Class XII Session 2024-25
Subject - Applied Mathematics
Sample Question Paper - 8

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.
3. **Section – A:** It comprises of 20 MCQs of 1 mark each.
4. **Section – B:** It comprises of 5 VSA type questions of 2 marks each.
5. **Section – C:** It comprises of 6 SA type of questions of 3 marks each.
6. **Section – D:** It comprises of 4 LA type of questions of 5 marks each.
7. **Section – E:** It has 3 case studies. Each case study comprises of 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.
8. Internal choice is provided in 2 questions in Section - B, 2 questions in Section – C, 2 questions in Section - D. You have to attempt only one of the alternatives in all such questions.

Section A

1. For what value of k inverse does not exist for the matrix $\begin{bmatrix} 1 & 2 \\ k & 6 \end{bmatrix}$? **[1]**

a) 6 b) 0

c) 3 d) 2

2. A machine makes car wheels and in a random sample of 26 wheels, the test statistic is found to be 3.07. As per t-distribution test (of 5% level of significance), what can you say about the quality of wheels produced by the machine? (Use t_{25} (0.05) = 2.06) **[1]**

a) Different quality b) Inferior quality

c) Same quality d) Superior quality

3. What amount should be deposited at the end of every 6 months to accumulate ₹ 50000 in 8 years, if money is worth 6% p.a. compounded semi-annually? (Given $(1.03)^{16}$ = 1.6047) **[1]**

a) ₹2149.93 b) ₹2783.08

c) ₹2480.57 d) ₹3432.53

4. The corner points of the feasible region determined by the system of linear constraints are: **[1]**

(0, 10), (5, 5), (15, 15), (0, 20). Let z = px + qy, where p, q > 0. Condition on p and q so that the maximum of z occurs at both the points (15, 15) and (0, 20) is

a) p = q
b) q = 3p
c) p = 2q
d) q = 2p

5. The function f(x) = $x^9 + 3x^7 + 64$ is increasing on: **[1]**

a) R
b) (-∞, 0)
c) (0, ∞)
d) R_0

6. A die is rolled thrice. If the event of getting an even number is a success, then the probability of getting atleast two successes is **[1]**

a) $\frac{1}{4}$
b) $\frac{1}{2}$
c) $\frac{7}{8}$
d) $\frac{2}{3}$

7. If Z is a standard normal variable, then P(0 < Z < 1.7) is equal to **[1]**

a) 1 - F(1.7)
b) F(1.7) - F(0)
c) F(1.7) - 1
d) F(0) - F(1.7)

8. The equation of the curve whose slope is given by $\frac{dy}{dx} = \frac{2y}{x}$; x > 0, y > 0 and which passes through the point (1, 1) is: **[1]**

a) $x^2 = 2y$
b) $y^2 = x$
c) $y^2 = 2x$
d) $x^2 = y$

9. In a game of 100 points, A can give B 10 points and C 18 points. Then, B can give C: **[1]**

a) 55 : 25
b) 45 : 41
c) 35 : 41
d) 35 : 12

10. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right)$ + 1 = 0, is: **[1]**

a) None of these
b) 3
c) 2
d) 1

11. [(3 × 7) + 5] (mod 4) is **[1]**

a) 4
b) 2
c) 5
d) 3

12. If $\frac{|x-2|}{x-2} \geq 0$, then **[1]**

a) x ∈ $(-\infty, 2)$
b) x ∈ $[2, \infty)$
c) x ∈ $(2, \infty)$
d) x ∈ $(-\infty, 2]$

13. Three pipes A, B and C can fill a tank in 10, 20 and 30 hours respectively. If A is open for all the time and B and C are open for one hour each alternately, then the tank will be full in **[1]**

a) 7 hours
b) 6 hours
c) $7\frac{1}{2}$ hours
d) $6\frac{1}{2}$ hours

14. In an L.P.P. if the objective function Z = ax + by has same maximum value on two corner points of the feasible region, then the number of points at which the maximum value of Z occurs is **[1]**

a) 0
b) finite
c) infinite
d) 2

15. If x is a negative integer, then the solution set of -12x > 30 is **[1]**

a) {-2, -1}
b) {-2, -1, 0, 1, 2 ...}
c) {..., -5, -4, -3, -2}
d) {..., -5, -4, -3}

16. Inferential statistics is a process that involves all of the following except **[1]**

a) test a hypothesis
b) analyse relationships
c) estimating a statistic
d) estimating a parameter

17. $\int \frac{x^3}{x+1} dx$ is equal to **[1]**

a) $x - \frac{x^2}{2} - \frac{x^3}{3}$ - log |1 + x| + C
b) $x + \frac{x^2}{2} - \frac{x^3}{3}$ - log |1 - x| + C
c) $x - \frac{x^2}{2} + \frac{x^3}{3}$ - log |1 + x| + C
d) $x + \frac{x^2}{2} + \frac{x^3}{3}$ - log |1 - x| + C

18. Which of the following is not an example of a time series model? **[1]**

a) Cross-sectional
b) Moving average
c) Exponential smoothing
d) Naive approach

19. **Assertion (A):** If A, B and C are the matrices of order 3×1, $2 \times m$ and $3 \times n$ respectively, then (CA + BA) is defined, if n = 3 and m = 3. **[1]**

Reason (R): Multiplication of two matrices A and B i.e. AB is defined if number of columns of A = number of rows of B.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. The function f be given by f(x) = $2x^3 - 6x^2 + 6x + 5$. **[1]**

Assertion (A): x = 1 is not a point of local maxima.

