13. Statistics

Exercise 13.1

1. Question

The marks scored by 40 candidates in an examination (out of 100) is given below:

75, 65, 57, 50, 32, 54, 75, 67, 75, 88, 80, 42, 40, 41, 34, 78, 43,

61, 42, 46, 68, 52, 43, 49, 59, 49, 67, 34, 33, 87, 97, 47, 46, 54,

48, 45, 51, 47, 41, 43.

Prepare a frequency distribution table with the class size 10.

Take the class intervals as (30-39), (40-49), ... and answer the following questions:

(i) Which class intervals have highest and lowest frequency?

(ii) Write the upper and lower limits of the class interval 30-39

(iii) What is the range of the given distribution?

Answer

Theory.

The number of times a particular observation occurs in data is called frequency.

Showing data in tabular form with showing frequency of each distribution. This representation is called Frequency distribution table.

Group of marks	Tally marks	Frequency
90-99	I	1
80-89	III	3
70-79	IIII	4
60-69	HHH	5
50-59	HHH II	7
40-49	HHH HHH HHH I	16
30-39	IIII	4

(i) As looking in Frequency distribution table

Highest Frequency is 16

And the group of marks having highest frequency is 40-49

∴ Maximum number of students have got marks between 40 to 49.

Lowest Frequency is 1

And the group of marks having lowest frequency is 90-99

∴ Minimum number of students have got marks between 90 to 99.

(ii) The given distribution is in inclusive form . It should be converted in exclusive form

Upper limit of 1st interval is 39

Lower limit of 2nd interval is 40

$$\frac{d}{2} = \frac{\text{Lower limit of class} - \text{Upper limit of class before it}}{2} = \frac{40-39}{2} = \frac{1}{2} = 0.5$$

Actual upper limit = Stated upper limit $+\dfrac{d}{2}$ = 39 + 0.5 = 39.5

Actual lower limit = Stated lower limit $-\dfrac{d}{2}$ = 30 – 0.5 = 29.5

∴ Upper limit of group 30-39 is 29.5

Lower limit is lower most value of group

∴ Lower limit of group 30-39 is 30.5

(iii) Range = highest – lowest

Highest marks in class is 97

Lowest marks in class is 32

Range = 97 – 32 = 65

2. Question

Prepare the frequency distribution table for the given set of scores:

39, 16, 30, 37, 53, 15, 16, 60, 58, 26, 28, 19, 20, 12, 14, 24, 59,

21, 57, 38, 25, 36, 34, 15, 25, 41, 52, 45, 60, 63, 18, 26, 43, 36,

18, 27, 59, 63, 46, 48, 25, 33, 46, 27, 46, 42, 48, 35, 64, 24.

Take class intervals as (10-20), (20-30), … and answer the following:

(i) What does the frequency corresponding to the third class interval mean?

(ii) What is the size of each class interval? Find the midpoint of the class interval 30-40.

(iii) What is the range of the given set of scores?

Answer

Theory.

The number of times a particular observation occurs in data is called frequency.

Showing data in tabular form with showing frequency of each distribution. This representation is called Frequency distribution table.

Group of marks	Tally marks	Frequency
60-70	IIII	4
50-60	⊬⊬ I	6
40-50	⊬⊬ IIII I	9
30-40	⊬⊬ ⊬⊬	10
20-30	⊬⊬ ⊬⊬ II	12
10-20	⊬⊬ IIII	9

