1. SETS AND RELATIONS

Let's study.

- Representation of a Set
- Intervals
- Types of sets
- Operations on sets
- Ordered pair
- Relations

Let's recall.

The concept of a set was developed by German mathematician George Cantor (1845–1918)

You have already learnt about sets and some basic operations involving them in the earlier standards.

In everyday life, we generally talk about group or a collection of objects. Surely you must have used the words such as team, bouquet, bunch, flock, family for collection of different objects.

It is very important to determine whether a given object belongs to a given collection or not. Consider the following collections:

i) Successful persons in your city.

ii) Happy people in your town.

iii) Clever students in your class.

iv) Days in a week.

v) First five natural numbers.

First three collections are not examples of sets, but last two collections represent sets. This is because in first three collections, we are not sure of the objects. The terms 'successful persons,' 'Happy people', 'Clever student' are all relative terms. Here, the objects are not well–defined. In the last two collections. We can determine the objects clearly. Thus, we can say that objects are well–defined.

1.1 SET:

Definition:

A Collection of well–defined objects is called a set.

Object in a set is called its element or member.

We denote sets by capital letters A,B,C. etc. The elements of a set are represented by small letters *a, b, c, x, y, z* etc. If x is an element of a set A we write $x \in A$, and read as '*x* belongs to A'. If *x* is not an element of a set A, we write $x \notin A$, and read as '*x* does not belong to A.'

For example, zero is a whole number but not a natural number.

$\therefore\ 0 \in W$ and $0 \notin N$

Representation of a set:

1) Roster method:

In the Roster method, we list all the elements of the set within brackets, and separate the elements by commas.

Example : State the sets using Roster method.

i) B is the set of all days in a week.

B = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

ii) C is the set of all vowels in English alphabets.

$C = \{a, e, i, o, u\}$

2) Set-Builder method:

In the set builder method, we describe the elements of the set by specifying the property which determines the elements of the set uniquely.

Example : State the sets using set–builder method.

i) Y is the set of all months of a year.

Y = $\{x \mid x$ is month of a year$\}$

ii) B is the set of perfect squares of natural numbers.

B = $\{x \in N / x$ is perfect square$\}$

3) Venn Diagram:

The pictorial representation of a set is called Venn diagram. Generally, the geometrical closed figures like circle, triangle or rectangle, are used to represent the sets, which are known as Venn diagrams and are named after the English logician John Venn.

In Venn diagram the elements of the sets are shown as points enclosed in the diagram representing set:

A = {1,2,3} B = $\{a,b,c,d,e,f\}$ C = {4,5,6}

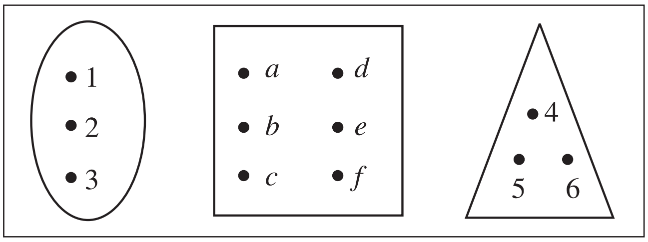


Fig. 1.1 Fig. 1.2 Fig. 1.3

Let's Note.

1) If the elements are repeated, write them once.

2) While listing the elements of a set, the order in which the elements are listed is immaterial.

Let's learn.

1.2 INTERVALS:

1) **Open Interval:** Let a, b $\in$R and a < b then the set $\{x / x \in$ R $a < x < b\}$ is called open interval and is denoted by (a,b). All the numbers between a and b belong to the open interval (a,b) but a, b themselves do not belong to this interval.

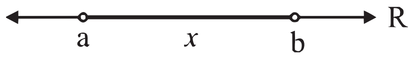


Fig. 1.4

$\therefore$ (a,b) = $\{x / x \in$ R, a < x < b$\}$

2) **Closed Interval:** Let a, b$\in$R and a<b then the set $\{x / x \in$ R $a \le x \le b\}$ is called closed interval and is denoted by [a,b]. All the numbers between a and b belong to the closed interval $[a, b]$. Also a and b belong to this interval.

Fig. 1.5

$[a, b] = \{x / x \in$ R, $a \le x \le b\}$

3) **Semi-closed Interval:**

$[a, b) = \{x / x \in$R, $a \le x < b\}$

Fig. 1.6

Note that $a \in [a, b]$ and $b \notin [a, b]$

4) **Semi–open Interval:**

$(a, b] = \{x / x \in$ R, $a < x \le b\}$

Fig. 1.7

$(a, b]$ excludes a but includes b.

5) i) The set of all real numbers greater than a i.e. $(a,\infty) = \{x/x\in R,\ x>a\}$

Fig. 1.8

ii) The set of all real numbers greater than or equal to a

i.e. $[a,\infty) = \{x/x\in R,\ x\geq a\}$

Fig. 1.9

6) i) The set of all real numbers less than b. ie. $(-\infty,\ b)$

Fig. 1.10

$\therefore (-\infty,\ b) = \{x/x\in R,\ x<b\}$

ii) The set of all real numbers less than or equal to b i.e. $(-\infty,\ b]$

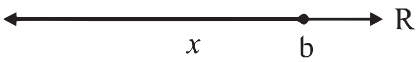


Fig. 1.11

$\therefore (-\infty,\ b] = \{x/x\in R,\ x\leq b\}$

7) The set of all real numbers i.e. $(-\infty,\ \infty)$

Fig. 1.12

$\therefore\ (-\infty,\infty) = \{x/x\in R,\ -\infty<x<\infty\}$

Number of elements of a set: (Cardinality)

The number of distinct elements contained in a finite set A is denoted by n (A).

Thus, if A = {1,2,3,4}, then n (A) = 4

1.3 TYPES OF SETS:

1) Empty Set:

A set containing no element is called an empty or a null set and is denoted by the symbol ϕ or { } or void set.

e.g. $A = \{x\ /\ x\in N,\ 1<x<2\},\ n(A) = 0$

2) Singleton set:

A Set containing only one element is called a singleton set.

e.g. Let A be the set of all integers which are neither positive nor negative.

$\therefore A = \{0\},\ \ n\ (A) = 1$

3) Finite set:

A set in which the process of counting of elements comes to an end is called a finite set.

e.g. the set of letters in the word 'beautiful'.

$A = \{b,e,a,u,t,i,f,l\}$, n (A) = 8

A is a infinite set

4) Infinite set:

A set which is not finite, is called an infinite set.

e.g. set of natural numbers.

Let's Note.

1) An empty set is a finite set.
2) N, Z, set of all points on a circle are infinite sets.

Some definitions :

1) Equality of sets:

Two sets are said to be equal if they contain the same elements i.e. if $A\subseteq B$ and $B\subseteq A$.

For example:

Let X be the set of letters in the word 'ABBA' and Y be the set of letters in the word 'BABA'.

