Level - I

Chapter 2

Electrostatic Potential and Capacitance

Solutions (Set-1)

SECTION - A

School/Board Exam. Type Questions

Very Short Answer Type Questions :

1. What are linear isotropic dielectrics?

Sol. In an external electric field, the positive and negative charges of a non- polar molecule are displaced in opposite directions. The displacement stops when the external force on the constituent charges of the molecule is balanced by the restoring force (due to internal fields in the molecule). The non-polar molecule thus develops an induced dipole moment. The dielectric is said to be polarized by the external field. We consider only the simple situation when the induced dipole moment is in the direction of the field and is proportional to the field strength. *(Substances for which this assumption is true are called **linear isotropic dielectrics.**)*

2. Define dielectric strength.

Sol. The maximum electric field that a dielectric medium can withstand without breaking down of its insulating property is called its dielectric strength.

3. What is the fringing effect of an electric field?

Sol. Electric field in the space between the plates of a parallel plate capacitor is non-uniform at the ends and the field lines are curved. This is known as fringing effect.

4. Write the relation between electric susceptibility and dielectric constant.

Sol. In either case, whether polar or non-polar, a dielectric develops a net dipole moment in the presence of an external field. The dipole moment per unit volume is called polarization and is denoted by P. For linear isotropic dielectrics,

$P = \chi_e E$

Here χ_e is a constant, characteristic of the dielectric and is known as the electric susceptibility of the dielelctric medium.

5. A charge q is moving along a straight line perpendicular to the bisector of an electric dipole, then what is the work done by charge against electric field?

Sol. Zero

6. When we move along the direction of electric field, then does electrostatic potential increases, decreases or remains same?

Sol. Decreases

7. Write the dimensional formula for polarisation.

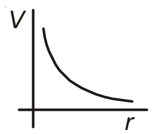
Sol. $[M^0L^{-2}T^1A^1]$

8. What is the electrostatic potential due to a charge *q* at its own location?

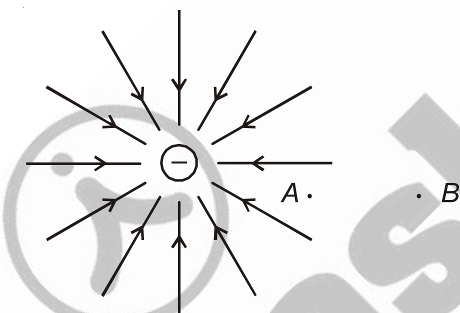
Sol. Infinite

9. Draw a graph between electrostatics potential and distance for a point charge *Q*.

Sol. $\because V = K\frac{Q}{r}$



10. Does the kinetic energy of a small positive charge increase or decrease in going from *A* to *B*?



Sol. Increases

Short Answer Type Questions :

11. Derive an expression for electrostatic potential at any point due to a point charge.

Sol. Consider a point charge *q* placed at point *O*. Consider any point *P* in the field of the above charge. Let us calculate the potential at point *P* due to the charge *q* kept at point *O*.

By definition, potential at *P* is equal to work done against electric field in moving a unit positive charge from a large distance (∞) to point *P*. For that matter, a force equal and opposite to the force exerted by electric field is to be applied on the unit positive charge.

$\therefore\ \overrightarrow{F_{ext}} = -(+1C)\ \vec{E} = -\vec{E}$.

For small displacement $\overrightarrow{dr}$,work done is

$dw = \vec{F}\cdot\overrightarrow{dr} = -\vec{E}.\overrightarrow{dr}$

Now $\vec{E}$ is radially outward. Taking $\overrightarrow{dr}$ to be radially outward, the angle between $\vec{E}$ and $\overrightarrow{dr}$ becomes 0°,

So, $dw = -E\,dr = -\frac{q}{4\pi\varepsilon_0 r^2}dr$

Now $w = \int dw = -\int_{\infty}^{r}\frac{q}{4\pi\varepsilon_0 r^2}dr$

$$\Rightarrow\ w = -\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{r} \qquad \Rightarrow\ w = \frac{q}{4\pi\varepsilon_0 r} \qquad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1}\right)$$

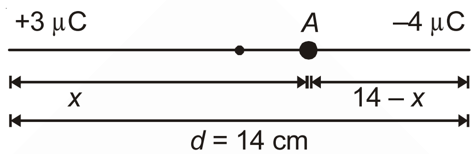
by defination is the potential at P due to the charge q

$$V = \frac{q}{4\pi\varepsilon_0 r} \text{ or } V = \frac{Kq}{r}, \text{ where } K = \frac{1}{4\pi\varepsilon_0}$$

12. Two charges +3 μC and –4 μC are placed 14 cm apart. Find the point on the line joining the two charges where electric potential is zero. (Taking zero potential at infinity).

Sol. Case 1:

Let at point A, potential is zero

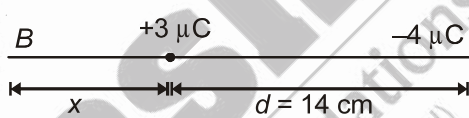


$$\frac{K\left(3\times10^{-6}\right)}{\left(\frac{x}{100}\right)} - \frac{K\left(4\times10^{-6}\right)}{\left(\frac{14-x}{100}\right)} = 0$$

$$\frac{3}{x} = \frac{4}{14-x} \Rightarrow x = 6 \text{ cm}$$

Case 2:

Let potential at B is zero



$$\frac{K\left(3\times10^{-6}\right)}{\left(\frac{x}{100}\right)} - \frac{K\left(4\times10^{-6}\right)}{\left(\frac{(14+x)}{100}\right)} = 0$$

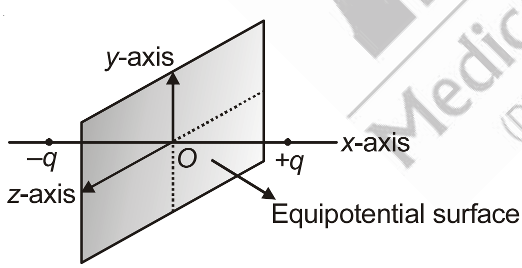
$$\frac{3}{x} = \frac{4}{14+x} \Rightarrow 42 + 3x = 4x$$

$$\Rightarrow\ x = 42 \text{ cm}$$

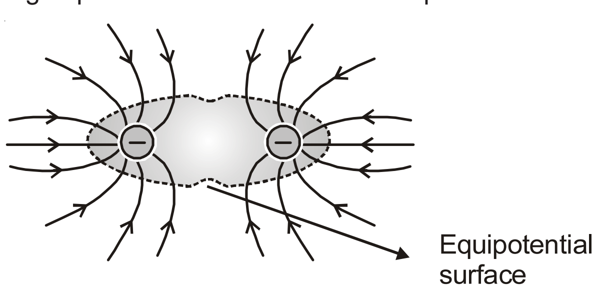
13. Draw the equipotential surface for (a) a dipole and (b) two identical negative charges placed at a certain distance apart.

Sol. Equipotential surface for

(a) a dipole

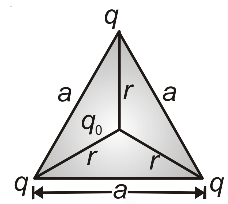


(b) Two identical negative charges placed at a certain distance apart.



14. Three identical charges (q) are arranged at the corners of an equilateral triangle of side a and a charge q_0 brought to the centre of the triangle, the three charges being held fixed at its corners. How much extra work is needed to this?

Sol. $W = 3\dfrac{Kqq_0}{r}$ $\qquad \left\{\because r = \dfrac{a}{\sqrt{3}}\right.$



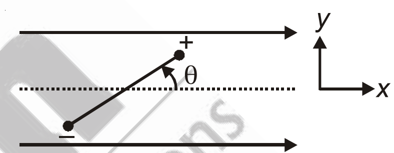
So, $W = \dfrac{3Kqq_0}{\left(a/\sqrt{3}\right)}$

$$\boxed{W = \dfrac{3\sqrt{3}\cdot q\cdot q_0}{4\pi\varepsilon_0\cdot a}}$$

15. Derive an expression of stored potential energy, when a dipole rotated from stable equilibrium position to any position in an external uniform electric field.

Sol. Potential energy can be associated with the orientation of an electric dipole in an electric field. Change in potential energy is related to the work done by electric field as

$$dU = -dW_E = -\vec{\tau}\,.\,\overrightarrow{d\theta}$$



(a) Potential energy is U

$\Rightarrow \; dU = -(-pE\sin\theta\hat{k})\,.\,(d\theta\,\hat{k}) \qquad [\because \; \overrightarrow{d\theta} = d\theta\hat{k}, \text{ anticlockwise}]$

$\Rightarrow \; dU = pE\sin\theta\, d\theta$

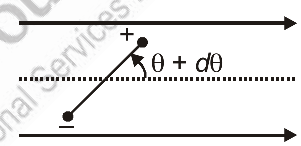
$\Rightarrow \; \int_{U_1}^{U_2} dU = \int_{\theta_1}^{\theta_2} pE\sin\theta d\theta \Rightarrow U_2 - U_1 = -pE[\cos\theta_2 - \cos\theta_1]$

Let $\theta = 90°$ is taken as reference position or zero of potential energy.

$\therefore \quad \theta_1 = 90° \Rightarrow U_1 = 0$

$\Rightarrow \quad U_2 - 0 = -pE[\cos\theta_2 - \cos 90°]$

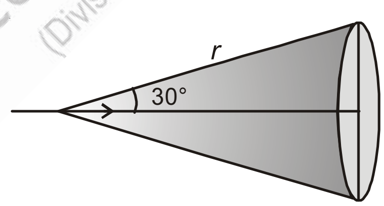
$\Rightarrow \quad U = -pE\cos\theta$



(b) Potential energy is $U + dU$

16. An electric dipole of dipole moment 10 μCm is placed along the axis of a right angled cone such that the vertex of cone is at the centre of dipole. If the semivertex angle of cone is 30° and slant height of curved surface is 20 cm, then find the electrostatic potential at any point of circumference of base of cone

Sol. $\boxed{V = \dfrac{K.P.\cos\theta}{r^2 - a^2\cos^2\theta}}$



$\theta = 30°$, $P = 10 \times 10^{-6}$ cm

$r = 20$ cm

For a short dipole $a \simeq 0$

$$V = \frac{9\times10^9\times10\times10^{-6}}{\left(\frac{20}{100}\right)^2\times100}\times\frac{\sqrt{3}}{2} = \frac{90\times10^3\times\sqrt{3}}{2\times20\times20}\times\frac{100}{100}\times100$$

$$\boxed{V = \frac{9\sqrt{3}}{8}\times10^4 \text{ volts}}$$

17. "Inside a conductor, electrostatic field is zero", why?

Sol. Due to equal and opposite induced electric field inside the conductor, the net electrostatic field inside conductor become zero.

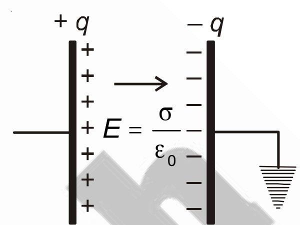
18. What is electrostatic shielding? Explain.

Sol. Electrostatic shielding/screening is the phenomenon of protecting a certain region of space from external electric field.

19. Derive an expression for capacitance of a parallel plate capacitor.

Sol. The arrangement consists of two thin conducting plates, each of area *A* and separated by *d* distance. When charge *q* is given to first plate, a charge – *q* is induced on the inner face of other plate and positive on the outer face of plate. As this face is connected to earth, a net negative charge is left on this plate. Thus, the arrangement is equivalent to two thin sheets of charge.

The electric field between the plates is



$E = \frac{\sigma}{\varepsilon_0}$ where σ is the charge density.

As $E = \frac{V}{d}$;

V is the potential difference between the plates.