(i) Third interval is 30-40

The frequency of the Third interval is 10

(ii) Class size = upper limit – lower limit

In class interval 30-40

Upper limit of interval is 40

Lower limit of interval is 30

Class size = 40 – 30 = 10

Midpoint = $\dfrac{\text{upper limit} + \text{lower limit}}{2}$

$$= \frac{40 + 30}{2} = \frac{70}{2} = 35$$

(iii) Range = highest – lowest

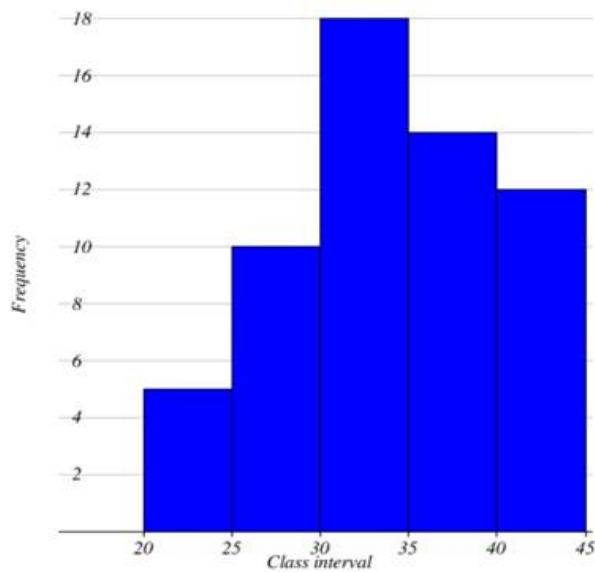Highest Score is 64

Lowest Score is 12

Range = 64 – 12 = 52

Exercise 13.2

1. Question

Draw a histogram to represent the following frequency distribution.

Class - Interval	Frequency
20 - 25	5
25 - 30	10
30 - 35	18
35 - 40	14
40 - 45	12

Answer

2. Question

Draw a histogram to represent the following frequency distribution.

Class - Interval	Frequency
10 - 19	7
20 - 29	10
30 - 39	20
40 - 49	5
50 - 59	15

Answer

The given distribution is in inclusive form. It should be converted in exclusive form

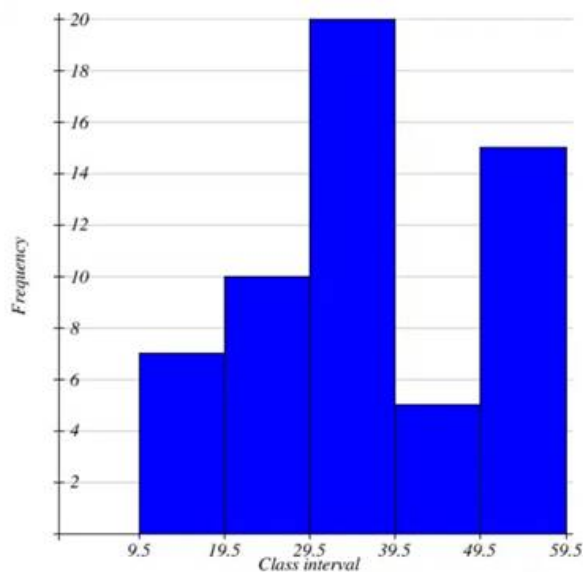Upper limit of 1st interval is 19

Lower limit of 2nd interval is 20

$$\frac{d}{2} = \frac{\text{Lower limit of class} - \text{Upper limit of class before it}}{2} = \frac{20-19}{2} = \frac{1}{2} = 0.5$$

Actual upper limit = Stated upper limit $+ \frac{d}{2}$

Actual lower limit = Stated lower limit $- \frac{d}{2}$

∴ Frequency distribution table

Class - Interval	Actual Class-Interval	Frequency
10 - 19	9.5 – 19.5	7
20 - 29	19.5 – 29.5	10
30 - 39	29.5 – 39.5	20
40 - 49	39.5 – 49.5	5
50 - 59	49.5 – 59.5	15

Exercise 13.3

1. Question

Runs scored by 10 batsmen in a one day cricket match are given.

Find the average runs scored.

23, 54, 08, 94, 60, 18, 29, 44, 05, 86

Answer

Theory.

$$\text{Average} = \frac{\text{Sum of all observation}}{\text{number of observation}}$$

Solution.

$$\text{Average} = \frac{\text{Sum of all observation}}{\text{number of observation}}$$

Sum of runs scored by 10 batsmen

= 23 + 54 + 8 + 94 + 60 + 18 + 29 + 44 + 5 + 86

= 421

There are 10 batsmen

Average = $\dfrac{\text{Total runs}}{\text{number of batsmen}}$

∴ Average = $\dfrac{421}{10}$ = 42.1 runs

2. Question

Find the mean weight from the following table.

Weight (Kg)	29	30	31	32	33
No. of children	02	01	04	03	05

Answer

To find the mean let us prepare frequency distribution table first. We observed that some values are repeated So, to find sum of all we have to multiply weight with number of children and then find the sum.