$\therefore$ X = {A, B}, Y = {B, A}

Thus the sets X and Y are equal sets and we denote it by X = Y

2) Equivalent sets:

Two finite sets A and B are said to be equivalent if n (A) = n (B)

A = {d, o, m, e}

B = {r, a, c, k}

Here n (A) = n (B)

There for A and B are equivalent sets

3) Subset:

A set A is said to be a subset of set B if every element of A is also an element of B and we write $A \subseteq B$.

4) Superset:

If $A \subseteq B$, then B is called a superset of A and we write, $B \supseteq A$.

5) Proper Subset:

A nonempty set A is said to be a proper subset of the set B, if all elements of set A are in set B and atleast one element of B is not in A.

i.e. If $A \subseteq B$ and $A \neq B$ then A is called a proper subset of B and we write $A \subset B$.

e.g. 1) Let A = {1,3,5} and B = {1,3,5,7}. Then, every element of A is an element of B but $A \neq B$.

$\therefore$ $A \subset B$, i.e. A is a proper subset of B.

Remark: If there exists even a single element in A which is not in B then A is not a subset of B and we write $A \not\subset B$.

6) Universal set:

If in a particular discussion all sets under consideration are subsets of a set, say U, then U is called the universal set for that discussion.

The set of natural numbers N, the set of integers Z are subsets of set of real numbers R. Thus, for this discussion R is a universal set.

In general universal set is denoted by 'U' or 'X'.

7) Power Set:

The set of all subsets of a given set A is called the power set of A and is denoted by P(A). Thus, every element of power set A is a set.

e.g. consider the set A={a,b}. Let us write all subsets of the set A. We know that ϕ is a subset of every set, so ϕ is a subset of A. Also {a}, {b}, {a,b} are also subsets of A. Thus, the set A has in all four subsets viz. ϕ, {a}, {b}, {a,b}

$\therefore$ P(A) = {ϕ, {a}, {b}, {a,b}}

Operations on sets:

1) Complement of a set:

Let A be a subset of universal set.

The complement of the set A is denoted by

A' or A^c. It is defined as

A' = {x / $x \in$ U $x \notin$ A} = set of all elements in U which are not in A.

Ex 1) Let X = {0, 1, 2, 3, 4, 5, 6, 7, 8} be the Universal Set and

A = {2, 4, 6, 8}

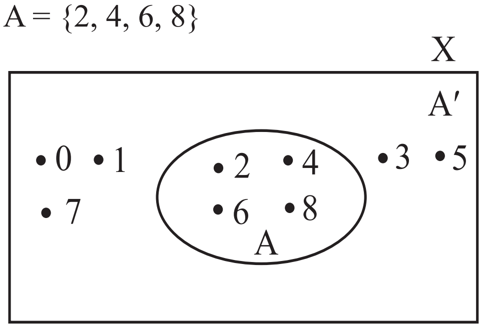


Fig. 1.13

$\therefore$ The complement of the set A is

A' = {0, 1, 3, 5, 7}

Properties:

i) (A')' = A

ii) ϕ' = U (U is the universal set)

iii) U' = ϕ

iv) If A $\subseteq$ B then B$'$ $\subseteq$ A$'$

2) Union of Sets:

Union of sets A and B is the set of all elements which are in A or in B. (Here 'or' is taken in the inclusive sense)

Thus, A $\cup$ B = {x | $x \in$A or $x \in$B}

The union of two sets A and B can be represented by a Venn-diagram Fig. 1.14 and fig. 1.15 represent A $\cup$ B

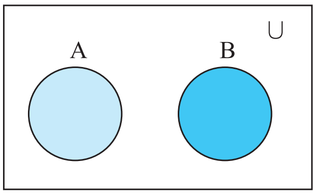


Fig. 1.14
A $\cup$ B

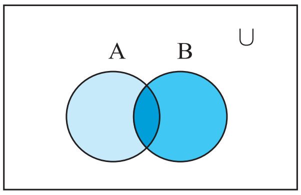


Fig. 1.15
A $\cup$ B

Properties:

i) A $\cup$ B = B $\cup$ A ..... (Commutativity)

ii) (A $\cup$ B) $\cup$ C = A $\cup$ (B $\cup$ C) .... (Associative Property)

iii) A $\cup$ ϕ = A ..... (Identity of Union)

iv) A $\cup$ A = A ...... (Idempotent law)

v) A$\cup$ A$'$ = U

vi) If A$\subset$B then A$\cup$B = B

vii) U$\cup$A = U

viii) A$\subset$(A$\cup$B), B$\subset$(A$\cup$B)

Ex. : let A = {x/x is a prime number less than 10}

B= {x | x is a factor of 8}

find A$\cup$B.

Solution : We have A = {2,3,5,7}

B = {1,2,4,8}

$\therefore$ A$\cup$B = {1, 2, 3, 4, 5, 7, 8}

3) Intersection of sets:

The intersection of two sets A and B is the set of all elements which are both in A and B. It is denoted by A$\cap$B.

Thus A$\cap$B = {x | $x \in$A **and** $x \in$B}

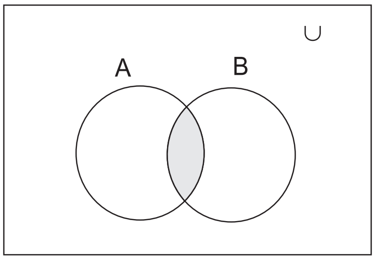


Fig. 1.16
A$\cap$B

The shaded portion in Fig. 1.16 represents the intersection of A and B i.e. A$\cap$B

Properties:

i) A$\cap$B = B $\cap$A .... Commutativity

ii) (A $\cap$ B) $\cap$ C = A $\cap$ (B $\cap$ C) .... Associativity

iii) ϕ $\cap$ A = ϕ

iv) A $\cap$ A = A .... Idempotent law

v) A $\cap$ A$'$ = ϕ

vi) If A$\subset$B then A$\cap$B =A

vii) U$\cap$A = A

viii) (A$\cap$B) $\subset$A, (A$\cap$B) $\subset$B

Remark : If A$\cap$B = ϕ, A and B are disjoint sets.

4) Distributive Property

a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

DeMorgan's law

If A and B are subsets of a universal set, then

i) $(A \cup B)' = A' \cap B'$

ii) $(A \cap B)' = A' \cup B'$

Ex. 1: If A={1,3,5,7,9} B = {1,2,3,4,5,6,7,8}
Find $A \cap B$.

Solution: $A \cap B = \{1,3,5,7\}$

Ex. 2: If $A = \{x / x$ is a factor of $12\}$

$B = \{x / x$ is a factor of $18\}$

Find $A \cap B$

Solution:

$A = \{1,2,3,4,6,12\}$

$B = \{1,2,3,6,9,18\}$

$\therefore A \cap B = \{1,2,3,6\}$

Ex. 3 : If A = {1,3,5,7,9}

B = {2,4,6,8, 10}. Find $A \cap B$

Solution: $A \cap B = \{\ \} = \phi$

5) Difference of sets:

Difference of Set A and Set B is the set of elements which are in A but not in B and is denoted by A–B.

