Q1: NTA Test 01 (Single Choice)

A partition wall has two layers of different materials A and B in contact with each other. They have the same thickness but the thermal conductivity of layer A is twice that of B. At steady state, if the temperature difference across the layer B is $50\ \mathrm{K}$, then the corresponding temperature difference across the layer A is

(A) 50 K

(B) 12.5 K

(C) 25 K

(D) 60 K

Q2: NTA Test 02 (Single Choice)

A sphere and a cube of same material and same total surface area are placed in the same evacuated space turn by turn after they are heated to the same temperature. Find the ratio of their initial rates of cooling in the enclosure.

(A) $\sqrt{\frac{\pi}{6}} : 1$

(B) $\sqrt{\frac{\pi}{3}} : 1$

(C) $\frac{\pi}{\sqrt{6}} : 1$

(D) $\frac{\pi}{\sqrt{3}} : 1$

Q3: NTA Test 03 (Single Choice)

A partition wall has two layers of different materials A and B in contact with each other. They have the same thickness but the thermal conductivity of layer A is twice that of B. At steady state, if the temperature difference across the layer B is $50\ \mathrm{K}$, then the corresponding temperature difference across the layer A is

(A) 50 K

(B) 12.5 K

(C) 25 K

(D) 60 K

Q4: NTA Test 04 (Single Choice)

Three very large plates of the same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures $2T$ and $3T$ respectively. The temperature of the middle plate under steady state condition is

(A) $\left(\frac{65}{2}\right)^{1/4}T$

(B) $\left(\frac{97}{4}\right)^{1/4}T$

(C) $\left(\frac{97}{2}\right)^{1/4}T$

(D) $(97)^{1/4}T$

Q5: NTA Test 06 (Single Choice)

One end of a copper rod of uniform cross-section and of length $1.5\ \mathrm{m}$ is kept in contact with ice at $0^{\circ}\mathrm{C}$ and the other end with water at $100^{\circ}\mathrm{C}$. If the whole system is insulated from its surroundings, then at what distance from the end of the rod which is in contact with the steam, the temperature should be maintained at $200^{\circ}\mathrm{C}$, so that in steady-state the rate of melting of ice be equal to the rate of production of steam?
$\left[L_{\text{fusion}} = 80\ \mathrm{cal\ g^{-1}}, L_{\text{vaporization}} = 540\ \mathrm{cal\ g^{-1}}\right]$

(A) $10.34\ \mathrm{cm}$

(B) $16.48\ \mathrm{cm}$

(C) $21.68\ \mathrm{cm}$

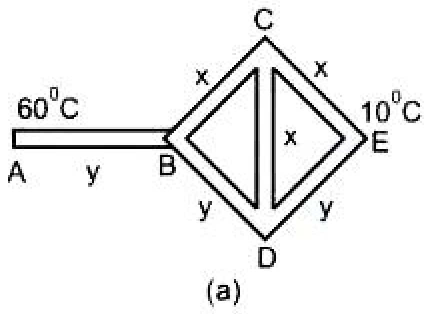
(D) $27.87\ \mathrm{cm}$

Q6: NTA Test 07 (Numerical)

Two uniform solid spheres made of copper have radii $15\ \mathrm{cm}$ and $20\ \mathrm{cm}$ respectively. Both of them are heated to a temperature of $70^{\circ}\mathrm{C}$ and then placed in a region of ambient temperature equal to $45^{\circ}\mathrm{C}$. What will be the ratio of the initial rates of cooling of both the spheres?

Q7: NTA Test 10 (Single Choice)

Three rods of material x and three rods of material y of identical length and cross sectional area are connected as shown in the figure. If the end A is maintained at $60^{\circ}\mathrm{C}$ and the junction E at $10^{\circ}\mathrm{C}$, then the temperature of junction C is [Given the thermal conductivity of x is $0.92\ \mathrm{W/m.K}$ and that of y is $0.46\ \mathrm{W/m.K}$]



(A) 10° C

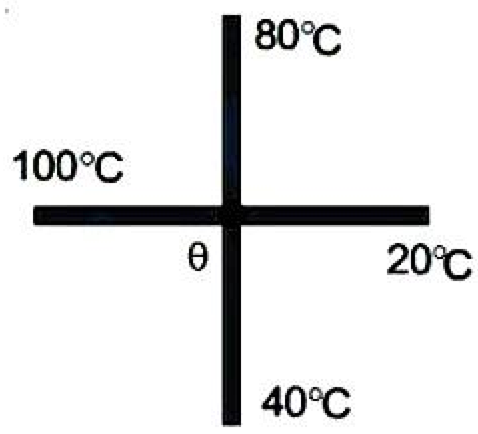
(B) 20° C

(C) 30° C

(D) 40° C

Q8: NTA Test 11 (Single Choice)

Four identical heat conductors are connected as shown in the figure. The temperature θ of the junction is [rods are insulated from the sides]



(A) 30°C

(B) 15°C

(C) 60°C

(D) 70°C

Q9: NTA Test 12 (Single Choice)

A liquid A of mass 100 g at 100°C is added to 50 g of a liquid B at temperature 75°C, the temperature of the mixture becomes 90°C. Now if 100 g of liquid A at 100°C is added to 50 g of liquid B at 50°C, temperature of the mixture will be

(A) 80°C

(B) 60°C

(C) 70°C

(D) 85°C

Q10: NTA Test 13 (Single Choice)