Reason (R): x = 1 is not a point of local minima.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Construct 5-yearly moving averages from the following data of the number of industrial failures in a country during 2003-2018: **[2]**

Year	**No. of failures**	**Year**	**No. of Failures**
2003	23	2011	9
2004	26	2012	13
2005	28	2013	11

2006	32	2014	14
2007	20	2015	12
2008	12	2016	9
2009	12	2017	3
2010	10	2018	1

22. Surjeet purchased a new house, costing ₹40,00,000 and made a certain amount of down payment so that he can pay the balance by taking a home loan from XYZ Bank. If his equated monthly installment is ₹30,000 at 9 % interest compounded monthly (reducing balance method) and payable for 25 years, then what is the initial down payment made by him? [Use $(1.0075)^{-300} = 0.1062$] **[2]**

OR

Find the effective rate of return equivalent to a nominal rate of 6% per annum compounded

i. quarterly

ii. continuously. (Given $(1.015)^4 = 1.06136$ and $e^{0.06} = 1.06183$)

23. Evaluate the definite integral: **[2]**

$$\int_2^e \left(\frac{1}{\log x} - \frac{1}{(\log x)^2}\right) dx$$

24. Amrita bought a car worth ₹ 12,50,000 and makes a down payment of ₹ 3,00,000. The balance amount is to be paid in 4 years by equal monthly instalments at an interest rate of 15% p.a. Find the EMI that Amrita has to pay for the car. **[2]**

{Given $(1.0125)^{-48} = 0.5508565$)}

OR

A machine being used by a company is estimated to have a life of 15 years. At that time a new machine would cost ₹75000 and the scrap of the old machine would yield ₹9600 only. A sinking fund is created for replacing the machine at the end of its life. What sum should be retained by the company at the end of every year to accumulate at 6% per annum?

25. How many kg of sugar costing ₹ 45 per kg must be mixed with 30 kg sugar costing ₹ 35 per kg so that there may be a gain of 12% by selling the mixture at ₹ 47.04 per kg? **[2]**

Section C

26. Solve the initial value problem: $x\frac{dy}{dx} + y = x \log x$, $y(1) = \frac{1}{4}$ **[3]**

OR

Find the differential equation of all the circles in the first quadrant which touch the coordinate axes.

27. Mr. M borrowed ₹10,00,000 from a bank to purchase a house and decided to repay by monthly equal instalments in 10 years. The bank charges interest at 9% compounded annually. The bank calculated his EMI as ₹12,668. Find the principal and interest paid in first year. [Given $a_{108\rceil 0.0075} = 73.83916$] **[3]**

28. Suppose that the demand function for a certain commodity is $p = 20 - 4x^2$ and the marginal cost is MC = 2x + 6, where x is the number of units produced. Find the consumer's surplus at the sales level x_0 where profit is maximized. **[3]**

29. In a normal distribution 31% of the articles are under 45 and 8% are over 64. Calculate the mean and standard deviation of the distribution. **[3]**

OR

If X is a normal variate with mean 30 and S.D. 5. Find:

i. $P(26 \leq X \leq 40)$
ii. $P(X \geq 45)$
iii. $P(|X - 30| > 5)$

30. The average number, in lakhs, of working days lost in strikes during each year of the period 2001-2010 was **[3]**

2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
1.5	1.8	1.9	2.2	2.6	3.7	2.2	6.4	3.6	5.4

Calculate the three-yearly moving averages and draw the moving averages graph.

31. The I.Q.'s (intelligence quotients) of 16 students from one area of a city showed a mean of 107 with a standard deviation of 10 while the I.Q.'s of 14 students from another area of the city showed a mean of 112 with a standard deviation of 8. Is there a significant difference between the I.Q.'s of the two groups at **[3]**

i. 1%
ii. 5% level of significance?

Section D

32. If A = $\begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{bmatrix}$, B = $\begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 4 & 2 \end{bmatrix}$ and C = $\begin{bmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$, verify that A(B + C) = AB + AC. **[5]**

OR

Following equations are consistent? If consistent, solve the:

$x + y - 2z = 4$

$x - 2y + z = -2$

$5x - 5y + z = -2$

33. A person can row a boat at 5 km/h in still water. It takes him thrice as long to row upstream as to row downstream. Find the rate at which the stream is flowing. **[5]**

34. A die is tossed twice. Success is defined as getting an odd number on a random toss. Find the mean and variance of the number of successes. **[5]**

OR

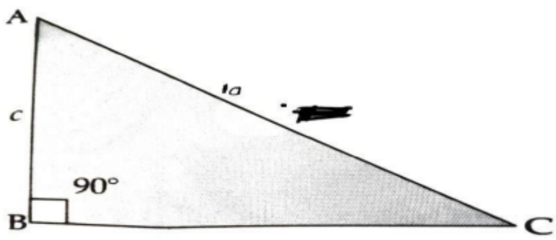
Two cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of aces drawn. Also find the mean and standard deviation of the distribution.

35. How much amount has to be deposited at the end of each year for 4 years at 6% p.a compounded annually so as to get a sum of ₹ 80,000. [Use $(1.06)^4 = 1.262$] **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

The sum of the length of hypotenuse and a side of a right-angled triangle is given by AC + BC = 10



(a) Base BC =?

(b) If **S** be the area of the triangle, then find the value of $\frac{dS}{dc}$?

(c) What is the values of c when $\frac{ds}{dc}$ = 0?

OR

Find the value of $\frac{d^2S}{dc^2}$ at C = $\frac{10\sqrt{3}}{3}$?

37. **Read the text carefully and answer the questions:** **[4]**

Flexible payment arrangements, in which the borrower might pay higher sums of his or her choosing, are not the same as EMIs. Borrowers on EMI programmes are usually only allowed to make one set payment per month. Borrowers profit from an EMI since they know exactly how much money they will have to pay towards their loan each month, making personal financial planning easier. Lenders benefit from the loan interest, as it provides a consistent and predictable stream of income.

Example:

A loan of ₹400000 at the interest rate of 6.75% p.a. compounded monthly is to be amortized by equal payments at the end of each month for 10 years.

(Given $(1.005625)^{120}$ = 1.9603, $(1.005625)^{60}$ - 1.4001)

(a) Find the size of each monthly payment.

(b) Find the principal outstanding at the beginning of 61st month.

(c) Find the interest paid in 61st payment.

OR

Find the principal contained in 61st payment.

