(\frac{3\varepsilon C - 2q}{3\varepsilon c}\right) = \frac{-2t}{RC}$$

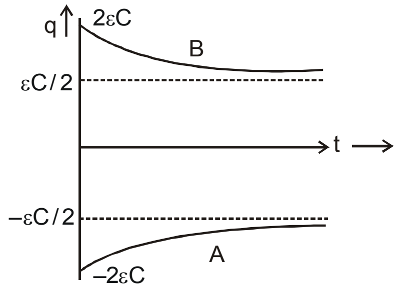
$$3\varepsilon C - 2q = 3\varepsilon C e^{-2t/RC}$$

$$\Rightarrow \quad q = \frac{3\varepsilon C}{2}(1 - e^{-2t/RC})$$

$$i = \frac{dq}{dt} = \frac{3\varepsilon}{R} e^{-2t/RC}$$

At plate A,

Charge $q_A = q - 2\varepsilon C = \frac{3}{2}\varepsilon C\,(1 - e^{-2t/RC}) - 2\varepsilon C$



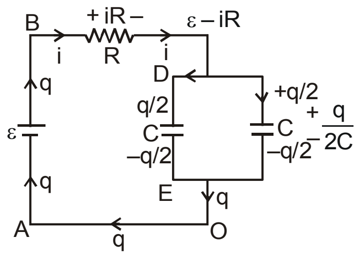
$$= -\frac{\varepsilon c}{2} - \frac{3\varepsilon c}{2}e^{-2t/RC}$$

$$q_A = -\frac{\varepsilon c}{2}(1+3e^{-2t/RC})$$

$$q_B = \frac{\varepsilon c}{2}(1+3e^{-2t/RC})$$

EXAMPLE 37

Without using the formula of equivalent find out the charge on capacitor and current in all the branches as a function of time.



Sol. Applying KVL in ABDEA

$$\varepsilon - iR = \frac{q}{2C} \quad \Rightarrow \quad i = \frac{\varepsilon}{R} - \frac{q}{2CR} = \frac{2C\varepsilon - q}{2CR}$$

$$\frac{dq}{2\varepsilon C - q} = \frac{dt}{2CR} \quad \Rightarrow \quad \int_0^q \frac{dq}{(2\varepsilon C - q)} = \frac{t}{2CR}$$

$$\frac{2\varepsilon C - q}{2\varepsilon C} = e^{-t/2RC} \quad \Rightarrow \quad q = 2\varepsilon C(1-e^{-t/2RC})$$

$$q_1 = \frac{q}{2} = \varepsilon C(1-e^{-t/RC}) \quad \Rightarrow i_1 = \frac{\varepsilon}{2R}e^{-t/2RC}$$

$$q_2 = \frac{q}{2} = \varepsilon C(1-e^{-t/2RC}) \quad \Rightarrow i_2 = \frac{\varepsilon}{2R}e^{-t/RC}$$

Alternate solution

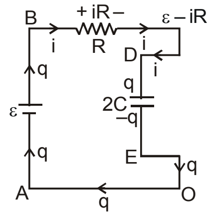
By equivalent

Time constant of circuit = 2C × R = 2RC

maximum charge on capacitor = 2C × ε = 2Cε

Hence equations of charge and current are as given below

$$q = 2\varepsilon C(1-e^{-t/2RC})$$



$$q_1 = \frac{q}{2} = \varepsilon C(1-e^{-t/2RC})$$

$$\Rightarrow \quad i_1 = \frac{\varepsilon}{2R}e^{-t/2RC}$$

$$q_2 = \frac{q}{2} = \varepsilon C\ (1 - e^{-t/2RC})$$

$$\Rightarrow \quad i_2 = \frac{\varepsilon}{2R}e^{-t/RC}$$

EXAMPLE 38

A capacitor is connected to a 12 V battery through a resistance of 10 Ω. It is found that the potential difference across the capacitor rises to 4.0 V in 1μs. Find the capacitance of the capacitor.

Sol. The charge on the capacitor during charging is given by $Q = Q_0(1-e^{-t/RC})$

Hence, the potential difference across the capacitor is $V = \frac{Q}{C} = \frac{Q_0}{C}(1-e^{-t/RC})$

Here, at t = 1 μs, the potential differences is 4V whereas the steady potential difference is $\frac{Q_0}{C} = 12V$.

So, $4V = 12V\ (1 - e^{-t/RC})$

or, $1-e^{-t/RC} = \frac{1}{3}$

or, $e^{-t/RC} = \frac{2}{3}$

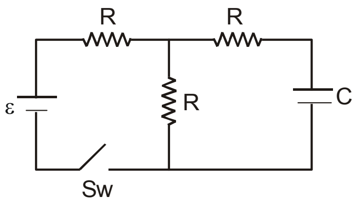
or, $\frac{t}{RC} = \ell n\left(\frac{3}{2}\right) = 0.405$

or, $RC = \frac{t}{0.405} = \frac{1\,\mu s}{0.405} = 2.469\ \mu s$

or, $C = \frac{2.469\ \mu s}{10\,\Omega} = 0.25\ \mu F.$

EXAMPLE 39

Initially the capacitor is uncharged find the charge on capacitor as a function of time, if switch is closed at t = 0.



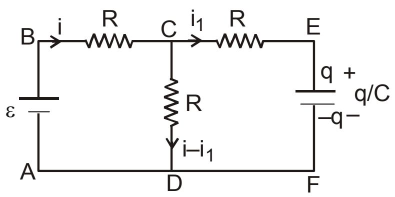
Sol. Applying KVL in loop ABCDA

$\varepsilon - iR - (i - i_1) R = 0$

$\varepsilon - 2iR + i_1R = 0$

Applying KVL in loop ABCEFDA

$$\varepsilon - iR - i_1R - \frac{q}{C} = 0$$



$$\frac{2\varepsilon - \varepsilon - i_1R - 2i_1R}{2} = \frac{q}{C}$$

$$\varepsilon C - 3i_1RC = 2q$$

$$\varepsilon C - 2q = 3\frac{dq}{dt}.RC$$

$$\int_0^q \frac{dq}{\varepsilon C - 2q} = \int_0^t \frac{dt}{3RC} \Rightarrow -\frac{1}{2}\ln\frac{\varepsilon C - 2q}{\varepsilon C} = \frac{t}{3RC}$$

$$q = \frac{\varepsilon C}{2}(1 - e^{-2t/3RC})$$

Method for objective :

In a circuit when there is only one capacitor then

$q = Q_{st}(1 - e^{-t/\tau})$: Q_{st} = steady state change on capacitor

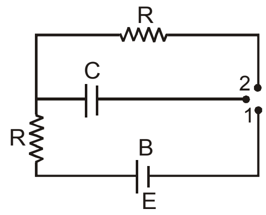
$\tau = R_{eff}C$

$R_{effective}$ is the resistance between the capacitor when battery is replaced by its internal resistance.

8.2 Discharging of a condenser :

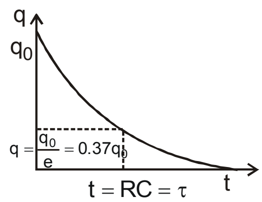
(i) In the given circuit if key 1 is opened and key 2 is closed then the condenser gets discharged.

(ii) The quantity of charge on the condenser at any instant of time t is given by $q = q_0\, e^{-(t/RC)}$ i.e. the charge falls exponentially.



(iii) If t = RC = τ = time constant, then $q = \frac{q_0}{e} = 0.37\, q_0 = 37\%$ of q_0

i.e. the time constant is that time during which the charge on condenser plates falls to 37%

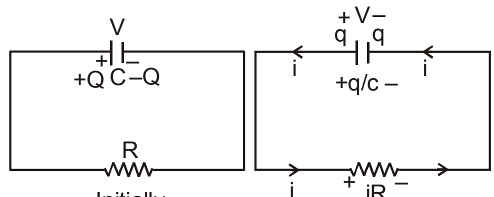


(iv) The dimensions of RC are those of time i.e. $M°L°T^1$ and the dimensions of $\frac{1}{RC}$ are those of frequency i.e., $M°L°T^{-1}$

(v) The potential difference across the condenser plates at any instant of time t is given by $V = V_0e^{-(t/RC)}$ volt

(vi) The transient current at any instant of time is given by $I = -I_0e^{-(t/RC)}$ ampere. i.e. the current in the circuit decreases exponentially but its direction is opposite to that of charging current.

Derivation of equation of discharging circuit :



Initially

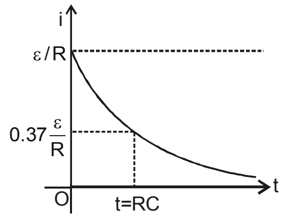
At any time t

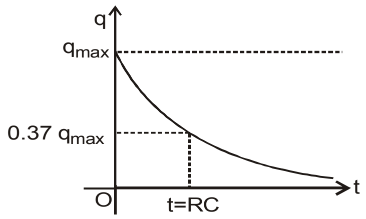
Applying K.V.L. $\quad +\frac{q}{C} - iR = 0 \Rightarrow i = \frac{q}{CR}$

$$\int_Q^q \frac{-dq}{q} = \int_0^t \frac{dt}{CR} \quad \Rightarrow \quad -\ln\frac{q}{Q} = +\frac{t}{RC}$$

$$q = Q.e^{-t/RC}$$

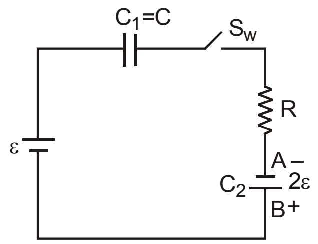
$$i = -\frac{dq}{dt} = \frac{Q}{RC}e^{-t/RC}$$





EXAMPLE 40

At $t = 0$ S_w is closed, if initially C_1 is uncharged and C_2 is charged to a potential difference 2ε then find out following (Given $C_1 = C_2 = C$)



(a) Charge on C_1 and C_2 as a function of time.

(b) Find out current in the circuit as a function of time.

(c) Also plot the graphs for the relations derived in part (a)

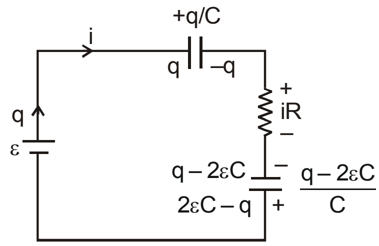
Sol. Let q charge flow in time 't' from the battery as shown. The charge on various plates of the capacitor is as shown in the figure.

Now applying KVL

$$\varepsilon - \frac{q}{C} - iR - \frac{q - 2\varepsilon C}{C} = 0$$

$$\varepsilon - \frac{q}{C} - \frac{q}{C} + 2\varepsilon - iR = 0$$

$$3\varepsilon = \frac{2q}{C} + iR$$



$$3\varepsilon - iR = \frac{2q}{C}$$

$$3\varepsilon - iRC = 2q$$

$$\frac{dq}{dt}RC = 3\varepsilon C - 2q$$

$$\int_0^q \frac{dq}{2\varepsilon C - 2q} = \int_0^t \frac{dt}{RC}$$

$$-\frac{1}{2}\ell n\left(\frac{3C\varepsilon - 2q}{3C\varepsilon}\right) = \frac{t}{RC}$$

$$\ln\left(\frac{3\varepsilon C - 2q}{3\varepsilon C}\right) = -\frac{2t}{RC}$$

$$3\varepsilon C - 2q = 3\varepsilon C\ e^{-2t/RC}$$

$$3\varepsilon C\ (1 - e^{-2t/RC}) = 2q$$

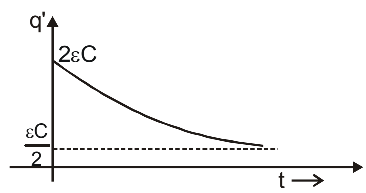
$$q = \frac{3}{2}\varepsilon C(1 - e^{-2t/RC})$$ **Ans.**

$$i = \frac{dq}{dt} = \frac{3\varepsilon}{R}e^{-2t/RC}$$ **Ans.**

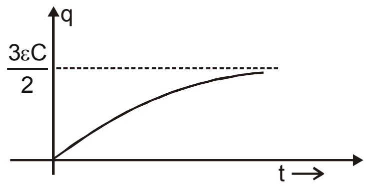
On the plate B

$$q' = 2\varepsilon C - q$$

$$= 2\varepsilon C - \frac{3}{2}\varepsilon C + \frac{3}{2}\varepsilon C e^{-2t/RC}$$



$$= \frac{\varepsilon C}{2} + \frac{3}{2}\varepsilon C e^{-2t/RC} \quad = \frac{\varepsilon C}{2}\left[1 + 3e^{-2t/RC}\right]$$



9. FORCE BETWEEN THE PLATES OF A CAPACITOR

In a capacitor as plates carry equal and opposite charges, there is a force of attraction between the plates. To calculate this force, we use the fact that the electric field is conservative and in a conservative field $F = -\frac{dU}{dx}$. In case of parallel plate capacitor:

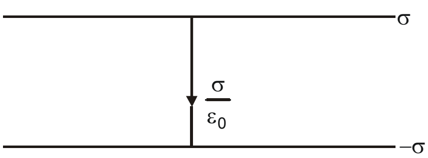
$$U = \frac{q^2}{2C} = \frac{1}{2}\frac{q^2 x}{\varepsilon_0 A} \quad [\text{as } C = \frac{\varepsilon_0 A}{x}]$$

So, $$F = -\frac{d}{dx}\left[\frac{q^2}{2\varepsilon_0 A}x\right] = \frac{-1}{2}\frac{q^2}{\varepsilon_0 A}$$

The negative sign implies that the force is attractive.

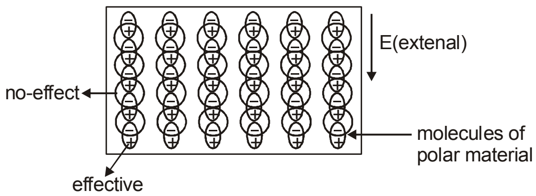
10. CAPACITORS WITH DIELECTRIC:

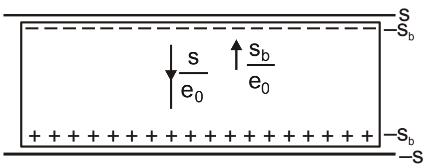
(i) In absence of dielectric



$$E = \frac{\sigma}{\varepsilon_0}$$

(ii) When a dielectric fills the sapce between the plates then molecules having dipole moment align themselves in the direction of electric field.



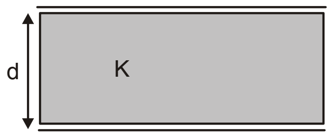


σ_b = induced charge density (called bound charge because it is not due to free electrons).

* For polar molecules dipole moment ≠ 0
* For non-polar molecules dipole meomet = 0

(iii) Capacitance in the presence of dielectric

$$C = \frac{\sigma A}{V} = \frac{\sigma A}{\frac{\sigma}{K\varepsilon_0}.d} = \frac{AK\varepsilon_0}{d} = \frac{AK\varepsilon_0}{d}$$

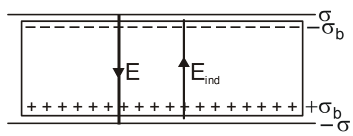


Here capacitance is increased by a factor K.

$$C = \frac{AK\varepsilon_0}{d}$$

(iv) Polarisation of material :

When a nonpolar substance is placed in an electric field then dipole moment is induced in the molecule. This induction of dipole moment is called polarisation of material. The induced charge also produces electric field.



