PART : MATHEMATICS

1. The middle term in the expansion of $\left(x^2+\frac{1}{x^2}+2\right)^n$ is

(1) $\frac{n}{\left(\frac{n}{2}!\right)^2}$ (2) $\frac{(2n)!}{\left(\frac{n}{2}!\right)^2}$ (3) $\frac{1.3.5.........(2n+1)}{n!}2^n$ (4) $\frac{(2n)!}{(n!)^2}$

Ans. **(4)**

Sol. $\left(x^2+\frac{1}{x^2}+2\right)^n = \left(x+\frac{1}{x}\right)^{2n}$

$\therefore$ middle term will be $(n + 1)^{th}$ term

$T_{n+1} = {}^{2n}C_n (x)^n \left(\frac{1}{x}\right)^n = {}^{2n}C_n = \frac{2n!}{n!\,n!}$

2. If $\frac{dy}{dx}+2y\tan x=\sin x$, $y\left(\frac{\pi}{3}\right)$ = 0. The maximum value of y(x) is

(1) $\frac{1}{8}$

(2) $\frac{1}{16}$

(3) $-\frac{15}{4}$

(4) $\frac{3}{8}$

Ans. **(1)**

Sol. $\frac{dy}{dx}+2y\tan x=\sin x$

I.F = $e^{\int 2\tan x\,dx} = e^{2\ell n\sec x} = \sec^2 x$

$y(\sec^2 x)=\int\frac{\sin x}{\cos^2 x}dx$

put $\cos x = t$

$\therefore -\sin x\,dx = dt$

$y(\sec^2 x) = -\int\frac{dt}{t^2} = \frac{1}{t} + c = \frac{1}{\cos x} + c$

$y = \cos x + c\cos^2 x \Rightarrow 0 = \frac{1}{2}+c\left(\frac{1}{4}\right)$

$c = -2$

$y = \cos x - 2\cos^2 x = -2\left[\left(\cos x-\frac{1}{4}\right)^2\right]+\frac{1}{8} = \frac{1}{8} - 2\left(\cos x-\frac{1}{4}\right)^2$

$\therefore y_{max} = \frac{1}{8}$

3. Which of the following is a tautology:
(1) $(p\wedge q)\vee(p\to q)$ (2) $(p\wedge q)\wedge(p\to q)$ (3) $(p\wedge q)\to(p\wedge q)$ (4) $(p\to q)\to(p\vee q)$

Ans. **(3)**

Sol. Using truth table diagram,

p	q	$p\vee q$	$p\wedge q$	$p\to q$	Option(1)	Option(2)	Option(3)	Option(4)
T	T	T	T	T	T	T	T	T
T	F	T	F	F	F	F	T	T
F	T	T	F	T	T	F	T	T
F	F	F	F	T	T	F	T	F

4. The number of solutions of the equation $81^{\sin^2 x}+81^{\cos^2 x}=30$ in $[0, \pi]$ is
(1) 1 (2) 2 (3) 3 (4) 4

Ans. **(4)**

Sol. $81^{\sin^2 x}+81^{\cos^2 x}=30$

Let $81^{\sin^2 x} = y$

$\therefore y + \frac{81}{y} = 30$

$\Rightarrow y^2 + 81 = 30y$

$y^2 - 30y + 81 = 0$

$y = 3, y = 27$

$81^{\sin^2 x} = 3, 27$

$3^{4\sin^2 x} = 3^1, 3^3$

$\therefore \sin^2 x = \frac{1}{4}, \frac{3}{4}$

$\therefore \sin x = \frac{1}{2}, \frac{\sqrt{3}}{2}$

4 solutions.