A body cools from 50 $^{\circ}$C to 40 $^{\circ}$C in 5 minutes. Its temperature comes down to 33.33 $^{\circ}$C in the next 5 minutes. The temperature of the surroundings is

(A) 15 °C

(B) 20 °C

(C) 25 °C

(D) 10 °C

Q11: NTA Test 16 (Single Choice)

The temperature of a body falls from 40 $^{\circ}$C to 36 $^{\circ}$C in $4\ \text{minutes}$, when placed in a constant ambient temperature of 20 $^{\circ}$C. The time it takes for the temperature to fall from 36 $^{\circ}$C to 32 $^{\circ}$C in the same ambient temperature is

(A) $5.1\ \text{min}$

(B) $6.8\ \text{min}$

(C) $9\ \text{min}$

(D) $7.2\ \text{min}$

Q12: NTA Test 18 (Single Choice)

Two electric lamps A and B radiate the same power. Their filaments have the same dimensions and have emissivities e_A and e_B respectively. Their surface temperatures are T_A and T_B. The ratio $\frac{T_A}{T_B}$ will be equal to

(A) $\left(\frac{e_B}{e_A}\right)^{1/4}$

(B) $\left(\frac{e_B}{e_A}\right)^{1/2}$

(C) $\left(\frac{e_A}{e_B}\right)^{1/2}$

(D) $\left(\frac{e_A}{e_B}\right)^{1/4}$

Q13: NTA Test 19 (Single Choice)

Ice starts forming in a lake where the water is at $0\ ^{\circ}C$ and the ambient temperature is $-10\ ^{\circ}C$. If the time taken for 1 cm of ice to be formed is 7 hours, then the time taken for the thickness of ice to change from 1 cm to 2 cm is

(A) 7 hours
(B) 14 hours
(C) 10.5 hours
(D) 21 hours

Q14: NTA Test 25 (Single Choice)

A seconds pendulum clock having steel wire is calibrated at $20\ ^{\circ}C$. When temperature is increased to $30\ ^{\circ}C$, then calculate how much time does the clock lose or gain in one week $\left[\alpha_{steel} = 1.2 \times 10^{-5}\ ^{\circ}C^{-1}\right]$

(A) 0.3628 s
(B) 3.626 s
(C) 362.8 s
(D) 36.28 s

Q15: NTA Test 30 (Single Choice)

Two rods, one of aluminium and the other made of steel, having initial lengths l_1 and l_2 are connected together to form a single rod of length $l_1 + l_2$. The coefficients of linear expansion for aluminium and steel are α_a and α_s respectively. If the length of each rod increases by the same amount when their temperature is raised by $t\ ^{\circ}C$, then find the ratio $\frac{l_1}{l_1+l_2}$

(A) $\frac{\alpha_s}{\alpha_a}$
(B) $\frac{\alpha_a}{\alpha_s}$
(C) $\frac{\alpha_s}{(\alpha_a+\alpha_s)}$
(D) $\frac{\alpha_a}{(\alpha_a+\alpha_s)}$

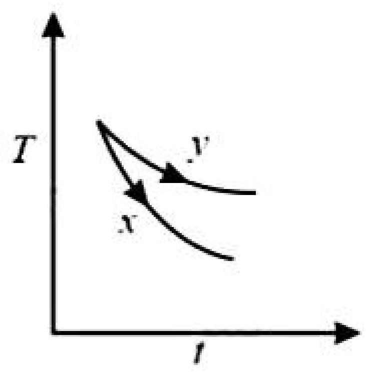
Q16: NTA Test 33 (Single Choice)

In an energy recycling process, X g of steam at $100\ ^{\circ}C$ becomes water at $100\ ^{\circ}C$ which converts Y g of ice at $0\ ^{\circ}C$ into water at $100\ ^{\circ}C$. The ratio of $\frac{X}{Y}$ will be (specific heat of water $= 4200\ J\ kg^{-1}\ K$, specific latent heat of fusion $= 3.36 \times 10^5\ J\ kg^{-1}$, specific latent heat of vaporization $= 22.68 \times 10^5\ J\ kg^{-1}$)

(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) 3
(D) 2

Q17: NTA Test 34 (Single Choice)

The graph shown in the adjacent diagram represents the variation of temperature (T) of two bodies x and y having the same surface area, with time (t). Both of these bodies lose heat only due to the emission of radiation. Find the correct relation between the emissive and absorptive power of the two bodies.



(A) $E_x > E_y$ and $a_x < a_y$
(B) $E_x < E_y$ and $a_x > a_y$
(C) $E_x > E_y$ and $a_x > a_y$
(D) $E_x < E_y$ and $a_x < a_y$

Q18: NTA Test 35 (Single Choice)

Two rods of the same length and material transfer a given amount of heat in 12 s, when they are joined end to end, i.e. in series. When they are joined in parallel, they will transfer the same heat under the same conditions in

(A) 24 s
(B) 3 s
(C) 48 s
(D) 1.5 s

Q19: NTA Test 36 (Single Choice)

The amount of heat energy required to freeze 4. 5 g of water at 6 $^\circ$C to ice at 0 $^\circ$C is $[S = 4190\ \text{J kg}^{-1}\ \text{K}^{-1}, L = 3.33 \times 10^5\ \text{J kg}^{-1}]$

(A) 1612 J
(B) 1512 J
(C) 1132 J
(D) 1499 J

Q20: NTA Test 38 (Single Choice)

Four rods, of different radii r and length l, are used to connect two reservoirs of heat at different temperatures. The rod that will conduct the heat fastest will be