σ_b = induced (bound) charge density.

$$E_{in} = E - E_{ind} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_b}{\varepsilon_0}$$

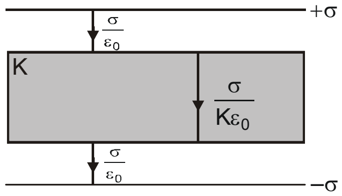
It is seen the ratio of electric field between the plates in absence of dielectric and in presence of dielectric is constant for a material of dielectric. This ratio is called 'Dielectric constant' of that material. It is represented by ε_r or k.

$$E_{in} = \frac{\sigma}{K\varepsilon_0} \Rightarrow \quad \sigma_b = \sigma\left(1 - \frac{1}{K}\right)$$

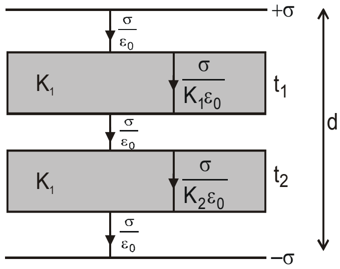
(v) If the medium is not filled between the plates completely then electric filed will be as shown in figure.

Case : (1)

The total electric field produced by bound induced charge on the dielectric outside the slab is zero because they cancel each other.



Case : (2)



so potential difference between plates

$$v = \frac{\sigma}{\varepsilon_0}[d - t_1 - t_2] + \frac{\sigma}{k_1\varepsilon_0}t_1 + \frac{\sigma}{k_2\varepsilon_0}t_2$$

so equivalent capacitance

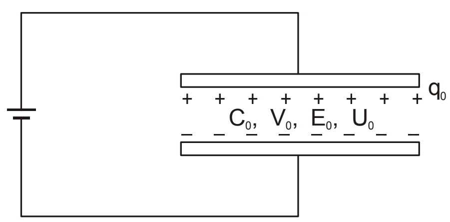
$$C = \frac{Q}{v}$$

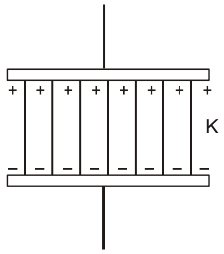
$$C = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0}\left[d - t_1 - t_2 + \frac{t_1}{k_1} + \frac{t_2}{k_2}\right]}$$

$$C = \frac{A\varepsilon_0}{d - t_1\left(1 - \frac{1}{k_1}\right) - t_2\left(1 - \frac{1}{k_2}\right)}$$

10.1 Introduction of a Dielectric slab of dielectric constant K between the plates

(a) When the battery is disconnected





Let q_0, C_0, V_0, E_0 and U_0 represents the charge, capacity, potential difference, electric field and energy associated with charged air capacitor respectively. With the introduction of a dielectric slab of dielectric constant K between the plates and the battery disconnected.

(i) Charge remains constant, i.e., $q = q_0$, as in an isolated system charge is conserved.

(ii) Capacity increases, i.e., $C = KC_0$, as by the presence of a dielectric capacity becomes K times.

(iii) Potential difference between the plates decreases, i.e., $V = \left(\frac{V_0}{K}\right)$, as

$$V = \frac{q}{C} = \frac{q_0}{KC_0} = \frac{V_0}{K} \quad [\because\ q = q_0 \text{ and } C = KC_0]$$

(iv) Field between the plates decreases, i.e.,

$$E = \frac{E_0}{K}, \text{ as}$$

$$E = \frac{V}{d} = \frac{V_0}{Kd} = \frac{E_0}{K} \quad [\text{as } V = \frac{V_0}{K}] \text{ and } E_0 = \frac{V_0}{d}$$

(v) Energy stored in the capacitor decreases i.e.

$$U = \left(\frac{U_0}{K}\right), \text{ as}$$

$$U = \frac{q^2}{2C} = \frac{q_0^2}{2KC_0} = \frac{V_0}{K} \quad (\text{as } q = q_0 \text{ and } C = KC_0]$$

(b) When the battery remains connected (potential is held constant)

(i) Potential difference remains constant, i.e., $V = V_0$, as battery is a source of constant potential difference.

(ii) Capacity increases, i.e., $C = KC_0$, as by presence of a dielectric capacity becomes K times.

(iii) Charge on capacitor increases, i.e., $q = Kq_0$, as

$q = CV = (KC_0)V = Kq_0$ [$\because q_0 = C_0V$]

(iv) Electric field remains unchanged, i.e., $E = E_0$, as

$$E = \frac{V}{d} = \frac{V_0}{d} = E_0 \text{ [as } V = V_0 \text{ and } \frac{V_0}{d} = E_0 \text{]}$$

(v) Energy stored in the capacitor increases,

i.e., $U = KU_0$, as

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)(V_0)^2 = \frac{1}{2}KU_0 \text{ [as } C = KC_0$$

$$\text{and } U_0 = \frac{1}{2}C_0V_0^2\text{]}$$

EXAMPLE 41

A parallel plate air capacitor is made using two square plates each of side 0.2 m, spaced 1 cm apart. It is connected to a 50V battery.

(a) What is the capacitance ?

(b) What is the charge on each plate ?

(c) What is the energy stored in the capacitor ?

(d) What is the electric field between the plates ?

(e) If the battery is disconnected and then the plates are pulled apart to a separation of 2 cm, what are the answers to the above parts ?

Sol. (a)

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12})\times0.2\times0.2}{0.01} = 3.54\times10^{-5}\ \mu F$$

(b) $Q_0 = C_0V_0 = 3.54 \times 10^{-5} \times 50 = 1.77 \times 10^{-3}\ \mu C$

(c) $U_0 = \frac{1}{2}C_0V_0^2 = \frac{1}{2}\times(3.54\times10^{-11})(50)^2 = 4.42\times10^{-8} J$

(d) $E_0 = \frac{V_0}{d} = \frac{50}{0.01} = 5000\ V/m$

(e) If the battery is disconnected the charge on the capacitor plates remains constant while the potential difference between the plates can change.

$$C = \frac{\varepsilon_0 A}{d} = \frac{C_0}{2} = 1.77\times10^{-5}\ \mu F$$

$Q = Q_0 = 1.77 \times 10^{-3}\ \mu C$

$$V = \frac{Q}{C} = \frac{Q_0}{C_0/2} = 2V_0$$

$$U = \frac{1}{2}CV^2 = C_0V_0^2 = 8.84\times10^{-8} J$$

$$E = \frac{V}{d} = \frac{2V_0}{2d_0} = E_0 = 5000\ V/m$$

EXAMPLE 42

In the last illustration, suppose that the battery is kept connected while the plates are pulled apart. What are the answers to the parts (a), (b), (c) and (d) in that case ?

Sol. If the battery is kept connected, the potential difference across the capacitor plates always remains equal to the emf of battery and hence is constant.

$V = V_0 = 50V$

$$C = \frac{\varepsilon_0 A}{d'} = \frac{\varepsilon_0 A}{2d} = \frac{C_0}{2} = 1.77\times10^{-5}\ \mu F$$

$$Q = CV = \frac{C_0V_0}{2} = \frac{Q_0}{2} = 8.85\times10^{-4}\mu C$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{C_0}{2}\right)V_0^2 = \frac{U_0}{2} = 2.21\times10^{-8} J$$

$$E = \frac{V}{d} = \frac{V_0}{2d_0} = \frac{E_0}{2} = 2500\ V/m$$

EXAMPLE 43

A parallel plate capacitor has plates of area 4 m^2 separated by distance of 0.5 mm. The capacitor is connected across a cell of emf 100 V.

(a) Find the capacitance, charge and energy stored in the capacitor.

(b) A dielectric slab of thickness 0.5 mm is inserted inside this capacitor after it has been disconnected from the cell. Find the answers to part (a) if K = 3.

Sol. (a) $C_0 = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 4}{0.5 \times 10^{-3}} = 7.08 \times 10^{-2}\ \mu F$

$Q_0 = C_0V_0 = (7.08 \times 10^{-2} \times 100)\mu C = 7.08\ \mu C$

$U_0 = \frac{1}{2} C_0 V_0^2 = 3.54 \times 10^{-4}\ J$

(b) As the cell has been disconnected

$Q = Q_0$

$C = \frac{K\varepsilon_0 A}{d} = KC_0 = 0.2124 \mu F$

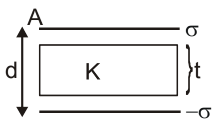
$V = \frac{Q}{C} = \frac{Q_0}{KC_0} = \frac{V_0}{K} = \frac{100}{3} V$

$U = \frac{Q_0^2}{2C} = \frac{Q_0^2}{2KC_0} = \frac{U_0}{K} = 118 \times 10^{-6}\ J$

EXAMPLE 44

If a dielectric slab of thickness t and area A is inserted in between the plates of a parallel plate capacitor of plate area A and distance between the plates d (d > t) then find out capacitance of system. What do you predict about the dependence of capacitance on location of slab ?

Sol.



$C = \frac{Q}{V} = \frac{\sigma A}{V}$

$V = \frac{\sigma t_1}{\varepsilon_0} + \frac{\sigma t}{K\varepsilon_0} + \frac{\sigma t_2}{\varepsilon_0}$ $(\because\ t_1 + t_2 = d - t)$

$= \frac{\sigma}{\varepsilon_0}[t_1 + t_2 + t/k]$

$\Rightarrow$ $V = \frac{\sigma}{\varepsilon_0}\left[d - t + \frac{t}{k}\right] = \frac{Q}{C} = \frac{\sigma A}{C}$

$\Rightarrow$ $C = \frac{\varepsilon_0 A}{d - t + t/K}$

☛

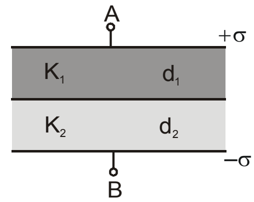
* Capacitance does not depend upon the position of dielectric (it can be shifted up or down & still capacitance does not change).

* If the slab is of metal then

$C = \frac{A\varepsilon_0}{d - t}$

EXAMPLE 45

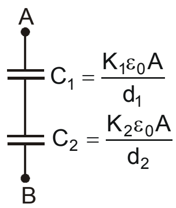
Find out capacitance between A and B if two dielectric slabs of dielectric constant K_1 and K_2 of thickness d_1 and d_2 and each of area A are inserted between the plates of parallel plate capacitor of plate area A as shown in figure.



Sol. $C = \frac{\sigma A}{V}$;

$V = E_1\ d_1 + E_2\ d_2 = \frac{\sigma d_1}{K_1\varepsilon_0} + \frac{\sigma d_2}{K_2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}\left(\frac{d_1}{k_1} + \frac{d_2}{k_2}\right)$

$\therefore$ $C = \frac{A\varepsilon_0}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$



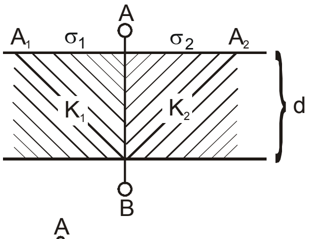
$\Rightarrow$ $\frac{1}{C} = \frac{d_1}{AK_1\varepsilon_0} + \frac{d_2}{AK_2\varepsilon_0}$

This formula suggests that the system between A and B can be

considered as series combination of two capacitors.

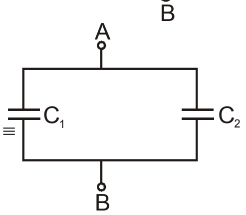
EXAMPLE 46

Find out capacitance between A and B if two dielectric slabs of dielectric constant K_1 and K_2 of area A_1 and A_2 and each of thickness d are inserted between the plates of parallel plate capacitor of plate area A as shown in figure.



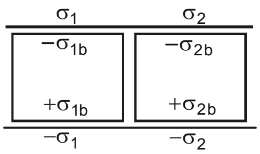
Sol.

≡



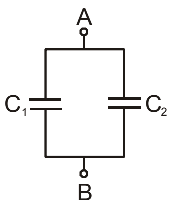
$$C_1 = \frac{A_1K_1\varepsilon_0}{d},\ C_2 = \frac{A_2K_2\varepsilon_0}{d}$$

$$E_1 = \frac{V}{d} = \frac{\sigma_1}{K_1\varepsilon_0}\ ;\ E_2 = \frac{V}{d} = \frac{\sigma_2}{K_2\varepsilon_0}$$



$$\sigma_1 = \frac{K_1\varepsilon_0 V}{d} \qquad \sigma_2 = \frac{K_2\varepsilon_0 V}{d}$$

$$C = \frac{Q_1 + Q_2}{V} = \frac{\sigma_1 A_1 + \sigma_2 A_2}{V} = \frac{K_1\varepsilon_0 A_1}{d} + \frac{K_2\varepsilon_0 A_2}{d}$$

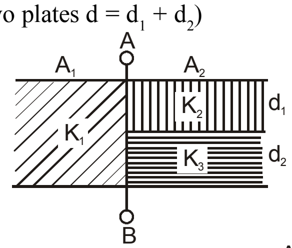


The combination is equivalent to :

$\therefore$ $C = C_1 + C_2$

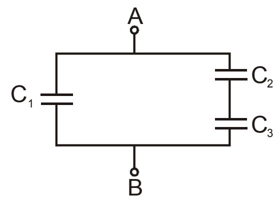
EXAMPLE 47

Find out capacitance between A and B if three dielectric slabs of dielectric constant K_1 of area A_1 and thickness d_1 K_2 of area A_2 and thickness d_2 and K_3 of area A_2 and thickness d_3 are inserted between the plates of parallel plate capacitor of plate area A as shown in figure. (Given distance between the two plates $d = d_1 + d_2$)



Sol. It is equivalent to

$$C = C_1 + \frac{C_2C_3}{C_2 + C_3}$$



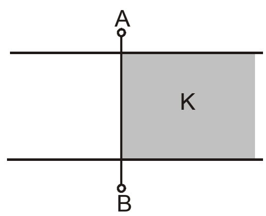
$$C = \frac{A_1K_1\varepsilon_0}{d_1 + d_2} + \frac{\frac{A_2K_2\varepsilon_0}{d_1} \cdot \frac{A_2K_3\varepsilon_0}{d_2}}{\frac{A_2K_2\varepsilon_0}{d_1} + \frac{A_2K_3\varepsilon_0}{d_2}}$$

$$= \frac{A_1K_1\varepsilon_0}{d_1 + d_2} + \frac{A_2^2K_2K_3\varepsilon_0^2}{A_2K_2\varepsilon_0 d_2 + A_2K_3\varepsilon_0 d_1}$$

$$= \frac{A_1K_1\varepsilon_0}{d_1 + d_2} + \frac{A_2K_2K_3\varepsilon_0}{K_2d_2 + K_3d_1}$$

EXAMPLE 48

A dielectric of constant K is slipped between the plates of parallel plate condenser in half of the space as shown in the figure. If the capacity of air condenser is C, then new capacitance between A and B will be



(A) $\frac{C}{2}$

(B) $\frac{C}{2K}$

(C) $\frac{C}{2}[1+K]$

(D) $\frac{2[1+K]}{C}$

Sol. This system is equivalent to two capacitors in parallel with area of each plate $\frac{A}{2}$.