The shaded portion in fig. 1.17 represents A–B. Thus, $A - B = \{x | x \in A, x \notin B\}$

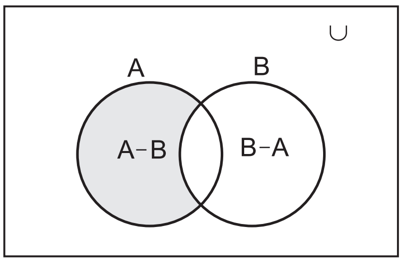


Fig. 1.17

Similary, $B - A = \{y | y \in B, y \notin A\}$

Let's note:

i) A–B is a subset of A and B–A is a subset of B.

ii) The sets A–B, $A \cap B$, B–A are mutually disjoint sets, i.e. the intersection of any of these two sets is the null (empty) set.

iii) $A - B = A \cap B'$

$B - A = A' \cap B$

iv) $A \cup B = (A - B) \cup (A \cap B) \cup (B - A)$

Shaded portion in fig. 1.18 represents $A \cup B$

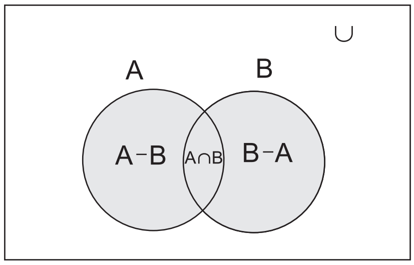


Fig. 1.18 $A \cup B$

Properties of Cardinality of Sets:

For given sets A, B

1) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

2) When A and B are disjoint sets

$n(A \cup B) = n(A) + n(B)$, as $A \cap B = \phi$,

$\therefore n(A \cap B) = 0$

3) $n(A \cap B') + n(A \cap B) = n(A)$

4) $n(A' \cap B) + n(A \cap B) = n(B)$

5) $n(A \cap B') + n(A \cap B) + n(A' \cap B) = n(A \cup B)$

For any sets A, B, C.

6) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$
$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

7) If $n(A) = m$, $n(P(A)) = 2^m$
where P(A) is the power set of A

8) $n(P') = n(x) - n(P)$

SOLVED EXAMPLES

Ex. 1: If A= $\{x / x$ is a factor of $6\}$

B = $\{x / x$ is a factor of $8\}$

find the A–B and B–A

Solution: $A = \{1,2,3,6\}; B = \{1,2,4,8\}$

$\therefore A-B = \{3,6\}$

$B-A = \{4,8\}$

Ex.2: $A = \left\{\frac{1}{x}\ x \in N, x < 8\right\}$

$B = \left\{\frac{1}{2x}\ x \in N, x \le 8\right\}$ Find A–B and B–A

Solution: $A = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}\right\}$

$B = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{14}, \frac{1}{16}\right\}$

$\therefore A-B = \left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}\right\}$

and $B-A = \left\{\frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{14}, \frac{1}{16}\right\}$

Ex. 3: If $A = \{1,2,3,4\}$, $B = \{3,4,5,6\}$

$C = \{5,6,7,8\}$, $D = \{7,8,9,10\}$;

find i) $A\cup B$ ii) $A\cup B\cup C$ iii) $B\cup C\cup D$

Are the sets A, B, C, D equivalent?

Solution: We have

i) $A\cup B = \{1,2,3,4,5,6\}$

ii) $A\cup B\cup C = \{1,2,3,4,5,6,7,8\}$

iii) $B\cup C\cup D = \{3,4,5,6,7,8,9,10\}$

As the number of elements in every set A, B, C, D is 4, the sets A, B, C, D are equivalent.

Ex. 4: Let $U = \{1,2,3,4,5,6,7,8,9,10\}$ be the universal set, $A = \{1,3,5,7,9\}$

$B = \{2,3,4,6,8,10\}$, $C = \{6,7,8,9\}$

Find i) A' ii) $(A\cap C)'$ iii) $(A')'$ iv) $(B-C)'$

Solution: We have

i) $A' = \{2,4,6,8,10\}$

ii) $(A\cap C) = \{7,9\}$

$\therefore (A\cap C)' = \{1,2,3,4,5,6,8,10\}$

iii) $(A')' = \{1,3,5,7,9\} = A$

iv) $B-C = \{2,3,4,10\}$

$\therefore (B-C)' = \{1,5,6,7,8,9\}$

Ex. 5: Let X be the universal set, for the non–empty sets A and B, verify the De Morgan's laws

i) $(A\cup B)' = A'\cap B'$

ii) $(A\cap B)' = A'\cup B'$

Where $X = \{1,2,3,4,5,6,7, 8,9,10\}$

$A = \{1,2,3,4,5\}$ $B = \{1,2,5,6,7\}$

Solution: $A\cup B = \{1,2,3,4,5,6,7\}$

$(A\cap B) = \{1,2,5\}$

$A' = \{6,7,8,9,10\}$

$B' = \{3,4,8,9,10\}$

i) $(A\cup B)' = \{8,9,10\}$ ...(1)

$A'\cap B' = \{8,9,10\}$ ...(2)

from (1) and (2), $(A\cup B)' = A'\cap B'$

ii) $(A\cap B)' = \{3,4,6,7,8,9,10\}$ ...(3)

$A'\cup B' = \{3,4,6,7,8,9,10\}$ ...(4)

from (3) and (4),

$(A\cap B)' = A'\cup B'$

Ex.6: If $P = \{x/x^2 + 14x + 40 = 0\}$

$Q = \{x/x^2 - 5x + 6 = 0\}$

$R = \{x/x^2 + 17x - 60 = 0\}$ and the universal set $X = \{-20, -10 - 4, 2, 3, 4\}$, find

i) $P \cup Q$ ii) $Q \cap R$ iii) $P \cup (Q \cap R)$

iv) $P \cap (Q \cup R)$

Solution: $P = \{x | x^2 + 14x + 40 = 0\}$

$\therefore P = \{-10, -4\}$

Similarly $Q = \{3, 2\}$, $R = \{-20, 3\}$ and

$X = \{-20, -10, -4, 2, 3, 4\}$

i) $P \cup Q = \{-10, -4, 3, 2\}$

ii) $Q \cap R = \{3\}$

iii) $P \cup (Q \cap R) = \{-10, -4, 3\}$

iv) $P \cap (Q \cup R) = \phi$

Ex. 7: If A and B are the subsets of X and

$n(X) = 50$, $n(A) = 35$, $n(B) = 22$ and

$n(A' \cap B') = 3$, find i) $n(A \cup B)$ ii) $n(A \cap B)$

$n(A' \cap B)$ iv) $n(A \cup B')$

Solution:

i) $n(A \cup B) = n(X) - n(A \cup B)'$

$= n(X) - n(A' \cap B')$

$= 50 - 3$

$= 47.$

ii) $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$n(A \cap B) = 35 + 22 - 47$

$= 10$

iii) $n(A' \cap B) = n(B) - n(A \cap B)$

$= 22 - 10$

$= 12$

iv) $n(A \cup B') = n(X) - n(A' \cap B)$

$= 50 - 12$

$= 38$

Ex.8: In an examination; 40 students failed in Physics, 40 in Chemistry and 35 in Mathematics; 20 failed in Mathematics and Physics, 17 in Physics and Chemistry, 15 in Mathematics and Chemistry and 5 in all the three subjects. If 350 students appeared for the examination, how many of them did not fail in any of the three subjects?