5. The number of solutions of the equation $(3 - |x|)\,|x + 4| = 6, x \in R$ are
(1) 4 (2) 3 (3) 2 (4) 1

Ans. **(3)**

Sol. **Case-I :** $x < -4$

$(3 + x)(x + 4) = -6$

$\Rightarrow 3x + x^2 + 12 + 4x = -6 \quad \Rightarrow \quad x^2 + 7x + 18 = 0$

$\because D < 0$

$\therefore$ no solution

Case-II : $-4 \le x < 0$

$(3 + x)(x + 4) = 6$

$\Rightarrow 3x + x^2 + 12 + 4x = 6 \quad \Rightarrow \quad x^2 + 7x + 6 = 0$

$\Rightarrow (x + 1)(x + 6) = 0$

$x = -1$, $x = -6$ (rejected)

Case-III : $x \geq 0$
$(3 - x)(x + 4) = 6$
$\Rightarrow 3x - x^2 + 12 - 4x = 6$
$\Rightarrow x^2 + x - 6 = 0$
$\Rightarrow (x + 3)(x - 2) = 0$
$x = 2$, $x = -3$ (rejected)
$\therefore$ Total number of solutions are 2

6. The value of the sum $\sum_{r=1}^{\infty} \tan^{-1}\left(\frac{6^r}{2^{2r+1} + 3^{2r+1}}\right)$ is

(1) $\pi - \tan^{-1}\frac{3}{2}$ (2) $\frac{\pi}{2} - \tan^{-1}\frac{3}{2}$ (3) $\frac{\pi}{2} - \cot^{-1}\frac{3}{2}$ (4) $\tan^{-1}\frac{3}{2}$

Ans. **(2)**

Sol. $S_n = \sum_{r=1}^{n} \tan^{-1}\left(\frac{\frac{3^r}{2^{r+1}}}{1+\left(\frac{3}{2}\right)^{2r+1}}\right) = \sum_{r=1}^{n} \tan^{-1}\left(\frac{\left(\frac{3}{2}\right)^{r+1} - \left(\frac{3}{2}\right)^{r}}{1+\left(\frac{3}{2}\right)^{r+1}.\left(\frac{3}{2}\right)^{r}}\right) = \sum_{r=1}^{n}\left(\tan^{-1}\left(\frac{3}{2}\right)^{r+1} - \tan^{-1}\left(\frac{3}{2}\right)^{r}\right)$

$= \left(\tan^{-1}\left(\frac{3}{2}\right)^{2} - \tan^{-1}\frac{3}{2}\right) + \left(\tan^{-1}\left(\frac{3}{2}\right)^{3} - \tan^{-1}\left(\frac{3}{2}\right)^{2}\right) + \ldots.. + \left(\tan^{-1}\left(\frac{3}{2}\right)^{n+1} - \tan^{-1}\left(\frac{3}{2}\right)^{n}\right)$

$= \tan^{-1}\left(\frac{3}{2}\right)^{n} - \tan^{-1}\frac{3}{2}$

$\Rightarrow \lim_{n\to\infty} \frac{\pi}{2} - \tan^{-1}\frac{3}{2}$

7. If $L = \lim_{x\to 0} \frac{\int_0^{x^2} \cos^2 t\,dt}{x \sin x}$. Find L

(1) 2 (2) 1 (3) 0 (4) None of these

Ans. **(2)**

Sol. It is of the form $\frac{0}{0}$

Using L.H' Rule,

$L = \lim_{x\to 0} \frac{2x.\cos^2 x^2}{\sin x + x\cos x}$

$\Rightarrow L = \lim_{x\to 0} \frac{2\cos^2 x^2}{\frac{\sin x}{x} + \cos x} \Rightarrow L = \frac{2}{1+1} \Rightarrow L = 1$

8. The equation of common tangent to the curve $y^2 = 4x$ and $xy = -1$ is:
(1) $2^{2/3}y = x+2^{4/3}$ (2) $2^{2/3}x = y+2^{4/3}$ (3) $y = 2^{2/3}x-2^{4/3}$ (4) $y = x-2^{2/3}$