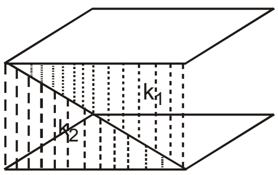
$$C = C_1 + C_2$$

$$= \frac{\varepsilon_0 A}{2d} + \frac{\varepsilon_0 AK}{2d}$$

$$= \frac{\varepsilon_0 A}{2d}[1+K] = \frac{C}{2}[1+K]$$

Hence the correct answer will be (C)

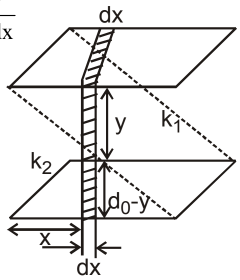
EXAMPLE 49



Find the equivalent capacitance of the given figure.

Sol.

$$\frac{1}{dc} = \frac{y}{\varepsilon_0 k_1 b dx} + \frac{d_0 - y}{\varepsilon_0 k_2 b\, dx}$$



$$\frac{1}{dc} = \frac{d_0 k_1 + y(k_2 - k_1)}{\varepsilon_0 k_1 k_2 b\, dx}$$

$$dc = \frac{\varepsilon_0 k_1 k_2 b\, dx}{d_0 k_1 + y(k_2 - k_1)}$$

All these capacitors (small) are parallel

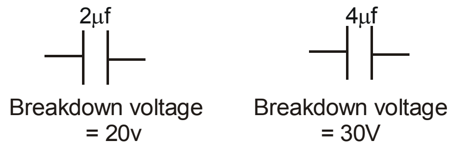
so $C_{eq} = \int_0^{C_{eq}} dC = \int_0^a \frac{\varepsilon_0 k_1 k_2 b\ dx}{d_0 k_1 + y(k_2 - k_1)}$

Now $\frac{d_0}{a} = \frac{y}{x} \Rightarrow C_{eq} = \int_0^a \frac{\varepsilon_0 k_1 k_2 b\, dx}{d_0 k_1 + \frac{d_0}{a} x(k_2 - k_1)}$

$$C_{eq} = \frac{a\,\varepsilon_0 k_1 k_2 b}{d_0(k_2 - k_1)} \ln\left[\frac{(k_2 - k_1)}{k_1}\right]$$

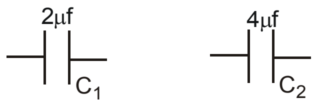
- **Breakdown voltage**

The voltage across the capacitor at which the current starts flowing through capacitor is called Breakdown voltage.



EXAMPLE 50

Find the break down voltage for the capacitors if they are connected in series



max. charge max. charge

$= 20 \times 2\mu C = 40\ \mu C = 30 \times 4\ \mu c = 120\ \mu C$

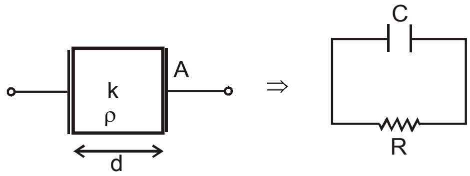
Charge will be same on both the capacitors so 2μF capacitor will reach at breakdown voltage first, so maximum charge possible = 40 μC

So Breakdown voltage $= V_{C_1} + V_{C_2} = \frac{Q_1}{C_1} + \frac{Q_1}{C_2}$

$$V = \frac{40}{2} + \frac{40}{4} \text{ volt}$$

$$V = \frac{120}{4} \text{volt} = 30 \text{ volt}$$

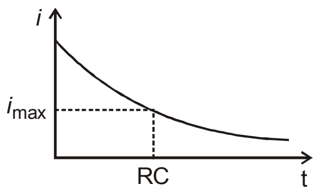
Leakage current :



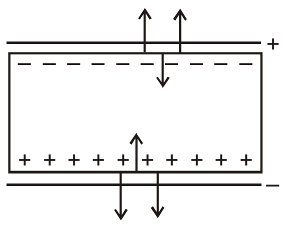
here $C = \frac{\varepsilon_0 kA}{d}$

Resistance (R) = $\frac{\rho d}{A}$

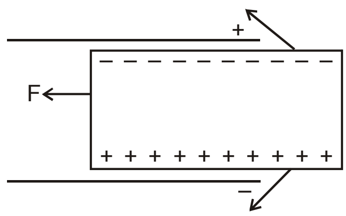
Theoritically after disconnection from battery charge should remain as it is but due to the material's resistance discharging takes place. This discharging current is called leakage current. So R should be high for a good capacitor so that leakage current is minimum.



(viii) Force on a dielectric due to charged capacitor



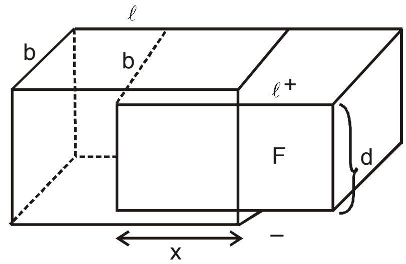
If dielectric is completely inside the capacitor then force is equal to zero.



Case I Voltage source remains connected

V = constant.

$$U = \frac{1}{2}CV^2$$



$$F = \left(\frac{dU}{dx}\right) = \frac{v^2}{2}\frac{dC}{dx}$$

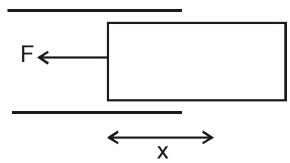
where $C = \frac{xb\varepsilon_0 K}{d} + \frac{\varepsilon_0(\ell - x)b}{d}$

$$\Rightarrow \quad C = \frac{\varepsilon_0 b}{d}[Kx + \ell - x]$$

$$\frac{dC}{dx} = \frac{\varepsilon_0 b}{d}(K-1)$$

$$\therefore \quad F = \frac{\varepsilon_0 b(K-1)V^2}{2d} = \text{constant (does not depends on x)}$$

Case II : When charge on capacitor is constant



$$C = \frac{xb\varepsilon_0 x}{d} + \frac{\varepsilon_0(\ell - x)b}{d}, \qquad U = \frac{Q^2}{2C}$$

$$F = \left(\frac{dU}{dx}\right) = \frac{Q^2}{2C^2}\cdot\frac{dC}{dx} \;\left[\text{where, } \frac{dC}{dx} = \frac{\varepsilon_0 b}{d}(K-\ell)\right]$$

$$= \frac{Q^2}{2C^2}\cdot\frac{dC}{dx} \qquad \text{(here force 'F' depends on x)}$$

Note

The student can now attempt section C , D from exercise.

Exercise - 1 Objective Problems | JEE Main

Section A - Capacitance Calculation (Sphere, Cylinderical, Parallel plate), Sharing of Charge, Combination of Capacitor

1. The capacitance of a metallic sphere will be 1μF, if its radius is nearly
(A) 9 km (B) 10 m
(C) 1.11 m (D) 1.11 cm

2. No current flows between two charged bodies connected together when they have the same
(A) capacitance or Q/V ratio
(B) charge
(C) resistance
(D) potential or Q/C ratio

3. Two spherical conductors A and B of radii R and 2R respectively are each given a charge Q. When they are connected by a metallic wire. The charge will
(A) flow from A to B
(B) flow from B to A
(C) remain stationary on conductor
(D) none of these

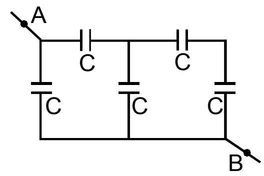
4. The capacity of a parallel plate condenser is C. Its capacity when the separation between the plates is halved will be
(A) 4C (B) 2C
(C) C/2 (D) C/4

5. Two spherical conductors of capacitance 3.0μF and 5.0μF are charged to potentials of 300volt and 500volt. The two are connected resulting in redistribution of charges. Then the final potential is -
(A) 300 volt (B) 500 volt
(C) 425 volt (D) 400 volt

6. N drops of mercury of equal radii and possessing equal charges combine to form a big spherical drop. Then the capacitance of the bigger drop compared to each individual drop is
(A) N times (B) $N^{2/3}$ times
(C) $N^{1/3}$ times (D) $N^{5/3}$ times

7. From a supply of identical capacitors rated 8 μ F ,250 V, the minimum number of capacitors required to form a composite 16 μ F,1000 V is :
(A) 2 (B) 4
(C) 16 (D) 32

8. What is the equivalent capacitance of the system of capacitors between A & B



(A) $\frac{7}{6}$ C (B) 1.6 C
(C) C (D) None

Section B - Circuit Problems (Switch, Energy etc.)

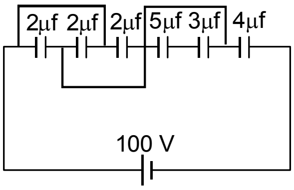
9. If the p.d. across the ends of a capacitor 4μF is 1.0 kilovolt. Then its electrical potential energy will be
(A) 4×10^{-3}ergs (B) 2 ergs
(C) 2 joules (D) 4 joules

10. A conductor of capacitance 0.5μF has been charged to 100volts. It is now connected to uncharged conductor of capacitance 0.2μF. The loss in potential energy is nearly -
(A) 7×10^{-4} J (B) 3.5×10^{-4} J
(C) 14×10^{-4} J (D) 7×10^{-3} J

11. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance 2C is similarly charged to a potential difference 2V. The charging battery is now disconnected and the capacitors are connect in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is
(A) zero (B) $\frac{3}{2}CV^2$
(C) $\frac{25}{6}CV^2$ (D) $\frac{9}{2}CV^2$

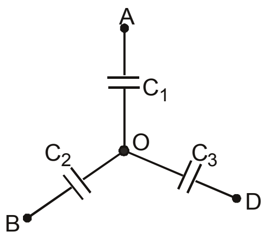
12. A 2 μ F capacitor is charged to a potential = 10 V. Another 4 μ F capacitor is charged to a potential = 20V. The two capacitors are then connected in a single loop, with the positive plate of one connected with negative plate of the other. What heat is evolved in the circuit?
(A) 300 μ J (B) 600 μ J
(C) 900 μ J (D) 450 μ J

13. In the circuit shown in figure charge stored in the capacitor of capacity $5\,\mu f$ is



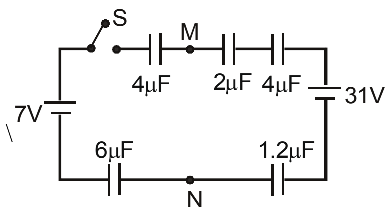
(A) $60\ \mu C$ (B) $20\ \mu C$
(C) $30\ \mu C$ (D) zero

14. Three uncharged capacitors of capacitane $C_1 = 1\mu F$, $C_2 = 2\mu F$ and $C_3 = 3\mu F$ are connected as shown in figure to one another and to points A, B and D potential ϕ_A = 10V, ϕ_B = 25V and ϕ_D = 20 V, Determine the potential (ϕ_0) at point O.



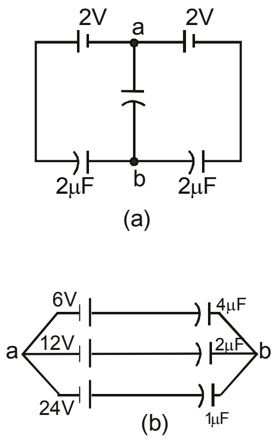
(A) 20 V (B) 30 V
(C) 40 V (D) 10 V

15. Five capacitors are connected as shown in the figure. Initially S is opened and all capacitors are uncharged. When S is closed, steady state is obtained. Then find out potential difference between the points M and N.



(A) 14 (B) 12
(C) 10 (D) 15

16. Find the potential difference $V_a - V_b$ between the points a and b shows in each parts of the figure.



(A) zero, $-\frac{7}{72}V = -10.3$ V

(B) one, $-\frac{72}{7}V = -10.3$ V

(C) one, $-\frac{7}{72}V = -10.3$ V

(D) zero, $-\frac{72}{7}V = -10.3$ V

17. Each plate of a parallel plate air capacitor has an area S. What amount of work has to be performed to slowly increase the distance between the plates from x_1 to x_2 If :

(i) the charge of the capacitor, which is equal to q or

(A) $\frac{q^2(x_2+x_1)}{2\in_0 S}$ (B) $\frac{q^2(x_2-x_1)}{2\in_0 S}$

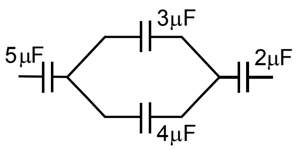
(C) $\frac{q^2(x_2-x_1)}{\in_0 S}$ (D) $\frac{q^2(x_1-x_2)}{2\in_0 S}$

(ii) the voltage across the capacitor, which is equal to V, is kept constant in the process.

(A) $\frac{\in_0 SV^2\left(\frac{1}{x_1}-\frac{1}{x_2}\right)}{2}$ (B) $\frac{\in_0 SV^2\left(\frac{1}{x_2}-\frac{1}{x_1}\right)}{4}$

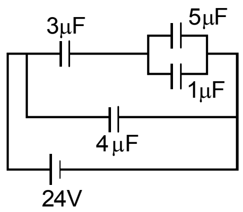
(C) $\frac{\in_0 SV^2\left(\frac{1}{x_2}-\frac{1}{x_1}\right)}{2}$ (D) $\frac{\in_0 SV^2\left(\frac{1}{x_2}+\frac{1}{x_1}\right)}{2}$

18. If charge on left plane of the 5 μ F capacitor in the circuit segment shown in the figure is-20 μ C, the charge on the right plate of 3 μ F capacitor is



(A) +8.57 μ C (B) -8.57 μ C
(C) + 11.42 μ C (D) -11.42 μ C

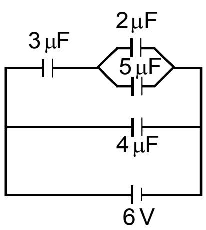
19. In the circuit shown, the energy stored in 1 μ F capacitor is



(A) 40 μ J (B) 64 μ J
(C) 32 μ J (D) none

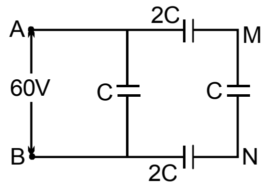
20. A capacitor C_1 = 4μF is connected in series with another capacitor C_2 = 1μF. The combination is connected across a d.c. source of voltage 200V. The ratio of potential across C_1 and C_2 is -
(A) 1 : 4 (B) 4 : 1
(C) 1 : 2 (D) 2 : 1

21. In the circuit shown in figure, the ratio of charges on 5 μ F and 4 μ F capacitor is :



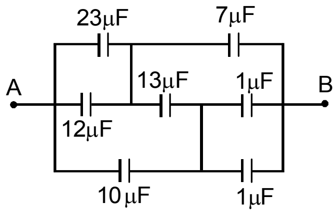
(A) 4/5 (B) 3/5
(C) 3/8 (D) 1/2

22. In the circuit shown, a potential difference of 60V is applied across AB. The potential difference between the point M and N is



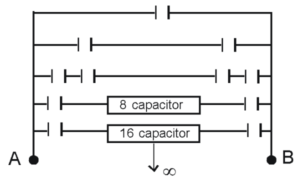
(A) 10 V (B) 15 V
(C) 20 V (D) 30 V

23. Find the equivalent capacitance across A & B



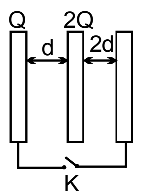
(A) $\frac{28}{3}$ μ F (B) $\frac{15}{2}$ μ F
(C) 15 μ F (D) none