Let Weight be x and number of children be f

Weight (x) kg	29	30	31	32	33	Σ
No. of children (f)	02	01	04	03	05	15
fx (kg)	58	30	124	96	165	473

Average = $\dfrac{\text{Total weight of children}}{\text{number of children}} = \dfrac{\sum fx}{\sum f}$

= $\dfrac{473}{15}$ = 31.53 kg

∴ Average weight of each child is 31.53 kg

3. Question

Calculate the mean for the following frequency distribution:

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	3	7	10	6	8	2	4

Answer

To find the mean let us prepare frequency distribution table first .

Calculate the mid-point of each interval

And put it as x

Mid-point = $\dfrac{\text{upperlimit} - \text{lower limit}}{2}$

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80	Σ
Midpoint(x)	15	25	35	45	55	65	75	
Frequency(f)	3	7	10	6	8	2	4	40
fx (kg)	45	175	350	270	440	130	300	1710

Average = $\dfrac{\text{Sum of all observation}}{\text{number of observation}}$

= $\dfrac{\sum fx}{\sum f} = \dfrac{1710}{40}$ = 42.75

4. Question

Calculate the mean for the following frequency distribution:

Marks	15-19	20-24	25-29	30-34	35-39	40-44
Frequency	6	5	9	12	6	2

Answer

To find the mean let us prepare frequency distribution table first.

Calculate the mid-point of each interval

And put it as x

Mid-point = $\dfrac{upperlimit - lower\ limit}{2}$

Marks	15-19	20-24	25-29	30-34	35-39	40-44	
Midpoint(x)	17	22	27	32	37	42	
Frequency(f)	6	5	9	12	6	2	40
fx (kg)	102	110	243	384	222	84	1145

Average = $\dfrac{Sum\ of\ all\ observation}{number\ of\ observation}$

$= \dfrac{\sum fx}{\sum f} = \dfrac{1145}{40} = 28.625$

5. Question

Find the median of the data: 15, 22, 9, 20, 6, 18, 11, 25, 14.

Answer

1st arrange data in ascending order

6, 9, 11, 14, 15, 18, 20, 22, 25

As we can count there are odd number of terms

∴ Median = $\dfrac{N+1}{2}$ term

Where N is number of terms

Median = $\dfrac{9+1}{2} = \dfrac{10}{2} = 5^{th}$ term = 15

6. Question

Find the median of the data: 22, 28, 34, 49, 44, 57, 18, 10, 33, 41, 66, 59.

Answer

1st arrange data in ascending order

10, 18, 22, 28, 33, 34, 41, 44, 49, 57, 59, 66

As we count there are odd numbers of terms

∴ Median = Average of $\dfrac{N}{2}$ term and $\dfrac{N}{2}+1$ term

Where N is number of terms which is 12

∴ Median = Average of $\dfrac{12}{2}$ term and $\dfrac{12}{2}+1$ term

= Average of 6th term and 7th term

= Average of 34 and 41

Average = $\dfrac{sum\ of\ terms}{number\ of\ terms}$

Sum of terms = 34 + 41 = 75

Average = $\dfrac{75}{2} = 37.5$

7. Question

Find the median for the following frequency distribution table:

Class Interval	110-119	120-129	130-139	140-149	150-159	160-169
Frequency	6	8	15	10	6	5

Answer

The given distribution is in inclusive form. It should be converted in exclusive form

Upper limit of 1st interval is 119

Lower limit of 2nd interval is 120

$$\frac{d}{2} = \frac{\text{Lower limit of class} - \text{Upper limit of class before it}}{2} = \frac{120-119}{2} = \frac{1}{2} = 0.5$$

Actual upper limit = Stated upper limit $+\frac{d}{2}$

Actual lower limit = Stated lower limit $-\frac{d}{2}$

As we have sum of frequency to be (N) 50

As it is an even number

It has 2 middle scores

$$\frac{N}{2}; \frac{N}{2} + 1$$

$$\frac{50}{2}; \frac{50}{2} + 1 = 25^{th}\text{ term}; 26^{th}\text{ term}$$

For finding 25th and 26th term we need to find cumulative frequency

Class Interval	110-119	120-129	130-139	140-149	150-159	160-169
Actual Class Interval	109.5-119.5	119.5-129.5	129.5-139.5	139.5-149.5	149.5-159.5	159.5-169.5
Frequency	6	8	15	10	6	5
Cumulative frequency	6	14	29	39	45	50
	6	6 + 8	14 + 15	29 + 10	39 + 6	45 + 5

