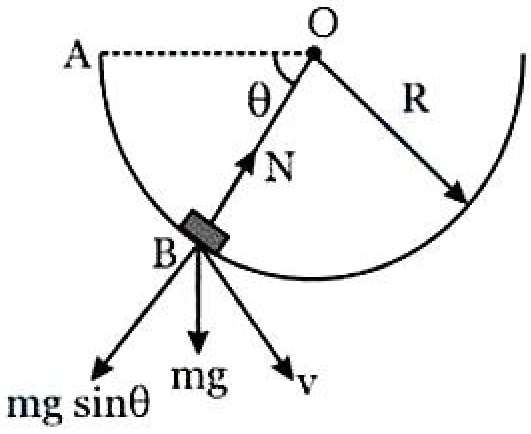
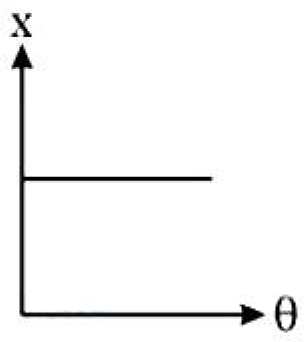
Q1: NTA Test 01 (Single Choice)

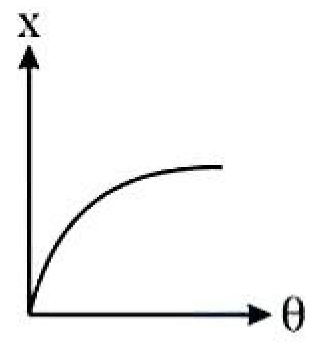
A small block of mass m is released from rest from point A inside a smooth hemisphere bowl of radius R, which is fixed on ground such that OA is horizontal. The ratio (x) of magnitude of centripetal force and normal reaction on the block at any point B varies with θ as:



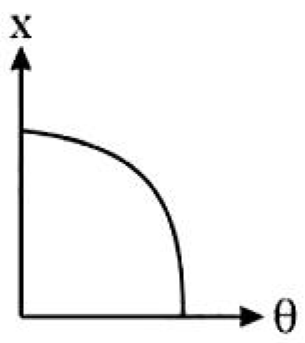
(A)



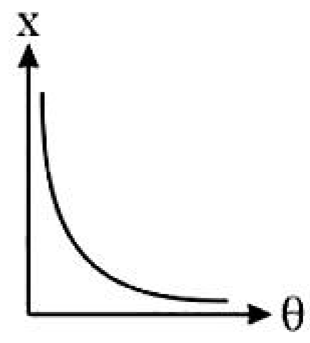
(B)



(C)



(D)



Q2: NTA Test 02 (Single Choice)

A road is banked at an angle of 30° to the horizontal for negotiating a curve of radius $10\sqrt{3}\,\text{m}$. At what velocity will a car experience no friction while negotiating the curve? Take $g = 10\text{ ms}^{-2}$

(A) 54 km/hr

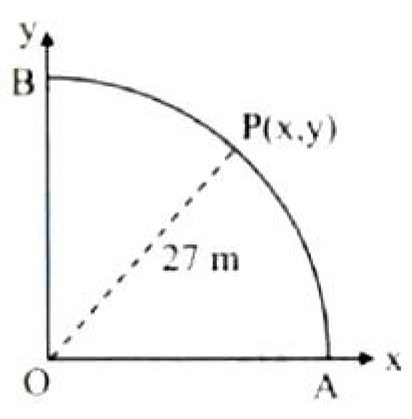
(B) 72 km/hr

(C) 36 km/hr

(D) 18 km/hr

Q3: NTA Test 03 (Numerical)

A point P moves in a counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$ where s is in metre and t is in seconds. The radius of the path is 27 m. The acceleration of P when $t = 3$ s is ____ m/s^2.
$(\text{Take } \sqrt{13} = 3.6)$



Q4: NTA Test 04 (Single Choice)

A long horizontal rod has a bead which can slide along its length, and initially it is at a distance L from the end A of the rod. The rod is set in angular motion about A with constant angular acceleration α in a gravity free space. If the coefficient of friction between the rod and the bead is μ, then the time after which the bead starts slipping is