$V = Ed = \frac{\sigma d}{\varepsilon_0}$

$\Rightarrow V = \frac{qd}{A\varepsilon_0}$ as $\sigma = \frac{q}{A}$

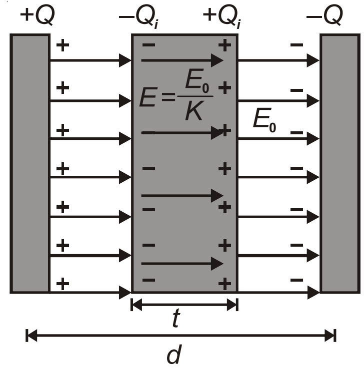
$\Rightarrow C = \frac{q}{V} = \frac{q}{\left(\frac{qd}{A\varepsilon_0}\right)}$, $C = \frac{\varepsilon_0 A}{d}$

20. When a dielectric medium (slab) is inserted between the plates of a capacitor, then how is the capacitance changed? Explain.

Sol. Capacitance becomes *k* times of original capacitance where "*k*" is dielectric constant of dielectric

Effect of Dielectric Slab (Inserted along the length of plates)

When a dielectric slab is placed, between the plates of capacitor its polarisation takes place. Thus a charge $-Q_i$ appears on its left face and $+Q_i$ appears on its right face, as shown in diagram.



$E_o = \frac{\sigma}{\varepsilon_0}$

$E = E_0 - E_p$

$E_p = \frac{\sigma_i}{\varepsilon_0}$

$\sigma_i = \frac{Q_i}{A}$

Potential difference between the plates can be calculated as,

$$V = E_0(d - t) + Et = E_0\left(d - t + \frac{t}{K}\right)$$

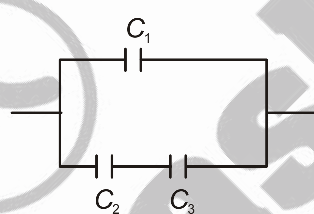
$$\therefore \quad V = \frac{\sigma}{\varepsilon_0}\left(d - t + \frac{t}{K}\right) = \frac{Q}{A\varepsilon_0}\left(d - t + \frac{t}{K}\right)$$

As, $C = \dfrac{Q}{V}$ $\Rightarrow$ $C = \dfrac{\varepsilon_0 A}{d - t + \dfrac{t}{K}}$

Note : *When t = d,* $C = \dfrac{K\varepsilon_0 A}{d}$

21. A slab of material of dielectric constant *k* has the area (1/4)th as that of the plates of a parallel plate capacitor and has thickness $\frac{3}{4}$ *d* where *d* is the separation of plates. How is the capacitance changed when the slab is inserted between the plates?

Sol. $C_1 = \dfrac{\left(\dfrac{3A}{4}\right)\varepsilon_0}{d}$



$$C_2 = \frac{\left(\dfrac{A}{4}\right)\varepsilon_0 \cdot K}{\dfrac{3d}{4}}, \quad C_3 = \frac{\left(\dfrac{A}{4}\right)\varepsilon_0}{\dfrac{d}{4}}$$

$$C_{23} = \frac{C_2 \cdot C_3}{C_2 + C_3} = \frac{\dfrac{A\varepsilon_0 \cdot K}{3d} \cdot \dfrac{A\varepsilon_0}{d}}{\dfrac{A\varepsilon_0 \cdot K}{3d} + \dfrac{A\varepsilon_0}{d}} = \frac{K \cdot A\varepsilon_0 \times 3}{3d(K+3)}$$

$$C_{23} = \frac{A\varepsilon_0}{d} \cdot \frac{K}{K+3}$$

$$C_{eq} = C_1 + C_{23}$$

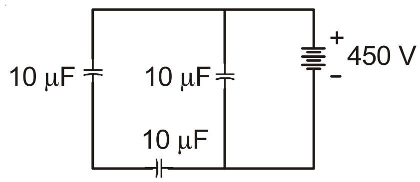
$$= \frac{3A\varepsilon_0}{4d} + \frac{A\varepsilon_0 \cdot K}{d(K+3)}$$

$$= \frac{A\varepsilon_0}{d}\left[\frac{3}{4} + \frac{K}{K+3}\right]$$

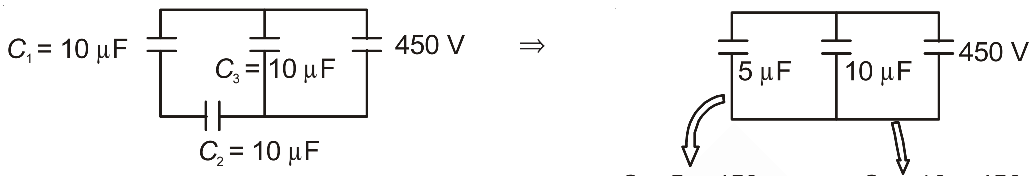
$$= \frac{A\varepsilon_0}{d}\left[\frac{3K + 9 + 4K}{4K + 12}\right]$$

$$\boxed{C_{eq} = \frac{A\varepsilon_0}{d}\left(\frac{7K+9}{4K+12}\right)}$$

22. A network of three 10 μF capacitors is connected to a 450 V supply, as shown in fig. Determine the charge on each capacitor.



Sol.



$Q = 5 \times 450$ $\quad$ $Q_3 = 10 \times 450$

$Q = 2250\ \mu C$ $\quad$ $Q_3 = 4500\ \mu C$

$Q_1 = Q_2 = 2250\ \mu C$

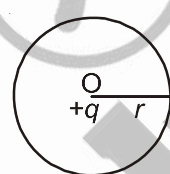
$Q_3 = 4500\ \mu C$

23. Prove that for an isolated spherical conductor the capacitance is directly proportional to the radius of conductor.

Sol. Capacity of an isolated spherical conductor

Consider a sphere with centre *O* and radius *r*, which is supplied with a charge = +*q*. This charge is distributed uniformly over the outer surface of the sphere. Thus, the potential at every point on the surface is same and is given by

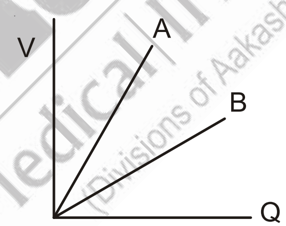
$$V = \frac{q}{4\pi\varepsilon_0 r}$$



As $C = \frac{q}{V}$

$C = 4\pi\varepsilon_0 r$

24. Graphs for two capacitors of capacitances C_1 and C_2 are shown in figure. The area of plates for both capacitors are same but separation between plates is double for C_1 to that of C_2. Which of the graph corresponds to C_1 and C_2 and why?



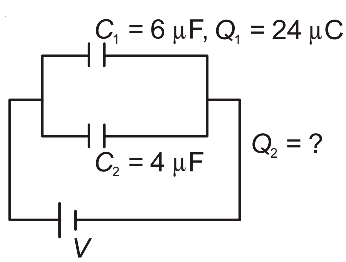
Sol. *A* for C_1 and *B* for C_2

25. Two capacitors of capacitance 6 μF and 4 μF are connected in parallel with a battery. The charge on 6μF capacitor is 24 μC, then find the charge on another capacitor and voltage of battery.

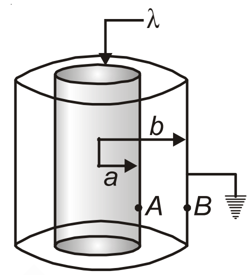
Sol. $\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

$\frac{24}{6} = \frac{Q_2}{4}$

$\boxed{Q_2 = 16\ \mu C}$



26. A long charged conducting cylinder of linear charge density λ is surrounded by a hollow co-axial conducting cylinder. The outer surface of hollow cylinder is earthed. Find the potential difference between cylinders, if radius of inner cylinder is a and outer hollow cylinder is *b*.



Sol. $\int_B^A dv = -\int_b^a \vec{E}\cdot\vec{dr}$

$$V_A - V_B = -\int_b^a \frac{\lambda}{2\pi\varepsilon_0 r}dr$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\left[\log_e r\right]_b^a$$

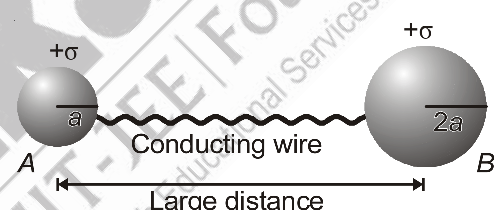
$$= \frac{-\lambda}{2\pi\varepsilon_0}\left(\log_e a - \log_e b\right)$$

$$\Rightarrow \quad V_A - V_B = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{b}{a}\right)$$

$\therefore$ Cylinder B is earthed so $V_B = 0$ then,

$$\boxed{V_A - 0 = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{b}{a}\right)}$$

27. Two conducting sphere of radius a and 2a having same charge density $+\sigma$ are placed far distance away. When they are connected by a conducting wire then due to charge re-distribution their potential change. Find the common potential of each sphere.



Sol. Now, $V_A = V_B$

$$\frac{K.q_A}{a} = \frac{K.q_B}{2a}$$

$$q_A = \frac{q_B}{2}$$

Also, $q_A + q_B = \left(\sigma 4\pi a^2 + \sigma 4\pi (2a)^2\right)$

$$\frac{q_B}{2} + q_B = \sigma 4\pi a^2 5$$

$$q_B = \frac{40}{3}\pi a^2 \cdot \sigma$$

$$\therefore \quad V = \frac{Kq}{r} = \frac{K\cdot\sigma\cdot 4\pi r^2}{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\sigma 4\pi r^2}{r}$$

$$V = \frac{\sigma r}{\varepsilon_0}$$

Common potential $V = \dfrac{K\cdot q_B}{2a} = \dfrac{1}{4\pi\varepsilon_0 \times 2a} \times \dfrac{40\pi a^2\cdot\sigma}{3}$

$$\boxed{V = \frac{5\sigma\cdot a}{3\varepsilon_0}}$$

28. Determine the electrostatic potential energy of a system consisting of two charges 5 μC and –3 μC placed in an external electric field $E = \dfrac{2\times10^5}{r^2}$ V/m at (-3 cm, 0, 0) and (3 cm, 0, 0) respectively, where r is distance from origin. What would the electrostatic energy of the configuration be?

Sol. Given electric field is due to a unit positive point charge q placed at origin.

$$\frac{K\cdot q}{r^2} = \frac{2\times10^5}{r^2} \Rightarrow \boxed{q = \frac{200}{9}\,\mu\text{C}}$$

$$\text{Energy} = \frac{K\left(\frac{200}{9}\times10^{-6}\right)\left(5\times10^{-6}\right)}{3\times10^{-2}} - \frac{K\left(\frac{200}{9}\times10^{-6}\right)\left(3\times10^{-6}\right)}{3\times10^{-2}} - \frac{K\left(5\times10^{-6}\right)\left(3\times10^{-6}\right)}{6\times10^{-2}}$$

$$= 9\times10^9\left(\frac{200}{9}\times\frac{5}{3} - \frac{200}{9} - \frac{5}{2}\right)\times\frac{10^{-12}}{10^{-2}}$$

$$= 9\left(\frac{200}{9}\times\left(\frac{5-3}{3}\right) - \frac{5}{2}\right)\frac{1}{10} = \frac{9}{10}\left(\frac{200}{9}\times\frac{2}{3} - \frac{5}{2}\right)$$

$$= \frac{40}{3} - \frac{9}{4} = \frac{160-27}{12} \Rightarrow \boxed{\text{Energy} = \frac{133}{12}\text{J}}$$

29. Define the polar and non-polar dielectric materials with examples.

Sol. In general the dielectric can be classified into Polar and Non-polar dielectrics. The molecules of a substance may be polar or non-polar. In a non-polar molecule, the centres of positive and negative charges coincide. The molecule thus has no permanent (or intrinsic) dipole moment. Examples of non-polar molecules are oxygen (O_2) and hydrogen (H_2) molecules which, because of their symmetry, have no dipole moment. On the other hand, a polar molecule is one in which the centres of positive and negative charges are separated (even when there is no external field). Such molecules have a permanent dipole moment. An ionic molecule such as HCl or a molecule of water (H_2O) are examples of polar molecules.

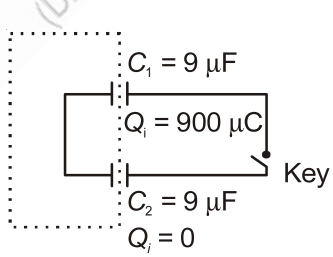
30. A 9 μF capacitor is charged by 100 V battery. The capacitor is disconnected from battery and connected with another identical uncharged capacitor. What is the electrostatic energy stored by the system?

