Polynomials And Factorisation

Exercise 2.1

Q. 1. Find the degree of each of the polynomials given below

(i) $x^5 - x^4 + 3$
(ii) $x^2 + x - 5$
(iii) 5
(iv) $3x^6 + 6y^3 - 7$
(v) $4 - y^2$
(vi) $5t - \sqrt{3}$

Answer : Degree of p(x) is the highest power of x in p(x).

(i) The highest power of x in $x^5 - x^4 + 3$ is 5.

∴ The degree of $x^5 - x^4 + 3$ is 5.

(ii) The highest power of x in $x^2 + x - 5$ is 2.

∴ The degree of $x^2 + x - 5$ is 2.

(iii) The highest power of x in 5 is 0(∵ there is no term of x).

∴ The degree of 5 is 0.

(iv) The highest power of x in $3x^6 + 6y^3 - 7$ is 6.

∴ The degree of $3x^6 + 6y^3 - 7$ is 6.

(v) The highest power of y in $4 - y^2$ is 2.

∴ The degree of $4 - y^2$ is 2.

(vi) The highest power of t in $5t – \sqrt{3}$ is 1.

∴ The degree of $5t – \sqrt{3}$ is 1.

Q. 2. Which of the following expressions are polynomials in one variable and which are not? Give reasons for your answer.

(i) $3x^2 - 2x + 5$
(ii) $x^2 + \sqrt{2}$
(iii) $p^2 - 3p + q$
(iv) $y + \frac{2}{y}$
(v) $5\sqrt{x} + x\sqrt{5}$
(vi) $x^{100} + y^{100}$

Answer : (i) $3x^2 - 2x + 5$ has only one variable that is x.

∴ yes, it is a polynomial in one variable.

(ii) $x^2 + \sqrt{2}$ has only one variable that is x.

∴ yes, it is a polynomial in one variable.

(iii) $p^2 – 3p + q$ has two variables that are p and q.

∴ no, it is not a polynomial in one variable.

(iv) $y + \frac{2}{y}$ has a negative exponent of y.

∴ no, it is not a polynomial.

(v) The exponent of x in $5\sqrt{x} + x\sqrt{5}$ is 1/2 which is not a non-negative integer

∴ no, it is not a polynomial.

(vi) $x^{100} + y^{100}$ has two variables that are x and y.

∴ no, it is not a polynomial in one variable.

Q. 3. Write the coefficient of x^3 in each of the following

(i) $x^3 + x + 1$ (ii) $2 - x^3 + x^2$

(iii) $\sqrt{2}x^3 + 5$ (iv) $2x^3 + 5$

(v) $\frac{\pi}{2}x^3 + x$ (vi) $-\frac{2}{3}x^3$

(vii) $2x^2 + 5$ (vi) 4

Answer : A coefficient is a multiplicative factor in some term of a polynomial. It is the constant written before the variable.

Therefore,

(i) The constant written before x^3 in $x^3 + x + 1$ is 1.

∴ The coefficient of x^3 in $x^3 + x + 1$ is 1.

(ii) The constant written before x^3 in $2 – x^3 + x^2$ is -1.

∴ The coefficient of x^3 in $2 – x^3 + x^2$ is -1.

(iii) The constant written before x^3 in $\sqrt{2}x^3 + 5$ is $\sqrt{2}$.

∴ The coefficient of x^3 in $\sqrt{2}x^3 + 5$ is $\sqrt{2}$.

(iv) The constant written before x^3 in $2x^3 + 5$ is 2.

∴ The coefficient of x^3 in $2x^3 + 5$ is 2.

(v) The constant written before x^3 in $\frac{\pi}{2}x^3 + x$ is $\frac{\pi}{2}$.

∴ The coefficient of x^3 in $\frac{\pi}{2}x^3 + x$ is $\frac{\pi}{2}$.

(vi) The constant written before x^3 in $\frac{-2}{3}x^3 + x$ is $\frac{-2}{3}$.

∴ The coefficient of x^3 in $\frac{-2}{3}x^3 + x$ is $\frac{-2}{3}$.

(vii) The term x^3 does not exist in $2x^2 + 5$.

∴ The coefficient of x^3 in $2x^2 + 5$ is 0.

(viii) The term x^3 does not exist in 4.

∴ The coefficient of x^3 in 4 is 0.

Q. 4. Classify the following as linear, quadratic and cubic polynomials

(i) $5x^2 + x - 7$ (ii) $x - x^3$
(iii) $x^2 + x + 4$ (iv) $x - 1$
(v) $3p$ (vi) πr^2

Answer : (i) A quadratic polynomial is a polynomial of degree 2

∵ the degree of $5x^2 + x – 7$ is 2

∴ $5x^2 + x – 7$ is a quadratic polynomial.

(ii) A cubic polynomial is a polynomial of degree 3

∵ the degree of $5x^2 + x – 7$ is 3

∴ $5x^2 + x – 7$ is a cubic polynomial.

(iii) A quadratic polynomial is a polynomial of degree 2

∵ the degree of $x^2 + x + 4$ is 2

∴ $x^2 + x + 4$ is a quadratic polynomial.

(iv) A linear polynomial is a polynomial of degree 1

∵ the degree of $x – 1$ is 1

∴ $x – 1$ is a linear polynomial.

(v) A linear polynomial is a polynomial of degree 1

∵ the degree of $3p$ is 1

∴ $3p$ is a linear polynomial.

(vi) A quadratic polynomial is a polynomial of degree 2

∵ the degree of πr^2 is 2

∴ πr^2 is a quadratic polynomial.

Q. 5. Write whether the following statements are True or False. Justify your answer

(i) A binomial can have at the most two terms
(ii) Every polynomial is a binomial
(iii) A binomial may have degree 3
(iv) Degree of zero polynomial is zero
(v) The degree of $x^2 + 2xy + y^2$ is 2
(vi) πr^2 is monomial.

Answer : (i) A polynomial with two terms is called a binomial.

∴ The statement is true.

(ii) A polynomial can have more than two terms.

∴ The statement is false.

(iii) A binomial should have two terms, the degree of those terms can be any integer.

∴ The statement is true.

(iv) The constant polynomial whose coefficients are all equal to 0, is called a zero polynomial. Its degree can be any integer.

∴ The statement is false.

(v) The highest power in $x^2 + 2xy + y^2$ is 2, therefore its degree is 2.

∴ The statement is true.

(vi) A monomial is a polynomial which has only one term.

∵ πr^2 has only one term

∴ The statement is true.

Q. 6. Give one example each of a monomial and trinomial of degree 10.

Answer : A monomial is a polynomial which has only one term, and the degree is the highest power of the variable. Therefore, an example of a monomial of degree 10 is $3x^{10}$.

A trinomial is a polynomial which has three terms, and the degree is the highest power of the variable. Therefore, example of a trinomial of degree 10 is $3x^{10} + 2x^2 + 5$.

Exercise 2.2

Q. 1. Find the value of the polynomial $4x^2 - 5x + 3$, when

(i) x = 0 (ii) x = -1

(iii) x = 2 (iv) $x = \frac{1}{2}$

Answer : (i) $p(x) = 4x^2 – 5x + 3$

⇒ $p(0) = 4(0)^2 – 5(0) + 3$

⇒ p(0) = 0– 0 + 3

⇒ p(0) = 3

(ii) $p(x) = 4x^2 – 5x + 3$

⇒ $p(-1) = 4(-1)^2 – 5(-1) + 3$

⇒ p(-1) = 4 × 1– (-5) + 3

⇒ p(-1) = 4 +5 + 3

⇒ p(-1) = 12

(iii) $p(x) = 4x^2 – 5x + 3$

⇒ $p(2) = 4(2)^2 – 5(2) + 3$

⇒ p(2) = 4 × 4– 10 + 3

⇒ p(2) = 16– 10 + 3

⇒ p(2) = 9

(iv) $p(x) = 4x^2 – 5x + 3$

$$\Rightarrow\ p\left(\frac{1}{2}\right) = 4 \times \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 3$$

$$\Rightarrow p\left(\frac{1}{2}\right) = 4 \times \frac{1}{4} - \frac{5}{2} + 3$$

