9

Mechanical Properties of Fluids

TOPIC 1 Pressure, Density, Pascal's Law and Archimedes' Principle

1. A hollow spherical shell at outer radius R floats just submerged under the water surface. The inner radius of the shell is r. If the specific gravity of the shell material is $\frac{27}{8}$ w.r.t water, the value of r is : **[5 Sep. 2020 (I)]**

(a) $\frac{8}{9}R$ (b) $\frac{4}{9}R$ (c) $\frac{2}{3}R$ (d) $\frac{1}{3}R$

2. An air bubble of radius 1 cm in water has an upward acceleration 9.8 cm s^{-2}. The density of water is 1 gm cm^{-3} and water offers negligible drag force on the bubble. The mass of the bubble is ($g = 980\ cm/s^2$) **[4 Sep. 2020 (I)]**

(a) 4.51 gm (b) 3.15 gm (c) 4.15 gm (d) 1.52 gm

3. Two identical cylindrical vessels are kept on the ground and each contain the same liquid of density d. The area of the base of both vessels is S but the height of liquid in one vessel is x_1 and in the other, x_2. When both cylinders are connected through a pipe of negligible volume very close to the bottom, the liquid flows from one vessel to the other until it comes to equilibrium at a new height. The change in energy of the system in the process is : **[4 Sep. 2020 (II)]**

(a) $gdS(x_2^2 + x_1^2)$ (b) $gdS(x_2 + x_1)^2$

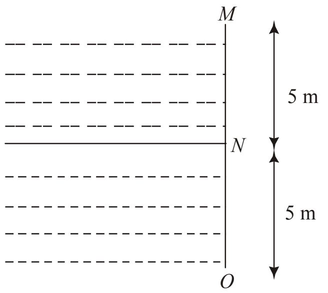
(c) $\frac{3}{4}gdS(x_2 - x_1)^2$ (d) $\frac{1}{4}gdS(x_2 - x_1)^2$

4. A leak proof cylinder of length 1 m, made of a metal which has very low coefficient of expansion is floating vertically in water at 0°C such that its height above the water surface is 20 cm. When the temperature of water is increased to 4°C, the height of the cylinder above the water surface becomes 21 cm. The density of water at $T = 4°C$, relative to the density at $T = 0°C$ is close to: **[8 Jan 2020 (I)]**

(a) 1.26 (b) 1.04 (c) 1.01 (d) 1.03

5. Consider a solid sphere of radius R and mass density $\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$, $0 < r \le R$. The minimum density of a liquid in which it will float is: **[8 Jan 2020 (I)]**

(a) $\frac{\rho_0}{3}$ (b) $\frac{\rho_0}{5}$ (c) $\frac{2\rho_0}{5}$ (d) $\frac{2\rho_0}{3}$

6. 

Two liquids of densities ρ_1 and $\rho_2(\rho_2 = 2\rho_1)$ are filled up behind a square wall of side 10 m as shown in figure. Each liquid has a height of 5 m. The ratio of the forces due to these liquids exerted on upper part MN to that at the lower part NO is (Assume that the liquids are not mixing): **[8 Jan 2020 (II)]**

(a) 1/3 (b) 2/3 (c) 1/2 (d) 1/4

7. A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water? [Take, density of water = 10^3 kg/m^3] **[10 April 2019 (II)]**

(a) 46.3 kg (b) 87.5 kg (c) 65.4 kg (d) 30.1 kg

8. A submarine experiences a pressure of 5.05×10^6 Pa at depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa. Then $d_1 - d_1$ is approximately (density of water=10^3 kg/m^3 and acceleration due to gravity = 10 ms^{-2}): **[10 April 2019 (II)]**

(a) 300 m (b) 400 m (c) 600 m (d) 500 m

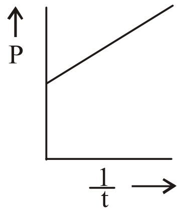
9. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is: **[9 April 2019 (II)]**

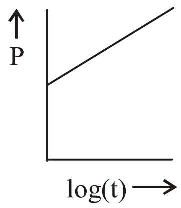
(a) 0.5 (b) 0.8 (c) 0.6 (c) 0.7

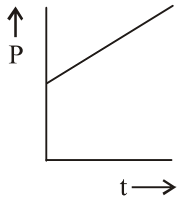
10. A load of mass M kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8. The new value of increase in length of the steel wire is : **[12 Jan. 2019 (II)]**

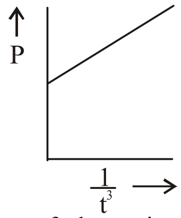
(a) 3.0 mm (b) 4.0 mm (c) 5.0 mm (d) Zero

11. A soap bubble, blown by a mechanical pump at the mouth of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by: **[12 Jan. 2019 (II)]**

(a) 

(b) 

(c) 

(d) 

12. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be: **[11 Jan. 2019 (I)]**

(a) $\frac{1}{4}\rho v^2$ (b) $\frac{3}{4}\rho v^2$ (c) $\frac{1}{2}\rho v^2$ (d) ρv^2

13. A thin uniform tube is bent into a circle of radius *r* in the vertical plane. Equal volumes of two immiscible liquids, whose densities are ρ_1 and ρ_1 $(\rho_1 > \rho_2)$ fill half the circle. The angle θ between the radius vector passing through the common interface and the vertical is **[Online April 15, 2018]**

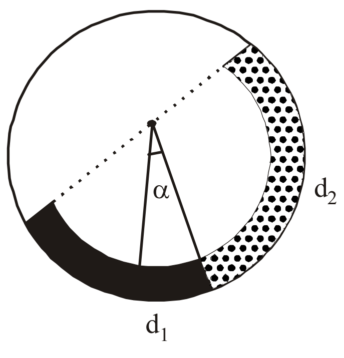
(a) $\theta = \tan^{-1}\left[\frac{\pi}{2}\left(\frac{\rho_1-\rho_2}{\rho_1+\rho_2}\right)\right]$

(b) $\theta = \tan^{-1}\frac{\pi}{2}\left(\frac{\rho_1-\rho_2}{\rho_1+\rho_2}\right)$

(c) $\theta = \tan^{-1}\pi\left(\frac{\rho_1}{\rho_2}\right)$

(d) None of above

14. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90º angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is: **[2014]**



(a) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (b) $\frac{1+\cos\alpha}{1-\cos\alpha}$

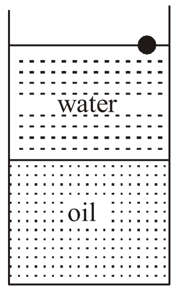
(c) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (d) $\frac{1+\sin\alpha}{1-\cos\alpha}$

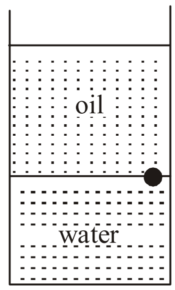
15. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is: **[2013]**

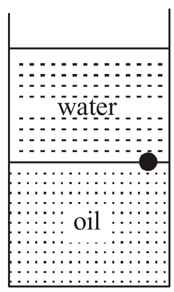
(a) $\frac{Mg}{k}$ (b) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$

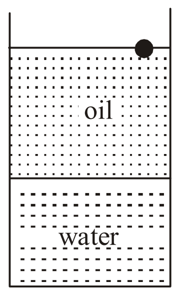
(c) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ (d) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$

16. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} represe-nting the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position ? **[2010]**

(a) 

(b) 

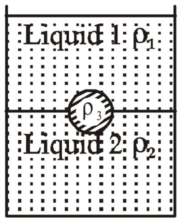
(c) 

(d) 

17. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is **[2010]**

(a) 4 (b) 3 (c) 2 (d) 1

18. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and, ρ_2 respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure.Which of the following is true for ρ_1, $\rho_2$and ρ_3? **[2008]**



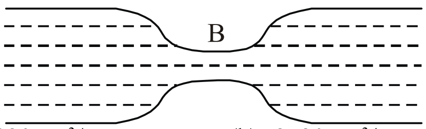
(a) $\rho_3 < \rho_1 < \rho_2$ (b) $\rho_1 > \rho_3 > \rho_2$
(c) $\rho_1 < \rho_2 < \rho_3$ (d) $\rho_1 < \rho_3 < \rho_2$

TOPIC 2 Fluid Flow, Reynold's Number and Bernoulli's Principle

19. A fluid is flowing through a horizontal pipe of varying cross-section, with speed v ms^{-1} at a point where the pressure is P Pascal. At another point where pressure is $\frac{P}{2}$ Pascal its speed is V ms^{-1}. If the density of the fluid is ρ kg m^{-3} and the flow is streamline, then V is equal to : **[6 Sep. 2020 (II)]**

(a) $\sqrt{\frac{P}{\rho}+v}$ (b) $\sqrt{\frac{2P}{\rho}+v^2}$
(c) $\sqrt{\frac{P}{2\rho}+v^2}$ (d) $\sqrt{\frac{P}{\rho}+v^2}$

20. Water flows in a horizontal tube (see figure). The pressure of water changes by 700 Nm^{-2} between A and B where the area of cross section are 40 cm^2 and 20 cm^2, respectively. Find the rate of flow of water through the tube.
(density of water = 1000 kgm^{-3}) **[9 Jan. 2020 (I)]**