Sol. For isolated system

Net initial charge = Net final charge

900 + 0 = 9 V + 9 V

V = 50 volts



$$U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2$$

$$= \frac{1}{2}\times18\,\mu\text{F}\times50\times50$$

$= 9 \times 2500 \times 10^{-6}$

$= 22400 \times 10^{-6}$ J

$= 2.2 \times 10^{-2}$ J

Long Answer Type Questions :

31. Discuss briefly the principle, construction and working of Van-de-Graaff electrostatic generator.

Sol. Van de Graaff Generator

Van de Graaff generator is a machine that can built up voltages in order of a few million volts. The resultant electric fields are used to accelerate charged particles (proton, electrons, ions) to high energies required for experiments to examine small scale structure of matter.

Principle : Let a small sphere of radius r having charge q is placed at the centre of a spherical conducting shell of radius R having charge Q.

Potential of the shell

$$V(R) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{R}\right) \quad \text{...(i)}$$

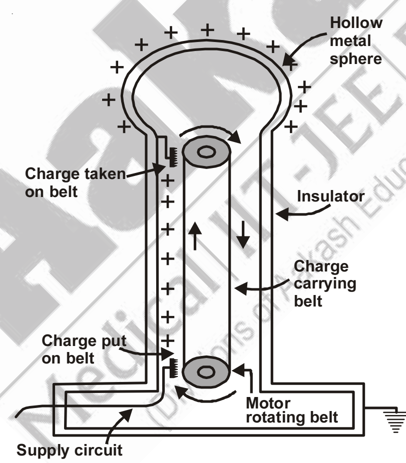
Potential of sphere at the centre of shell

$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right) \quad \text{...(ii)}$$

So, $V(r) - V(R) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$

If $q > 0$, $v(r) - V(R) > 0$, i.e. inner sphere is always at higher potential w.r.t. shell, irrespective of amount of charge Q.

So, when inner sphere and outer shell is connected with a wire, the charge q is transferred to the shell, as positive charge moves from higher potential to lower potential. We can keep increasing amount of charge on outer sphere. The potential of outer sphere keeps increasing until electric field reaches dielectric breakdown field of air (3×10^6 v/m). This is the principle of van de Graaff generator.



A large spherical conducting shell of few metre radius is supported on an insulating column. A long insulated belt of rubber of silk, is wound over two pulleys, one at ground level and other at the centre of the shell. This belt continuously carries positive charge, sprayed on it by a metallic brush at ground level, to the top. At the top its positive charge is transferred to the spherical shell, where it is distributed uniformly. A potential deference of 6×10^6 volt to 8×10^6 volt w.r.t. ground can be achieved.

32. (i) When a capacitor of capacitance C charged upto a potential V, then derive an expression for energy stored in the capacitor.

(ii) If electric field between plates of capacitor is E then derive the expression of energy density.

Sol. (i) Energy stored in a capacitor during the charging of a capacitor, work has to be done to add charge to the capacitor against its potential. This work is stored in the capacitor as electrical energy.

Suppose during the charging of capacitor its potential at any instant is given by

$V = \dfrac{q}{C}$ small amount of work done in adding a charge dq is given by

$dW = \dfrac{q}{C}\, dq$

Total work done in giving a charge Q to the condenser is

$$W = \int_0^Q \frac{q}{C} dq$$

$$\therefore\ W = \left[\frac{q^2}{2C}\right]_0^Q$$

$$\therefore\ W = \frac{Q^2}{2C}$$

$$\therefore\ U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

(ii) Energy density in parallel plate capacitor

The volume of a parallel plate capacitor is Ad

$$\therefore \text{ Energy density } u = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad} \text{ where } C = \varepsilon_0 \frac{A}{d},\ V = Ed$$

$$\therefore\ u = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)\left(\frac{E^2 d^2}{Ad}\right) = \frac{1}{2}\varepsilon_0 E^2$$

33. Derive an expression for electrostatic potential due to an electric dipole at any point $p(r,\theta)$.

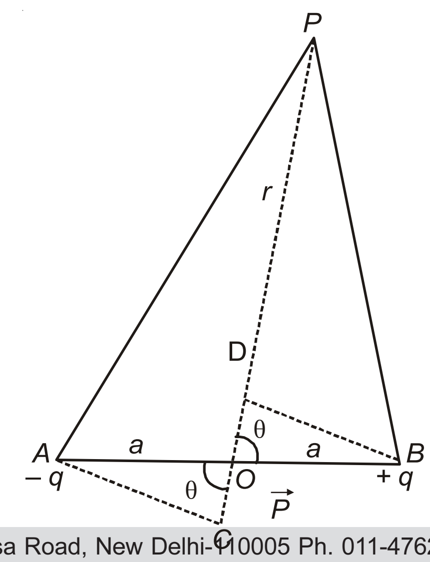
Sol. Potential due to an electric dipole

In the previous chapter on electric charges and field, we have already calculated the electric field due to an electric dipole and seen that for an ideal (short) dipole, the electric field varies inversely as r^3, we now determine the potential due to an electric dipole.

AB be an electric dipole of length $2a$ and let P be any point where $OP = r$.

Let θ be the angle between r and the dipole axis.

$AB = 2a$, $AO = OB = a$, $OP = r$



In Δ OAC, $\cos\theta = \dfrac{OC}{OA} = \dfrac{OC}{a}$

$\therefore\ OC = a\cos\theta$

Also $OD = a\cos\theta$

If $r >> a$, $PA \approx PC = OP + OC = r + a\cos\theta$

$PB \approx PD = OP - OD = r - a\cos\theta$

V is the potential due to electric dipole,

$$V = \left(\frac{1}{4\pi\varepsilon_0}\right)\left[\frac{q}{PB} - \frac{q}{PA}\right]$$

$$V = \left(\frac{1}{4\pi\varepsilon_0}\right) q \left[\frac{1}{(r - a\cos\theta)} - \frac{1}{(r + a\cos\theta)}\right]$$

$$V = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2a\,q\cos\theta}{\left(r^2 - a^2\cos^2\theta\right)} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{p\cos\theta}{\left(r^2 - a^2\cos^2\theta\right)}$$

where p is dipole moment.

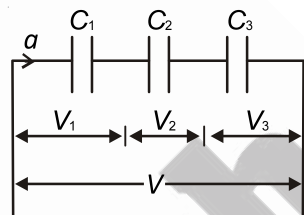
34. Three capacitors of capacitance C_1, C_2 and C_3 are connected (i) in series and (ii) in parallel. Find the energy stored in each combination when they are connected to the same battery of emf V one by one.

Sol. Capacitors in series

Consider three capacitors connected in series. Consider the shown arrangement of three capacitors in series. Let a V potential is applied to the circuit and $+Q$ be the charge, which will became on all the capacitors. Let V_1, V_2, V_3 be the individual potentials of the capacitors and their respective capacitance is C_1, C_2, C_3.

Therefore, $Q = C_1 V_1 = C_2 V_2 = C_3 V_3$

But $V = V_1 + V_2 + V_3 = \frac{Q}{C}$



where C is the effective capacitance .

$$\therefore\ \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

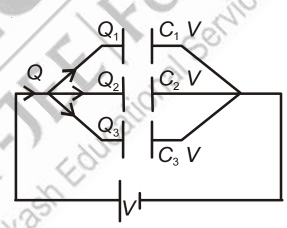
$$\therefore\ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Capacitors in parallel

Consider three capacitors connected in parallel. A voltage V is applied across the combination. Q_1, Q_2, Q_3 be the charge on each capacitor and C_1, C_2, C_3 be their respective capacitance.

$Q_1 = C_1 V$, $Q_2 = C_2 V$, $Q_3 = C_3 V$

As $Q = CV$



where C is the net capacitance of the combination,

$\therefore\ Q = Q_1 + Q_2 + Q_3$

$\therefore\ CV = C_1 V + C_2 V + C_3 V$

$\therefore\ C = C_1 + C_2 + C_3$

Energy stored :-

In series combination

$$U = \frac{1}{2}\left(\frac{Q^2}{C_1} + \frac{Q^2}{C_2} + \frac{Q^2}{C_3}\right)$$

$$\boxed{U = \frac{Q^2}{2}\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)}$$

In parallel combination

$$U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \frac{1}{2}C_3V^2$$

$$\boxed{U = \frac{1}{2}(C_1 + C_2 + C_3)V^2}$$

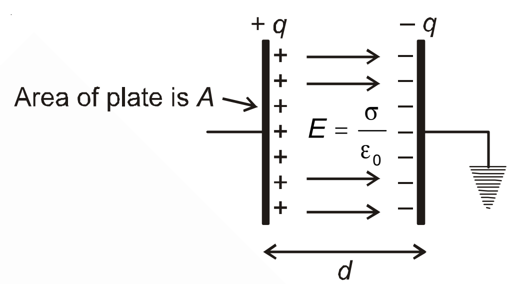
35. What is a parallel plate capacitor? Explain the principle of a capacitor. Derive an expression for the capacitance of a parallel plate capacitor.

Sol. Parallel plate capacitor

The arrangement consists of two thin conducting plates, each of area *A* and separated by *d* distance. When charge *q* is given to first plate, a charge – *q* is induced on the inner face of other plate and positive on the outer face of plate. As this face is connected to earth, a net negative charge is left on this plate. Thus, the arrangement is equivalent to two thin sheets of charge.

The electric field between the plates is

$E = \frac{\sigma}{\varepsilon_0}$ where σ is the charge density.



As $E = \frac{V}{d}$;

V is the potential difference between the plates.

$$V = Ed = \frac{\sigma d}{\varepsilon_0}$$

$$\Rightarrow V = \frac{qd}{A\varepsilon_0} \text{ as } \sigma = \frac{q}{A}$$

$$\Rightarrow C = \frac{q}{V} = \frac{q}{\left(\frac{qd}{A\varepsilon_0}\right)} \qquad C = \frac{\varepsilon_0 A}{d}$$

36. (i) What is the dimensional formula of capacitance?

(ii) Why does the electric field inside a dielectric decrease when it is placed in an external electric field?

(iii) A parallel plate capacitor has a capacitance of 3 μF in air and 24 μF when dielectric medium is introduced. What is dielectric constant of the medium?

Sol. (i) $[M^{-1} L^{-2} A^2 T^4]$

(ii) Due to induced electric field in opposite directions

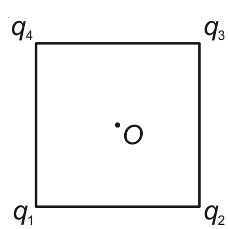
(iii) $C_0 = 3\ \mu F$

$C = 24\ \mu F = C_0 K$

$$\Rightarrow K = \frac{24}{3}$$

$$\Rightarrow K = 8$$

37. (i) In the given figure, there are four point charges placed at the vertex of a square of side 7 cm. If q_1 = +18 μC, q_2 = –24 μC, q_3 = +35 μC and q_4 = +16 μC, then find the electric potential at the centre *O* of the square, assume the potential to be zero at infinity.



(ii) An electric field $\vec{E} = (2\hat{i} + 3\hat{j})$ N/C exists in the space. If potential at the origin is taken to be 10 volt, then find the potential at (2 m, 1 m).