$$\Rightarrow p\left(\frac{1}{2}\right) = 1 - \frac{5}{2} + 3$$

$$\Rightarrow p\left(\frac{1}{2}\right) = \frac{2 - 5 + 6}{2}$$

$$\Rightarrow p\left(\frac{1}{2}\right) = \frac{3}{2}$$

Q. 2. A. Find p(0), p(1) and p(2) for each of the following polynomials.

$p(x) = x^2 - x + 1$

Answer : $p(x) = x^2 – x + 1$

$\Rightarrow p(0) = (0)^2 – 0 + 1$

$\Rightarrow p(0) = 1$

And,

$\Rightarrow p(1) = (1)^2 – 1 + 1$

$\Rightarrow p(1) = 1– 1 + 1$

$\Rightarrow p(1) = 1$

And,

$\Rightarrow p(2) = (2)^2 – 2 + 1$

$\Rightarrow p(2) = 4– 2 + 1$

$\Rightarrow p(2) = 3$

Q. 2. B. Find p(0), p(1) and p(2) for each of the following polynomials.

$p(y) = 2 + y + 2y^2 - y^3$

Answer : $p(y) = 2 + y + 2y^2 – y^3$

$\Rightarrow p(0) = 2 + 0 + 2(0)^2 – (0)^3$

$\Rightarrow p(0) = 2 + 0 + 0– 0$

$\Rightarrow p(0) = 2$

And,

$\Rightarrow p(1) = 2 + 1 + 2(1)^2 – (1)^3$

$\Rightarrow p(1) = 2 + 1 + 2– 1$

$\Rightarrow p(1) = 4$

And,

$\Rightarrow p(2) = 2 + 2 + 2(2)^2 – (2)^3$

$\Rightarrow p(2) = 2 + 2 + 8– 8$

$\Rightarrow p(2) = 4$

Q. 2. C. Find p(0), p(1) and p(2) for each of the following polynomials.

$p(z) = z^3$

Answer : $p(z) = z^3$

$\Rightarrow p(0) = 0^3$

$\Rightarrow p(0) = 0$

And,

$\Rightarrow p(1) = 1^3$

$\Rightarrow p(1) = 1$

And,

$\Rightarrow p(2) = 2^3$

$\Rightarrow p(2) = 8$

Q. 2. D. Find p(0), p(1) and p(2) for each of the following polynomials.

p(t) = (t - 1) (t + 1)

Answer : p(t) = (t - 1) (t + 1)

$p(t) = t^2 + t - t – 1$

$\Rightarrow p(t) = t^2 – 1$

$\Rightarrow p(0) = (0)^2 – 1$

$\Rightarrow p(0) = 0– 1$

$\Rightarrow p(0) = -1$

And,

$\Rightarrow p(1) = (1)^2 – 1$

$\Rightarrow p(1) = 1– 1$

$\Rightarrow p(1) = 0$

And,

$\Rightarrow p(2) = (2)^2 – 1$

$\Rightarrow p(2) = 4– 1$

$\Rightarrow p(2) = 3$

Q. 2. E. Find p(0), p(1) and p(2) for each of the following polynomials.

$p(x) = x^2 - 3x + 2$

Answer : $p(t) = x^2 - 3x + 2$

$\Rightarrow p(0) = (0)^2 – 3(0) + 2$

$\Rightarrow p(0) = 0– 0 + 2$

$\Rightarrow p(0) = 2$

And,

⇒ $p(1) = (1)^2 – 3(1) + 2$

⇒ p(1) = 1– 3 + 2

⇒ p(0) = 0

And,

⇒ $p(2) = (2)^2 – 3(2) + 2$

⇒ p(2) = 4– 6 + 2

⇒ p(2) = 0

Q. 3. A. Verify whether the values of x given in each case are the zeroes of the polynomial or not?

$$p(x) = 2x + 1;\ x = -\frac{1}{2}$$

Answer : p(x) = 2x + 1

⇒ p(-1/2) = 2(-1/2) + 1

⇒ p(-1/2) = -1 + 1

⇒ p(-1/2) = 0

∴ Yes x = -1/2 is the zero of polynomial 2x + 1.

Q. 3. B. Verify whether the values of x given in each case are the zeroes of the polynomial or not?

$$p(x) = 5x - \pi;\ x = \frac{-3}{2}$$

Answer : p(x) = 5x - π

$$\Rightarrow p\left(-\frac{3}{2}\right) = 5\left(-\frac{3}{2}\right) - \pi$$

$$\Rightarrow p\left(-\frac{3}{2}\right) = -\frac{15}{2} - \pi$$

$\Rightarrow p\left(-\frac{3}{2}\right) \neq 0$

∴ No x = $-\frac{3}{2}$ is not the zero of polynomial 5x – π.

Q. 3. C. Verify whether the values of x given in each case are the zeroes of the polynomial or not?

p(x) = x^2 - 1; x = ±1

Answer : p(x) = x^2 - 1

⇒ p(-1) = $(-1)^2$ – 1

⇒ p(-1) = 1 – 1

⇒ p(-1) = 0

∴ Yes x = -1 is the zero of polynomial x^2 – 1.

And,

⇒ p(1) = $(1)^2$ – 1

⇒ p(1) = 1 – 1

⇒ p(1) = 0

∴ Yes x = 1 is the zero of polynomial x^2 – 1.

Q. 3. D. Verify whether the values of x given in each case are the zeroes of the polynomial or not?

p(x) = (x - 1)(x + 2); x = -1, -2

Answer : p(x) = (x - 1)(x + 2)

⇒ p(-1) = (-1 - 1)(-1 + 2);

⇒ p(-1) = -2 × 1

⇒ p(-1) = -2

∴ No x = -1 is not the zero of polynomial (x - 1)(x + 2).

And,

⇒ p(-2) = (-2 - 1)(-2 + 2);

⇒ p(-2) = -3 × 0

⇒ p(-2) = 0

∴ Yes x = -2 is not the zero of polynomial (x - 1)(x + 2).

Q. 3. E. Verify whether the values of x given in each case are the zeroes of the polynomial or not?

$p(y) = y^2$; y = 0

Answer : $p(y) = y^2$

⇒ $p(0) = 0^2$

⇒ p(0) = 0

∴ Yes y = 0 is the zero of polynomial y^2.

Q. 3. F. Verify whether the values of x given in each case are the zeroes of the polynomial or not?

p(x) = ax + b ; $x = -\frac{b}{a}$

Answer : p(x) = ax + b

$$\Rightarrow p\left(-\frac{b}{a}\right) = a\left(-\frac{b}{a}\right) + b$$

$$\Rightarrow p\left(-\frac{b}{a}\right) = -b + b$$

$$\Rightarrow p\left(-\frac{b}{a}\right) = 0$$

∴ Yes $x = -\frac{b}{a}$ is the zero of polynomial ax + b.

Q. 3. G. Verify whether the values of x given in each case are the zeroes of the polynomial or not?

$f(x) = 3x^2 - 1$; $x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

Answer : $f(x) = 3x^2 - 1$

$\Rightarrow f\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1$

$\Rightarrow f\left(-\frac{1}{\sqrt{3}}\right) = 3\left(\frac{1}{3}\right) - 1$

$\Rightarrow f\left(-\frac{1}{\sqrt{3}}\right) = 1 - 1$

$\Rightarrow f\left(-\frac{1}{\sqrt{3}}\right) = 0$

$\therefore$ Yes $x = -\frac{1}{\sqrt{3}}$ is the zero of polynomial $3x^2 - 1$.

$\Rightarrow f\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1$

$\Rightarrow f\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{4}{3}\right) - 1$

$\Rightarrow f\left(\frac{2}{\sqrt{3}}\right) = 4 - 1$

$\Rightarrow f\left(\frac{2}{\sqrt{3}}\right) = 3$

$\therefore$ No $x = \frac{2}{\sqrt{3}}$ is not the zero of polynomial $3x^2 - 1$.

