14. HEAT TRANSFER

1. INTRODUCTION

Heat can be transformed from one place to another place by the three processes - conduction, convection and radiation. In conduction, the heat flows from a place of higher temperature to a place of lower temperature through a stationary medium. The molecules of the medium oscillate about their equilibrium positions more violently at a place of higher temperature and collide with the molecules of adjacent position, thus transferring a part of their energy to these molecules which now vibrate more violently. Thus heat can be transmitted by collision of molecules. In metals, the conduction of heat takes place by the movement of free electrons. In the cases of liquids and gases, the heat is transferred not only by collision but also by motion of heated molecules which carry the heat in such media. This process is called convection. When a liquid in a vessel is heated, the lighter molecules present in the lower layer of the liquid get heated which rise to the surface of the liquid and cold molecules at the surface go towards the bottom of vessel. These are convection currents and are the major means of heat transport in fluids. Radiation is mode of transfer of heat in which the heat travels directly from one place to another without the role of any intervening medium. The heat from the sun propagates mostly through vacuum to reach the earth by the process of radiation.

2. CONDUCTION

The figure shows a rod whose ends are in thermal contact with a hot reservoir at temperature T_1 and a cold reservoir at temperature T_2 . The sides of the rod are insulated, hence heat transfer is only along the rod and not through its sides. The molecules at the hot reservoir have greater vibrational energy. This energy is transferred by collisions to the atoms at the end face of the rod. These atoms in turn transfer the energy to their neighbors further along the rod. Such transfer of heat through a substance in which heat is transported without direct mass transport is called conduction.

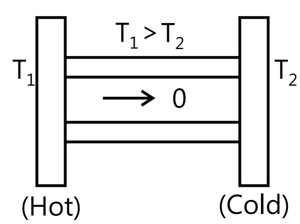


Figure 14.1

The quantity of heat conducted Q in time t across a slab of length L, area of cross-section A and steady state temperature θ_1 and θ_2 at respective hot and cold ends is given by $Q = \frac{kA(\theta_1 - \theta_1)t}{L}$, where k is the coefficient of thermal conductivity which is equal to the quantity of heat flowing per unit time through unit area of cross-section of a material per unit length along the direction of flow of heat.

Units of k are kilocalorie/meter second degree centigrade or $J.m^{-1}sec^{-1}\ K^{-1}$. In C.G.S. units,

k is expressed in $cal cm^{-1}\ (°C)^{-1}\ sec^{-1}$

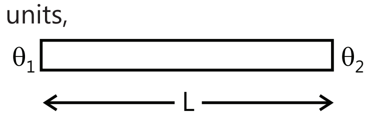


Figure 14.2

The temperature Gradient / (unit distance) = $-\frac{d\theta}{dx}$

$$\therefore \quad Q = -kA\left(\frac{d\theta}{dx}\right)t; \qquad \frac{\Delta Q}{\Delta t} = -kA\frac{dT}{dx}$$

The quantity dT/dx is called the temperature gradient. The minus sign indicates that dT/dx is negative along the direction of the heat flow, i.e., heat flows from a higher temperature to a lower one.

$$\frac{dT}{dx} = H = \frac{\Delta t}{L/kA} = \frac{\Delta T}{R}$$

Here ΔT = temperature difference (TD) and $R = \frac{L}{kA}$ = Thermal resistance of the rod.

PLANCESS CONCEPTS

This relation is mathematically equivalent to Ohm's Law and can be used very effectively in solving problems effectively by considering temperature analogous to potential and heat transferred per unit time as current.

Nivvedan (JEE 2009, AIR 113)

Heat flow through a conducting rod	**Current flow through a resistance**
Heat current $H = \frac{dQ}{dt}$ =Rate of heat flow $H = \frac{\Delta T}{R} = \frac{T(\text{temp diff})}{R}$ where $R = \frac{L}{kA}$ and k = Thermal conductivity	Electric current $i = \frac{dq}{dt}$ = Rate of charge flow $i = \frac{\Delta V}{R} = \frac{PD(\text{potential diff})}{R}$; $\quad R = \frac{i}{\sigma A}$ σ = Electrical conductivity.

3. GROWTH OF ICE ON PONDS

When temperature of the atmosphere falls below 0°C, the water in the pond starts freezing. Let at time t thickness of ice in the pond is y and atmospheric temperature is -T°C. The temperature of water in contact with the lower surface of ice will be 0°C.

Using $\frac{dQ}{dt} = L\left(\frac{dm}{dt}\right)$; $\qquad \frac{TD}{R} = L\frac{d}{dt}\{A\rho y\}$ $\qquad$ (A = Area of pond)

$$\therefore \frac{\left[0-(-T)\right]}{(y/kA)} = LA\rho.\frac{dy}{dt} \quad \therefore -\frac{dy}{dt} = \frac{kT}{\rho}.\frac{1}{Ly}$$ where L -> Latent heat of fusion

And hence time taken by ice to grow a thickness y $\quad t = \frac{\rho L}{kT}\int_0^y y\,dy$ or $t = \frac{1}{2}\frac{\rho L}{kT}y^2$

Time does not depend on the area of pond.

PLANCESS CONCEPTS

Time taken by ice to grow on ponds is independent of area of the pond and it is only dependent only the thickness of ice sheet.

Vaibhav Krishnan (JEE 2009, AIR 22)

4. SERIES AND PARALLEL CONNECTION OF RODS

4.1 Series Connection

Consider two rods of thermal resistances R_1 and R_2 joined one after the other as shown in figure. The free ends are kept at temperatures T_1 and T_2 with $T_1>T_2$. In steady state, any heat that goes through the first rod also goes through the second rod. Thus, the same heat current passes through the two rods. Such a connection of rods is called a series connection.

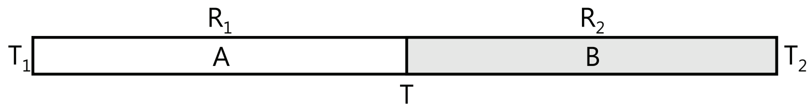


Figure 14.3

Suppose, the temperature of the junction is T, the heat current through the first rod is,

$$i=\frac{\Delta Q}{\Delta t}=\frac{T_1-T}{R_1} \text{ or } T_1-T=R_1 i \qquad \text{... (i)}$$

and that through the second rod is $i=\dfrac{\Delta Q}{\Delta t}=\dfrac{T-T_2}{R_2}$ or $T-T_2=R_2 i$ ... (ii)

Adding (i) and (ii) $\quad T_1-T_2=\left(R_1+R_2\right)i$ or $i=\dfrac{T_1-T_2}{R_1+R_2}$

Thus, the two rods together is equivalent to a single rod of thermal resistance R_1+R_2.

If more than two rods are joined in series, the equivalent thermal resistance is given by, $R=R_1+R_2+R_3+\ldots$

4.2 Parallel Connection

Now, suppose the two rods are joined at their ends as shown in figure. The left end of both the rods are kept at temperature T_1 and the right ends are kept at temperature T_2.

So the same temperature difference is maintained between the ends of each rod. Such a connection of rods is called a parallel connection. The heat current going through the first rod is $i_1=\dfrac{\Delta Q_1}{\Delta t}=\dfrac{T_1-T_2}{R_1}$

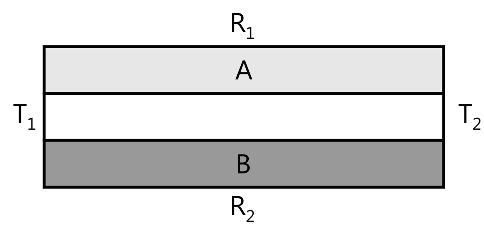


Figure 14.4

and that through the second rod is $i_2=\dfrac{\Delta Q_2}{\Delta t}=\dfrac{T_1-T_2}{R_2}$

The total heat current going through the left end is $i=i_1+i_2=(T_1-T_2)\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)$

or $i=\dfrac{T_1-T_2}{R}$

Where $\dfrac{1}{R}=\dfrac{1}{R_1}+\dfrac{1}{R_2}$ ... (i)

5. RADIAL FLOW OF HEAT THROUGH A CYLINDRICAL TUBE

Consider a cylindrical tube of length l and respective inner and outer radii as r_1 and r_2. If the heat flows radially i.e., perpendicular to the axis of the tube from the steady state temperatures θ_1 at the inner surface to the temperature θ_2 at the outer surface, then the rate of heat flowing through an element of shell lying between radius r and r+ dr is given by $\Delta Q=-k\left(2\pi r\ell\right)\dfrac{d\theta}{dr}$ where $d\theta$ is temperature difference across the shell.

It can be integrated for total heat flow per second.

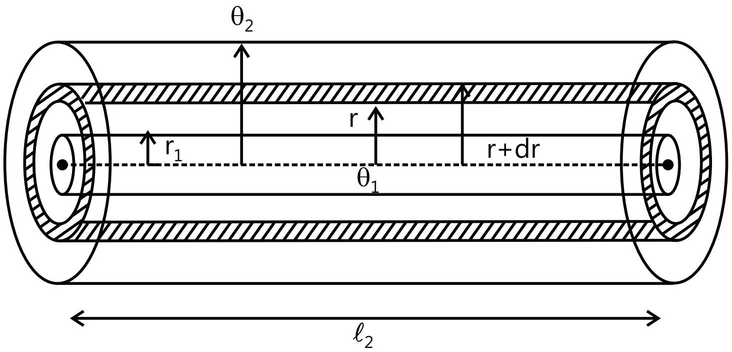


Figure 14.5

$\therefore$ Total heat flowing per second,

$$Q=\frac{2\pi k\ell(\theta_1-\theta_2)}{\int_{r_1}^{r_2}\frac{dr}{r}};\quad Q=\frac{2\pi k\ell(\theta_1-\theta_2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

PLANCESS CONCEPTS

No mass movement of matter occurs in conduction. Solids are better conductors than liquids, liquids are better conductors than gases.

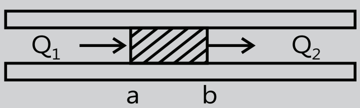


Figure 14.6

Consider a section ab of a rod as shown in figure. Suppose Q_1 heat enters into the section at 'a' and Q_2 leaves at 'b', then $Q_2 < Q_1$.

Part of the energy Q_2 - Q_1 is utilized in raising the temperature of section ab and the remaining is lost to the atmosphere through ab. If heat is continuously supplied from the left end of the rod, a stage comes when temperature of the section becomes constant. In that case Q_1=Q_2 if rod is insulated from the surroundings (or loss through ab is zero). This is called the steady state condition. Thus, in steady state temperature of different sections of the rod becomes constant (but not same).

Nitin Chandrol (JEE 2012, AIR 134)

Illustration 1: One face of a copper cube of edge 10 cm is maintained at 100°C and the opposite face is maintained at 0°C. All other surfaces are covered with an insulating material. Find the amount of heat flowing per second through the cube. Thermal conductivity of copper is 385 Wm^{-1} $°C^{-1}$. **(JEE MAIN)**

Sol: Always consider the A which perpendicular to the flow of heat.

The heat flows from the hotter face towards the colder face. The area of cross section perpendicular to the heat flow is $A=(10cm)^2$

The amount of heat flowing per second is $\frac{\Delta Q}{\Delta t}=KA\frac{T_1-T_2}{X}=(385Wm^{-1}°C^{-1})\times(0.1m)^2\times\frac{(100°C-0°C)}{0.1m}=3850W.$

Illustration 2: A cylindrical block of length 0.4 m and area of cross-section 0.04m^2 is placed coaxially on a thin metallic disc of mass 0.4 kg and of the same cross-section. The upper face of the cylinder is maintained at the constant temperature of 400 K and initial temperature of the disc is 300 K. If the thermal conductivity of the material of the cylinder is 100 watt/m-K and the specific heat of the material of the disc is 600 J/kg-K, how long will it take for the temperature of the disc to increase to 350 K? Assume, for the purpose of calculation, the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder.

(JEE ADVANCED)

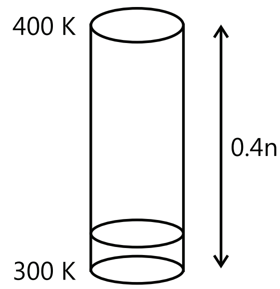


Figure 14.7

Sol: Write the equation rate of heat transfer at any temperature 'T' for the disc. Rate of heat transfer proportional to rate of change in temperature.

As heat is conducted from the cylinder to the disc, the temperature of the disc increases. If the temperature of the disc at some instant is T, then rate of flow of heat through the cylinder at that instant is $\frac{dQ}{dt} = \frac{KA(400-T)}{L}$ ... (i)

If dT is the further increase in the temperature of the disc in the infinitesimal time interval dt,

then $\frac{dQ}{dt} = ms\frac{dT}{dt}$ ... (ii)

Where m is the mass of the disc and c is its specific heat.

From equations (i) and (ii)

$$\frac{KA(400-T)}{L} = ms\frac{dT}{dt};\ dt = \frac{msL}{KA}\left[\frac{dT}{400-T}\right]$$

Integrating we get, $t = \frac{msL}{KA}\int_{300}^{350}\frac{dT}{400-T} = \frac{msL}{KA}\times 2.303\log_{10}\left[\frac{400-300}{400-350}\right]$

$$= \frac{0.4\times 600\times 0.4}{10\times 0.04}\times 2.303\times 0.3010 = 166s.$$

6. CONVECTION

In this process, actual motion of heated material results in transfer of heat from one place to another. For example, in a hot air blower, air is heated by a heating element and is blown by a fan. The air carries the heat wherever it goes. When water is kept in a vessel and heated on a stove, the water at the bottom gets heated due to conduction through the vessel's bottom. Its density decreases and consequently it rises. Thus, the heat is carried from bottom to the top by the actual movement of the parts of the water. If the heated material is forced to move, say by a blower or by a pump, the process of heat transfer is called forced convection. If the material moves due to difference in density, it is called natural or free convection.

PLANCESS CONCEPTS

The convection currents created in a room by a radiator means that the warm air is circulated around and the warming is more uniform than just being the air around the radiator. When heating water on a stove, the convection currents created by the rising hot water means that all the water gets heated instead of just the water at the very bottom of the pan. Some rainfall is also caused by moist air being heated and rising, then cooling quickly and allowing the water vapor to condense into rain.

Anand K (JEE 2011, AIR 47)

7. RADIATION

The third means of energy transfer is radiation which does not require a medium. The best known example of this process is the radiation from Sun. All objects radiate energy continuously in the form of electromagnetic waves. The rate at which an object radiates energy is proportional to the fourth power of its absolute temperature. This is known as the **Stefan's law** and is expressed in equation form as $P = \sigma AeT^4$

Here P is the power in watts(J/s) radiated by object, A is the surface area in m^2,e lies between 0 and 1 and is called **emissivity** of the object and σ is universal constant called Stefan's constant, which has the value, $\sigma = 5.67 \times 10^{-8}\, W/m^2 - K^4$.

8. PERFECTLY BLACK BODY

A body that absorbs all the radiation incident upon it and has as emissivity equal to 1 is called a perfectly black body. A black body is also an ideal radiator. It implies that if a black body and an identical another body is kept at the same temperature, then the black body will radiate maximum power as is obvious from equation $P = \sigma AeT^4$

This is also because e=1 for a perfectly black body while for any other body, e<1.

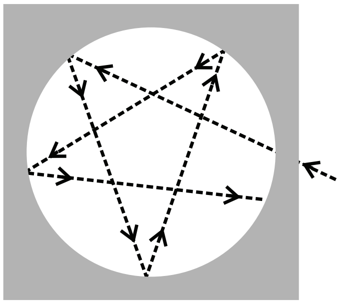


Figure 14.8

PLANCESS CONCEPTS

Always remember that black body is a perfect absorber and emitter of light. At temperatures higher than the surrounding, it is the most shining thing and at lower temperatures it is the darkest thing.

There is no perfect black body. Materials like black velvet or lamp black come close to being ideal black bodies, but the best practical realization of an ideal black body is a small hole leading into a cavity, as this absorbs 98% of the radiation incident on them.

GV Abhinav (JEE 2012, AIR 329)

Illustration 3: A solid copper sphere of density ρ, specific heat c and radius r is at temperature T_1. It is suspended inside a chamber whose walls are at temperature 0K. What is the time required for the temperature of sphere to drop to T_2? Take the emissivity of the sphere to be equal to e. **(JEE MAIN)**

Sol: Heat lost by radiation cause temperature to fall.

The rate of loss of energy due to radiation, $P = \sigma AeT^4$. This rate must be equal to $mc\frac{dT}{dt}$ Hence, $-mc\frac{dT}{dt} = \sigma AeT^4$

Negative sign is used as temperature decreases with time. In this equation,

$$m = \left(\frac{4}{3}\pi r^3\right)\rho \text{ and } A = 4\pi r^2 \therefore -\frac{dT}{dt} = \frac{3e\sigma}{\rho cr}T^4 \text{ or } -\int_0^1 dt = \frac{\rho cr}{3e\sigma}\int_{T_1}^{T_2}\frac{dT}{T^4}; \quad t = \frac{\rho cr}{9e\sigma}\left(\frac{1}{T_2^3} - \frac{1}{T_1^3}\right)$$

9. ABSORPTIVE POWER ‘a’

“It is defined as the ratio of the radiant energy absorbed by a body in a given time to the total radiant energy incident on it in the same interval of time.”

$$a = \frac{\text{Energy absorbed}}{\text{Energy incident}}$$

As a perfectly black body absorbs all radiations incident on it, the absorptive power of perfectly black body is maximum and unity.

10. SPECTRAL ABSORPTIVE ‘a_λ’

This absorptive power ‘a’ refers to radiations of all wavelengths (or the total energy) while the spectral absorptive power is the ratio of radiant energy absorbed by a surface to the radiant energy incident on it for a particular wavelength λ. It may have different values for different wavelengths for a given surface. Let us take an example, suppose a = 0.6, a_λ = 0.4 for 1000 Å and a_λ = 0.7 for 2000 Å for a given surface. Then it means that this surface will absorb only 60% of the total radiant energy incident on it. Similarly it absorbs 40% of the energy incident on it corresponding to 1000 Å and 70% corresponding to 2000 Å. The spectral absorptive power a_λ is related to absorptive power a through the relation $a = \int_0^\infty a_\lambda d\lambda$

11. EMISSIVE POWER ‘e’

(Don’t confuse it with the emissivity e which is different from it, although both have the same symbols e).

“For a given surface it is defined as the radiant energy emitted per second per unit area of the surface.” It has the units of W/m^2 or J/s-m^2, for a black body $e = \sigma T^4$

Note: Absorptive power is dimensionless quantity where emissive power is not.

12. SPECTRAL EMISSIVE POWER

Similar to the definition of the spectral absorptive power, it is emissive power for a particular wavelength λ.

Thus, $e = \int_0^\infty e_\lambda d\lambda$

13. KIRCHHOFF’S LAW

The ratio of emissive power to absorptive power is the same for all bodies at a given temperature and is equal to the emissive power E of a blackbody at that temperature. Thus,

$$\frac{E(\text{body})}{a(\text{body})} = E(\text{blackbody})$$

Kirchhoff's law tells that if a body has high emissive power, it should also have high absorptive power to have the ratio e/a same. Similarly, a body having low emissive power should have low absorptive power. Kirchhoff's law may be easily proved by a simple argument as described below.