Sol. (i) $V_0 = \frac{Kq_1}{r} + \frac{Kq_2}{r} + \frac{Kq_3}{r} + \frac{Kq_4}{r}$

$$= \frac{1}{4\pi\varepsilon_0 r}[+18 - 24 + 35 + 16] \times 10^{-6}$$

$$= \frac{9 \times 10^9}{\left(7/\sqrt{2}\right) \times 10^{-2}} \times 45 \times 10^{-6} \text{ volt}$$

$$\Rightarrow V_0 = \frac{9 \times 45 \times 10^5}{\left(7/\sqrt{2}\right)} \text{ volt}$$

(ii) $\vec{E} = 2\hat{i} + 3\hat{j}$

$$\int dV = \int -\vec{E} \cdot \vec{dr}$$

$$\int_{10}^{V} dV = -\int_{(0,0)}^{(2,1)} \left(2\hat{i} + 3\hat{j}\right) \cdot \left(dx\,\hat{i} + dy\,\hat{j}\right)$$

$$V - 10 = -\int_{(0,0)}^{(2,1)} 2dx + 3dy$$

$$V - 10 = -\left(2x + 3y\right)_{(0,0)}^{(2,1)}$$

$V = 10 - (2 \times 2 + 3 \times 1) = 10 - 7$

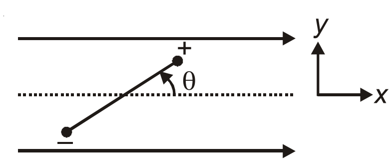
$V = 3$ volt

38. (i) Derive an expression for potential energy of an electric dipole in an uniform electric field.

(ii) Three identical charges q are at the vertices of an equilateral triangle of side L. How much work is done in bringing them closer to an equilateral triangle of side $\frac{L}{2}$?

Sol. (i) Potential Energy of a dipole in an electric field

Potential energy can be associated with the orientation of an electric dipole in an electric field. Change in potential energy is related to the work done by electric field as

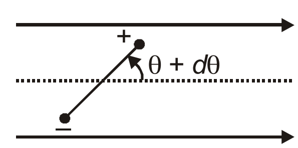


(a) Potential energy is U

$$dU = -dW_E = -\vec{\tau} \cdot \vec{d\theta}$$

$\Rightarrow \quad dU = -(-pE\sin\theta\hat{k}) \cdot (d\theta\,\hat{k}) \quad [\because \ \vec{d\theta} = d\theta\hat{k}, \text{ anticlockwise}]$

$\Rightarrow \quad dU = pE \sin\theta \, d\theta$



(b) Potential energy is $U + dU$

$$\Rightarrow \quad \int_{U_1}^{U_2} dU = \int_{\theta_1}^{\theta_2} pE\sin\theta d\theta \Rightarrow U_2 - U_1 = -pE[\cos\theta_2 - \cos\theta_1]$$

Let $\theta = 90°$ is taken as reference position or zero of potential energy.

$\therefore \quad \theta_1 = 90° \Rightarrow U_1 = 0$

$\Rightarrow \quad U_2 - 0 = -pE[\cos\theta_2 - \cos 90°]$

$\Rightarrow \quad U = -pE\cos\theta$

(ii) $W_{ext} = U_f - U_i$

$$W = \frac{3q^2}{4\pi\varepsilon_0}\left(\frac{1}{\left(L/2\right)} - \frac{1}{L}\right)$$

$$= \frac{3q^2}{4\pi\varepsilon_0 L}$$

39. (i) "At the surface of a charged conductor electrostatics field must be normal to the surface at every point." Explain it.

(ii) An electrical technician requires a capacitance of 4 μF in a circuit across a potential difference of 500 V. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 150 V. Suggest a possible arrangement that requires the minimum number of capacitors.

Sol. (i) A charge conductor behaves as an equipotential surface. The potential at each point on the surface of it is same. So, work done to move a unit positive over the surface from one point to another point is zero.

As, $W = q\,\Delta V = 0$

Also, $W = 0 = q\int \vec{E}\cdot\vec{ds}$

Hence, $\vec{E} \perp \vec{ds}$

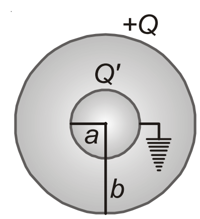
So, elelctric field lines must be normal to the surface at every point.

(ii) The circuit consists of 16 rows with each row containing 4 capacitors each of value 1 μF. This circuit is connected to a 500 volts battery.

40. A spherical capacitor consists of two concentric spherical conductors of radius a and b respectively ($b > a$). Then inner sphere is earthed and outer sphere is given a positive charge of Q, then find the capacitance of the system.

Sol. $V_{inner} = 0$ (as grounded)

$$\frac{KQ'}{a} + \frac{KQ}{b} = 0$$



$$\boxed{Q' = -\frac{Qa}{b}}$$

$$\Delta V = \frac{K}{b}\left(-Q\frac{a}{b}\right) + \frac{KQ}{b} = \frac{KQ}{b}\left(\frac{b-a}{b}\right)$$

$$C = \frac{Q}{\Delta V} \Rightarrow C = \frac{Qb^2}{KQ(b-a)} \Rightarrow \boxed{C = \frac{4\pi\varepsilon_0 \cdot b^2}{b-a}}$$

41. (i) Derive the relation between electric field intensity and electrostatic potential.

(ii) In a parallel plate capacitor with a dielectric medium of dielectric constant 8, each plate has an area of $2\times10^{-3}\,m^2$ and distance between the plates is 2 mm. Calculate the capacitance of capacitor, if this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Sol. (i) Relation between Electric Field and Potential

As we know that $\Delta V = \frac{-W_E}{q_0}$

$\Rightarrow \quad V_f - V_i = \frac{-W_E}{q_0}$

Now, $W_E = \int \vec{F}\cdot\overrightarrow{dr}$, where $\vec{F}$ is the force due to electric field.

$\Rightarrow \quad V_f - V_i = \frac{-\int \vec{F}\cdot\overrightarrow{dr}}{q_0}$

or, $\quad V_f - V_i = -\int_{r_i}^{r_f} \vec{E}\cdot\overrightarrow{dr}$ $\quad \left[\text{If } V_\infty = 0, \text{ then } V = -\int_\infty^r E dr\right]$

The above result can also be expressed in differential form as, $dV = -\vec{E}\cdot\overrightarrow{dr}\left[\text{as } dV = \frac{-dW_E}{q_0}\right]$

The negative sign in the expression $dV = -\vec{E}\cdot\overrightarrow{dr}$ signifies that as one moves in the direction of electric field, potential decreases.

In Cartesian form, $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

$$\overrightarrow{dr} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$\Rightarrow \quad dV = -E_x dx - E_y dy - E_z dz$

$\Rightarrow \quad E_x = \frac{-\partial V}{\partial x}; E_y = \frac{-\partial V}{\partial y}; E_z = \frac{-\partial V}{\partial z}$

(ii) $K = 8$, $A = 2\times10^{-3}\ m^2$, $d = 2\times10^{-3}$ mm

$V = 100$ volts

$C = \frac{A\varepsilon_0 K}{d}$

$= \frac{2\times10^{-3}\times8.85\times10^{-12}\times8}{2\times10^{-3}}$

$= 70.8\times10^{-12}$ F

$Q = CV = 70.8\times10^{-12}\times100$

$Q = 70.8\times10^{-8}$ C.

42. (i) What is capacitance?

(ii) Two charges q_1 = +4 μC and q_2 = –4 μC are placed at a distance 90 cm. Estimate the potential energy of system in eV taking the potential energy as zero when charges q_1 and q_2 are at infinite separation.

Sol. (i) Any conducting object that carries a charge is characterized by an electric potential that is constant everywhere on and within that object. If two such conductors have a potential difference between them then, as any potential difference is able to accelerate charges, the system effectively stores energy. Such a device that can maintain a potential difference, storing energy by storing charge is called capacitor.

(ii) $U = \frac{K \cdot q_1 q_2}{r} = \frac{9\times10^9 \times 4\times10^{-6} \times -4\times10^{-6}}{90} \times 100$

$U = 160 \times 10^{-2}$ J = 1.6 J

$\because$ 1 eV = 1.6×10^{-19} J

so, $U = 10^{19}$ eV

43. Derive an expression for force on each plate of parallel plate capacitor if magnitude of charge on each plate is *Q*, Area of each plate is *A* and separation between plates is *d*.

Sol. $F = qE$

$F = q\frac{\sigma}{2\varepsilon_0}$

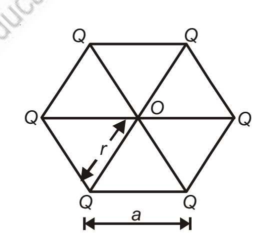
$F = \frac{1}{2}\frac{\sigma q}{\varepsilon_0} \Rightarrow F = \frac{q^2}{2A\varepsilon_0}$

44. (i) A regular hexagon of side a has a charge *Q* at each of its vertices. Determine the electrostatic potential due to this charge array at the centre of Hexagon.

(ii) The plates of a parallel plate capacitor has an area of 30 cm² each and separation 2 mm is charged by a battery of emf 200 V. Calculate the electrostatic energy stored in the capacitor.

Sol. (i) $V_0 = \frac{KQ}{r} \times 6$

$\boxed{V_0 = \frac{6KQ}{a}}$

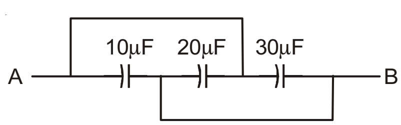


(ii) $U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{A\varepsilon_0}{d}V^2 = \frac{30}{10000} \times \frac{8.85\times10^{-12}}{2\times2\times10^{-3}} \times 200 \times 200$

$U = 26.55 \times 10^{-9}$ J = 2.7×10^{-8} J

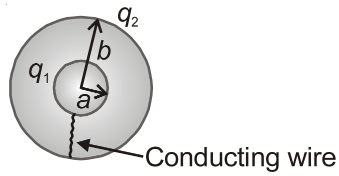
45. (i) Two concentric spherical conductors of radius *a* and *b* (*b* > *a*) inner sphere has q_1 charge and outer sphere has q_2. When they are connected by a conducting wire then prove that charge on inner sphere must be zero.

(ii) Find the equivalent capacitance between *A* and *B.*

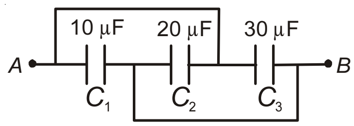


(iii) If the given combination is connected with a battery of emf 200 volt then find the charge on each capacitor.

Sol. (i) Charge on inner sphere will become zero, as all the charges flow to outer sphere through conducting wire.



(ii) C_{eq} = 60 μF



Here C_1, C_2,C_3 are in parallel.

(iii) $Q = CV$

$Q_1 = 10 \times 200 = 2000\ \mu C$

$Q_2 = 20 \times 200 = 4000\ \mu C$

$Q_3 = 30 \times 200 = 6000\ \mu C$

SECTION - B

Model Test Paper

1. Define 1 farad.

Sol. Capacitance of a capacitor is defined to be as 1 farad if amount of charge required to raise the potential of a conductor by one volt, is unity.

2. State the principle of a capacitor.

Sol. Capacitor works on the principle of storing large amounts of electric charge and hence electrical energy in a small space.

3. What is electrostatic shielding?

Sol. Electrostatic shelding/screening is the phenomenon of protecting a certain region of space from external electric field.

4. Write the ratio of electric potential at a distance *r* on equatorial position to the electric potential at a distance *r* on axial position of an electric dipole.

Sol. Zero

5. Calculate the radius of a spherical conductor of capacitance 1 pF.

Sol. For a isolated spherical conductor

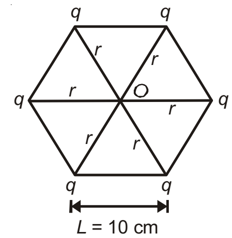
$$C = 4\pi\varepsilon_0 R$$

$$R = \frac{C}{4\pi\varepsilon_0}$$

$$R = 9 \times 10^9 \times 10^{-12}$$

$$\Rightarrow \quad R = 9 \times 10^{-3}\ \text{m}$$

6. A regular hexagon of side 10 cm has a charge of 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.



Sol. $q = 5\ \mu C$

$$V_0 = \frac{Kq}{r} \times 6$$

$$= \frac{9 \times 10^9 \times 5 \times 10^{-6} \times 6 \times 100}{10} = 2.7 \times 10^6 \text{ volts}$$

7. Derive the expression for energy stored in a capacitor

Sol. During the charging of a capacitor, work has to be done to add charge to the capacitor against its potential. This work is stored in the capacitor as electrical energy.