Q. 3. H. Verify whether the values of x given in each case are the zeroes of the polynomial or not?

f (x) = 2x - 1, x = $\frac{1}{2}, \frac{-1}{2}$

Answer : f(x) = 2x - 1

$$\Rightarrow f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) - 1$$

$$\Rightarrow f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) - 1$$

$$\Rightarrow f\left(\frac{1}{2}\right) = 1 - 1$$

$$\Rightarrow f\left(\frac{1}{2}\right) = 0$$

∴ Yes $x = \frac{1}{2}$ is the zero of polynomial 2x - 1.

$$\Rightarrow f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right) - 1$$

$$\Rightarrow f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right) - 1$$

$$\Rightarrow f\left(-\frac{1}{2}\right) = -1 - 1$$

$$\Rightarrow f\left(-\frac{1}{2}\right) = -2$$

∴ No $x = -\frac{1}{2}$ is not the zero of polynomial 2x - 1.

Q. 4. A. Find the zero of the polynomial in each of the following cases.

f(x) = x + 2

Answer : f(x) = x + 2

f(x) = 0

⇒ x + 2 = 0

⇒ x = 0 – 2

⇒ x = -2

∴ x = -2 is the zero of the polynomial x + 2.

Q. 4. B. Find the zero of the polynomial in each of the following cases.

f(x) = x - 2

Answer : f(x) = x - 2

f(x) = 0

⇒ x – 2 = 0

⇒ x = 0 + 2

⇒ x = 2

∴ x = 2 is the zero of the polynomial x – 2.

Q. 4. C. Find the zero of the polynomial in each of the following cases.

f(x) = 2x + 3

Answer : f(x) = 2x + 3

f(x) = 0

⇒ 2x + 3 = 0

⇒ 2x = 0 – 3

⇒ 2x = -3

$\Rightarrow\ x = -\frac{3}{2}$

$\therefore x = -\frac{3}{2}$ is the zero of the polynomial 2x + 3.

Q. 4. D. Find the zero of the polynomial in each of the following cases.

f(x) = 2x - 3

Answer : f(x) = 2x – 3

f(x) = 0

⇒ 2x – 3 = 0

⇒ 2x = 0 + 3

⇒ 2x = 3

$\Rightarrow x = \frac{3}{2}$

$\therefore x = \frac{3}{2}$ is the zero of the polynomial 2x – 3.

Q. 4. E. Find the zero of the polynomial in each of the following cases.

f(x) = x^2

Answer : f(x) = x^2

f(x) = 0

⇒ $x^2 = 0$

⇒ x = 0

∴ x = 0 is the zero of the polynomial x^2.

Q. 4. F. Find the zero of the polynomial in each of the following cases.

f(x) = px, p ≠ 0

Answer : f(x) = px, p ≠ 0

f(x) = 0

⇒ px = 0

⇒ x = 0

∴ x = 0 is the zero of the polynomial px.

Q. 4. G. Find the zero of the polynomial in each of the following cases.

f(x) = px + q, p ≠ 0, p q are real numbers.

Answer : f(x) = px + q, p ≠ 0, p q are real numbers.

f(x) = 0

⇒ px + q = 0

⇒ px = -q

$$\Rightarrow x = \frac{-q}{p}$$

$\therefore x = \frac{-q}{p}$ is the zero of the polynomial px + q.

Q. 5. If 2 is a zero of the polynomial $p(x) = 2x^2 - 3x + 7a$, find the value of a.

Answer : ∵ 2 is the zeroes of the polynomial $p(x) = 2x^2 - 3x + 7a$

∴ p(2) = 0

Now,

$p(x) = 2x^2 - 3x + 7a$

$\Rightarrow p(2) = 2(2)^2 – 3(2) + 7a$

⇒ 2 × 4– 3 × 2 + 7a = 0

⇒ 8– 6 + 7a = 0

⇒ 2 + 7a = 0

⇒ 7a = -2

$$\Rightarrow a = -\frac{2}{7}$$

Q. 6. If 0 and 1 are the zeroes of the polynomial $f(x) = 2x^3 - 3x^2 + ax + b$, find the values of a and b.

Answer : ∵ 0 and 1 are the zeroes of the polynomial $f(x) = 2x^3 - 3x^2 + ax + b$

∴ f(0) = 0 and f(1) = 1

Now,

$f(x) = 2x^3 - 3x^2 + ax + b$

⇒ $f(0) = 2(0)^3 – 3(0)^2 + a(0) + 0$

⇒ 2 × 0– 3 × 0 + a × 0 + b = 0

⇒ 0– 0 + 0 + b = 0

⇒ b = 0

And,

⇒ $f(1) = 2(1)^3 – 3(1)^2 + a(1) + 1$

⇒ 2 × 1– 3 × 1 + a × 1 + b = 0

⇒ 2– 3 + a + b = 0

⇒ 2– 3 + a + 0 = 0 [∵ b = 0]

⇒ -1 + a = 0

⇒ a = 1

Exercise 2.3

Q. 1. A. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by the following Linear polynomials:

x + 1

Answer : Let $p(x) = x^3 + 3x^2 + 3x + 1$

As we know by Remainder Theorem,

If a polynomial p(x) is divided by a linear polynomial (x – a) then, the remainder is p(a)

⇒ Remainder of p(x) when divided by x+1 is p(–1)

$p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1$

$\Rightarrow p(-1) = -1 + 3 - 3 + 1 = 0$

∴ Remainder of $x^3 + 3x^2 + 3x + 1$ when divided by $x+1$ is 0

Q. 1. B. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by the following Linear polynomials:

$$x - \frac{1}{2}$$

Answer : Let $p(x) = x^3 + 3x^2 + 3x + 1$

As we know by Remainder Theorem,

If a polynomial p(x) is divided by a linear polynomial (x – a) then, the remainder is p(a)

⇒ Remainder of p(x) when divided by $x - \frac{1}{2}$ is p(1/2)

$$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$$

$$\Rightarrow p\left(\frac{1}{2}\right) = \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1$$

∴ Remainder of $x^3 + 3x^2 + 3x + 1$ when divided by $x - \frac{1}{2}$ is $\frac{27}{8}$

Q. 1. C. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by the following Linear polynomials:

x

Answer : Let $p(x) = x^3 + 3x^2 + 3x + 1$

As we know by Remainder Theorem,

If a polynomial p(x) is divided by a linear polynomial (x – a) then, the remainder is p(a)

⇒ Remainder of p(x) when divided by x is p(0)

$P(0) = (0)^3 + 3(0)^2 + 3(0) + 1$

$= 1$

∴ Remainder of $x^3 + 3x^2 + 3x + 1$ when divided by x is 1

Q. 1. D. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by the following Linear polynomials:

x + π

Answer : Let $p(x) = x^3 + 3x^2 + 3x + 1$

As we know by Remainder Theorem,

If a polynomial p(x) is divided by a linear polynomial (x – a) then, the remainder is p(a)

⇒ Remainder of p(x) when divided by x + π is p(–π)

$p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$

$\Rightarrow p(-\pi) = -\pi^3 + 3\pi^2 - 3\pi + 1$

∴ Remainder of $x^3 + 3x^2 + 3x + 1$ when divided by x+ π is $-\pi^3 + 3\pi^2 - 3\pi + 1$

Q. 1. E. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by the following Linear polynomials:

5 + 2x

Answer : Let $p(x) = x^3 + 3x^2 + 3x + 1$

As we know by Remainder Theorem,

If a polynomial p(x) is divided by a linear polynomial (x – a) then, the remainder is p(a)

⇒ Remainder of p(x) when divided by 5 + 2x is $p(-\frac{5}{2})$

$$p\left(\frac{-5}{2}\right) = \left(-\frac{5}{2}\right)^3 + 3\left(-\frac{5}{2}\right)^2 + 3\left(-\frac{5}{2}\right) + 1$$

$$\Rightarrow p\left(-\frac{5}{2}\right) = -\frac{125}{8} + \frac{75}{4} - \frac{15}{2} + 1$$

Q. 2. Find the remainder when $x^3 - px^2 + 6x - p$ is divided by $x - p$.

Answer : Let $q(x) = x^3 - px^2 + 6x - p$

As we know by Remainder Theorem,

If a polynomial p(x) is divided by a linear polynomial (x – a) then, the remainder is p(a)

⇒ Remainder of q(x) when divided by x – p is q(p)

$q(p) = (p)^3 - p(p)^2 + 6(p) - p$

$\Rightarrow q(p) = p^3 - p^3 + 6p - p$

∴ Remainder of $x^3 - px^2 + 6x - p$ when divided by x – p is 5p

Q. 3. Find the remainder when $2x^2 - 3x + 5$ is divided by $2x - 3$. Does it exactly divide the polynomial? State reason.