Consider two bodies A and B of similar geometrical shapes placed in an enclosure. Suppose A is any random body and B is a blackbody. In thermal equilibrium, both the bodies will have the same temperature as the temperature of the enclosure. Suppose an amount ΔU of radiation falls on the body A in a given time Δt. As A and B have the same geometrical shapes, the radiation falling on the blackbody B is also ΔU. The blackbody absorbs all of this ΔU. As the temperature of the blackbody remains constant, it also emits an amount ΔU of radiation in that time. If the emissive power of the blackbody is e_0, we have $\Delta U \propto E_0$ or $\Delta U = kE_0$ ... (i)

where k is constant.

Let the absorptive power of A be a. Thus, it absorbs an energy of a ΔU of the radiation falling on it in time Δt. As its temperature remains constant, it must also emit the same energy a ΔU in that time. If the emissive power of the body A is e, we have a $\Delta U = ke$ ... (ii)

The same proportionality constant k is used in (i) and (ii) because the two bodies have identical geometrical shapes and radiation emitted in the same time Δt is considered.

From (i) and (ii),

$$a = \frac{E}{E_0} \text{ or } \frac{E}{a} = E_0 \text{ or } \frac{E(\text{body})}{a(\text{body})} = E(\text{blackbody})$$

PLANCESS CONCEPTS

It can be thought like, good absorber is a good emitter because at some point of time, it might have stored energy because it is a good absorber. Now as soon as the temperature of the surrounding becomes low than that of the body, this energy starts decreasing until the steady state is reached. Hence, it must be a good emitter too.

Good absorbers for a particular wavelength are also good emitters of the same wavelength.

Anurag Saraf (JEE 2011, AIR 226)

14. STEFANS-BOLTZMANN LAW

The energy of thermal radiation emitted per unit time by a blackbody of surface area A is given by $u = \sigma AT^4$ ... (i)

Where is a universal constant known as Stefan Boltzmann constant and T is its temperature on absolute scale. The measured value of σ is 5.67×1^{-8} Wm^{-2} K^{-4}. Equation (i) itself is called the Stefan-Boltzmann law. Stefan had suggested this law based on his experimental data on radiation and Boltzmann derived it from thermo dynamical analysis. The law is also quoted as Stefan's law and the constant σ as Stefan constant.

A body which is not a blackbody, emits less radiation than given by equation (i). It is, however, proportional to T^4. The energy emitted by such a body per unit time is written as $u = e\sigma AT^4$ ... (ii)

Where e is a constant for the given surface having a value between 0 and 1. This constant is called the emissivity of the surface. It is zero for completely reflecting surface and is unity for a blackbody.

Using Kirchhoff's law $\frac{E(\text{body})}{E(\text{blackbody})} = a$ ... (i)

Where a is the absorptive power of the body. The emissive power E is proportional to the energy radiated per unit time, that is, proportional to u. Using above equations, $\frac{e\sigma AT^4}{\sigma AT^4} = a$ or e=a.

Thus, emissivity and absorptive power have the same value.

Consider a body of emissivity e kept in thermal equilibrium in a room at temperature T_0.

The energy of radiation absorbed by it per unit time should be equal to the energy emitted by it per unit time. This is because the temperature remains constant. Thus, the energy of the radiation absorbed per unit time is $u = e\sigma A T_0^4$.

Now suppose the temperature of the body is changed to T but room temperature remains T_0. The energy of the thermal radiation emitted by the body per unit time is $u = e\sigma A T^4$.

The energy absorbed per unit time by the body is $u_0 = e\sigma A T_0^4$.

Thus, the net loss of thermal energy per unit time is $\Delta u = u - u_0 = e\sigma A(T^4 - T_0{}^4)$ ... (iii)

Illustration 4: A blackbody of surface area $10cm^2$ is heated to 127°C and is suspended in a room at temperature 27°C. Calculate the initial rate of loss of heat from the body to the room. **(JEE MAIN)**

Sol: Heat lost by radiation and gained by absorption.

For a blackbody at temperature T, the rate of emission is $u = \sigma A T^4$. When it is kept in a room at temperature T_0, the rate of absorption is $u_0 = \sigma A T_0^4$.

The net rate of loss of heat is $u - u_0 = \sigma A(T^4 - T_0{}^4)$

Here $A = 10 \times 10^{-4}\, m^2$ $T = 400K$ $T_0 = 300K$

Thus, $u - u_0 = (5.67 \times 10^{-8}\, Wm^{-2}K^{-4})(10 \times 10^{-4}\, m^2)(400^4 - 300^4)K^4 = 0.99W$

Illustration 5: Energy falling on 1.0 area placed at right angles to a sun beam just outside the earth's atmosphere is 1.35 K joule in one second. Find sun's surface temperature. Mean distance of earth from sun is $1.50 \times 10^8 km$, mean distance of sun= $1.39 \times 10^6 km$ and Stefan's constant= 5.67×10^{-8} watt $m^{-2}K^{-4}$. **(JEE MAIN)**

Sol: $\sigma A_{sun}\, T^4 = S \times A_{earth}$

The temperature of the sun is given by

$$T^4 = \frac{S}{\sigma}\left(\frac{R}{r}\right)^2$$

$$\frac{S}{\sigma} = \frac{1.35 kJ/m^2 - sec}{5.67 \times 10^{-8}\, watt/m^2 - K^2} = \frac{135 \times 10^3\, watt/m^2}{5.67 \times 10^{-8}\, watt/m^2 - K^4} = 2.38 \times 10^{10} K^4$$

$$\frac{R}{r} = \frac{1.50 \times 10^8 km}{0.695 \times 10^6 km} = 215.8$$

$$\therefore T^4 = (2.38 \times 10^{10} K^4)(215.8)^2 = 1108 \times 10^{12} K^4$$

$$T = 5.770 \times 10^3 K \text{ or } T = 5770K$$

15. NEWTON'S LAW OF COOLING

The rate of cooling of a body is directly proportional to the difference of temperature of the body over its surroundings.

If a body at temperature θ_1 is placed in surroundings at lower temperature θ_2, the rate of cooling is given by $\frac{dQ}{dt} \propto (\theta_1 - \theta_2)$ where dQ is the quantity of heat lost in time dt.

Newton's law of cooling gives $\frac{dQ}{dt} = -k\left(\theta_1 - \theta_2\right)$ where k is constant.

If a body of mass m and specific heat s loses a temperature $d\theta$ in time dt, then $\frac{dQ}{dt} = ms\frac{d\theta}{dt} = -k(\theta_1 - \theta_2)$

Illustration 6: A liquid cools from 70°C to 60°C in 5 minutes. Calculate the time taken by the liquid to cool from 60°C to 50°C, if the temperature of the surrounding is constant at 30°C. **(JEE MAIN)**

Sol: Use newton's law cooling and taking temperature of the body is average of initial and final value.

The average temperature of the liquid in the first case is $\theta_1 = \frac{70°C + 60°C}{2} = 65°C$

The average temperature difference from the surrounding is $\theta_1 - \theta_0 = 65°C - 30°C = 35°C$.

The rate of fall of temperature is $-\frac{d\theta_1}{dt} = \frac{70°C - 60°C}{5\text{mins}} = 2°C\text{min}^{-1}$.

From Newton's law of cooling, $2°C\text{min}^{-1} = bA(35°C)$ Or $bA = \frac{2}{35\text{min}}$ ... (i)

In the second case, the average temperature of the liquid is $\theta_2 = \frac{60°C + 50°C}{2} = 55°C$

So that, $\theta_2 - \theta_0 = 55°C - 30°C = 25°C$

If it takes a time t to cool down from 60°C to 50°C, the rate of fall in temperature is $-\frac{d\theta_2}{dt} = \frac{60°C - 50°C}{t} = \frac{10°C}{t}$.

From Newton's law of cooling and (i), $\frac{10°C}{t} = \frac{2}{35\text{min}} \times 25°C$ Or $t = 7\text{min}$.

Illustration 7: At midnight, with the temperature inside your house at 70°F and the temperature outside at 20°F, your furnace breaks down. Two hours later, the temperature in your house has fallen to 50°F. Assume that the outside temperature remains constant at 20°F. At what time will the inside temperature of your house reach 40°F? **(JEE ADVANCED)**

Sol: Newton's law of cooling, follow logarithm curve in cooling.

The boundary value problem that models this situation is

$$\frac{dT}{dt} = k(20 - T) \qquad \begin{array}{l} T(0) = 70 \\ T(2) = 50 \end{array}$$

Where time 0 is midnight. The solution of this boundary value problem is $T = 20 + 50\left(\frac{3}{5}\right)^{t/2}$

This is obtained by solving above differential equation.

Note (for the purpose of a reasonableness check) that this formula given us

$$T(0) = 20 + 50\left(\frac{3}{5}\right)^{0/2} = 70. \text{ and } T(2) = 20 + 50\left(\frac{3}{5}\right)^{2/2} = 50.$$

To find when the temperature in the house will reach 40°F, we must solve equation $20 + 50\left(\frac{3}{5}\right)^{t/2} = 40$

The solution of this equation is $t = 2\left(\frac{\ln(2/5)}{\ln(3/5)}\right) \approx 3.6$

Thus, the temperature in the house will reach 40°F a little after 3.30 a.m.

PLANCESS CONCEPTS

Newton's law of cooling can also be thought in the context of Stefan-Boltzmann law by considering the temperature difference between the body and the surroundings very close to zero, i.e. it can be considered as a special case of the latter.

Vijay Senapathi (JEE 2011, AIR 71)

16. WIEN'S DISPLACEMENT LAW

At ordinary temperatures (below about 600°C), the thermal radiation emitted by bodies is invisible, most of them lie in wavelengths longer than visible light. The figure shows how the energy of a black body radiation varies with temperature and wavelength. As the temperature of the black body increases, two different behaviors are observed. The first effect is that the peak of the distribution shifts to shorter wavelengths. This shift is found to satisfy the following relationship called Wien's displacement law.

$\lambda_{max}T=b$. Here b is a constant called Wien's constant. The value of this constant in SI unit is 2.898×10^{-3} m-K. Thus, $\lambda_{max}\alpha 1/T$

Here λ_{max} is the wavelength corresponding to the maximum spectral emissive power e_λ.

The second effect is that the total amount of energy the black body emits per unit area per unit time ($=\sigma T^4$) increases with fourth power of absolute temperature T.

This is also known as emissive power. We know

$e=\int_0^\infty e_\lambda d\lambda=$ Area under graph, e_λ Vs $\lambda=\sigma T^4$

Area $\propto T^4$ $\;A_2=(2)^4=16A_1$

Thus, if the temperature of the black body is made two fold, λ_{max} remains half while the area becomes 16 times.

PLANCESS CONCEPTS

Have you ever wondered how do scientists calculate the temperature of sun and other stars? It is through this law.

Ankit Rathore (JEE Advanced 2013, AIR 158)

Illustration 8: The light from the sun is found to have a maximum intensity near the wavelength of 470 nm. Assuming that the surface of the sun emits as a blackbody, calculate the temperature of the surface of the sun. **(JEE MAIN)**

Sol: Formula of Wien's displacement law.

For a blackbody, λ_m T=0.288 cmK. Thus, $T=\dfrac{0.288\,\text{cmK}}{470\,\text{nm}}=6130\text{K}$

Illustration 9: What is the wavelength of the brightest part of the light from our next closest star, Proxima Centauri? Proxima Centauri is a red dwarf star about 4.2 light years away from us with an average surface temperature of 3,042 Kelvin? **(JEE MAIN)**

Sol: $\lambda_{max}\ T=b$

We don't really need the distance to solve this. All we need is the surface temperature to plug into our Wien's law equation

Wavelength λ_{max} in meters = $\frac{0.0029\text{meters}-K}{3.042K}$ which is 0.000000953 meters.

We can convert this to nanometers and we get a peak wavelength of 953 nm.

Illustration 10: Two bodies A and B have thermal emissivity of 0.1 and 0.81 respectively. The outer surface areas of the two bodies are identical. These two bodies emit total radiative power at the same rate. The wavelength λ_B corresponding to the maximum spectral radiancy in the radiation from B is 1.0 μm larger than the wavelength λ_A corresponding to the maximum spectral radiancy in the radiation from A. If the temperature of body A is 5802 K, find (a) temperature of (B) and (b) λ_B. **(JEE MAIN)**

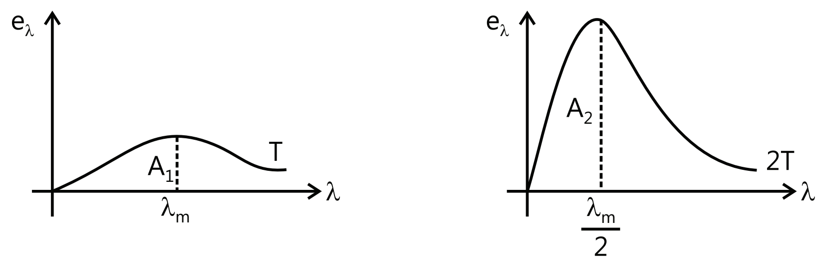


Figure 14.9

Sol: By equating their emissive power, ratio of temperatures (a) could be calculated.

(a) Power radiated from A $= P_A = E_A A = e_A \sigma T_A^4 A$

Power radiated from B $= P_B = E_A A = e_B \sigma T_B^4 A$

Where A is surface area of both the bodies as $P_1 = P_2$, $e_A T_A^4 = e_B T_B^4$

$$\therefore 0.01T_A^4 = 0.81T_B^4 \ \therefore \left[\frac{T_B}{T_A}\right]^4 = \left[\frac{0.01}{0.81}\right] = \left[\frac{1}{81}\right]; \frac{T_B}{T_A} = \left[\frac{1}{3}\right] \qquad \text{or } T_B = \frac{1}{3} \times T_A = \frac{1}{3} \times 5802 = 1934K$$

(b) $\lambda_m T$ = constant as per Wien's law

$$\therefore \lambda_A T_A = \lambda_B T_B \text{ or } \frac{\lambda_B}{\lambda_A} = \frac{T_A}{T_B} = 3 \ ; \ \lambda_A = \frac{\lambda_B}{3}; \qquad \lambda_B - \lambda_A = 1\mu m, \lambda_B - \frac{\lambda_B}{3} = \frac{2\lambda_B}{3} = 1\mu m$$

$$\therefore \lambda_B = \frac{1 \times 3}{2} = 1.5\mu m$$

17. SOLAR CONSTANT AND TEMPERATURE OF SUN

Solar constant is defined as the amount of radiation received from the sun at the earth per minute per cm^2 of a surface placed at right angle to the solar radiation at a mean distance of the earth from the sun. Assuming that the absorption of solar radiation by the atmosphere near the earth is negligible, the value of solar constant, S, is equal to 1.94 $\text{cal.cm}^{-2}\,\text{min}^{-1}$.

The temperature of the sun, T, is given as follows $T^4 = \frac{S}{\sigma}\left(\frac{R}{r}\right)^2$

Where S is solar constant, σ is Stefan's constant, R is mean distance of earth from sun and r is radius of sun.

PROBLEM-SOLVING TACTICS

1. Problems of conduction can be easily solved by making analogy with current electricity (Problems like calculation of net conductance of series and parallel connection. Actually, the way in which steady state is achieved in heat transfer and current electricity is very similar. At steady state considering a cylindrical rod, potential at each point becomes constant in current electricity and so does temperature in heat transfer. The amount of charge transferred per unit time is related in same way to potential as that of heat energy transferred relates to temperature difference and the constant of proportionality have similar properties.)

2. Most of the problems involve concepts of integration, so be careful with infinitesimal elements. Basically, try to be physically involved in the problem and understand it event by event so that you learn more. Toughness in most of the questions is involved only in its mathematical analysis.

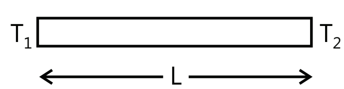
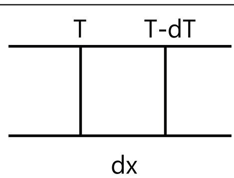
3. Problems from radiation and law of cooling also generally involve integration which becomes necessary to do at times. However an approximate approach is also available in case of law of cooling useful in solving problems without involving integration.

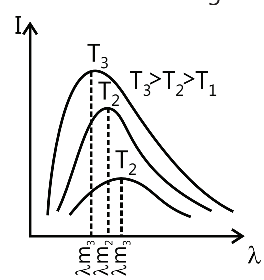
4. Laws must be carefully known because many questions directly focus on understanding of laws rather than involving calculations (Example - If temperature of a body is doubled, find the ratio of maximum wavelength for final and initial state.)

5. Noting down the known and asked quantities and thinking of a link between them will always prove to be a good way.

6. Questions from this topic usually come in a hybrid involving concepts of other topics like thermodynamics, gaseous state and calorimetry. So one must be strong in their concepts too!!

FORMULAE SHEET

S. No.	Term	Descriptions
1.	Conduction	Due to vibration and collision of medium particles.
2.	Steady state	In this state heat absorption stops and temperature gradient throughout the rod becomes constant i.e. $\frac{dT}{dx} = \text{constant}$.
3.	Before steady state	Temp of rod at any point changes. **Note:** If specific heat of any substance is zero, it can be considered always to be in steady state.
4.	Ohm's law for thermal Conduction in Steady state	Let the two ends of rod of length L is maintained At temp T_1 and $T_2\left(T_1 > T_2\right)$ Thermal Current $\frac{dQ}{dT} = \frac{T_1 - T_2}{R_{Th}}$. Where $R_{Th} = \frac{L}{KA}$ (L is length of material, K is coefficient of thermal conductivity, A is area of cross- section) 
5.	Differential form of Ohm's law	$\frac{dQ}{dT} = KA\frac{dT}{dx}$ $\frac{dT}{dx}$ = Temperature gradient 
6.	Convection	Heat transfer due to movement of medium particles.

7.	Radiation	Every body radiates electromagnetic radiation of all possible wavelength at all temp>0 K
8.	Stefan's Law	Rate of heat emitted by a body at temp T K from per unit area $E=\sigma T^4 J/sec/m^2$ Radiation power $\frac{dQ}{dT}=P=\sigma AT^4 watt$ If body is placed in a surrounding of temperature T_s $\frac{dQ}{dT}=\sigma A(T^4-T_s^{\,4})$ valid only for black body Emissivity or emmisive power $e=\frac{\text{heat from general body}}{\text{heat from black body}}$ If temp of body falls by dT in time dt $\frac{dT}{dt}=\frac{eA\sigma}{ms}\left(T^4-T_s^{\,4}\right)$ (dT/dt=Rate of cooling)
9.	Newton's law of cooling	If temp difference of body with surrounding is small i.e. $T=T_s$ Then, $\frac{dT}{dt}=\frac{4eA\sigma}{ms}T_s^{\,3}\left(T-T_s\right)$ So $\frac{dT}{dt}\propto\left(T-T_s\right)$
10.	Average form of Newton's law of cooling	If a body cools from T_1 to T_2 in time δt $\frac{T_1-T_2}{\delta t}=\frac{K}{mS}\left(\frac{T_1+T_2}{2}-T_S\right)$ (Used generally in objective questions) $\frac{dT}{dt}=\frac{K}{mS}(T-T_S)$ (For better results use this generally in subjective)
11.	Wien's black body radiation	At every temperature (>0K) a body radiates energy radiations of all wavelengths. According to Wien's displacement law if the wavelength corresponding to maximum energy is λ_m then λ_m T=b where b= is a constant(Wien's Constant) T=Temperature of body 

Solved Examples

JEE Main/Boards

Example 1: A copper rod 2 m long has a circular cross section of radius 1 cm. One end is kept at 100°C and other at 0°C, and the surface is insulated so that negligible heat is lost through the surface. Find

(a) The thermal resistance of bar

(b) The thermal current H

(c) The temperature gradient $\frac{dT}{dx}$

(d) The temperature 25 cm from hot end. Thermal conductivity of copper is 401 W/m-K

Sol: Recall the formula of heat transfer.