24. An infinite number of identical capacitors each of capacitance 1μF are connected as in adjoining figure. Then the equivalent capacitance between A and B is



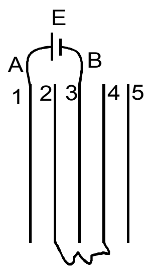
(A) 1μF (B) 2μF
(C) 1/2μF (D) ∞

25. Three large plates are arranged as shown. How much charge will flow through the key k if it is closed?



(A) $\frac{5Q}{6}$ (B) $\frac{4Q}{3}$
(C) $\frac{3Q}{2}$ (D) none

26. Five conducting parallel plates having area A and separation between them d, are placed as shown in the figure. Plate number 2 and 4 are connected wire and between point A and B, a cell of emf E is connected. The charge flown through the cell is



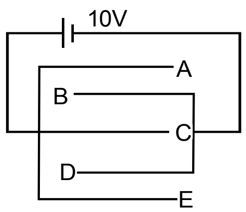
(A) $\frac{3}{4}\frac{\varepsilon_0 AE}{d}$

(B) $\frac{2}{3}\frac{\varepsilon_0 AE}{d}$

(C) $\frac{4\varepsilon_0 AE}{d}$

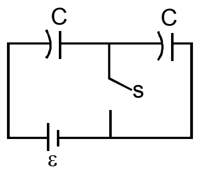
(D) $\frac{\varepsilon_0 AE}{2d}$

27. Five identical capacitor plates are arranged such that they make capacitors each of 2 μ F. The plates are connected to a source of emf 10 V. The charge on plate C is



(A) +20 μ C (B) +40 μ C
(C) +60 μ C (D) +80 μ C

28. Consider the situatino shown in the figure. The switch S is open for a long time and then closed.



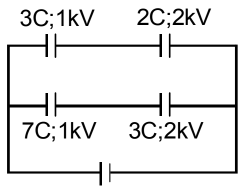
(i) Find the charge flown through the battery when the switch S is closed.
(A) $C\varepsilon/2$ (B) $C\varepsilon/4$
(C) $C\varepsilon$ (D) none of these

(ii) Find the work done by the battery.
(A) $C\varepsilon^2$ (B) $C\varepsilon^2/2$
(C) $C\varepsilon^2/4$ (D) none of these

(iii) Find the change in energy stored in the capacitors.
(A) $C\varepsilon^2$ (B) $C\varepsilon^2/2$
(C) $C\varepsilon^2/4$ (D) none of these

(iv) Find the heat developed in the system.
(A) $C\varepsilon^2$ (B) $C\varepsilon^2/2$
(C) $C\varepsilon^2/4$ (D) none of these

29. The diagram shows four capacitors with capacitances and break down voltages as mentioned. What should be the maximum value of the external emf source such that no capacitor breaks down?[Hint: First of all find out the break down voltages of each branch. After that compare them.]



(A) 2.5 KV (B) 10/3 KV
(C) 3 KV (D) 1 KV

30. Three capacitors 2 μ F, 3 μ F and 5 μ F can withstand voltages to 3 V, 2V and 1V respectively. Their series combination can withstand a maximum voltage equal to
(A) 5 Volts (B) (31/6) Volts
(C) (26/5) Volts (D) None

Section C - Dielectric

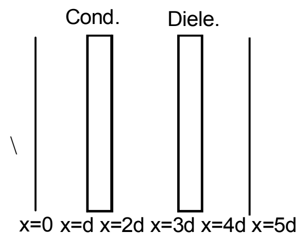
31. A metallic plate of thickness (t) and face area of one side (A) is inserted between the plates of a parallel plate air capacitor with a separation (d) and face are (A). Then the equivalent capacitance is :

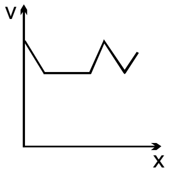
(A) $\frac{\epsilon_0 A}{d}$ (B) $\frac{\epsilon_0 A}{(d \times t)}$

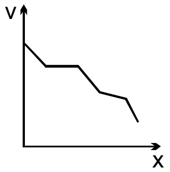
(C) $\frac{\epsilon_0 A}{(d - t)}$ (D) $\frac{\epsilon_0 A}{(d + t)}$

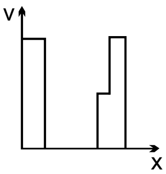
32. Two metal plates form a parallel plate condenser. The distance between the plates in d. Now a metal plate of thickness d/2 and of same area is inserted completely between the plates, the capacitance -
(A) remains unchanged
(B) is doubled
(C) is halved
(D) reduced to one fourth

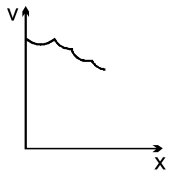
33. The distance between plates of a parallel plate capacitor is 5d. Let the positively charged plate is at x=0 and negatively charged plate is at x=5d.Two slabs one of conductor and other of a dielectric of equal thickness d are inserted between the plates as shown in figure. potential versus distance graph will look like:



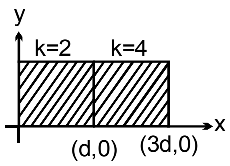
(A) 

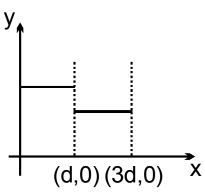
(B) 

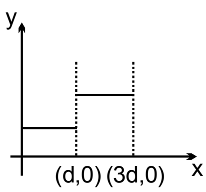
(C) 

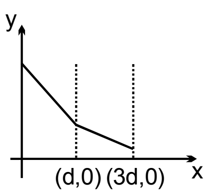
(D) 

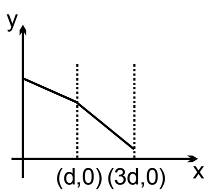
34. A parallel plate capacitor has two layers of dielectric as shown in figure. This capacitor is connected across a battery. The graph which shows the variation of electric field (E) and distance (x) from left plate.



(A) 

(B) 

(C) 

(D) 

35. The parallel plates of a capacitor have an area 0.2 m^2 and are 10^{-2} m apart. The original potential difference between them is 3000 V, and it decreases to 1000 V when a sheet of dielectric is inserted between the plates filling the full space. Compute :

($\in_0 = 9 \times 10^{-12}$ S.I. units)

(i) Permittivity $\in$ of the dielectric.

(A) $25 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

(B) $37 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

(C) $27 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

(D) $28 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

36. A parallel plate isolated condenser consists of two metal plates of area A and separation 'd'. A slab of thickness 't' and dielectric constant K is inserted between the plates with its faces parallel to the plates and having the same surface area as that of the plates. Find the capacitance of the system. If K = 2, for what value of t/d will the capacitance of the system be 3/2 times that of the condenser with air filling the full space ?

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) 3

37. Hard rubber has a dielectric constant of 2.8 and a dielectric strength of 18×10^6 volts/meter. If it is used as the dielectric material filling the full space in a parallel plate capacitor. What minimum area may the plates of the capacitor have in order that the capacitance be 7.0×10^{-2} μf and that the capacitor be able to withstand a potential difference of 4000 volts.

(A) 0.62 m^{-2} (B) 0.32 m^{-2}

(C) 0.42 m^{-2} (D) 0.52 m^{-2}

38. Two parallel plate air capacitors each of capacitance C were connected in series to a battery with e.m.f ε. Then one of the capacitors was filled up with uniform dielectric with relative permittivity k. What amount of charge flows through the battery ?

(A) $\Delta q = \frac{1}{2}CE\frac{k+1}{k-1}$ (B) $\Delta q = \frac{1}{2}CE\frac{k-1}{k+1}$

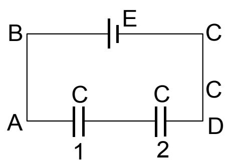
(C) $\Delta q = \frac{1}{2}CE\frac{k-1}{k+1}$ (D) none of these

39. A parallel-plate capacitor of plate area A and plate separation d is charged to a potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the whole space between the plates. Find the work done on the system in the process of inserting the slab.

(A) $\frac{\varepsilon_0 AV^2}{2d}\left(\frac{1}{K}-1\right)$ (B) $\frac{\varepsilon_0 AV^2}{d}\left(\frac{1}{K}-1\right)$

(C) $\frac{\varepsilon_0 AV^2}{2d}\left(\frac{1}{K}+1\right)$ (D) $\frac{\varepsilon_0 AV^2}{d}\left(\frac{1}{K}+1\right)$

40. In the adjoining figure, capacitor (1) and (2) have a capacitance 'C' each When the dielectric of dielectric constant K is inserted between the plates of one of the capacitor, the total charge flowing through battery is :



(A) $\frac{KCE}{K+1}$ from B to C

(B) $\frac{KCE}{K+1}$ from C to B

(C) $\frac{(K-1)CE}{2(K+1)}$ from B to C

(D) $\frac{(K-1)CE}{2(K+1)}$ from C to B

41. The distance between the plates of a charged parallel plate capacitor is 5 cm and electric field inside the plates is 200 Vcm^{-1}. An uncharged metal bar of width 2 cm is fully immersed into the capacitor. The length of the metal bar is same as that of plate of capacitor. The voltage across capacitor after the immersion of the bar is :

(A) zero (B) 400 V

(C) 600 V (D) 100 V

42. The capacity and the energy stored in a parallel plate condenser with air between its plates are respectively C_0 and W_0. If the air is replaced by glass (dielectric constant = 5) between the plates at constant charge, the capacity of the plates and the energy stored in it will respectively be -

(A) $5C_0$, $5W_0$ (B) $5C_0$, $\frac{W_0}{5}$

(C) $\frac{C_0}{5}$, $5W_0$ (D) $\frac{C_0}{5}$, $\frac{W_0}{5}$

43. By inserting a plate of dielectric material between the plates of a parallel plate capacitor at constant potential, the energy is increased five times. The dielectric constant of the material is

(A) 1/25 (B) 1/5

(C) 5 (D) 25

44. A glass slab is put with in the plates of a charged parallel plate condenser. Which of the following quantities does not change?

(A) energy of the condenser

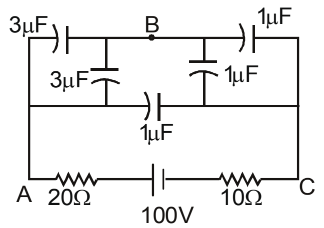
(B) capacity

(C) intensity of electric field

(D) charge

Section D - R-C Circuit

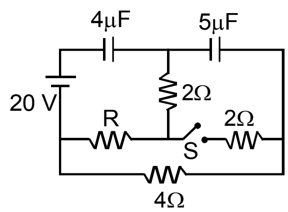
45. Find the potential difference between the points A and B and between the points B and C figure in steady



(A) 75 V and 25 V (B) 35 V and 65 V

(C) 25 V and 75 V (D) 65 V and 35 V

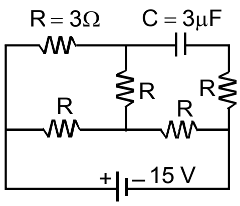
46. Find heat produced in the capacitors after long time on closing the switch S



(A) 0.0002 J (B).0005 J

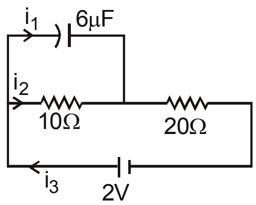
(C) 0.00075 J (D) zero

47. In the circuit shown, the cell is ideal, with emf=15 V. Each resistance is of 3 Ω. The potential difference across the capacitor is (after long time)



(A) zero (B) 9 V
(C) 12 V (D) 15 V

48.



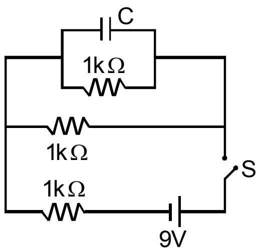
(i) In steady state, find the charge on the capacitor shown in figure.

(A) 4 μC (B) 5 μC
(C) 6 μC (D) 7 μC

(ii) Find out values of i_1, i_2 and i_3

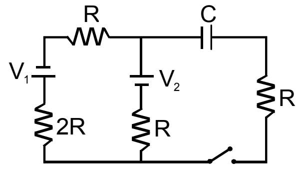
(A) 0, 1/15A, 1/15A (B) 1/15A, 0, 1/15A
(C) 0, 1/15A, 0 (D) 1/15A, 1/15A, 0

49. A capacitor C = 100 μ F is connected to three resistor each of resistance 1 kΩ and a battery of emf 9V. The switch S has been closed for long time so as to charge the capacitor. When switch S is opened, the capacitor. Discharges with time constant



(A) 33 ms (B) 5 ms
(C) 3.3 ms (D) 50 ms

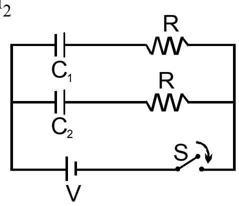
50. In the transient shown the time constant of the circuit is :



(A) $\frac{5}{3}$RC (B) $\frac{5}{2}$RC

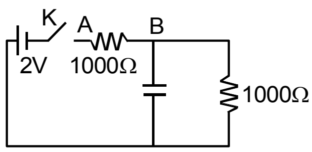
(C) $\frac{7}{4}$RC (D) $\frac{7}{3}$RC

51. In the circuit shown in figure $C_1 = 2C_2$. Switch S is closed at time t=0. Let i_1 and i_2 be the currents flowing through C_1 and C_2 at any time t, then the ratio i_1 / i_2



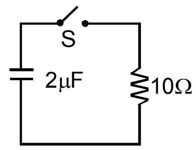
(A) is constant
(B) increases with increase in time t
(C) decreases with increase in time t.
(D) first increases then decreases

52. In the circuit shown, when the key k is pressed at time t = 0, which of the following statements about current I in the resistor AB is true



(A) I = 2mA at all t
(B) I oscillates between 1 mA and 2mA
(C) I = I mA at all t
(D) At t=0, I=2mA and with time it goes to 1 mA

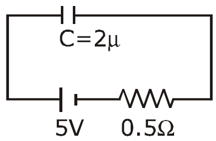
53. In the R-C circuit shown in the figure the total energy of 3.6×10^{-3} J is dissipated in the $10\,\Omega$ resistor when the switch S is closed. The initial charge on the capacitor is



(A) 60 μC (B) 120 μC

(C) $60\sqrt{2}$ μC (D) $\frac{60}{\sqrt{2}}$ μC

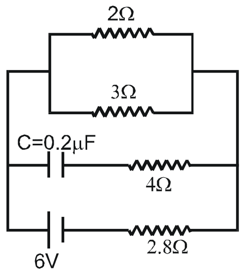
54. A capacitor of capacitance 5 μF is connected as shown in the fig. The internal resistance of the cell is 0.5 Ω. The amount of charge on the capacitor plate is :-



(A) 0 μC (B) 5 μC

(C) 10 μC (D) 25 μC

55. In the given figure the steady state current is



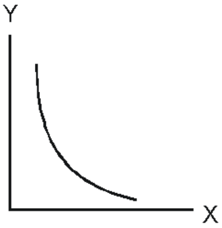
(A) zero (B) 0.6 A

(C) 0.9 A (D) 1.5 A

Exercise - 2 (Level-I) Objective Problems | JEE Main

Section A - Capacitance Calculation (Sphere, Cylinderical, Parallel plate), Sharing of Charge, Combination of Capacitor

1. What physical quantities may X and Y represent ?



(A) pressure v/s temperature of a given gas (constant volume)
(B) kinetic energy v/s velocity of a particle
(C) capacitance v/s charge at a constant potential
(D) potential v/s capacitance at a constant charge

2. The capacitance of a parallel plate condenser does not depend upon
(A) the distance between the plates
(B) area of the plates
(C) medium between the plates
(D) metal of the plates

3. Two capacitances of capacity C_1 and C_2 are connected in series and potential difference V is applied across it. Then the potential difference acros C_1 will be

(A) $V\dfrac{C_2}{C_1}$ (B) $V\dfrac{C_1+C_2}{C_1}$
(C) $V\dfrac{C_2}{C_1+C_2}$ (D) $V\dfrac{C_1}{C_1+C_2}$

4. Three long concentric conducting cylindrical shells have radii R, 2R and $2\sqrt{2}$ R. Inner and outer shells are connected to each other. The capacitance across middle and inner shells per unit length is:

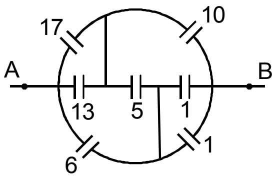
(A) $\dfrac{\frac{1}{3}\in_0}{\ln 2}$ (B) $\dfrac{6\pi\in_0}{\ln 2}$
(C) $\dfrac{\pi\in_0}{2\ln 2}$ (D) None

5. Two capacitor having capacitances 8 μF and 16 μF have breaking voltages 20 V and 80 V. They are combined in series. The maximum charge they can store individually in the combination is
(A) 160 μC (B) 200 μC
(C) 1280 μC (D) none of these

6. A capacitor of capacitance 1 μF withstands the maximum voltage 6 kV while a capacitor of 2 μF withstands the maximum voltage 4 kV. What maximum voltage will the system of these two capacitor withstands if they are connected in series?
(A) 10 kV (B)12 kV
(C) 8 kV (D) 9 kV

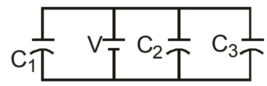
Section B - Circuit Problems (Switch, Energy etc.)