(A) $r = 2\ \text{cm},\ l = 0.5\ \text{m}$
(B) $r = 1\ \text{cm},\ l = 0.5\ \text{m}$
(C) $r = 2\ \text{cm},\ l = 2\ \text{m}$
(D) $r = 1\ \text{cm},\ l = 1\ \text{m}$

Q21: NTA Test 39 (Single Choice)

The temperature of a liquid drops from 365 K to 361 K in 2 minutes. The time during which temperature of the liquid drops from 344 K to 342 K is (Room temperature is 293 K) -

(A) 84 s
(B) 72 s
(C) 66 s
(D) 60 s

Q22: NTA Test 40 (Single Choice)

The power radiated by a black-body is P_0 and it radiates maximum energy around the wavelength λ_0. If the temperature of the black-body is now changed so that it radiates maximum energy around a wavelength $^{3\lambda_0}/_4$, then the power radiated by it will be

(A) $\frac{4}{3}P_0$
(B) $\frac{16}{9}P_0$
(C) $\frac{64}{27}P_0$
(D) $\frac{256}{81}P_0$

Q23: NTA Test 41 (Numerical)

A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2\left(\frac{P}{P_0}\right)$, where P_0 is a constant. When the metal surface is at a temperature of 487 $^\circ$C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 $^\circ$C?

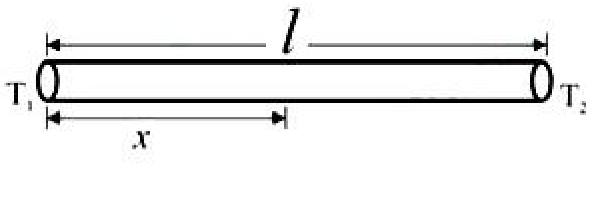
Q24: NTA Test 42 (Single Choice)

A point source of heat of power P is placed at the centre of a thin spherical shell of mean radius R. The material of the shell has thermal conductivity K. Calculate the thickness of the shell if the temperature difference between the outer and inner surfaces of the shell in steady-state is T.

(A) $\frac{4\pi R^2 KT}{2P}$
(B) $\frac{3\pi R^4 KT}{P}$
(C) $\frac{4\pi R^2 KT}{P}$
(D) $\frac{2\pi R^2 KT}{3P}$

Q25: NTA Test 43 (Single Choice)

A rod of length l with thermally insulated lateral surface consists of material whose heat conductivity coefficient varies with temperature as $K(x) = \frac{\alpha}{T}$ (here α is a positive constant). The ends of the rod are kept at temperature T_1 and T_2 $(T_1 > T_2)$.



Find the heat current per unit cross-sectional area in the rod

(A) $\frac{\alpha}{l}\ln\left(\frac{T_1+T_2}{T_1}\right)$
(B) $\frac{\alpha}{l}\ln\left(\frac{T_1}{T_2}\right)$
(C) $\frac{\alpha}{l}\ln\left(\frac{2T_1}{T_1+T_2}\right)$
(D) $\frac{\alpha}{l}\ln\left(\frac{T_1+T_2}{T_2}\right)$

Q26: NTA Test 44 (Single Choice)

Three discs, A, B and C, having radii 2 m, 4 m and 6 m, respectively, are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm, respectively. The power radiated by them are Q_A, Q_B and Q_C respectively. The correct statement, among the following, is

(A) Q_A is maximum

(B) Q_B is maximum

(C) Q_C is maximum

(D) $Q_A = Q_B = Q_C$

Q27: NTA Test 45 (Single Choice)

If the filament of a 100 W bulb has an area $0.25\ cm^2$ and behaves as a perfect black body, find the wavelength corresponding to the maximum in its energy distribution. [Given $\sigma = 5.67 \times 10^{-8}\ J\ m^{-2}\ s\ K^{-4}$, $b = 2.89 \times 10^{-3}$ m K]

(A) 8751.23 Å

(B) 2898.14 Å

(C) 9971.9 Å

(D) 7055.5 Å

Q28: NTA Test 45 (Single Choice)

An earthen pitcher loses 1 g of water per minute due to evaporation. If the water equivalent of pitcher is 0.5 kg and the pitcher contains 9.5 kg of water, calculate the time required for the water in the pitcher to cool to 28 °C from its original temperature of 30 °C. Neglect radiation effects. Latent heat of vapourization of water in this range of temperature is 580 cal g^{-1} and specific heat of water is 1 kcal g^{-1} $°C^{-1}$.

(A) 38.6 min

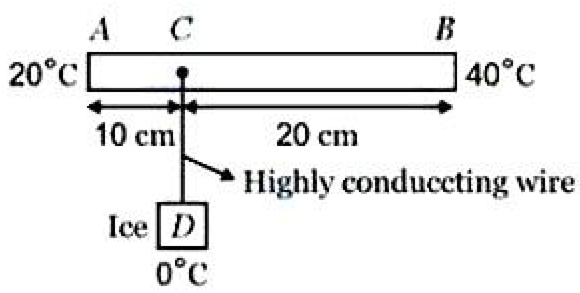
(B) 30.5 min

(C) 34.5 min

(D) 41.2 min

Q29: NTA Test 47 (Single Choice)

As shown in diagram, AB is a rod of length 30 cm and area of cross section $1.0\ cm^2$ and thermal conductivity 336 SI units. The ends A and B are maintained at temperatures 20 °Cand 40 °C, respectively. A point C of this rod is connected to a box D, containing ice at 0 °C, through a highly conducting wire of negligible heat capacity. The rate at which ice melts in the box is (assume latent heat of fusion for ice $L_f = 80$ cal g^{-1})