(A) $\sqrt{\frac{\mu}{\alpha}}$

(B) $\frac{\mu}{\sqrt{\alpha}}$

(D) infinitesimal

(C) $\frac{1}{\sqrt{\mu\alpha}}$

Q5: NTA Test 05 (Single Choice)

The angular velocity of a particle is $\vec{\omega} = 4\hat{i} + \hat{j} - 2\hat{k}$ about the origin. If the position vector of the particle is $2\hat{i} + 3\hat{j} - 3\hat{k}$, then its linear velocity is

(A) $5\hat{i} + 8\hat{j} - 14\hat{k}$

(B) $3\hat{i} + 8\hat{j} + 10\hat{k}$

(C) $8\hat{i} + 3\hat{j} - 10\hat{k}$

(D) $-8\hat{i} + 3\hat{j} - 2\hat{k}$

Q6: NTA Test 05 (Single Choice)

An aircraft loops a vertical loop of radius $R = 500\ \text{m}$ with a constant velocity $v = 360\ \text{kmh}^{-1}$. Find the weight of the flier of mass $m = 70\ \text{kg}$ in the lower, upper, and middle points of the loop.

(A) $2.1\ \text{kN},\ 0.7\ \text{kN},\ 1.5\ \text{kN}$

(B) $0.7\ \text{kN},\ 1.5\ \text{kN},\ 2.1\ \text{kN}$

(C) $1.5\ \text{kN},\ 2.1\ \text{kN},\ 0.7\ \text{kN}$

(D) none of these

Q7: NTA Test 08 (Single Choice)

When a ceiling fan is switched off, its angular velocity reduces to 50% of its initial value while it makes 36 rotations. If angular retardation of the fan is uniform, then how many more rotations will it make before coming to rest?

(A) 18

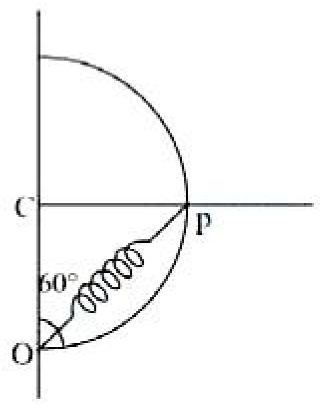
(B) 12

(C) 36

(D) 48

Q8: NTA Test 11 (Single Choice)

A smooth semicircular wire track of radius R is fixed in a vertical plane. One end of a massless spring of natural length $\frac{3R}{4}$ is attached to the lowest point O of the wire track. A small ring of mass m which can slide on the track is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle of 60° with the vertical. The spring constant $K = \frac{mg}{R}$. Consider the instant when the ring is released. The normal reaction on the ring by the track is



(A) $\frac{3mg}{8}$

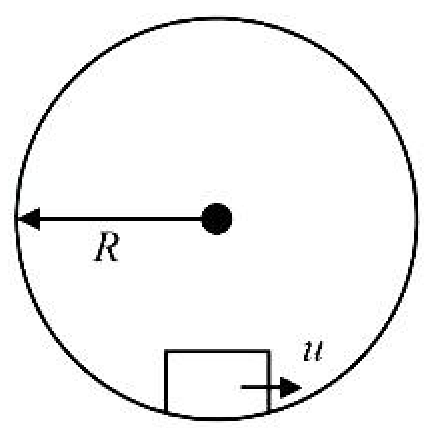
(B) mg

(C) $\frac{mg}{4}$

(D) $\frac{3mg}{4}$

Q9: NTA Test 16 (Single Choice)

A block of mass m is placed at the lowest point of a smooth vertical track of radius R. In this position, the block is given a horizontal velocity u such that the block is just able to perform a complete vertical circular motion. The acceleration of block, when its velocity is vertical is