Solution:

P = set of students failed in Physics

C = Set of students failed in Chemistry

M = set of students failed in Mathematics

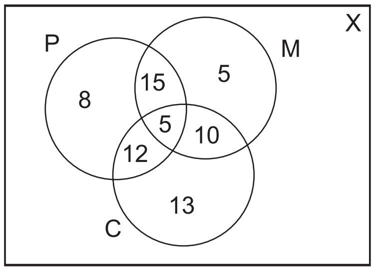


Fig. 1.19

From figure 1.19, we have

$n(X) = 350$, $n(P) = 40$, $n(C) = 40$, $n(M) = 35$

$n(M \cap P) = 20$, $n(P \cap C) = 17$, $n(M \cap C) = 15$

and $n(M \cap P \cap C) = 5$

The number of students who failed in at least one subject $= n(M \cup P \cup C)$

we have,

$n\,(M \cup P \cup C) = n(M) + n(P) + n(C) + n(M \cap P \cap C)$

$-n\,(M \cap P) - (P \cap C) - n\,(M \cap C)$

$= 35 + 40 + 40 + 5 - 20 - 17 - 15$

$= 68$

The number of students who did not fail in any subject

$= 350 - 68$

$= 282$

Ex. 9: A company produces three kinds of products A, B and C. The company studied the preference of 1600 consumers and found that the product A was liked by 1250, the product B was liked by 930 and product C was liked by 1000. The proudcts A and B were liked by 650, the products B and C were liked by 610 and the products C and A were liked by 700 consumers. 30 consumers did not like any of these three products

Find number of consumers who liked.

i) all the three products

ii) only two of these products.

Solution: Given that totally 1600 consumers were studied.

$\therefore n(X) = 1600$,

Let A be the set of all consumers who liked product A. Let B the set of all consumers who liked product B and C be the set of all consumers who liked product C.

$n(A) = 1250$,

$n(B) = 930$, $n(C) = 1000$.

$n(A \cap B) = 650$, $n(B \cap C) = 610$

$n(A \cap C) = 700$, $n(A' \cap B' \cap C') = 30$

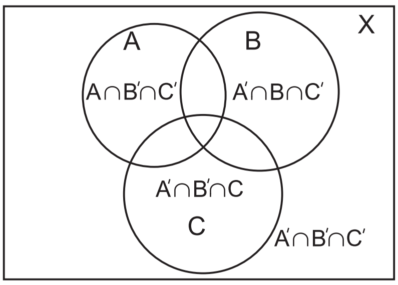


Fig. 1.20

i) $n(A \cup B \cup C) = n(X) - n[(A \cup B \cup C)]'$

$= n(X) - n(A' \cap B' \cap C')$

$= 1600 - 30$

$= 1570$

$\therefore n(A \cup B \cup C) = n(A) + n(B) + n(C)$

$- n(A \cap B) - n(B \cap C) - n(C \cap A)$

$+ n(A \cap B \cap C)$

$\therefore 1570 = 1250 + 930 + 1000 - 650 - 610$

$- 700 + n(A \cap B \cap C)$

$\therefore n(A \cap B \cap C) = 1570 + 1960 - 3180$

$= 350$

$\therefore$ The number of consumers who liked all the three products is 350.

ii) $n[(A \cap B) \cap C'] = n(A \cap B) - n[(A \cap B) \cap C]$

$= 650 - 350$

$= 300$

Similarly $n(A' \cap B \cap C) = 610 - 350 = 260$

and $n(A \cap B' \cap C) = 700 - 350 = 350$

$\therefore$ The number of consumers who liked only two of the three products are

$= n(A \cap B \cap C') + n(A \cap B' \cap C) + n(A' \cap B \cap C)$

$= 300 + 350 + 260$

$= 910$

EXERCISE 1.1

1) Describe the following sets in Roster form

i) $\{x/x$ is a letter of the word 'MARRIAGE'$\}$

ii) $\{x/x$ is an integer, $-\frac{1}{2} < x < \frac{9}{2}\}$

iii) $\{x/x = 2n, n \in N\}$

2) Describe the following sets in Set–Builder form

i) $\{0\}$

ii) $\{0, \pm 1, \pm 2, \pm 3\}$

iii) $\left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\right\}$

3) If $A = \{x/6x^2+x-15 = 0\}$

$B = \{x/2x^2-5x-3 = 0\}$

$C = \{x/2x^2-x-3 = 0\}$ then

find i) $(A \cup B \cup C)$ ii) $(A \cap B \cap C)$

4) If A, B, C are the sets for the letters in the words 'college', 'marriage' and 'luggage' respectively, then verify that

$A-(B\cup C) = (A-B)\cap(A-C)$

5) If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{4, 5, 6, 7, 8\}$ and universal set $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then verify the following:

i) $A\cup(B\cap C) = (A\cup B) \cap(A\cup C)$

ii) $A\cap(B\cup C) = (A\cap B) \cup(A\cap C)$

iii) $(A\cup B)' = (A'\cap B')$

iv) $(A\cap B)' = A'\cup B'$

v) $A = (A\cap B)\cup (A\cap B')$

vi) $B = (A\cap B)\cup (A'\cap B)$

vii) $n (A\cup B) = n(A) + n(B) - n(A\cap B)$

6) If A and B are subsets of the universal set X and $n(X) = 50$, $n(A) = 35$, $n(B) = 20$, $n(A'\cap B') = 5$, find i) $n (A\cup B)$ ii) $n (A\cap B)$ iii) $n(A'\cap B)$ iv) $n(A\cap B')$

7) Out of 200 students; 35 students failed in MHT–CET, 40 in AIEEE and 40 in IIT entrance, 20 failed in MHT–CET and AIEEE, 17 in AIEEE and IIT entrance, 15 in MHT–CET and IIT entrance and 5 failed in all three examinations. Find how many students.

i) did not fail in any examination.

ii) failed in AIEEE or IIT entrance.

8) From amongst 2000 literate individuals of a town, 70% read Marathi newspapers, 50% read English newspapers and 32.5% read both Marathi and English newspapers. Find the number of individuals who read.

i) at least one of the newspapers.

ii) neither Marathi nor English newspaper.

iii) Only one of the newspapers.