38. **Read the following text carefully and answer the questions that follow:** **[4]**

A factory manufactures tennis rackets and cricket bats. A tennis racket takes $1\frac{1}{2}$ hours of machine time and 3 hours of craftsmanship in its making; while a cricket bat takes 3 hours of machine time and 1 hour of craftsmanship. In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsmanship. Profit on a racket and on a bat are ₹ 20 and ₹ 10 respectively.

i. If x and y are the numbers of bats and rackets manufactured by the factory, then write the expression of total profit. (1)

ii. Write the constraint that relates the number of craftsmanship hours. (1)

iii. Determine the maximum profit (in ₹) earned by the factory. (2)

OR

How many bats and rackets respectively, are to be manufactured to earn maximum profit? (2)

OR

Read the following text carefully and answer the questions that follow:

A company started airlines business and for running business it bought aeroplanes. Now an aeroplane can carry maximum of 200 passengers. A profit of ₹ 400 is made on each first-class ticket and a profit of ₹300 is made on each second class ticket. The airline reserves at least 20 seats for first class. However, at least four times as many passengers prefer to travel by second class than by first class. Company wants to make maximum profit by selling tickets of first-class (x) and second class (y).

i. To get maximum profit how many first-class tickets should be sold? (1)

ii. What is the difference between the maximum profit and minimum profit value? (1)

iii. Write any two corner points of feasible region. (2)

OR

What will be the minimum profit value? (2)

Solution

Section A

1.

(c) 3

Explanation: $\begin{vmatrix} 1 & 2 \\ k & 6 \end{vmatrix}$ = 0 ⇒ 6 - 2k = 0 ⇒ k = 3

2.

(b) Inferior quality

Explanation: Given n = 26 and t = 3.0

⇒ degree of freedom = n - 1 = 26 - 1 = 25

Now, level of significance = 5% = 0.05

$t_{25}(0.05) = 2.06$

∵ t = 3.07 > 2.06

So, the wheels produced by the machine are of inferior quality.

3.

(c) ₹2480.57

Explanation: ₹2480.57

4.

(b) q = 3p

Explanation: Since Z occurs maximum at (15, 15) and (0, 20), therefore, 15p + 15q = 0p + 20q

⇒ q = 3p

5. **(a)** R

Explanation: $f(x) = x^9 + 3x^7 + 64$

$f'(x) = 9x^8 + 21x^6$

$= 3x^6 (3x^2 + 7)$

∴ function is increasing

$3x^6 (3x^2 + 7) > 0$

⇒ function is increasing on R.

6.

(b) $\frac{1}{2}$

Explanation: Let X denote the number of successes in 3 trials. Then, X is a binomial variate with n = 3, p = $\frac{3}{6} = \frac{1}{2}$ such that

$P(X = r) = {}^3C_r\left(\frac{1}{2}\right)^3$, r = 0, 1, 2, 3

∴ Required Probability = $P(X \geq 2) = P(X = 2) + P(X = 3)$

$= {}^3C_2\left(\frac{1}{2}\right)^3 + {}^3C_3\left(\frac{1}{2}\right)^3 = \frac{1}{2}$

7.

(b) F(1.7) - F(0)

Explanation: P(0 < Z < 1.7) = F(1.7) - F(0)

8.

(d) $x^2 = y$

Explanation: We have,

$\frac{dy}{dx} = \frac{2y}{x}$

$\Rightarrow \frac{1}{2} \times \frac{1}{y}dy = \frac{1}{x}dx$

Integrating both sides, we get

$\frac{1}{2}\int \frac{1}{y}dy = \int \frac{1}{x}dx$

$\Rightarrow \frac{1}{2}$ log y = log x + log C

$\Rightarrow \log y^{\frac{1}{2}}$ -log x = log C

$\Rightarrow \log\left(\frac{\sqrt{y}}{x}\right)$ = log C

$\Rightarrow \frac{\sqrt{y}}{x}$ = C

$\Rightarrow \sqrt{y}$ = Cx ...(i)

As (i) passes through (1, 1), we get

$\therefore$ 1 = C

Putting the value of C in (i), we get

$\sqrt{y}$ = x

$\Rightarrow$ x^2 = y

9.

(b) 45 : 41

Explanation: A : B = 100 : 90

A : C = 100 : 82

$\therefore \frac{B}{C} = \left(\frac{B}{A} \times \frac{A}{C}\right)$ = $\frac{90}{100} \times \frac{100}{82}$ = 45 : 41

10. **(a)** None of these

Explanation: It is given that equation is $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right)$ + 1 = 0

The given differential equation is not a polynomial equation in its derivative

Therefore, its degree is not defined.

11.

(b) 2

Explanation: (3 $\times$ 7 + 5) (mod 4) = 26 (mod 4) = 2

12.

(c) x $\in$ (2, ∞)

Explanation: Since $\frac{|x-2|}{x-2} \geq 0$, for |x - 2| $\geq$ 0, and x - 2 $\neq$ 0 solution set (2,∞)

13. **(a)** 7 hours

Explanation: (A + B)'s one hour work = $\left(\frac{1}{10} + \frac{1}{20}\right)$ = $\frac{3}{20}$

(A + C)'s one hour work = $\left(\frac{1}{10} + \frac{1}{30}\right)$ = $\frac{4}{30}$ = $\frac{2}{15}$

So total part filled in 2 hours = $\frac{3}{20} + \frac{2}{15}$

= $\frac{9+8}{60}$ = $\frac{17}{60}$

Part filled in 6 hours = 3 $\times$ $\frac{17}{60}$ = $\frac{17}{20}$

Remaining part = 1 - $\frac{17}{20}$ = $\frac{3}{20}$

Now its the turn of A & B and they both can fill $\frac{3}{20}$ part in one hour.

$\therefore$ total time = 6 hour + 1 hour

= 7 hour

14.

(c) infinite

Explanation: infinite

15.

(d) {..., -5, -4, -3}

Explanation: {..., -5, -4, -3}

16.

(c) estimating a statistic

Explanation: estimating a statistic

17.

(c) $x - \frac{x^2}{2} + \frac{x^3}{3}$ - log |1 + x| + C

Explanation: Given: $\int \frac{x^3}{x+1}dx$

$\Rightarrow \frac{x^3}{x+1} = \frac{x^3+1-1}{x+1}$

$\Rightarrow \frac{x^3+1}{x+1} - \frac{1}{x+1} = \frac{(x+1)(x^2-x+1)}{x+1} - \frac{1}{x+1}$
$\Rightarrow \int \frac{x^3}{x+1} dx = \int \left((x^2 - x + 1) - \frac{1}{x+1} \right) dx$
$\Rightarrow \int (x^2 - x + 1) dx - \int \frac{1}{x+1} dx = \frac{x^3}{3} - \frac{x^2}{2} + x - \ln |1 + x| + C$

18.

(d) Naive approach

Explanation: Naive approach

19.

(d) A is false but R is true.