(A) g

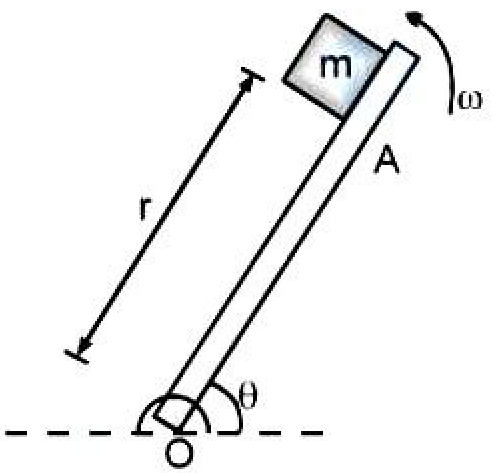
(B) 3g

(C) $g\sqrt{10}$

(D) $2\sqrt{2}g$

Q10: NTA Test 19 (Single Choice)

A rough platform OA rotates in a vertical plane about a horizontal axis through the point O with a constant counterclockwise velocity $\omega = 3 \text{ rad s}^{-1}$. As it passes through the position $\theta = 0$, a small mass m is placed upon it at a radial distance. If the mass is observed to slip at $\theta = 37^{\circ}$, then the coefficient of friction between the mass & the member is



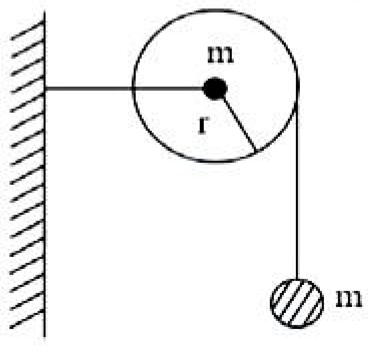
(A) $\frac{3}{16}$

(B) $\frac{9}{16}$

(C) $\frac{4}{9}$

(D) $\frac{5}{9}$

Q11: NTA Test 28 (Single Choice)



As shown in the figure, a bob of mass m is tied to a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be:

(A) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$

(B) $r\sqrt{\frac{3}{2gh}}$

(C) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$

(D) $r\sqrt{\frac{3}{4gh}}$

Q12: NTA Test 33 (Single Choice)

A stone tied to a string is rotated in a vertical circle. The minimum speed of the stone during a complete vertical circular motion

(A) is independent of the mass of the stone

(B) is independent of the length of the string

(C) decreases with increasing mass of the stone

(D) decreases with increasing length of the string

Q13: NTA Test 34 (Single Choice)

For a particle in a uniform circular motion, the acceleration $\vec{a}$ at a point P (R, θ) on the circle of radius R is (here θ is measured from the y-axis)

(A) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

(B) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$

(C) $-\frac{v^2}{R}\sin\theta\,\hat{i} - \frac{v^2}{R}\cos\theta\,\hat{j}$

(D) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$

Q14: NTA Test 40 (Single Choice)

A car is moving along the circle $x^2 + y^2 = a^2$ in the anti-clockwise direction with a constant speed. The $x - y$ plane is a rough horizontal stationary surface. When the car is at the point $(a\cos\theta,\ a\sin\theta)$, the unit vector in the direction of the friction force acting on the car is

(A) $\cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

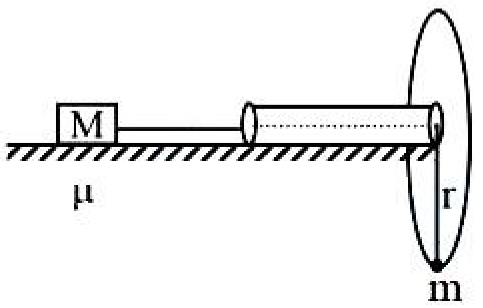
(B) $\cos\theta\,\hat{i} - \sin\theta\,\hat{j}$

(C) $-\cos\theta\,\hat{i} - \sin\theta\hat{j}$

(D) $-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

Q15: NTA Test 41 (Single Choice)

The figure below shows a block of mass M connected to an ideal string which passes through a thin fixed smooth pipe. On the other end, a particle of mass m is connected which revolves in a vertical circle of radius r. If the coefficient of friction between M and the surface is $\mu = \frac{2}{3}$, then for what minimum value of M, the block of mass m can undergo complete vertical circular motion?