Ans. **(1)**

Sol. Let equation of tangent to $y^2 = 4x$ be

$$y = mx + \frac{1}{m}$$

It is also a tangent to the equation $\Rightarrow xy = -1$

$$\Rightarrow x\left(mx + \frac{1}{m}\right) = -1$$

$\Rightarrow m^2x^2 + x + m = 0$

$\Rightarrow D = 0 \Rightarrow 1-4m^3 = 0$

$$\Rightarrow m^3 = \frac{1}{4} \Rightarrow m = \left(\frac{1}{2}\right)^{2/3}$$

So equation of tangent will be $y = \dfrac{x}{2^{2/3}} + 2^{2/3}$

$\Rightarrow 2^{2/3}\,y = x+2^{4/3}$

9. If a variable chord of the circle $x^2 + y^2 = 25$ touching the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$ then the locus of mid-point of the variable chord is
(1) $(x^2+y^2)^2 = 16x^2 - 9y^2$ (2) $(x^2+y^2)^2 = 9x^2 + 16y^2$
(3) $(x^2+y^2)^2 = 9x^2 - 16y^2$ (4) $(x^2+y^2)^2 = 16x^2 + 9y^2$

Ans. **(3)**

Sol. Let mid-point be (h, k)

Equation of chord of circle $\Rightarrow T = S_1$,

$\Rightarrow hx + ky = h^2 + k^2$

$$\Rightarrow y = \left(-\frac{x}{k}\right) + \left(\frac{h^2+k^2}{k}\right)$$

This is also a tangent to hyperbola

$$\text{So, } \left(\frac{h^2+k^2}{k}\right)^2 = 9\left(\frac{-h}{k}\right)^2 - 16$$

$$\Rightarrow \frac{(h^2+k^2)^2}{k^2} = \frac{9h^2-16k^2}{k^2}$$

$\Rightarrow (h^2 + k^2)^2 = 9h^2 - 16h^2$

Locus of (h, k) is $(x^2+y^2)^2 = 9x^2 - 16y^2$

10. Three normals are drawn to $y^2 = 2x$ intersect at point (a, 0), Then a must be greater than
(1) 1 (2) 1/2 (3) –1 (4) 1/4

Ans. **(1)**

Sol. Using standard result
h > 2a
$\Rightarrow a > 2\left(\frac{1}{2}\right) \Rightarrow a > 1$

11. A 3 × 3 matrix is formed from {0, 1, 2, 3}. If sum of diagonal elements of AA^T is 9 then number of such matrices is:
(1) 766 (2) 767 (3) 768 (4) 769

Ans. **(1)**

Sol. Let A be $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

Sum of diagonal elements of $AA^T = 9$
Then $a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 9$
from 0, 1, 2, 3 is squares will be 0, 1, 4, 9
Case-1 :One 9 eight 0 $\rightarrow$ 9 matrix = 9
Case-2 Nine 1 $\rightarrow$ 1 matrix = 1
Two 4 one 1 six 0 $\rightarrow {}^9C_2 . {}^7C_1 = 252$
One 4 five 1 Three 0 $\rightarrow {}^9C_1 . {}^8C_5 = 504$
Then total matrix will be 766.

12. A matrix A = $\begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$, where i = $\sqrt{-1}$ and $A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$, Then (x, y) has
(1) Unique solution (2) No solution
(3) Infinite solutions (4) Two solutions

Ans. **(2)**

Sol. $A^2 = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}\begin{bmatrix} i & -i \\ -i & i \end{bmatrix} = \begin{bmatrix} -1-1 & 1+1 \\ 1+1 & -1-1 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = 2\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$A^4 = 4\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = 4\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 8\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$A^8 = 64\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 64\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

Given, $A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$\Rightarrow 128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ $\Rightarrow 128\begin{bmatrix} x-y \\ -x+y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

So, x – y = 8/128 and x – y = –64/128
$\Rightarrow$ (x,y) has no Solution.

13. A card is missing from a well shuffled pack of 52 card. Two cards are randomly picked and found that both are spade. Find the probability that missing card must not be spade.

(1) $\frac{11}{24}$ (2) $\frac{13}{24}$ (3) $\frac{13}{50}$ (4) $\frac{39}{50}$

Ans. (4)

Sol. Missing card spade + Missing card not spade

$$= \frac{^{13}C_1}{^{52}C_1}\cdot\frac{^{12}C_2}{^{51}C_2} + \frac{^{39}C_1}{^{52}C_1}\cdot\frac{^{13}C_2}{^{51}C_2}$$

$$\text{Required Probability} = \frac{\frac{39}{52}\times\frac{^{13}C_2}{^{51}C_2}}{\frac{13}{52}\cdot\frac{^{12}C_2}{^{51}C_2}+\frac{39}{52}\cdot\frac{^{13}C_2}{^{51}C_2}}$$

$$= \frac{39\times13\times12}{13\times12\times11+39\times13\times12} = \frac{39}{11+39} = \frac{39}{50}$$