Explanation: For multiplication of C and A
number of columns of C = number of rows of A
⇒ n = 3,
order of CA = 3 × 1
For multiplication of B and A
number of columns of B = number of rows of A
⇒ m = 3,
order of BA = 2 × 1.
For addition of CA and BA
∵ order of CA ≠ order of BA
∴ (CA + BA) is not defined.
∴ Assertion is false.
Reason is ture.
∴ Option (A is false but R is true) is the correct answer.

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: We have,
$f(x) = 2x^3 - 6x^2 + 6x + 5$
$\Rightarrow f'(x) = 6x^2 - 12x + 6 = 6(x - 1)^2$
and f''(x) = 12(x - 1)
Now, f'(x) = 0 gives x = 1.
Also, f''(1) = 0.
Therefore, the second derivative test fails in this case.
So, we shall go back to the first derivative test.
Using first derivatives test, we get x = 1 is neither a point of local maxima nor a point of local minima and so it is a point of inflexion.

Section B

21. Computation of moving averages

Year	**No. of failures**	**5-yearly moving totals**	**5-yearly moving averages**
2003	23	-	-
2004	26	-	-
2005	28	129	25.8
2006	32	118	23.6
2007	20	104	20.8
2008	12	86	17.2
2009	12	63	12.6
2010	10	56	11.2
2011	9	55	11.0
2012	13	57	11.4

2013	11	59	11.8
2014	14	59	11.8
2015	12	49	9.8
2016	9	39	7.8
2017	3	-	-
2018	1	-	-

22. Let the initial down payment made by Surjeet be ₹ x, then P = ₹(4000000 - x)
Given EMI = ₹ 30000, i = $\frac{9}{12\times100}$ = 0.0075, n = 25 $\times$12 = 300 months
Using formula EMI = P$\frac{i}{1-(1+i)^{-n}}$
$\Rightarrow$ 30000 = (4000000 - x) $\times$ $\frac{0.0075}{1-(1.0075)^{-300}}$
$\Rightarrow$ 30000 = (4000000 - x) $\times$ $\frac{0.0075}{1-0.1062}$
$\Rightarrow$ 4000000 - x = $\frac{30000\times0.8938}{0.0075}$
$\Rightarrow$ 4000000 - x = 3575200
x = ₹424800

OR

i. Given r = 6% p.a.
p = 4 quarters.
So, effective rate (per rupee) = $\left(1+\frac{6}{400}\right)^4-1$ = $(1.015)^4$ - 1
= 1.06136 - 1 = 0.06136 = 0.0614
Hence, effective rate = 0.0614 $\times$ 100% = 6.14%

ii. Given r = 6% p.a.
So, i = $\frac{6}{100}$ = 0.06
When compounded continuously, then
effective rate (per rupee) = e^i - 1 = $e^{0.06}$ - 1
= 1.06183 - 1 = 0.06183 = 0.0618
Hence, effective rate = 0.0618 $\times$ 100% = 6.18%

23. $\int_2^e\left(\frac{1}{\log x}-\frac{1}{(\log x)^2}\right)dx=\int_2^e(\log x)^{-1}\cdot 1\,dx-\int_2^e\frac{1}{(\log x)^2}dx$
(integrate the first integral by parts, taking $(\log x)^{-1}$ as the first function)
$=\left[(\log x)^{-1}\cdot x\right]_2^e-\int_2^e(-1)(\log x)^{-2}\cdot\frac{1}{x}\cdot x\,dx-\int_2^e\frac{1}{(\log x)^2}dx$
$=\left[\frac{x}{\log x}\right]_2^e+\int_2^e\frac{1}{(\log x)^2}dx$ - $\int_2^e\frac{1}{(\log x)^2}dx$
$=\frac{e}{\log e}-\frac{2}{\log 2}=\frac{e}{1}-\frac{2}{\log 2}=e-\frac{2}{\log 2}$

24. Here P = ₹ 9,50,000, i = $\frac{15}{1200}$ = 0.0125
n = 48 months
Using the reducing balancing method,
$E=\frac{Pi}{1-(1+i)^{-n}}=\frac{9,5,0000\times0.0125}{1-(1+0\cdot0125)^{-48}}$
$=\frac{11875}{1-(1.0125)^{-48}}=\frac{11875}{1-0.5508565}$
= ₹ 26,439.21

OR

Cost of new machine = ₹75000
Scrap value of old machine = ₹9600
Hence, the money required for new machine after 15 years
= ₹75000 - ₹9600 = ₹65400
So, we have A = ₹65400, r = 6% p.a. $\Rightarrow$ i = 0.06 and n = 15 years.
Using formula,

A = R $\left[\frac{(1+i)^n-1}{i}\right]$

$\Rightarrow$ 65400 = $\left[\frac{(1.06)^{15}-1}{0.06}\right]$

$\Rightarrow R = \frac{65400\times 0.06}{2.396-1}$

$\Rightarrow$ R = $\frac{3924}{1.396}$ = ₹ 2810.89

Let x = $(1.06)^{15}$ Taking logarithm on both sides, we get

log x = 15 log 1.06

$\Rightarrow$ log x = 15 $\times$ 0.0253

$\Rightarrow$ log x = 0.3795

$\Rightarrow$ x = antilog 0.3795

$\Rightarrow$ x = 2.396

25. Let the CP. of mixture be ₹ x per kg

Given S.P.= ₹ 47.04 and profit = 12%

$\because$ S.P. = C.P. + profit

$\Rightarrow$ 47.04 = x + 12% of x

$\Rightarrow$ x = $\frac{4704}{112}$ $\Rightarrow$ x = 42

Given c = ₹ 35 per kg, d = ₹ 45 per kg, m = ₹ 42 per kg

and quantity of cheaper sugar = 30 kg

So, $\frac{\text{quantity of cheaper sugar}}{\text{quantity of dearer sugar}} = \frac{45-42}{42-35}$

$\Rightarrow \frac{30}{\text{quantity of dearer sugar}} = \frac{3}{7}$

$\Rightarrow$ quantity of dearer sugar = 70 kg

Hence, 70 kg of sugar costing ₹ 45 per kg should be mixed.