25 and 26 can be covered under Cumulative frequency 29

$\therefore$ 129.5 – 139.5 is Median class

$\Rightarrow$ Low real limit (LRL) = 129.5

$\Rightarrow$ Frequency of median class (f_m) = 15

$\Rightarrow$ Cumulative Frequency of above median class (f_c) = 14

$\Rightarrow$ Size of class interval (i) = 10

Median = LRL $+\dfrac{\frac{N}{2} - f_c}{f_m} \times i$

$= 129.5 + \dfrac{\frac{50}{2} - 14}{15} \times 10$

$= 129.5 + \dfrac{25 - 14}{15} \times 10$

$= 129.5 + \dfrac{110}{15} = 129.5 + 7.33 = 136.83$

8. Question

Find the median for the following frequency distribution table:

Class-Interval	0 - 5	5 - 10	10 -15	15 –20	20 –25	25 –30
Frequency	5	3	9	10	8	5

Answer

As we have sum of frequency to be (N) 40

As it is an even number

It has 2 middle scores

$$\frac{N}{2} ; \frac{N}{2} + 1$$

$$\frac{40}{2} ; \frac{40}{2} + 1 = 20^{th} \text{ term} ; 21^{th} \text{ term}$$

For finding 20th and 21th term we need to find cumulative frequency

Class-Interval	0 - 5	5 - 10	10 –15	15 -20	20 –25	25 –30
Frequency	5	3	9	10	8	5
Cumulative Frequency(f_c)	5	8	17	27	35	40
	5	5 + 3	8 + 9	17 + 10	27 + 8	35 + 5

20 and 21 can be covered under Cumulative frequency 27

∴ 15-20 is Median class

⇒ Low real limit (LRL) = 15

⇒ Frequency of median class (f_m) = 10

⇒ Cumulative Frequency of above median class (f_c) = 17

⇒ Size of class interval (i) = 5

$$\text{Median} = LRL + \frac{\frac{N}{2} - f_c}{f_m} \times i$$

$$= 15 + \frac{\frac{40}{2} - 17}{10} \times 5$$

$$= 15 + \frac{20 - 17}{10} \times 5$$

$$= 15 + \frac{3}{2} = 15 + 1.66 = 16.66$$

9 A. Question

Find the mode for the following data:

4, 3, 1, 5, 3, 7, 9, 6

Answer

In the given data

Only 3 is repeater is twice

∴ 3 is mode of given data

9 B. Question

Find the mode for the following data:

22, 36, 18, 22, 20, 34, 22, 42, 46, 42

Answer

In the given data

22 is repeater thrice and 42 is repeated twice

∴ 22 is mode of given data

10. Question

Find the mode for the following data:

x	5	10	12	15	20	30	40
f	4	8	11	13	16	12	9

Answer

In the given data

The maximum frequency is 16

Which is of number 20

Hence;

Number 20 is repeated maximum times

∴ 20 is the mode of the data

Additional Problems 13

1 A. Question

The size or width of the Class interval (0 - 4) is :

A. 4

B. 5

C. 3

D. 0

Answer

The width of this interval is 5, i.e., 0,1,2,3,4,5.

1 B. Question

The midpoint of the class interval (10 -19) is:

A. 10

B. 14

C. 15D. 14.5

Answer

We know that,

$$\text{midpoint} = \frac{\text{lower limit} + \text{upper limit}}{2}$$

$$\Rightarrow \text{midpoint} = \frac{10 + 19}{2}$$

$$\Rightarrow \text{midpoint} = \frac{29}{2}$$

$\Rightarrow$ midpoint = 14.5

1 C. Question

The difference between the highest and lowest score of a distribution gives:

A. class interval

B. class width

C. range

D. class limit

Answer

By Definition,

In a set of data, the **range** is the difference between the highest and the lowest observation.