14. A vector $\sqrt{3}\,\hat{i}+\hat{j}$ is rotated by an angle of 45° in anticlock wise direction, After rotation a new vector is formed $\alpha\hat{i}+\beta\hat{j}$. Then find the area formed by (0,0) , (0,β) & (α,β) is :

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{1}{8}$

Ans. (2)

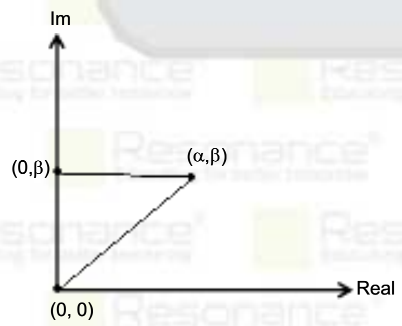
Sol. $\sqrt{3}\,\hat{i}+\hat{j} \equiv (\sqrt{3}+i)$

Rotated by angle 45°

$(\sqrt{3}+i)$ (cos45° + i sin45°)

$$= (\sqrt{3}+i)\,\frac{1}{\sqrt{2}}\,(1+i) = \frac{1}{\sqrt{2}}\left(\sqrt{3}+\sqrt{3}i+i-1\right) = \frac{1}{\sqrt{2}}\left(\sqrt{3}-1+i(\sqrt{3}+1)\right)$$

So, $\alpha = \frac{\sqrt{3}-1}{\sqrt{2}}$; $\beta = \frac{\sqrt{3}+1}{\sqrt{2}}$



Area $= \frac{1}{2}\alpha\beta$

$$= \frac{1}{2}\cdot\frac{\sqrt{3}-1}{\sqrt{2}}\cdot\frac{\sqrt{3}+1}{\sqrt{2}} = \frac{1}{2}$$

15. If n is the number of irrational terms $\left(3^{1/4}+5^{1/8}\right)^{60}$, then (n–1) is divisible by

(1) 7 (2) 8 (3) 36 (4) 26

Ans. **(4)**

Sol. $T_{r+1} = {}^{60}C_r \left(3^{1/4}\right)^{60-r}\left(5^{1/8}\right)^r$

$= {}^{60}C_r 3^{\frac{60-r}{4}} 5^{\frac{r}{8}}$

for rational term r = 0,8,16, ……56

Number of rational terms = 8

Irrational terms = 61– 8 = 53 = n

So, n –1 = 52 is divisible by 26

16. The value of n for the given set of equations

$\log_{10} \sin x + \log_{10} \cos x = -1$

$\log_{10} (\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$ is

(1) 10 (2) 12 (3) 18 (4) 16

Ans. **(2)**

Sol. $\log_{10} \sin x + \log_{10} \cos x = -1$ ....(1)

$\log_{10} (\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$ ....(2)

from (1),

$\log_{10} (\sin x . \cos x) = -1$

$\Rightarrow \sin x . \cos x = \frac{1}{10}$

from (2),

$\log_{10} (\sin x+ \cos x) = \frac{1}{2}(\log_{10} n - \log_{10} 10)$

$\Rightarrow \log_{10} (\sin x + \cos x)^2 = \log_{10} \frac{n}{10}$

$\Rightarrow 1 + 2 \sin x . \cos x = \frac{n}{10}$

$\Rightarrow \frac{12}{10} = \frac{n}{10}$

$\Rightarrow n = 12$

17. If $f(x) = \log_2\left(1+\tan\left(\frac{\pi x}{4}\right)\right)$ then find $\lim\limits_{n\to\infty}\frac{2}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+.........+f\left(\frac{n}{n}\right)\right)$

(1) 2 (2) 3 (3) 1 (4) 0

Ans. (3)

Sol. $\lim\limits_{n\to\infty}\frac{2}{n}\sum\limits_{r=1}^{n} f(r/n)$

$\lim\limits_{n\to\infty}\frac{2}{n}\sum\limits_{r=1}^{n}\log_2\left(1+\tan\left(\frac{\pi}{4}.\frac{r}{n}\right)\right)$