Section C

26. We have, x$\frac{dy}{dx}$ + y = x log x

$\Rightarrow \frac{dy}{dx} + \frac{1}{x}y$ = log x ...(i)

This is linear differential equation of the form $\frac{dy}{dx}$ + Py = Q with P = $\frac{1}{x}$ and Q = log x

$\therefore$ I.F. = $e^{\int \frac{1}{x}dx}$ = $e^{\log x}$ = x [$\because$ x > 0]

Multiplying both sides of (i) by I.F. = x, we get

x$\frac{dy}{dx}$ + y = x log x

Integrating with respect to x, we ge

yx = $\int \underset{II}{x} \underset{I}{\log x}\, dx$ [Using: y (I.F.) = $\int$ Q (I.F.) dx + C]

$\Rightarrow$ yx = $\frac{x^2}{2}$(log x)$\frac{1}{2}\int$ x dx

$\Rightarrow$ yx = $\frac{x^2}{2}$(log x) - $\frac{x^2}{4}$ + C ...(ii)

It is given that y (1) = $\frac{1}{4}$ i.e. y = $\frac{1}{4}$ where x = 1. Putting x = 1 and y = $\frac{1}{4}$ in (ii), we get

$\frac{1}{4}$ = 0 -$\frac{1}{4}$ + C $\Rightarrow$ C = $\frac{1}{2}$

Putting C = $\frac{1}{2}$ in (ii), we get

xy = $\frac{x^2}{2}$(log x) -$\frac{x^2}{2}$ + $\frac{1}{2}$ $\Rightarrow$ y = $\frac{1}{2}$x log x - $\frac{x}{4} + \frac{1}{2x}$

Hence, y = $\frac{1}{2}$ x log x - $\frac{x}{4} + \frac{1}{2x}$ is the solution of the given differential equation.

OR

The equation of the family of circles in the first quadrant which touch the coordinate axes is

$(x - a)^2 + (y - a)^2 = a^2$ ...(i)

where a is a parameter.

This equation contains one arbitrary constant, so we shall differentiate it once only to get a differential equation of first order.

Differentiating (i) with respect to x, we get

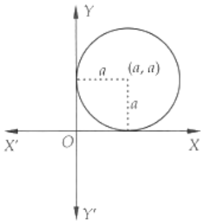
2 (x - a) + 2(y - a)$\frac{dy}{dx}$ = 0

$\Rightarrow$ x - a + (y - a)$\frac{dy}{dx}$ = 0

$\Rightarrow$ a = $\frac{x+y\frac{dy}{dx}}{1+\frac{dy}{dx}}$

$\Rightarrow$ a = $\frac{x+py}{1+p}$, where p = $\frac{dy}{dx}$

Substituting the value of a in (i), we get



$\left(x - \frac{x+py}{1+p}\right)^2 + \left(y - \frac{x+py}{1+p}\right)^2 = \left(\frac{x+py}{1+p}\right)^2$

$\Rightarrow (xp - py)^2 + (y - x)^2 = (x + py)^2$

$\Rightarrow (x - y)^2 p^2 + (x - y)^2 = (x + py)^2$

$\Rightarrow (x - y)^2 (p^2 + 1) = (x + py)^2$

$\Rightarrow (x - y)^2 \left\{1 + \left(\frac{dy}{dx}\right)^2\right\} = \left(x + y\frac{dy}{dx}\right)^2$, which is the required differential equation.

27. Principal left unpaid after one year (12 payments)

= present value of remaining 108 payments = $Ra_{n\rceil i}$

Where R = 12,668, n = 108 and i = $\frac{0.09}{12}$ = 0.0075

$= 12,668 \times a_{108\rceil 0.0075}$

= 12,668 $\times$ 73.83916 = ₹ 9,35,395

Principal paid during first year = 10,00,000 - 9,35,395 = ₹64,605

Interest paid during first year

= (12,668 $\times$ 12) - 64,605

= ₹ 87,651

28. Let P be the profit function and R be the revenue function. Then,

R = px = 20x - 4x^3 and MR = $\frac{dR}{dx}$ = 20 - 12x^2

Now, P = R - C

$\Rightarrow \frac{dP}{dx} = \frac{dR}{dx} - \frac{dC}{dx}$

$\Rightarrow \frac{dP}{dx}$ = MR - MC

$\Rightarrow \frac{dP}{dx} = (20 - 12x^2) - (2x + 6)$

$\Rightarrow \frac{dP}{dx} = 14 - 2x - 12x^2$ and $\frac{d^2P}{dx^2} = -2 - 24x$

For maximum value of P, we must have

$\frac{dP}{dx} = 0$

$\Rightarrow 14 - 2x - 12x^2 = 0$

$\Rightarrow 6x^2 + x - 7 = 0$

$\Rightarrow 6x^2 + 7x - 6x - 7 = 0 \Rightarrow (6x - 7)(x - 1) = 0 \Rightarrow x - 1 = 0 \Rightarrow x = 1$

Clearly, $\left(\frac{d^2p}{dx^2}\right)_{x=1} = -2 - 24 = -26 < 0$

Hence, the profit is maximum when x = 1. Therefore, $x_0 = 1$. Putting $x_0 = 1$ in $p = 20 - 4x^2$, we obtain $p_0 = 16$.

The consumer's surplus at $x_0 = 1$ is given by

CS = $\int_0^1 p\,dx - p_0 x_0$

$\Rightarrow$ CS = $\int_0^1 \left(20 - 4x^2\right) dx - 16 \times 1$

$\Rightarrow$ CS = $\left[20x - \frac{4}{3}x^3\right]_0^1 - 16 = 20 - \frac{4}{3} - 16 = \frac{8}{3}$

29. Let X be a normal distribution random variable, then

P(X < 45) = 31% i.e. 0.31 and P(X > 64) = 8% i.e. 0.08

Let the mean and the standard deviation of the distribution be μ and σ respectively, then

for X = 45, Z = $\frac{45-\mu}{\sigma}$ and for X = 64, Z = $\frac{64-\mu}{\sigma}$

$\therefore$ P(X < 45) = P$\left(Z < \frac{45-\mu}{\sigma}\right)$ = 0.31

$\Rightarrow \text{P}\left(\text{Z} < \frac{45-\mu}{\sigma}\right)$ = P(Z < -0.5) (using table)

$\Rightarrow \frac{45-\mu}{\sigma}$ = -0.5 ...(i)

and P(X > 64) = $\text{P}\left(\text{Z} > \frac{64-\mu}{\sigma}\right)$ = 0.08 = 1 - 0.92

$\Rightarrow \text{P}\left(\text{Z} > \frac{64-\mu}{\sigma}\right)$ = 1 - P(Z $\leq$ 1.4) = P(Z > 1.4) (using table)

$\Rightarrow \frac{64-\mu}{\sigma}$ = 1.4 ...(ii)

Dividing equation (i) by (ii), we get

$\frac{45-\mu}{64-\mu} = \frac{-0.5}{1.4} \Rightarrow 63 - 1.4\mu$ = -32 + 0.5 μ

$\Rightarrow 95 = 1.9\mu \Rightarrow \mu$ = 50 .