1 D. Question

The number of times a particular observation (score) occurs in a data is called its:

A. frequency

B. range

C. class interval

D. class limit

Answer

By definition,

The number of times a particular observation (score) occurs in a data is called its **frequency**.

1 E. Question

In inclusive form, the actual upper limit and lower limit of class interval (0-4) are:

A. -0.5 & 3.5

B. 0.5 & 4.5

C. –1& 5

D. 1 & 5

Answer

In inclusive form,

Actual lower limit = lower limit -0.5

= 0-0.5

= -0.5

And, Actual upper limit = upper limit-0.5

= 4-0.5

= 3.5

1 F. Question

The height of a rectangle in a histogram represents:

A. class interval

B. midpoint

C. frequency density

D. frequency

Answer

During Representation,

The height of a rectangle in a histogram represents the frequency.

1 G. Question

In a histogram, the width of the rectangle indicates:

A. class interval

B. midpoint

C. frequency density

D. frequency

Answer

During the representation,

In a histogram, the width of the rectangle indicates the class interval.

1 H. Question

The mean of scores 10, 15, 12, 15, 15 is:

A. 15

B. 13

C. 13.4

D. 14.3

Answer

$$\text{Mean} = \frac{\text{sum of all the observations}}{\text{No.of observations}}$$

$$\Rightarrow \text{Mean} = \frac{10 + 15 + 12 + 15 + 15}{5}$$

⇒ Mean = 13.4

1 I. Question

Class interval grouping of data is done when:

A. the range of data is small

B. the range of data is large

C. the class intervals are small

D. class intervals are large

Answer

Class interval grouping of data is done when the range of data is large.

1 J. Question

The mean of 6, 4, 7, x and 10 is 8. The value of x is:

A. 10

B. 12

C. 14

D. 13

Answer

$$\text{Mean} = \frac{\text{sum of all the observations}}{\text{No. of observations}}$$

$$\Rightarrow \text{Mean} = \frac{6 + 4 + 7 + x + 10}{5}$$

$$\Rightarrow 8 = \frac{6 + 4 + 7 + x + 10}{5}$$

$\Rightarrow 40 = x + 27$

$\Rightarrow x = 13$

1 K. Question

If n = 10 and Mean = 12, then Σ fx is:

A. 120

B. 1200

C. 12

D. 13

Answer

We know that,

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\Rightarrow 12 = \frac{\Sigma fx}{10}$$

$\Rightarrow \Sigma fx = 120$

1 L. Question

The mean of first three multiples of 5 is :

A. 5

B. 10

C. 15

D. 30

Answer

The first three multiples of 5 are-5, 10, 15.

And, their mean will be-

$$\text{Mean} = \frac{\text{sum of all the observations}}{\text{No. of observations}}$$

$$\Rightarrow \text{Mean} = \frac{5 + 10 + 15}{3}$$

$\Rightarrow$ Mean = 5

1 M. Question

The median of 37, 83, 70, 29, 32, 42, 40 is:

A. 29

B. 30

C. 40

D. 42

Answer

Arranging the data in ascending order, we get-

29, 32, 37, 40, 42, 70, 83

And, since the no. of observations(n) is 7 and which is odd

$$\Rightarrow \text{Median} = \frac{n + 1}{2} \text{ th term}$$

$\Rightarrow$ Median = 4th term

$\Rightarrow$ Median = 40

1 N. Question

In an inclusive class interval (10 -14), the lower real limit is:

A. 9.5

B. 10.5

C. 13.5

D. 14.5

Answer

In inclusive form,

Lower real limit = lower Limit-0.5

= 10-0.5

= 9.5

1 O. Question

In an exclusive class interval (10 -20), the lower real limit is:

A. 20

B. 10

C. 10.5

D. 20.5

Answer

In inclusive form,

Lower real limit = lower Limit

= 10

1 P. Question

The mode of 2, 3, 3, 5, 3, 5, 7, 3,5is:

A. 3

B. 5

C. 3 and 5

D. 3,5,7

Answer

Mode of observations is the data with highest frequency.