(a) Thermal resistance

$$R=\frac{1}{kA}=\frac{1}{k(\pi r^2)} \text{ or } R=\frac{2}{(401)(\pi)(10^{-2})^2}=15.9K/W$$

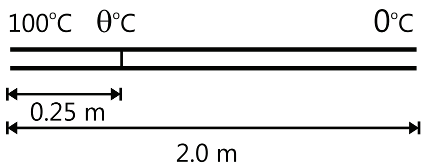
(b) Thermal current, $H=\frac{\Delta T}{R}=\frac{\Delta\theta}{R}=\frac{100}{15.9}$ or

$H=6.3W$

(c) Temperature gradient

$$=\frac{0-100}{2}=-50K/m=-50^\circ C/m$$

(d) Let be θ°C the temperature at 25 cm from hot end then



$(\theta - 100)$=(Temperature gradient) × (Distance)

$\theta - 100 = (-50)(0.25)$

$\theta = 87.5°C$

Example 2: In a murder investigation, a corpse was found by a detective at exactly 8 P.M. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and it found to be 60°F. If the room temperature is 50°F, and assuming that the body temperature of the person before death was 98.6°F, at what time did the murder occur?

Sol: Newton's law of cooling is used.

With time 0 taken to be 8 P.M., we have the boundary value problem

$$\frac{dT}{dt} = k(50 - T); \qquad T(0) = 70, \quad T(2) = 60$$

Whose solution is $T = 50 + 20\left(\frac{1}{2}\right)^{t/2}$

We would like to find the value of t for which T(t)=98.6. Solving the equation

$$50 + 20\left(\frac{1}{2}\right)^{t/2} = 98.6$$

Given us $t = 2\left(\frac{\ln(48.6/20)}{\ln(1/2)}\right) \approx -2.56.$

It appears that this person was murdered at about 530 P.M. or so.

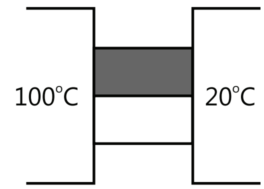
From the function $T = 50 + 20\left(\frac{1}{2}\right)^{t/2}$

Over the time interval $-2.56. \le t \le 2.56.$

Example 3: Two metal cubes with 3 cm edges of copper and aluminium are arranged as shown in fig. find

(a) The total thermal current from one reservoir to the other

(b) The ratio of the thermal current carried by the copper cube to that of the aluminium cube. Thermal conductivity of copper is 401 W/m-K and that of aluminium is 237 W/m-K



Sol: This is parallel combination and thermal current would be sum of both cubes.

(a) Thermal resistance of aluminum cube

$$R_1 = \frac{1}{kA} \text{ or } R_1 = \frac{(3\times10^{-2})}{(237)(3\times10^{-2})^2} = 0.14 K/W$$

and Thermal resistance of aluminum cube

$$R_2 = \frac{(3\times10^{-2})}{(401)(3\times10^{-2})^2} = 0.08 K/W$$

As these two resistances are in parallel, their equivalent resistance will be

$$R = \frac{R_1R_2}{R_1+R_2} = \frac{(0.14)(0.08)}{(0.14)+(0.08)} = 0.05 K/W$$

$$\text{Thermal Current } H = \frac{\text{Temperature difference}}{\text{Thermal resistance}}$$

$$= \frac{(100-20)}{0.05} = 1.6\times10^3 W$$

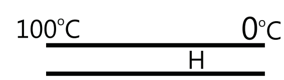
(b) In parallel thermal current distributes in the inverse ratio of resistance.

$$\text{Hence, } \frac{H_{Cu}}{H_{Al}} = \frac{R_{Al}}{R_{Cu}} = \frac{R_1}{R_2} = \frac{0.14}{0.08} = 1.75$$

Example 4: One end of a copper rod of length 1 m and area of cross section $4.0\times10^{-4} m^2$ is maintained at 100°C. At the end of rod ice is kept at 0°C. Neglecting the loss of heat from the surroundings, find the mass of ice melted in 1 h. Given $k_{cu} = 401 W/m-K$ and $L_f = 3.35\times10^5 J/kg$.

Sol: Find total heat transfer in 1 hr time through rod and hence, melted ice can be found.

Thermal resistance of the rod,



$$R = \frac{1}{kA} = \frac{1.0}{(401)(4\times10^{-4})} = 6.23 K/W$$

$$\text{Heat Current } H = \frac{\text{Temperature difference}}{\text{Thermal resistance}}$$

$$= \frac{(100-0)}{6.23} = 16\text{W}$$

Heat transferred in 1 h,

$$Q = Ht \; = (16)(3600) = 57600\,\text{J} \;\left(H = \frac{Q}{T}\right)$$

Now, let m mass of ice melts in 1 h, then

$$m = \frac{Q}{L} \; (Q=mL)$$

$$= \frac{57600}{3.35\times 10^5} = 0.172\text{kg} \;\text{ or }\; 172\text{g}$$

Example 5: A body cools in 10 minutes from 60°C to 40°C. What will be its temperature after next 10 minutes? The temperature of the surrounding is 10°C

Sol: Think of Newton's law of cooling.

According to Newton's law of cooling

$$\left(\frac{\theta_1-\theta_2}{t}\right) = \alpha\left[\left(\frac{\theta_1+\theta_2}{2}\right)-\theta_0\right]$$

For the given conditions,

$$\frac{60-40}{10} = \alpha\left[\frac{60+40}{2}-10\right] \quad \text{... (i)}$$

Let be the temperature after next 10 minutes.

$$\text{Then } \frac{40-\theta}{10} = \alpha\left[\frac{40+\theta}{2}-10\right] \quad \text{... (ii)}$$

Solving Eqs. (i) and (ii), we get $\theta = 28°\text{C}$

Example 6: Two bodies A and B have thermal emissivity of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are same. The two bodies emit total radiant power at the same rate. The wavelength corresponding to maximum spectral radiancy from B is shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from A by 1.0 μm. If the temperature of A is 5802 K. calculate (a) The temperature of B,

(b) Wavelength λ_B

Sol: Compare the emissive power of both and then temperature and λ_m of B can be calculated, Use $\lambda_B - \lambda_A = 1\mu\text{m}$.

(a) $P_A = P_B \qquad \therefore e_A\sigma A_A T_A^4 = e_B\sigma A_B T_B^4$

$$\therefore T_B = \left(\frac{e_A}{e_B}\right)^{\frac{1}{4}} T_A \quad \text{... } (A_A = A_B)$$

Substituting the values

$$T_B = \left(\frac{0.01}{0.81}\right)^{\frac{1}{4}}(5802) = 1934\text{K}$$

(b) According to Wein's displacement law,

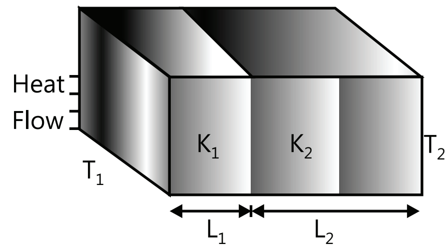
$$\lambda_A T_A = \lambda_B T_B$$

$$\therefore \quad \lambda_B = \left(\frac{5802}{1934}\right)\lambda_A \;\text{ or }\; \lambda_B = 3\lambda_A$$

Also, $\lambda_B - \lambda_A = 1\mu\text{m}$ or $\lambda_B - \left(\frac{1}{3}\right)\lambda_B = 1\mu\text{m}$

Or $\lambda_B = 1.5\mu\text{m}$

Example 7: Two plates each of area A, thickness L_1 and L_2 thermal conductivities K_1 and K_2 respectively are joined to form a single plate of thickness $L_1 + L_2$. If the temperatures of the free surfaces are T_1 and T_2, calculate



(a) Rate of flow of heat

(b) Temperature of interface

(c) Equivalent thermal conductivity

Sol: Consider as thermal current where thermal resistors in series.

(a) If the thermal resistances of the two plates are R_1 and R_2 respectively then as plates are in series.

$$R_S = R_1 + R_2 = \frac{L_1}{AK_1} + \frac{L_2}{AK_2}$$

As $R = \frac{L}{AK}$ and so

$$H = \frac{dQ}{dt} = \frac{\Delta Q}{R} = \frac{(T_1-T_2)}{(R_1+R_2)} = \frac{A(T_1-T_2)}{\left[\frac{L_1}{K_1}+\frac{L_2}{K_2}\right]}$$

(b) If T is the common temperature of interface then as in series, rate of flow of heat remains same. i.e. $H = H_1 (= H_2)$

$$\frac{T_1-T_2}{R_1+R_2} = \frac{T_1-T}{R_1} \;\text{ i.e. } T = \frac{T_1R_2+T_2R_1}{(R_1+R_2)}$$

or $T = \dfrac{\left[T_2 \dfrac{L_1}{K_1} + T_1 \dfrac{L_2}{K_2}\right]}{\left[\dfrac{L_1}{K_1} + \dfrac{L_2}{K_2}\right]}$

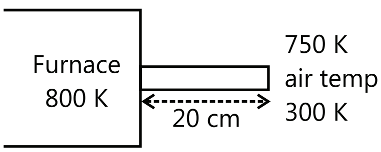
(c) If K is the equivalent conductivity of composite slab i.e. slab of thickness $L_1 + L_2$ and cross sectional area A, then as in series

$$R_S = R_1 + R_2 \text{ or } \frac{(L_1 + L_2)}{AK_{eq}} = R_1 + R_2$$

$$K_{eq} = \frac{(L_1 + L_2)}{A(R_1 + R_2)} = \frac{L_1 + L_2}{\left[\dfrac{L_1}{K_1} + \dfrac{L_2}{K_2}\right]} \text{ As } R = \frac{L}{AK}$$

Example 8: One end of a rod of length 20cm is inserted in a furnace at 800K. The sides of the rod are covered with an insulating material and the other end emits radiation like a black body. The temperature of this end is 750K in the steady state. The temperature of the surrounding air is 300K. Assuming radiation is the only important mode of energy transfer between the surrounding and the open end of the rod. Find the thermal conductivity of the rod. Stefan constant

$\sigma = 6.0 \times 10^{-3} W/m^2 - K^4$



Sol: Rate of heat through radiation would be equal to rate of heat transfer through rod.

Quantity of heat flowing though the rod per second in steady state

$$\frac{dQ}{dt} = \frac{K.A.d\theta}{x} \quad \text{... (i)}$$

Quantity of heat radiated from the end of the rod per second in steady state

$$\frac{dQ}{dt} = A\sigma(T^4 - T_0^4) \quad \text{... (ii)}$$

From Eqs. (i) and (ii),

$$\frac{K.d\theta}{x} = \sigma(T^4 - T_0^4)$$

$$\frac{K \times 50}{0.2} = 6.0 \times 10^{-8}\left[(7.5)^4 - (3)^4\right] \times 10^8$$

$$K = 74\,W/mK$$

Example 9: The lower surface of a slab of stone of face-area 3600 cm and thickness 10 cm is exposed to steam at 100°C. A block of ice at 0°C rests on the upper surface of slab. 4.8 g of ice melts in one hour. Calculate the thermal conductivity of the stone. Latent heat of fusion of ice = $3.36 \times 10^5 Jkg^{-1}$.

Sol: Amount of heat transfer per second would be used to melt the mass of ice per second.

The amount of heat transferred through the slab to the ice in one hour is

$$Q = (4.8 \times 10^{-3}\,kg) \times (3.36 \times 10^5\,Jkg^{-1})$$

$$= 4.8 \times 336J.$$

Using the equation $Q = \dfrac{KA(\theta_1 - \theta_2)t}{x}$

$$4.8 \times 336J = \frac{K(3600\,cm)^2(100°C)(3600\,s)}{10cm}$$

or $K = 1.24 \times 10^{-3}\,Wm^{-1}°C^{-1}$

Example 10: An icebox made of 1.5 cm thick Styrofoam has dimensions $60cm \times 60cm \times 30cm$. It contains ice at 0°C and kept in a room at 40°C. Find the rate at which ice is melting. Latent heat of fusion of ice = $3.36 \times 10^5 Jkg^{-1}$ and thermal conductivity of Styrofoam = $0.04 Wm^{-1}°C^{-1}$.

Sol: Heat transfer through Styrofoam will melt the ice.

The total surface area of the walls

$$= 2(60cm \times 60cm + 60cm \times 30cm + 60cm \times 30cm)$$

$$= 1.44m^2$$

The rate of heat flow into the box is

$$\frac{\Delta Q}{\Delta t} = \frac{KA(\theta_1 - \theta_2)}{x}$$

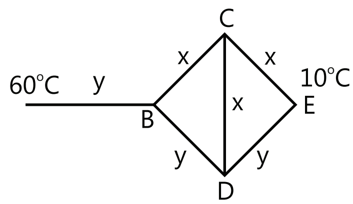
$$= \frac{(0.04\,Wm^{-1}°C^{-1})(1.44\,m^2)(40°C)}{0.015m} = 154W$$

The rate at which the ice melts is

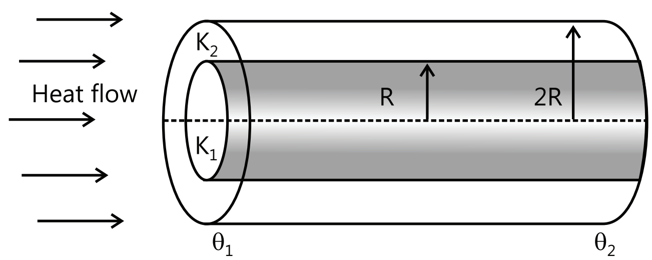
$$= \frac{154W}{3.36 \times 10^5\,Jkg^{-1}} = 0.46gs^{-1}$$

JEE Advanced/Boards

Example 1: Three rods of the material x and three rods of material y are connected as shown in figure. All the rods are of identical length and cross sectional area. If the end A is maintained at 60°C and the junction E is at 10°C Calculate temperature of junction B, C, D. the thermal conductivity of x is 0.92 cal/cm-s°C and that of y is 0.46 cal/cm-s°C.



Sol: Think of temperature drop across BCE and across BDE, temperature of C and D would be same as similar drop across BC and CE,



also across BD and DE.Thermal resistance $R = \dfrac{1}{kA}$

$$\therefore \frac{R_x}{R_y} = \frac{k_y}{k_x} \qquad (\text{as } l_x = l_y \text{ and } A_x = A_y)$$

$$\therefore \frac{R_x}{R_y} = \frac{0.46}{0.92} = \frac{1}{2}$$

So, if $R_x = R$ then $R_y = 2R$

CEDB forms a balanced Wheatstone bridge i.e.

$T_C = T_D$ and no heat flows through CD

$$\therefore \frac{1}{R_{BE}} = \frac{1}{R+R} + \frac{1}{2R+2R} \text{ or } R_{BE} = \frac{4}{3}R$$

The total resistance between A and E will be,

$$R_{AE} = R_{AB} + R_{BE} = 2R + \frac{4}{3}R = \frac{10}{3}R$$

$\therefore$ Heat current between A and E is

$$H = \frac{(\Delta T)}{R_{AE}} = \frac{(60-10)}{(10/3)R} = \frac{15}{R}$$

Now, if T_B is the temperature at B,

$$H_{AB} = \frac{(\Delta T)_{AB}}{R_{AB}} \text{ or } \frac{15}{R} = \frac{60 - T_B}{2R} \text{ or } T_B = 30°C$$

Further, $H_{AB} = H_{BC} + H_{BD}$ or $\dfrac{15}{R} = \dfrac{30 - T_c}{R} + \dfrac{30 - T_D}{2R}$

(Say $T_C = T_D = T$)

Or $15 = (30 - T) + \dfrac{(30-T)}{2}$

Solving this we get $T = 20°C$ or $T_C = T_D = 20°C$

Example 2: A cylinder of radius R made of a thermal conductivity K_1 is surrounded by cylindrical shell of inner radius R and another radius 2R made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and system is in steady state. What is the effective thermal conductivity of system?

Sol: Assume this to parallel combination of thermal resistors. As both have same temperature across their ends.

In this situation a rod of length L and area of cross section πR^2 and another of same length L and area of cross-section $\pi\left[(2R)^2 - R^2\right] = 3\pi R^2$ will conduct heat simultaneously so total heat flowing per second will be,

$$\frac{dQ}{dt} = \frac{dQ_1}{dt} + \frac{dQ_2}{dt}$$

$$= \frac{K_1 \pi R^2 (\theta_1 - \theta_2)}{L} + \frac{K_2 3\pi R^2 (\theta_1 - \theta_2)}{L} \qquad \text{...(i)}$$

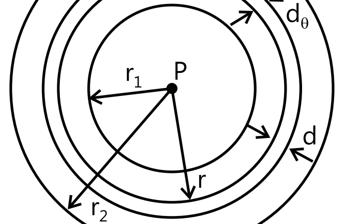
Now, if the equivalent conductivity is K then,

$$\frac{dQ}{dt} = K\frac{4\pi R^2(\theta_1 - \theta_2)}{L} \; [\text{As } A = \pi(2R)^2] \qquad \text{...(ii)}$$

So, from Eqs. (i) and (ii), we have

$$4K = K_1 + 3K_2 \quad \text{i.e. } K = \frac{(K_1 + 3K_2)}{4}$$

Example 3: A point source of heat of power P is placed at the center of a spherical shell of mean radius R. the material of the shell has thermal conductivity k. calculate the thickness of the shell if temperature difference between the outer and inner surfaces of the shell in steady state is T.



Sol: Total thermal resistance $\int \frac{dr}{k\,4\pi r^2} = \left(\frac{l}{KA}\right)$. Power of source equal to rate of heat transfer at steady state.