Substituting μ = 50 in equation (i), we get

$\frac{45-50}{\sigma} = -0.5 \Rightarrow \sigma$ = 10

Hence, mean 50 and standard deviation = σ = 10

OR

We have, μ = 30 and σ = 5

Let Z be the standard normal variate. Then,

Z = $\frac{X-\mu}{\sigma} \Rightarrow Z = \frac{X-30}{5}$

i. When X = 26, we obtain: Z = $\frac{26-30}{5}$ = -0.8

When X = 40, we obtain : Z = $\frac{40-30}{5}$ = 2

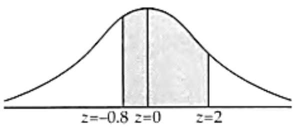
$\therefore$ P(26 $\leq$ X $\leq$ 40)

= P (-0.8 $\leq$ Z $\leq$ 2)

= P (-0.8 $\leq$ Z $\leq$ 0) + P (0 $\leq$ Z $\leq$ 2)

= P (0 $\leq$ Z $\leq$ 0.8) + P (0 $\leq$ 2 $\leq$ 2)

= 0.2881 + 0.4772 = 0.7563 [See table]



ii. When X = 45, we obtain

Z = $\frac{45-30}{5}$ = 3

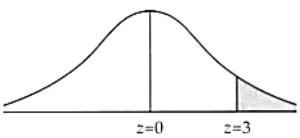
$\therefore$ P(X $\geq$ 45)

= P (Z $\geq$ 3)

= 0.5 - P(0 $\leq$ Z $\leq$ 3)

= 0.5 - 0.49865 [See Table]

= 0.00135



iii. P(|X - 30| > 5)

= 1 - P(|X - 30| $\leq$ 5)

= 1 - P(30 - 5 $\leq$ X $\leq$ 30 + 5)

= 1 - P(25 $\leq$ X $\leq$ 35)

Now, X = 25 $\Rightarrow$ Z = $\frac{25-30}{5}$ = -1 and, X = 35 $\Rightarrow$ Z = $\frac{35-30}{5}$ = 1.

$\therefore$ P(|X - 30| > 5)

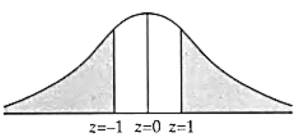
= 1 - P(-1 $\leq$ Z $\leq$ 1)

= 1 - 2. P(0 $\leq$ Z $\leq$ 1)

= 1 - 2 $\times$ 0.3413

= 1 - 0.6826

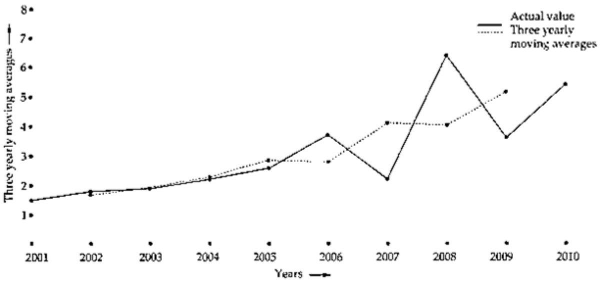
= 0.3174



30. In order to calculate three-yearly moving averages, we first compute three-yearly moving totals and place each total against the middle year of the three-year span from which the moving totals are calculated. These moving totals are given in the third column of the following table. From these three yearly moving totals, we calculate three-yearly moving averages by dividing each moving total by 3 as shown in the following table.

Year	**Working days lost in strikes (in lakhs)**	**Three yearly moving totals**	**Three yearly moving averages**
2001	1.5	-	-
2002	1.8	5.2	1.73
2003	1.9	5.9	1.96
2004	2.2	6.7	2.23
2005	2.6	8.5	2.83
2006	3.7	8.5	2.83
2007	2.2	12.3	4.1
2008	6.4	12.2	4.06
2009	3.6	15.4	5.13
2010	5.4	-	-

The graph of these moving averages is shown in Fig.



31. Performing independent samples t-test,not assuming equal variances.

Assumptions: both populations must be normal.

The null hypothesis: the mean IQs are equal.

The alternative hypothesis: the mean IQs are different.

Degrees of freedom: df= min(N1, N2) - 1 = 13

The standard error:

SE = $\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}=\sqrt{\frac{10^2}{16}+\frac{8^2}{14}}$ = 3.2896

The test statistics:

t = $\frac{\left(\bar{X}_1-\bar{X}_2\right)-0}{SE}=\frac{107-112}{3.2896}$ = -1.52

The two-tailed cumulative probability value associated with the given t-statistic can be determined from the Student's t-distribution table or calculated using the technology (function T.DIST.2T() of MS Excel).

For df = 13 and t = -1.52, p = 0.152

Since the p-value is greater than both α values, fail to reject the null hypothesis at both significance levels.

The samples do not provide sufficient evidence to conclude the difference between the mean IQs.