7. Find equivalent capacitance across AB (all capacitances in μF)



(A) $\dfrac{20}{3}$ μF (B) 9 μF
(C) 48 μF (D) None

8. **(i)** Find the charges on the three capacitors connected to a battery as shown in figure. Take $C_1 = 2.0\ \mu F$, $C_2 = 4.0\ \mu F$, $C_3 = 6.0\ \mu F$ and V = 10 volt.

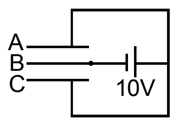


(A) 20 μC, 40 μC, 60 μC
(B) 40 μC, 20 μC, 60 μC
(C) 60 μC, 40 μC, 20 μC
(D) 40 μC, 60 μC, 20 μC

(ii) Find out work done by the battery during he process of charging
(A) 1500 μJ (B) 1200 μJ
(C) 1600 μJ (D) 1300 μJ

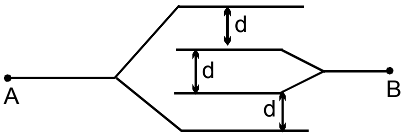
(iii) Find out total energy stored in the capacitors.
(A) 500 μJ (B) 300 μJ
(C) 600 μJ (D) 800 μJ

9. Three plates A, B and C each of area 0.1 m^2 are separated by 0.885mm from each other as shown in the figure. A 10 V battery is used to charge the system. The energy stored in the system is



(A) 1 μJ (B) 10^{-1} μJ
(C) 10^{-2} μJ (D) 10^{-3} μJ

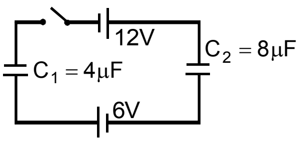
10. Four metallic plates are arranged as shown in the figure. If the distance between each plate then capacitance of the given system between points A and B is (Given d <<A)



(A) $\frac{\varepsilon_0 A}{d}$ (B) $\frac{2\varepsilon_0 A}{d}$

(C) $\frac{3\varepsilon_0 A}{d}$ (D) $\frac{4\varepsilon_0 A}{d}$

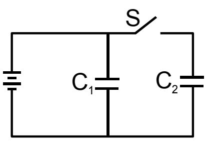
11. In the circuit shown initially C_1, C_2 are uncharged. After closing the switch



(A) The charge on C_2 is greater that on C_1
(B) The charge on C_1 and C_2 are the same
(C) The potential drops across C_1 and C_2 are the same
(D) The potential drops across C_2 is greater than that across C_1

12. Two capacitors of equal capacitance (C_1 = C_2) are shown in the figure. Initially, while the switch S is open, one of the capacitors is uncharged and the other carries charge Q_0. The energy stored in the charged capacitor is U_0. Sometimes after the switch is colsed, the capacitors C_1 and C_2 carry charges Q_1 and Q_2, respectively, the voltages across the capacitors are V_1 and V_2, and the energies stored in the capacitors are U_1 and U_2. Which of the following statements is INCORRECT ?

(A) $Q_0 = \frac{1}{2}(Q_1 + Q_2)$
(B) $Q_1 = Q_2$
(C) $V_1 = V_2$
(D) $U_0 = U_1 + U_2$



Question No.13 to 16 (4 questions)

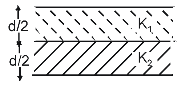
13. A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. A charge +Q is now given to its positive plate. The potential difference across the capacitor is now

(A) V (B) $V + \frac{Q}{C}$

(C) $V + \frac{Q}{2C}$ (D) $V - \frac{Q}{C}$, if $V < CV$

14. A capacitor of capacitance C is initially charged to a potential difference of V volt. Now it is connected to a battery of 2V with opposite polarity. The ratio of heat generated to the final energy stored in the capacitor will be
(A) 1.75 (B) 2.25
(C) 2.5 (D) 1/2

15. Two dielectric slabs of constant K_1 and K_2 have been filled in between the plates of a capacitor as shown below. What will be the capacitance of the capacitor



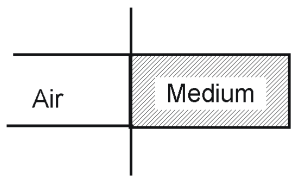
(A) $\frac{2\varepsilon_0 A}{d}(K_1 + K_2)$ (B) $\frac{2\varepsilon_0 A}{d}\left(\frac{K_1 + K_2}{K_1 \times K_2}\right)$

(C) $\frac{d}{2\varepsilon_0 A}$ (D) $\frac{2\varepsilon_0 A}{d}\left(\frac{K_1 \times K_2}{K_1 + K_2}\right)$

16. A capacitor C is charged to a potential difference V and battery is disconnected. Now if the capacitor plates are brought close slowly by some distance
(A) some + ve work is done by external agent
(B) energy of capacitor will decrease
(C) energy of capacitor will increase
(D) none of the above

Section C - Dielectric

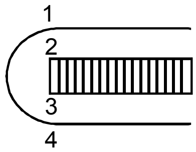
17. The capacitance of a parallel plate capacitor is 2.5μF when it is half filled with a dielectric as shown in the figure, Its capacitance becomes 5μF, the dielectric constant of the dielectric is



(A) 7.5 (B) 3.0
(C) 0.33 (D) 4.0

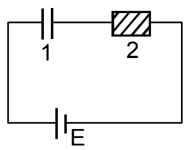
18. Four identical plates 1, 2, 3 and 4 are placed parallel to each other at equal distance as shown in the figure. Plates 1 and 4 are joined together and the space between 2 and 3 is filled with a dielectric of dielectric constant k = 2. The capacitance of the system between 1 and 3 & 2 and 4 are C_1 and C_2 respectively. The ratio $\frac{C_1}{C_2}$ is :

(A) $\frac{5}{3}$
(B) 1
(C) 3/5
(D) $\frac{5}{7}$



19. The capacity of a parallel plate condenser is 5μF. When glass plate is placed between the plates of the conductor, its potential becomes 1/8th of the original value. The value of dielectric constant will be -
(A) 1.6 (B) 5
(C) 8 (D) 40

20. Two identical capacitors 1 and 2 are connected in series to a battery as shown in figure. Capacitor 2 contains a dielectric slab of dielectric constant k as shown. Q_1 and Q_2 are the charges stored in the capacitors. Now the dielectric slab is removed and the corresponding charges are Q'_1 and Q'_2. Then



(A) $\frac{Q'_1}{Q_1}=\frac{K+1}{K}$ (B) $\frac{Q'_2}{Q_2}=\frac{K+1}{2}$
(C) $\frac{Q'_2}{Q_2}=\frac{K+1}{2K}$ (D) $\frac{Q'_1}{Q_1}=\frac{K}{2}$

21. A capacitor stores 60 μ C Charge when connected across a battery. When the gap between the plates is filled with a dielectric, a charge of 120 μ C flows through the battery. The dielectric constant of the material inserted is
(A) 1 (B) 2
(C) 3 (D) None

22. In the **above question,** if the initial capacitance of the capacitor was 2 μ F, the amount of heat produced when the dielectric is inserted.
(A) 3600 μ J (B) 2700 μ J
(C) 1800 μ J (D) none

23. A parallel plate capacitor is connected to a battery and inserted a dielectric plate between the place of plates then which quantity increase.
(A) potential difference
(B) electric field
(C) stored energy
(D) E. M . F of battery

24. A parallel plate capacitor has a capacity C. The separation between plates is doubled and a dielectric medium is inserted between plates. The new capacity is 3C. The dielectric constant of medium is
(A) 1.5 (B) 3.0
(C) 6.0 (D) 12.0

25. An air capacitor of 1μμF is immersed in a transformer oil of dielectric constant 3. The capacitance of the oil capacitor is
(A) 1μμF (B) $\frac{1}{3}$ μμF
(C) 3μμF (D) 2μμF

26. A parallel plate condenser is immersed in an oil of dielectric constant 2. The field between the plates is
(A) increased proportional to 2.
(B) decreased proportional to 1/2
(C) increased proportional to √2
(D) decreased proportional to 1/√2

Section D - R-C Circuit

27. A capacitor of capacitance 100 μF is connected across a battery of emf 6.0 V through a resistance of 20 kΩ for 4.0s. The battery is then replaced by a thick wire. What will be the charge on the capacitor 4.0 s after the battery is diconnected ?
(A) 70 μC (B) 80 μC
(C) 60 μC (D) none of these

28. A capacitor of capacitance 0.1 μF is charged to certain potential and allow to discharge through a resistance of 10 MΩ. How long will it take for the potential to fall to one half of its original value-
(A) 0.1s (B) 0.2346 s
(C) 1.386 s (D) 0.693 s

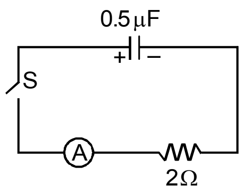
29. The electric field between the plates of a parallel-plate capacitance 2.0 μF drops to one third of its initial value in 4.4 μs when the plates are connected by a thin wire. Find the resistance of the wire.
(A) 3.0 Ω (B) 2.0 Ω
(C) 4.0 Ω (D) 1.0 Ω

30. A 5.0 μF capacitor having a charge of 20 μC is discharged through a wire of resistance of 5.0 Ω. Find the heat dissipated in the wire between 25 to 50 μs after the capactions are made.

(A) 4.7 μJ (B) 3.7 μJ

(C) 5.7 μJ (D) 2.7 μJ

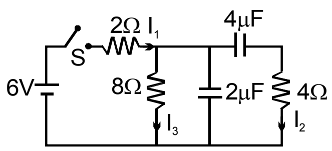
31. A charged capacitor is allowed to discharge through a resistance 2 Ω by closing the switch S at the instant t = 0. At time t = $ln2$ μ s, the reading of the ammeter falls half of its initial value. The resistance of the ammeter equal to



(A) 0 (B) 2 Ω

(C) ∞ (D) 2 MΩ

32. In the circuit shown in the figure, the switch S is initially open and the capacitor is initially uncharged. I_1, I_2 and I_3 represent the current in the resistance 2Ω, 4Ω and 8Ω respectively.



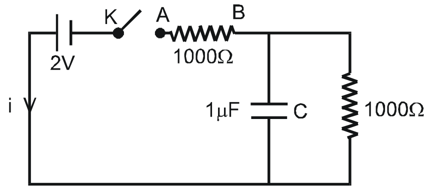
(A) Just after the switch S is closed,
$I_1 = 3A$, $I_2 = 3A$ and $I_3 = 0$

(B) Just after the switch S is closed,
$I_1 = 3A$, $I_2 = 0$ and $I_3 = 0$

(C) long time after the switch S is closed,
$I_1 = 0.6A$, $I_2 = 0$ and $I_3 = 0$

(D) long after the switch S is closed,
$I_1 = I_2 = I_3 = 0.6A$

33. When the key is pressed at time t = 0 then which of the following statement about the current i in the resistor AB of the given circuit is true:



(A) at t = 0, i = 2mA and with time it goes to 1 mA

(B) i oscillatets between 1 mA and 2 mA

(C) i = 2mA at all t

(D) i = 1 m A at all t

34. Time constant of a series R–C circuit is :-

(A) +RC (B) –RC

(C) $\frac{R}{C}$ (D) $\frac{C}{R}$

Exercise - 2 (Level-II) Multiple Correct | JEE Advanced

Section A - Capacitance Calculation (Sphere, Cylinderical, Parallel plate), Sharing of Charge, Combination of Capacitor

1. Two capacitors of capacitances $1\mu F$ and $3\mu F$ are charged to the same voltages 5V. They are connected in parallel with oppositely charged plates connected together. Then

(A) Final common voltage will be 5V

(B) Final common voltage will be 2.5 V

(C) Heat produced in the circuit will be zero

(D) Heat produced in the circuit will be $37.5\mu J$

2. A parallel-plate capacitor is connected to a cell. Its positive plate A and its negative plate B have charges +Q and –Q respectively. A third plate C, identical to A and B, with charge +Q, is now introduced midway between A and B, parallel to them. Which of the following are correct ?

(A) The charge on the inner face of B is now $-\frac{3Q}{2}$

(B) There is no change in the potential difference between A and B

(C) The potential difference between A and C is one-third of the potential difference between B and C

(D) The charge on the inner face of A is now Q/2

3. When two identical capacitors are charged individually to different potentials and connected parallel to each other, after disconnecting them from the source :

(A) net charge on connected plates is less than the sum of initial individual charges.

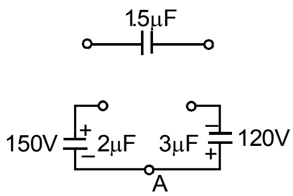
(B) net charge on connected plates equals the sum of initial charges.

(C) the net potential difference across them is different from the sum of the individual initial potential differences.

(D) the net energy stored in the two capacitors is less than the sum of the initial individual energies.

Section B - Circuit Problems (Switch, Energy etc.)

4. Two capacitors of $2\mu F$ and $3\mu F$ are charged to 150 volt and 120 volt respectively. The plates of capacitor are connected as shown in the figure. A discharged capacitor of capacity $1.5\ \mu F$ falls to the free ends of the wire. Then



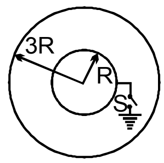
(A) charge on the $1.5\ \mu F$ capacitor is $180\ \mu C$

(B) charge on the $2\ \mu F$ capacitor is $120\ \mu C$

(C) charge flows through A from right to left

(D) charge flows through A from left to right

5. Two thin conducting shells of radii R and 3R are shown in the figure. The outer shell carries a charge +Q and the inner shell is neutral. The inner shell is earthed with the help of a switch S.