Suppose during the charging of capacitor its potential at any instant is given by

$V = \frac{q}{C}$ small amount of work done in adding a charge dq is given by

$$dW = \frac{q}{C}\ dq$$

Total work done in giving a charge Q to the condenser is

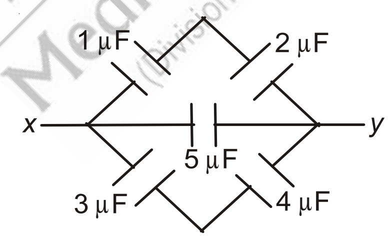
$$W = \int_0^Q \frac{q}{C} dq$$

$$\therefore\ W = \left[\frac{q^2}{2C}\right]_0^Q$$

$$\therefore\ W = \frac{Q^2}{2C}$$

$$\therefore\ U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

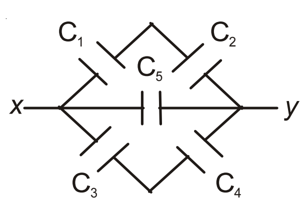
8. Five capacitors are connected as shown in the diagram. Calculate equivalent capacitance between the points x and y.



Sol. $C_{eq} = \frac{2}{3} + 5 + \frac{12}{7}$

$$= \frac{14 + 105 + 36}{21}$$

$$\boxed{C_{eq} = \frac{155}{21} \mu F}$$



9. Keeping the voltage of the charging source constant, what would be the percentage change in the energy stored in a parallel plate capacitor if the separation between its plates were to be decreased by 10% ?

Sol. V = constant.

% change in energy = $\left|\dfrac{E_f - E_i}{E_i}\right| \times 100\,\%$

$$= \left|\dfrac{\frac{1}{2}C'V^2 - \frac{1}{2}CV^2}{\frac{1}{2}CV^2}\right| \times 100\%$$

$$= \left|\dfrac{C'}{C} - 1\right| \times 100\%$$

$$= \left(\dfrac{10}{9} - 1\right) \times 100\%$$

$$= \dfrac{100}{9}\%$$

10. At the surface of a charged conductor, electrostatic field must be normal to the surface at every point. Explain.

Sol. The surface of a charged conductor acts as a equipotential surface. So, work done in moving a charge from one point to another point over such a surface is zero, as potential on each point of such a surface is same

$W = q\ \Delta V = 0$

Also, $W = \int \vec{F} \cdot \vec{ds} = 0$

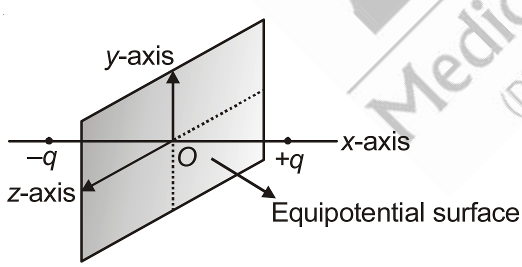
So, it suggest electric field lines must be perpendicular to surface at every point.

11. Draw equipotential surfaces for

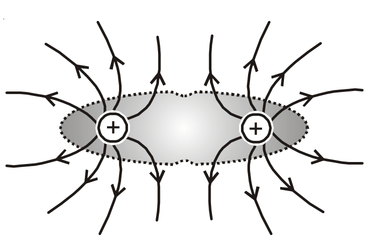
(i) an electric dipole

(ii) two identical positive charges

Sol. (i) Equipotential surface for a dipole



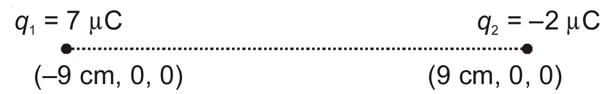
(ii) Two identical positive charge



Equipotential Surface

12. Determine the electrostatic potential energy of the system consisting of two charges 7 μC and –2 μC placed at (– 9 cm, 0, 0) and (9 cm, 0, 0). Also find the work required to separate the two charges infinitely away from each other.

Sol. Electrostatic potential energy = U



$$U = \frac{K.q_1q_2}{r}$$

$$U = \frac{9\times10^9\times7\times10^{-6}\times-2\times10^{-6}}{18}\times100$$

$$\boxed{U = 0.7\,\text{J}}$$

$$\boxed{W = 0.7\,\text{J}}$$

13. A parallel plate capacitor with air between the plates has a capacitance of 8 pF. What will be the capacitance if the distance between the plates is reduced by half and the space between them is filled with a substance of dielectric constant K = 6 ?

Sol. C = 8 pF, distance = d

C' = ? , distance = $\frac{d}{2}$, K = 6

$$C = 8\text{ pF} = \frac{A\varepsilon_0}{d}$$

$$C' = K.A.\frac{\varepsilon_0}{d/2}$$

$$\frac{C'}{C} = K.\frac{A\varepsilon_0 2\cdot d}{d\cdot A\varepsilon_0}$$

$C' = C \times 2K$

$= 8\text{ pF} \times 2 \times 6$

$$\boxed{C' = 96\,\text{pF}}$$

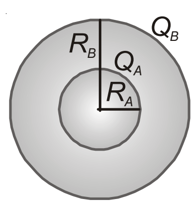
14. A and B are two concentric hollow metallic shells of radius R_A and R_B. A is given a charge Q_A while B is given a charge $Q_{B.}$ Find the electric potential at a distance R from the centre such that

(a) $R < R_{A,}$

(b) $R_A < R < R_B$

(c) $R > R_B$

Sol. (a) $V = K\left(\frac{Q_A}{R_A}+\frac{Q_B}{R_B}\right)$



(b) $V = K\left(\frac{Q_A}{R}+\frac{Q_B}{R_B}\right)$

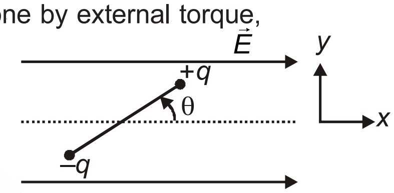
(c) $V = K\left(\frac{Q_A+Q_B}{R}\right)$

15. Derive an expression for electric potential energy of an electric dipole in an external electric field.

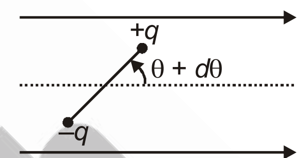
Sol. Consider a dipole placed in a uniform electric field $\vec{E}$ as shown in figure (a). We know, in a uniform electric field, torque experienced by dipole

$$\vec{\tau} = \vec{p} \times \vec{E}$$

which will tend to rotate it, if $\vec{p}$ is not parallel or antiparallel to $\vec{E}$. Now if an external torque τ_{ext} is applied in such a way that it rotates the dipole in the plane of paper from angle θ_1 to angle θ_2 at an infinitesimal angular speed without angular acceleration. The amount of work done by external torque,



(a) Potential energy is $U(\theta)$



(b) Potential energy is $U(\theta + d\theta)$

$$W = \int_{\theta_1}^{\theta_2} \tau_{ext} d\theta = \int_{\theta_1}^{\theta_2} pE \sin\theta d\theta$$

$$= pE (\cos\theta_1 - \cos\theta_2)$$

This work done by external torque is stored as potential energy of the system.

$$U(\theta_2) - U(\theta_1) = pE (\cos\theta_1 - \cos\theta_2)$$

Let potential energy is taken zero at $\theta = 90°$

$(\theta_1 = 90°$ and $U_1 = 0)$

$\Rightarrow \quad U_2 - 0 = -pE[\cos\theta_2 - \cos 90°]$

$\Rightarrow \quad U = -pE\cos\theta$

$$\boxed{U = -\vec{p}\cdot\vec{E}}$$

16. Explain the difference in behaviour of a conductor and a dielectric in an external electric field.

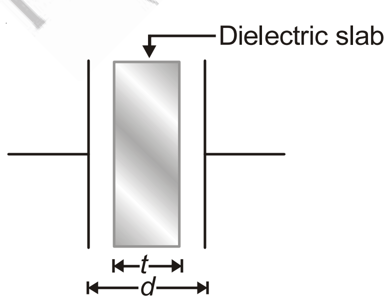
Sol. When a conductor is placed in an external electric field, then net electric field inside conductor becomes zero due to exactly equal and opposite induced fields.

But when a dielectric is placed in an external electric field then net electric field inside dielectric get reduced due to induced opposite electric field. Net electric field inside dielectric is given by

$$E_{net} = \frac{\text{Electric field in air}}{\text{Dielectric constant}}$$

17. A slab of material of dielectric constant K having same area as the plates of a parallel plate capacitor but has a thickness 't' ($t < d$), where d is the separation of the plates. What is the capacitance when the slab is inserted between the plates?

Sol. $V = \dfrac{\sigma}{\varepsilon_0}\left[(d-t) + \dfrac{t}{K}\right]$



$$C = \frac{Q}{V} = \frac{\sigma A}{V}$$

$$C = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0}\left[(d-t) + \frac{t}{K}\right]}$$

$$\boxed{C = \frac{A \cdot \varepsilon_0}{\left[(d-t) + \frac{t}{K}\right]}}$$

18. When two capacitors of capacitance C_1 and C_2 are connected in series, the net capacitance is 3 μF. When connected in parallel, the effective capacitance is 16 μF. Calculate the values of C_1 and C_2

Sol. C_{eq} = 3 μF (in series)

$$\frac{C_1 \cdot C_2}{C_1 + C_2} = 3\ \mu\text{F} \quad \text{...(i)}$$

$$C_1 + C_2 = 16\ \mu\text{F} \quad \text{....(ii)}$$

from equation (i) and (ii), we get

C_1 = 12 μF and C_2 = 4 μF   or C_1 = 4 μF and C_2 = 12 μF

19. Give proper explanation of the statement "Electric field is in the direction in which the potential decreases steepest".

Sol. $E_r = -\frac{dV}{dr}$, E_r is the component of this electric field in direction of '*dr*'. A rectangular component of a vector is always less than the magnitude of the vector itself. Thus, the direction in which E_r is maximum is the actual direction of the electric field. Maximum E_r implies that $-\frac{dV}{dr}$ is maximum or potential decreases at the maximum rate.

20. With the help of a diagram, explain the principle of working of a Van De Graaff generator.

Sol. Van de Graaff Generator

Van de Graaff generator is a machine that can built up voltages in order of a few million volts. The resultant electric fields are used to accelerate charged particles (proton, electrons, ions) to high energies required for experiments to examine small scale structure of matter.

Principle : Let a small sphere of radius *r* having charge *q* is placed at the centre of a spherical conducting shell of radius *R* having charge *Q*.

Potential of the shell

$$V(R) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{R}\right) \quad \text{...(i)}$$

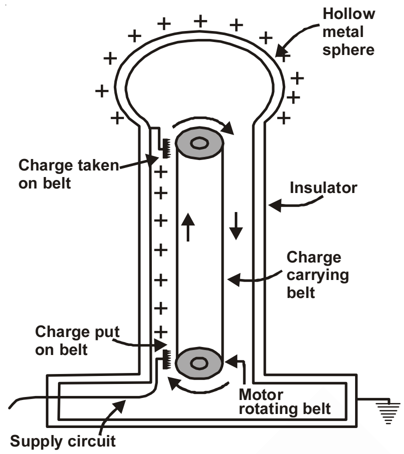
Potential of sphere at the centre of shell

$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right) \quad \text{...(ii)}$$

So, $V(r) - V(R) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$

If $q > 0$, $v(r) - V(R) > 0$, i.e. inner sphere is always at higher potential w.r.t. shell, irrespective of amount of charge *Q*.

So, when inner sphere and outer shell is connected with a wire, the charge *q* is transferred to the shell, as positive charge moves from higher potential to lower potential. We can keep increasing amount of charge on outer sphere. The potential of outer sphere keeps increasing until electric field reaches dielectric breakdown field of air (3×10^6 v/m). This is the principle of van de Graaff generator.



A large spherical conducting shell of few metre radius is supported on an insulating column. A long insulated belt of rubber of silk, is wound over two pulleys, one at ground level and other at the centre of the shell. This belt continuously carries positive charge, sprayed on it by a metallic brush at ground level, to the top. At the top its positive charge is transferred to the spherical shell, where it is distributed uniformly. A potential deference of 6×10^6 volt to 8×10^6 volt w.r.t. ground can be achieved.