(a) 3020 cm^3/s (b) 2720 cm^3/s
(c) 2420 cm^3/s (d) 1810 cm^3/s

21. An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is: **[7 Jan. 2020 (II)]**

(a) $\frac{9}{16}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{3}{4}$ (d) $\frac{81}{256}$

22. Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be : [Take g = 10 ms^{-2}) **[10 April 2019 (II)]**

(a) 2×10^{-5} m^2 (b) 5×10^{-5} m^2
(c) 5×10^{-4} m^2 (d) 1×10^{-5} m^2

23. Water from a pipe is coming at a rate of 100 liters per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of : (density of water = 1000 kg/m^3, coefficient of viscosity of water = 1 mPa *s*) **[8 April 2019 I]**

(a) 10^3 (b) 10^4 (c) 10^2 (d) 10^6

24. Water flows into a large tank with flat bottom at the rate of 10^{-4} m^3 s$^{-(}$1) Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is: **[10 Jan. 2019 I]**

(a) 5.1 cm (b) 7 cm (c) 4 cm (d) 9 cm

25. The top of a water tank is open to air and its water lavel is maintained. It is giving out 0.74m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to: **[9 Jan. 2019 (II)]**

(a) 6.0 m (b) 4.8 m (c) 9.6 m (d) 2.9 m

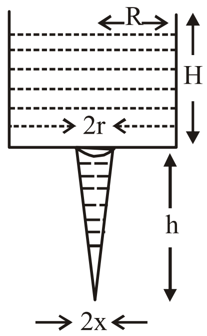
26. When an air bubble of radius r rises from the bottom to the surface of a lake, its radius becomes $\frac{5r}{4}$. Taking the atmospheric pressure to be equal to 10m height of water column, the depth of the lake would approximately be (ignore the surface tension and the effect of temperature): **[Online April 15, 2018]**

(a) 10.5m (b) 8.7m (c) 11.2m (d) 9.5m

27. Two tubes of radii r_1 and r_2, and lengths l_1 and l_2, respectively, are connected in series and a liquid flows through each of them in streamline conditions. P_1 and P_2 are pressure differences across the two tubes. If P_2 is $4P_1$ and l_2 is $\frac{l_1}{4}$, then the radius r_2 will be equal to : **[Online April 9, 2017]**

(a) r_1 (b) $2r_1$ (c) $4r_1$ (d) $\frac{r_1}{2}$

28. Consider a water jar of radius R that has water filled up to height H and is kept on a stand of height h (see figure). Through a hole of radius r (r << R) at its bottom, the water leaks out and the stream of water coming down towards the ground has a shape like a funnel as shown in the figure. If the radius of the cross–section of water stream when it hits the ground is x. Then : **[Online April 9, 2016]**



(a) $x = r\left(\frac{H}{H+h}\right)^{\frac{1}{4}}$ (b) $x = r\left(\frac{H}{H+h}\right)$
(c) $x = r\left(\frac{H}{H+h}\right)^2$ (d) $x = r\left(\frac{H}{H+h}\right)^{\frac{1}{2}}$

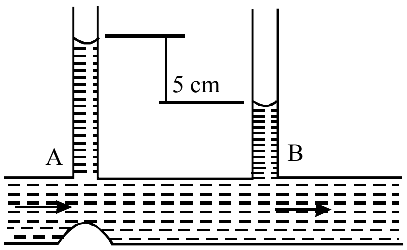
29. If it takes 5 minutes to fill a 15 litre bucket from a water tap of diameter $\frac{2}{\sqrt{\pi}}$ cm then the Reynolds number for the flow is (density of water = 10^3 kg/m^3) and viscosity of water = 10^{-3} Pa.s) close to : **[Online April 10, 2015]**

(a) 1100 (b) 11,000 (c) 550 (d) 5500

30. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg) **[2014]**

(a) 16 cm (b) 22 cm (c) 38 cm (d) 6 cm

31. In the diagram shown, the difference in the two tubes of the manometer is 5 cm, the cross section of the tube at A and B is 6 mm^2 and 10 mm^2 respectively. The rate at which water flows through the tube is ($g = 10\ ms^{-2}$) **[Online April 19, 2014]**



(a) 7.5 cc/s (b) 8.0 cc/s (c) 10.0 cc/s (d) 12.5 cc/s

32. A cylindrical vessel of cross-section A contains water to a height h. There is a hole in the bottom of radius 'a'. The time in which it will be emptied is: **[Online April 12, 2014]**

(a) $\frac{2A}{\pi a^2}\sqrt{\frac{h}{g}}$ (b) $\frac{\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$

(c) $\frac{2\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$ (d) $\frac{A}{\sqrt{2}\pi a^2}\sqrt{\frac{h}{g}}$

33. Water is flowing at a speed of 1.5 ms^{-1} through horizontal tube of cross-sectional area 10^{-2} m^2 and you are trying to stop the flow by your palm. Assuming that the water stops immediately after hitting the palm, the minimum force that you must exert should be (density of water = 10^3 kgm^{-3}) **[Online April 9, 2014]**

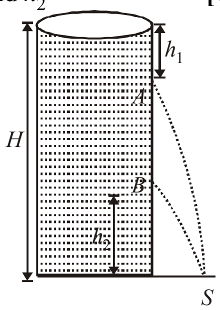
(a) 15 N (b) 22.5 N (c) 33.7 N (d) 45 N

34. Air of density 1.2 kg m^{-3} is blowing across the horizontal wings of an aeroplane in such a way that its speeds above and below the wings are 150 ms^{-1} and 100 ms^{-1}, respectively. The pressure difference between the upper and lower sides of the wings, is : **[Online April 22, 2013]**

(a) 60 Nm^{-2} (b) 180 Nm^{-2}

(c) 7500 Nm^{-2} (d) 12500 Nm^{-2}

35. In a cylindrical water tank, there are two small holes A and B on the wall at a depth of h_1, from the surface of water and at a height of h_2 from the bottom of water tank. Surface of water is at heigh H from the bottom of water tank. Water coming out from both holes strikes the ground at the same point S. Find the ratio of h_1 and h_2 **[Online May 26, 2012]**



(a) Depends on H (b) 1 : 1

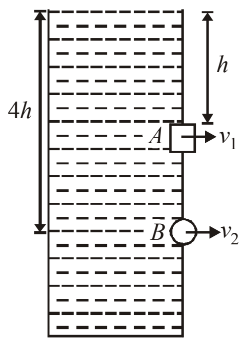
(c) 2 : 2 (d) 1 : 2

36. Water is flowing through a horizontal tube having cross-sectional areas of its two ends being A and A' such that the ratio A/A' is 5. If the pressure difference of water between the two ends is 3×10^5 N m^{-2}, the velocity of water with which it enters the tube will be (neglect gravity effects) **[Online May 12, 2012]**

(a) 5 m s^{-1} (b) 10 m s^{-1}

(c) 25 m s^{-1} (d) $50\sqrt{10}$ m s^{-1}

37. A square hole of side length ℓ is made at a depth of h and a circular hole of radius r is made at a depth of 4h from the surface of water in a water tank kept on a horizontal surface. If $\ell << h$, $r << h$ and the rate of water flow from the holes is the same, then r is equal to **[May 7, 2012]**



(a) $\frac{\ell}{\sqrt{2\pi}}$ (b) $\frac{\ell}{\sqrt{3\pi}}$ (c) $\frac{\ell}{3\pi}$ (d) $\frac{\ell}{2\pi}$

38. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to: **[2011]**

(a) 7.5×10^{-3} m (b) 9.6×10^{-3} m

(c) 3.6×10^{-3} m (d) 5.0×10^{-3} m

39. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom is **[2002]**

(a) 10 (b) 20 (c) 25.5 (d) 5

TOPIC 3 Viscosity and Terminal Velocity

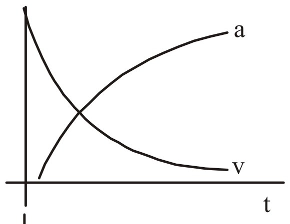
40. In an experiment to verify Stokes law, a small spherical ball of radius r and density ρ falls under gravity through a distance h in air before entering a tank of water. If the terminal velocity of the ball inside water is same as its velocity just before entering the water surface, then the value of h is proportional to : (ignore viscosity of air) **[5 Sep. 2020 (II)]**

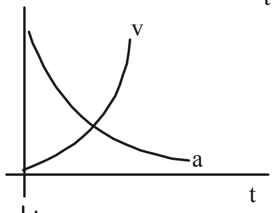
(a) r^4 (b) r (c) r^3 (d) r^2

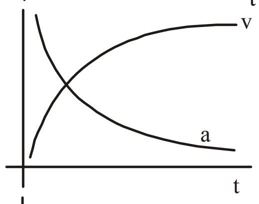
41. A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ratio (v_1/v_2) equals: **[12 April 2019 (II)]**

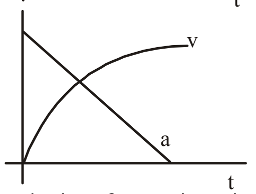
(a) 9 (b) 1/27 (c) 1/9 (d) 27

42. Which of the following option correctly describes the variation of the speed v and acceleration 'a' of a point mass falling vertically in a viscous medium that applies a force F = –kv, where 'k' is a constant, on the body? (Graphs are schematic and not drawn to scale)
[Online April 9, 2016]