Consider a concentric spherical shell of radius r and thickness dr as shown in figure. In steady state, the rate of heat flow (heat current) through this shell will be,

$$H = \frac{\Delta T}{R} = \frac{(-d\theta)}{\frac{dr}{(k)(4\pi r^2)}} \quad \left(R = \frac{1}{kA}\right)$$

or $H = -(4\pi kr^2)\frac{d\theta}{dr}$

Here, negative sign is used because with increase in r, decreases.

$$\therefore \int_{r_1}^{r_2} \frac{dr}{r^2} = -\frac{4\pi k}{H} \int_{\theta_1}^{\theta_2} d\theta$$

This equation gives, $H = \frac{4\pi k r_1 r_2 (\theta_1 - \theta_2)}{(r_2 - r_1)}$

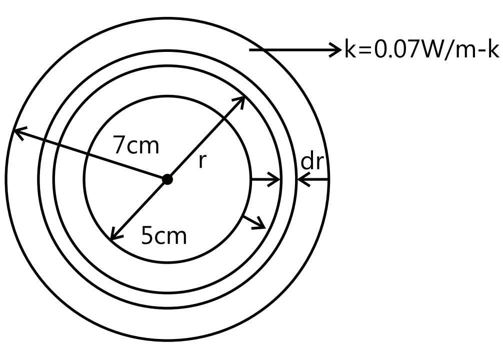
In steady state, $H = P, r_1 r_2 = R^2$ and $\theta_1 - \theta_2 = T$

$\therefore$Thickness of shell, $r_2 - r_1 = \frac{4\pi k R^2 T}{P}$

Example 4: A steam pipe of radius 5cm carries steam at 100°C. The pipe is covered by a jacket of insulating material 2cm thick having a thermal conductivity 0.07 W/m-K. If the temperature at the outer wall of the pipe jacket is 20°C, how much heat is lost through the jacket per meter length in an hour?

Sol: Heat lost through curved surface of the pipe. $R_{thermal} = \int \frac{dr}{K\,2\pi rl}$ for pipe of length l.

Thermal resistance per meter length of an element at distance r of thickness dr is



$$dR = \frac{dr}{k(2\pi r)} \quad (R = \frac{1}{kA})$$

$$\therefore \text{Total resistance } R = \int_{r_1=5cm}^{r_2=7cm} dR$$

$$= \frac{1}{2\pi k}\int_{5\times10^{-2}m}^{7\times10^{-2}m} \frac{dr}{r} = \frac{1}{2\pi k}\ln\left(\frac{7}{5}\right)$$

$$= \frac{1}{2\pi(0.07)}\ln(1.4) = 0.765K/W$$

$$\text{Heat current } H = \frac{\text{Temperature difference}}{\text{Thermal resistance}}$$

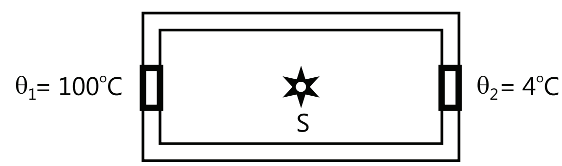
$$= \frac{(100-20)}{0.765} = 104.6W$$

$\therefore$ Heat lost in one hour $=$ Heat current $\times$ time

$= (104.6)(3600)\,J = 3.76 \times 10^5\,J$

Example 5: A closed cubical box is made of perfectly insulating material and the only way for heat to enter or leave the box is through two solid cylindrical metal plugs, each of cross sectional area 12 cm^2 and length 8 cm fixed in the opposite walls of the box. The outer surface of the plug is kept at a temperature of 100°C while the outer surface of the other plug is maintained at a temperature of 4°C. The thermal conductivity of the material of the plug is 2.0Wm^{-1} °C^{-1}. A source of energy generating 13 W is enclosed inside the box. Find the equilibrium temperature of the inner surface of the box assuming that it is the same at all points on the inner surface.

Sol: At steady state, rate of heat transfer through both plugs would be same.



The situation is shown in figure. Let the temperature inside the box be θ. The rate at which heat enters the box through the left plug is $\frac{\Delta Q_1}{\Delta t} = \frac{KA(\theta_1 - \theta)}{x}$

The rate of heat generation in the box=13 W. The rate at which heat flows out of the box through the right plug is

$$\frac{\Delta Q_2}{\Delta t} = \frac{KA(\theta - \theta_2)}{x}$$

In the steady state $\frac{\Delta Q_1}{\Delta t} + 13W = \frac{\Delta Q_2}{\Delta t}$

or, $\frac{KA}{x}(\theta_1 - \theta) + 13\,W = \frac{KA}{x}(\theta - \theta_2)$

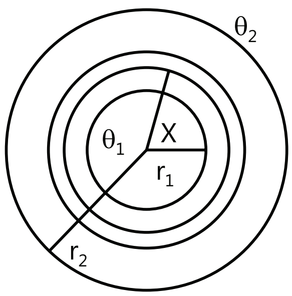
or, $2\frac{KA}{x}\theta = \frac{KA}{x}(\theta_1 + \theta_2) + 13W$

or, $\theta = \frac{\theta_1 + \theta_2}{2} + \frac{(13W)x}{2KA}$

$= \frac{100°C + 4°C}{2} + \frac{(13W) \times 0.08m}{2 \times (2.0 Wm^{-1}°C^{-1})(12 \times 10^{-4} m^2)}$

$= 52°C + 216.67°C = 269°C$

Example 6: Two thin metallic spherical shells of radii r_1 and r_2 ($r_1 < r_2$) are placed with their centres coinciding. A material of thermal conductivity K is filled in the space between the shells. The inner shell is maintained at temperature and the outer shell at temperature θ_1 ($\theta_1 < \theta_2$). Calculate the rate at which heat flows radially through the material.



Sol: Heat flowing radially outward through spherical shells. Both connected in series.

Let us draw two spherical shells of radii x and x+dx concentric with the given system. Let the temperatures at these shells be θ and $\theta + d\theta$ respectively. The amount of heat flowing radially inward through the material between x and x+dx is

$$\frac{\Delta Q}{\Delta t} = \frac{K4\pi x^2}{dx}.dQ$$

Thus,

$$K4\pi\int_{\theta_1}^{\theta_2} d\theta = \frac{\Delta Q}{\Delta t}\int_{r_1}^{r_2}\frac{dx}{x^2}$$

or, $K4\pi(\theta_2 - \theta_1) = \frac{\Delta Q}{\Delta t}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

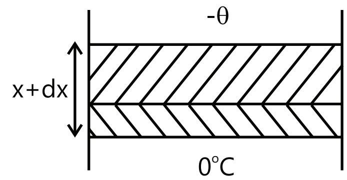
or, $\frac{\Delta Q}{\Delta t} = \frac{K4\pi r_1 r_2(\theta_2 - \theta_1)}{r_2 - r_1}$

Example 7: The temperature of air above a lake is -10°C. At some instant, the thickness of ice in the lake is 2 cm. calculate the time required for the thickness to be doubled. Thermal conductivity ice = 0.004cal/cm/s/°C, density of ice = 0.92 g/cm³ and latent heat of ice = 80cal/g.

Sol: Amount of Heat transfer through ice at any time would result in freezing the water of lake. Proceed with assuming Area of lake = A, eventually it will cancel out.

As the temperature of air is below 0°C, water begins to freeze to form a layer of ice. The thickness of the layer gradually increases.

Consider that a layer of thickness x has already been formed on a lake at 0°C. Let A be the area of the layer, L the latent heat of ice and ρ its density. The amount of heat required when the thickness of ice increases by dx is



$Q = mL = (Adx\rho)L$

This quantity of heat is conducted upwards through the layer in time dt when the temperature of air is $-\theta$.

$\therefore A\rho Ldx = \frac{KA(0-(-\theta))}{x}dt;\ \frac{dx}{dt} = \frac{K\theta}{\rho Lx};\ dt = \frac{\rho Lxdx}{K\theta}$

Time taken t for the thickness to increase from x_1 and x_2 to is obtained by integrating

$$t = \int_0^t dt = \frac{\rho L}{K\theta}\int_{x_1}^{x_2} xdx \text{ Or}$$

$$t = \frac{\rho L}{2K\theta}(x_2^2 - x_1^2)\ \therefore t = \frac{0.92 \times 80}{2 \times 0.004 \times 10}(4^2 - 2^2)$$

$= 11040 s = 3.07 hr$

Example 8: A liquid placed in a container open to atmosphere takes 5 minutes to cool from 80°C to 50°C. How much time will it take to cool from 60°C to 30°C? The temperature of the surroundings is 20°C.

Sol: Newton's law of cooling.

The rate of cooling of a body at temperature T is given by Newton's law of cooling as $\frac{dT}{dt} = -K(T - T_0)$

Where K is a constant for the body and T_0 is the temperature of the surroundings.

$\frac{T - T_0}{dT} = -Kdt$

The negative sign indicates that the temperature is falling.

Integrating, we get $\int_{T_1}^{T_2} \frac{dT}{T-T_0} = -K\int_0^t dt$

$$\log_e\left(\frac{T_2 - T_0}{T_1 - T_0}\right) = -Kt$$

As $t = 5, T_1 = 80°C, T_2 = 50°C, T_0 = 20°C$

$$\therefore 5 = \frac{1}{K}\log_e\left(\frac{80-20}{50-20}\right)$$

or $5K = \log_e(2)$ ... (i)

If t is time taken when

$T_1 = 60°C \text{ and } T_2 = 30°C$

$$Kt = \log_e\left(\frac{60-20}{30-20}\right) \quad \text{... (ii)}$$

or $Kt = \log_e(4)$

Dividing equation (ii) by equation (i)

$$\frac{t}{5} = \frac{\log_e 4}{\log_e 2} = \frac{1.386}{0.693} = 2 \quad \text{or } t = 10 \text{ minutes}$$

Example 9: A solid copper sphere cools at the rate of 2.8°C per minute, when its temperature is 127°C. Find the rate at which another copper sphere of twice the radius will lose its temperature at 327°C, if in both the cases, the room temperature is maintained at 27°C.

Sol: Get the rate of heat loss through radiations.

The rate of loss of heat $= \frac{dQ}{dt} = ms\frac{dT}{dt}$

$$= \sigma A(T^4 - T_0^4) \text{ or } \frac{dT}{dt} = \frac{\sigma A}{ms}(T^4 - T_0^4)$$

If r is radius of sphere is r, then $m = \frac{4}{3}\pi r^3 \times \rho$

Where ρ is density and s is specific heat

$$\frac{dT}{dt} = \frac{\sigma \times 4\pi r^2}{\frac{4}{3}\pi r^3 \rho \times s}(T^4 - T_0^4) = \frac{3\sigma}{r\rho \times s}(T^4 - T_0^4)$$

$$\left(\frac{dT}{dt}\right)_{127°C} = 2.8 = \frac{3\sigma}{r\rho \times s}(400^4 - 300^4) \quad \text{...(i)}$$

For the second sphere of radius 2r

$$\left(\frac{dT}{dt}\right)_{327°C} = \frac{3\sigma}{(2r)\rho \times s}(600^4 - 300^4) \quad \text{...(ii)}$$

Dividing equation (ii) by equation (i), we get

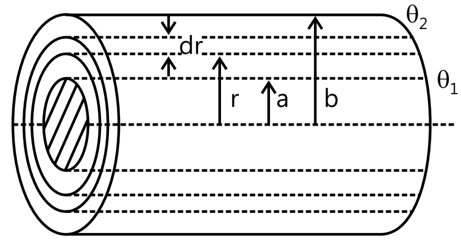
$$\left(\frac{dT}{dt}\right)_{327°C} = \frac{2.8}{2}\left[\frac{6^4 - 3^4}{4^4 - 3^4}\right] = 9.72°C / \text{minute}$$

Example 10: A 2m long wire of resistance 4 ohm and diameter 0.64 mm is coated with plastic insulation of thickness 0.06 mm. When a current of 5 ampere flows through the wire, find the temperature difference across insulation in steady state if

$$\left[K = 0.16 \times 10^{-2} \text{cal/cm} - °\text{Cs}\right]$$

Sol: Tricky one! Rate of heat generation in the wire due to flow of current must be same as rate of heat transfer through plastic insulation.

Considering a concentric cylindrical shell of radius r and thickness dr as shown in figure. The radial rate of flow of heat through this shell in steady state will be



$$H = \frac{dQ}{dt} = -KA\frac{d\theta}{dr}$$

Negative sign is used as with increase in r, θ decrases

Now as for cylindrical shell $A = 2\pi rL$

$$H = -2\pi rLK\frac{d\theta}{dr}$$

$$\text{or } \int_a^b \frac{dr}{r} = -\frac{-2\pi rLK}{H}\int_{\theta_1}^{\theta_2} d\theta$$

Which on integration and simplification gives

$$H = \frac{dQ}{dt} = -\frac{2\pi LK(\theta_1 - \theta_2)}{\ln(b/a)} \quad \text{... (i)}$$

Here, $H = \frac{I^2R}{4.2} = \frac{(5)^2 \times 4}{4.2} = 24\frac{cal}{s}$

$L = 2m = 200cm$

$r_1 = (0.64/2) = 0.032\,cm$

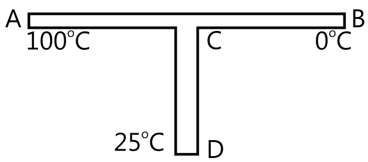
and $R_2 = r_1 + d = 0.032 + 0.006 = 0.038$

$$\text{So } (\theta_1 - \theta_2) = \frac{24 \times \ln(38/32)}{2 \times 2.3026[\log_{10} 38 - \log_{10} 32]}$$

$$= \frac{24 \times 2.3026\left[\log_{10} 38 - \log_{10} 32\right]}{3.14 \times 0.64}$$

$$\text{or } (\theta_1 - \theta_2) = \frac{55 \times \left[1.57 - 1.50\right]}{2} = 2°C.$$

Example 11: A rod CD of thermal resistance $5.0 KW^{-1}$ is joined at the middle of an identical rod AB as shown in figure. The ends A, B and D are maintained at 100°C, 0°C, and 25°C respectively. Find the heat current in CD.



Sol: At point C, total thermal current inflow equal to total thermal current out flow.

The thermal resistance of AC is equal to that of CB and is equal to $2.5 KW^{-1}$. Suppose, the temperature at C is θ. The heat current through AC, CB, and CD are

$$\frac{\Delta Q_1}{\Delta t} = \frac{100°C - \theta}{2.5KW^{-1}};$$

$$\frac{\Delta Q_2}{\Delta t} = \frac{\theta - 0°C}{2.5KW^{-1}} \text{ and } \frac{\Delta Q_3}{\Delta t} = \frac{\theta - 25°C}{5.0KW^{-1}}$$

We also have

$$\frac{\Delta Q_1}{\Delta t} = \frac{\Delta Q_2}{\Delta t} + \frac{\Delta Q_3}{\Delta t}$$

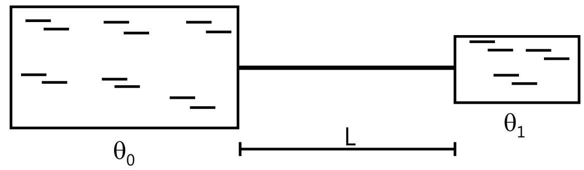
$$\text{or, } \frac{100°C - \theta}{2.5} = \frac{\theta - 0°C}{2.5} + \frac{\theta - 25°C}{5.0}$$

$$\text{or, } 225°C = 5\theta$$

$$\text{or, } \theta = 45°C$$

$$\text{Thus, } \frac{\Delta Q_3}{\Delta t} = \frac{45°C - 25°C}{5.0KW^{-1}} = \frac{20K}{5.0KW^{-1}} = 4.0W.$$

Example 12: Figure shows a large tank of water at a constant temperature θ_0 and a small vessel containing a mass m of water at an initial temperature θ_1 ($<\theta_0$). A metal rod of length L, area cross section A and thermal conductivity K connects the two vessels. Find the time taken for the temperature of the water in the smaller vessel become θ_2 ($\theta_1 < \theta_2 < \theta_0$). Specific heat capacity of water is s and all other heat capacities are negligible.



Sol: Rate of heat transfer is variable as temperature of small vessel will be changing.

Suppose, the temperature of the water in the smaller vessel is at time t. In the next time interval dt, a heat dQ is transferred to it where

$$\Delta Q = \frac{KA}{L}(\theta_0 - \theta)\,dt. \quad \text{... (i)}$$

This heat increases the temperature of the water of mass m to $\theta + d\theta$ where

$$\Delta Q = ms\,d\theta \quad \text{... (ii)}$$

From (i) and (ii),

$$\frac{KA}{L}(\theta_0 - \theta)\,dt = ms\,d\theta$$

$$\text{or, } dt = \frac{Lms}{KA}\frac{d\theta}{\theta_0 - \theta} \quad \text{or, } \int_0^T dt = \frac{Lms}{KA}\int_{\theta_1}^{\theta_2}\frac{d\theta}{\theta_0 - \theta}$$

Where T is the time required for the temperature of the water to become.

$$\text{Thus, } T = \frac{Lms}{KA}\ln\frac{\theta_0 - \theta_1}{\theta_0 - \theta_2}$$

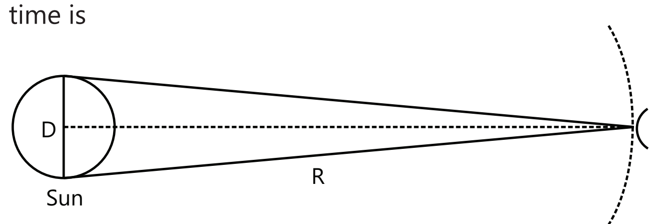
Example 13: The earth receives solar radiation at a rate of. $8.2\ J\,cm^{-2}\,min^{-1}$. Assuming that the sun radiates like a blackbody, calculate surface temperature of the sun. The angle subtended by the sun on the earth is 0.53^0 and Stefan constant $\sigma = 5.67 \times 10^{-8} Wm^{-3}K^{-4}$.

Sol: Think of intensity of thermal heat out a distance R from the source.

Let the diameter of the sun be D and its distance from the earth be R. From the question,

$$\frac{D}{R} = 0.53 \times \frac{\pi}{180} = 9.25 \times 10^{-3} \text{ ...(i)}$$

The radiation emitted by the surface of the sun per unit time is



$$4\pi\left(\frac{D}{2}\right)^2 \sigma T^4 = \pi D^2 \sigma T^4$$

At distance R, this radiation falls on an area of $4\pi R^2$ in unit time. The radiation received at the earth's surface per unit time per unit area is, therefore,

$$\frac{\pi D^2 \sigma T^4}{4\pi R^2} = \frac{\sigma T^4}{4}\left(\frac{D}{R}\right)^2$$

Thus, $\frac{\sigma T^4}{4}\left(\frac{D}{R}\right)^2 = 8.2 Jcm^{-2}\min^{-1}$

or, $\frac{1}{4}\times(5.67\times10^{-8}\,Wm^{-2}K^{-4})T^4\times$

$(9.25\times10^{-3})^2 \, x \, T^4$

$= \frac{8.2}{10^{-4}\times 60} Wm^{-2}$

or, $T = 5794K \approx 5800K$

Example 14: On a cold winter day, the atmospheric temperature is θ (on Celsius scale) which is below 0°C. A cylindrical drum of height h made of a bad conductor is completely filled with water at 0°C and is kept outside without any lid. Calculate the time taken for the whole mass of water to freeze. Thermal conductivity of ice is K and its latent heat of fusion is L. Neglect expansion of water on freezing.

Sol: Rate of heat transfer would be dependent on thickness of layer of ice. Write equation of heat transfer at any time 't' when thickness of ice is 'x'.