Answer : Let $p(x) = 2x^2 - 3x + 5$

As we know by Remainder Theorem,

If a polynomial p(x) is divided by a linear polynomial (x – a) then, the remainder is p(a)

⇒ Remainder of p(x) when divided by 2x – 3 is $p(\frac{3}{2})$

$$p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^2 - 3\left(\frac{5}{2}\right) + 5$$

$$\Rightarrow p\left(\frac{3}{2}\right) = \frac{9}{4} - \frac{15}{2} + 5 = -\frac{1}{4}$$

⇒ Remainder of $2x^2 - 3x + 5$ when divided by 2x – 3 is $-\frac{1}{4}$

As on dividing the given polynomial by 2x – 3, we get a non–zero remainder, therefore, 2x – 3 does not complete divide the polynomial.

∴ It is not a factor.

Q. 4. Find the remainder when $9x^3 - 3x^2 + x - 5$ is divided by $X - \frac{2}{3}$

Answer : Let $p(x) = 9x^3 - 3x^2 + x - 5$

As we know by Remainder Theorem,

If a polynomial p(x) is divided by a linear polynomial (x – a) then, the remainder is p(a)

⇒ Remainder of p(x) when divided by $\boldsymbol{X} - \frac{2}{3}$ is $p(\frac{2}{3})$

$$p\left(\frac{2}{3}\right) = 9\left(\frac{2}{3}\right)^3 - 3\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right) - 5$$

$$\Rightarrow p\left(\frac{3}{2}\right) = \frac{8}{3} - \frac{4}{3} + \frac{2}{3} - 5 = -3$$

Q. 5. If the polynomials $2x^3 + ax^2 + 3x – 5$ and $x^3 + x^2 – 4x + a$ leave the same remainder when divided by x – 2, find the value of a.

Answer : Let $p(x) = 2x^3 + ax^2 + 3x – 5$ and $q(x) = x^3 + x^2 – 4x + a$

As we know by Remainder Theorem,

If a polynomial p(x) is divided by a linear polynomial (x – a) then, the remainder is p(a)

⇒ Remainder of p(x) when divided by x – 2 is p(2). Similarly, Remainder of q(x) when divided by x – 2 is q(2)

⇒ $p(2) = 2(2)^3 + a(2)^2 + 3(2) – 5$

⇒ p(2) = 16 + 4a +6 – 5

⇒ p(2) = 17 + 4a

Similarly, $q(2) = (2)^3 + (2)^2 + –4(2) + a$

⇒ q(2) = 8 + 4 –8 + a

⇒ q(2) = 4 + a

Since they both leave the same remainder, so p(2) = q(2)

⇒ 17 + 4a = 4 + a

⇒ 13 = 3a

$$\Rightarrow a = -\frac{13}{3}$$

∴ The value of a is –13/3

Q. 6. If the polynomials $x^3 + ax^2 + 5$ and $x^3 - 2x^2 + a$ are divided by (x + 2) leave the same remainder, find the value of a.

Answer : Let $p(x) = x^3 + ax^2 + 5$ and $q(x) = x^3 - 2x^2 + a$

As we know by Remainder Theorem,

If a polynomial p(x) is divided by a linear polynomial (x – a) then, the remainder is p(a)

⇒ Remainder of p(x) when divided by x + 2 is p(–2). Similarly, Remainder of q(x) when divided by x + 2 is q(–2)

$\Rightarrow p(-2) = (-2)^3 + a(-2)^2 + 5$

$\Rightarrow p(-2) = -8 + 4a + 5$

$\Rightarrow p(-2) = -3 + 4a$

Similarly, $q(-2) = (-2)^3 - 2(-2)^2 + a$

$\Rightarrow q(-2) = -8 - 8 + a$

$\Rightarrow q(-2) = -16 + a$

Since they both leave the same remainder, so p(–2) = q(–2)

$\Rightarrow -3 + 4a = -16 + a$

$\Rightarrow -13 = 3a$

$$\Rightarrow a = -\frac{13}{3}$$

∴ The value of a is –13/3

Q. 7. Find the remainder when $f(x) = x^4 - 3x^2 + 4$ is divided by g(x)= x – 2 and verify the result by actual division.

Answer : As we know by Remainder Theorem,

If a polynomial p(x) is divided by a linear polynomial (x – a) then, the remainder is p(a)

Therefore, remainder when f(x) is divided by g(x) is f(2)

$f(2) = 2^4 – 3(2)^2 + 4$

⇒ $f(2) = 16 – 12 + 4 = 8$

∴ The remainder when $x^4 – 3x^2 + 4$ is divided by $x – 2$ is 8

Q. 8. Find the remainder when $p(x) = x^3 – 6x^2 + 14x – 3$ is divided by $g(x) = 1 – 2x$ and verify the result by long division.

Answer : Given: $p(x) = x^3 – 6x^2 + 14x – 3$ and $g(x) = 1 – 2x$

As we know by Remainder Theorem,

If a polynomial p(x) is divided by a linear polynomial (x – a) then, the remainder is p(a)

Therefore, remainder when p(x) is divided by g(x) is $p(\frac{1}{2})$

$$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 6\left(\frac{1}{2}\right)^2 + 14\left(\frac{1}{2}\right) - 3$$

$$\Rightarrow p\left(\frac{1}{2}\right) = \frac{1}{8} - \frac{6}{4} + 7 - 3$$

$$\Rightarrow p\left(\frac{1}{2}\right) = \frac{21}{8}$$

∴ the remainder when $x^3 – 6x^2 + 14x – 3$ is divided by $1 – 2x$ is $\frac{21}{8}$

Q. 9. When a polynomial $2x^3 +3x^2 + ax + b$ is divided by (x – 2) leaves remainder 2, and (x + 2) leaves remainder –2. Find a and b.

Answer : Let $p(x) = 2x^3 +3x^2 + ax + b$

As we know by Remainder Theorem,

If a polynomial p(x) is divided by a linear polynomial (x – a) then, the remainder is p(a)

⇒ Remainder of p(x) when divided by x – 2 is p(2)

$p(2) = 2(2)^3 +3(2)^2 + a(2) + b$

⇒ $p(2) = 16 + 12 + 2a + b$

Also, it is given that p(2) = 2, on substituting value above, wev get,

$2 = 28 + 2a + b$

$\Rightarrow 2a + b = -26$ ---------- (A)

Similarly,

Remainder of p(x) when divided by x + 2 is p(–2)

$p(-2) = 2(-2)^3 + 3(-2)^2 + a(-2) + b$

$\Rightarrow p(-2) = -16 + 12 - 2a + b$

Also, it is given that p(–2) = –2, on substituting value above, we get,

$-2 = -4 - 2a + b$

$\Rightarrow -2a + b = 2$ ---------- (B)

On solving the above two equ. (A) and (b), we get,

a = –7 and b = –12

∴ Value of a and b is –7 and –12 respectively.

Exercise 2.4

Q. 1. A. Determine which of the following polynomials has (x + 1) as a factor.

$x^3 - x^2 - x + 1$

Answer : Let $f(x) = x^3 - x^2 - x + 1$

By Factor Theorem, we know that,

If p(x) is a polynomial and a is any real number, then (x – a) is a factor of p(x), if p(a) = 0

For checking (x+1) to be a factor, we will find f(–1)

$\Rightarrow f(-1) = (-1)^3 - (-1)^2 - (-1) + 1$

$\Rightarrow f(-1) = -1 -1 +1 +1$

$\Rightarrow f(-1) = 0$

As, f(–1) is equal to zero, therefore (x+1) is a factor $x^3 - x^2 - x + 1$

Q. 1. B. Determine which of the following polynomials has (x + 1) as a factor.

$x^4 - x^3 + x^2 - x + 1$

Answer : Let $f(x) = x^4 - x^3 + x^2 - x + 1$

By Factor Theorem, we know that,

If p(x) is a polynomial and a is any real number, then (x – a) is a factor of p(x), if p(a) = 0

For checking (x+1) to be a factor, we will find f(–1)

$\Rightarrow f(-1) = (-1)^4 - (-1)^3 + (-1)^2 - (-1) + 1$

$\Rightarrow f(-1) = 1 + 1 + 1 + 1 + 1$

$\Rightarrow f(-1) = 5$

As, f(–1) is not equal to zero, therefore (x+1) is not a factor $x^4 - x^3 + x^2 - x + 1$