Here, 3 appears 3 times -

⇒ mode = 3

1 Q. Question

For given two values of x, 16, 18 the frequencies are respectively 12 and 20. Then the mode is:

A. 16

B. 18

C. 12

D. 20

Answer

Mode of observations is the data with highest frequency and here 18 has highest frequency i.e., 20.

⇒ Mode = 18

1 R. Question

A collection of data having more than 3 modes is said to be:

A. uni-mode

B. bi-mode

C. tri-mode

D. multi-mode

Answer

A collection of data having more than 3 modes is said to be multimode.

2. Question

Prepare a frequency distribution table for the scores given:

42,22,55,18,50,10,33,29,17,29,29,27,34,15,40,42,40,41,35,27,

44,31,38,19,54,55,38,19,20,30,42,59,15,19,27,23,40,32,28,51.

Take the class intervals as 10-20, 20-30, 30-40, 40-50, 50-60. From

the frequency distribution table answer the following questions:

(i) What does the frequency corresponding to the class interval 20-30 indicate?

(ii) In which class intervals are the scores 10, 20 and 30 included?

(iii) Find the range of the scores.

Answer

X	f
10-20	8
20-30	10
30-40	8
40-50	8
50-60	6
Total	40

(i) The frequency corresponding to the class interval 20-30 indicate that there are 10 values lying between 20 and 30 and which is highest of all.

(ii) 10 will be included in 10-20, 20 will be included in 20-30 and 30 will be included in 30-40.

(iii) Since the intervals includes data values from 10 to 59.

⇒ Range = 59-10

⇒ Range = 49

3. Question

The following are the marks scored in a unit test (out of 25). Prepare a frequency distribution table, taking the class intervals as 0-4,

5-9, 10-14, 15-19, 20-24:

21,14,3,7,23,18,24,16,18,17,20,10,17,18,21,23,19,12,14,9,16,1 8,12,14,11.

From the table (i) find the mid-points of each class interval (ii) find the class interval having a maximum frequency (iii) find the range of the scores.

Answer

X	f
0-4	1
5-9	2
10-14	7
15-19	9
20-24	6
Total	25

(i) Mid-points of 0-4, 5-9, 10-14, 15-19, 20-24 are 2, 7, 12, 17, 22 respectively.

(ii) Here maximum frequency is 12 which is corresponding to the 15-19 class interval.

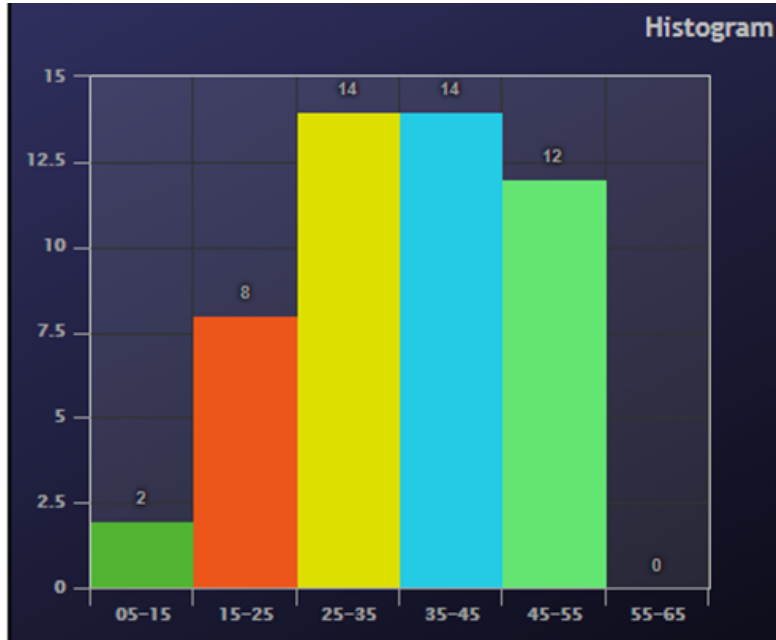(iii) Range = Highest data value-lowest data value

⇒ Range = 24-3 = 21

4. Question

Draw a histogram for the following frequency distribution.

Class - Interval	Frequency
5-15	2
15-25	8
25-35	14
35-45	14
45-55	12

Answer

5. Question

Draw a histogram for the following frequency distribution.