9) In a hostel, 25 students take tea, 20 students take coffee, 15 students take milk, 10 student take both tea and coffee, 8 students take both milk and coffee. None of them take tea and milk both and everyone takes atleast one beverage, find the number of students in the hostel.

10) There are 260 persons with a skin disorder. If 150 had been exposed to the chemical A, 74 to the chemical B, and 36 to both chemicals A and B, find the number of persons exposed to

i) Chemical A but not Chemical B

ii) Chemical B but not Chemical A

iii) Chemical A or Chemical B

11) If $A = \{1,2,3\}$ write the set of all possible subsets of A.

12) Write the following intervals in set–builder form.

i) $(-3, 0)$ ii) $[6,12]$

iii) $(6, 12)$ iv) $(-23, 5)$

1.4 RELATIONS:

1.4.1 Ordered Pair:

A pair (a,b) of numbers, such that the order, in which the numbers appear is important, is called an ordered pair. In general, ordered pairs (a,b) and (b, a) are different. In ordered pair (a,b), 'a' is called the first component and 'b' is called the second component.

Two ordered pairs (a,b) and (c, d) are equal, if and only if $a = c$ and $b = d$.

Also, $(a, b) = (b, a)$ if and only if $a = b$

Ex. 1: Find x and y when $(x + 3, 2) = (4, y - 3)$

Solution: Using the definition of equality of two ordered pairs, we have

$(x +3, 2) = (4, y - 3)$

$\therefore \ x + 3 = 4$ and $2 = y - 3$

$\therefore \ x = 1$ and $y = 5$

1.4.2 Cartesian Product of two sets:

Let A and B be two non–empty sets then the cartesian product of A and B is defined as the set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$. It is denoted as $A \times B$ and read as 'A cross B'

Thus, $A \times B = \{(a, b) \,/\, a \in A, b \in B\}$

For example,

If $A = \{1, 2\}$ and $B = \{a, b, c\}$ then

$A \times B = \{(1, a)\ (1, b), (1, c), (2, a), (2, b), (2, c)\}$

Let's Note.

If $A = \phi$ or $B = \phi$, then

$\therefore \ A \times B = \phi$

1.4.3 Number of elements in the Cartesian product of two finite sets:

Let A and B be any two finite sets with $n(A) = m_1$, and $n(B) = m_2$, then the number of elements in the Cartesian product of A and B is given by

$$n(A \times B) = m_1 \bullet m_2 = n(A) \bullet n(B)$$

Ex. 1) : Let $A = \{1, 3\}$; $B = \{2, 3, 4\}$ Find the number of elements in the Cartesian product of A and B.

Solution: Given $A = \{1, 3\}$ and $B = \{2, 3, 4\}$

$\therefore \quad n(A) = 2$ and $n(B) = 3$

$\therefore \quad n(A \times B) = 2 \times 3 = 6$

1.4.4 Relation (Definition): If A and B are two non empty sets then any subset of $A \times B$ is called relation from A to B and is denoted by capital letters P, Q, R etc..

If R is a relation and $(x, y) \in R$ then it is denoted by xRy, read as x is related to y under the relation R or $R : x \rightarrow y$.

y is called 'image' of x under R and x is called 'pre–image' of y under R.

Ex. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 4, 9\}$

Let R be a relation such that $(x, y) \in R$ if $x^2 = y$. List the elements of R.

Solution: Here $A = \{1, 2, 3, 4, 5\}$

and $B = \{1, 4, 9\}$

$\therefore \ R = \{(1, 1), (2, 4), (3, 9)\}$

Arrow diagram fot this relation R is given by

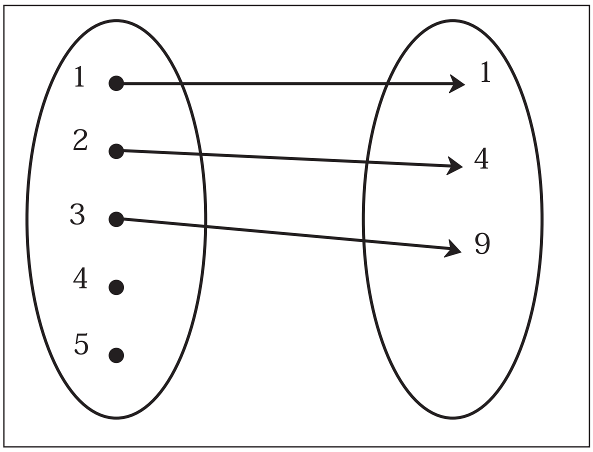


Fig. 1.21

1) Domain:

The set of all first components of the ordered pairs in a relation R is called the domain of the relation R.

i.e. domain $(R) = \{a \in A \mid (a, b) \in R\}$

2) Co-domain:

If R is a relation from A to B then set B is called the co–domain of the relation R

3) Range:

The set of all second components of all ordered pairs in a relation R is called the range of the relation.

i.e. Range $(R) = \{b \in B \mid (a, b) \in R\}$

1.5 TYPES OF RELATIONS:

i) One-One Relation(Injective): A relation R from A to B is said to be one–one if every element of A has at most one image in B under R and distinct elements in A have distinct images in B under R.

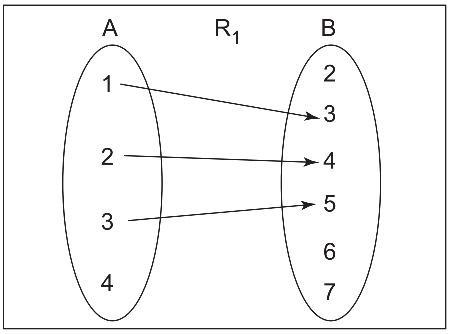


Fig. 1.22

For example: Let A= {1,2,3,4}

B = {2,3,4,5,6,7}

and R_1 = {(1, 3), (2, 4), (3, 5)}

Then R_1 is a one – one relation. (fig 1.22)

Here, domain of R_1 = {1, 2, 3} and range is {3, 4, 5}

ii) Many-one relation: A relation R from A to B is said to be many – one if two or more than two elements in A have same image in B.