Q. 1. C. Determine which of the following polynomials has (x + 1) as a factor.

$x^4 + 2x^3 + 2x^2 + x + 1$

Answer : Let $f(x) = x^4 + 2x^3 + 2x^2 + x + 1$

By Factor Theorem, we know that,

If p(x) is a polynomial and a is any real number, then (x – a) is a factor of p(x), if p(a) = 0

For checking (x+1) to be a factor, we will find f(–1)

$\Rightarrow f(-1) = (-1)^4 + 2(-1)^3 + 2(-1)^2 + (-1) + 1$

$\Rightarrow f(-1) = 1 - 2 + 2 - 1 + 1$

$\Rightarrow f(-1) = 1$

As, f(–1) is not equal to zero, therefore (x+1) is not a factor $x^4 + 2x^3 + 2x^2 + x + 1$

Q. 1. D. Determine which of the following polynomials has (x + 1) as a factor.

$x^3 - x^2 - (3 - \sqrt{3})x + \sqrt{3}$

Answer : Let $f(x) = x^3 - x^2 - (3 - \sqrt{3}\ ) x + \sqrt{3}$

By Factor Theorem, we know that,

If p(x) is a polynomial and a is any real number, then (x – a) is a factor of p(x), if p(a) = 0

For checking (x+1) to be a factor, we will find f(–1)

$\Rightarrow f(-1) = (-1)^3 - (-1)^2 - (3 - \sqrt{3})(-1) + \sqrt{3}$

$\Rightarrow f(-1) = -1 - 1 + 3 - \sqrt{3} + \sqrt{3}$

$\Rightarrow f(-1) = 1$

As, f(–1) is not equal to zero, therefore (x+1) is not a factor $x^3 - x^2 - (3 - \sqrt{3}) x + \sqrt{3}$

Q. 2. A. Use the Factor Theorem to determine whether g(x) is factor of f(x) in the following cases:

$f(x) = 5x^3 + x^2 - 5x - 1$, $g(x) = x + 1$

Answer : By Factor Theorem, we know that,

If p(x) is a polynomial and a is any real number, then g(x) = (x – a) is a factor of p(x), if p(a) = 0

For checking (x+1) to be a factor, we will find f(–1)

$\Rightarrow f(-1) = 5\ (-1)^3 + (-1)^2 - 5(-1) - 1$

$\Rightarrow f(-1) = -5 + 1 + 5 - 1$

$\Rightarrow f(-1) = 0$

As, f(–1) is equal to zero, therefore, g(x) = (x+1) is a factor f(x)

Q. 2. B. Use the Factor Theorem to determine whether g(x) is factor of f(x) in the following cases:

$f(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 1$

Answer : By Factor Theorem, we know that,

If p(x) is a polynomial and a is any real number, then g(x) = (x – a) is a factor of p(x), if p(a) = 0

For checking (x+1) to be a factor, we will find f(–1)

⇒ $f(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1$

⇒ $f(-1) = -1 + 3 - 3 + 1$

⇒ $f(-1) = 0$

As, f(–1) is equal to zero, therefore, g(x) = (x+1) is a factor f(x)

Q. 2. C. Use the Factor Theorem to determine whether g(x) is factor of f(x) in the following cases:

$f(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 2$

Answer : By Factor Theorem, we know that,

If p(x) is a polynomial and a is any real number, then g(x) = (x – a) is a factor of p(x), if p(a) = 0

For checking (x – 2) to be a factor, we will find f(2)

⇒ $f(2) = (2)^3 - 4(2)^2 + (2) + 6$

⇒ $f(-1) = 8 - 16 + 2 + 6$

⇒ $f(-1) = 0$

As, f(–1) is equal to zero, therefore, g(x) = (x – 2) is a factor of f(x)

Q. 2. D. Use the Factor Theorem to determine whether g(x) is factor of f(x) in the following cases:

$f(x) = 3x^3 + x^2 - 20x + 12$, $g(x) = 3x-2$

Answer : By Factor Theorem, we know that,

If p(x) is a polynomial and a is any real number, then g(x) = (x – a) is a factor of p(x), if p(a) = 0

For checking (3x – 2) to be a factor, we will find f(2/3)

$$f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 - 20\left(\frac{2}{3}\right) + 12$$

$$\Rightarrow f\left(\frac{2}{3}\right) = \frac{8}{9} + \frac{4}{9} - \frac{40}{3} + 12$$

$$\Rightarrow f\left(\frac{2}{3}\right) = 0$$

As, f(–1) is equal to zero, therefore, g(x) = (3x – 2) is a factor of f(x)

Q. 2. E. Use the Factor Theorem to determine whether g(x) is factor of f(x) in the following cases:

f(x) = $4x^3 + 20x^2 + 33x + 18$, g(x) = 2x + 3

Answer : By Factor Theorem, we know that,

If p(x) is a polynomial and a is any real number, then g(x) = (x – a) is a factor of p(x), if p(a) = 0

For checking (2x + 3) to be a factor, we will find f(–3/2)

$$f\left(-\frac{3}{2}\right) = 4\left(-\frac{3}{2}\right)^3 + 20\left(-\frac{3}{2}\right)^2 + 33\left(-\frac{3}{2}\right) + 18$$

$$\Rightarrow f\left(-\frac{3}{2}\right) = -\frac{27}{2} + 45 - \frac{99}{2} + 18$$

$$\Rightarrow f\left(-\frac{3}{2}\right) = 0$$

As, f(–3/2) is equal to zero, therefore, g(x) = (3x – 2) is a factor of f(x)

Q. 3. Show that (x – 2), (x + 3) and (x – 4) are factors of $x^3 - 3x^2 - 10x + 24$.

Answer : Let f(x) = $x^3 - 3x^2 - 10x + 24$

By Factor Theorem, we know that,

If p(x) is a polynomial and a is any real number, then g(x) = (x– a) is a factor of p(x), if p(a) = 0

For checking (x – 2) to be a factor, we will find f(2)

$\Rightarrow f(2) = (2)^3 - 3(2)^2 - 10(2) + 24$

⇒ f(2) = 8 – 12 – 20 + 24

⇒ f(2) = 0

So, (x–2) is a factor.

For checking (x + 3) to be a factor, we will find f(–3)

⇒ $f(-3) = (-3)^3 - 3(-3)^2 - 10(-3) + 24$

⇒ f(–3) = –27 – 27 + 30 + 24

⇒ f(–3) = 0

So, (x+3) is a factor.

For checking (x – 4) to be a factor, we will find f(4)

⇒ $f(4) = (4)^3 - 3(4)^2 - 10(4) + 24$

⇒ f(4) = 64 – 48 – 40 + 24

⇒ f(4) = 0

So, (x–4) is a factor.

∴ (x – 2), (x + 3) and (x – 4) are factors of $x^3 - 3x^2 - 10x + 24$

Q. 4. Show that (x + 4), (x – 3) and (x – 7) are factors of $x^3 - 6x^2 - 19x + 84$.

Answer : Let $f(x) = x^3 - 6x^2 - 19x + 84$

By Factor Theorem, we know that,

If p(x) is a polynomial and a is any real number, then g(x) = (x– a) is a factor of p(x), if p(a) = 0

For checking (x + 4) to be a factor, we will find f(–4)

⇒ $f(-4) = (-4)^3 - 6(-4)^2 - 19(-4) + 84$

⇒ f(–4) = –64 – 96 + 76 + 84

⇒ f(–4) = 0

So, (x+4) is a factor.

For checking (x – 3) to be a factor, we will find f(3)

⇒ $f(3) = (3)^3 – 6(3)^2 – 19(3) + 84$

⇒ f(3) = 27 – 54 – 57 + 84

⇒ f(3) = 0

So, (x–3) is a factor.

For checking (x – 7) to be a factor, we will find f(7)

⇒ $f(7) = (7)^3 – 6(7)^2 – 19(7) + 84$

⇒ f(7) = 343 – 294 – 133 + 84

⇒ f(7) = 0

So, (x–7) is a factor.