(a)


(b)


(c)


(d)


43. The velocity of water in a river is 18 km/hr near the surface. If the river is 5 m deep, find the shearing stress between the horizontal layers of water. The co-efficient of viscosity of water = 10^{-2} poise. **[Online April 19, 2014]**
(a) 10^{-1} N/m^2 (b) 10^{-2} N/m^2
(c) 10^{-3} N/m^2 (d) 10^{-4} N/m^2

44. The average mass of rain drops is 3.0×10^{-5} kg and their avarage terminal velocity is 9 m/s. Calculate the energy transferred by rain to each square metre of the surface at a place which receives 100 cm of rain in a year.
[Online April 11, 2014]
(a) 3.5×10^5 J (b) 4.05×10^4 J
(c) 3.0×10^5 J (d) 9.0×10^4 J

45. A tank with a small hole at the bottom has been filled with water and kerosene (specific gravity 0.8). The height of water is 3m and that of kerosene 2m. When the hole is opened the velocity of fluid coming out from it is nearly: (take g = 10 ms^{-2} and density of water = 10^3 kg m^{-3})
[Online April 11, 2014]
(a) 10.7 ms^{-1} (b) 9.6 ms^{-1}
(c) 8.5 ms^{-1} (d) 7.6 ms^{-1}

46. In an experiment, a small steel ball falls through a liquid at a constant speed of 10 cm/s. If the steel ball is pulled upward with a force equal to twice its effective weight, how fast will it move upward ? **[Online April 25, 2013]**
(a) 5 cm/s (b) Zero (c) 10 cm/s (d) 20 cm/s

47. The terminal velocity of a small sphere of radius a in a viscous liquid is proportional to **[Online May 26, 2012]**
(a) a^2 (b) a^3 (c) a (d) a^{-1}

48. If a ball of steel (density $\rho = 7.8$ g cm^{-3}) attains a terminal velocity of 10 cm s^{-1} when falling in water (Coefficient of viscosity $\eta_{water} = 8.5 \times 10^{-4}$ Pa.s), then, its terminal velocity in glycerine ($\rho = 1.2$ g cm^{-3}, $\eta = 13.2$ Pa.s) would be, nearly **[2011 RS]**
(a) 6.25×10^{-4} cm s^{-1} (b) 6.45×10^{-4} cm s^{-1}
(c) 1.5×10^{-5} cm s^{-1} (d) 1.6×10^{-5} cm s^{-1}

49. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_1 ($\rho_2 < \rho_1$). Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, i.e., $F_{viscous} = -kv^2$ ($k > 0$). The terminal speed of the ball is **[2008]**

(a) $\sqrt{\dfrac{Vg(\rho_1 - \rho_2)}{k}}$ (b) $\dfrac{Vg\rho_1}{k}$

(c) $\sqrt{\dfrac{Vg\rho_1}{k}}$ (d) $\dfrac{Vg(\rho_1 - \rho_2)}{k}$

50. If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3), find the terminal speed of a sphere of silver (density = 10.5 kg/m^3) of the same size in the same liquid
[2006]
(a) 0.4 m/s (b) 0.133 m/s
(c) 0.1 m/s (d) 0.2 m/s

51. Spherical balls of radius 'R' are falling in a viscous fluid of viscosity 'η' with a velocity 'v'. The retarding viscous force acting on the spherical ball is **[2004]**
(a) inversely proportional to both radius 'R' and velocity 'v'
(b) directly proportional to both radius 'R' and velocity 'v'
(c) directly proportional to 'R' but inversely proportional to 'v'
(d) inversely proportional to 'R' but directly proportional to velocity 'v'

TOPIC 4 Surface Tension, Surface Energy and Capillarity

52. When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it. If the contact angle between the liquid and glass to close to 0°, the surface tension of the liquid, in milliNewton m^{-1}, is [$\rho_{(liquid)} = 900$ kgm^{-3}, $g = 10$ ms^{-2}] (Give answer in closest integer) __________.
[NA 3 Sep. 2020 (I)]

53. Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is :
[3 Sep. 2020 (I)]
(a) 4 : 1 (b) 0.8 : 1 (c) 8 : 1 (d) 2 : 1

54. A capillary tube made of glass of radius 0.15 mm is dipped vertically in a beaker filled with methylene iodide (surface tension = 0.05 Nm^{-1}, density = 667 kg m^{-3}) which rises to height h in the tube. It is observed that the two tangents drawn from liquid-glass interfaces (from opp. sides of the capillary) make an angle of 60° with one another. Then h is close to (g = 10 ms^{-2}). **[2 Sep. 2020 (II)]**

(a) 0.049 m (b) 0.087 m
(c) 0.137 m (d) 0.172 m

55. A small spherical droplet of density d is floating exactly half immersed in a liquid of density ρ and surface tension T. The radius of the droplet is (take note that the surface tension applies an upward force on the droplet): **[9 Jan. 2020 (II)]**

(a) $r=\sqrt{\frac{2T}{3(d+\rho)g}}$ (b) $r=\sqrt{\frac{T}{(d-\rho)g}}$

(c) $r=\sqrt{\frac{T}{(d+\rho)g}}$ (d) $r=\sqrt{\frac{3T}{(2d-\rho)g}}$

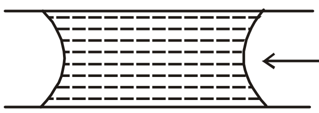
56. The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio, (r_1/r_2), is then close to : **[10 April 2019 (I)]**

(a) 4/5 (b) 2/5 (c) 3/5 (d) 2/3

57. If 'M' is the mass of water that rises in a capillary tube of radius 'r', then mass of water which will rise in a capillary tube of radius '2r' is : **[9 April 2019 I]**

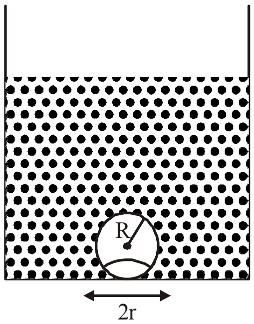
(a) M (b) $\frac{M}{2}$ (c) 4 M (d) 2 M

58. If two glass plates have water between them and are separated by very small distance (see figure), it is very difficult to pull them apart. It is because the water in between forms cylindrical surface on the side that gives rise to lower pressure in the water in comparison to atmosphere. If the radius of the cylindrical surface is R and surface tension of water is T then the pressure in water between the plates is lower by : **[Online April 10, 2015]**



(a) $\frac{2T}{R}$ (b) $\frac{4T}{R}$ (c) $\frac{T}{4R}$ (d) $\frac{T}{R}$

59. On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If $r \ll R$ and the surface tension of water is T, value of r just before bubbles detach is: (density of water is ρ_w) **[2014]**



(a) $R^2\sqrt{\frac{2\rho_W g}{3T}}$ (b) $R^2\sqrt{\frac{\rho_W g}{6T}}$

(c) $R^2\sqrt{\frac{\rho_W g}{T}}$ (d) $R^2\sqrt{\frac{3\rho_W g}{T}}$

60. A large number of liquid drops each of radius r coalesce to from a single drop of radius R. The energy released in the process is converted into kinetic energy of the big drop so formed. The speed of the big drop is (given, surface tension of liquid T, density ρ) **[Online April 19, 2014, 2012]**

(a) $\sqrt{\frac{T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$ (b) $\sqrt{\frac{2T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$

(c) $\sqrt{\frac{4T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$ (d) $\sqrt{\frac{6T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$

61. Two soap bubbles coalesce to form a single bubble. If V is the subsequent change in volume of contained air and S change in total surface area, T is the surface tension and P atmospheric pressure, then which of the following relation is correct? **[Online April 12, 2014]**

(a) $4PV+3ST=0$ (b) $3PV+4ST=0$
(c) $2PV+3ST=0$ (d) $3PV+2ST=0$

62. An air bubble of radius 0.1 cm is in a liquid having surface tension 0.06 N/m and density 10^3 kg/m^3. The pressure inside the bubble is 1100 Nm^{-2} greater than the atmospheric pressure. At what depth is the bubble below the surface of the liquid? ($g = 9.8$ ms^{-2}) **[Online April 11, 2014]**

(a) 0.1 m (b) 0.15 m (c) 0.20 m (d) 0.25 m

63. A capillary tube is immersed vertically in water and the height of the water column is x. When this arrangement is taken into a mine of depth d, the height of the water column is y. If R is the radius of earth, the ratio $\frac{x}{y}$ is: **[Online April 9, 2014]**

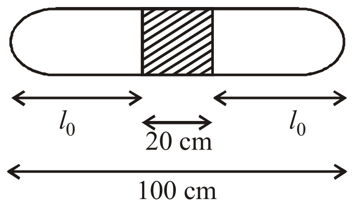
(a) $\left(1-\frac{d}{R}\right)$ (b) $\left(1-\frac{2d}{R}\right)$

(c) $\left(\frac{R-d}{R+d}\right)$ (d) $\left(\frac{R+d}{R-d}\right)$

64. Wax is coated on the inner wall of a capillary tube and the tube is then dipped in water. Then, compared to the unwaxed capillary, the angle of contact θ and the height h upto which water rises change. These changes are : **[Online April 23, 2013]**

(a) θ increases and h also increases
(b) θ decreases and h also decreases
(c) θ increases and h decreases
(d) θ decreases and h increases

65. A thin tube sealed at both ends is 100 cm long. It lies horizontally, the middle 20 cm containing mercury and two equal ends containing air at standard atmospheric pressure. If the tube is now turned to a vertical position, by what amount will the mercury be displaced? **[Online April 23, 2013]**



(Given : cross-section of the tube can be assumed to be uniform)

(a) 2.95 cm (b) 5.18 cm (c) 8.65 cm (d) 0.0 cm

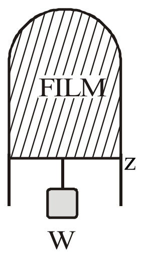
66. This question has Statement-1 and Statement-2. Of the four choices given after the Statements, choose the one that best describes the two Statetnents.