(A) $M_{\min} = 6m$

(B) $M_{\min} = 9m$

(C) $M_{\min} = 3m$

(D) $M_{\min} = 15m$

Q16: NTA Test 42 (Numerical)

The bob of the simple pendulum is given a vertically downward velocity of magnitude $v_0 = 5 \text{ m s}^{-1}$, as shown in the figure. The string can withstand a maximum tension equal to twice the weight of the bob and during motion the string breaks at an angle θ with the vertical. If $\cos\theta = \frac{m}{n}$, where m and n are smallest integers, then, the value of $m + n$ is

Q17: NTA Test 43 (Single Choice)

A stone hanging from a massless string of length 15 m is projected horizontally with speed 12 m s^{-1}. The speed of the particle, at the point where the tension in the string is equal to the weight of the particle, is close to

(A) 10 m s^{-1}

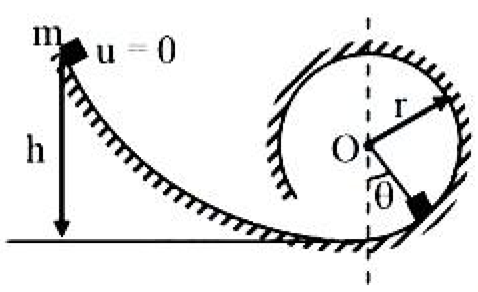
(B) 7 m s^{-1}

(C) 12 m s^{-1}

(D) 5 m s^{-1}

Q18: NTA Test 44 (Single Choice)

A particle of mass m is released from height h on a smooth curved surface which ends into a vertical loop of radius r, as shown. If $h = 2r$, then



(A) the particle reaches the top of the loop with zero velocity

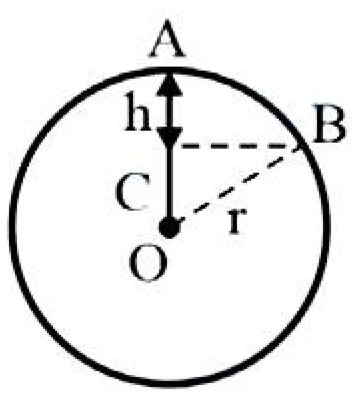
(B) the particle reaches the top of the loop with a non-zero velocity

(C) the particle breaks off at a height $h = r$ from base

(D) the particle breaks off at a height $r < h < 2r$

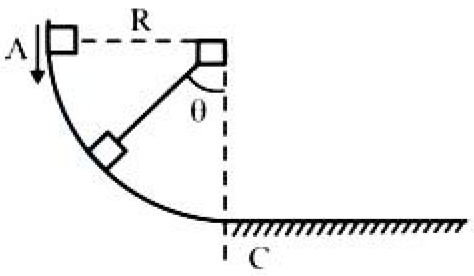
Q19: NTA Test 45 (Single Choice)

In figure, a particle is placed at the highest point A of a smooth sphere of radius r. It is given slight push, and it leaves the sphere at B, at a depth h vertically below A. The value of h is



(A) $\frac{r}{6}$
(B) $\frac{1}{4}r$
(C) $\frac{1}{3}r$
(D) $\frac{1}{2}r$

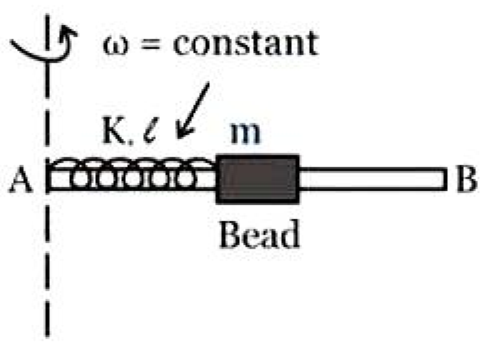
Q20: NTA Test 46 (Single Choice)
A block of mass m is placed on a vertical fixed circular track and then it is given velocity v along the track at position A on track. The coefficient of friction between the block and the track varies with the angle θ. If the block moves on track with constant speed then the coefficient of friction is