Class - Interval	Frequency
0-10	4
11-20	18
21-30	12
31-40	6
41-50	20
51-60	10

Answer

Histogram
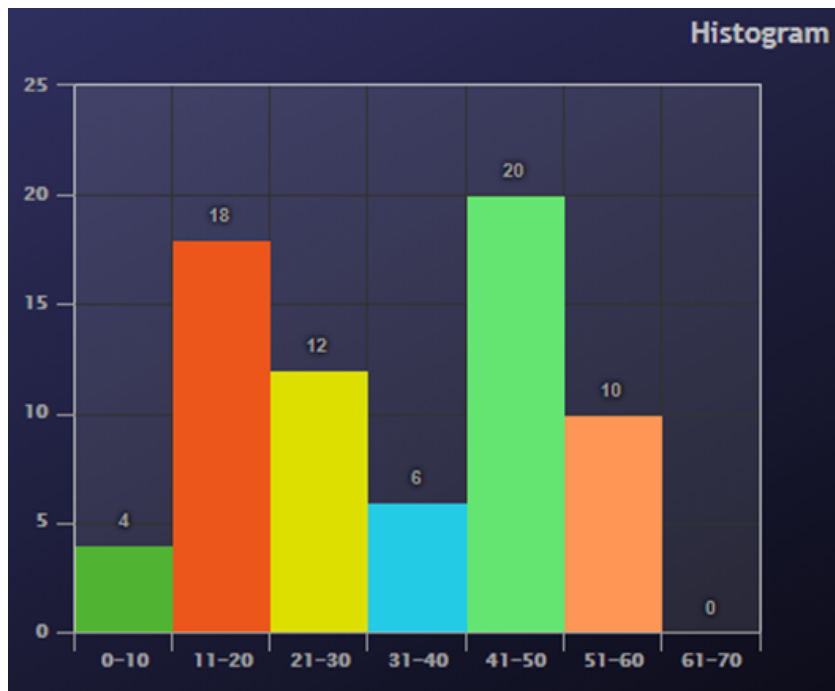
6. Question

The marks obtained by 12 students in a mathematics examination are given below.

48,78,93,90,66,54,83,58,60,75,89,84.

Find (i) the mean of the marks (ii) the mean mark of the students if each student is given 4 grace marks.

Answer

(i) $\text{Mean} = \frac{\text{sum of all the observations}}{\text{No. of observations}}$

$\Rightarrow \text{Mean} = \frac{48 + 78 + 93 + 90 + 66 + 54 + 83 + 58 + 60 + 75 + 89 + 84}{12}$

$\Rightarrow \text{Mean} = \frac{878}{12}$

$\Rightarrow \text{Mean} = 73.17$

(ii) $\text{Mean} = \frac{\text{sum of all the observations}}{\text{No. of observations}}$

Since, each student got 4 grace marks implies we need to add 4 marks 12 times.

$\Rightarrow \text{Mean}$
$= \frac{48 + 78 + 93 + 90 + 66 + 54 + 83 + 58 + 60 + 75 + 89 + 84 + (12 \times 4)}{12}$

$\Rightarrow \text{Mean} = \frac{926}{12}$

$\Rightarrow \text{Mean} = 77.17$

7. Question

If the mean of 8,12,21,42, x is 20, find the value of x.

Answer

$\text{Mean} = \frac{\text{sum of all the observations}}{\text{No. of observations}}$

$\Rightarrow \text{Mean} = \frac{8 + 12 + 21 + 42 + x}{5}$

$$\Rightarrow 20 = \frac{8 + 12 + 21 + 42 + x}{5}$$

⇒ 100 = x+83

⇒ x = 17

8. Question

Find the mean for the following distribution:

12,14,10,12,15,12,18,10,15,11,19,20,12,15,19,10,18,16,20,17.

Answer

$$\text{Mean} = \frac{\text{sum of all the observations}}{\text{No.of observations}}$$

⇒ Mean

$$= \frac{12 + 14 + 10 + 12 + 15 + 12 + 18 + 10 + 15 + 11 + 19 + 20 + 12 + 15 + 19 + 10 + 18 + 16 + 20 + 17}{20}$$

$$\Rightarrow \text{Mean} = \frac{281}{20}$$

⇒ Mean = 14.75