∴ (x + 4), (x – 3) and (x – 7) are factors of $x^3 – 3x^2 – 10x + 24$

Q. 5. If both (x – 2) and $\left(x - \frac{1}{2}\right)$ are factors of $px^2 + 5x + r$, show that p = r.

Answer : Let $f(x) = px^2 + 5x + r$

By Factor Theorem, we know that,

If p(x) is a polynomial and a is any real number, then g(x) = (x– a) is a factor of p(x), if p(a) = 0 and vice versa.

So, if (x – 2) is a factor of f(x)

⇒ f(2) = 0

⇒ $p(2)^2 + 5(2) + r = 0$

⇒ 4p + r = –10 -------- (A)

Also as $\left(x - \frac{1}{2}\right)$ is also a factor,

$$\Rightarrow f\left(\frac{1}{2}\right) = 0$$

$$\Rightarrow p\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r = 0$$

$$\Rightarrow \frac{p}{4} + \frac{5}{2} + r = 0$$

⇒ p + 4r = –10 ----- (B)

Subtract B from A to get,
4p + r - (p + 4r)= –10 - (-10)
4p + r - p - 4r = -10 + 10
3p - 3r = 03p = 3rp = r

Q. 6. If (x^2 – 1) is a factor of $ax^4 + bx^3 + cx^2 + dx + e$, show that a + c + e = b + d = 0

Answer : Let $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

By Factor Theorem, we know that,

If p(x) is a polynomial and a is any real number, then g(x) = (x– a) is a factor of p(x), if p(a) = 0 and vice versa.

Also we can write, (x^2 – 1) = (x + 1)(x – 1)

Since (x^2 – 1) is a factor of f(x), this means (x + 1) and (x – 1) both are factors of f(x).

So, if (x – 1) is a factor of f(x)

⇒ f(1) = 0

⇒ $a(1)^4 + b(1)^3 + c(1)^2 + d(1) + e = 0$

⇒ a + b + c + d + e = 0 ----- (A)

Also as (x + 1) is also a factor,

⇒ f(–1) = 0

⇒ $a(-1)^4 + b(-1)^3 + c(-1)^2 + d(-1) + e = 0$

⇒ a – b + c – d + e = 0

$\Rightarrow$ a + c + e = b + d ---- (B)

On solving equations (A) and (B), we get,

a + c + e = b + d = 0

Q. 7. A. Factorize

$x^3 – 2x^2 – x + 2$

Answer : Let $p(x) = x^3 – 2x^2 – x + 2$

By trial, we find that p(1) = 0, so by Factor theorem,

(x – 1) is the factor of p(x)

When we divide p(x) by (x – 1), we get $x^2 – x – 2$.

Now, $(x^2 – x – 2)$ is a quadratic and can be solved by splitting the middle terms.

We have $x^2 – x – 2 = x^2 – 2x + x – 2$

$\Rightarrow$ x (x – 2) + 1 (x – 2)

$\Rightarrow$ (x + 1)(x – 2)

So, $x^3 – 2x^2 – x + 2 = (x – 1)(x + 1)(x – 2)$

Q. 7. B. Factorize

$x^3 – 3x^2 – 9x – 5$

Answer : Let $p(x) = x^3 – 3x^2 – 9x – 5$

By trial, we find that p(–1) = 0, so by Factor theorem,

(x + 1) is the factor of p(x)

When we divide p(x) by (x + 1), we get $x^2 – 4x – 5$.

Now, $(x^2 – 4x – 5)$ is a quadratic and can be solved by splitting the middle terms.

We have $x^2 – 4x – 5 = x^2 – 5x + x – 5$

$\Rightarrow$ x (x – 5) + 1 (x – 5)

⇒ (x + 1)(x – 5)

So, $x^3 - 3x^2 - 9x - 5 = (x + 1)(x + 1)(x - 5)$

Q. 7. C. Factorize

$x^3 + 13x^2 + 32x + 20$

Answer : Let $p(x) = x^3 + 13x^2 + 32x + 20$

By trial, we find that p(–1) = 0, so by Factor theorem,

(x + 1) is the factor of p(x)

When we divide p(x) by (x + 1), we get $x^2 + 12x + 20$.

Now, $(x^2 + 12x + 20)$ is a quadratic and can be solved by splitting the middle terms.

We have $x^2 + 12x + 20 = x^2 + 10x + 2x + 20$

⇒ x (x + 10) + 2 (x + 10)

⇒ (x + 2)(x + 10)

So, $x^3 + 13x^2 + 32x + 20 = (x + 1)(x + 2)(x + 10)$

Q. 7. D. Factorize

$y^3 + y^2 - y - 1$

Answer : Let $p(y) = y^3 + y^2 - y - 1$

On taking y^2 common from first two terms in p(y), we get,

$p(y) = y^2(y + 1) - 1(y + 1)$

Now, taking (y + 1) common, we get,

⇒ $p(y) = (y^2 - 1)(y + 1)$

As we know the identity, $(y^2 - 1) = (y + 1)(y - 1)$

⇒ p(y) = (y – 1)(y + 1)(y + 1)

Q. 8. If $ax^2 + bx + c$ and $bx^2 + ax + c$ have a common factor x + 1 then show that c = 0 and a = b.

Answer : Let $f(x) = ax^2 + bx + c$ and $p(x) = bx^2 + ax + c$

As (x + 1) is the common factor of f(x) and p(x) both, and as by Factor Theorem, we know that,

If p(x) is a polynomial and a is any real number, then g(x) = (x– a) is a factor of p(x), if p(a) = 0 and vice versa.

$\Rightarrow$ f(–1) = p (–1) = 0

$\Rightarrow a(-1)^2 + b(-1) + c = b(-1)^2 + a(-1) + c$

$\Rightarrow$ a – b + c = b – a + c

$\Rightarrow$ 2a = 2b

$\Rightarrow$ a = b ------ (A)

Also, we discussed that,

f(–1) = 0

$\Rightarrow a(-1)^2 + b(-1) + c = 0$

$\Rightarrow$ a – b + c = 0

From equation (A), we see that a = b,

$\Rightarrow$ c = 0 -------- (B)

∴ Equations (A) and (B) show us the required result.

Q. 9. If $x^2 - x - 6$ and $x^2 + 3x - 18$ have a common factor (x – a) then find the value of a.

Answer : Let $f(x) = x^2 - x - 6$ and $p(x) = x^2 + 3x - 18$

As (x – a) is the common factor of f(x) and p(x) both, and as by Factor Theorem, we know that,

If p(x) is a polynomial and a is any real number, then g(x) = (x– a) is a factor of p(x), if p(a) = 0 and vice versa.

$\Rightarrow$ f(a) = p (a)

$\Rightarrow (a)^2 - (a) - 6 = (a)^2 + 3(a) - 18$

⇒ $4a = 12$

⇒ $a = 3$

∴ The value of a is 3.

Q. 10. If (y – 3) is a factor of $y^3 - 2y^2 - 9y + 18$ then find the other two factors.

Answer : Let $f(x) = y^3 - 2y^2 - 9y + 18$

Taking y^2 common from the first two terms of f(x) and 9 from the last two terms of f(x), we get,

⇒ $f(x) = y^2(y - 2) - 9(y - 2)$

Now, taking (y – 2) common from above,

⇒ $f(x) = (y^2 - 9)(y - 2)$ -------- (A)

We know the identity as,

$a^2 - b^2 = (a - b)(a + b)$

So, using above identity on equation (A), we get,

⇒ $f(x) = (y + 3)(y - 3)(y - 2)$

∴ the other two factors of $y^3 - 2y^2 - 9y + 18$ besides (y – 3) are (y + 3) and (y – 2).