(A) $\mu=\frac{\sin\theta}{\cot\theta+\frac{v^2}{Rg}}$
(B) $\mu=\frac{\sin\theta}{\cos\theta+\frac{v^2}{Rg}}$
(C) $\mu=\frac{\tan\theta}{\cos\theta+\frac{v^2}{Rg}}$
(D) $\mu=\frac{\cos\theta}{\tan\theta+\frac{v^2}{Rg}}$

Q21: NTA Test 47 (Single Choice)
A hollow vertical cylinder of radius R and height h has a smooth internal surface. A small particle is held in contact with the inner side of the upper rim at a point P. It is given a horizontal speed v_0 tangential to the rim. It leaves the lower rim at point Q, vertically below P. The number of revolutions made by the particle will be [Take acceleration due to gravity g]
(A) $\frac{h}{2\pi R}$
(B) $\frac{v_0}{h}\sqrt{\frac{h}{2g}}$
(C) $\frac{v_0}{2\pi R}\sqrt{\frac{2h}{g}}$
(D) $\frac{v_0\pi}{g}\sqrt{\frac{2gh}{2R}}$

Q22: NTA Test 48 (Single Choice)
AB is a light rigid rod, which is rotating about a vertical axis passing through end A. A spring of force constant k and natural length l is attached at A and its other end is attached to a small bead of mass m. The bead can slide without friction on the rod. At the initial moment, the bead is at rest (w.r.t the rod) and the spring is unstretched. Select incorrect options :



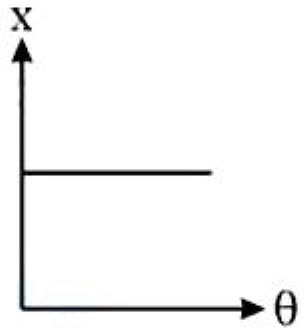
(A) $V_{\max}=\sqrt{\frac{m\omega^2l^2}{k-m\omega^2}}$
(B) $V_{\max}=\sqrt{\frac{m\omega^4l^2}{k-m\omega^2}}$
(C) $V_{\max}=\sqrt{\frac{m\omega^4l^2}{m\omega^2-k}}$
(D) $V_{\max}=\sqrt{\frac{m\omega^2l^2}{m\omega^2-k}}$

Answer Keys

Q1: (A)
Q2: (C)
Q3: 32.40
Q4: (A)
Q5: (B)
Q6: (A)
Q7: (B)
Q8: (A)
Q9: (C)
Q10: (A)
Q11: (A)
Q12: (A)
Q13: (C)
Q14: (C)
Q15: (B)
Q16: 5
Q17: (B)
Q18: (D)
Q19: (C)
Q20: (B)
Q21: (C)
Q22: (B)

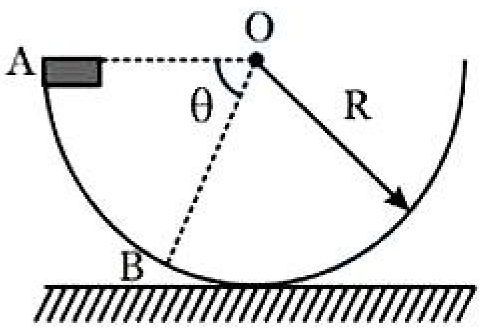
Solutions

Q1: (A)



$\frac{mv^2}{R} = N - mg\sin\theta$

$N = \frac{mv^2}{R} + mg\sin\theta$



By energy conservation,

$mgR\ sin\theta = \frac{1}{2}mv^2$

$\frac{mv^2}{R} = 2mg\sin\theta$

$N = 3mg\sin\theta$

$Ratio = \frac{mv^2}{RN} = \frac{2}{3}$ (constant)