For example: Let R_2 = {(1, 4), (3, 7), (4, 4)}

Then R_2 is many – one relation from A to B

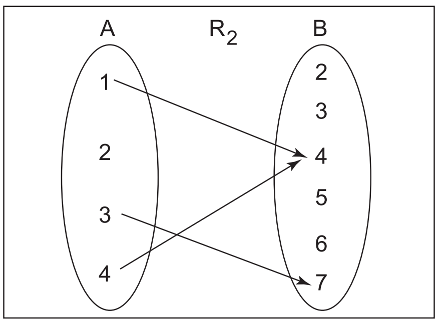


Fig. 1.23

Domain of R_2 = {1, 3, 4}

Range of R_2 = {4, 7}

iii) Into relation: A relation R from A to B is said to be into relation if there exists at least one element in B which has no pre–image in A. i.e. Range of R is proper subset of B

For example: Let A = {–2, –1, 0, 1, 2, 3}

B = {0, 1, 2, 3, 4}

Let R_3 = {(–2, 4), (–1, 1), (0, 0), (1, 1), (2, 4)}

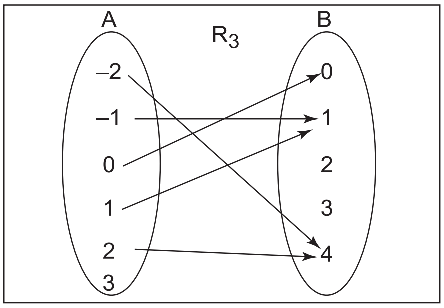


Fig. 1.24

∴ Range of R_3 = {0, 1, 4}

∴ Range of $R_3 \subset$ B

Then R_3 is an into relation from A into B

iv) Onto relation (Surjective) : A relation R from A to B is said to be onto relation, if every element of B is the image of some element of A.

For Example: Let A = {–3 , –2, –1, 1, 3, 4}

B = {1, 4, 9}. Let

R_4 = {(–3, 9), (–2, 4), (–1, 1), (1, 1), (3, 9)}

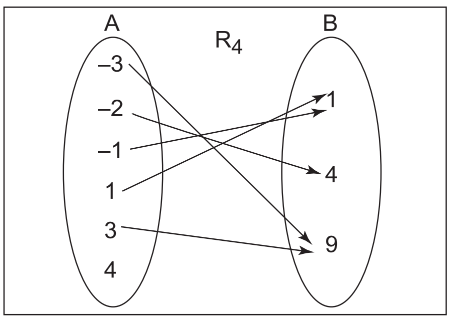


Fig. 1.25

$\therefore$ Range or $R_4 = \{1, 4, 9\}$

$\therefore$ Range = co–domain (B)

Thus R_4 is an onto relation from A to B.

Binary relation on a set:

Let A be non–empty set then every subset of $A \times A$ is called a binary relation on A.

Let's Note.

A relation having the same set as domain and co-domain is a binary relation on that set.

SOLVED EXAMPLES

Ex. 1.: Let $A = \{1, 2, 3\}$ and

$R = \{(1, 2), (2, 2), (3, 1), (3, 2)\}$

$AxA = \{(1, 1), (1, 2), 1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

Clearly $R \subset A \times A$ and therefore, R is a binary relation on A.

Ex. 2: Let N be the set of all natural numbers and $R = \{a, b) / a, b \in N$ and $2a + b = 10\}$

Since $R \subset N \times N$, R is binary relation on N. Clearly, $R = \{(1, 8), (2, 6), (3, 4), (4, 2)\}$

We can state domain, range and co–domain of the relation R as follows :

Domain (R) = $\{1, 2, 3, 4\}$

Range (R) = $\{2, 4, 6, 8\}$

Co–domain = N.

i) $\phi \subset A \times A$ is a relation on A and is called the empty or void relation on A.

ii) $A \times A \subseteq A \times A$. So it is a relation on A called the universal relation i.e. $R = A \times A$

Ex. 3: If $A = \{2, 4, 6\}$

then $R = A \times A = \{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)\}$

and $R = A \times A$ is the universal relation on A.

Note: The total number of relations that can be defined from a set A to a set B is the number of possible subsets of $A \times B$

If $n(A) = m_1$ and $n(B) = m_2$

then $n(A \times B) = m_1 m_2$

and the total number of relations is $2^{m_1 \cdot m_2}$

Properties of relations:

Let A be a non–empty set. Then a relation R on A is said to be

(i) Reflexive, if $(a, a) \in R$ for every $a \in A$ i.e. aRa for every $a \in A$

(ii) Symmetric, if $(a, b) \in R$
$\Rightarrow (b, a) \in R$ for all $a, b, \in A$
i.e. aRb $\Rightarrow$ bRa for all $a, b \in A$

(iii) Transitive, if $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow (a, c) \in R$ for all $a, b, c \in A$.

Note: Read the symbol " $\Rightarrow$" as "implies".

Equivalence relation:

A relation which is reflexive, symmetric and transitive is called an equivalence relation.

SOLVED EXAMPLES

Ex. 1: Let R be a relation on Q, defined by
$R = \{(a, b)/a, b \in Q$ and $a - b \in Z\}$

Show that R is an equivalence relation.

Solution: Given

$R = \{(a, b)/a, b \in Q$ and $a - b \in Z\}$

i) Let a $\in Q$ then $a - a = 0 \in Z$

$\therefore (a, a) \in R$

So, R is reflexive.

ii) $(a, b) \in R \Rightarrow (a-b) \in Z$

i.e. $(a-b)$ is an integer

$\Rightarrow -(a-b)$ is an integer

$\Rightarrow (b-a)$ is an integer

$\Rightarrow (b, a) \in R$

Thus $(a, b) \in R \Rightarrow (b, a) \in R$

$\therefore$ R is symmetric.

iii) $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow (a-b)$ is an integer and $(b-c)$ is an integer

$\Rightarrow \{(a-b) + (b-c)\}$ is an integer

$\Rightarrow (a-c)$ is an integer

$\Rightarrow (a, c) \in R$

Thus $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

$\therefore$ R is transitive.

Thus, R is reflexive, symmetric and transitive.

$\therefore$ R is an equivalence relation.

SOLVED EXAMPLES

Ex. 1: If $(x+1, y-2) = (3, 1)$ find the value of x and y.

Solution: Since the order pairs are equal, the corresponding elements are equal.

$\therefore x + 1 = 3$ and $y-2 = 1$

$\therefore x = 2$ and $y = 3$

Ex. 2: If A = {1, 2}, find A×A

Solution: We have A = {1, 2}

$\therefore$ A×A = {(1, 1), (1, 2), (2, 1), (2, 2)}

Ex. 3: If A = {1, 3, 5} and B = {2, 3}. Find A×B and B×A. Is A×B = B×A?

Solution: We have

A×B = {(1, 2), (1, 3), (3, 2), (3, 3), (5, 2), (5, 3)} and B×A = {(2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5)} All elements in A×B, B×A (except (3, 3) are different.

$\therefore A\times B \neq B\times A$

Ex. 4: If A×B = {(3, 2), (3, 4), (5, 2), (5, 4)} then find A and B

Solution: Clearly, we have

A = Set of all first components of A×B

$\therefore$ A = {3, 5}

B = Set of all second components of A×B

$\therefore$ B = {2, 4}

Thus A = {3, 5} and B = {2, 4}

Ex. 5: Express $\{(x, y)/x^2 + y^2 = 25$ where $x, y \in W\}$ as a set of ordered pairs.