(A) 84 mg s^{-1}

(B) 84 g s^{-1}

(C) 20 mg s^{-1}

(D) 40 mg s^{-1}

Q30: NTA Test 48 (Single Choice)

A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius $2R$ made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is–

(A) $K_1 + K_2$

(B) $\frac{K_1+3K_2}{4}$

(C) $\frac{K_1K_2}{K_1+K_2}$

(D) $\frac{3K_1+K_2}{4}$

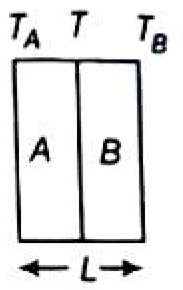
Answer Keys

Q1: (C)	**Q2:** (A)	**Q3:** (C)
Q4: (C)	**Q5:** (A)	**Q6:** 01.33
Q7: (B)	**Q8:** (C)	**Q9:** (A)
Q10: (B)	**Q11:** (A)	**Q12:** (A)
Q13: (D)	**Q14:** (D)	**Q15:** (C)

Q16: (A)	**Q17:** (C)	**Q18:** (B)
Q19: (A)	**Q20:** (A)	**Q21:** (A)
Q22: (D)	**Q23:** 9	**Q24:** (C)
Q25: (B)	**Q26:** (B)	**Q27:** (C)
Q28: (C)	**Q29:** (D)	**Q30:** (B)

Solutions

Q1: (C) 25 K



Let T be the junction temperature

Here, $K_A = 2K_B,\ T - T_B = 50K$

At the steady state $H_A = H_B$

$$\Rightarrow \quad \frac{K_A A(T_A - T)}{\frac{L}{2}} = \frac{K_B A\ (T - T_B)}{\frac{L}{2}}$$

$$\Rightarrow \quad 2K_B\ (T_A - T) = K_B\ (T - T_B)$$

$$\Rightarrow \quad T_A - T = \frac{T - T_B}{2} = \frac{50}{2} = 25\ K$$

Q2: (A) $\sqrt{\frac{\pi}{6}} : 1$

Rate of emission of energy $= \sigma T^4 S$

Let m_1 be the mass of sphere, C is specific heat and $(d\theta/dt)$, the rate of cooling.

For sphere

$$\sigma T^4 S = m_1 C\left(\frac{d\theta}{dt}\right)_S \qquad ...(i)$$

Let m_2 be the mass of cube, C its specific heat and $(d\theta/dt)$, the rate of cooling

For cube

$$\sigma T^4 S = m_2 C\left(\frac{d\theta}{dt}\right)_C \qquad ...(ii)$$

From Eqs.(i) and (ii)

$$\frac{(d\theta/dt)_s}{(d\theta/dt)_c} = \frac{m_2}{m_1}$$

$$= \frac{a^3 \rho}{(4/3)\pi r^2 \rho}$$

where a is the side of cube and r is the radius of sphere, ρ is the density.

Required ratio $\frac{R_s}{R_c} = \frac{3a^3}{4\pi r^3}$

But since S (surface area) is the same,

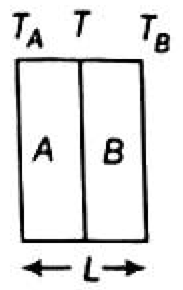
$$6a^2 = 4\pi r^2$$

or $a^2 = (2/3)\pi r^2$

$$\therefore \quad \frac{R_s}{R_c} = \frac{3\left(2\pi r^2/3\right)^{3/2}}{4\pi r^3} = \frac{2\pi\sqrt{2\pi}}{\sqrt{3}(4\pi)}$$

$$= \sqrt{\frac{2\pi}{12}} = \sqrt{\frac{\pi}{6}}$$

Q3: (C) 25 K



Let T be the junction temperature

Here, $K_A = 2K_B$, $T - T_B = 50K$

At the steady state $H_A = H_B$

$$\Rightarrow \quad \frac{K_A A(T_A - T)}{\frac{L}{2}} = \frac{K_B A\,(T - T_B)}{\frac{L}{2}}$$

$$\Rightarrow \quad 2K_B\ (T_A - T) = K_B\ (T - T_B)$$

$$\Rightarrow \quad T_A - T = \frac{T - T_B}{2} = \frac{50}{2} = 25\ K$$

Q4: (C) $\left(\frac{97}{2}\right)^{1/4} T$

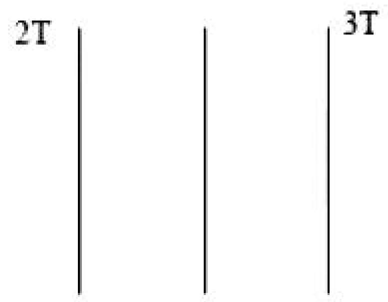
$$\sigma A\ (2T)^4 + \sigma A\ (3T)^4 = \sigma\ 2A\left(T'\right)^4$$