Statement-1: A capillary is dipped in a liquid and liquid rises to a height h in it. As the temperature of the liquid is raised, the height h increases (if the density of the liquid and the angle of contact remain the same).

Statement-2: Surface tension of a liquid decreases with the rise in its temperature. **[Online April 9, 2013]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not** the correct explanation for Statement-1.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation for Statement-1.

67. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight is negligible. The surface tension of the liquid film is **[2012]**



(a) 0.0125 Nm^{-1} (b) 0.1 Nm^{-1}
(c) 0.05 Nm^{-1} (d) 0.025 Nm^{-1}

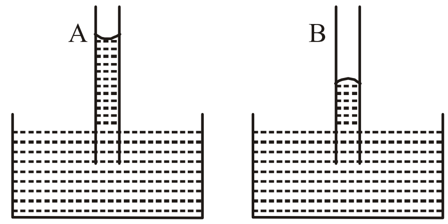
68. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = 0.03 Nm^{-1}) **[2011]**

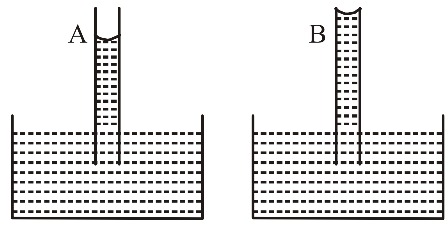
(a) 0.2πmJ (b) 2πmJ (c) 0.4πmJ (d) 4πmJ

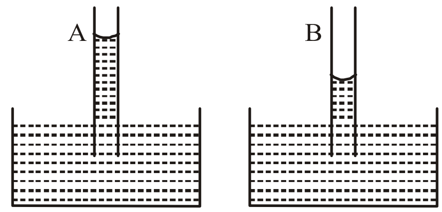
69. Two mercury drops (each of radius 'r') merge to form bigger drop. The surface energy of the bigger drop, if T is the surface tension, is : **[2011 RS]**

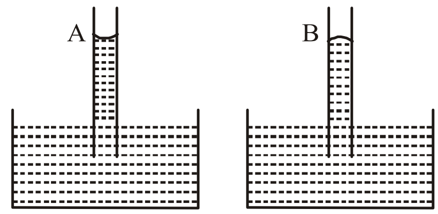
(a) $4\pi r^2 T$ (b) $2\pi r^2 T$
(c) $2^{8/3}\pi r^2 T$ (d) $2^{5/3}\pi r^2 T$

70. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes? **[2008]**

(a)


(b)


(c)


(d)


71. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator the length of water column in the capillary tube will be **[2005]**

(a) 10 cm (b) 8 cm (c) 20 cm (d) 4 cm

72. If two soap bubbles of different radii are connected by a tube **[2004]**

(a) air flows from the smaller bubble to the bigger
(b) air flows from bigger bubble to the smaller bubble till the sizes are interchanged
(c) air flows from the bigger bubble to the smaller bubble till the sizes become equal
(d) there is no flow of air.

Hints & Solutions

1. **(a)** In equilibrium, $mg = F_e$

$F_B = V\rho_0 g$ and mass = volume × density

$$\frac{4}{3}\pi(R^3 - r^3)\rho_0 g = \frac{4}{3}\pi R^3 \rho_w g$$

Given, relative density, $\frac{\rho_0}{\rho_w} = \frac{27}{8}$

$$\Rightarrow \left[1 - \left(\frac{r}{R}\right)^3\right]\frac{27}{8}\rho_w = \rho_w$$

$$\Rightarrow 1 - \frac{r^3}{R^3} = \frac{9}{27} \Rightarrow 1 - \frac{1}{3} = \frac{r^3}{R^3} \Rightarrow \frac{2}{3} = \frac{r^3}{R^3}$$

$$\Rightarrow \frac{r}{R} = \left(\frac{2}{3}\right)^{1/3} \Rightarrow 1 - \frac{r^3}{R^3} = \frac{8}{27}$$

$$\Rightarrow \frac{r^3}{R^3} = 1 - \frac{8}{27} = \frac{19}{27}$$

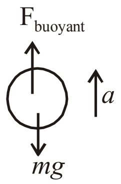
$$\therefore r = 0.89R = \frac{8}{9}R.$$

2. **(c)** Given :

Radius of air bubble = 1 cm,

Upward acceleration of bubble, $a = 9.8$ cm/s^2,

$\rho_{water} = 1$ g cm^{-3}

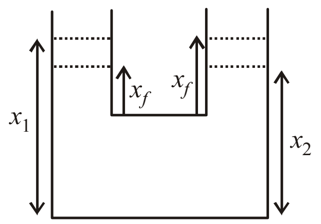


Volume $V = \frac{4\pi}{3}r^3 = \frac{4\pi}{3} \times (1)^3 = 4.19$ cm^3

$$F_{buoyant} - mg = ma \Rightarrow m = \frac{F_{buoyant}}{g + a}$$

$$\therefore m = \frac{(V\rho_\omega g)}{g + a} = \frac{V\rho_\omega}{1 + \frac{a}{g}} = \frac{(4.19) \times 1}{1 + \frac{9.8}{980}} = \frac{4.19}{1.01} = 4.15\,\text{g}$$

3. **(d)**



Initial potential energy,

$$U_1 = (\rho S x_1)g \cdot \frac{x_1}{2} + (\rho S x_2)g \cdot \frac{x_2}{2}$$

Final potential energy,

$$U_f = (\rho S x_f)g \cdot \frac{x_f}{2} \times 2$$

By volume conservation,

$$Sx_1 + Sx_2 = S(2x_f)$$

$$x_f = \frac{x_1 + x_2}{2}$$

When valve is opened loss in potentail energy occur till water level become same.

$$\Delta U = U_i - U_f$$

$$\Delta U = \rho S g\left[\left(\frac{x_1^2}{2} + \frac{x_2^2}{2}\right) - x_f^2\right]$$

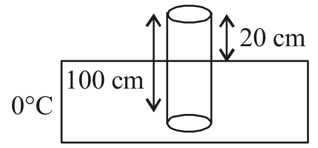
$$= \rho S g\left[\frac{x_1^2}{2} + \frac{x_2^2}{2} - \left(\frac{x_1 + x_2}{2}\right)^2\right]$$

$$= \frac{\rho S g}{2}\left[\frac{x_1^2}{2} + \frac{x_2^2}{2} - x_1 x_2\right] = \frac{\rho S g}{4}(x_1 - x_2)^2$$

4. **(c)** When cylinder is floating in water at 0°C

Net thrust $= A(h_2 - h_1)\rho_{0°c}g$

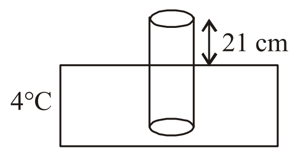
$= A(100 - 80)\rho_{0°c}g$



When cylinder is floating in water at 4° C

Net thrust $= A(h_2 - h_1)\rho_{4°c}g$

$= A(100 - 21)\rho_{4°c}g$



$$\therefore \frac{\rho_{4°c}}{\rho_{0°c}} = \frac{80}{79} = 1.01$$

5. **(c)** For minimum density of liquid, solid sphere has to float (completely immersed) in the liquid.

$mg = F_B$ (also $V_{immersed} = V_{total}$)

or $\int \rho dV = \frac{4}{3}\pi R^3 \rho_\ell$

$$\left[\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right) 0 < r \le R \text{ given}\right]$$

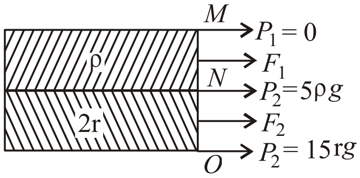
$$\Rightarrow \int_0^R \rho_0 4\pi\left(1 - \frac{r^2}{R^2}\right). r^2 dr = \frac{4}{3}\pi R^3 \rho_\ell$$

$$\Rightarrow 4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^5}{5R^2}\right]_0^R=\frac{4}{3}\pi R^3\rho_\ell$$

$$\frac{4\pi\rho_0 R^3}{3}\times\frac{2}{5}=\frac{4}{3}\pi R^3\rho_\ell$$

$$\therefore \rho_\ell=\frac{2\rho_0}{5}$$

6. **(d)**



Let P_1, P_2 and P_3 be the pressure at points M, N and O respectively.
Pressure is given by $P = \rho gh$
Now, $P_1 = 0$ ($\because h = 0$)
$P_2 = \rho g(5)$
$P_3 = \rho g(15)$
$= 15\,\rho g$