$x = \frac{2}{3}$

Q2: (C) 36 km/hr

For negotiating curve without friction $\tan\theta = \frac{V^2}{rg}$

$V = \sqrt{rg\tan\theta}$

= 10 m/s

= 36 km/hr

Q3: 32.40

As $s = t^3 + 5$

$\frac{ds}{dt} = 3t^2 = v$

$\therefore \quad a_t = \frac{dv}{dt} = 6t$

$\left|\vec{a}\right| = \sqrt{a_c^2 + a_t^2}$

At $t = 3s$

$$\left|\vec{a}\right| = \sqrt{\left(\frac{v^2}{R}\right)^2 + a_t^2} = \sqrt{\left(\frac{9t^4}{R}\right)^2 + (6t)^2}$$

$$= \sqrt{(27)^2 + 324} = \sqrt{1053} = 9\sqrt{13}$$

$$\therefore \left|\vec{a}\right| = 32.4\ \text{m/s}^2$$

Q4: (A) $\sqrt{\frac{\mu}{\alpha}}$

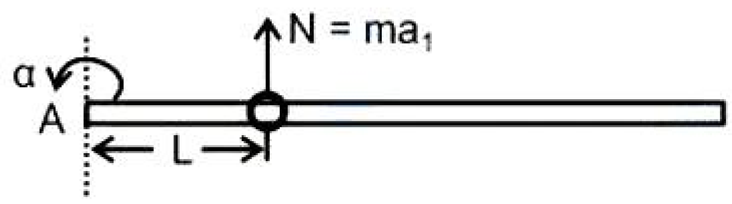
$N = m\alpha L$

When the bead starts slipping

$f_{\max} = \mu N = m\omega^2 L$

$\mu m\alpha L = m(\alpha t)^2 L$

$\Rightarrow t = \sqrt{\frac{\mu}{\alpha}}$



Q5: (B) $3\hat{i} + 8\hat{j} + 10\hat{k}$

$\vec{v} = \vec{\omega} \times \vec{r}$

$= 3\hat{i} + 8\hat{j} + 10\hat{k}$

Q6: (A) $2.1\ \text{kN},\ 0.7\ \text{kN},\ 1.5\ \text{kN}$

Forces on the flier in the frame of aircaraft (non inertial reference frame) is:

(i) Normal reaction (N)

(ii)Pseudo force $\left(\frac{mv^2}{R}\right)$

(iii) Gravitational force d(mg)

According to problem,

$$v_T = v_b = v_m = v$$

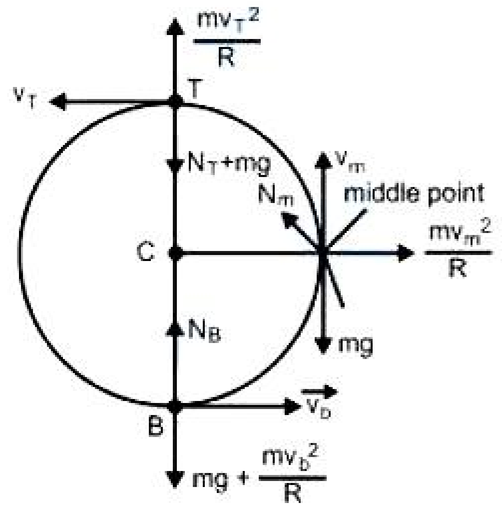
The net force on flier at every point should be zero in the frame of aircraft. For top point T,

$$N_T + mg = \frac{mv_t^2}{R} \quad \therefore N_T = \frac{mv^2}{R} - mg$$

$= 0.7$ kN

For bottom point, $N_B = mg + \frac{mv_b^2}{R} = mg + \frac{mv^2}{R}$

$= 2.1$ kN.



For middle point, $N_m = \sqrt{\left(\frac{mv^2}{R}\right)^2 + (mg)^2} = 1.5\ kN$

Q7: (B) 12

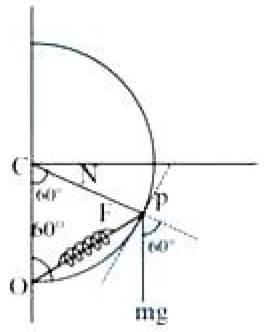
$\left(\frac{\omega_0}{2}\right)^2 = \omega_0^2 - 2\alpha\theta_1 \Rightarrow 2\alpha\,\theta_1 = \frac{3}{4}\,\omega_0^2$ ....(i)