$I = 2\int_0^1 \log_2\left(1+\tan\left(\frac{\pi}{4}x\right)\right)dx$ Let $\frac{\pi}{4}.x = t$ $\Rightarrow dx = \frac{4}{\pi}dt$

$I = \frac{8}{\pi}\int_0^{\frac{\pi}{4}} \log_2(1+\tan t)dt$ ......(1) $t \to \frac{\pi}{4} - t$

$I = \frac{8}{\pi}\int_0^{\frac{\pi}{4}} \log_2\left(1+\tan\left(\frac{\pi}{4}-t\right)\right)dt$

$I = \frac{8}{\pi}\int_0^{\frac{\pi}{4}} \log_2\left(1+\frac{1-\tan t}{1+\tan t}\right)dt$

$I = \frac{8}{\pi}\int_0^{\frac{\pi}{4}} \log_2\left(\frac{2}{1+\tan t}\right)dt$

$I = \frac{8}{\pi}\int_0^{\frac{\pi}{4}} \log_2(2)dt - \frac{8}{\pi}\int_0^{\frac{\pi}{4}} \log_2(1+\tan t)dt$ ......(2)

(1) and (2) equation

$2I = \frac{8}{\pi}\int_0^{\frac{\pi}{4}} 1.dt = \frac{8}{\pi}\times\frac{\pi}{4}$

$\Rightarrow$ $I = 1$

18. If three number a, b, c are related with equation b = a + c and d is standard deviation of a + 2, b + 2, c + 2, then:

(1) $b^2 = 3(a^2 + c^2) - 9d^2$ (2) $b^2 = 5(a^2 + c^2) - 9d^2$

(3) $b^2 = a(a + c)^2 - d^2$ (4) $b^2 = 5(a^2 + ac + c^2) - 9d^2$

Ans. **(1)**

Sol. $\sigma^2 = \frac{a^2+b^2+c^2}{3} - \left(\frac{a+b+c}{3}\right)^2$

$$d^2 = \frac{a^2+b^2+c^2}{3} - \left(\frac{2b}{3}\right)^2$$

$$d^2 = \frac{a^2+b^2+c^2}{3} - \frac{4b^2}{9}$$

$$d^2 = \frac{3(a^2+b^2+c^2)-4b^2}{9}$$

$9d^2 = 3(a^2 + c^2) - b^2$

$b^2 = 3(a^2 + c^2) - 9d^2$

19. $\lim\limits_{x\to 0}\frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$. Then a + b + c =?

Ans. **(04.00)**

Sol. $$\lim_{x\to 0}\frac{a\left(1+x+\frac{x^2}{2!}+\ldots\right)-b\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}\ldots\right)+c\left(1-x+\frac{x^2}{2!}+\ldots\right)}{x^2\left(\frac{\sin x}{x}\right)} = 2$$

$$\Rightarrow \lim_{x\to 0}\frac{(a-b+c)+(a-c)x+\left(\frac{a}{2}+\frac{b}{2}+\frac{c}{2}\right)x^2+\ldots}{x^2\left(\frac{\sin x}{x}\right)} = 2$$

$$\Rightarrow \left.\begin{array}{l} a-b+c=0 \\ \&\ \ a-c=0 \\ \&\ \ \frac{a+b+c}{2}=2 \end{array}\right\} \Rightarrow a+b+c=4$$

20. A function f(x) is defined by $\int_0^x (2t^2 - 15t + 10)$ at and normal drawn to f(x) at point (a, b) ; a > 1 is parallel to line x + 3y = –5, then the value of |a + 6b|

Ans. **(406.00)**

Sol. $f(x) = \int_0^x (2t^2 - 15t + 10)dt$

$f(x) = \frac{2x^3}{3} - \frac{15x^2}{2} + 10x$

$f'(x) = 2x^2 - 15x + 10$

$f'(x)|_a = 2a^2 - 15a + 10$

Since slope of normal at (a, b) = $\frac{-1}{f'(x)|_a}$

$\Rightarrow \quad \frac{-1}{(2a^2 - 15a + 10)} = \frac{-1}{3} \quad \Rightarrow \quad 2a^2 - 15a + 10 = 3$