Force on upper part, $F_1=\frac{(P_1+P_2)}{2}A$

Force on lower part, $F_2=\frac{(P_2+P_3)}{2}A$

$$\therefore \frac{F_1}{F_2}=\frac{5\rho g}{20\rho g}=\frac{5}{20}=\frac{1}{4}$$

7. **(b)** When a body floats then the weight of the body = upthrust

$\therefore (50)^3\times\frac{30}{100}\times(1)\times g=M_{cube}g$ ...(i)

Let m mass should be placed, then
$(50)^3 \times (1) \times g = (M_{cube} + m)g$ ...(ii)
Subtracting equation (i) from equation (ii), we get
$\Rightarrow mg = (50)^3 \times g\,(1 - 0.3) = 125 \times 0.7 \times 10^3 g$
$\Rightarrow m = 87.5$ kg

8. **(a)** $P_1 = P_0 + \rho gd_1$
$P_2 = P_0 + \rho gd_2$
$\Delta P = P_2 - P_1 = \rho g\Delta d$
$3.03 \times 10^6 = 10^3 \times 10 \times \Delta d$
$\Rightarrow \Delta d \simeq 300$ m

9. **(c)** $Mg=\left(\frac{4V}{5}\right)\rho\omega g$

or $\left(\frac{M}{V}\right)=\frac{4\rho\omega}{5}$ or $\rho=\frac{4\rho\omega}{5}$

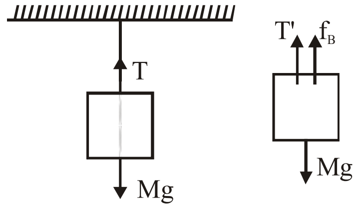
When block floats fully in water and oil, then

$Mg = F_{b_1} + F_{b_2}$

$(\rho V)g=\left(\frac{V}{2}\right)\rho_{oil}\,g+\frac{V}{2}\rho\omega g$

or $\rho_{oil}=\frac{3}{5}\rho\omega=0.6\rho\omega$

10. **(a)** Using $\frac{F}{A}=Y\cdot\frac{\Delta\ell}{\ell}$



$\Rightarrow \Delta\ell \propto F$ ...(i)
$T = Mg$

$T=Mg-f_B=Mg-\frac{M}{\rho_b}\cdot\rho_\ell\cdot g$

$=\left(1-\frac{\rho_\ell}{\rho_b}\right)Mg=\left(1-\frac{2}{8}\right)Mg$

$T=\frac{3}{4}Mg$

From eqn (i)

$\frac{\Delta\ell'}{\Delta\ell}=\frac{T'}{T}=\frac{3}{4}$ [Given: $\Delta\ell = 4$ mm]

$\therefore \Delta\ell'=\frac{3}{4}\cdot\Delta\ell=\frac{3}{4}\times 4=3$ mm

11. **(d)**

$V = ct$ or, $\frac{4}{3}\pi r^3=ct$

$\Rightarrow r=kt^{\frac{1}{3}}$

$P=P_0+\frac{4T}{k\,t^{1/3}}$

$P=P_0+c\left(\frac{1}{t^{1/3}}\right)$

12. **(b)** Mass per unit time of the liquid = ρav
Momentum per second carried by liquid
$= \rho av \times v$
Net force due to bounced back liquid,

$F_1=2\times\left[\frac{1}{4}\rho av^2\right]$

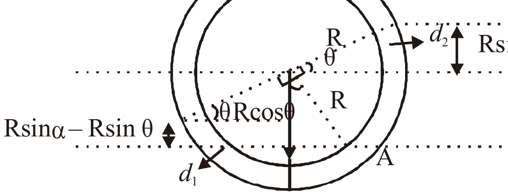
Net force due to stopped liquid, $F_2=\frac{1}{4}\rho av^2$

Total force,

$F=F_1+F_2=\frac{1}{2}\rho av^2+\frac{1}{4}\rho av^2=\frac{3}{4}\rho av^2$

Net pressure $=\frac{3}{4}\rho v^2$

13. **(d)** Pressure at interface A must be same from both the sides to be in equilibrium.



$\therefore (R\cos\theta+R\sin\theta)\rho_2 g=(R\cos\theta-R\sin\theta)\rho_1 g$

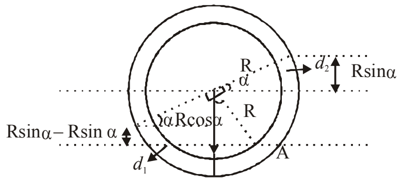
$$\Rightarrow \frac{d_1}{d_2} = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{1+\tan\theta}{1-\tan\theta}$$

$\Rightarrow \rho_1 - \rho_1 \tan\theta = \rho_2 + \rho_2 \tan\theta$

$\Rightarrow (\rho_1 + \rho_2)\tan\theta = \rho_1 - \rho_2$

$$\therefore \theta = \tan^{-1}\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)$$

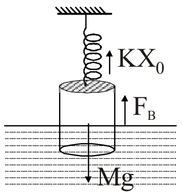
14. **(c)** Pressure at interface A must be same from both the sides to be in equilibrium.



$$\therefore (R\cos\alpha + R\sin\alpha)d_2 g = (R\cos\alpha - R\sin\alpha)d_1 g$$

$$\Rightarrow \frac{d_1}{d_2} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{1+\tan\alpha}{1-\tan\alpha}$$

15. **(c)** From figure, $kx_0 + F_B = Mg$



$$kx_0 + \sigma\frac{L}{2}Ag = Mg$$

[$\because$ mass = density × volume]

$$\Rightarrow kx_0 = Mg - \sigma\frac{L}{2}Ag$$

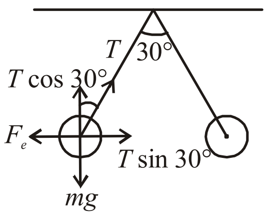
$$\Rightarrow x_0 = \frac{Mg - \frac{\sigma LAg}{2}}{k} = \frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$$

Hence, extension of the spring when it is in equilibrium is,

$$x_0 = \frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$$

16. **(b)** Oil will float on water so, (b) or (d) is the correct option. But density of ball is more than that of oil,, hence it will sink in oil.

17. **(c)**



$F_e = T\sin 30°$

$mg = T\cos 30°$

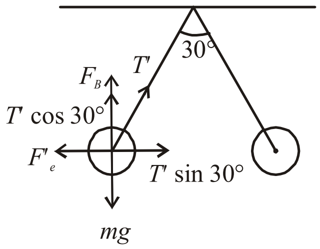
$$\Rightarrow \tan 30° = \frac{F_e}{mg} \quad ...(1)$$

In liquid ,

$F_e' = T'\sin 30° \quad ...(A)$

$mg = F_B + T'\cos 30°$

But F_B = Buoyant force



$= V(d-\rho)g = V(1.6-0.8)g = 0.8\,Vg$

$$= 0.8\frac{m}{d}g = \frac{0.8mg}{1.6} = \frac{mg}{2}$$

$$\therefore mg = \frac{mg}{2} + T'\cos 30°$$

$$\Rightarrow \frac{mg}{2} = T'\cos 30° \quad ...(B)$$

From (A) and (B), $\tan 30° = \frac{2F_e'}{mg}$

From (1) and (2)

$$\frac{F_e}{mg} = \frac{2F_e'}{mg} \quad (2)$$

$\Rightarrow F_e = 2F_e'$

If K be the dielectric constant, then

$$F_e' = \frac{F_e}{K}$$

$$\therefore F_e = \frac{2F_e}{K} \Rightarrow K = 2$$

18. **(d)** As liquid 1 floats over liquid 2. The lighter liquid floats over heavier liquid. So, $\rho_1 < \rho_2$

Also $\rho_3 < \rho_2$ because the ball of density ρ_3 does not sink to the bottom of the jar.

Also $\rho_3 > \rho_1$ otherwise the ball would have floated in liquid 1. we conclude that

$\rho_1 < \rho_3 < \rho_2$.

19. **(d)** Using Bernoulli's equation

$$P_1 + \frac{1}{2}\rho\upsilon_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho\upsilon_2^2 + \rho g h_2$$

For horizontal pipe, $h_1 = 0$ and $h_2 = 0$ and taking

$P_1 = P, P_2 = \frac{P}{2}$, we get

$$\Rightarrow P + \frac{1}{2}\rho\upsilon^2 = \frac{P}{2} + \frac{1}{2}\rho V^2$$

$$\Rightarrow \frac{P}{2} + \frac{1}{2}\rho\upsilon^2 = \frac{1}{2}\rho V^2$$

$$\Rightarrow V = \sqrt{\upsilon^2 + \frac{P}{\rho}}$$

20. **(b)** According to question, area of cross-section at A, $aA = 40\text{ cm}^2$ and at B, $aB = 20\text{ cm}^2$
Let velocity of liquid flow at A, $= V_A$ and at B, $= V_B$
Using equation of continuity $a_A V_A = a_B V_B$
$40V_A = 20V_B$
$\Rightarrow 2V_A = V_B$
Now, using Bernoulli's equation

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2 \Rightarrow P_A - P_B = \frac{1}{2}\rho\left(V_B^2 - V_A^2\right)$$

$$\Rightarrow \Delta P = \frac{1}{2}1000\left(V_B^2 - \frac{V_B^2}{4}\right) \Rightarrow \Delta P = 500 \times \frac{3V_B^2}{4}$$

$$\Rightarrow V_B = \sqrt{\frac{(\Delta P)\times 4}{1500}} = \sqrt{\frac{(700)\times 4}{1500}}\text{m/s} = 1.37\times 10^2\text{ cm/s}$$