$$16T^4 + 81T^4 = 2\left(T'\right)^4$$

$$97T^4 = 2\left(T'\right)^4$$

$$\left(T'\right)^4 = \frac{97}{2}T^4$$

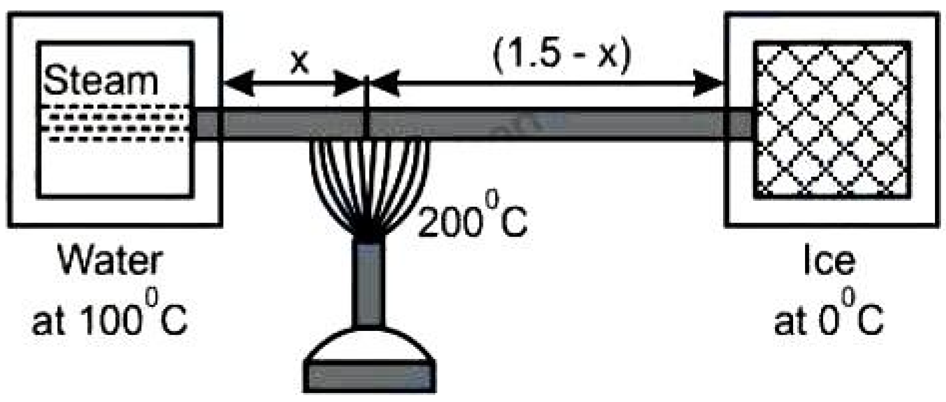
$$\therefore \quad T' = \left(\frac{97}{2}\right)^{1/4}\ T$$



Q5: (A) 10. 34 cm

If the point is at a distance x from water at 100^oC, heat conducted to ice in time t,

$$Q_{ice} = KA\frac{(200-0)}{(1.5-x)} \times t$$



So ice melted by this heat

$$m_{ice} = \frac{Q_{ice}}{L_F} = \frac{KA}{80}\frac{(200-0)}{(1.5-x)} \times t$$

Similarly heat conducted by the rod to the water at 100^oC in time t,

$$Q_{water} = KA\frac{(200-100)}{x}t$$

So steam formed by this heat

$$m_{steam} = \frac{Q_{water}}{L_V} = KA\frac{(200-100)}{540 \times x}t$$

According to given problem $m_{ice} = m_{steam}$, i.e.,

$$\frac{200}{80(1.5-x)} = \frac{100}{540 \times x} \text{ or } x = \frac{6}{58}\text{m} = 10.34 \text{ cm}$$

i.e., 200^oC temperature must be maintained at a distance 10.34 cm from water at 100^oC.

Q6: 01.33

For two bodies made up of same material, rate of loss of heat will depend only on their surface area for same temperature change.

$$\therefore \frac{\left(\frac{dQ}{dt}\right)_1}{\left(\frac{dQ}{dt}\right)_2} = \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} \quad \ldots\ldots \text{(i)}$$

Ratio of initial rates of cooling$= \frac{\left(\frac{d\theta}{dt}\right)_1}{\left(\frac{d\theta}{dt}\right)_2}$

As, $\frac{dQ}{dt} = mc\frac{d\theta}{dt}$

$$\therefore \quad \frac{m_1 c_1 (d\theta/dt)_1}{m_2 c_2 (d\theta/dt)_2} = \frac{r_1^2}{r_2^2} \qquad \text{...[from (i)]}$$

since, both the spheres are made up of copper,

$c_1 = c_2 = c.$

$$\therefore \quad \frac{\left(\frac{d\theta}{dt}\right)_1}{\left(\frac{d\theta}{dt}\right)_2} = \frac{r_1^2 \times m_2}{r_2^2 \times m_1}$$

As, spheres have same densities,

$$\frac{m_1}{m_2} = \frac{V_1}{V_2} = \frac{r_1^3}{r_2^3}$$

$$\therefore \quad \frac{(d\theta/dt)_1}{(d\theta/dt)_2} = \frac{r_1^2}{r_2^2} \times \frac{r_2^3}{r_1^3} = \frac{r_2}{r_1} = \frac{20}{15} = \frac{4}{3} = 1.33$$

Q7: (B) 20° C

Treating the given network of rods in terms of thermal resistance R_x and R_y with

$$R_x = \frac{L}{A \times 0.92} \text{ and } R_y = \frac{L}{A \times 0.46} \quad \left[\text{as } R = \frac{L}{AK}\right]$$

so that if $R_x = R$, $R_y = 2R_x = 2R$

Now as in this bridge [(P/Q) = (R/S)], so the bridge is balanced, i.e., the temperature at junctions C and D is equal and the rod CD becomes ineffective as no heat will flow through it.

Now as the thermal resistance of the bridge between junctions B and E is

$$\frac{1}{R_{BE}} = \frac{1}{(R+R)} + \frac{1}{(2R+2R)}, \quad \text{i.e.,} \quad R_{BE} = \frac{4}{3}R$$

The total resistance of bridge between A and E will be

$$R_{eq} = R_{AB} + R_{BE} = 2R + (4/3)\,R = (10/3)\,R$$

So the net rate of flow of heat through the bridge will be

$$\frac{dQ}{dt} = \frac{\Delta\theta}{R_{eq}} = \frac{(60-10)}{(10/3)R} = \frac{15}{R}$$

Now if T_B is the temperature at B,

$$\left[\frac{dQ}{dt}\right]_{AB} = \frac{\Delta\theta}{R_{AB}} = \frac{60-T_B}{2R}$$

But $\left[\frac{dQ}{dt}\right]_{AB} = \frac{dQ}{dt}$, i.e., $\frac{60-T_B}{2R} = \frac{15}{R}$, i.e., $T_B = 30^\circ C$

Also at B

$$\left[\frac{dQ}{dt}\right]_{AB} = \left[\frac{dQ}{dt}\right]_{BC} + \left[\frac{dQ}{dt}\right]_{BD}$$

i.e., $\frac{15}{R} = \frac{30-T_C}{R} + \frac{30-T_D}{2R}$

and as $T_C = T_D = T$, $30 = 3\,(30 - T)$,

i.e., $T_C = T_D = T = 20^\circ$ C

Q8: (C) $60^{\circ}C$

The thermal resistance of each rod is same as all are identical

$\frac{\theta-20}{R}+\frac{\theta-40}{R}+\frac{\theta-80}{R}+\frac{\theta-100}{R}=0$

$4\theta-240=0$

$4\theta=240$

$\theta=60$

Q9: (A) $80^{\circ}C$

Heat loss will always be equal to heat gain.