Exercise 2.5

Q. 1. A. Use suitable identities to find the following products

(x + 5) (x + 2)

Answer : Using the identity $(x + a) \times (x + b) = x^2 + (a + b)x + ab$

Here a = 5 and b = 2

⇒ $(x + 5)(x + 2) = x^2 + (5 + 2)x + 5 \times 2$

Therefore $(x + 5)(x + 2) = x^2 + 7x + 10$

Q. 1. B. Use suitable identities to find the following products

(x - 5) (x - 5)

Answer : $(x - 5)(x - 5) = (x – 5)^2$

Using identity $(a – b)^2 = a^2 – 2ab + b^2$

Here a = x and b = 5

$\Rightarrow (x - 5)(x - 5) = x^2 – 2 \times x \times 5 + 5^2$

$\Rightarrow (x - 5)(x - 5) = x^2 – 10x + 25$

Therefore $(x - 5)(x - 5) = x^2 – 10x + 25$

Q. 1. C. Use suitable identities to find the following products

(3x + 2)(3x - 2)

Answer : Using the identity $(a + b) \times (a – b) = a^2 – b^2$

Here a = 3x and b = 2

$\Rightarrow (3x + 2)(3x - 2) = (3x)^2 – 2^2$

Therefore $(3x + 2)(3x - 2) = 9x^2 – 4$

Q. 1. D. Use suitable identities to find the following products

$$\left(x^2 + \frac{1}{x^2}\right)\left(x^2 - \frac{1}{x^2}\right)$$

Answer : Using the identity $(a + b) \times (a – b) = a^2 – b^2$

Here $a = x^2$ and $b = \frac{1}{x^2}$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)\left(x^2 - \frac{1}{x^2}\right) = (x^2)^2 - \left(\frac{1}{x^2}\right)^2$$

$$\text{Therefore } \left(x^2 + \frac{1}{x^2}\right)\left(x^2 - \frac{1}{x^2}\right) = x^4 - \frac{1}{x^4}$$

Q. 1. E. Use suitable identities to find the following products

(1 + x) (1 + x)

Answer : $(1 + x)(1 + x) = (1 + x)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

Here a = 1 and b = x

$\Rightarrow (1 + x)(1 + x) = 1^2 + 2(1)(x) + x^2$

Therefore $(1 + x)(1 + x) = 1 + 2x + x^2$

Q. 2. A. Evaluate the following products without actual multiplication.

101 × 99

Answer : 101 can be written as (100 + 1) and

99 can be written as (100 - 1)

$\Rightarrow 101 \times 99 = (100 + 1) \times (100 - 1)$

Using the identity $(a + b) \times (a - b) = a^2 - b^2$

Here a = 100 and b = 1

$\Rightarrow 101 \times 99 = 100^2 - 1^2$

$\Rightarrow 101 \times 99 = 10000 - 1$

$\Rightarrow 101 \times 99 = 9999$

Q. 2. B. Evaluate the following products without actual multiplication.

999 × 999

Answer : 999 can be written as (1000 – 1)

⇒ 999 × 999 = (1000 – 1) × (1000 – 1)

⇒ 999 × 999 = $(1000 – 1)^2$

Using identity $(a – b)^2 = a^2 – 2ab + b^2$

Here a = 1000 and b = 1

⇒ 999 × 999 = $1000^2 – 2(1000)(1) + 1^2$

⇒ 999 × 999 = 1000000 – 2000 + 1

⇒ 999 × 999 = 998000 + 1

⇒ 999 × 999 = 998001

Q. 2. C. Evaluate the following products without actual multiplication.

$$50\frac{1}{2} \times 49\frac{1}{2}$$

Answer :

$\Rightarrow 50\frac{1}{2} = \frac{50 \times 2 + 1}{2}$ and $49\frac{1}{2} = \frac{49 \times 2 + 1}{2}$

$\Rightarrow 50\frac{1}{2} = \frac{100 + 1}{2}$ and $49\frac{1}{2} = \frac{98 + 1}{2}$

$\Rightarrow 50\frac{1}{2} = \frac{101}{2}$ and $49\frac{1}{2} = \frac{99}{2}$

$$\Rightarrow 50\frac{1}{2} \times 49\frac{1}{2} = \frac{101}{2} \times \frac{99}{2}$$

$$\Rightarrow 50\frac{1}{2} \times 49\frac{1}{2} = \frac{101 \times 99}{4}$$

Consider 101 × 99

101 can be written as (100 + 1) and

99 can be written as (100 - 1)

⇒ 101 × 99 = (100 + 1) × (100 - 1)

Using the identity (a + b) × (a – b) = $a^2 - b^2$

Here a = 100 and b = 1

⇒ 101 × 99 = $100^2 - 1^2$

⇒ 101 × 99 = 10000 – 1

⇒ 101 × 99 = 9999

Q. 2. D. Evaluate the following products without actual multiplication.

501 × 501

Answer : 501 can be written as (500 + 1)

⇒ $501 \times 501 = (500 + 1) \times (500 + 1)$

⇒ $501 \times 501 = (500 + 1)^2$

⇒ $501 \times 501 = (500 + 1) \times (500 + 1)$

⇒ $501 \times 501 = (500 + 1)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

Here a = 500 and b = 1

⇒ $501 \times 501 = 500^2 + 2(500)(1) + 1^2$

⇒ $501 \times 501 = 250000 + 1000 + 1$

⇒ $501 \times 501 = 251001$

Q. 2. E. Evaluate the following products without actual multiplication.

30.5 × 29.5

Answer :

$30.5 = \frac{61}{2}$ and $29.5 = \frac{59}{2}$

$\Rightarrow 30.5 \times 29.5 = \frac{61}{2} \times \frac{59}{2}$

$\Rightarrow 30.5 \times 29.5 = \frac{61}{2} \times \frac{59}{2}$

$\Rightarrow 30.5 \times 29.5 = \frac{61 \times 59}{2}$ ...(i)

Consider 61×59

$61 = (60 + 1)$

$59 = (60 - 1)$

⇒ $61 \times 59 = (60 + 1)(60 - 1)$

Using the identity $(a + b) \times (a - b) = a^2 - b^2$

Here a = 60 and b = 1

$\Rightarrow 61 \times 59 = 60^2 - 1^2$

$\Rightarrow 61 \times 59 = 3600 - 1$

$\Rightarrow 61 \times 59 = 3599$

From (i)

$\Rightarrow 30.5 \times 29.5 = \frac{3599}{4}$

Therefore $30.5 \times 29.5 = 899.75$

Q. 3. A. Factorise the following using appropriate identities.

$16x^2 + 24xy + 9y^2$

Answer : $16x^2$ can be written as $(4x)^2$

24xy can be written as 2(4x)(3y)

$9y^2$ can be written as $(3y)^2$

$\Rightarrow 16x^2 + 24xy + 9y^2 = (4x)^2 + 2(4x)(3y) + (3y)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

Here a = 4x and b = 3y

$\Rightarrow 16x^2 + 24xy + 9y^2 = (4x + 3y)^2$

Therefore $16x^2 + 24xy + 9y^2 = (4x + 3y)(4x + 3y)$

Q. 3. B. Factorise the following using appropriate identities.

$4y^2 - 4y + 1$

Answer : $4y^2$ can be written as $(2y)^2$

4y can be written as 2(1)(2y)

1 can be written as 1^2

$\Rightarrow 4y^2 - 4y + 1 = (2y)^2 - 2(1)(2y) + 1^2$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

Here a = 2y and b = 1

$\Rightarrow 4y^2 - 4y + 1 = (2y - 1)^2$

Therefore $4y^2 - 4y + 1 = (2y - 1)(2y - 1)$

Q. 3. C. Factorise the following using appropriate identities.

$$4x^2 - \frac{y^2}{25}$$

Answer : $4x^2$ can be written as $(2x)^2$

$\frac{y^2}{25}$ can be written as $\left(\frac{y}{5}\right)^2$

$\Rightarrow 4x^2 - \frac{y^2}{25} = (2x)^2 - \left(\frac{y}{5}\right)^2$

using the identity $(a + b) \times (a - b) = a^2 - b^2$

here a = 2x and b = $\frac{y}{5}$

Therefore $4x^2 - \frac{y^2}{25} = (2x + \frac{y}{5})(2x - \frac{y}{5})$

Q. 3. D. Factorise the following using appropriate identities.

$18a^2 - 50$

Answer : Take out common factor 2

⇒ $18a^2 – 50 = 2 (9a^2 - 25)$

Now

$9a^2$ can be written as $(3a)^2$

25 can be written as 5^2

⇒ $18a^2 – 50 = 2 ((3a)^2 – 5^2)$

Using the identity $(a + b) \times (a – b) = a^2 – b^2$

Here a = 3a and b = 5

Therefore $18a^2 – 50 = 2 (3a + 5) (3a – 5)$

Q. 3. E. Factorise the following using appropriate identities.

$x^2 + 5x + 6$

Answer : Given is quadratic equation which can be factorised by splitting the middle term as shown