Volume flow rate $Q = a_B \times v_B$
$= 20 \times 100 \times V_B = 2732\text{ cm}^3/\text{s} \approx 2720\text{ cm}^3/\text{s}$

21. **(a)** From the equation of continuity
$A_1 v_1 = A_2 v_2$
Here, v_1 and v_2 are the velocities at two ends of pipe.
A_1 and A_2 are the area of pipe at two ends

$$\Rightarrow \frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{\pi(4.8)^2}{\pi(6.4)^2} = \frac{9}{16}$$

22. **(b)** Using Bernoullie's equation

$$P + \frac{1}{2}(v_1^2 - v_2^2) + \rho gh = P$$

$$\Rightarrow v_2^2 = v_1^2 + 2gh$$

$$\Rightarrow v_2 = \sqrt{v_1^2 + 2gh}$$

Equation of continuity
$A_1v_1 = A_2v_2$

$$(1\text{ cm}^2)(1\text{ m/s}) = (A_2)\left(\sqrt{(1)^2 + 2\times 10\times\frac{15}{100}}\right)$$

$$10^{-4}\times 1 = A_2 \times 2$$

$$\therefore A_2 = \frac{10^{-4}}{2} = 5\times 10^{-5}\text{ m}^2$$

23. **(b)** Rate of flow of water $(V) = 100$ lit/min

$$= \frac{100\times 10^{-3}}{60} \times \frac{5}{3}\times 10^{-3}\text{m}^3$$

$$\therefore \text{ Velocity of flow of water } (v) = \frac{V}{A} = \frac{5\times 10^{-3}}{3\times\times(5\times 10^{-2})^2}$$

$$= \frac{10}{15\pi} = \frac{2}{3\pi}\text{ m/s}$$

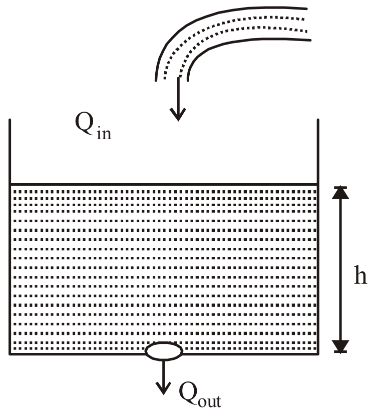
$= 0.2$ m/s

$$\therefore \text{ Reynold number } (N_R) = \frac{Dv\rho}{\eta}$$

$$= \frac{(10\times 10^{-2})\times\frac{2}{3\pi}\times 1000}{1} = 2\times 10^4$$

Order of $N_R = 10^4$

24. **(a)**



Since height of water column is constant therefore, water inflow rate (Q_{in})
= water outflow rate
$Q_{in} = 10^{-4}\text{ m}^3\text{s}^{-1}$
$Q_{out} = Au = 10^{-4} \times \sqrt{2gh}$
$\therefore 10^{-4} = 10^{-4} \times \sqrt{20\times h}$

$$\therefore h = \frac{1}{20}\text{ m} = 5\text{ cm}$$

25. **(b)** Here, volume tric flow rate

$$= \frac{0.74}{60} = \pi r^2 v = \left(\pi\times 4\times 10^{-4}\right)\times\sqrt{2gh}$$

$$\Rightarrow \sqrt{2gh} = \frac{74\times 100}{240\pi} \Rightarrow \sqrt{2gh} = \frac{740}{24\pi}$$

$$\Rightarrow 2gh = \frac{740\times 740}{24\times 24\times 10}\left(\because \pi^2 = 10\right)$$

$$\Rightarrow h = \frac{74\times 74}{2\times 24\times 24} \approx 4.8\text{m}$$

i.e., The depth of the centre of the opening from the level of water in the tank is close to 4.8 m

26. **(d)** Using $P_1V_1 = P_2V_2$

$$(P_1)\frac{4}{3}\pi r^3 = (P_2)\frac{4}{3}\pi\frac{125r^3}{64}$$

$$\frac{\rho g(10) + \rho gh}{\rho g(10)} = \frac{125}{64}$$

$640 + 64h = 1250$
On solving we get $h = 9.5$ m

27. **(d)** The volume of liquid flowing through both the tubes i.e., rate of flow of liquid is same.
Therefore, $V = V_1 = V_2$

$$\text{i.e., } \frac{\pi P_1 r_1^4}{8\eta l_1} = \frac{\pi P_2 r_2^4}{8\eta l_2}$$

$$\text{or } \frac{P_1 r_1^4}{l_1} = \frac{P_2 r_2^4}{l_2}$$

$\because$ $P_2 = 4P_1$ and $l_2 = l_1/4$

$$\frac{P_1 r_1^4}{l_1} = \frac{4P_1 r_2^4}{l_1/4} \Rightarrow r_2^4 = \frac{r_1^4}{16}$$

$r_2 = r_1/2$

28. **(a)** According to Bernoulli's Principle,

$$\frac{1}{2}\rho v_1^2 + \rho gh = \frac{1}{2}\rho v_2^2$$

$$v_1^2 + 2gh = v_2^2$$

$$2gH + 2gh = v_2^2 \quad ...(i)$$

$a_1 v_1 = a_2 v_2$

$$\pi r^2 \sqrt{2gh} = \pi x^2 v_2$$

$$\frac{r^2}{x^2}\sqrt{2gh} = v_2$$

Substituting the value of v_2 in equation (i)

$$2gH + 2gh = \frac{r^4}{x^4} 2gh \text{ or, } x = r\left[\frac{H}{H+h}\right]^{\frac{1}{4}}$$

29. **(d)** Given: Diameter of water tap $= \frac{2}{\sqrt{\pi}}$ cm

$\therefore$ Radius, $r = \frac{1}{\sqrt{\pi}} \times 10^{-2}$ m

$$\frac{dm}{dt} = \rho AV$$

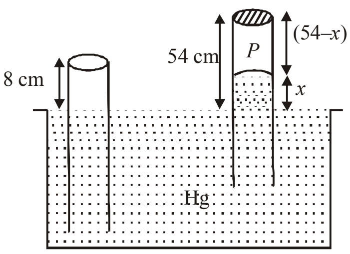
$$\frac{15}{5 \times 60} = 10^3 \times \pi\left(\frac{1}{\sqrt{\pi}}\right)^2 \times 10^{-4} V$$

$\Rightarrow$ V = 0.05 m/s

Reynold's number, $R_e = \frac{\rho V r}{n}$

$$= \frac{10^3 \times 0.5 \times \frac{2}{\sqrt{\pi}} 10^{-2}}{10^{-3}} \cong 5500$$

30. **(a)**



Length of the air column above mercury in the tube is,

$P + x = P_0$

$\Rightarrow \; P = (76 - x)$

$\Rightarrow \; 8 \times A \times 76 = (76 - x) \times A \times (54 - x)$

$\therefore \; x = 38$

Thus, length of air column

$= 54 - 38 = 16$ cm.

31. **(a)** According to Bernoulli's theorem,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\therefore \; v_2^2 - v_1^2 = 2gh \quad ...(1)$$

According to the equation of continuty

$$A_1 v_1 = A_2 v_2 \quad ...(2)$$

$$\frac{A_1}{A_2} = \frac{6 \text{ mm}^2}{10 \text{ mm}^2}$$

From equation (2), $\frac{A_1}{A_2} = \frac{v_2}{v_1} = \frac{6}{10}$

or, $v_2 = \frac{6}{10} v_1$

Putting this value of v_2 in equation (1)

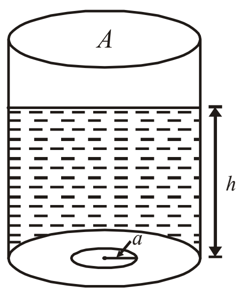
$$\left(\frac{6}{10} v_1\right)^2 - (v_1)^2 = 2 \times 10^3 \times 5$$

$$\left[\begin{array}{l}\because g = 10 m/s^2 = 10^3 cm/s^2 \\ \text{and } h = 5 \text{ cm}\end{array}\right]$$

Solving we get $v_1 = \frac{10}{8}$

Therefore the rate at which water flows through the tube $= A_1 v_1 = A_2 v_2 = \frac{6 \times 10}{8} = 7.5$ cc / s

32. **(b)** Let the rate of falling water level be $-\frac{dh}{dt}$



Initially at $t = 0 ; h = h$

$t = t ; h = 0$

Then, $A\left(-\frac{dh}{dt}\right) = \pi a^2 . v$

$$dt = -\frac{A}{\pi a^2 \sqrt{2gh}} dh$$

[$\because$ velocity of efflux of liquid $v = \sqrt{2gh}$]

Integrating both sides

$$\int_0^t dt = -\frac{A}{\sqrt{2g}\pi a^2} \int_h^0 h^{-1/2} dh$$

$$[t]_0^t = -\frac{A}{\sqrt{2g}\pi a^2} \cdot \left[\frac{h^{1/2}}{1/2}\right]_h^0$$

$$t = \frac{\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$$

33. **(a)** For 1 m length of horizontal tube

Mass of water M = density × volume

$= 10^3 \times$ area × length

$= 10^3 \times 10^{-2} \times 1 = 10$ kg

Therefore minimum force $= \frac{\Delta p}{\Delta t}$ (rate of change of momentum)