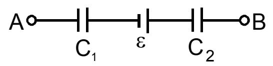
(A) With the switch S open, the potential of the inner sphere is equal to that of the outer

(B) When the switch S is closed, the potential of the inner sphere becomes zero

(C) With the switch S closed, the charge attained by the inner sphere is –q/3

(D) By closing the switch the capacitance of the system increases

6. A circuit shown in the figure consists of a battery of emf 10V and two capacitance C_1 and C_2 of capacitances $1.0\mu F$ and $2.0\mu F$ respectively. The potential difference $V_A - V_B$ is 5V



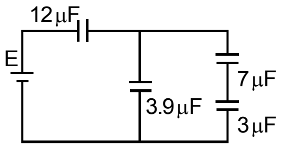
(A) charge on capacitor C_1 is equal to charge on capacitor C_2

(B) Voltage across capacitor C_1 is 5V

(C) Voltage across capacitor C_2 is 10 V

(D) Energy stored in capacitor C_1 is two times the energy stored in capacitor C_2

7. Four capacitors and a battery are connected as shown. The potential drop across the 7μF capacitor is 6V. Then the

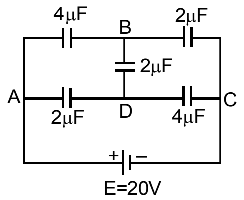


(A) potential difference across the 3μF capacitor is 10 V
(B) charge on the 3μF capacitor is 42μC
(C) e.m.f. of the battery is 30V
(D) potential difference across the 12μF capacitor is 10V

Question No.8 to 11 (4 questions)

The figure shows a diagonal symmetric arrangement of capacitors and a battery

8. Identify the correct statements.

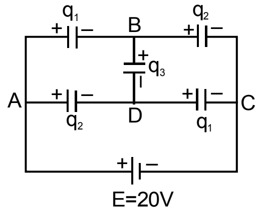


(A) Both the 4μF capacitors carry equal charges in opposite sense
(B) Both the 4μF capacitors carry equal charges in same sense
(C) $V_B - V_D > 0$
(D) $V_D - V_B > 0$

9. If the potential of C is zero, then
(A) $V_A = +20V$
(B) $4(V_A - V_B) + 2(V_D - V_B) = 2V_B$
(C) $2(V_A - V_D) + 2(V_B - V_D) = 4V_D$
(D) $V_A = V_B + V_D$

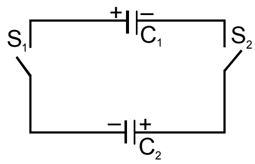
10. The potential of the point B and D are
(A) $V_B = 8V$ (B) $V_B = 12V$
(C) $V_D = 8V$ (D) $V_D = 12V$

11. The value of charge q_1, q_2 and q_3 as shown in the figure are



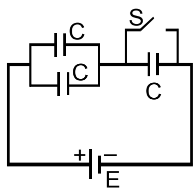
(A) $q_1 = 32\ \mu C$; $q_2 = 24\ \mu C$; $q_3 = -8\ \mu C$
(B) $q_1 = 48\ \mu C$; $q_2 = 16\ \mu C$; $q_3 = +8\ \mu C$
(C) $q_1 = 32\ \mu C$; $q_2 = 24\ \mu C$; $q_3 = +8\ \mu C$
(D) $q_1 = 3\ \mu C$; $q_2 = 4\ \mu C$; $q_3 = +2\ \mu C$

12. Two capacitors $C_1 = 4\mu F$ and $C_2 = 2\mu F$ are charged to same potential V = 500 Volt, but with opposite polarity as shown in the figure. The switches S_1 and S_2 are closed.



(A) The potential difference across the two capacitors are same and is given by 500/3V
(B) The potential difference across the two capacitors are same and is given by 1000/3V
(C) The ratio of final energy to initial energy of the system is 1/9
(D) The ratio of final energy to initial energy of the system is 4/9

13. In the circuit shown, each capacitor has a capacitance C.The emf of the cell is E. If the switch S is closed



(A) positive charge will flow out of the positive terminal of the cell
(B) positive charge will enter the positive terminal of the cell
(C) the amount of charge flowing through the cell will be CE
(D) the amount of charge flowing through the cell will be 4/3 CE

14. When a parallel plates capacitor is connected to a source of constant potential difference,
(A) all the charge drawn from the source is stored in the capacitor.
(B) all the energy drawn from the source is stored in the capacitor.
(C) the potential difference across the capacitor grows very rapidly initially and this rate decreases to zero eventually.
(D) the capacity of the capacitor increases with the increase of the charge in the capacitor

Section C - Dielectric

15. Following operations can be performed on a capacitor X – connect the capacitor to a battery of emf E Y – disconnect the battery
Z – reconnect the battery with polarity reversed
W – insert a dielectric slab in the capacitor
(A) In XYZ (perform X, then Y, then Z) the stored electric energy remains unchanged and no thermal energy is developed
(B) The charge appearing on the capacitor is greater after the action XWY than after the action XYW.
(C) The electric energy stored in the capacitor is greater after the action WXY than after the action XYW.
(D) The electric field in the capacitor after the action XW is the same as that after WX

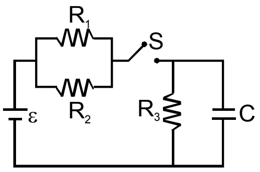
16. The capacitance of a parallel plate capacitor is C when the region between the plate has air. This region is now filled with a dielectric slab of dielectric constant k. The capacitor is connected to a cell of emf E, and the slab is taken out
(A) charge CE(k –1) flows through the cell
(B) energy $E^2C(k-1)$ is absorbed by the cell
(C) the energy stored in the capacitor is reduced by $E^2C(k-1)$
(D) the external agent has to do $\frac{1}{2}E^2C(k-1)$ amount of work to take the slab out

17. The plates of a parallel plate capacitor with no dielectric are connected to a voltage source. Now a dielectric of dielectric constant K is inserted to fill the whole sapce between the plates with voltage source remaining connected to the capacitor.
(A) the energy stored in the capacitor will become K-times
(B) the electric field inside the capacitor will decrease to K-times
(C) the force of attraction between the plates will increase to K^2 - times
(D) the charge on the capacitor will increase to K-times

18. A parallel plate capacitor A is filled with a dielectric whose dielectric constant varies with applied voltage as K = V. An identical capacitor B of capacitance C_0 with air as dielectric is connected to voltage source V_0 = 30 V and then connected to the first capacitor after disconnecting the voltage source. The charge and voltage on capacitor.
(A) A are 25 C_0 and 25 V (B) A are 25 C_0 and 5 V
(C) B are 5c_0 and 5V (D) B are 5C_0 and 25 V

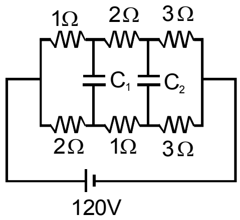
Section D - R-C Circuit

19. The circuit shown in the figure consists of a battery of emf ε = 10 V; a capacitor of capacitance C = 1.0μF and three resistor of values $R_1 = 2\Omega$, $R_2 = 2\Omega$ and $R_3 = 1\Omega$. Initially the capacitor is completely uncharged and the switch S is open. The switch S is closed at t = 0.

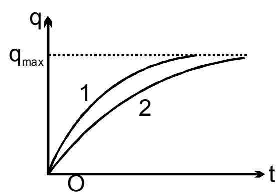


(A) The current through resistor R_3 at the moment the switch closed is zero
(B) The current through resistor R_3 a long time after the switch closed is 5A
(C) The ratio of current through R_1 and R_2 is always constant
(D) The maximum charge on the capacitor during the operation is 5μC

20. In the circuit shown in figure $C_1 = C_2 = 2\mu F$. Then charge stored in
(A) capacitor C_1 is zero
(B) capacitor C_2 is zero
(C) both capacitor is zero
(D) capacitor C_1 is 40μC



Question No. 21 to 22 (2 question)
The charge across the capacitor in two different RC circuits 1 and 2 are plotted as shown in figure.



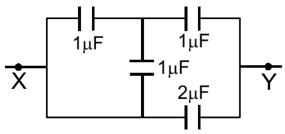
21. Choose the correct statement (s) related to the two circuits
(A) Both the capacitors are charged to the same charge
(B) The emf's of cells in both the circuit are equal
(C) The emf's of the cells may be different
(D) The emf E_1 is more than E_2

22. Identify the correct statement(s) related to the R_1, R_2, C_1 and C_2 of the two RC circuits.
(A) $R_1 > R_2$ if $E_1 = E_2$ (B) $C_1 < C_2$ if $E_1 = E_2$
(C) $R_1C_1 > R_2C_2$ (D) $\frac{R_1}{R_2} < \frac{C_2}{C_1}$

Exercise - 3 | Level-I   Subjective | JEE Advanced

Section A - Capacitance Calculation (Sphere, Cylinderical, Parallel plate), Sharing of Charge, Combination of Capacitor

1. The figure shows a circuit consisting of four capacitors. Find the effective capacitance between X and Y.

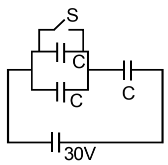


2. The plates of a parallel plate capacitor are given charges +4Q and –2Q. The capacitor is then connected across an uncharged capacitor of same capacitance as first one (= C). Find the final potential difference between the plates of the first capacitor.

3. A parallel plate capacitor has an electric field of 10^5V/m between the plates. If the charge on the capacitor plate is $1\,\mu$C, then the force on each capacitor plate is

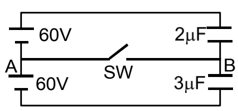
Section B - Circuit Problems (Switch, Energy etc.)

4. The capacitor each having capacitance C = 2μF are connected with a battery of emf 30 V as shown in figure. When the switch S is closed. Find

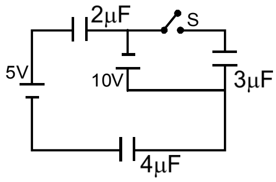


(A) the amount of charge flown through the battery
(B) the heat generated in the circuit
(C) the energy supplied by the battery
(D) the amount of charge flown through the switch S

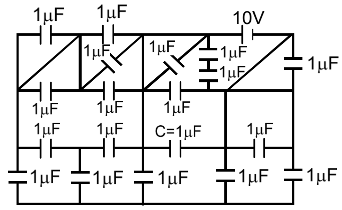
5. In the circuit shown in the figure, initially SW is open. When the switch is closed, the charge passing through the switch ____________ in the direction ____________ to __________.



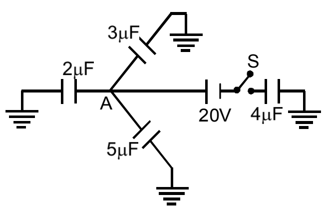
6. In the circuit shown in figure, find the amount of heat generated when switch s is closed.



7. Find the charge on the capacitor C = 1μF in the circuit shown in the figure.

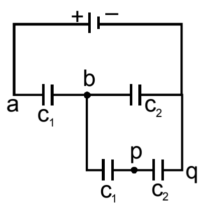


8. Three capacitors of 2μF, 3μF and 5μF are independently charged with batteries of emf's 5V, 20V and 10V respectively. After disconnecting from the voltage sources. These capacitors are connected as shown in figure with their positive polarity plates are connected to A and negative polarity is earthed. Now a battery of 20V and an uncharged capacitor of 4μF capacitance are connected to the junction A as shown with a switch S. When switch is closed, find :

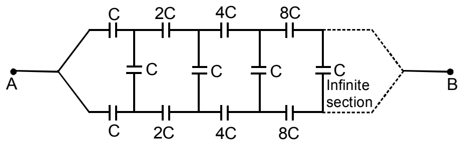


(a) the potential of the junction A.
(b) final charges on all four capacitors.

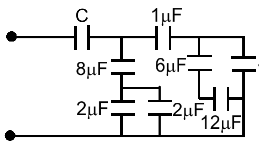
9. In the given network if potential difference between p and q is 2V and $C_2 = 3C_1$. Then find the potential difference between a & b.



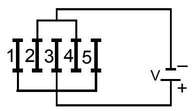
10. Find the equivalent capacitance of the circuit between point A and B.



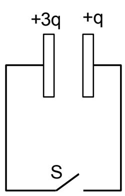
11. In the following circuit, the resultant capacitance between A and B is 1μF. Find the value of C.



12. Five identical capacitor plates, each of area A, are arranged such that adjacent plates are at a distance 'd' apart, the plates are connected to a source of emf V as shown in figure. The charge on plate 1 is ____________ and that on plate 4 is ____________.



13. The two identical parallel plates are given charges as shown in figure. If the plate area of either face of each plate is A and separation between plates is d, then find the amount of heat liberate after closing the switch.

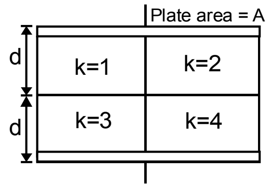


14. A solid conducting sphere of radius 10 cm is enclosed by a thin metallic shell of radius 20 cm. A charge q = 20 μC is given to the inner sphere. Find the heat generated in the process, the inner sphere is connected to the shell by a conducting wire -

15. A parallel plate capacitor is charged by connecting it to a battery. The battery is disconnected and the plates of the capacitor are pulled apart to make the separation between the plates twice. Again the capacitor is connected to the battery (with same polarity) then the potential difference between the plates increases when the plates are pulled apart (yes or not)

Section C - Dielectric

16. Find the capacitance of the system shown in figure.



17. The two parallel plates of a capacitor have equal and opposite charges Q. The dielectric has a dielectric constant K and resistivity ρ. Find that the "leakage" current carried by the dielectric is given by the relationship $i = \dfrac{Q}{K \epsilon_0 \rho}$

18. A parallel plate air-core capacitor is connected across a source of constant potential difference. When a dielectric plate is introduced between the two plates then prove that electric field intensity does not change

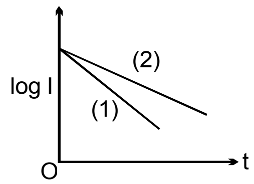
19. A parallel plate capacitor has a parallel sheet of copper inserted between and parallel to the two plates, without touching the plates. For what position of copper plate the capacity of the capacitor after the introduction of the copper sheet is minimum, maximum and invariant

20. The area of the plates of a parallel plate capacitor is A and the gap between them is d. The gap is filled with a non homogeneous dielectric whose dielectric constant varies with the distance 'y' from one plate as: $K = \lambda \sec(\pi y/2d)$, where λ is a dimensionless constant. The capacitance of this capacitor is

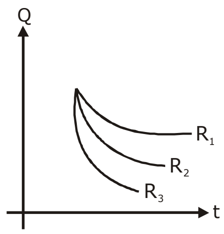
21. Condenser A has a capacity of 15 μF when it is filled with a medium of dielectric constant 15. Another condenser B has a capacity 1 μF with air between the plates. Both are charged separately by a battery of 100 V. After charging, both are connected in parallel without the battery and the dielectric material being removed. The common potential now is :

Section D - R-C Circuit

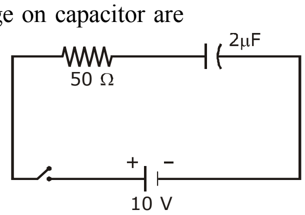
22. A capacitor of capacity C is charged to a steady potential difference V and connected in series with an open key and a pure resistor 'R'. At time t = 0, the key is closed. If I = current at time t, a plot of log I against 't' is as shown in (1) in the graph. Later one of the parameters i.e. V, R or C is changed keeping the other two constant, and the graph (2) is recorded. Then C increase or not.