As per 1^{st} condition,

$100\times S_A\times(100-90)=50\times S_B\times(90-75)$ ___(i)

As per 2^{nd} condition,

$100\times S_A(100-\theta)=50\times S_B(\theta-50)$ ___(ii)

Dividing (ii) by (i), we get
$\frac{100-\theta}{100-90}=\frac{\theta-50}{90-75}$

$300-3\theta=2\theta-100$

$\theta=80^{\circ}C$

Q10: (B) 20 °C
Using Newton's law of cooling,

$\log\frac{\theta_2-\theta_0}{\theta_1-\theta_0}=-Kt$

$\log\frac{40-\theta_0}{50-\theta_0}=-K\times5$ .....(i)

$\log\frac{33.33-\theta_0}{40-\theta_0}=-K\times5$ .....(ii)

From Eqs.(i) and (ii),

$\frac{40-\theta_0}{50-\theta_0}=\frac{33.33-\theta_0}{40-\theta_0}$

On solving, we get

$\theta_0=19.95^{\circ}C\approx20^{\circ}C$

Q11: (A) $5.1\ \text{min}$

Let θ be instantaneous temperature, θ

For $40\ ^{\circ}C\rightarrow36\ ^{\circ}C$
$\frac{36-40}{4}=-k\left(38-20\right)\ldots\left(\text{i}\right)$

$\theta_{avg} = 38\ ^{\circ}\mathrm{C}$

For $36\ ^{\circ}\mathrm{C} \to 32\ ^{\circ}\mathrm{C}$

$\frac{32-36}{t} = -k\left(34 - 20\right) \ldots \left(\mathrm{ii}\right)$

$\theta_{avg} = 34\ ^{\circ}\mathrm{C}$

$\frac{(\mathrm{i})}{(\mathrm{ii})}$ Gives

$t = 4\left(\frac{18}{14}\right) \Rightarrow t = 5.1\ \mathrm{min}$

Q12: (A) $\left(\frac{e_B}{e_A}\right)^{1/4}$

The power radiated by a filament is $P = eA\left(\sigma T^4\right)$ (where e =emissivity, σ = Stefan's constant and T = surface temperature)
Here, eT^4 =constant or $e_A T_A^4 = e_B \mathrm{T}_B^4$

Q13: (D) $21\ \mathrm{hours}$

If at some time t, the thickness of ice is x and at time $\mathrm{t} + \mathrm{dt}$, the thickeness is $\mathrm{x} + \mathrm{dx}$, then

$(\rho \mathrm{Adx})\mathrm{L}_{\text{fusion}} = \frac{\mathrm{kA}(0-(-10))}{\mathrm{x}}\mathrm{dt}$

$\Rightarrow \Delta \mathrm{t} = \frac{\rho \mathrm{L}_{\text{fusion}}}{20\mathrm{k}}\left(\mathrm{x}_2^2 - \mathrm{x}_1^2\right)$

$\Rightarrow \Delta \mathrm{t} \propto \left(\mathrm{x}_2^2 - \mathrm{x}_1^2\right)$

$\Rightarrow \frac{\Delta \mathrm{t}}{\Delta \mathrm{t'}} = \frac{\mathrm{x}_2^2 - \mathrm{x}_1^2}{(\mathrm{x'}_2)^2 - (\mathrm{x'}_1)^2}$

$\Rightarrow \frac{7}{\Delta \mathrm{t'}} = \frac{(1^2 - 0^2)}{(2^2 - 1^2)}$

$\Rightarrow \Delta \mathrm{t'} = 21\ \mathrm{hours}$

Q14: (D) 36.28 s

The time period of the second's pendulum = 2 s

Change in the time period, $\Delta \mathrm{T} = \frac{1}{2}\mathrm{T}\alpha\Delta\theta$

$\Delta \mathrm{T} = \left(\frac{1}{2}\right)(2)\left(1.2 \times 10^{-5}\right)\left(30^{\circ} - 20^{\circ}\right)$

$= 1.2 \times 10^{-4}\ \mathrm{s}$

New time period, $\mathrm{T}' = \mathrm{T} + \Delta \mathrm{T}$

$\mathrm{T}' = 2.00012\ \mathrm{s}$

Time lost in a week,

$\Delta \mathrm{t} = \frac{\Delta \mathrm{T}}{\mathrm{T}'} \times \mathrm{t}$

$= \frac{1.2 \times 10^{-4}}{2.00012} \times (7 \times 24 \times 3600)$

$= 36.28 \text{ s}$

Q15: (C) $\frac{\alpha_s}{(\alpha_a + \alpha_s)}$

Given, $\Delta l_1 = \Delta l_2$

or, $\quad l_1 \alpha_a t = l_2 \alpha_s t$

$\therefore \frac{l_1}{l_2} = \frac{\alpha_s}{\alpha_a}$

or, $\quad \frac{l_1}{l_1 + l_2} = \frac{\alpha_s}{\alpha_a + \alpha_s}$

Q16: (A) $\frac{1}{3}$

Heat loss = Heat gain

$m_1 L_v = m_2 . L_f + m_2 . S . \Delta T$

$X \times 10^{-3} \times 22.68 \times 10^5 = Y \times 10^{-3} \times 3.36 \times 10^5 + Y \times 10^{-3} \times 4200 \times 100$