Suppose, the ice starts forming at time t=0 and a thickness x is formed at time t. The amount of heat flown from the water to the surrounding in the time interval t to t+dt is

$$\Delta Q = \frac{KA\theta}{x} dt.$$

The mass of the ice formed due to the loss of this amount of heat is

$$dm = \frac{\Delta Q}{L} = \frac{KA\theta}{xL} dt.$$

The thickness dx of ice formed in time dt is

$$dx = \frac{dm}{A\rho} = \frac{KA\theta}{\rho x L} dt \quad \text{or,} \qquad dt = \frac{\rho L}{K\theta} x dx.$$

Thus, the time T taken for the whole mass of water to freeze is given by

$$\int_0^T dt = \frac{\rho L}{K\theta}\int_0^h x dx \quad \text{or,} \qquad T = \frac{\rho L h^2}{2K\theta}$$

Example 15: A thermometer is taken from a room that is 20°C to the outdoors where the temperature is 5°C. After one minute, the thermometer reads 12°C. Use Newton's law of cooling to answer following questions. (a) What will the reading on the thermometer be after one more minute?

(b)When will the thermometer read 6°C.?

Sol: Get the 'k' for Newton's law of cooling by given condition, then the all desired value.

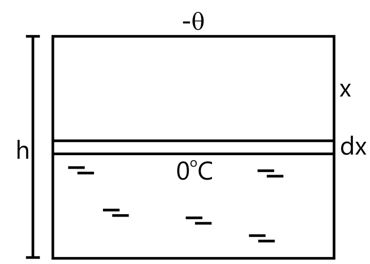
If T is the thermometer temperature, then Newton's law of cooling tells us that

$$\frac{dT}{dt} = k(5 - T); \quad T(0) = 20.$$

The solution of this initial value problem is

$$T = 5 + 15e^{-kt}.$$

We still need to find the value of k. We can do this by using the given information that T(1)=12. In fact, let us pause here to consider the general problem of finding the value of k. We will obtain some facts that can be used in the rest of the problems involving Newton's law of cooling.



Suppose that we have the model

$$\frac{dT}{dt} = k(T_s - T); \quad \begin{matrix} T(0) = T_0 \\ T(t_1) = T_1 \end{matrix}$$

Where t_1 is some time other than O. then the first two equations in the model, we obtain $T = T_s + (T_0 - T_s)e^{-kt}$ and from the third equation we obtain

$$T_s + (T_0 - T_s)e^{-kt_1} = T_1$$

Thus, $(T_0 - T_s)e^{-kt_1} = T_1 - T_s$

which gives us

$$e^{-kt_1} = \frac{T_1 - T_s}{T_0 - T_s} \qquad \text{or } e^{kt_1} = \frac{T_0 - T_s}{T_1 - T_s}$$

$$\text{or } k = \frac{1}{t_1}\ln\left(\frac{T_0 - T_s}{T_1 - T_s}\right)$$

The latter equations give us the value of k. However, note that, in most problems that we deal with, it is not really necessary to find the value of k. Since the term

e^{-kt} that appears in the solution of Newton's Law of cooling can be written as $e^{-kt} = (e^{-kt_1})^{t/t_1}$

We really just need (in most situations) to know the value of e^{-kt_1}, and this value has been obtained in the work done above. In particular, the solution of Newton's Law of Cooling,

$$T = T_s + (T_0 - T_S)e^{-kt},$$

Can be written as

$$T = T_s + (T_0 - T_S)(e^{-kt_1})^{t/t_1}$$

or as $T = T_s + (T_0 - T_S)\left(\frac{T_1 - T_s}{T_0 - T_s}\right)^{t/t_1}$

$$T = T_s + (T_0 - T_S)\left(\frac{T_1 - T_s}{T_0 - T_s}\right)^{t/t_1}$$

Returning now to the problem at hand (with the thermometer), we see that temperature function for the thermometer is $T = 5 + 15\left(\frac{7}{15}\right)^t$.

Note that this makes sense because this formula gives us $T(0) = 5 + 15\left(\frac{7}{15}\right)^0 = 20.$

And $T(1) = 5 + 15\left(\frac{7}{15}\right)^1 = 12.$

To find what the thermometer will read two minutes after being taken outside, we compute

$$T(2) = 5 + 15\left(\frac{7}{15}\right)^2 \approx 8.3.$$

This tells us that the thermometer will read about 8.3°C two minutes after being taken outside.

Finally, to determine when the thermometer will read 6°C, we solve the equation

$$5 + 15\left(\frac{7}{15}\right)^t = 6$$

The step-by-step solution of this equation is

$$15\left(\frac{7}{15}\right)^t = 1 \left(\frac{7}{15}\right)^t = \frac{1}{15}$$

$$\ln\left(\left(\frac{7}{15}\right)^t\right) = \ln\left(\frac{1}{15}\right);\ t\ln\left(\frac{7}{15}\right) = \ln\left(\frac{1}{15}\right)$$

$$t = \frac{\ln(1/15)}{\ln(7/15)} \approx 3.5.$$

Thus, the thermometer will reach 6°C after being outside for about 3.5 minutes.

JEE Main/Boards

Exercise 1

Q.1 Which metal is the best conductor of heat?

Q.2 Which mode of transfer of heat is quickest?

Q.3 What is temperature gradient?

Q.4 How can heat be transferred from one place to other?

Q.5 What are the basic differences between conduction, convection and radiation?

Q.6 What are the thermal radiations? From where do you obtain them? How do they transfer from one place to another?

Q.7 Discuss the variation of temperature of the hot body with time during cooling process. What do you conclude from this?

Q.8 What is meant by thermal conductivity and its coefficient? What are its SI units and CGS units?

Q.9 Explain Newton's law of cooling and discuss its experimental verification.

Q.10 Thickness of ice on a lake is 5 cm. and the temperature of air is -20°C. If the rate of cooling of water inside the lake be 20000 cal min^{-1} through each square meter surface, find K of ice?

Q.11 A metal plate 4 mm thick has a temp difference of 32°C between its faces. It transmits 200kcal h^{-1} through an area of 5 cm^2. Calculate thermal conductivity of the material of the plate.

Q.12 Estimate the rate at which ice would melt in a wooden box 2.5 cm thick and of inside measurements 100×60×40 cm, assuming that the external temperature is 32°C and coefficient of thermal conductivity of wood is 0.168 Wm^{-1} K^{-1}. Given L=80cal/g.

Q.13 A pan filled with hot food cools from 94°C to 86°C in 2 minutes when the room temperature is at 20°C. How long will it take to cool from 71°C to 69°C? Here cooling takes place according to Newton's law of cooling.

Q.14 A liquid initially at 70°C cools to 55°C in 5 minutes and 45°C in 10 minutes. What is the temperature of the surroundings?

Exercise 2

Single Correct Choice Type

Q.1 Four rods of same material with different radii r and length l are used to connect two reservoirs of heat at different temperatures. Which one will conduct most heat?

(A) r=2cm, l=0.5 cm (B) r=2cm, l=2m

(C) r=0.5cm,l=0.5m (D)r=1cm, l=1m

Q.2 A wall has two layers A and B each made of different materials, both the layers have same thickness. The thermal conductivity of the material A is twice of that of B. Under thermal equilibrium, the temperature difference across the wall B is 36°C. The temperature difference across wall A is

(A) 6°C (B) 12°C (C) 18°C (D) 72°C

Q.3 A black metal foil is warmed by radiation from a small sphere at temperature 'T' and at a distance'd'. It is found that the power received by the foil is P. If both the temperature and distance are doubled, the power received by the foil will be

(A) 16 P (B) 4 P (C) 2 P (D) P

Q.4 The rate of emission of radiation of a black body at 273°C is E, then the rate of emission of radiation of this body at 0°C will be

(A) $\frac{E}{16}$ (B) $\frac{E}{4}$ (C) $\frac{E}{8}$ (D) 0

Q.5 The power radiated by a black body is P and it radiates maximum energy around the wavelength λ_0. If the temperature of the black body is now changed so that it radiates maximum energy around wavelength $3/4\,\lambda_0$, the power radiated by it will increase by a factor of

(A) 4/3 (B) 16/9 (C) 64/27 (D) 256/81

Q.6 Star S1 emits maximum radiation of wavelength 420 nm and the star S2 emits maximum radiation of wavelength 560 nm, what is the ratio of the temperature of S1 and S2

(A) $4/3$ (B) $(4/3)^{1/4}$ (C) $3/4$ (D) $(3/4)^{1/2}$

Q.7 Spheres P and Q are uniformly constructed from the same material which is a good conductor of heat and the radius of Q is thrice the radius of P. the rate of fall of temperature of P is x times that of Q when both are at the same surface temperature. The value of x is

(A) 1/4 (B) 1/3 (C) 3 (D) 4

Q.8 A black body calorimeter filled with hot water cools from 60°C to 50°C in 4 min and 40°C to 30°C in 8 min. The approximate temperature of surrounding is

(A) 10°C (B) 15°C (C) 20°C (D) 25°C

Q.9 A system S receives heat continuously from an electrical heater of power 10W. The temperature of S becomes constant at 50°C. When the surrounding temperature is 20°C. After the heater is switched off, S cools from 35.1°C to 34.9°C in 1 minute. The heat capacity of S is

(A) 100 J/°C (B) 300 J/°C

(C) 750 J/°C (D) 1500 J/°C

Previous Years' Questions

Q.1 A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius 2R made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is________. ***(1988)***

Q. 2 Two metallic spheres S_1 and S_2 are made of the same material and have got identical surface finish. The mass of S_1 is thrice that of S_2. Both the spheres are heated to the same high temperature and placed in the same room having lower temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of S_1 to that S_2 is_______. ***(1995)***

Q.3 The intensity of radiation emitted by the sun has its maximum value at a wavelength of 510 nm and that emitted by the North Star has the maximum value at 350 nm. If these stars behave like blackbodies, then the ratio of the surface temperature of the North Star is_____. ***(1997)***

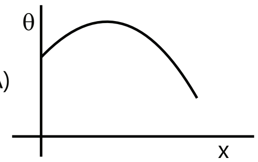
Q. 4 A spherical black body with radius of 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, the power radiated would be___________. ***(1997)***

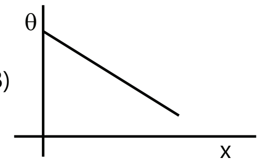
Q.5 A black body is at temperature of 2880 K. The energy of radiation emitted by this body with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wien constant, $b = 2.88 \times 10^6$ nm$-$K. Then, what can be inferred about the relation between the energies? ***(1998)***

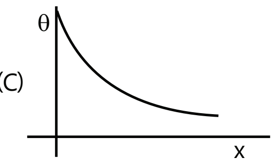
Q.6 Two identical conducting rods are first connected independently to two vessels, one containing water at 100°C and the other containing ice at 0°C. In the second case, rods are joined end to end and connected to the same vessels. Let q_1 and q_2 gram per second be the rate of melting of ice in the two cases respectively. The ratio $\frac{q_1}{q_2}$ is _____. ***(2004)***

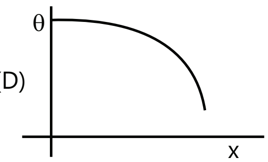
Q.7 Three discs, A, B and C having radii 2m, 4m and 6m respectively are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm respectively. The power radiated by them are Q_A, Q_B and Q_C respectively. Which is the maximum power radiated? ***(2004)***

Q.8 A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figure. ***(2009)***

(A) 

(B) 

(C) 

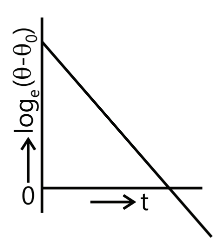
(D) 

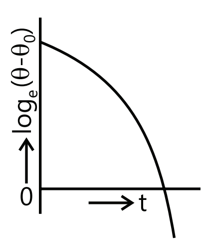
Q.9. 100g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4148 J/kg/K): ***(2011)***

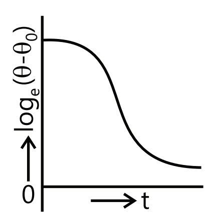
(A) 8.4 kJ (B) 84 kJ

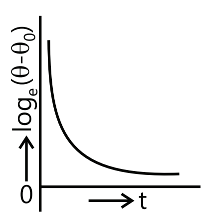
(C) 2.1 kJ (D) 4.2 kJ

Q.10. A liquid in a beaker has temperature θ(t) at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is ***(2012)***

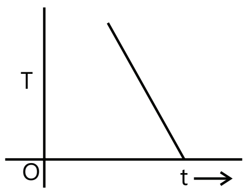
(A) 

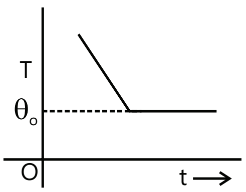
(B) 

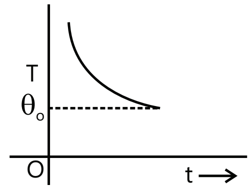
(C) 

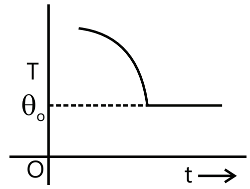
(D) 

Q.11. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closest to: ***(2013)***

(A) 

(B) 

(C) 

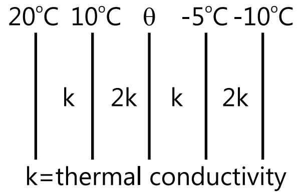
(D) 

JEE Advanced/Boards

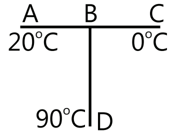
Exercise 1

Q.1 A thin walled metal tank of surface area 5 m^2 is filled with water and contains an immersion heater dissipating 1kW. The tank is covered with 4 cm thick layer of insulation whose thermal conductivity is 0.2 W/m/K. The outer face of the insulation is 25°C. Find the temperature of the tank in the steady state.

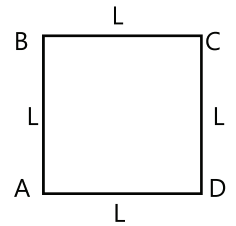
Q.2 The figure shows the face and interface temperature of a composite slab containing of four layers of two materials having identical thickness. Under stady state condition, find the value of temperature θ



Q.3 Three conducting rods of same material and cross-section are shown in figure. Temperature of A,D and C are maintained at 20°C, 90°C and 0°C. Find the ratio of length BD and BC if there is no heat flow in AB

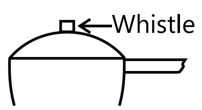


Q.4 In the square frame of side L of metallic rods, the corners A and C are maintained at T_1 and T_2 respectively. The rate of heat flow from A to C is W. If A and D are instead maintained at T_1 and T_2 respectively, find the total rate of heat flow.



Q.5 One end of copper rod of uniform cross-section and of length 1.5 meters is in contact with melting ice and the other end with boiling water. At what point along the length should a temperature of 200°C be maintained, so that in steady state, the mass of ice melting is equal to that of steam produced in the same interval of time? Assume that the whole system is insulated from the surroundings.

Q.6 An empty pressure cooker of volume 10 liters contains air at atmospheric pressure 10^5 Pa and temperature of 27°C. It contains a whistle which has area of 0.1 cm^2 and weight of 100 gm. What should be temperature of air inside so that the whistle is just lifted up?



Exercise 2

Multiple Correct Choice Type

Q.1 Two metallic spheres A and B are made of same material and have got identical surface finish. The mass of sphere A is four times that of B. Both the spheres are heated to the same temperature and placed in a room having lower temperature but thermally insulated from each other.

(A) The ratio of heat loss of A to that of B is $2^{4/3}$

(B) The ratio of heat loss of A to that of B is $2^{2/3}$

(C) The ratio of the initial rate of cooling of A to that of B is $2^{-2/3}$

(D) The ratio of the initial rate of cooling of A to that of B is $2^{-4/3}$

Q.2 Two bodies A and B have thermal emissivity of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies radiate energy at the same rate. The wavelength λ_B, corresponding to the maximum special radiancy in the radiation from B, is shifted from the wavelength corresponding to the maximum spectral radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5802 K,

(A) The temperature of B is 1934 K

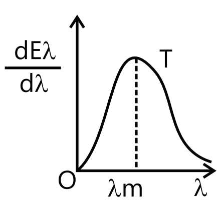
(B) $\lambda_B = 1.5\ \mu m$

(C) The temperature of B is 11604 K

(D) The temperature of B is 2901 K

Comprehension Type

Paragraph 1:



Q.3 The figure shows a radiant energy spectrum graph for a black body at a temperature T.

Choose the correct statement(s)

(A) The radiant energy is not equally distributed among all the possible wavelengths

(B) For a particular wavelength the spectral intensity is maximum

(C) The area under the curve is equal to the rate at which heat is radiated by the body at that temperature

(D) None of these

Q.4 If the temperature of the body is raised to higher temperature T′, then choose the correct statement(s)

(A) The intensity of radiation for every wavelength increases

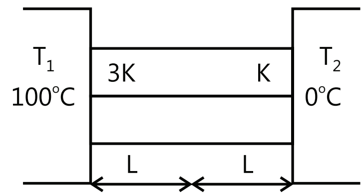
(B) The maximum intensity occurs at a shorter wavelength

(C) The area under the graph increases

(D) The area under the graph is proportional to the fourth power of temperature

Paragraph 2:

Two rods A and B of same cross-sectional area A and length l connected in series between a source (T_1 = 100 °C) and a sink (T_2 = 0 °C) as shown in figure. The rod is laterally insulated



Q.5 The ratio of the thermal resistance of the rod is

(A) $\frac{R_A}{R_B} = \frac{1}{3}$ (B) $\frac{R_A}{R_B} = 3$ (C) $\frac{R_A}{R_B} = \frac{3}{4}$ (D) $\frac{4}{3}$

Q.6 If T_A and T_B are the temperature drops across the rod A and B, then

(A) $\frac{T_A}{T_B} = \frac{3}{1}$ (B) $\frac{T_A}{T_B} = \frac{1}{3}$ (C) $\frac{T_A}{T_B} = \frac{3}{4}$ (D) $\frac{T_A}{T_B} = \frac{4}{3}$

Q.7 If G_A and G_B are the temperature gradients across the rod A and B, then

(A) $\frac{G_A}{G_B} = \frac{3}{1}$ (B) $\frac{G_A}{G_B} = \frac{1}{3}$ (C) $\frac{G_A}{G_B} = \frac{3}{4}$ (D) $\frac{G_A}{G_B} = \frac{4}{3}$

Paragraph 3:

In fluids heat transfer takes place and molecules of the medium take very active part. The molecules take energy from high temperature zone and move towards low temperature zone. This method is known as convection, when we require heat transfer with fast phase, we use some mechanism to make the flow of fluid on the body fast. The rate of loss of heat is proportional to velocity of fluid (v), and temperature difference (ΔT) between the body and fluid, of course more surface area of body , more rate of loss of heat. We can write the rate of loss of heat as

$\frac{dQ}{dt} = KAv\Delta T$ where K is Positive constant.