Solution: We have $x^2 + y^2 = 25$

$\therefore x = 0, y = 5 \Rightarrow x^2 + y^2 = 0^2 + 5^2 = 25$

$x = 3, y = 4 \Rightarrow x^2 + y^2 = (3)^2 + (4)^2 = 25$

$x = 4, y = 3 \Rightarrow x^2 + y^2 = (4)^2 + (3)^2 = 25$

$x = 5, y = 0 \Rightarrow x^2 + y^2 = (5)^2 + (0)^2 = 25$

$\therefore$ The given set = {(0, 5), (3, 4), (4, 3), (5, 0)}

Ex. 6: Let A = {1, 2, 3} and B = {2, 4, 6}

Show that R = {(1, 2), (1, 4), (3, 2), (3, 4)} is a relation from A to B find

i) domain (R) ii) Co–domain (R)

iii) Range (R)

Also represent this relation by arrow diagram

Solution: Here A = {1, 2, 3}, B = {2, 4, 6}

and R = {(1, 2), (1, 4), (3, 2), (3, 4)}

Since $R \subset A\times B$, R is a relation from A to B

i) Domain (R) = Set of first components of R = {1, 3}

ii) Co–domain (R) = B = {2, 4, 6}

iii) Range (R) = Set of second components of R = {2, 4}

Arrow Diagram:

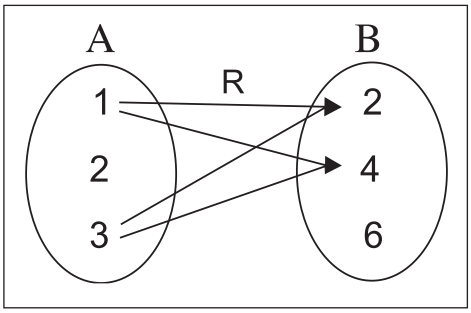


Fig. 1.26

Ex. 7 : Let A = {1, 2, 3, 4, 5} and B = {1, 4, 5}

Let R be a relation from A to B such that $(x, y) \in R$ if $x < y$

i) List the elements of R.

ii) Find the domain, co–domain and range of R.

iii) Draw the above relation by an arrow diagram.

Solution: A = {1, 2, 3, 4, 5} and B = {1, 4, 5}

i) The elements of R are as follow:

R = {(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5) (4, 5)}

ii) Domain (R) = {1, 2, 3, 4}

Range (R) = {4, 5}

Co–domain (R) = {1, 4, 5} = B

iii) An Arrow diagram

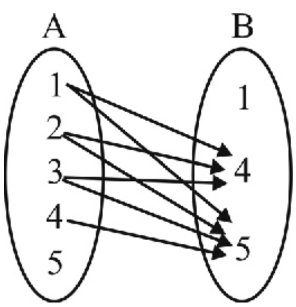


Fig. 1.27

Ex. 8 : Let A = {1, 2, 3, 4, 5, 6 } define a relation R from A to A by R = $\{(x, y) / y = x + 1\}$

i) Draw this relation using an arrow diagram.

ii) Write down the domain, co-domain and range or R.

Solution :

1) A relation R from A to A by

R = $\{(x,y) / y=x+1\}$ is given by

R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

The corresponding arrow diagram is

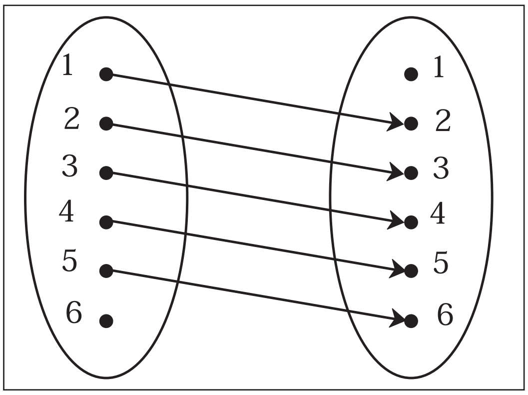


Fig. 1.28

ii) Domain = {1, 2, 3, 4, 5}

Range = {2 , 3, 4, 5, 6}

Co-domain = { 1, 2, 3, 4, 5, 6}

EXERCISE 1.2

1) If $(x - 1, y + 4) = (1, 2)$ find the values of x and y.

2) If $\left(x+\frac{1}{3}, \frac{y}{3}-1\right)=\left(\frac{1}{3}, \frac{3}{2}\right)$, find x and y.

3) If A = $\{a, b, c\}$, B = $(x , y\}$ find

A × B, B× A, A × A, B× B.

4) If P = { 1, 2, 3} and Q = {6, 4},

find the sets $P \times Q$ and $Q \times P$

5) Let A = { 1, 2, 3, 4}, B = { 4, 5, 6}, C = { 5, 6}.

Find i) $A \times (B \cap C)$

ii) $(A \times B) \cap (A \times C)$

iii) $A \times (B \cup C)$

iv) $(A \times B) \cup (A \times C)$

6) Express $\{(x, y) / x^2 + y^2 = 100$ where $x, y \in W\}$ as a set of ordered pairs.

7) Write the domain and range of the following relations.

i) $\{(a, b) / a \in N, a<6$ and $b = 4\}$

ii) $\{(a, b) / a, b \in N, a+b = 12\}$

iii)(2, 4), (2, 5), (2,6), (2, 7)}

8) Let A = {6, 8} and B = {1, 3, 5}

Let R = $\{(a, b)/a\in A, b\in B, a\text{-}b$ is an even number$\}$

Show that R is an empty relation from A to B.

9) Write the relation in the Roster form and hence find its domain and range.

i) $R_1 = \{(a, a^2)$ / a is prime number less than 15}

ii) $R_2 = \{(a, \frac{1}{a}) / 0<a \leq 5, a\in N\}$

10) R = {(a, b) / b = a + 1, a∈z, 0<a<5}

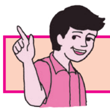
Find the Range of R

11) Find the following relation as sets of ordered pairs.

i) $\{(x, y) / y = 3x, x\in \{1,2,3\}, y\in \{3,6,9,12\}\}$

ii) $\{(x, y) / y > x + 1, x \in \{1, 2\}$ and $y \in \{2, 4, 6\}\}$

iii) $\{(x, y) / x+y = 3, x, y\in \{0, 1, 2, 3\}\}$



Let's remember!

- A set is a collection of well defined objects.
- A set which does not contain any element is called an empty set and is denoted by ϕ.
- The power set of a set A is the sets of all subsets of A and is denoted by P(A).
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
- For any sets A and B
  $(A \cup B)' = A' \cap B'$
- If A and B are finite sets such that :
  $A \cap B = \phi$, then
  $n(A \cup B) = n(A) + n(B)$
- If $n(A \cap B) \neq \phi$, then
  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $A \times B = \{(a, b)/a\in A, b\in B\}$
  In particular, $R \times R = \{(x, y) / x, y \in R\}$
- If $(a, b) = (x, y)$ then $a = x$ and $b = y$
- If $n(A) = p$ and $n(B) = q$ then $n(A \times B) = pq$
- $A \times \phi = \phi$
- The image of an element x under a relation R is given by y, where $(x, y) \in R$. (relation).
- The domain of R is the set of all 1st components of the ordered pairs in R (relation).
- The range of R (relation) is the set of all second components of the ordered pairs in R
- Let A be any non-empty set, then every subset of $A \times A$ is binary relation on A.