$\therefore \frac{X}{Y} = \frac{7.56}{22.68} = \frac{1}{3}$

Q17: (C) $E_x > E_y$ and $a_x > a_y$

Rate of cooling $\left(-\frac{dT}{dt}\right) \propto$ emissivity (e)

From the graph,

$\left(-\frac{dT}{dt}\right)_x > \left(-\frac{dT}{dt}\right)_y$

$\therefore e_x > e_y$

Further emissivity (e)$\propto$ absorptive power (a) (good absorbers are good emitters also)

$\therefore a_x > a_y$

Q18: (B) 3 s
$\Delta t = \frac{\Delta Q(\Delta x)}{KA(\Delta T)}$

When two rods of same length are joined in parallel,

$A \rightarrow 2$ and $(\Delta x) \rightarrow \frac{1}{2}$ times

$\therefore \ \Delta t$ becomes $\frac{1}{4}$ times i.e., $\frac{1}{4} \times 12 \text{ s} = 3 \text{ s}$

Q19: (A) 1612 J

Total heat loss $Q = ms\,(\Delta T) + mL$

$\Rightarrow \ Q = \left(4.5 \times 10^{-3}\right)\ (4190)\,(6-0) + \left(4.5 \times 10^{-3}\right)\left(3.33 \times 10^5\right)$

$= 113 + 1499 = 1612 \text{ J}$

Q20: (A) $r = 2 \text{ cm}, \ l = 0.5 \text{ m}$

$\frac{\Delta T}{\Delta t} = KA\left(\frac{\Delta T}{\Delta x}\right) = K\left(\pi r^2\right)\frac{\Delta T}{(l)}$

$\therefore \left(\frac{\Delta Q}{\Delta t}\right) \propto \frac{r^2}{l}$, which is maximum in this case.

Q21: (A) 84 s

From Newton's law of cooling,

$\frac{365-361}{2} = K\left(\frac{365+361}{2} - 293\right)$ ..... (1)

$\frac{344-342}{t} = K\left(\frac{344+342}{2} - 293\right)$ .... (2)

Equation $(1) \div (2)$

$t = \frac{365+361-586}{344+342-586} = \frac{140}{100} \text{ min}$

$= \frac{140\times60}{100} \text{ s} = 84 \text{ s}$

Q22: (D) $\frac{256}{81}P_0$

Let T_0 be the initial temperature of the black body

$\therefore \ \lambda_0 T_0 = b$ (Wien's law)

Power radiated, $P_0 = CT_0^4$, where, C is constant.

If T is new temperature of black body, then

$\frac{3\lambda_0}{4}T = b = \lambda_0 T_0 \ \text{ or } \ T = \frac{4}{3}\, T_0$

Power radiated, $P = CT^4 = CT_0^4\left(\frac{4}{3}\right)^4$

$P = \frac{256}{81}P_0$

Q23: 9

$P = eA\sigma T^4$ where T is in kelvin

$\log_2 \frac{eA\sigma(487+273)^4}{P_0} = 1$ .....(i)

$\log_2 \frac{eA\sigma(2767+273)^4}{P_0} = x$ ......(ii)

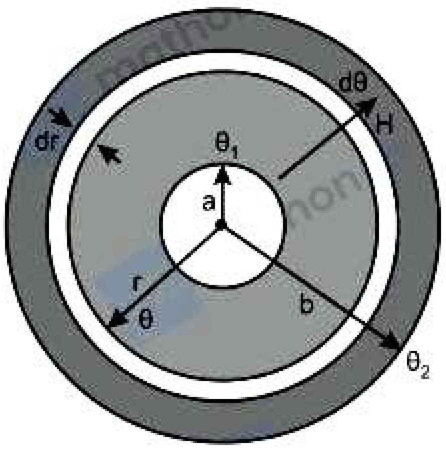
(ii) - (i)

$\log_2\left(\frac{3040}{760}\right)^4 = x - 1$

$\therefore x = 9$

Q24: (C) $\frac{4\pi R^2 KT}{P}$

Consider a concentric spherical shell of radius r and thickness dr as shown in diagram. The radial rate of flow of heat through this shell in steady state will be,



$H = \frac{dQ}{dt} = -KA\frac{d\theta}{dr}$

[Negative sign is used because, with an increase in r, θ decreases.]