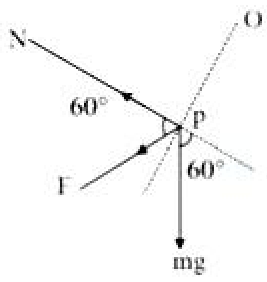
$0 = \left(\frac{\omega_0}{2}\right)^2 - 2\alpha\theta_2 \Rightarrow 2\alpha\theta_2 = \frac{\omega_0^2}{4}$ ....(ii)

from (i) & (ii) $\frac{\theta_1}{\theta_2} = 3$

$\Rightarrow \theta_2 = \frac{\theta_1}{3} = \frac{36}{3} = 12$ rotation

Q8: (A) $\frac{3mg}{8}$





In ΔOCP, OC = CP = R.

$\therefore$ The triangle is isosceles

$\therefore$ $\angle COP = \angle CPO = 60^\circ \Rightarrow \angle OCP = 60^\circ$

$\therefore$ ΔOCP is an equilateral triangle

$\Rightarrow$ $OP = R$

$\therefore$ Extension of string $= R - \frac{3R}{4} = \frac{R}{4} = x$

The forces acting are shown in the figure (i)
The free-body diagram of the ring is shown in fig (ii)

Force in the tangential direction

$= F\cos 30^\circ + mg\cos 30^\circ = [kx + mg]\ \cos 30^\circ$

$\therefore\ F_t = \frac{5mg}{8}\sqrt{3}$ $\qquad\qquad \therefore\ F_t = ma_t$

$\Rightarrow \quad \frac{5mg\sqrt{3}}{8} = ma_t \qquad \Rightarrow \quad a_t = \frac{5\sqrt{3}}{8}g$

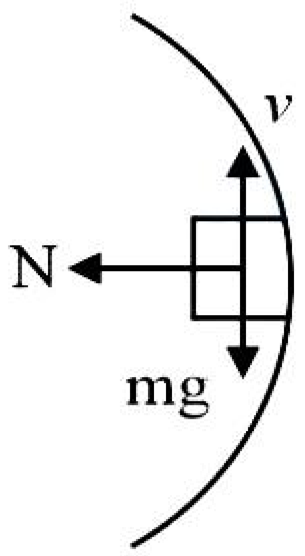
Also when the ring is just released

$N + F\sin 30^{o} = mg\sin 30^{o}$

$= \left(mg - \frac{mg}{4}\right) \times \frac{1}{2} = \frac{3mg}{8}$

Q9: (C) $g\sqrt{10}$

The initial velocity of the block to undergo a complete vertical circular motion is $u = \sqrt{5gR}$



Let us assume that the speed of the block when it is moving in the vertical direction is v, then using conservation of mechanical energy we get

$\frac{1}{2}m\left(\sqrt{5gR}\right)^2 = \frac{1}{2}mv^2 + mgR$

$\Rightarrow v^2 = \sqrt{3gR}$

The tangential and centripetal accelerations of the block are

$a_t = g\ (\downarrow)$ and $a_c = 3g\ (\leftarrow)$

$a_{net} = \sqrt{a_c^2 + a_t^2} = g\sqrt{10}$

Q10: (A) $\frac{3}{16}$

As the mass is at the verge of slipping

$\therefore \text{mg}\sin 37 - \mu\ \text{mg}\cos 37 = \text{m}\omega^2\text{r}$

$6 - 8\mu = 4.5$

$\therefore \mu = \frac{3}{16}$

Q11: (A) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$

Using work-energy theorem

$\text{mgh} = \frac{1}{2}\text{mv}^2 + \frac{1}{2}\text{I}\omega^2$

$\text{mgh} = \frac{1}{2}\text{m}\omega^2\text{r}^2 + \frac{1}{2}\frac{\text{mr}^2}{2}\omega^2 \quad (\text{ since I} = \frac{\text{mr}^2}{2} \text{ and v} = \text{r}\omega\ )$