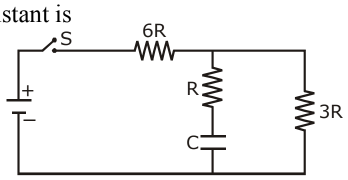
23. Three identical capacitors are charged by connecting them in parallel across a battery of V volt. They are then allowed to discharge via resistor R_1, R_2 and R_3. Here, Q charge versus t time graphs are shown. Then R_1 is smallest or not.



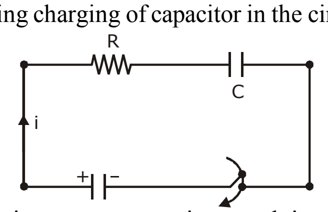
24. For the circuit shown in figure below, At t = 0, switch is closed, the initial current through resistor and final charge on capacitor are



25. For the given circuit shown in figure below, Time constant is



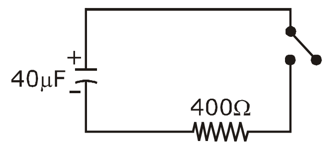
26. During charging of capacitor in the circuit shown,



Circuit current versus time graph is

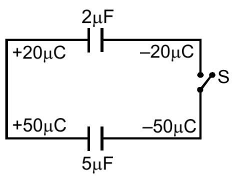
27. In an R-C circuit capacitor lost 63% of initial charge in 5 ms when it discharges through a resistor of R Ω. If it discharges through a resistor of 2R Ω, then it lost 63% of its initial chargein K ms. Find the value of K.

28. In the given circuit, At $t = 0^-$, before the switch closed, V_C was measured as 100 V. If energy liberated in resistor in t second is $k \times 10^{-1}(1 - e^{-125t})$, then find the value of k.

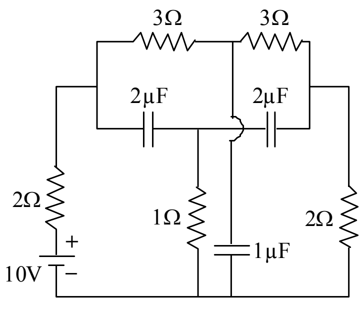


29. An RC transient has a power transient given by $P = 360\, e^{-t \times 10^{5}}$. Value of resistor in circuit is 10Ω and initial charge on capacitor is 60 k μC. Find the value of K.

30. Find heat produced in the circuit shown in figure on closing the switch S.



31 In steady state, calculate energy stored in capacitors shown in **Fig.** and the rate at which battery supplies energy.

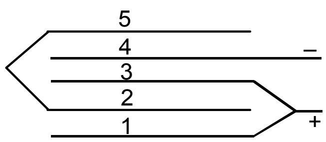


Section A - Capacitance Calculation (Sphere, Cylinderical, Parallel plate), Sharing of Charge, Combination of Capacitor

1. An insolated conductor initially free from charge is charged by repeated contacts with a plate which after each contact has a charge Q due to some mechanism. If q is the charge on the conductor after the first operation, prove that the maximum charge which can be given to the conductor in this way is $\frac{Qq}{Q-q}$.

Section B - Circuit Problems (Switch, Energy etc.)

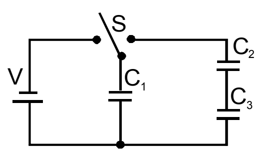
2. Five identical conducting plates 1, 2, 3, 4 & 5 are fixed parallel to and equdistant from each other (see figure). Plates 2 & 5 are connected by a conductor while 1 & 3 are joined by another conductor. The junction of 1 & 3 and the plate 4 are connected to a source of constant e.m.f. V_0. Find :



(i) the effective capacity of the system between the terminals of the source.

(ii) the charges on plates 3 & 5 Given d = distance between any 2 successive plates & A = area of either face of each plate.

3. When the switch S in the figure is thrown to the left, the plates of capacitors C_1 acquire a potential difference V. Initially the capacitors C_2C_3 are uncharged. The switch is now thrown to the right. What are the final charges q_1, q_2 & q_3 on the corresponding capacitors.

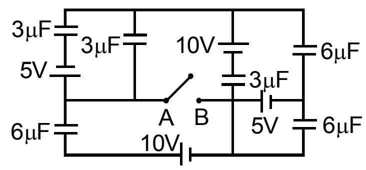


4. A capacitor consists of two air spaced concentric cylinders. The outer of radius b is fixed, and the inner is of radius a. If breakdown of air occurs at field strengths greater than E_b, show that the inner cylinder should have

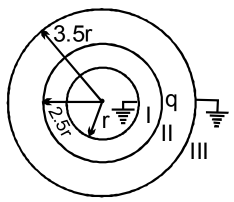
(i) radius a = b/e if the potential of the capacitor is to be maximum

(ii) radius $a = b/\sqrt{e}$ if the energy per unit length of the system is to be maximum.

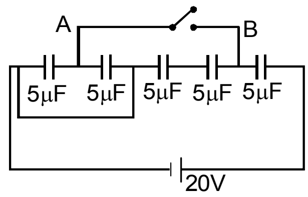
5. Find the charge flown through the switch from A to B when it is closed.



6. Figure shows three concentric conducting spherical shells with inner and outer shells earthed and the middle shell is given a charge q. Find the electrostatic energy of the system stored in the region I and II.



7. Find the charge which flows from point A to B, when switch is closed.



Section C - Dielectric

8. A potential difference of 300 V is applied between the plates of a plane capacitor spaced 1cm apart. A plane parallel glass plate with a thickness of 0.5 cm and a plane parallel paraffin plate with a thickness of 0.5 cm are placed in the space between the capacitor plates find :

(i) Intensity of electric field in each layer.

(ii) The drop of potential in each layer.

(iii) The surface charge density of the charge on capacitor the plates. Given that : $k_{glass} = 6$, $k_{paraffin} = 2$

9. A parallel plate capacitor has plates with area A & separation d. A battery charges the plates to a potential difference of V_0. The battery is then disconnected & a di-electric slab of constant K & thickness d is introduced. Calculate the positive work done by the system (capacitor + slab) on the man who introduces the slab.

10. A parallel plate capacitor is filled by a di-electric whose relative permittivity varies with the applied voltage according to the law = αV, where $\alpha = 1$ per volt. The same (but containing no di-electric) capacitor charged to a voltage V = 156 volt is connected in parallel to the first "non-linear" uncharged capacitor. Determine the final voltage V_f across the capacitors.

11. Two parallel plate capacitors of capacitance C and 2C are connected in parallel then following steps are performed.

(i) A battery of voltage V is connected across the capacitors.

(ii) A dielectric slab of relative permittivity k is slowly inserted in capacitor C.

(iii) Battery is disconnected.

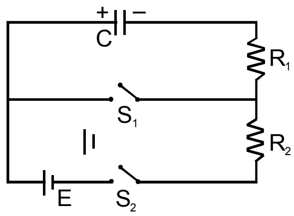
(iv) Dielectric slab is slowly removed from capacitor.

Find the heat produced in (i) and work done by external agent in step (ii) & (iv).

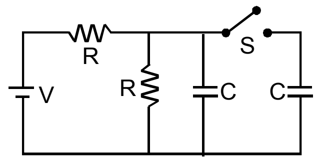
12. The gap between the plates of a plane capacitor is filled with an isotropic insulator whose di-electric constant varies in the direction perpendicular to the plates according to the law $K = K_1\left[1+\sin\frac{\pi}{d}X\right]$, where d is the separation, between the plates & K_1 is a constant. The area of the plates is S. Determine the capacitance of the capacitor.

Section D - R-C Circuit

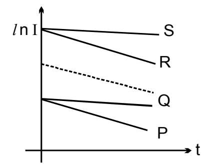
13. The capacitors shown in figure has been charged to a potential difference of V volts, so that it carries a charge CV with both the switches S_1 and S_2 remaining open. Switch S_1 is closed at t = 0. At t = R_1C switch S_1 is opened and S_2 is closed. Find the charge on the capacitor at t = $2R_1C + R_2C$.



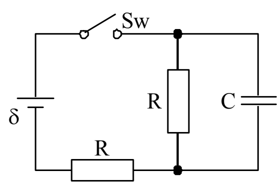
14. In the figure shown initially switch is open for a long time. Now the switch is closed at t = 0. Find the charge on the rightmost capacitor as a function of time given that it was initially unchanged.



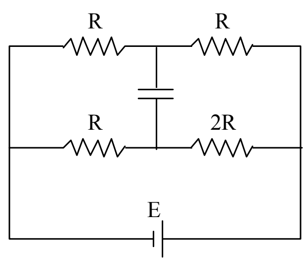
15. In an RC circuit while charging, the graph of *l*n I versus time is as shown by the dotted line in the adjoining diagram where I is the current. When the value of the resistance is doubled, which of the solid curves best represents the variation of *l*n I versus time?



16. Find how the voltage across the capacitor C varies with time **t (Fig.)** after the shorting of the switch Sw at the moment t = 0.



17 Find the time constant for the following circuit.



Exercise - 4 | Level-I Previous Year | JEE Main

1. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is C, then the resultant capacitance is **(AIEEE 2005)**

(A) $(n-1)C$ (B) $(n+1)C$

(C) C (D) nC

2. A full charged capacitor has a capacitance C. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity s and mass m. If the temperture of the block is raised by ΔT, the potential difference V across the capacitance is **(AIEEE 2005)**

(A) $\sqrt{\frac{2mC\Delta T}{s}}$ (B) $\frac{mC\Delta T}{s}$

(C) $\frac{ms\Delta T}{C}$ (D) $\sqrt{\frac{2ms\Delta T}{C}}$

3. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volts. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is **(AIEEE 2007)**

(A) $\frac{1}{2}(K-1)CV^2$ (B) $CV^2(K-1)/K$

(C) $(K-1)CV^2$ (D) zero

4. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be **(AIEEE 2007)**

(A) 1 (B) 2

(C) $\frac{1}{4}$ (D) $\frac{1}{2}$

5. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is d. The space between the plate is now filled with two dielectrics. One of the dielectrics has dielectric constant $K_1 = 3$ and thicknes $\frac{d}{3}$ while the other one has dielectric constant $K_2 = 6$ and thickness $\frac{2d}{3}$. Capacitance of the capacitor is now **(AIEEE 2008)**

(A) 1.8 pF (B) 45 pF

(C) 40.5 pF (D) 20.25 pF

6. Let C be that capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio $\frac{t_1}{t_2}$ will be. **(AIEEE 2010)**

(A) 1 (B) $\frac{1}{2}$

(C) $\frac{1}{4}$ (D) 2

7. A resistor R and $2\mu F$ capacitor in series in connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed $(\log_{10} 2.5 = 0.4)$ **(AIEEE 2011)**

(A) $1.7\times10^5\Omega$ (B) $2.7\times10^6\Omega$

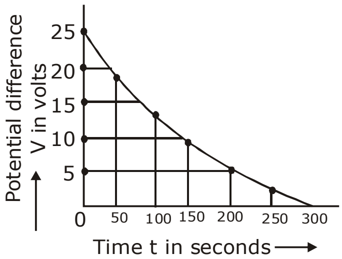
(C) $3.3\times10^7\Omega$ (D) $1.3\times10^4\Omega$

8. Combination of two identical capacitors, a resistor R and a DC voltage source of voltage 6V is used in an experiment on C–R circuit. It is found that for a parallel combination of the capacitor, the time in which the voltage of the fully charged combination reduces to half its original voltage is 10 s. For series combination, the time needed for reducing the voltage of the fully charged series combination by half is **[AIEEE 2011]**

(A) 20 s (B) 10 s

(C) 5 s (D) 2.5 s

9.



The figure shows an experimental plot for discharging of a capacitor in an R-C circuit. The time constant τ of this circuit lies between. **[AIEEE 2012]**

(A) 50 sec and 100 sec
(B) 100 sec and 150 sec
(C) 150 sec and 200 sec
(D) 0 and 50 sec

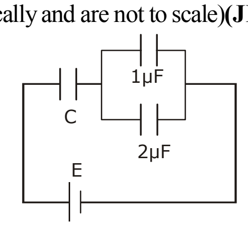
10. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then: **[AIEEE 2013]**

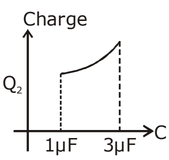
(A) $3C_1 + 5C_2 = 0$ (B) $9C_1 = 4C_2$
(C) $5C_1 = 3C_2$ (D) $3C_1 = 5C_2$

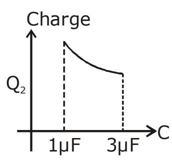
11. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the change density of the positive plate will be close to : **(JEE MAIN 2014)**

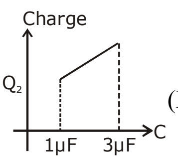
(A) 3×10^4 C/m^2 (B) 6×10^4 C/m^2
(C) 6×10^{-7} C/m^2 (D) 3×10^{-7} C/m^2

12. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1 μF to 3 μF. Q_2 as a function of 'C' is given properly by : (figure are drawn shematically and are not to scale) **(JEE MAIN 2015)**



(A)


(B)


(C)


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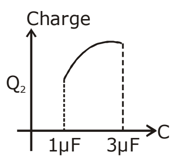
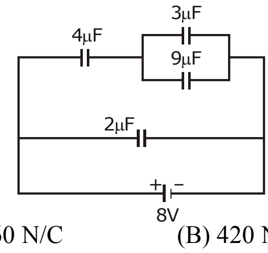


13. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4μF and 9μF capacitors), at a point distant 30 m from it, would equal : **(JEE MAIN 2016)**



(A) 360 N/C (B) 420 N/C
(C) 480 N/C (D) 240 N/C

14. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V.