$= 10 \times 1.5 = 15\,N$

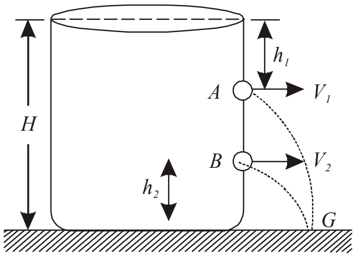
34. **(c)** Pressure difference

$$P_2 - P_1 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \quad = \frac{1}{2}\times 1.2\left((150)^2 - (100)^2\right)$$

$$= \frac{1}{2}\times 1.2(22500 - 10000)$$

$$= 7500 \text{ Nm}^{-2}$$

35. **(b)**



i.e. $R_1 = R_2 = R$

or, $v_1\, t_1 = v_2\, t_2$ ... (i)

Where v_1 = velocity of efflux at $A = \sqrt{(2gh_1)}$ and

v_2 = velocity of efflux at $B = \sqrt{(2g(H - h_2)}$

t_1 = time of fall water stream through A

$$= \sqrt{(\frac{2(H - h_1}{g})}$$

t_2 = time of fall of the water stream through $B = \sqrt{\frac{2h_2}{g}}$

Putting these values is eqn (i) we get

$(H - h_1)h_1 = (H - h_2)h_2$

or $[H - (h_1 + h_2)][h_1 - h_2] = 0$

Here, $H = h_1 + h_2$ is irrelevant because the holes are at two different heights. Hence $h_1 = h_2$ *or,* $\frac{h_1}{h_2} = 1$

36. **(a)** According to Bernoulli's theorem

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \quad ...(i)$$

From question,

$$P_1 - P_2 = 3\times 10^5, \frac{A_1}{A_2} = 5$$

According to equation of continuity

$A_1 v_1 = A_2 v_2$

or, $\frac{A_1}{A_2} = \frac{v_2}{v_1} = 5$

$\Rightarrow\ v_2 = 5v_1$

From equation (i)

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$\text{or } 3\times 10^5 = \frac{1}{2}\times 1000\left(5v_1^2 - v_1^2\right)$$

$\Rightarrow\ 600 = 6v_1 \times 4v_1$

$\Rightarrow\ v_1^2 = 25$

$\therefore\ v_1 = 5$ m/s

37. **(a)** As $A_1 v_1 = A_2 v_2$ (Principle of continuity)

or, $\ell^2\sqrt{2gh} = \pi r^2\sqrt{2g\times 4h}$

(Efflux velocity = $\sqrt{2gh}$)

$$\therefore\ r^2 = \frac{\ell^2}{2\pi} \text{ or } r = \sqrt{\frac{\ell^2}{2\pi}} = \frac{\ell}{\sqrt{2\pi}}$$

38. **(c)** Using Bernoulli's theorem, for horizontal flow

$$P_0 + \frac{1}{2}\rho v_1^2 + \rho gh = P_0 + \frac{1}{2}\rho v_2^2 + 0$$

$$v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{0.16 + 2\times 10\times 0.2} = 2.03 \text{ m/s}$$

According to equation of continuity

$A_2 v_2 = A_1 v_1$

$$\pi\frac{D_2^2}{4}\times v_2 = \pi\frac{D_1^2}{4}v_1$$

$$\Rightarrow D_2 = D_1\sqrt{\frac{v_1}{v_2}} = 3.55\times 10^{-3}\text{ m}$$

39. **(b)** Given, Height of cylinder, h=20 cm Acceleration due to gravity, g=10 ms^{-2}

Velocity of efflux

$v = \sqrt{2gh}$

Where h is the height of the free surface of liquid from th e hole

$\Rightarrow\ v = \sqrt{2\times 10\times 20} = 20\text{ m/s}$

40. **(a)** Using, $v^2 - u^2 = 2gh$

$\Rightarrow v^2 - 0^2 = 2gh \Rightarrow v = \sqrt{2gh}$

Terminal velocity,

$$V_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

After falling through h the velocity should be equal to terminal velocity

$$\therefore \sqrt{2gh} = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

$$\Rightarrow 2gh = \frac{4}{81}\frac{r^4 g^2(\rho-\sigma)^2}{\eta^2}$$

$$\Rightarrow h = \frac{2r^4 g(\rho-\sigma)^2}{81\eta^2} \Rightarrow h \propto r^4$$

41. **(a)** $27\times\frac{4}{3}\pi r^3 = \frac{4}{3}\pi r^3$

or $r = \frac{R}{3}$.

Terminal velocity, $v \propto r^2$

$$\therefore\ \frac{v_1}{v_2} = \frac{r_1^2}{r_2^2}$$

$$\text{or } v_2 = \left(\frac{r_2}{r_1}\right)^2 v_1 = \left(\frac{R/3}{R}\right)^2 v_1 = \frac{1}{9}$$

or $\frac{v_1}{v_2} = 9.$

42. **(c)** When a point mass is falling vertically in a viscous medium, the medium or viscous fluid exerts drag force on the body to oppose its motion and at one stage body falling with constant terminal velocity.

43. **(b)** $\eta = 10^{-2}$ poise

$v = 18 \text{ km/h} = \frac{18000}{3600} = 5 \text{ m/s}$

$l = 5 \text{ m}$

Strain rate $= \frac{v}{l}$

Coefficient of viscosity, $\eta = \frac{\text{shearing stress}}{\text{strain rate}}$

$\therefore$ Shearing stress = $\eta \times$ strain rate

$= 10^{-2} \times \frac{5}{5} = 10^{-2} \text{ Nm}^{-2}$

44. **(b)** Total volume of rain drops, received 100 cm in a year by area 1 m^2

$= 1\text{m}^2 \times \frac{100}{100}\text{m} = 1 \text{ m}^3$

As we know, density of water,
$d = 10^3 \text{ kg/m}^3$
Therefore, mass of this volume of water
$M = d \times v = 10^3 \times 1 = 10^3$ kg
Average terminal velocity of rain drop
$v = 9$ m/s (given)
Therefore, energy transferred by rain,

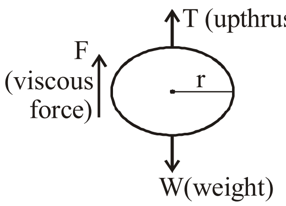
$E = \frac{1}{2} mv^2$

$= \frac{1}{2} \times 10^3 \times (9)^2$

$= \frac{1}{2} \times 10^3 \times 81 = 4.05 \times 10^4$ J

45. **(b)** According to Toricelli's theorem,
Velocity of efflex,
$V_{\text{eff}} = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 5} \cong 9.8 \text{ ms}^{-1}$

46. **(c)**



Weight of the body

$W = mg = \frac{4}{3}\pi r^3 \rho g$

$T = \frac{4}{3}\pi r^3 \sigma g$

and $F = 6\pi\eta vr$

When the body attains terminal velocity net force acting on the body is zero. i.e.,

$W - T - F = 0$

And terminal velocity $v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$

As in case of upward motion upward force is twice its effective weight, therefore, it will move with same speed 10 cm/s

47. **(a)** Terminal velocity in a viscous medium is given by:

$V_T = \frac{2a^2(\rho - \sigma)g}{9\eta}$

$\therefore V_T \propto a^2$

48. **(a)** When the ball attains terminal velocity
Weight of the ball = viscous force + buoyant force

$\therefore V\rho g = 6\pi\eta r v + V\rho_\ell g$

$\Rightarrow \; Vg(\rho - \rho_\ell) = 6\pi\eta r v$

Also $\; Vg(\rho - \rho_i') = 6\pi\eta' r v'$

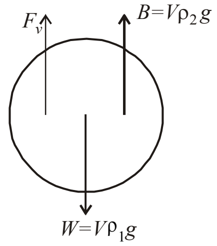
$\therefore \quad v'\eta' = \frac{(\rho - \rho_\ell')}{(\rho - \rho_\ell)} \times v\eta$

$\Rightarrow \quad v' = \frac{(\rho - \rho_\ell')}{(\rho - \rho_\ell)} \times \frac{v\eta}{\eta'}$

$= \frac{(7.8 - 1.2)}{(7.8 - 1)} \times \frac{10 \times 8.5 \times 10^{-4}}{13.2}$

$\therefore \quad v' = 6.25 \times 10^{-4}$ cm/s

49. **(a)** When the ball attains terminal velocity
Weight of the ball = Buoyant force + Viscous force



$\therefore \; V\rho_1 g = V\rho_2 g + kv_t^2 \Rightarrow Vg(\rho_1 - \rho_2)g = kv_t^2$

$\Rightarrow v_t = \sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$

50. **(c)** Given,
Density of gold, $\rho_G = 19.5 \text{ kg/m}^3$
Density of silver, $\rho_5 = 10.5 \text{kg/m}^3$
Density of liquid, $\sigma = 1.5 \text{kg/m}^3$

Terminal velocity, $v_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$

$\therefore \; \frac{v_{T_2}}{0.2} = \frac{(10.5 - 1.5)}{(19.5 - 1.5)} \Rightarrow v_{T_2} = 0.2 \times \frac{9}{18}$

$\therefore v_{T_2} = 0.1$ m/s

51. **(b)** From Stoke's law, force of viscosity acting on a spherical body is

$F = 6\pi\eta r v$

hence F is directly proportional to radius & velocity.