Section D

32. A(B + C) = A $\left(\begin{bmatrix}3 & 1\\ -1 & 0\\ 4 & 2\end{bmatrix}+\begin{bmatrix}4 & 7\\ 2 & 1\\ 1 & -1\end{bmatrix}\right)$ = A $\begin{bmatrix}3+4 & 1+7\\ -1+2 & 0+1\\ 4+1 & 2+(-1)\end{bmatrix}$ = $\begin{bmatrix}2 & 0 & -3\\ 1 & 4 & 5\end{bmatrix}\begin{bmatrix}7 & 8\\ 1 & 1\\ 5 & 1\end{bmatrix}$

= $\begin{bmatrix}2.7+0.1+(-3).5 & 2.8+0.1+(-3).1\\ 1.7+4.1+5.5 & 1.8+4.1+5.1\end{bmatrix}$ = $\begin{bmatrix}-1 & 13\\ 36 & 17\end{bmatrix}$ ...(i)

AB = $\begin{bmatrix}2 & 0 & -3\\ 1 & 4 & 5\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 0\\ 4 & 2\end{bmatrix}$

$= \begin{bmatrix} 2.3+0.(-1)+(-3).4 & 2.1+0.0+(-3)\cdot 2 \\ 1.3+4.(-1)+5.4 & 1.1+4.0+5.2 \end{bmatrix} = \begin{bmatrix} -6 & -4 \\ 19 & 11 \end{bmatrix}$

and AC = $\begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{bmatrix} \begin{bmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 2.4+0.2+(-3)\cdot 1 & 2.7+0.1+(-3)\cdot(-1) \\ 1.4+4.2+5.1 & 1.7+4.1+5\cdot(-1) \end{bmatrix} = \begin{bmatrix} 5 & 17 \\ 17 & 6 \end{bmatrix}$

$\therefore$ AB + AC = $\begin{bmatrix} -6 & -4 \\ 19 & 11 \end{bmatrix} + \begin{bmatrix} 5 & 17 \\ 17 & 6 \end{bmatrix}$

$= \begin{bmatrix} -6+5 & -4+17 \\ 19+17 & 11+6 \end{bmatrix} = \begin{bmatrix} -1 & 13 \\ 36 & 17 \end{bmatrix}$ ...(ii)

From (i) and (ii), we get

A(B + C) = AB + AC

OR

Here, D = $\begin{vmatrix} 2 & 1 & -2 \\ 1 & -2 & 1 \\ 5 & -5 & 1 \end{vmatrix}$ = 2(-2 + 5)-1(1 - 5)-2(-5 + 10) = 0

$D_1 = \begin{vmatrix} 4 & 1 & -2 \\ -2 & -2 & 1 \\ -2 & -5 & 1 \end{vmatrix}$ = 4(-2 + 5)-1(-2 + 2)-2(+10 - 4) = 0

$D_2 = \begin{vmatrix} 2 & 4 & -2 \\ 1 & -2 & 1 \\ 5 & -2 & 1 \end{vmatrix}$ = 2(-2 + 2)-4(1 - 5)-2(-2 + 10) = 0

and $D_3 = \begin{vmatrix} 2 & 1 & 4 \\ 1 & -2 & -2 \\ 5 & -5 & -2 \end{vmatrix}$ = 2(4 - 10)-1(-2 + 10)+4(-5 + 10) = 0.

Thus, $D = D_1 = D_2 = D_3 = 0$, therefore, the given system may or may not be consistent. Let us solve first two equations for x and y in terms of z.

These equations can be written as

2x + y = 4 + 2z

x - 2y = -2 - z

To solve these equations we use Cramer's rule,

D = $\begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix}$ = -4 - 1 = -5 $\neq$ 0

$D_1 = \begin{vmatrix} 4+2z & 1 \\ -2-z & -2 \end{vmatrix}$ = -8 - 4z + 2 + z = -6 - 3z

$D_2 = \begin{vmatrix} 2 & 4+2z \\ 1 & -2-z \end{vmatrix}$ = -4 - 2z - 4 - 2z = -8 - 4z

By Cramer's rule, x = $\frac{D_1}{D} = \frac{6+3z}{5}$, y = $\frac{D_2}{D} = \frac{8+4z}{5}$.

Let z = k where k is any real number, then we get

x = $\frac{6+3k}{5}$, y = $\frac{8+4k}{5}$, z = k, where k $\in$ R.

Note that these values satisfy the third equation i.e. 5x - 5y + z = -2 of the given system. Hence, the given system is consistent and it has infinitely many solutions given by x = $\frac{3}{5}(2+k)$, y = $\frac{4}{5}(2+k)$, z = k, where k $\in$ R.

33. Let the distance covered be d km. and y be speed of stream

speed of boat = 5 km/h

speed of stream = y km/h

speed of boat in upstram(u): x - y km/h

= 5 - y km/h

speed of boat in downstream (v) = x + y km/h

= 5 + y km/h

ATQ.

$\frac{d}{5-y} = 3\left(\frac{d}{5+y}\right) \quad \left[\because T = \frac{D}{S}\right]$

$\frac{1}{5-y} = \frac{3}{5+y}$

5 + y = 3(5 - y)

5 + y = 15 - 3y

y + 3y = 15 - 5

4y = 10

y = $\frac{10}{4}$

y = $\frac{5}{2}$ km/h

y = $2\frac{1}{2}$ km/h

speed of stream is 2.5 km/h

34. Let x be the random variable denoting the number of times an odd number (the number of successes) when a die is tossed twice.

Then x takes the values 0, 1, 2

Let P(X = 0) be probability of getting no odd number (both times showing even).

$\therefore$ P(X = 0) = $\frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$

Let P(X = 1) be probability of getting odd number once.

$\therefore$ P(X = 1) = $^2C_1\ \frac{3}{6} \times \frac{3}{6} = \frac{6}{6} \times \frac{3}{6} = \frac{1}{2}$

Let P(X = 2) be probability of getting odd number twice.

$\therefore$ P(X = 2) = $\frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$

Thus the probability distribution of X is given by

X = x: x = 0, x = 1, x = 2

P(X = x) $\frac{1}{4}\,\frac{1}{2}\,\frac{1}{4}$

We know that mean E(X) = $\sum x_i p_i = 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4}$

$\therefore$ E(X) = 0 + $\frac{1}{2}$ + $\frac{1}{2}$ = 1

Thus mean E(X) = 1

We know that var(X) = $E(X^2) - [E(X)]^2$

$E(X^2) = \sum x_i^2 p_i = 0 \times \frac{1}{4} + 1^2 \times \frac{1}{2} + 2^2 \times \frac{1}{4}$

$\therefore E(X^2) = 0 + \frac{1}{2} + 4 \times \frac{1}{4} = \frac{3}{2}$

Thus var(X) = $\frac{3}{2} - [1]^2 = \frac{3}{2} - 1 = \frac{1}{2}$

Hence mean is 1 and variance is $\frac{1}{2}$

OR

Let E be the event 'drawing an ace from a pack of 52 cards'.