21. A parallel plate capacitor of plate area *A* and separation *d* is charged to a potential *V*. The battery is then disconnected and a dielectric slab of thickness *d* and dielectric constant *K* is inserted in the capacitor. What change, if any, will take place in

(a) Charge on the plates

(b) Electric field between the plates

(c) Energy stored

(d) Voltage across the capacitor

(e) Capacitance of the capacitor

Sol. (a) Charge remain same

(b) Electric field between the plates becomes $\frac{1}{K}$ times of original electric field

(c) Energy stored will becomes $\frac{1}{K}$ times of original energy stored

(d) Voltage across capacitor becomes $\frac{1}{K}$ times of original voltage (As, $V = Ed$)

(e) Capacitance of the capacitor becomes '*K*' time the original capacitance.

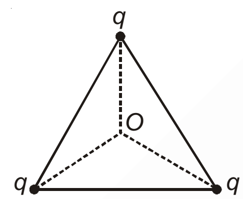
❑ ❑ ❑

Solutions (Set-2)

Objective Types Questions

(Electric Potential due to a Point Charge, Electric Dipole, System of Charges and Charged Bodies)

1. Three isolated equal charges are placed at the three corners of an equilateral triangle as shown in figure. The statement which is true for net electric potential V and net electric field intensity E at the centre of the triangle is

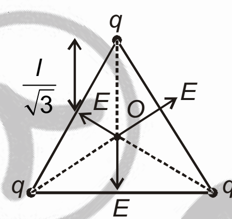


(1) $E = 0,\ V = 0$
(2) $V = 0,\ E \neq 0$
(3) $V \neq 0,\ E = 0$
(4) $V \neq 0,\ E \neq 0$

Sol. Answer (3)

$E_{net} = 0$

$$V_{net} = 3\left(\frac{kq\sqrt{3}}{l}\right) = 3\sqrt{3}\frac{kq}{l}$$



2. The potential at a point 0.1 m from an isolated point charge is + 100 volt. The nature of the point charge is

(1) Positive
(2) Negative
(3) Zero
(4) Either positive or zero

Sol. Answer (1)

As potential is +ve, so point charge is also +ve.

3. A charge of 10μC is placed at the origin of x-y coordinate system. The potential difference between two points (0, a) and (a, 0) in volt will be

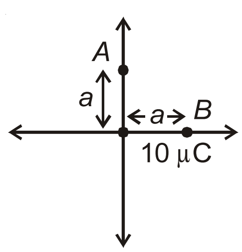
(1) $\dfrac{9\times10^4}{a}$
(2) $\dfrac{9\times10^4}{a\sqrt{2}}$
(3) $\dfrac{9\times10^4}{2a}$
(4) Zero

Sol. Answer (4)

$$V_A = \frac{kq}{a}$$

$$V_B = \frac{kq}{a}$$

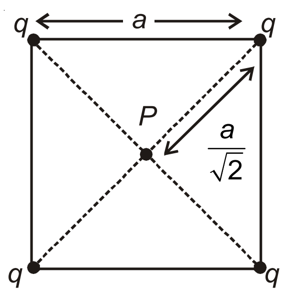
$\Delta V = V_A - V_B = 0$



4. Four charges of same magnitude q are placed at four corners of a square of side a. The value of electric potential at the centre of the square will be (Where $k = \frac{1}{4\pi\varepsilon_0}$)

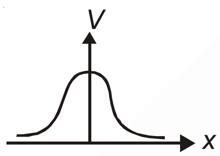
(1) $\frac{4kq}{a}$ (2) $4\sqrt{2}\frac{kq}{a}$ (3) $\frac{4kq}{\sqrt{2}a}$ (4) $\frac{kq}{a\sqrt{2}}$

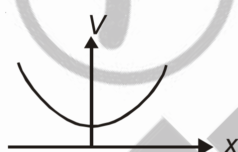
Sol. Answer (2)

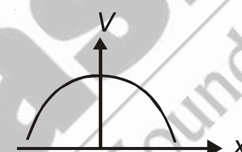


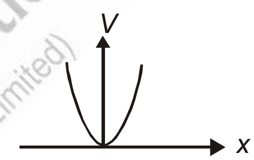
$$V_P = 4\left(\frac{kq\sqrt{2}}{a}\right) = 4\sqrt{2}\frac{kq}{a}$$

5. Two identical positive charges are placed on the y-axis at $y = -a$ and $y = +a$. The variation of V (electric potential) along x-axis is shown by graph

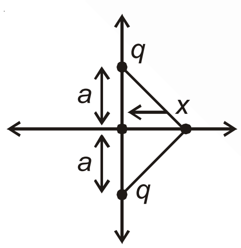
(1) 

(2) 

(3) 

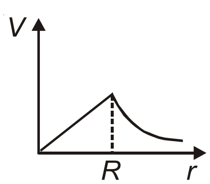
(4) 

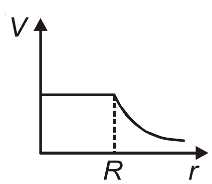
Sol. Answer (1)

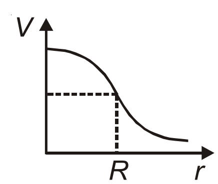


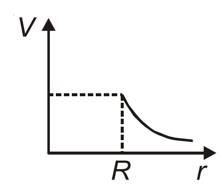
$$V = \frac{2kq}{\sqrt{a^2 + x^2}}$$

6. Which graph best represents the variation of electric potential as a function of distance from the centre of a uniformly charged solid sphere of charge of radius R?

(1) 

(2) 

(3) 

(4) 

Sol. Answer (3)

$$\left[V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2R^3}\left(3R^2 - r^2\right)\right]$$

7. When a test charge is brought in from infinity along the perpendicular bisector of an electric dipole, the work done is

(1) Positive (2) Zero (3) Negative (4) None of these

Sol. Answer (2)

$W = q(V_f - V_i) = q(0 - 0) = 0$

8. The electric potential at a distance of 3 m on the axis of a short dipole of dipole moment 4×10^{-12} coulomb-meter is

(1) 1.33×10^{-3} V (2) 4 mV (3) 12 mV (4) 27 mV

Sol. Answer (2)

$$V = \frac{9\times10^{9}\times4\times10^{-12}}{9} = 4\times10^{-3}\,\text{V} = 4\,\text{mV}$$

9. The electric potential in volts due to an electric dipole of dipole moment 2×10^{-8} coulomb-metre at a distance of 3m on a line making an angle of 60° with the axis of the dipole is

(1) Zero (2) 10 (3) 20 (4) 40

Sol. Answer (2)

$$V = \frac{9\times10^{9}\times2\times10^{-8}\times\frac{1}{2}}{9} = 10\ \text{V}$$

10. An electric dipole of length 2 cm is placed with its axis making an angle of 30° to a uniform electric field 10^5 N/C. If it experiences a torque of $10\sqrt{3}$ Nm, then potential energy of the dipole

(1) – 10 J (2) – 20 J (3) – 30 J (4) – 40 J

Sol. Answer (3)

$$10\sqrt{3} = P10^{5}\frac{1}{2}$$

$$2\sqrt{3}\times10^{-4} = P$$

$$U = -2\sqrt{3}\times10^{-4}\times10^{5}\times\frac{\sqrt{3}}{2}$$

$U = -3 \times 10$

$U = -30$ J

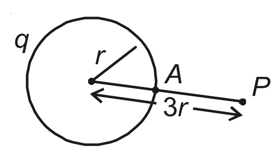
(Equipotential Surfaces, Potential Energy of a System of Charges, Potential Energy in an External Field)

11. A hollow charged metal sphere has radius r. If the potential difference between its surface and a point at a distance $3r$ from the centre is V, then the electric field intensity at distance $3r$ from the centre is

(1) $\frac{V}{3r}$ (2) $\frac{V}{4r}$ (3) $\frac{V}{6r}$ (4) $\frac{V}{2r}$

Sol. Answer (3)

$$V_A = \frac{kq}{r} \qquad V_P = \frac{kq}{3r}$$

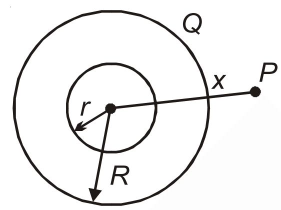


$$V = V_A - V_P = \frac{2kq}{3r}$$

$$kq = \frac{3Vr}{2}$$

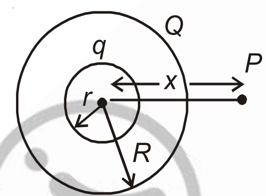
$$E = \frac{kq}{9r^2} = \frac{3Vr}{2(9r^2)} = \frac{V}{6r}$$

12. Two concentric hollow conducting spheres of radius *r* and *R* are shown. The charge on outer shell is *Q*. What charge should be given to inner sphere so that the potential at any point *P* outside the outer sphere is zero?



(1) $-\frac{Qr}{R}$ (2) $-\frac{QR}{r}$ (3) $-Q$ (4) $-\frac{2QR}{r}$

Sol. Answer (3)



$$V_P = \frac{kq}{x} + \frac{KQ}{x} = 0$$

$q = -Q$

13. The potential gradient is a

(1) Vector quantity (2) Scalar quantity (3) Conversion factor (4) Constant

Sol. Answer(1)

Potential gradient = $\frac{dV}{dr} = -E$ (Vector)

14. The electric potential *V* at a point *P*(*x, y, z*) in space is given by $V = 4\,x^2$ volt. Electric field at a point (1m, 0, 2m) in V/m is

(1) 8 along –ve *x*-axis (2) 8 along +ve *x*-axis (3) 16 along –ve *x*-axis (4) 16 along +ve *x*-axis

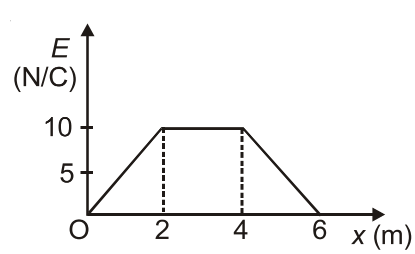
Sol. Answer (1)

$V = 4x^2$

$$E_x = \frac{-dV}{dx} = -8x$$

$E_x = -8$

15. Figure shows the variation of electric field intensity *E* versus distance *x*. What is the potential difference between the points at *x* = 2 m and at *x* = 6 m from *O*?

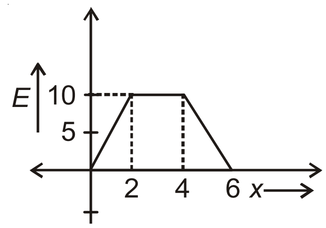


(1) 30 V (2) 60 V (3) 40 V (4) 80 V

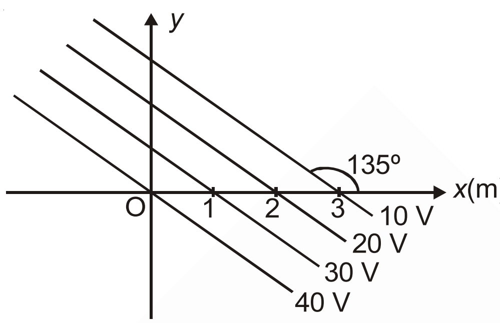
Sol. Answer(1)

$V_2 - V_6 = -\int E dr$

$V_2 - V_6 = (10)(2) + \frac{1}{2}(10)(2) = 30$



16. Figure shows a set of equipotential surfaces. The magnitude and direction of electric field that exists in the region is

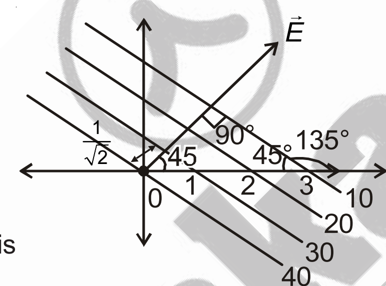


(1) $10\sqrt{2}$ V/m at 45º with *x*-axis
(2) $10\sqrt{2}$ V/m at –45º with *x*-axis
(3) $5\sqrt{2}$ V/m at 45º with *x*-axis
(4) $5\sqrt{2}$ V/m at –45º with *x*-axis