$\Rightarrow \quad 2a^2 - 14a - a + 7 = 0 \quad \Rightarrow \quad (2a - 1)(a - 7) = 0$

$\Rightarrow \quad a = 7, a = \frac{1}{2}$ (Rejected) $\quad \Rightarrow \quad a = 7$

So, $b = \frac{2 \times 7^3}{3} - \frac{15 \times 7^2}{2} + 10 \times 7$

$b = -\frac{413}{6} \quad \Rightarrow \quad |a + 6b| = |7 - 413| = 406$

21. If f(x) + f(x+1) = 2 ∀ x ∈ R and $I_1 = \int_{-1}^{3} f(x)dx$ and $I_2 = \int_0^8 f(x)dx$. Then the value of $I_2 + 2I_1$:

Ans. **(16.00)**

Sol. f(x) + f(x + 1) = 2 .....(i)

x → x + 1

f(x + 1) + f(x + 2) = 2 .....(ii)

equation (i) – (ii)

f(x) = f(x + 2)

f(x) is periodic with period 2

$I_1 = 2.\int_0^2 f(x)dx, \quad I_2 = 4.\int_0^2 f(x)dx$

$I_2 = 2.I_1$

$I_1 = 2\left[\int_0^1 f(x)dx + \int_1^2 f(x)dx\right] = 2\left[\int_0^1 f(x)dx + \int_0^1 f(x+1)dx\right] = 2\left[\int_0^1 f(x)dx + \int_0^1 (2 - f(x))dx\right]$

$= 2\left[\int_0^1 f(x)dx + \int_0^1 2dx - \int_0^1 f(x)dx\right] = 4$

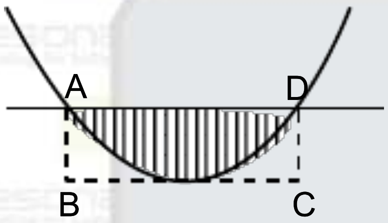
$\Rightarrow \quad I_2 + 2I_1 = 2I_1 + 2I_1 = 4I_1 = 16.$

22. If a differential equation is given by $\frac{dy}{dx} = 2(x + 1)$ and area bounded by y(x) with x axis is $\frac{4\sqrt{8}}{3}$ then y(1) is:

Ans. **(02.00)**

Sol. $\frac{dy}{dx} = 2(x + 1)$

$\Rightarrow \quad y = x^2 + 2x + c$



Area of shaded region = $\frac{2}{3}$ × area of rectangle ABCD = $\frac{2}{3}$ (AD × BC)

AD = Difference of root = $\frac{\sqrt{D}}{|1|} = \sqrt{D}$

BC = $-\frac{D}{4\times 1} = -\frac{D}{4}$ D is discriminant of y(x).

$\Rightarrow$ Area of shaded region = $\left|\frac{2}{3} \times -\frac{D}{4}\sqrt{D}\right| = \frac{4\sqrt{8}}{3}$

$\Rightarrow$ D = 8, $\Rightarrow$ $(2)^2 - 4c = 8$

$\Rightarrow$ c = – 1.

$\Rightarrow$ $y(x) = x^2 + 2x - 1$

So, y(1) = 2.

23. If $\log_{\frac{1}{\sqrt{2}}} \frac{|z|+11}{(|z|-1)^2} \geq 2$, Find $|z|_{min}$:

Ans. **(07.00)**

sol. $\log_{\frac{1}{\sqrt{2}}} \frac{|z|+11}{(|z|-1)^2} \geq 2$

$\Rightarrow \frac{|z|+11}{(|z|-1)^2} \leq \frac{1}{2}$ $\Rightarrow$ $2|z| + 22 \leq |z|^2 - 2|z| + 1$ $\Rightarrow 0 \leq |z|^2 - 4|z| - 21$

$\Rightarrow 0 \leq (|z| - 7)(|z| + 3)$ $\Rightarrow$ $0 \leq (|z| - 7)$ $\therefore |z| \geq 0$

So, $|z|_{min} = 7$