⇒ $x^2 + 5x + 6 = x^2 + 3x + 2x + 6$

$= x (x + 3) + 2 (x + 3)$

$= (x + 3) (x + 2)$

Therefore $x^2 + 5x + 6 = (x + 3) (x + 2)$

Q. 3. F. Factorise the following using appropriate identities.

$3p^2 - 24p + 36$

Answer : Take out common factor 3

⇒ $3p^2 - 24p + 36 = 3 (p^2 – 8p + 12)$

Now splitting the middle term of quadratic $p^2 – 8p + 12$ to factorise it

⇒ $3p^2 - 24p + 36 = 3 (p^2 – 6p – 2p + 12)$

$= 3 [p (p – 6) – 2 (p – 6)]$

$= 3 (p – 2) (p – 6)$

Q. 4. A. Expand each of the following, using suitable identities

$(x + 2y + 4z)^2$

Answer : Using $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Here a = x, b = 2y and c = 4z

$\Rightarrow (x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$

Therefore

$(x + 2y + 4z)^2 = x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

Q. 4. B. Expand each of the following, using suitable identities

$(2a - 3b)^3$

Answer : Using identity $(x – y)^3 = x^3 - y^3 – 3x^2y + 3xy^2$

Here x = 2a and y = 3b

$\Rightarrow (2a - 3b)^3 = (2a)^3 – (3b)^3 – 3(2a)^2(3b) + 3(2a)(3b)^2$

$= 8a^3 – 27b^3 – 18a^2b + 18ab^2$

Therefore $(2a - 3b)^3 = 8a^3 – 27b^3 – 18a^2b + 18ab^2$

Q. 4. C. Expand each of the following, using suitable identities

$(-2a + 5b - 3c)^2$

Answer : $(-2a + 5b - 3c)^2 = [(-2a) + (5b) + (-3c)]^2$

Using $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here x = -2a, y = 5b and z = -3c

$\Rightarrow (-2a + 5b - 3c)^2 = (-2a)^2 + (5b)^2 + (-3c)^2 + 2(-2a)(5b) + 2(5b)(-3c) + 2(-3c)(-2a)$

$\Rightarrow (-2a + 5b - 3c)^2 = 4a^2 + 25b^2 + 9c^2 + (-20ab) + (-30bc) + 12ac$

$\Rightarrow (-2a + 5b - 3c)^2 = 4a^2 + 25b^2 + 9c^2 - 20ab - 30bc + 12ac$

Therefore

$(-2a + 5b - 3c)^2 = 4a^2 + 25b^2 + 9c^2 - 20ab - 30bc + 12ac$

Q. 4. D. Expand each of the following, using suitable identities

$$\left(\frac{a}{4}-\frac{b}{2}+1\right)^2$$

Answer :

$$\left(\frac{a}{4}-\frac{b}{2}+1\right)^2 = [\tfrac{a}{4} + \left(-\tfrac{b}{2}\right) + 1]^2$$

Using $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here $x = \frac{a}{4}$, $y = -\frac{b}{2}$ and $z = 1$

$$\Rightarrow \left(\frac{a}{4}-\frac{b}{2}+1\right)^2 = \left(\frac{a}{4}\right)^2 + \left(-\frac{b}{2}\right)^2 + 1^2 + 2\left(\frac{a}{4}\right)\left(-\frac{b}{2}\right) + 2\left(-\frac{b}{2}\right)(1) + 2(1)\left(\frac{a}{4}\right)$$

$$\Rightarrow \left(\frac{a}{4}-\frac{b}{2}+1\right)^2 = \frac{a^2}{16} + \frac{b^2}{4} + 1 + \left(-\frac{ab}{4}\right) + (-b) + \frac{a}{2}$$

$$\Rightarrow \left(\frac{a}{4}-\frac{b}{2}+1\right)^2 = \frac{a^2}{16} + \frac{b^2}{4} + 1 - \frac{ab}{4} - b + \frac{a}{2}$$

Therefore $\left(\frac{a}{4}-\frac{b}{2}+1\right)^2 = \frac{a^2}{16} + \frac{b^2}{4} + 1 - \frac{ab}{4} - b + \frac{a}{2}$

Q. 4. E. Expand each of the following, using suitable identities

$(p + 1)^3$

Answer : Using identity $(x + y)^3 = x^3 + y^3 + 3x^2y + 3xy^2$

Here x = p and y = 1

$\Rightarrow (p + 1)^3 = p^3 + 1^3 + 3(p)^2(1) + 3(p)(1)^2$

$= p^3 + 1^3 + 3p^2 + 3p$

Therefore $(p + 1)^3 = p^3 + 1^3 + 3p^2 + 3p$

Q. 4. F. Expand each of the following, using suitable identities

$$\left(x - \frac{2}{3}y\right)^3$$

Answer : Using identity $(a – b)^3 = a^3 - b^3 – 3a^2b + 3ab^2$

Here $a = x$ and $b = \frac{2}{3}y$

$$\Rightarrow \left(x - \frac{2}{3}y\right)^3 = x^3 - \left(\frac{2}{3}y\right)^3 - 3(x)^2\left(\frac{2}{3}y\right) + 3(x)\left(\frac{2}{3}y\right)^2$$

$$\Rightarrow \left(x - \frac{2}{3}y\right)^3 = x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$$

Therefore $\left(x - \frac{2}{3}y\right)^3 = x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$

Q. 5. A. Factorise

$25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20xz$

Answer : $25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20xz$

$25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20xz = 25x^2 + 16y^2 + 4z^2 + (-40xy) + 16yz + (-20xz)$

$25x^2$ can be written as $(-5x)^2$

$16y^2$ can be written as $(4y)^2$

$4z^2$ can be written as $(2z)^2$

$-40xy$ can be written as $2(-5x)(4y)$

$16yz$ can be written as $2(4y)(2z)$

$-20xz$ can be written as $2(-5x)(2z)$

$\Rightarrow 25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20xz = (-5x)^2 + (4y)^2 +$

$(2z)^2 + 2(-5x)(4y) + 2(4y)(2z) + 2(-5x)(2z)$ …(i)

Using $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Comparing $(-5x)^2 + (4y)^2 + (2z)^2 + 2(-5x)(4y) + 2(4y)(2z) + 2(-5x)(2z)$ with $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ we get

$a = -5x$, $b = 4y$ and $c = 2z$

Therefore

$(-5x)^2 + (4y)^2 + (2z)^2 + 2(-5x)(4y) + 2(4y)(2z) + 2(-5x)(2z) = (-5x + 4y + 2z)^2$

From (i)

$25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20xz = (-5x + 4y + 2z)^2$

Q. 5. B. Factorise

$9a^2 + 4b^2 + 16c^2 + 12ab - 16bc - 24ca$

Answer : $9a^2 + 4b^2 + 16c^2 + 12ab - 16bc - 24ca$

$9a^2 + 4b^2 + 16c^2 + 12ab - 16bc - 24ca = 9a^2 + 4b^2 + 16c^2 + 12ab + (-16bc) + (-24ca)$

$9a^2$ can be written as $(3a)^2$

$4b^2$ can be written as $(2b)^2$

$16c^2$ can be written as $(-4c)^2$

12ab can be written as 2(3a)(2b)

-16bc can be written as 2(2b)(-4c)

-24ca can be written as 2(-4c)(3a)

$\Rightarrow 9a^2 + 4b^2 + 16c^2 + 12ab - 16bc - 24ca = (3a)^2 + (2b)^2 + (-4c)^2 + 2(3a)(2b) + 2(2b)(-4c) + 2(-4c)(3a)$ …(i)

Using $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Comparing $(3a)^2 + (2b)^2 + (-4c)^2 + 2(3a)(2b) + 2(2b)(-4c) + 2(-4c)(3a)$ with $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ we get

x = 3a, y = 2b and z = -4c

Therefore

$(3a)^2 + (2b)^2 + (-4c)^2 + 2(3a)(2b) + 2(2b)(-4c) + 2(-4c)(3a) = (3a + 2b + (-4c))^2$

From (i)

$9a^2 + 4b^2 + 16c^2 + 12ab - 16bc - 24ca = (3a + 2b – 4c)^2$