Sol. Answer (1)

$E = \frac{10\sqrt{2}}{1}$

$E = 10\sqrt{2}$ at 45° with *x*-axis



17. Determine the electric field strength vector if the potential of this field depends on *x, y* coordinates as *V* = 10 *axy*

(1) $10a(y\hat{i} + x\hat{j})$
(2) $-10a[y\hat{i} + x\hat{j}]$
(3) $-a[y\hat{i} + x\hat{j}]$
(4) $-10a[x\hat{i} + y\hat{k}]$

Sol. Answer (2)

$V = 10axy$

$E_x = \frac{-dV}{dx} = -10ay, E_y = \frac{-dV}{dy} = -10ax$

$\vec{E} = -10a\left(y\hat{i} + x\hat{j}\right)$

18. If on the x-axis electric potential decreases uniformly from 60 V to 20 V between x = –2 m to x = +2 m, then the magnitude of electric field at the origin

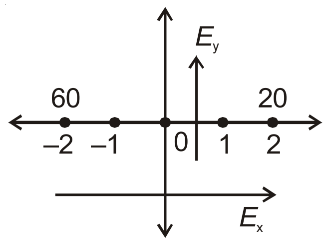
(1) Must be 10 V/m
(2) May be greater than 10 V/m
(3) Is zero
(4) Is 5 V/m

Sol. Answer (2)

$E_x = \dfrac{40}{4} = 10\ \dfrac{\text{V}}{\text{m}}$

$E_y \neq 0$

$E = \sqrt{E_x^2 + E_y^2} > 10$



19. An infinite conducting sheet has surface charge density σ. The distance between two equipotential surfaces is *r*. The potential difference between these two surfaces is

(1) $\dfrac{\sigma r}{2\varepsilon_0}$ (2) $\dfrac{\sigma r}{\varepsilon_0}$ (3) $\dfrac{\sigma}{\varepsilon_0 r}$ (4) $\dfrac{\sigma}{2\varepsilon_0 r}$

Sol. Answer (2)

$\Delta V = Ed = \dfrac{\sigma r}{\varepsilon_0}$

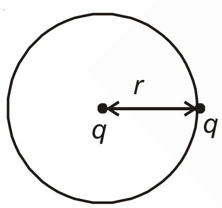
$\Delta V = \dfrac{\sigma r}{\varepsilon_0}$

20. Two small spheres each carrying a charge *q* are placed, distance *r* apart. If one of the spheres is taken around the other in a circular path, the work done will be equal to

(1) Force between them × *r* (2) $\dfrac{\text{Force between them}}{2\pi r}$

(3) Force between them × 2π*r* (4) Zero

Sol. Answer (4)



W = 0 as whole path is equipotential.

21. Work done in moving a charge *q* coulomb on the surface of a given charged conductor of potential *V* is

(1) $\dfrac{V}{q}$ joule (2) *Vq* joule (3) $\dfrac{q}{V}$ joule (4) Zero

Sol. Answer (4)

As the surface of a conductor is equipotential, So *w* = 0.

22. If an α-particle and a proton are accelerated from rest by a potential difference of 1 megavolt then the ratio of their kinetic energy will be

(1) $\dfrac{1}{2}$ (2) 1 (3) 2 (4) 4

Sol. Answer (3)

$\Delta \text{KE} = qV$

$\dfrac{\Delta \text{KE}_\alpha}{\Delta \text{KE}_P} = \dfrac{q_\alpha V}{q_p V} = \dfrac{q_\alpha}{q_p} = 2$

23. The work done in moving an electric charge q in an electric field does not depend upon

(1) Mass of the particle
(2) Potential difference between two points
(3) Magnitude of charge
(4) All of these

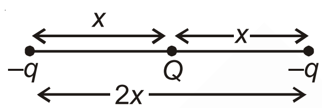
Sol. Answer (1)

Work done does not depend on mass of the particle.

24. Three charges $-q$, Q and $-q$ are placed respectively at equal distances on a straight line. If the potential energy of the system of three charges is zero, then what is the ratio of $Q : q$?

(1) 1 : 1 (2) 1 : 2 (3) 1 : 3 (4) 1 : 4

Sol. Answer (4)



$$U = \frac{-kqQ}{x} - \frac{kqQ}{x} + \frac{kq^2}{2x} = 0$$

$$\frac{kq^2}{2x} = \frac{2kqQ}{x}$$

$$q = 4Q, \qquad \frac{Q}{q} = \frac{1}{4}$$

(Applications of change in Electrostatic Potential Energy)

25. A particle A has charge $+q$ and particle B has charge $+4q$ with each of them having the same mass m. When allowed to fall from rest through the same electric potential difference, the ratio of their speeds $\frac{V_A}{V_B}$ will become

(1) 1 : 2 (2) 2 : 1 (3) 1 : 4 (4) 4 : 1

Sol. Answer (1)

$$qV = \frac{1}{2} m V_A^2 \qquad V_B = \sqrt{\frac{8qV}{m}}$$

$$V_A = \sqrt{\frac{2qV}{m}}$$

$$\frac{V_A}{V_B} = \frac{1}{2}$$

26. If 50 joule of work must be done to move an electric charge of 2 C from a point, where potential is –10 volt to another point, where potential is V volt, the value of V is

(1) 5 V (2) – 15 V (3) + 15 V (4) + 10 V

Sol. Answer (3)

$50 = 2(V - (-10))$

$25 = V + 10$

$V = 15$ V

27. A proton has a mass 1.67×10^{-27} kg and charge $+1.6 \times 10^{-19}$ C. If the proton is accelerated through a potential difference of million volts, then the kinetic energy is

(1) 1.6×10^{-15} J (2) 1.6×10^{-13} J (3) 1.6×10^{-21} J (4) 3.2×10^{-13} J

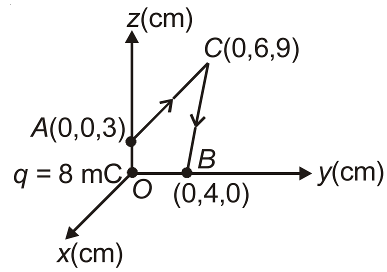
Sol. Answer (2)

$$(1.6 \times 10^{-19})(10^6) = \frac{1}{2}(1.67 \times 10^{-27})\, v^2$$

$$1.6 \times 10^{-13} = \frac{1.67}{2} \times 10^{-27} v^2 = \text{KE}$$

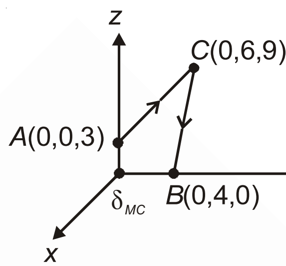
KE = 1.6×10^{-13} J

28. Calculate the work done in taking a charge -2×10^{-9} C from A to B via C (in diagram)



(1) 0.2 joule (2) 1.2 joule (3) 2.2 joule (4) Zero

Sol. Answer (2)



$$V_A = \frac{9\times10^9\times8\times10^{-3}}{3\times10^{-2}} = 24\times10^8\,\text{V}$$

$$V_B = \frac{9\times10^9\times8\times10^{-3}}{4\times10^{-2}} = 18\times10^8\,\text{V}$$

$\Delta W = -2 \times 10^{-9} (-6 \times 10^8)$

$\Delta W = 12 \times 10^{-1}$ J

$\Delta W = 1.2$ J

29. Two electrons are moving towards each other, each with a velocity of 10^6 m/s. What will be closest distance of approach between them?

(1) 1.53×10^{-8} m (2) 2.53×10^{-10} m (3) 2.53×10^{-6} m (4) Zero

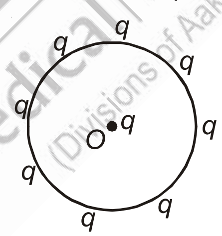
Sol. Answer (2)

$$2.\frac{1}{2}\left(9.1\times10^{-31}\right)\left(10^6\right)^2 = \frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{r}$$

$$9.1\times10^{-19} = \frac{9\times10^9\times2.56\times10^{-38}}{r}$$

$r = 2.56 \times 10^{-10}$ m

30. A point charge q is surrounded by eight identical charges at distance r as shown in figure. How much work is done by the forces of electrostatic repulsion when the point charge at the centre is removed to infinity?



(1) Zero (2) $\dfrac{8q^2}{4\pi\varepsilon_0 r}$ (3) $\dfrac{8q}{4\pi\varepsilon_0 r}$ (4) $\dfrac{64q^2}{4\pi\varepsilon_0 r}$

Sol. Answer (2)

$W = -q\,(V_f - V_i) = -q(V_\infty - V_i) = +qV_i$

$$V_i = 8.\frac{kq^2}{r} = \frac{8q^2}{4\pi\varepsilon_0 r}$$

$$W = \frac{+8q^2}{4\pi\varepsilon_0 r}$$

(Electrostatics of Conductors, Dielectrics and Polarization, Capacitors and Capacitance)

31. 1000 small water drops each of capacitance *C* join together to form one large spherical drop. The capacitance of bigger sphere is

(1) *C* (2) 10 *C* (3) 100 *C* (4) 1000 *C*

Sol. Answer (2)

$C = 4\pi\varepsilon_0 r$

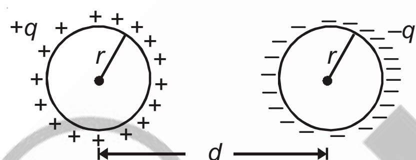
$1000.\dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi R^3$

$R = 10r$

$C' = 4\pi\varepsilon_0 R = 10\,(4\pi\varepsilon_0 r)$

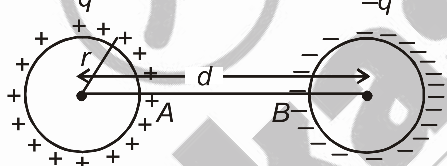
$C' = 10C$

32. Two similar conducting balls having charges +*q* and –*q* are placed at a separation *d* from each other in air. The radius of each ball is *r* and the separation between their centres is *d*(*d* >>*r*). Calculate the capacitance of the two ball system



(1) $4\pi\varepsilon_0 r$ (2) $2\pi\varepsilon_0 r$ (3) $4\pi\log_e\dfrac{\varepsilon_0 r}{d}$ (4) $4\pi\log_e\dfrac{r}{d}$

Sol. Answer (2)



$V_A = \dfrac{kq}{r} + \dfrac{k(-q)}{d-r}$

$V_B = \dfrac{-kq}{r} + \dfrac{kq}{d-r}$

$V = V_A - V_B = \dfrac{2kq}{r} - \dfrac{2kq}{d-r} = \dfrac{2q}{4\pi\varepsilon_0}\left[\dfrac{1}{r} - \dfrac{1}{d-r}\right]$

$V = \dfrac{q}{2\pi\varepsilon_0}\left[\dfrac{d-2r}{(r)(d-r)}\right]$

$d >> r$

$V = \dfrac{q}{2\pi\varepsilon_0}$

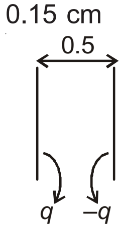
$\dfrac{q}{V} = 2\pi\varepsilon_0 r = C$

(The Parallel Plate Capacitor, Effect of Dielectrics on Capacitance, Combination of Capacitors)

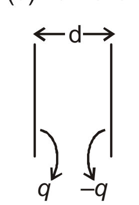
33. Two parallel plate capacitors have their plate areas 100 cm^2 and 500 cm^2 respectively. If they have the same charge and potential and the distance between the plates of the first capacitor is 0.5 mm, then the distance between the plates of the second capacitor is

(1) 0.10 cm (2) 0.15 cm (3) 0.20 cm (4) 0.25 cm

Sol. Answer (4)



$A_1 = 100\ \text{cm}^2$



$A_2 = 500\ \text{cm}^2$

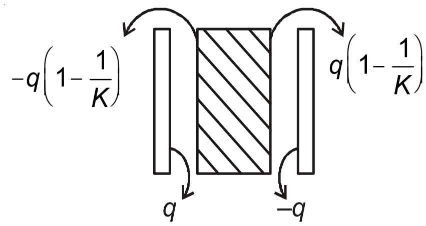
$\dfrac{100\varepsilon_0}{0.5} = \dfrac{500\varepsilon_0}{d}$