MISCELLANEOUS EXERCISE - 1

1) Write the following sets in set builder form

i) {10, 20, 30, 40, 50},

ii) { a, e, i, o, u)

iii) {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

2) If $U = \{x / x \in N, 1 \le x \le 12\}$

$A = \{1, 4, 7, 10\}$ $B = \{2, 4, 6, 7, 11\}$

$C = \{3, 5, 8, 9, 12\}$

Write the sets

i) $A \cup B$ ii) $B \cap C$ iii) A-B

iv) B-C v) $A \cup B \cup C$ vi) $A \cap (B \cup C)$

3) In a survey of 425 students in a school, it was found that 115 drink apple juice, 160 drink orange juice and 80 drink both apple as well as orange juice. How many drink neither apple juice nor orange juice?

4) In a school there are 20 teachers who teach Mathematics or Physics, of these, 12 teach Mathematics and 4 teach both Physics and Mathematics. How many teachers teach Physics?

5) i) If $A = \{1, 2, 3\}$ and $B = \{2, 4\}$, state the elements of $A \times A$, $A \times B$, $B \times A$, $B \times B$, $(A \times B) \cap (B \times A)$

ii) If $A = \{-1, 1\}$, find $A \times A \times A$

6) If $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$ which of following are relations from A to B

i) $R_1 = \{(1, 4), (1, 5), (1, 6)\}$

ii) $R_2 = \{(1, 5), (2, 4), (3, 6)\}$

iii) $R_3 = \{(1, 4), (1, 5), (3, 6), (2, 6), (3, 4)\}$

iv) $R_4 = \{(4, 2), (2, 6), (5, 1), (2, 4)\}$

7) Determine the Domain and range of the following relations.

$R = \{ (a, b) / a \in N, a < 5, b = 4\}$

ACTIVITIES

Activity 1.1 :

Take Universal set X having 10 elements and take two unequal subsets A and B of set X. Write A', B', A – B, B – A, $A \cup B$, $A \cap B$, $(A \cup B)'$, $(A \cap B)'$.

Activity 1.2 :

Give an example of nonempty sets A and B and universal set such that

i) $A \cup B = A \cap B$ ii) $(A \cup B)' = A' \cup B'$

iii) $A' \cap B' = (A \cap B)'$

Activity 1.3 :

By taking suitable example verify the $A \times B \neq B \times A$. But $n(A \times B) = n(B \times A)$.

Activity 1.4 :

What conclusion will you draw about two sets A and B if $A \subseteq B$ and $B \subseteq A$.

Activity 1.5 :

If $A = \left\{ \frac{-1}{2}, \frac{2}{3} \right\}$ then it can also be expressed as

$A = \{x / \boxed{\cdots} x^2 + \boxed{\cdots} x + \boxed{\cdots} = 0\}$.

Activity 1.6 :

Write the domain and range for the following relation.

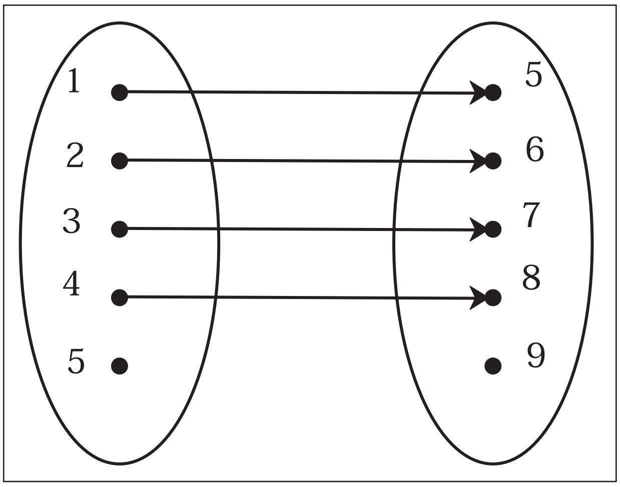


Fig.1.29

1) Write the relation in term of the ordered pairs.
2) Write the image of 5. (if it exists)
3) Write the pre-image of 8.
4) Write the relation in terms of the formula.

Activity 1.7 :

Write the following sets in Roster form. Also draw Venn diagram.

i) A - set of all factors of 24.

ii) B - set of all prime numbers less than 30.

iii) C - Set of all letters in the word 'MATHEMATICS'.

Activity 1.8 :

In a survey of 400 students. It was found that 150 drink milk, 250 drink Tea, 50 drink both. How many drink neither Tea nor milk. Solve using formula & Venn diagram.

Let M is set of students who drink milk.

Let T is set of students who drink Tea.

$n(M) = \boxed{}$

$n(T) = \boxed{}$

$n(M \cup T) = n(M) + n(T) - n(M \cap T)$

$= \boxed{} + \boxed{} - \boxed{}$

$= \boxed{}$

Number of students neither drink Tea nor milk = Total number of students $- n(M \cup T)$

$= \boxed{} - \boxed{} = \boxed{}$

Activity 1.9 :

Complete the following activity.

$A = \{\frac{1}{3x} / x \in N \;\&\; x < 8\}$

$B = \{\frac{1}{2x} / x \in N \;\&\; x \leq 8\}$

Find $A \cup B$, $A \cap B$, $A - B$, $B - A$

Solution :

Write set A & set B in list form

A = {..................}

B = {..................}

For $A \cup B$, [consider all elements from A as well as B, don't repeat elements]

$\therefore$ $A \cup B$ = {.....................}

For $A \cap B$, [Take all the elements that are common in A and B]

$\therefore$ $A \cap B$ = {......................}

For A – B [Take all the elements that are present in A but not in B]

$\therefore$ A – B = {...................}

B – A = {...................}

Activity 1.10 :

U = {1,2,3,4,5,6,7,8}

A = {1,2,3,4,,5} , A' = {.................}

B = {4,5,6,7,8} , B' = {.................}

Complete the following activity.

$A \cup B$ = {......................}, $n(A \cup B) = \boxed{}$

$A - B$ = {......................}, $n(A - B) = \boxed{}$

$B - A$ = {......................}, $n(B - A) = \boxed{}$

$A \cap B$ = {......................}, $n(A \cap B) = \boxed{}$

i) $n(A-B) + n(A \cap B) + n(B-A)$

$= \boxed{} + \boxed{} + \boxed{}$

$= \boxed{}$

ii) $A \cap B'$ = {......................}

$A \cap B' = A - B$

iii) $A' \cap B$ = {......................}

$A' \cap B = B - A$

❖ ❖ ❖