Now answer the following questions:-

Q.8 A body is being cooled with fluid. When we increase the velocity of fluid 4 times and decrease the temperature difference ½ time, the rate of loss of heat increases

(A) Four times (B) Two times

(C) Six times (D) No change

Q.9 In the above question, if mass of the body increased two times, without change in any of the other parameters, the rate of cooling

(A) Decreases

(B) Increases

(C) No effect of change of mass

(D) None of these

Previous Years' Questions

Q.1 A solid sphere of copper of radius R and a hollow sphere of the same material of inner radius r and outer radius R are heated to the same temperature and allowed to cool in the same environment. Which of them cools faster? ***(1982)***

Q.2 An electric heater is used in a room of total wall area 137 m^2 to maintain a temperature of +20°C inside it, when the outside temperature is -10°C. The walls have three different layers. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 1.0 cm and the outer most layer is of brick of thickness 25.0 cm. Find the power of the electrical heater. Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 W/m/°C respectively. ***(1986)***

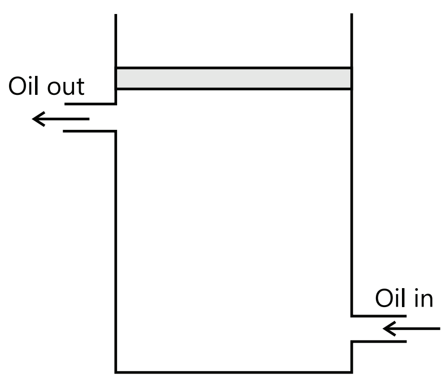
Q.3 A cylindrical block of length 0.4 m and area of cross-section 0.04 m^2 is placed coaxially on a thin metal disc of mass 0.4 kg and of the same cross-section. The upper face of the cylinder is maintained at a constant temperature of 400K and initial temperature of the disc is 300K. If the thermal conductivity of the material of the cylinder is 10W/mK and specific heat capacity of the material of the disc is 600 J/kg-K, how long will it take for the temperature of the disc to increase to 350 K? Assume, for purpose of calculation, the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder. ***(1992)***

Q.4 A double-pane window used for insulating a room thermally from outside consists of two glass sheets each of area 1 m^2 and thickness 0.01 m separated by a 0.05 m thick stagnant air space. In the steady state, the room glass interface and glass-outdoor interface are at constant temperatures of 27°C and 0°C respectively. Calculate the rate of heat of flow through window pane. Also find the temperatures of other interfaces. Given thermal conductivities of glass and air as 0.8 and 0.08 $W m^{-1} K^{-1}$ respectively. ***(1997)***

Q.5 A solid body X of heat capacity C is kept in an atmosphere whose temperature is $T_A = 300$ K. At time t=0, the temperature of X is $T_0 = 400$ K. It cools according to Newton's law of cooling. At time t_1 its temperature is found to be 350 K.

At this time (t_1), the body X is connected to a large body Y at atmospheric temperature T_A through a conducting rod of length L, cross-sectional area A and thermal conductivity K. The heat capacity of Y is so large that any variation in its temperature may be neglected. The cross-sectional area A of the connecting rod is small compared to the surface area of X. Find the temperature of X at time $t = 3t_1$. ***(1998)***

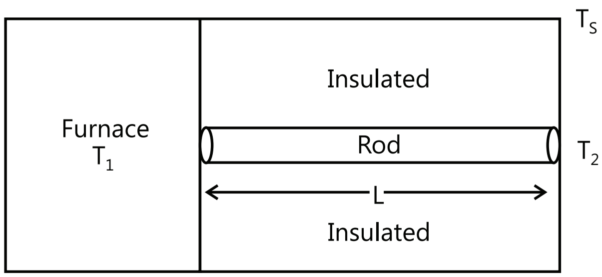
Q.6 The top of an insulated cylindrical container is covered by a disc having emissivity 0.6 and conductivity 0.167 W/Km and thickness 1 cm. The temperature is maintained by circulating oil as shown



(a) Find the radiation loss to the surroundings in W/m^2 if temperature of the upper surface of the disc is 127°C and temperature of surroundings is 27°C.

(b) Also find the temperature of the circulating oil. Neglect the heat loss due to convection ***(2003)***

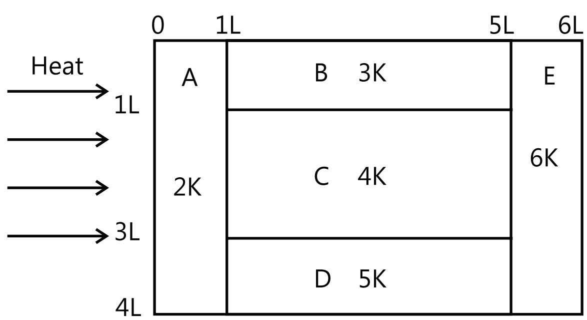
Q.7 One end of a rod of length L and cross-sectional area A is kept in a furnace of temperature T_1. The other end of the rod is kept at temperature T_2. The thermal conductivity of the material of the rod is K and emissivity of the rod is e.



It is given that $T_2 = T_S + \Delta T$, where $\Delta T << T_S$, T_S being the temperature of the surroundings. If $\Delta T \propto (T_1 - T_S)$, find he proportionality constant. Consider that heat is lost only by radiation at the end where the temperature of the rod is T_2. ***(2004)***

Q.8 Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B? ***(2010)***

Q.9 A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat 'Q' flows only from left to right through the blocks. Then in steady state



(A) Heat flow through A and E slabs are same.

(B) Heat flow through slab E is maximum.

(C) Temperature difference across slab E is smallest.

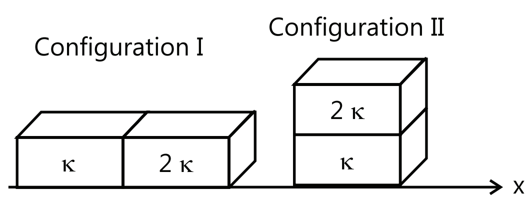
(D) Heat flow through C = heat flow through B + heat flow through D. ***(2011)***

Q.10 Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures 2T and 3T respectively. The temperature of the middle (i.e. second) plate under steady state condition is ***(2012)***

(A) $\left(\frac{65}{2}\right)^{1/4} T$ (B) $\left(\frac{97}{4}\right)^{1/4} T$

(C) $\left(\frac{97}{2}\right)^{1/4} T$ (D) $(97)^{1/4} T$

Q.11 Two rectangular blocks, having identical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure. One of the blocks has thermal conductivity κ and the other 2κ. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9 s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is ***(2013)***



(A) 2.0 s (B) 3.0 s (C) 4.5 s (D) 6.0

Q.12 Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio (λ_A / λ_B) of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is ***(2015)***

Q.13. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has scale that displays $\log_2(P/P_0)$, whre P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767°C? ***(2016)***

Q.14 Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly (take R =8.31 J/mol.K) ***(2012)***

(A) 62 J (B) 104 J (C) 124 J (D) 208 J

Q.15 One mole of mono-atomic ideal gas is taken along two cyclic processes $E \rightarrow F \rightarrow G \rightarrow E$ and $E \rightarrow F \rightarrow H \rightarrow E$ as shown in the PV diagram.

The processes involved are purely isochoric, isobaric, isothermal or adiabatic.

Match the paths in list I with the magnitudes of the work done in list II and select the correct answer using the codes given below the lists. ***(2013)***

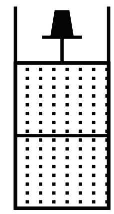
	List I		List II
P.	$G \rightarrow E$	1.	$160\ P_0V_0 \ln 2$
Q.	$G \rightarrow H$	2.	$36P_0\ V_0$
R.	$F \rightarrow H$	3.	$24P_0\ V_0$
S.	$F \rightarrow G$	4.	$31P_0\ V_0$

Codes:

	P	Q	R	S
(A)	4	3	2	1
(B)	4	3	1	2
(C)	3	1	2	4
(D)	1	3	2	4

Paragraph 1:

In the figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_V = \frac{3}{2}R, C_P = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \frac{5}{2}R, C_P = \frac{7}{2}R$.



Q.16 Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be ***(2014)***

(A) 550 K (B) 525 K (C) 513K (D) 490 K

Q.17 Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be ***(2014)***

(A) 250 R (B) 200 R (C) 100 R (D) –100 R

PlancEssential Questions

JEE Main/Boards

Exercise 1

Q.10 Q.11 Q.12 Q.13

Exercise 2

Q.1 Q.2 Q.5 Q.6

Q.9

JEE Advanced/Boards

Exercise 1

Q.2 Q.3 Q.4 Q.6

Exercise 2

Q. 1 Q.2 Q.5 Q.6

Q.7 Q.12 Q.15

Answer Key

JEE Main/Boards

Exercise 1

Q.1 Silver is the best conductor of heat

Q.2 Radiation is the quickest mode of transfer of heat.

Q.3 The fall in temperature in a body per unit distance is called temperature gradient.

Q.10 $3.5\,Wm^{-1\circ}C^{-1}$ **Q.11** $58.33\,Wm^{-1\circ}C^{-1}$

Q.12 $1.587\,gms^{-1}$ **Q.13** 42 s

Q.14 25°C

Exercise 2

Single Correct Choice Type

Q.1 A	**Q.2** C	**Q.3** B	**Q.4** A	**Q.5** D	**Q.6** A
Q.7 C	**Q.8** B	**Q.9** D			

Previous Years' Questions

Q.2 $(1/3)^{1/3}$	**Q.3** 0.69	**Q.4** 1800 W	**Q.5** $U_2>U_1$	**Q. 6** 4/1	**Q.7** Q_B
Q.8 B	**Q.9** A	**Q.10** A	**Q.11** C		

JEE Advanced/Boards

Exercise 1

Q.1 65°C	**Q.2** 5°C	**Q.3** 7/2	**Q.4** (4/3) W	**Q.5** 10.34 cm	**Q.6** 327 °C

Exercise 2

Multiple Correct Choice Type

Q.1 A, C **Q.2** A, B

Comprehension Type

Paragraph 1:	**Q.3** A, B	**Q.4** A, B, C, D	
Paragraph 2:	**Q.5** A	**Q.6** B	**Q.7** B
Paragraph 3:	**Q.8** B	**Q.9** A	

Previous Years' Questions

Q.1 Hollow Sphere

Q.2 9091 W

Q.3 T=166.32 s

Q.4 41.6 W, 26.48 °C, 0.52 °C

Q.5 $T_2 = \left(300 + 12.5e^{\frac{-2AKt_1}{CL}}\right)$

Q.6 (a) 595 W/m^2, (b) 162.6°C

Q.7 $\dfrac{K}{4e\sigma LT_S^3 + K}$

Q.8 9

Q 9: A, C, D or A, B, C, D

Q.10 C

Q.11 A

Q.12 2

Q.13 9

Q.14 D

Q.15 A

Q.16 D

Q.17 D

Solutions

JEE Main/Boards

Exercise 1

Sol 1: Silver is the best conductor of heat

Sol 2: Radiation

Sol 3: Temperature gradient → Fall in temperature in a body per unit distance is called the temperature gradient.

Sol 4: Three Methods:

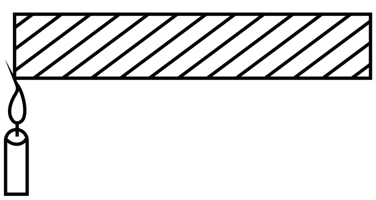
(i) Conduction

(ii) Convection

(iii) Radiation – fastest one because heat travels without any intervening medium.

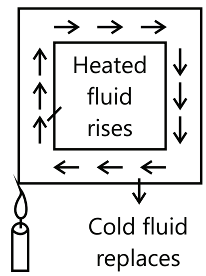
Sol 5: Conduction:- Heat flows from a place of higher temperature to a place of lower temperature with the medium remaining stationary.

Eg. A metal rod heated from one end



Convection: When a fluid in a vessel is heated, lighter molecules present in the lower layer of the fluid get heated, which rise and cold molecules go to the bottom of the vessel. i.e. by movement of the molecules of fluid.

E.g. A gas vessel filled with fluid being heated from bottom.



Radiation:- Heat travels directly from one place to another without any intervening medium.

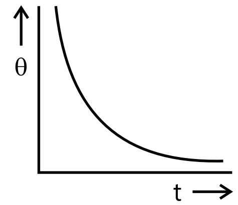
E.g. Heat from the sun to the earth.

Sol 6: Thermal radiations are electromagnetic waves which are invisible. These are radiated from a heated surface in all directions. These travel with velocity of light in a straight line and does not require an intervening medium to carry it.

Sol 7: If a body at temperature θ_1 is placed in surroundings at lower temperature θ_2, then it is observed that magnitude of temperature gradient decreases with time

i.e. $-\dfrac{d\theta}{dt} \propto +(\theta - \theta_2)$ [Newton's law of cooling]

$\Rightarrow \dfrac{d\theta}{dt} = -k(\theta - \theta_2)$ k → a constant



Sol 8: Thermal conductivity is the property of a material to conduct heat.

Coefficient of thermal conductivity (k) is the measure of thermal conductivity which is equal to the quantity of heat flowing per unit time through area of cross-section of a material per unit length along the direction of flow of heat.

S.I. Units: - $J.m^{-1}\ sec^{-1}\ K^{-1}$

C.G.S. Units: - cal. $cm^{-1}\ (°C)^{-1}$

Sol 9: The rate of cooling of a body is directly proportional to the difference of temperature of the body over its surrounding.

Body temperature at any time 't' $\rightarrow \theta$

Body initial temperature $\rightarrow \theta_1$

Surrounding temperature $\rightarrow \theta_2$

$\therefore$ Rate of cooling i.e. $\frac{-dQ}{dt} \propto (\theta - \theta_2)$

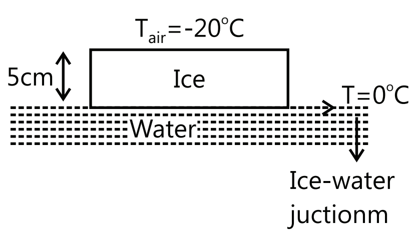
$$\therefore \frac{dQ}{dt} = -k\,(\theta - \theta_2)$$

$$\Rightarrow ms\,\frac{d\theta}{dt} = -k\,(\theta - \theta_2)$$

$$\Rightarrow \frac{d\theta}{dt} = -\left(\frac{k}{ms}\right)(\theta - \theta_2)$$

Sol 10: $\frac{dQ}{dt} = 20000$ cal min^{-1}

$$= \frac{20000 \times 4.2}{60}\ J\ sec^{-1}$$



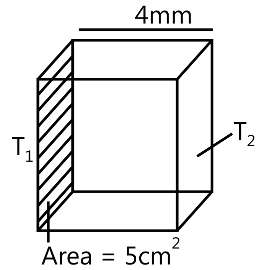
$$\Rightarrow \frac{-K\Delta\theta}{\Delta x} = \frac{20000 \times 4.2}{60}$$

$$\Rightarrow \frac{-K[(-20)-(0)]}{5/100} = 1400 \Rightarrow K = 3.5\ Wm^{-1}\ °C^{-1}$$

Sol 11: ℓ = 4mm = 4×10^{-3} m

$\Delta T = 32°$ C

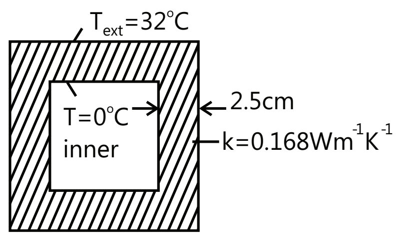
$\frac{dQ}{dt} = 200$ k cal $h^{-1} \simeq 233.33$ J/sec



$$\Rightarrow \frac{KA\Delta T}{\ell} = 233.33\ J/sec$$

$$\Rightarrow K = \frac{233.33 \times 4 \times 10^{-3}}{32 \times 5 \times 10^{-4}} = 58.33\ J/m\ °C\ sec$$

Sol 12: Area of surface perpendicular to direction of flow of heat $\simeq$ surface area of inner rectangle



$\simeq$ [2 [100 × 60]+2 [60 × 40]+2 [100×40]×10^{-4}

= 2.48 m^2

$$\therefore \frac{dQ}{dt} = \frac{-kA}{\ell}(T_{ext} - T_{inner})$$

$$= -\frac{0.168 \times 2.48 \times 32}{2.5 \times 10^{-2}} = -533.29\ J/sec$$

Therefore, rate at which ice melts

$$= \frac{533.29}{80 \times 4.2}\ gm/sec = 1.587\ gm/sec$$

Sol 13: $T = T_{surrounding} + (T_{initial} - T_{surrounding})\,e^{-kt}$

$$\Rightarrow \ln\left(\frac{T - T_{surrounding}}{T_{initial} - T_{surrounding}}\right) = -kt$$

Let when t = 0, $T_{initial} = 94°C$, $T_{surrounding} = 20°C$

$$\therefore -k \times 2 = \ln\left(\frac{86-20}{94-20}\right)$$

$$\Rightarrow -2k = \ln\left(\frac{66}{74}\right) \Rightarrow -k = \frac{-0.114}{2}$$

and let t = 0 when, $T_{initial}$ = 71 °C

$\therefore$ $-kt = \ln\left(\dfrac{69-20}{71-20}\right) = \ln\left(\dfrac{49}{51}\right)$

$\Rightarrow t \simeq 0.70$ min = 42 sec

Sol 14: $T_i = 70°C$

$T_f = 55°C \rightarrow t = 5$ min

$T'_f = 45°\,C \rightarrow t' = 10$ min

$T_0 \rightarrow$ Temperature of surrounding

$\therefore$ From Newton's law

$T - T_0 = (T_i - T_0)\,e^{-kt}$

We have following equation.

$\Rightarrow 55 - T_0 = (70 - T_0)\,e^{-5k}$ ...(i)

And $45 - T_0 = (70 - T_0)\,e^{-10k}$ ...(ii)

Dividing equations (ii)/(i)

$\Rightarrow \dfrac{45-T_0}{55-T_0} = e^{-5k}$

Substitute value of e^{-5k} in (i)

$\Rightarrow (55 - T_0) = (70 - T_0)\left(\dfrac{45-T_0}{55-T_0}\right)$

$\Rightarrow \dfrac{55-T_0}{70-T_0} - 1 = \dfrac{45-T_0}{55-T_0} - 1$

$\Rightarrow \dfrac{-15}{70-T_0} = \dfrac{-10}{55-T_0}$

$\Rightarrow T_0 = 25°\,C$

Exercise 2

Single Correct Choice Type

Sol 1: (A) $\dfrac{dQ}{dt} = -\dfrac{kA}{L}\,\Delta T$

The greater the value of $\dfrac{A}{L}$, more the heat will be conducted.

(A) $\dfrac{A}{L} = \dfrac{\pi(2)^2}{0.5} = 8\pi$

(B) $\dfrac{A}{L} = 2\pi$

(C) $\dfrac{A}{L} = \dfrac{\pi}{2}$

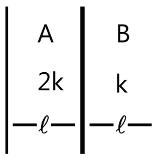
(D) $\dfrac{A}{L} = \pi$

Therefore (A) will conduct more heat.