$d = 2.5\ \text{cm} = 0.25\ \text{cm}$

34. A dielectric slab of dielectric constant K is placed between the plates of a parallel plate capacitor carrying charge q. The induced charge q' on the surface of slab is given by

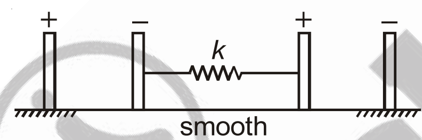
(1) $q' = q - \dfrac{q}{K}$ (2) $q' = -q + \dfrac{q}{K}$ (3) $q' = q\left[\dfrac{1}{K} + 1\right]$ (4) $q' = -q\left(1 + \dfrac{1}{K}\right)$

Sol. Answer (2)



$$q' = -q\left(1 - \frac{1}{K}\right)$$

35. Two charged capacitors have their outer plates fixed and inner plates connected by a spring of force constant 'k'. The charge on each capacitor is q. Find the extension in the spring at equilibrium



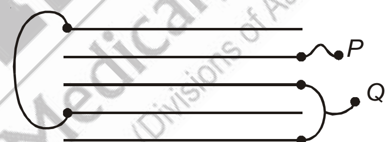
(1) $\dfrac{q^2}{2A\varepsilon_0 k}$ (2) $\dfrac{q^2}{4A\varepsilon_0 k}$ (3) $\dfrac{q^2}{A\varepsilon_0 k}$ (4) Zero

Sol. Answer (1)

$$F = kx = \frac{q^2}{2A\varepsilon_0}$$

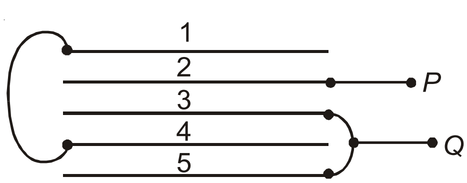
$$x = \frac{q^2}{2A\varepsilon_0 k}$$

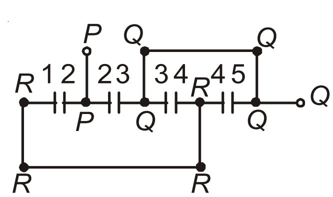
36. The following arrangement consists of five identical metal plates parallel to each other. Area of each plate is A and separation between the successive plates is d. The capacitance between P and Q is

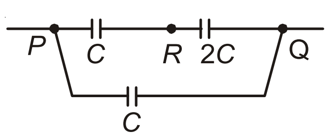


(1) $\dfrac{5\varepsilon_0 A}{d}$ (2) $\dfrac{7}{3}\varepsilon_0\dfrac{A}{d}$ (3) $\dfrac{4}{3}\dfrac{\varepsilon_0 A}{d}$ (4) $\dfrac{5}{3}\dfrac{\varepsilon_0 A}{d}$

Sol. Answer (4)







$$C_{\text{net}} = \frac{5C}{3}$$

37. A capacitor is half filled with a dielectric (K = 2) as shown in figure A. If the same capacitor is to be filled with same dielectric as shown, what would be the thickness of dielectric so that capacitor still has same capacity?

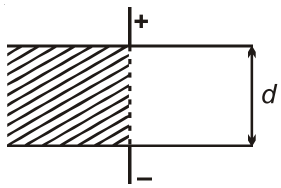


Fig. A

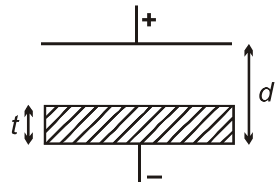
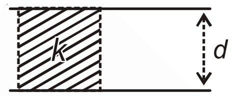


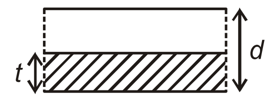
Fig. B

(1) $2d/3$ (2) $3d/2$ (3) $3d/4$ (4) $4d/3$

Sol. Answer (1)

$$C_{net} = \frac{k+1}{2}\frac{A\varepsilon_0}{d} \qquad C_{net} = \frac{A\varepsilon_0}{d - t\left(1-\frac{1}{2}\right)} = \frac{A\varepsilon_0}{d - \frac{t}{2}}$$





$$C_{net} = \frac{3A\varepsilon_0}{2d}$$

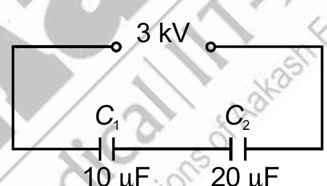
$$\frac{3A\varepsilon_0}{2d} = \frac{A\varepsilon_0}{d - \frac{t}{2}}$$

$$3d - \frac{3t}{2} = 2d$$

$$\frac{3t}{2} = d$$

$$t = \frac{2d}{3}$$

38. Capacitors C_1(10 μF) and C_2(20 μF) are connected in series across a 3 kV supply, as shown. What is the charge on the capacitor C_1?



(1) 45000 μC (2) 20000 μC

(3) 15000 μC (4) 10000 μC

Sol. Answer (2)

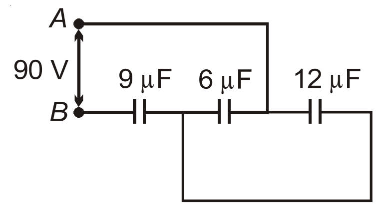
$$\frac{1}{C} = \frac{1}{10} + \frac{1}{20} = \frac{3}{20}$$

$$C = \frac{20}{3}\mu\text{F}$$

$$q = \frac{20}{3}.3000 = 20000 \times 10^{-6}$$

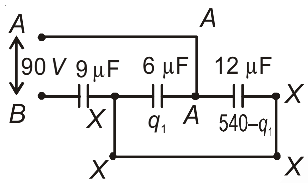
q = 2 × 10^{-4} C

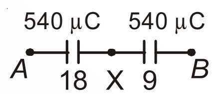
39. The charge on the 6 μF capacitor in the circuit shown is



(1) 540 μC (2) 270 μC (3) 180 μC (4) 90 μC

Sol. Answer (3)





$$\frac{1}{C} = \frac{1}{18} + \frac{1}{9} = \frac{3}{18}$$

$C = 6\ \mu F$

$q = 6 \times 90 = 540\ \mu C$

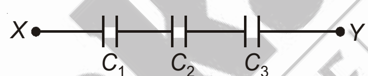
$$\frac{q_1}{540 - q_1} = \frac{6}{12}$$

$2q_1 = 540 - q_1$

$3q_1 = 540$

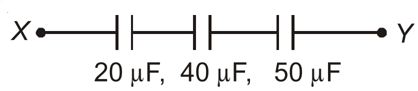
$q_1 = 180\ \mu C$

40. In the circuit below $C_1 = 20\,\mu F$, $C_2 = 40\,\mu F$ and $C_3 = 50\,\mu F$. If no capacitor can sustain more than 50 V, then maximum potential difference between X and Y is



(1) 95 V (2) 75 V (3) 150 V (4) 65 V

Sol. Answer (1)



q_{max} = 1000 μC, 2000 μC, 2500 μC

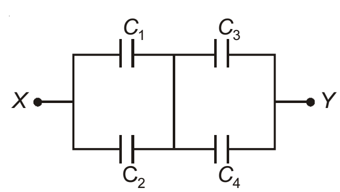
q = 1000 μC, 1000 μC, 1000 μC

V = 50 V, 25 V, 20 V

$V_{net} = 50 + 20 + 25$

$V_{net} = 95$ V

41. In the circuit shown below $C_1 = 10$ μF, $C_2 = C_3 = 20$ μF, and $C_4 = 40$ μF. If the charge on C_1 is 20 μC then potential difference between X and Y is



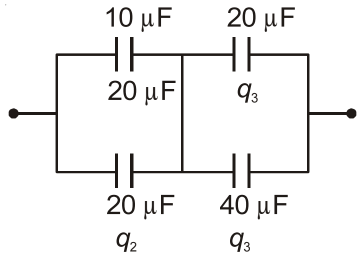
(1) 2 V (2) 3 V (3) 6 V (4) 3.5 V

Sol. Answer (2)

$q_3 = 20\,\mu C$

$q_4 = q_4 = 40\,\mu C$

$\Delta V = \frac{20}{10} + \frac{20}{20} = 3\text{ V}$



42. A parallel plate capacitor after charging is kept connected to a battery and the plates are pulled apart with the help of insulating handles. Now which of the following quantities will decrease ?

(1) Charge (2) Capacitance (3) Energy stored (4) All of these

Sol. Answer (4)

V remains constant

$C = \frac{A\varepsilon_0}{d} \Rightarrow d$ increases

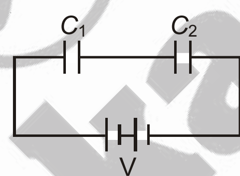
C decreases

$q = CV$

q decreases $U = \frac{1}{2}CV^2$

U decreases

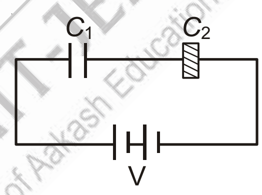
43. In the circuit below, if a dielectric is inserted into C_2 then the charge on C_1 will



(1) Increase (2) Decrease (3) Remain same (4) Be halved

Sol. Answer (1)

$q_1 = \frac{C_1C_2V}{C_1 + C_2}$ $q_1' = \frac{KC_1C_2V}{C_1 + KC_2}$



$q_1' > q_1$, so charge increases.

44. A capacitor with plate separation d is charged to V volts. The battery is disconnected and a dielectric slab of thickness $\frac{d}{2}$ and dielectric constant '2' is inserted between the plates. The potential difference across its terminals becomes

(1) V (2) $2V$ (3) $\frac{4V}{3}$ (4) $\frac{3V}{4}$

Sol. Answer (4)

$q = CV$

$$C' = \frac{A\varepsilon_0}{d - \frac{d}{2}\left(1 - \frac{1}{2}\right)} = \frac{4A\varepsilon_0}{3d} = \frac{4C}{3}$$

$$q = \frac{4CV'}{3}$$

$$CV = \frac{4CV'}{3}$$

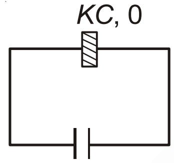
$$V' = \frac{3V}{4}$$

(Energy Stored in a Capacitor, Sharing of Charges)

45. An uncharged parallel plate capacitor having a dielectric of constant *K* is connected to a similar air cored parallel capacitor charged to a potential *V*. The two capacitors share charges and the common potential is *V′*. The dielectric constant *K* is

(1) $\frac{V'-V}{V'+V}$ (2) $\frac{V'-V}{V'}$ (3) $\frac{V'-V}{V}$ (4) $\frac{V-V'}{V'}$

Sol. Answer (4)



$$V' = \frac{(KC)(0)+(C)(V)}{KC+C}$$

$$V' = \frac{CV}{KC+C}$$

$KV' + V' = V$

$KV' = V' - V'$

$$K = \frac{V-V'}{V'}$$

46. A battery does 200 J of work in charging a capacitor. The energy stored in the capacitor is

(1) 200 J (2) 100 J (3) 50 J (4) 400 J

Sol. Answer (2)

$$U = \frac{1}{2}Cv^2$$

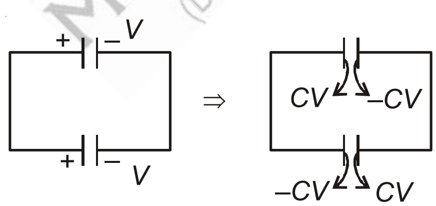
$W = Cv^2$

$U = \frac{W}{2} = 100$ J (half of work is lost in heat)

47. Two identical capacitors are connected in parallel across a potenial difference *V*. After they are fully charged, the positive plate of first capacitor is connected to negative plate of second and negative plate of first is connected to positive plate of other. The loss of energy will be

(1) $\frac{1}{2}CV^2$ (2) CV^2 (3) $\frac{1}{4}CV^2$ (4) Zero

Sol. Answer (2)



$$U_i = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$$

$U_f = 0$

$\Delta U = CV^2$

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