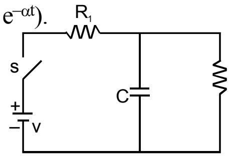
The minimum number of capacitors required to achieve this is : **(JEE MAIN 2017)**

(A) 32 (B) 2
(C) 16 (D) 24

15. A parallel plate capacitor of capacitance 90pF is connected to a battery of emf 20 V. If a dielectric material of dielectric $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be : **(JEE MAIN 2018)**

(A) 0.9 n C (B) 1.2 n C
(C) 0.3 nC (D) 2.4 nC

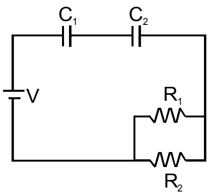
1. In the given circuit, the switch S is closed at time t = 0. The charge Q on the capacitor at any instant t is given by $Q(t) = Q_0\ (1 - e^{-\alpha t})$. Find the value of Q_0 and α in terms of given parameters shown in the circuit. **[JEE' 2005 (Scr)]**

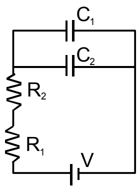


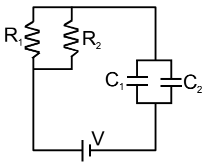
2. An uncharged capacitor of capacitance 4μF, a battery of emf 12 volt and a resistor of 2.5 MΩ are connected in series. The time after which $v_C = 3v_R$ is (take $ln\ 2 = 0.693$) **[JEE' 2005 (Scr)]**

(A) 6.93 sec. (B) 13.86 sec
(C) 20.52 sec. (D) none of these

3. Given : $R_1 =- 1\Omega$, $R_2 = 2\Omega$, $C_1 = 2\ \mu F$, $C_2 = 4\mu F$
The time constants (in μS) for the circuits I, II, III are respectively **[JEE 2006]**

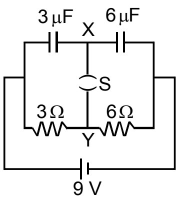
(I)


(II)


(III)


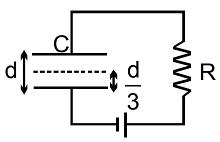
(A) 18, 8/9, 4 (B) 18, 4, 8/9
(C) 4, 8/9, 18 (D) 8/9, 18, 4

4. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is **[JEE 2007]**



(A) 0 (B) 54 μC
(C) 27 μC (D) 81 μC

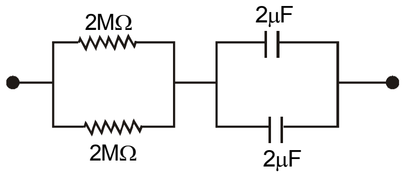
5. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant K = 2. The level of liquid is $\frac{d}{3}$ initially. Suppose the liquid level decreases at a constant speed V, the time constant as a function of time t is **[JEE 2008]**



(A) $\frac{6\varepsilon_0 R}{5d + 3Vt}$ (B) $\frac{(15d + 9Vt)\varepsilon_0 R}{2d^2 - 3dVt - 9V^2t^2}$

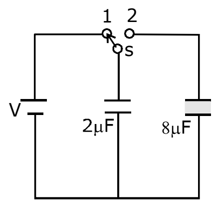
(C) $\frac{6\varepsilon_0 R}{5d - 3Vt}$ (D) $\frac{(15d - 9Vt)\varepsilon_0 R}{2d^2 + 3dVt - 9V^2t^2}$

6. At time t = 0, a battery of 10 V is connected across point A and B in the given circuit. It the capacitors have no charge initially, at what time (in seconds) does the voltage across them become 4 V ? **[JEE 2010]**



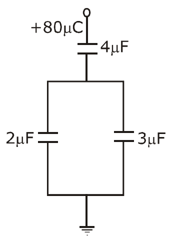
[Take : $\ell n\ 5 = 1.6$, $\ell n\ 3 = 1.1$]

7. A 2 μF capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is **[JEE 2011]**



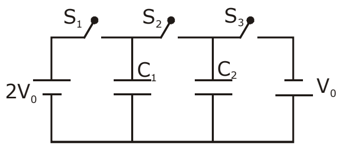
(A) 0% (B) 20%
(C) 75% (D) 80%

8. In the given circuit a charge of +80 μC is given to the upper plate of the 4μF capacitor. Then in the steady state, the charge on the upper plate of the 3μf capacitor is. **[JEE 2012]**



(A) +32μC (B) +40μC
(C) +48μC (D) +80μC

9. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first fo fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time, **[JEE 2013]**



(A) the charge on the upper plate C_1 is $2CV_0$.
(B) the charge on the upper plate of C_1 is CV_0.
(C) the charge on the upper plate of C_2 is 0.
(D) the charge on the upper plate of C_2 is $-CV_0$.

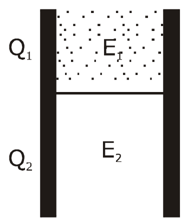
10. A parallel plate capacitor has a dielectric slab of dielectric constant K between it plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between C_1. When the capacitor is charged, the plate area covered by the dielectric get charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options. ignoring edge effects. **[JEE 2014]**

(A) $\frac{E_1}{E_2}=1$

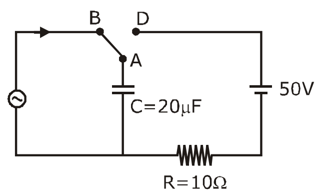
(B) $\frac{E_1}{E_2}=\frac{1}{K}$

(C) $\frac{Q_1}{Q_2}=\frac{3}{K}$

(D) $\frac{C}{C_1}=\frac{2+K}{K}$



11. At time t = 0, terminal A is the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0\cos(\omega t)$ with I_0=1A and ω=500 rad s^{-1} starts flowing in it with the initial direction shown in the figure. At $t=\frac{7\pi}{6\omega}$. the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If C=20μF, R=10Ω and the battery is ideal with emf of 50V, identify the correct statement (s).



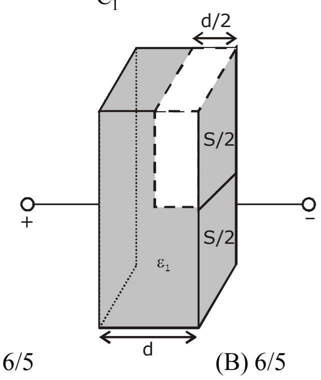
(A) Magnitude of the maximum charge on the capacitor before $t=\frac{7\pi}{6\omega}$ is 1×10^{-3} C.

(B) The current in the left part of the circuit just before $t=\frac{7\pi}{6\omega}$ is clockwise.

(C) Immediately after A is connected to D, the current in R is 10A.

(D) $Q = 2\times10^{-3}$C. **[JEE 2014]**

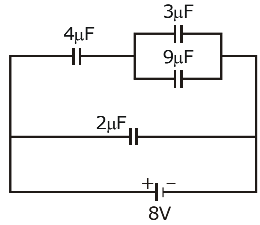
12. A parallel plate capaitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\frac{C_2}{C_1}$ is- **[JEE Advanced 2015]**



(A) 6/5 (B) 6/5
(C) 7/5 (D) 7/3

13. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4μF and 9μF capacitors), at a point distant 30 m from it, would equal:

[JEE Advanced 2016]



(A) 360 N/C (B) 420 N/C
(C) 480 N/C (D) 240 N/C

PARAGRAPH (14 to 15)

Consider an evacuted cylindrical chamber of height h having rigid conducting plates at the end and an insulting curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plated. The balls have a radius r << h. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will eventually collide with the top plate, where the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity) **[JEE Advanced 2016]**

14. Which one of the following statements is correct?
(A) The balls will stick to the top plate and remain there
(B) The balls will bounce back to the bottom plate carrying the same charge they went up with
(C) The balls will bounce back to the bottom plate carrying the opposite charge they went up with
(D) The balls will execute simple harmonic motion between the two plates

15. The average current in the steady state registered by the ammeter in the circuit will be
(A) zero
(B) proportional to the potential V_0
(C) proportional to $V_0^{1/2}$
(D) proportional to V_0^2

PARAGRAPH - (16 To 17) [JEE Advanced 2017]

Consider a simple RC circuit as shown in figure 1.

Process-1: In the circuit the switch S is closed at t = 0 and the capcitor is fully charged to voltage V_0 (i.e., charging continues for time T >> RC). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process-2: In a different process the voltage is first set set to $\frac{V_0}{3}$ and maintained for a charging time T >> RC. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time T >> RC. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in process 1.

These two processes are depicted in figure 2

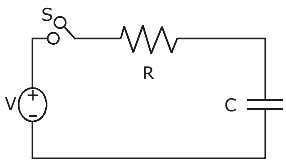


Figure 1

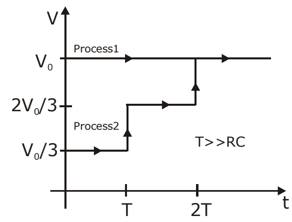


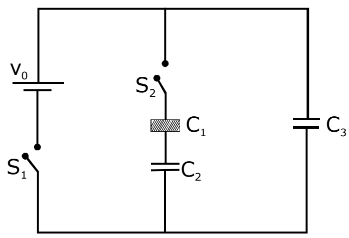
Figure 2

16. In process 2, total energy dissipated across the resistance E_D is - **[JEE Advanced 2017]**
(A) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$ (B) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$
(C) $E_D = 3\ CV_0^2$ (D) $E_D = \frac{1}{2}\ CV_0^2$

17. In process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by - **[JEE Advanced 2017]**
(A) $E_C = E_D$ (B) $E_C = E_D \ln 2$
(C) $E_C = 2E_D$ (D) $E_C = \frac{1}{2}\ E_D$

18. Three identical capacitors C_1,C_2 and C_3 have a capacitance of 1.0 μF. each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity $\in_r$. The cell electromotive force (emf) V_0=8V. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5 μC The value of $\in_r$ =____________.

[JEE Advanced 2018]



ANSWER KEYS

Exercise - 1 Objective Problems | JEE Main

1.	**A**	**2.**	**D**	**3.**	**A**	**4.**	**B**	**5.**	**C**	**6.**	**C**	**7.**	**D**
8.	**B**	**9.**	**C**	**10.**	**A**	**11.**	**B**	**12.**	**B**	**13.**	**D**	**14.**	**A**
15.	**B**	**16.**	**D**	**17.**	**(i) B (ii) A**			**18.**	**A**	**19.**	**C**	**20.**	**A**
21.	**C**	**22.**	**D**	**23.**	**B**	**24.**	**B**	**25.**	**A**	**26.**	**B**	**27.**	**B**
28.	(i) A	(ii) B	(iii) C	(iv) C		**29.**	**A**	**30.**	**B**	**31.**	**C**	**32.**	**B**
33.	**B**	**34.**	**A**	**35.**	**C**	**36.**	**B**	**37.**	**A**	**38.**	**B**	**39.**	**A**
40.	**D**	**41.**	**C**	**42.**	**B**	**43.**	**C**	**44.**	**D**	**45.**	**C**	**46.**	**D**
47.	**C**	**48.**	(i) A	(ii) A		**49.**	**D**	**50.**	**C**	**51.**	**B**	**52.**	**D**
53.	**B**	**54.**	**C**	**55.**	**D**								

Exercise - 2 (Level-I) Objective Problems | JEE Main

1.	**D**	**2.**	**D**	**3.**	**C**	**4.**	**B**	**5.**	**A**	**6.**	**D**	**7.**	**B**
8.	(i) A	(ii) B	(iii) C	**9.**	**B**	**10.**	**B**	**11.**	**B**	**12.**	**D**	**13.**	**C**
14.	**B**	**15.**	**D**	**16.**	**B**	**17.**	**B**	**18.**	**B**	**19.**	**C**	**20.**	**C**
21.	**C**	**22.**	**C**	**23.**	**C**	**24.**	**C**	**25.**	**C**	**26.**	**B**	**27.**	**A**
28.	**D**	**29.**	**B**	**30.**	**A**	**31.**	**A**	**32.**	**B**	**33.**	**A**	**34.**	**A**

Exercise - 2 (Level-II) Multiple Correct | JEE Advanced

1.	**BD**	**2.**	**ABCD**	**3.**	**BCD**	**4.**	**ABC**	**5.**	**ABCD**
6.	**AD**	**7.**	**BCD**	**8.**	**BC**	**9.**	**ABCD**	**10.**	**BC**
11.	**C**	**12.**	**AC**	**13.**	**AD**	**14.**	**AC**	**15.**	**BCD**
16.	**ABD**	**17.**	**ACD**	**18.**	**BC**	**19.**	**ABCD**	**20.**	**BD**
21.	**AC**	**22.**	**D**						

Exercise - 3 | Level-I Subjective | JEE Advanced

1. $\frac{8}{3}\mu F$ **2.** 3Q/2C **3.** **0.05 Nt** **4.** (a) 20 μC, (b) 0.3 mJ, (c) 0.6 mJ. (d) 60 μC

5. 60 μc, A to B **6.** 150 μJ **7.** 10 μC

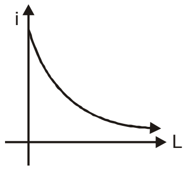
8. (a) $\frac{100}{7}$ volts ; (b) 28.56 μC, 42.84 μC, 71.4 μC, 22.88 μC **9.** 30 V **10.** C

11. $\frac{32}{23}\mu F$ **12.** $\frac{A\epsilon_0 V}{d}, -\frac{2A\epsilon_0 V}{d}$ **13.** $\frac{1}{2}\frac{q^2 d}{\epsilon_0 A}$ **14.** 9 J **15.** **proof**

16. $\frac{25}{24}\frac{\varepsilon_0 A}{d}$ 17. Proof 18. **proof** 19. **proof** 20. **$\pi\varepsilon_0\lambda$ A/2d**

21. **800 volt** 22. **C is increases** 23. **R_1 is smallest** 24. **0.2 A, 20 μc**

25. **τ = 3 RC** 26.



27. **10** 28. **2** 29. **2** 30. **0**

31. **$= 80.5 \times 10^{-6}$ J ,10 W**

Exercise - 3 | Level-II Subjective | JEE Advanced

1. Proof

2. (i) $\frac{5}{3}\left(\frac{\epsilon_0 A}{d}\right)$; (ii) $Q_3 = \frac{4}{3}\left(\frac{\epsilon_0 AV}{d}\right)$, $Q_5 = \frac{2}{3}\left(\frac{\epsilon_0 AV}{d}\right)$

3. $q_1 = \frac{C_1^2 V(C_2+C_3)}{C_1C_2+C_2C_3+C_1C_3}$ $q_2 = q_3$ $\frac{C_1C_2C_3V}{C_1C_2+C_2C_3+C_3C_1}$

4. Proof

5. 69 μC

6. $U_I = \frac{3kq_1^2}{10r}$ where $q_1 = -\frac{4q}{25}$; $U_{II} = 2K(q+q_1)^2/35r$

7. $-\frac{400}{7}\mu C$

8. (i) 1.5×10^4 V/m, 4.5×10^4 V/m, (ii) 75 V, 225 V, (iii) 8×10^{-7} C/m^2

9. $W = \frac{1}{2}C_0 V_0^2 \left(1-\frac{1}{K}\right)$

10. 12 volt

11. (i) $\frac{3}{2}CV^2$; (ii) $-\frac{1}{2}CV^2(K-1)$; $\frac{1}{6}(K+2)\ (K-1)\ CV^2$;

12. $C = \frac{\varepsilon_0 S\pi K_1}{2d}$

13. $q = CE\left(1-\frac{1}{e}\right) + \frac{CV}{e^2}$

14. $q = \frac{CV}{2}\left(1-\frac{1}{2}e^{-t/RC}\right)$

15. **R**

16. **$V = \frac{\xi}{2}(1 - e^{-2t/RC})$**

17. **Time constant = 7RC/6.**

Exercise - 4 | Level-I Previous Year | JEE Main

1.	**A**	**2.**	**D**	**3.**	**D**	**4.**	**D**	**5.**	**C**	**6.**	**C**	**7.**	**B**
8.	**D**	**9.**	**B**	**10.**	**D**	**11.**	**C**	**12.**	**D**	**13.**	**B**	**14.**	**A**
15.	**B**												

Exercise - 4 | Level-II Previous Year | JEE Advanced

1. **$Q_0 = \frac{CVR_2}{R_1+R_2}$ and $a = \frac{R_1+R_2}{CR_1R_2}$** 2. **B** 3. **D** 4. **C** 5. **A**

6.	**2 sec**	**7.**	**D**	**8.**	**C**	**9.**	**B,D**	**10.**	**A,D**	**11.**	**C,D**
12.	**D**	**13.**	**B**	**14.**	**C**	**15.**	**D**	**16.**	**B**	**17.**	**A**
18.	**1.50**										