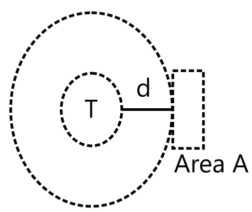
Sol 2: (C) $\left(\dfrac{dQ}{dt}\right)_{across\,A} = \left(\dfrac{dQ}{dt}\right)_{across\,B}$

$\Rightarrow \dfrac{-(2k)\times A}{\ell}\,\Delta T_A = \dfrac{-(k)\times A}{\ell} \times 36$

$\Rightarrow \Delta T_A = 18°C$



Sol 3: (B) P = Area of foil× Intensity (A_{foil})



Intensity $= \dfrac{\text{Power emitted}}{\text{Area of sphere}} = \dfrac{eA\sigma T^4}{4\pi d^2}$

$\therefore P = A_{foil} \times \dfrac{eA\sigma T^4}{4\pi d^2}$

When temperature and distance are double

$\therefore P' = A_{foil} \times \dfrac{eA\sigma(2T)^4}{4\pi(2d)^2} = 4P$

Sol 4: (A) At T = 273°C = (273 + 273) K

$E = eA\sigma\,(273 + 273)^4 = 16eA\sigma\,(273)^4$

At T = 273 K; $E' = eA\sigma\,(273)^4 = \dfrac{E}{16}$

$\therefore E' = \dfrac{E}{16}$

Sol 5: (D) $\lambda_m T$ = const. = 2.93×10^{-3} mK

$P = eA\,\sigma\,T^4$

Given: $\lambda_0 T$= const. = c (say)

When $\lambda_m = \dfrac{3}{4}\lambda_0$ then $\dfrac{3}{4}\lambda_0 \times T' = c$

$\Rightarrow T' = \dfrac{4}{3}\,T$

$\therefore P' = eA\sigma\left(\frac{4}{3}T\right)^4$

$\Rightarrow P' = \frac{256}{81} P$

Sol 6: (A) By Wien's displacement law:-

$\lambda_m T$ = constant = C

$\therefore \lambda_{m1} T_1 = \lambda m_2 T_2$

$\Rightarrow 420\, T_1 = 560\, T_2$

$\Rightarrow \frac{T_1}{T_2} = \frac{560}{420} = \frac{4}{3}$

Sol 7: (C) $r_\theta = 3r_p$

$P = eA\sigma\, T^4$

$\Rightarrow mc\,\frac{dT}{dt} = eA\sigma T^4$

(c: specific heat, m: mass of sphere)

$\Rightarrow \frac{dT}{dt} = \frac{-eA\sigma T^4}{mc} = \frac{-eA\sigma T^4}{(\rho V)c} = \frac{-e4\pi r^2\sigma T^4}{\frac{4}{3}\pi r^3 c}$

(V: volume of sphere)

$\Rightarrow \frac{dT}{dt} = \frac{1}{r} \times \left[\frac{-3e\sigma T^4}{c}\right]$

[Quantity in parenthesis is Constant for both spheres]

$\therefore \frac{\left(\frac{dT}{dt}\right)_P}{\left(\frac{dT}{dt}\right)_Q} = \frac{\frac{1}{r_P}}{\frac{1}{r_Q}} = 3 = x$

Sol 8: (B) By Newton's law of cooling:-

$(T - T_a) = (T_0 - T_a)\, e^{-kt}$

T_a : Surrounding temperature

T_0 : Initial temperature

When $T_0 = 60°$ C, T = 50° C, t = 4 min

$\therefore (50 - T_a) = (60 - T_a)\, e^{-4k}$ ....(i)

When $T_0 = 40$, T = 30 then t = 8 min

$\therefore (30 - T_a) = (40 - T_a)\, e^{-8k}$ ......(ii)

Dividing (ii)/(i) gives

$\frac{30 - T_a}{50 - T_a} = \frac{40 - T_a}{60 - T_a}\, e^{-4k}$ ......(iii)

On substituting value of e^{-4k} from (i) into (iii) we get:

$\frac{30 - T_a}{50 - T_a} = \frac{40 - T_a}{60 - T_a} \times \left(\frac{50 - T_a}{60 - T_a}\right)$

$\Rightarrow (30 - T_a)(60 - T_a)^2 = (40 - T_a)(50 - T_a)^2$

$\Rightarrow (T_a - 60)(T_a - 60)(T_a - 30)$

$= (T_a - 50)(T_a - 50)(T_a - 40)$

$\Rightarrow T_a^3 - [60 + 60 + 30]\, T_a^2 + [60 \times 60 + 60 \times 30 + 60 \times 30]\, T_a - 60 \times 60 \times 30 = T_a^3 - [50 + 50 + 40]\, T_a^2 + [50 \times 50 + 50 \times 40 + 50 \times 40]T_a - 50 \times 50 \times 40$

$\Rightarrow -10T_a^2 + 700\, T_a - 8000 = 0$

$\Rightarrow T_a^2 - 70\, T_a + 800 = 0$

$\Rightarrow T_a = 55.61$ or 14.38

Sol 9: (D) $P = \frac{dQ}{dt} = S \times \frac{d\theta}{dt} = k(\theta_1 - \theta_0)$

$10\,W = k(50 - 20)$ $\quad k = \frac{10W}{30\,°C}$

$S \times \frac{d\theta}{dt} = k(\theta - \theta_0)$

$\Rightarrow S\frac{\Delta\theta}{\Delta t} = k(\theta - \theta_0)$ $\quad \theta = \frac{35.1 + 34.9}{2} = 35$

$S\left(\frac{0.2}{60\text{ sec}}\right) = \frac{10}{30}(35 - 20)$

$S = 1500\text{ J}/°C$

Previous Years' Questions

Sol 1: Let R_1 and R_2 be the thermal resistances of inner and outer portions. Since, temperature difference at both ends is same, the resistances are in parallel. Hence,

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$\because \frac{K(4\pi R^2)}{\ell} = \frac{K_1(\pi R^2)}{\ell} + \frac{K_2(3\pi R^2)}{\ell}$

$\therefore K = \frac{3K_2 + K_1}{4}$

Sol 2: The rate at which energy radiates from the object is

$$\frac{\Delta Q}{\Delta t} = e\sigma AT^4$$

Since, $\Delta Q = mc\Delta T$, we get

$$\frac{\Delta T}{\Delta t} = \frac{e\sigma AT^4}{mc}$$

Also, since $m = \frac{4}{3}\pi r^3\rho$ for a sphere, we get

$$A = 4\pi r^2 = 4\pi\left(\frac{3m}{4\pi\rho}\right)^{2/3}$$

Hence, $\frac{\Delta T}{\Delta t} = \frac{e\sigma T^4}{mc}\left[4\pi\left(\frac{3m}{4\pi\rho}\right)^{2/3}\right] = K\left(\frac{1}{m}\right)^{1/3}$

For the given two bodies

$$\frac{(\Delta T/\Delta t)_1}{(\Delta T/\Delta t)_2} = \left(\frac{m_2}{m_1}\right)^{1/3} = \left(\frac{1}{3}\right)^{1/3}$$

Sol 3: From Wien's displacement law

$\lambda_m T$ = constant

or $T = \frac{1}{\lambda_m}$

$$\therefore \frac{T_{sun}}{T_{north\,star}} = \frac{(\lambda_m)_{north\,star}}{(\lambda_m)_{sun}} = \frac{350}{510} \approx 0.69$$

Sol 4: Power radiated $\propto$ (surface area)$(T)^4$. The radius is halved, hence, surface area will become $\frac{1}{4}$ times. Temperature is doubled, therefore, T^4 becomes 16 times.

New power = $(450)\left(\frac{1}{4}\right)(16) = 1800$ W.

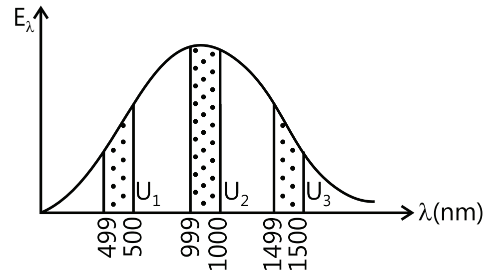
Sol 5: Wien's displacement law is

$\lambda_m T = b$ (b = Wien's constant)

$$\therefore \lambda_m = \frac{b}{T} = \frac{2.88\times10^6\,\text{nm}-\text{K}}{2880\ \text{K}}$$

$\therefore \lambda = 1000$ nm

Energy distribution with wavelength will be as follows:



From the graph it is clear that

$U_2 > U_1$ (In fact U_2 is maximum)

Sol 6: $q = \frac{dm}{dt} \propto \frac{1}{\text{Thermal Resistance}}$

In the first case rods are in parallel and thermal resistance is $\frac{R}{2}$ while in second case rods are in series and thermal resistance is 2R.

$$\frac{q_1}{q_2} = \frac{2R}{R/2} = \frac{4}{1}$$

Sol 7: $Q \propto AT^4$ and $\lambda_m T$ = constant.

Hence, $Q \propto \frac{A}{(\lambda_m)^4}$ or $Q \propto \frac{r^2}{(\lambda_m)^4}$

$$Q_A : Q_B : Q_C = \frac{(2)^2}{(3)^4} : \frac{(4)^2}{(4)^4} : \frac{(6)^2}{(5)^4}$$

$$= \frac{4}{81} : \frac{1}{16} : \frac{36}{625}$$

$= 0.05 : 0.0625 : 0.0576$

i.e., Q_B is maximum.

So 8: (B) We know that $\frac{dQ}{dt} = kA\frac{d\theta}{dx}$

In steady state flow of heat

$$d\theta = \frac{dQ}{dt}\cdot\frac{1}{kA}dx$$

$\Rightarrow \theta_H - \theta = k\,x' \Rightarrow \theta = \theta_H - k\,x'$

Equation $\theta = \theta_H - k'x$ represents a straight line.

Sol 9: (A) $\Delta Q = \Delta U + \Delta W$ (ignoring expansion)

$\Delta U = ms\Delta T = 0.1\times 4.184\times 20 = 8.368$kJ

Sol 10: (A) According to Newtons law of cooling.

$$\frac{d\theta}{dx} \propto (\theta - \theta_0)$$

$$\Rightarrow \frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\int \frac{d\theta}{\theta - \theta_0} = \int -k dt$$

$$\Rightarrow \ln(\theta - \theta_0) = -kt + c$$

Hence the plot of $\ln(\theta - \theta_0)$ vs t should be a straight line with negative slope.

Sol 11: (C) According to Newtons cooling law, option C is the correct option.

JEE Advanced/Boards

Exercise 1

Sol 1: Continuously 1 kW of heat is being dissipated from 25°C tank.

$$\therefore \frac{dQ}{dt} = 10^3 = \frac{-KA[25 - T]}{\ell}$$

$$\Rightarrow \frac{dQ}{dt} = 10^3 = \frac{-0.2 \times 5 \times [25 - T]}{4 \times 10^{-2}}$$

$\Rightarrow 25 - T = -40$; $\Rightarrow T = 65°C$

Sol 2: For 1st layer

$$\frac{dQ}{dt} = \frac{-KA\Delta T}{\ell} = \frac{-KA(10-20)}{\ell} = \frac{+KA \times 10}{\ell}$$

For 2nd layer

$$\frac{dQ}{dt} = \frac{-(2k)A(\theta - 10)}{\ell} = \frac{-2AK}{\ell}[\theta - 10]$$

Rate for both layers must be equal

$$\therefore \frac{kA \times 10}{\ell} = \frac{-2kA}{\ell}(\theta - 10) ; \Rightarrow \theta = 5°C$$

Sol 3: If $\left(\frac{dQ}{dt}\right)_{AB} = 0$

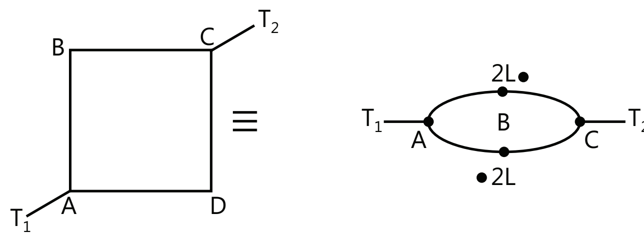
Then rate of heat flow from D to B must be equal to rate of heat flow from B to C.

i.e. $\left(\frac{dQ}{dt}\right)_{DB} = \left(\frac{dQ}{dt}\right)_{BC}$

$$\Rightarrow -\frac{kA(T_B - T_D)}{\ell_{DB}} = \frac{-kA(T_C - T_B)}{\ell_{BC}}$$

$$\Rightarrow \frac{(20-90)}{\ell_{DB}} = \frac{(0-20)}{\ell_{BC}} ; \Rightarrow \frac{\ell_{BD}}{\ell_{BC}} = \frac{-70}{-20} = 3.5$$

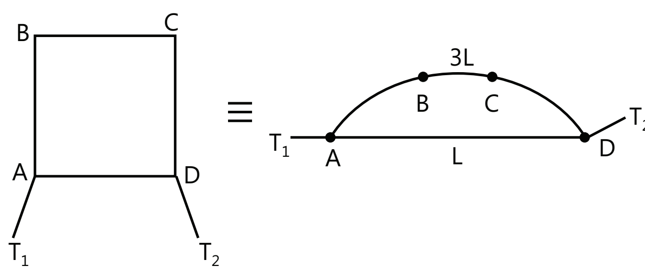
Sol 4:



$$\therefore \left(\frac{dQ}{dt}\right)_{AC} = \left(\frac{dQ}{dt}\right)_{ABC} + \left(\frac{dQ}{dt}\right)_{ADC}$$

$$\Rightarrow W = \frac{-kA(T_2 - T_1)}{2L} + \frac{-kA(T_2 - T_1)}{2L}$$

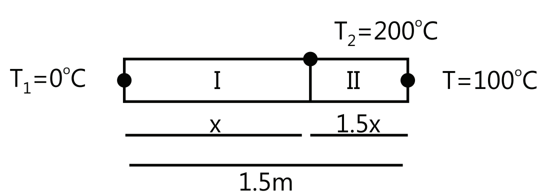
$$\Rightarrow W = \frac{-kA(T_2 - T_1)}{L}$$



$$\therefore \left(\frac{dQ}{dt}\right)_{AD} = \frac{-kA(T_2 - T_1)}{3L} + \frac{-kA(T_2 - T_1)}{L}$$

$$= \frac{-4kA(T_2 - T_1)}{L} = \frac{4}{3}W$$

Sol 5:



Mass of ice melting per second = mass of steam produced per sec

$$\Rightarrow \frac{\frac{-kA(0-200)}{x}}{80} = \frac{\frac{-kA(100-200)}{1.5-x}}{540}$$

$$\Rightarrow \frac{1.5-x}{x} = \frac{1}{2} \times \frac{80}{540}$$

$$\Rightarrow x = \frac{27 \times 1.5}{29} \simeq 1.3966 \text{ m}$$

$\therefore$ 1.5 – x = 0.1034 m = 10.34 cm

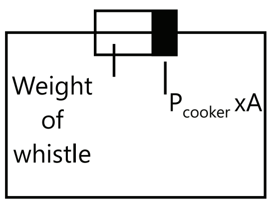
Sol 6: Mass of whistle = 100 gm = 0.1 kg

$\therefore$ Weight of whistle = 1 N

To just lift the whistle, pressure in pressure cooker must be equal to = Atmospheric pressure + Pressure due to weight of whistle

$$= 10^5 + \frac{1\text{N}}{0.1 \times 10^{-4} \text{m}^2} = 2 \times 10^5 \text{ Pa}$$

Free-body diagram of whistle P_{atm} A



By Force-balance

$P_{atm} \times A$ + weight of whistle = $P_{cooker} \times A$

$$\Rightarrow P_{cooker} = P_{atm} + \frac{\text{weight of whistle}}{A}$$

Initially, it is given P = 10^5 Pa, V = 10 L, T = 300K

$$\therefore \text{P.V.} = n\text{RT} ; \Rightarrow n\text{R} = \frac{10 \times 10^5}{300} \text{ Pa.L/K} \qquad \text{....(i)}$$

Finally, we require P = 2 × 10^5 Pa, V = 10 L ,T =?

$\therefore$ By gas equation:- PV = nRT

$$\Rightarrow 2 \times 10^5 \times 10 = \frac{10 \times 10^5}{300} \times T \text{ [using (i)]}$$

$\Rightarrow$ T = 600 K = 327 °C

Exercise 2

Multiple Correct Choice Type

Sol 1: (A, C) $\rho = \dfrac{m}{V}$

$$\Rightarrow \rho \times \frac{4}{3} \pi r^3 = m ; \Rightarrow r \propto (m)^{1/3}$$

and Area of sphere (A) $\propto r^2$

$$\therefore A \propto (m)^{2/3}$$

$$\therefore \frac{A_A}{A_B} = (4)^{2/3} = (2)^{4/3}$$

$$\therefore \text{Ratio of heat loss} = \frac{eA_A\sigma(T-T_0)^4}{eA_B\sigma(T-T_0)^4} = \frac{A_A}{A_B} = (2)^{4/3}$$

By Newton's law of cooling:

$$\frac{dQ}{dt} = ms\frac{dT}{dt} = -k\,(T - T_0)$$

$$\Rightarrow \frac{dT}{dt} = \frac{-k}{ms}(T - T_0)$$

where k = 4e A σ T_0^3

$$\therefore \frac{dT}{dt} \propto \frac{A}{m}$$

$$\therefore \frac{\left(\frac{dT}{dt}\right)_A}{\left(\frac{dT}{dt}\right)_B} = \frac{\frac{A_A}{m_A}}{\frac{A_B}{m_B}} = \frac{(2)^{4/3}}{4} = 2^{-2/3}$$

Sol 2: (A, B) e_A = 0.01 and e_B = 0.81

$A_A = A_B$

$E_A = E_B$

$\Rightarrow e_A \sigma A_A T_A^4 = e_B \sigma A_B T_B^4$

$\Rightarrow 0.01\, T_A^4 = 0.81\, T_B^4$

$$\Rightarrow \quad T_B = \frac{1}{3} \times T_A$$

$$\Rightarrow T_B = \frac{1}{3} \times 5802 = 1934 \text{ K}$$

By Wien's displacement law

λ_m T = const. = 2.93 × 10^{-3} mK

$\therefore \quad \lambda_{m_A}$ = 0.5 μm

Since, it is given in the question that

λ_{m_B} = 1 μm + λ_{m_A}

$\therefore \quad \lambda_{m_B}$ = 1.5 μm

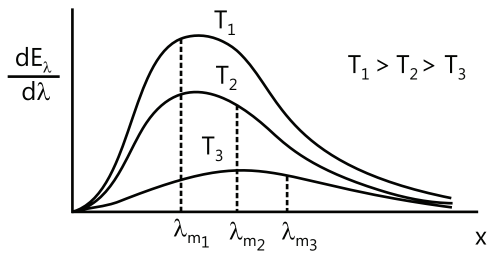
Comprehension Type

Paragraph 1

Sol 3: (A, B) Area under the curve gives the rate at which heat per unit surface is radiated by the body i.e. total rate of heat radiation = (Area under the curve) × (surface area of the body)

Sol 4: (A, B, C, D) $\lambda_m T$ = const. [By Wien's displacement law]

Area under graph = $E_\lambda = e\,\sigma\,T^4 \propto T^4$



Paragraph 2

Sol 5: (A) $A_A = A$; $A_B = A$

$\ell_A = l$; $\ell_B = l$

$k_A = 3k$; $k_B = k$

$\therefore R_A = \frac{\ell_A}{k_A A_A} = \frac{\ell}{3kA}$; $R_B = \frac{\ell_B}{k_B A_B} = \frac{\ell}{k_A}$

$\therefore \frac{R_A}{R_B} = \frac{1}{3}$

Sol 6: (B) Rate at which heat flows from A

= Rate at which heat flows from B

$\Rightarrow \left(\frac{dQ}{dt}\right)_A = \left(\frac{dQ}{dt}\right)_B$

$\Rightarrow \frac{T_A}{R_A} = \frac{T_B}{R_B} \Rightarrow \frac{T_A}{T_B} = \frac{R_A}{R_B} = \frac{1}{3}$

Sol 7: (B) $G_A = \frac{T_A}{L_A} = \frac{T_A}{L}$ and $G_B = \frac{T_B}{L_B} = \frac{T_B}{L}$

$\therefore \frac{G}{G_B} = \frac{T_A}{T_B} = \frac{1}{3}$

Paragraph 3

Sol 8: (B) $\left(\frac{dQ}{dt}\right)_{\text{initially}} = KAv\,\Delta T$

$\left(\frac{dQ}{dt}\right)_{\text{finally}} = KA(4v)\left(\frac{\Delta T}{2}\right) = 2\left(\frac{dQ}{dt}\right)_{\text{initially}}$

Sol 9: (A) If all the parameters are kept constant then

$\frac{dQ}{dt} = ms\frac{dT}{dt} = kA\,v\,\Delta T$

$\therefore \frac{dT}{dt} = \frac{kAv\Delta T}{ms}$

Previous Years' Questions

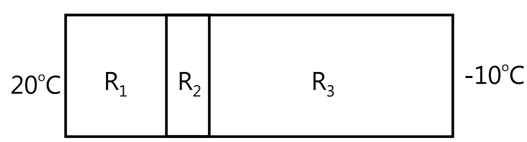
Sol 1: Net rate of heat radiation $\left(\frac{dQ}{dt}\right)$ will be same in both the cases, as temperature and area are same.