Now as for spherical shell $A = 4\pi r^2$

So $H = -4\pi r^2 K\frac{d\theta}{dr}$ or $\int_a^b \frac{dr}{r^2} = -\frac{4\pi K}{H}\int_{\theta_1}^{\theta_2} d\theta$

which on integration and simplification gives

$H = \frac{dQ}{dt} = K\frac{4\pi ab(\theta_1 - \theta_2)}{(b-a)}$ ...(i)

Now in steady state, as no heat is absorbed, the rate of loss of heat by conduction is equal to that of supply, i.e., $H = P$, and here

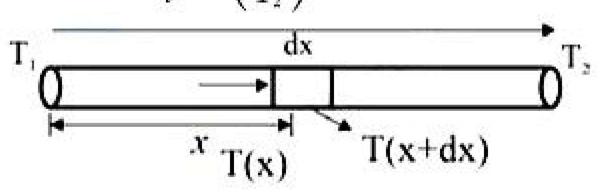
$\theta_1 - \theta_2 = T$ and $a \simeq b = R$

So Equation (i) becomes,

$P = \frac{4\pi R^2 KT}{(b-a)}$

i.e thickness of shell is $(b - a) = \frac{4\pi R^2 KT}{P}$

Q25: (B) $\frac{\alpha}{l}\ln\left(\frac{T_1}{T_2}\right)$



$\frac{dQ}{dt} = -KA\frac{(T(x+dx)-T(x))}{dx}$

$\Rightarrow I_0 = -\frac{\alpha}{T}A\frac{dT}{dx}$

$\Rightarrow \int_0^1 \frac{I_0}{A\alpha}\, dx = -\int_{T_1}^{T_2} \frac{dT}{T}$

$\therefore \left|\frac{I_0 l}{A\alpha}\right| = \ln\frac{T_1}{T_2}$

$\Rightarrow \left|\frac{I_0}{A}\right| = \frac{\alpha}{l}\ln\frac{T_1}{T_2}$

Q26: (B) Q_B **is maximum**

$Q \propto AT^4$ and $\lambda_m T$ =Constant

Hence,

$$Q \propto \frac{A}{(\lambda_m)^4} \text{ or } Q \propto \frac{r^2}{(\lambda_m)^4}$$

$$Q_A : Q_B : Q_C = \frac{(2)^2}{(3)^4} : \frac{(4)^2}{(4)^4} : \frac{(6)^2}{(5)^4}$$

$$= \frac{4}{81} : \frac{1}{16} : \frac{36}{625} = 0.05 : 0.0625 : 0.0576$$

i.e., Q_B is maximum.

Q27: (C) 9971. 9 Å

In the bulb filament given, energy radiated per second per m^2 of its surface area is given as

$$E = \frac{P}{A} = \frac{100}{0.25 \times 10^{-4}} = 4 \times 10^6 \text{ J s}^{-1} \text{ m}^{-2}$$

If T is the temperature of the filament then according to Stefan's law, we have

$$E = \sigma T^4$$

or $4 \times 10^6 = 5.67 \times 10^{-8} \times T^4$

or $T^4 = \frac{4 \times 10^6}{5.67 \times 10^{-8}} = 7.055 \times 10^{13}$

or $T = [7.055 \times 10^{13}]^{1/4} = 2898.14 \text{ K}$

If the filament radiates the maximum energy at a wavelength λ_m, from Wein's displacement law, we have

$$\lambda_m T = b$$

or $\lambda_m = \frac{b}{T}$

$$= \frac{2.89 \times 10^{-3}}{2898.14} = 9971.9 \text{ Å}$$

Q28: (C) 34. 5 min

As water equivalent of pitcher is 0.5 kg, i..e., pitcher is equivalent to 0.5 kg of water, heat to be extracted from the system of water and pitcher for decreasing its temperature from 30 to 28°C is

$$Q_1 = (m + M)c\Delta T$$

$$= (9.5 + 0.5)\text{kg} \left(1 \text{ k cal/kg C}^\circ\right)(30 - 28)^\circ\text{C}$$

$$= 20 \text{ kcal}$$

And heat extracted from the pitcher through evaporation in t minutes

$$Q_2 = mL = \left[\frac{dm}{dt} \times t\right] L = \left[\frac{1g}{min} \times t\right] 580 \frac{cal}{g}$$

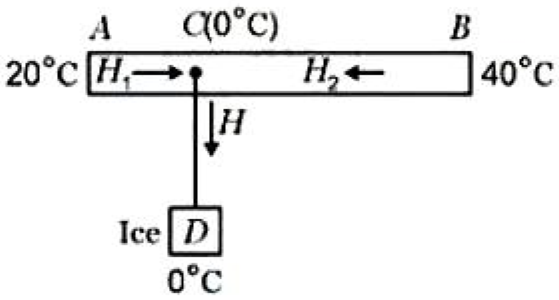
$= 580 \times t$ cal

According to given problem $Q_2 = Q_1$, i.e., $580 \times t = 20 \times 10^3$

$t = 34.5$ min

Q29: (D) $40 \ \text{mg s}^{-1}$

Thermal resistance of AC $= \frac{L}{KA} = \frac{0.1}{336 \times 10^{-4}} = \frac{10^3}{336} = R$ (let)



Thermal resistance of BC $= \frac{0.2}{336 \times 10^{-4}} = 2R$

Heat flow rates are

$H_1 = \frac{20}{R};\ H_2 = \frac{40}{2R} = \frac{20}{R}$

undefined

$= \frac{13440}{10^3} = 13.44$ W

Rate of melting of ice

$= \frac{H}{L_f} = \frac{13.44/4.2}{80}$ g s^{-1} $= 40 \ \text{mg s}^{-1}$

Q30: (B) $\frac{K_1+3K_2}{4}$

Parallel combination of cross - section area

πR^2 and $\pi\left[(2R)^2 - R^2\right] = 3\pi R^2$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$ with $R = \frac{L}{KA}$

$\frac{K_{eq} 4\pi R^2}{L} = \frac{K_1 \pi R^2}{L} + \frac{K_2(3\pi R^2)}{L}$

$\Rightarrow \quad K_{eq} = \frac{K_1+3K_2}{4}$