Then p = P(E) $= \frac{4}{52} = \frac{1}{13}$, so $q = 1 - \frac{1}{13} = \frac{12}{13}$.

As two cards are drawn successively with replacement, events are independent, therefore, it is a problem of binomial distribution.

As 2 cards are drawn, n = 2.

Let X denote the number of aces drawn, then X can take values 0, 1, 2.

P(0) $= {}^2C_0 q^2 = 1 \cdot \left(\frac{12}{13}\right)^2 = \frac{144}{169}$,

P(1) $= {}^2C_1 pq = 2 \cdot \frac{1}{13} \cdot \frac{12}{13} = \frac{24}{169}$,

P(2) $= {}^2C_2 p^2 = 1 \cdot \left(\frac{1}{13}\right)^2 = \frac{1}{169}$.

$\therefore$ Required probability distribution is $\begin{pmatrix} 0 & 1 & 2 \\ \frac{144}{169} & \frac{24}{169} & \frac{1}{169} \end{pmatrix}$.

Mean $= \mu = \Sigma p_i x_i = \frac{1}{169}$ (144 $\times$ 0 + 24 $\times$ 1 + 1 $\times$ 2) $= \frac{26}{169} = \frac{2}{13}$.

Now, $\Sigma p_i x_i^2 = \frac{1}{169}$ (144 $\times$ 0^2 + 24 $\times$ 1^2 + 1 $\times$ 2^2) $= \frac{28}{169}$.

Variance $= \Sigma p_i x_i^2 - \mu^2 = \frac{28}{169} - \left(\frac{2}{13}\right)^2 = \frac{28}{169} - \frac{4}{169} = \frac{24}{169}$.

Standard deviation $= \sqrt{\text{Variance}} = \sqrt{\frac{24}{169}} = \frac{2}{13}\sqrt{6}$.

35. Sum of annuity = ₹ 80,000, Each annuity = ₹ x

r = 6% p.a. $\Rightarrow$ i = 0.06

n = 4 years

$\therefore$ 80000 = $\frac{x}{0.06}$ $[(1 + 0.06)^4 - 1]$

$\Rightarrow$ 4800 = $x[(1.06)^4 - 1]$

$\Rightarrow$ 4800 = x(1.262 - 1) $\Rightarrow$ 4800 = 0.262x

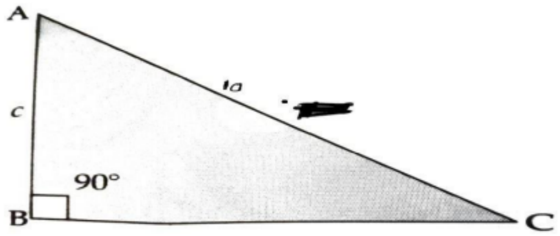
$\Rightarrow x = \frac{4800}{0.262} = 18320.61$

Amount of each annuity = ₹ 18320.61 ~ ₹ 18321

Section E

36. **Read the text carefully and answer the questions:**

The sum of the length of hypotenuse and a side of a right-angled triangle is given by AC + BC = 10



(i) $\frac{100-c^2}{20}$

(ii) $\frac{100-3c^2}{40}$

(iii) $\frac{10\sqrt{3}}{3}$

OR

$\frac{-\sqrt{3}}{2}$

37. **Read the text carefully and answer the questions:**

Flexible payment arrangements, in which the borrower might pay higher sums of his or her choosing, are not the same as EMIs. Borrowers on EMI programmes are usually only allowed to make one set payment per month. Borrowers profit from an EMI since they know exactly how much money they will have to pay towards their loan each month, making personal financial planning easier. Lenders benefit from the loan interest, as it provides a consistent and predictable stream of income.

Example:

A loan of ₹400000 at the interest rate of 6.75% p.a. compounded monthly is to be amortized by equal payments at the end of each month for 10 years.

(Given $(1.005625)^{120}$ = 1.9603, $(1.005625)^{60}$ - 1.4001)

(i) ₹ 4593

(ii) ₹ 233336.89

(iii) ₹ 1312.52

OR

₹ 3280.48

38. i. Z = 10x + 20y

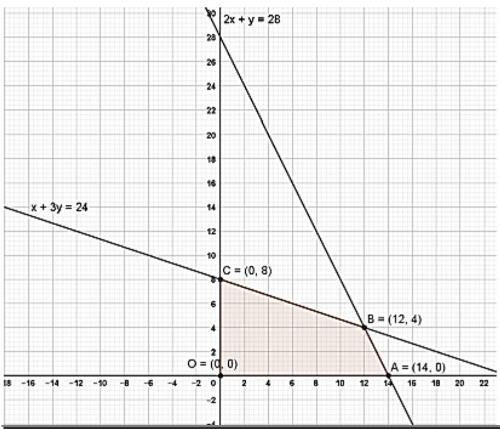
ii. $x + 3y \leq 24$

iii. Other constraints are

$2x + y \leq 28$

$x \geq 0$

$y \geq 0$

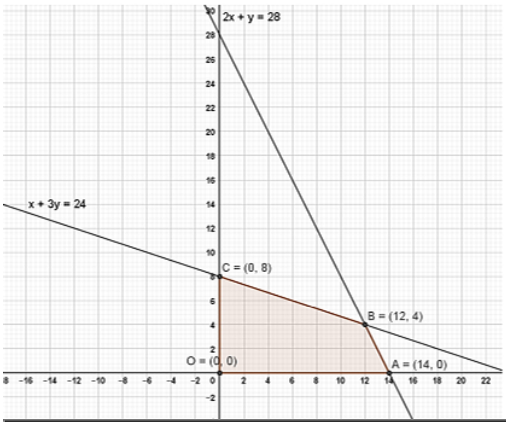


Corner Points	Value of Z
O (0, 0)	0
A (14, 0)	140

B (12, 4)	200 $\rightarrow$ Max value
C (0, 8)	160

$\therefore$ P is maximum at B (12, 4); which is ₹ 200.

OR



Corner Points	**Value of Z**
O (0, 0)	0
A (14, 0)	140
B (12, 4)	200 $\rightarrow$ Max value
C (0, 8)	160

OR

i. 40
ii. 8000
iii. (20,180), (20,0), (40,0)

OR

8000