52. **(101)**
Given : Radius of capillary tube,
$r = 0.015$ cm $= 15 \times 10^{-5}$ mm
$h = 15$ cm $= 15 \times 10^{-2}$ mm

Using, $h = \frac{2T\cos\theta}{\rho g r}$ $\quad [\cos\theta = \cos 0° = 1]$

Surface tension,

$$T = \frac{rh\rho g}{2} = \frac{15\times10^{-5}\times15\times10^{-2}\times900\times10}{2} = 101 \text{ milli newton m}^{-1}$$

53. **(c)** According to question, pressure inside, 1st soap bubble,

$$\Delta P_1 = P_1 - P_0 = 0.01 = \frac{4T}{R_1} \quad ...(i)$$

And $\Delta P_2 = P_2 - P_0 = 0.02 = \frac{4T}{R_2}$ ...(ii)

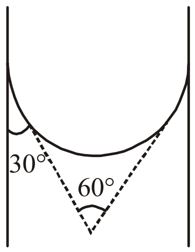
Dividing, equation (ii) by (i),

$$\frac{1}{2} = \frac{R_2}{R_1} \Rightarrow R_1 = 2R_2$$

Volume $V = \frac{4}{3}\pi R^3$

$$\therefore \frac{V_1}{V_2} = \frac{R_1^3}{R_2^3} = \frac{8R_2^3}{R_2^3} = \frac{8}{1}$$

54. **(b)** Given,
Angle of contact $\theta = 30°$
Surface tension, $T = 0.05$ Nm^{-1}
Radius of capillary tube, $r = 0.15$ mm $= 0.15 \times 10^{-3}$m
Density of methylene iodide, $\rho = 667$ kg m^{-3}



Capillary rise, $h = \frac{2T\cos\theta}{\rho g r}$

$$= \frac{2\times0.05\times\frac{\sqrt{3}}{2}}{667\times10\times0.15\times10^{-3}} = 0.087 \text{ m}$$

55. **(d)** For the drops to be in equilibrium upward force on drop = downward force on drop

$$T.2\pi R = \frac{4}{3}\pi R^3 dg - \frac{2}{3}\pi R^3 \rho g$$

$$\Rightarrow T(2\pi R) = \frac{2}{3}\pi R^3(2d-\rho)g$$

$$\Rightarrow T = \frac{R^2}{3}(2d-\rho)g \Rightarrow R = \sqrt{\frac{3T}{(2d-\rho)g}}$$

56. **(b)** As we know that

$$\frac{2T\cos\theta}{r\rho g} = R\ h$$

$$\frac{T_{Hg}}{T_{Water}} = 7.5$$

$$\frac{\rho_{Hg}}{\rho_W} = 13.6 \ \& \ \frac{\cos\theta_{Hg}}{\cos\theta_W} = \frac{\cos135°}{\cos0°} = \frac{1}{\sqrt{2}}$$

$$\frac{R_{Hg}}{R_{Water}} = \left(\frac{T_{Hg}}{T_W}\right)\left(\frac{\rho_W}{\rho_{Hg}}\right)\left(\frac{\cos\theta_{Hg}}{\cos\theta_W}\right)$$

$$= 7.5\times\frac{1}{13.6}\times\frac{1}{\sqrt{2}} = 0.4 = \frac{2}{5}$$

57. **(d)** We have, $h = \frac{2T\cos\theta}{r\rho g}$

Mass of the water in the capillary

$$m = \rho V = \rho\times\pi r^2 h = \rho\times\pi r^2\times\frac{2T\cos\theta}{r\rho g}$$

$\Rightarrow m \propto r$

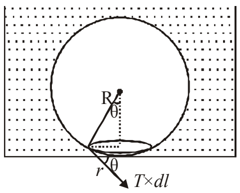
$$\therefore \frac{m_1}{m_2} = \frac{r}{2r}$$

or $m_2 = 2m_1 = 2m$.

58. **(d)** Here excess pressure, $P_{excess} = \frac{T}{r_1} + \frac{T}{r_2}$

$$P_{excess} = \frac{T}{R} \quad \because \begin{pmatrix} r_1 = R \\ r_2 = O \end{pmatrix}$$

59. **(a)** When the bubble gets detached,
Buoyant force = force due to surface tension



Force due to excess pressure = upthrust

Access pressure in air bubble = $\frac{2T}{R}$

$$\frac{2T}{R}(\pi r^2) = \frac{4\pi R^3}{3T}\rho_w g$$

$$\Rightarrow r^2 = \frac{2R^4\rho_w g}{3T} \Rightarrow r = R^2\sqrt{\frac{2\rho_w g}{3T}}$$

60. **(d)** When drops combine to form a single drop of radius R.

Then energy released, $E = 4\pi TR^3\left[\frac{1}{r} - \frac{1}{R}\right]$

If this energy is converted into kinetic energy then

$$\frac{1}{2}mv^2 = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right]$$

$$\frac{1}{2}\times\left[\frac{4}{3}\pi R^3\rho\right]v^2 = 4\pi R^3 T\left[\frac{1}{r}-\frac{1}{R}\right]$$

$$v^2 = \frac{6T}{\rho}\left[\frac{1}{r}-\frac{1}{R}\right]$$

$$v = \sqrt{\frac{6T}{\rho}\left[\frac{1}{r}-\frac{1}{R}\right]}$$

61. **(b)**

62. **(a)** **Given:** Radius of air bubble,
$r = 0.1$ cm $= 10^{-3}$ m
Surface tension of liquid,
$S = 0.06$ N/m $= 6 \times 10^{-2}$ N/m
Density of liquid, $\rho = 10^3$ kg/m^3
Excess pressure inside the bubble,
$\rho_{exe} = 1100$ Nm^{-2}
Depth of bubble below the liquid surface,
$h = ?$
As we know,

$$\rho_{Excess} = h\rho g + \frac{2s}{r}$$

$$\Rightarrow\ 1100 = h \times 10^3 \times 9.8 + \frac{2\times 6\times 10^{-2}}{10^{-3}}$$

$$\Rightarrow\ 1100 = 9800\,h + 120$$

$$\Rightarrow\ 9800h = 1100 - 120$$

$$\Rightarrow\ h = \frac{980}{9800} = 0.1\text{ m}$$

63. **(a)** Acceleration due to gravity changes with the depth,

$$g' = g\left(1-\frac{d}{R}\right)$$

Pressure, P = ρgh

Hence ratio, $\frac{x}{y}$ is $\left(1-\frac{d}{R}\right)$

64. **(c)** Angle of contact θ

$$\cos\theta = \frac{T_{SA} - T_{SL}}{T_{LA}}$$

when water is on a waxy or oily surface
$T_{SA} < T_{SL}$ cos θ is negative i.e.,
$90° < \theta < 180°$
i.e., angle of contact θ increases
And for $\theta > 90°$ liquid level in capillary tube fall. i.e., h decreases

65. **(b)**

66. **(b)** Surface tension of a liquid decreases with the rise in temperture. At the boiling point of liquid, surface tension is zero.

Capillary rise $h = \frac{2T\cos\theta}{rdg}$

As surface tension T decreases with rise in temperature hence capillary rise also decreases.

67. **(d)** Let T is the force due to surface tension per unit length, then
$F = 2lT$
l = length of the slider.
At equilibrium, $F = W$
$\therefore\ 2Tl = mg$

$$\Rightarrow\ T = \frac{mg}{2l} = \frac{1.5\times 10^{-2}}{2\times 30\times 10^{-2}} = \frac{1.5}{60} = 0.025\text{ Nm}^{-1}$$

68. **(c)** Work done = increase in surface area × surface tension
$\Rightarrow\ W = 2T4\pi[(5^2) - (3)^2] \times 10^{-4}$
$= 2 \times 0.03 \times 4\pi\,[25 - 9] \times 10^{-4}$ J
$= 0.4\pi \times 10^{-3}$ J $= 0.4\pi$ mJ

69. **(c)** As volume remains constant
∴ Sum of volumes of 2 smaller drops
= Volume of the bigger drop

$$2.\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \ \Rightarrow\ R = 2^{1/3}\,r$$

Surface energy = Surface tension × Surface area = $T.4\pi R^2$

$= T4\pi 2^{2/3} r^2 = T.2^{8/3}\pi r^2$.

70. **(c)** In case of water, the meniscus shape is concave upwards. From ascent formula $h = \frac{2\sigma\cos\theta}{r\rho g}$

The surface tension (σ) of soap solution is less than water. Therefore height of capillary rise for soap solution should be less as compared to water. As in the case of water, the meniscus shape of soap solution is also concave upwards.

71. **(c)** Water fills the tube entirely in gravityless condition i.e., 20 cm.

72. **(a)** Let pressure outside be P_0 and r and R be the radius of smaller bubble and bigger bubble respectively.

$\therefore$ Pressure P_1 For smaller bubble $= P_0 + \frac{2T}{r}$

P_2 For bigger bubble $= P_0 + \frac{2T}{R}$ $\quad (R > r)$

$\therefore P_1 > P_2$

hence air moves from smaller bubble to bigger bubble.