Therefore, from equation

$ms\left(-\frac{d\theta}{dt}\right) = \frac{dQ}{dt}$ or $-\frac{d\theta}{dt} \propto \frac{1}{m}$

The hollow sphere will cool faster as its mass is less.

Sol 2: Let R_1, R_2 and R_3 be the thermal resistances of wood, cement and brick. All the resistances are in series. Hence,



$R = R_1 + R_2 + R_3$

$= \frac{2.5\times10^{-2}}{0.125\times137} + \frac{1.0\times10^{-2}}{1.5\times137} + \frac{25\times10^{-2}}{1.0\times137}$

$= 0.33 \times 10^{-2}$ °C/W $\left(\text{as } R = \frac{\ell}{KA}\right)$

$\therefore$ Rate of heat transfer,

$\frac{dQ}{dt} = \frac{\text{Temperature difference}}{\text{thermal resistance}} = \frac{30}{0.33\times10^{-2}}$

≈ 9091 W

$\therefore$ Power of heater should be 9091 W.

Sol 3: Let at any time temperature of the disc be θ.

At this moment rate of heat flow,

$$\frac{dQ}{dt} = \frac{KA(\Delta\theta)}{\ell} = \frac{KA}{\ell}(\theta_0 - \theta) \quad \text{..... (i)}$$

This heat is utilised in increasing the temperature of the disc.

$$\text{Hence, } \frac{dQ}{dt} = ms\frac{d\theta}{dt} \quad \text{..... (ii)}$$

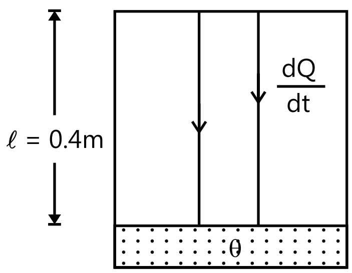
Equating Eqs. (i) and (ii), we have

$$ms\frac{d\theta}{dt} = \frac{KA}{\ell}(\theta_0 - \theta)$$

Therefore, $\dfrac{d\theta}{\theta_0 - \theta} = \dfrac{KA}{ms\ell}dt$

or $\displaystyle\int_{300K}^{350K}\frac{d\theta}{\theta_0 - \theta} = \frac{KA}{ms\ell}\int_0^t dt$

or $\left[-\ln(\theta_0 - \theta)\right]_{300K}^{350K} = \dfrac{KA}{ms\ell}t$



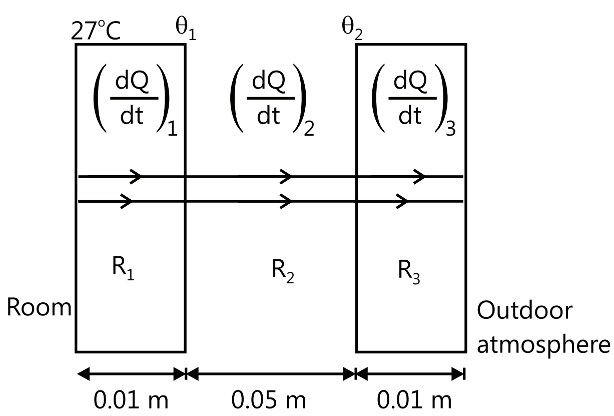
$$\therefore\ t = \frac{ms\ell}{KA}\lambda n\left(\frac{\theta_0 - 300}{\theta_0 - 350}\right)$$

Substituting the values, we have

$$T = \frac{(0.4)(600)(0.4)}{(10)(0.04)}\ln\left(\frac{400-300}{400-350}\right)$$

T = 166.32 s

Sol 4: Let θ_1 and θ_2 be the temperatures of the two interfaces as shown in figure.



Thermal resistance, $R = \dfrac{\ell}{KA}$

$\therefore\ R_1 = R_3 = \dfrac{(0.01)}{(0.8)(1)} = 0.0125$ K/W or °C/W

and $R_2 = \dfrac{(0.05)}{(0.08)(1)} = 0.625$ °C/W

Now the rate of heat flow $\left(\dfrac{dQ}{dt}\right)$ will be equal from all the three sections and since rate of heat flow is given by

$$\frac{dQ}{dt} = \frac{\text{Temperature difference}}{\text{Thermal resistance}}$$

and $\left(\dfrac{dQ}{dt}\right)_1 = \left(\dfrac{dQ}{dt}\right)_2 = \left(\dfrac{dQ}{dt}\right)_3$

Therefore, $\dfrac{27-\theta_1}{0.0125} = \dfrac{\theta_1 - \theta_2}{0.625} = \dfrac{\theta_2 - \theta}{0.0125}$

Solving this equation, we get

$\theta_1 = 26.48°C$

and $\theta_2 = 0.52°C$

and $\dfrac{dQ}{dt} = \dfrac{27-\theta_1}{0.0125}$

$$\frac{dQ}{dt} = \frac{(27-26.48)}{0.0125} = 41.6\text{ W}$$

Sol 5: In the first part of the question ($t \le t_1$)

At t = 0, $T_X = T_0 = 400$ K and at $t = t_1$,

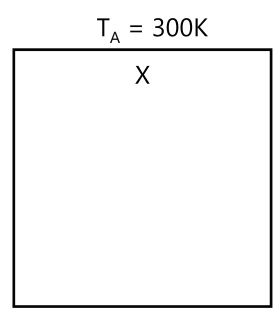
$T_X = T_1 = 350$ K

Temperature of atmosphere,

$T_A = 300$ K (constant)

This cools down according to Newton's law of cooling. Therefore,

rate of cooling ∝ temperature difference.



$$\therefore\ \left(-\frac{dT}{dt}\right) = k(T - T_A)$$

$$\Rightarrow \frac{dT}{T - T_A} = -k\,dt$$

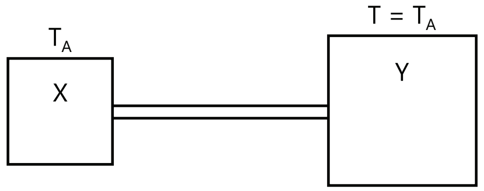
$$\Rightarrow \int_{T_0}^{T_1}\frac{dT}{T - T_A} = -k\int_0^{t_1}dt$$

$$\Rightarrow \ln\left(\frac{T_1 - T_A}{T_0 - T_A}\right) = -kt_1$$

$$\Rightarrow kt_1 = -\ln\left(\frac{350-300}{400-300}\right)$$

$$\Rightarrow kt_1 = \ln(2)$$

In the second part ($t > t_1$), body X cools by radiation (according to Newton's law) as well as by conduction.



Therefore, rate of cooling

= (cooling by radiation) + (cooling by conduction)

$$\therefore \left(-\frac{dT}{dt}\right) = k(T - T_A) + \frac{KA}{CL}(T - T_A) \qquad \text{.... (ii)}$$

In conduction, $\frac{dQ}{dt} = \frac{KA(T - T_A)}{L} = C\left(-\frac{dT}{dt}\right)$

$$\therefore \left(-\frac{dT}{dt}\right) = \frac{KA}{LC}(T - T_A)$$

where, C = heat capacity of body X

$$\left(-\frac{dT}{dt}\right) = \left(k + \frac{KA}{CL}\right)(T - T_A) \qquad \text{..... (iii)}$$

Let at $t = 3t_1$ temperature of X becomes T_2

Then from eq. (iii)

$$\int_{T_1}^{T_2} \frac{dT}{T - T_A} = -\left(k + \frac{KA}{LC}\right)\int_{t_1}^{3t_1} dt$$

$$\ln\left(\frac{T_2 - T_A}{T_1 - T_A}\right) = -\left(k + \frac{KA}{LC}\right)(2t_1)$$

$$= -\left(2kt_1 + \frac{2KA}{LC}t_1\right)$$

$$\text{or } \ln\left(\frac{T_2 - 300}{350-300}\right) = -2\ln(2) - \frac{2KAt_1}{LC};$$

$kt_1 = \ln(2)$ from Eq. (i)

This gives equation :-

$$T_2 = \left(300 + 12.5e^{-\frac{2KAt_1}{CL}}\right)K$$

Sol 6: (a) Rate of heat loss per unit area due to radiation

$$I = e\sigma(T^4 - T_0^4)$$

Here, $T = 127 + 273 = 400$ K

and $T_0 = 27 + 273 = 300$ K

$$\therefore I = 0.6 \times \frac{17}{3} \times 10^{-8}[(400)^4 - (300)^4]$$

$= 595$ W/m^2

(b) Let θ be the temperature of the oil. Then, rate of heat flow through conduction = rate of heat loss due to radiation

$$\therefore \frac{\text{temperature difference}}{\text{thermal resistance}} = (595)A$$

$$\frac{(\theta - 127)}{\left(\frac{\ell}{KA}\right)} = (595)A$$

Here, A = area of disc; K = Thermal conductivity and ℓ = thickness (or length) of disc

$$\therefore (\theta - 127)\frac{K}{\ell} = 595$$

$$\therefore \theta = 595\left(\frac{\ell}{K}\right) + 127$$

$$= \frac{595 \times 10^{-2}}{0.167} + 127 = 162.6°\text{C}$$

Sol 7: Rate of heat conduction through rod = rate of the heat lost from right end of the rod.

$$\therefore \frac{KA(T_1 - T_2)}{L} = eA\sigma(T_2^4 - T_s^4) \qquad \text{...... (i)}$$

Given that $T_2 = T_s + \Delta T$

$$\therefore \quad T_2^4 = (T_s + \Delta T)^4 = T_s^4\left(1 + \frac{\Delta T}{T_s}\right)^4$$

Using binomial expansion, we have

$$T_2^4 = T_s^4\left(1 + 4\frac{\Delta T}{T_s}\right) \quad (\text{as } \Delta T << T_s)$$

$$\therefore T_2^4 - T_s^4 = 4(\Delta T)(T_s^3)$$

Substituting in Eq. (i), we have

$$\frac{K(T_1 - T_s - \Delta T)}{L} = 4e\sigma T_s^3.\Delta T$$

$$\text{or } \frac{K(T_1 - T_s)}{L} = \left(4e\sigma T_s^3 + \frac{K}{L}\right)\Delta T$$

$\therefore \quad \Delta T = \dfrac{K(T_l - T_s)}{(4e\sigma LT_s^3 + K)}$

Comparing with the given relation,

proportionality constant $= \dfrac{K}{4e\sigma LT_s^3 + K}$

Sol 8: $\lambda_m \propto \dfrac{1}{T}$

$\therefore \quad \dfrac{\lambda_A}{\lambda_B} = \dfrac{T_B}{T_A} = \dfrac{500}{1500} = \dfrac{1}{3}$

$E \propto T^4 A$

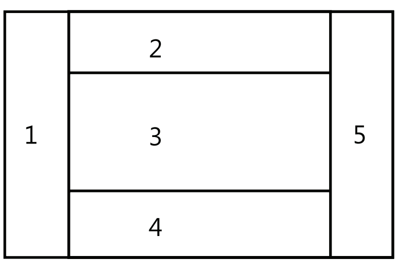
(Where A = surface area = $4\pi R^2$)

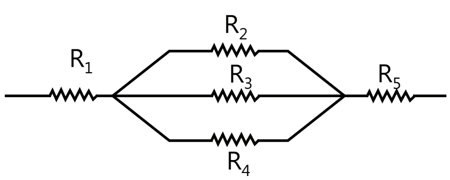
$\therefore \quad E \propto T^4R^2$

$\dfrac{E_A}{E_B} = \left(\dfrac{T_A}{T_B}\right)^4 \left(\dfrac{R_A}{R_B}\right)^2 = (3)^4\left(\dfrac{6}{18}\right)^2 = 9$

$\therefore$ Answer is 9.

Sol 9: **(A, C, D) or (A, B, C, D)**





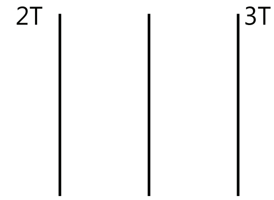
Let width of each rod is d

$R_1 = \dfrac{1}{8kd}, R_2 = \dfrac{1}{3kd}$

$R_3 = \dfrac{1}{2kd}, R_4 = \dfrac{1}{5kd},$

$R_5 = \dfrac{1}{24kd}$

Sol 10: (C)



$\sigma A(2T)^4 + \sigma A(3T)^4 = \sigma 2A(T)^4$

$16T^4 + 81T^4 = 2(T')^4$

$97T^4 = 2(T')^4$

$(T')^4 = \dfrac{97}{2}T^4$

$\therefore T' = \left(\dfrac{97}{2}\right)^{1/4} T$

Sol 11: (A) $R_1 = \dfrac{L}{\kappa A} + \dfrac{L}{2\kappa A} = \dfrac{3L}{2\kappa A}$

$\dfrac{1}{R_2} = \dfrac{1}{\left(\dfrac{L}{\kappa A}\right)} + \dfrac{1}{\left(\dfrac{L}{2\kappa A}\right)} = \dfrac{3\kappa A}{L}$

$R_2 = \dfrac{L}{3\kappa A}$

$\Delta Q_1 = \Delta Q_2$

$\dfrac{\Delta T}{R_1}t_1 = \dfrac{\Delta T}{R_2}t_2$

$\Rightarrow t_2 = \dfrac{R_2}{R_1}t_1 = 2 \text{ sec}$

Sol 12: (2) $\left(\dfrac{dQ}{dt}\right)_A = 10^4\left(\dfrac{dQ}{dt}\right)_B$

$(400R)^2T_A^4 = 10^4(R^2T_B^4)$

$\text{So}, 2T_A = T_B \text{ and } \dfrac{\lambda_A}{\lambda_B} = \dfrac{T_B}{T_A} = 2$

Sol 13: (9) At (T_1 = 487+ 273 = 760K) $P_1 \propto (760)^4$

i.e. $P_1 = c(760)^4$ where c = constant

$\log_2 \dfrac{P_1}{P_0} = 1 \Rightarrow P_1 = 2P_0 \Rightarrow P_0 \dfrac{P_1}{2}$

at (T_2 = 2767 + 273 = 3040)

$P_2 = c(3040)^4$

Reading of the sensor at $T_2 = \log_2\left(\dfrac{P_2}{P_0}\right)$

$= \log_2\left[2.\dfrac{P_2}{P_0}\right] = \log_2\left[2\left(\dfrac{3040}{760}\right)^4\right] = \log_2\left[2^1.2^8\right] = 9$

$\therefore$ Reading of T_2 = 9

Sol 14: (D) $\Delta Q = {}_nC_P\Delta T$

$$= 2\left(\frac{f}{2}R + R\right)\Delta T = 2\left[\frac{3}{2} + R + R\right] \times 5$$

$$= 2 \times \frac{5}{2} \times 8.31 \times 5 = 208\,J$$

Sol 15: (A) P. → 4 ; Q. → 3; R. → 2; S. → 1

Apply $PV^{1+2/3}$ = constant for F to H.

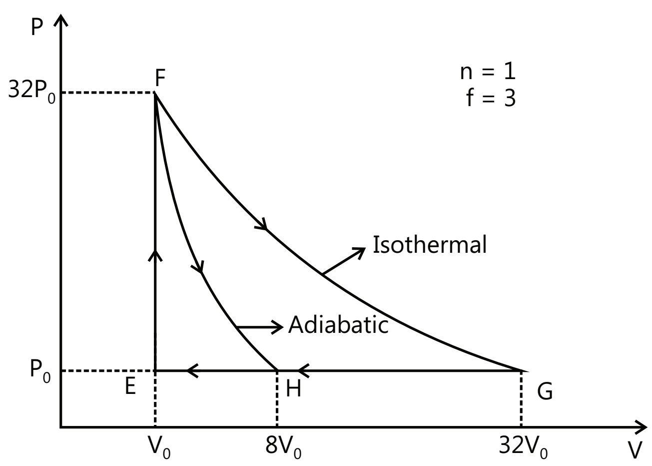
$$(32P_0)V_0^{5/3} = P_0V_H^{5/3} \Rightarrow V_H = 8V_0$$

For path FG PV = constant

$$\Rightarrow (32P_0)V_0 = P_0V_G \Rightarrow V_G = 32V_0$$

Work done in GE = 31 P_0V_0

Work done in GH = 24 P_0V_0



Work done in $FH = \dfrac{P_HV_H - P_FV_F}{(-2/f)} = 36P_0V_0$

Work done in $FG = RT\ln\left(\dfrac{V_G}{V_F}\right) = 160P_0V_0\ln 2$

Sol 16: (D) 13. Heat given by lower compartment

$$= 2 \times \frac{3}{2}R \times (700 - T) \qquad \text{... (i)}$$

Heat obtained by upper compartment

$$= 2 \times \frac{7}{2}R \times (T - 400) \qquad \text{... (ii)}$$

equating (i) and (ii)

3 (700 – T) = 7 (T – 400)

2100 – 3T = 7 T – 2800

4900 = 10 T ⇒ T = 490 K

Sol 17: (D) Heat given by lower compartment

$$= 2 \times \frac{5}{2}R \times (700 - T) \qquad \text{... (i)}$$

Heat obtained by upper compartment

$$= 2 \times \frac{7}{2}R \times (T - 400) \qquad \text{... (ii)}$$

By equating (i) and (ii)

5(700 - T) = 7(T - 400)

3000 – 5T = 7T– 2800

6300 = 12 T

T = 525K

∴ Work done by lower gas = $nR\Delta T$ = – 350 R

Work done by upper gas = $nR\Delta T$ = +250 R

Net work done - 100 R