$\text{mgh} = \frac{3}{4}\text{m}\omega^2\text{r}^2$

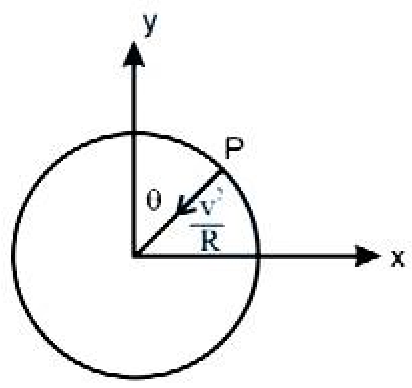
Hence, $\omega = \sqrt{\frac{4gh}{3r^2}} = \frac{1}{r}\sqrt{\frac{4gh}{3}}$

Q12: (A) is independent of the mass of the stone

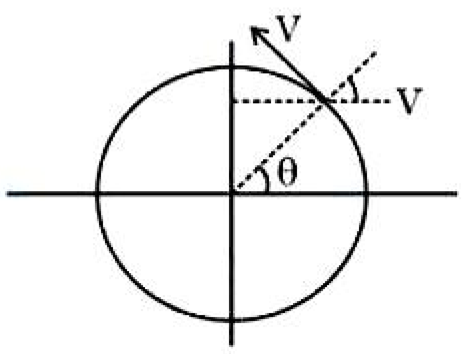
Minimum speed with which the string is rotating in a vertical circle, $(v) = \sqrt{gr}$

i.e, the minimum speed of stone is independent of the mass of stone.

Q13: (C) $-\frac{v^2}{R}\sin\theta\,\hat{i} - \frac{v^2}{R}\cos\theta\,\hat{j}$



Q14: (C) $-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}$



The direction of the friction force should be towards the centre of the circle which is

$-\hat{r} = -\cos\theta\,\hat{i} - \sin\theta\,\hat{j}$

Q15: (B) $M_{\min} = 9m$

For the particle to undergo complete vertical circular motion, its speed at the lowest point should be at least $v = \sqrt{5gr}$. Also, the tension in the string is maximum when the particle is at the lowest point. The tension at the lowest position of the particle is

$T = mg + \frac{mv^2}{r}$

$\Rightarrow T = mg + \frac{m(5gr)}{r} = 6mg$

The friction force acting on the block should be able to balance the maximum tension.

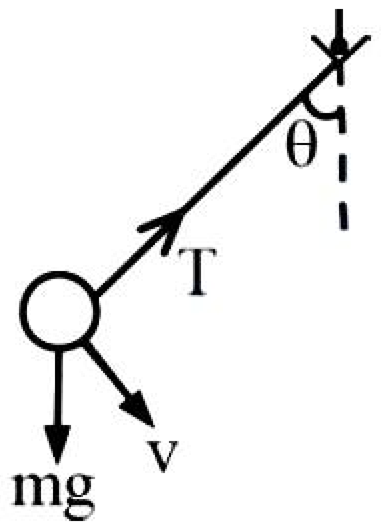
$f = T = 6mg$

$f = 6mg \leq \mu Mg$

$\Rightarrow M \geq \frac{6m}{2/3} = 9m$

$M_{\min} = 9m$

Q16: 5



Let us assume that the length of the string is l

$T - mg\cos\theta = \frac{mv^2}{l}$ .... (i)

$mgl\cos\theta = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$ .... (ii)

$T = 2\,mg$ .... (iii)

$\therefore \ \cos\theta = \frac{1}{4}$

$\Rightarrow m + n = 5$

Q17: (B) $7\ \mathrm{m\ s^{-1}}$

$u = 12\ \mathrm{m\ s^{-1}}$

$T - mg\ \cos\theta = \frac{mv^2}{l}$

As, $T = mg$ we get

$mg\,(1 - \cos\theta) = \frac{mv^2}{l}$

Also we know that

$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = -\ \mathrm{m}gl\,(1 - \cos\theta)$

$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = -mv^2$

$\Rightarrow v = \frac{u}{\sqrt{3}} = \frac{12}{\sqrt{3}} = 6.93\ \mathrm{m\ s^{-1}} \approx 7\ \mathrm{m\ s^{-1}}$

Q18: (D) the particle breaks off at a height $r < h < 2r$

To reach the top of loop, particle must have a minimum speed of $v = \sqrt{5gr}$ at the bottom of the loop.

but $v^2 = 2gh = 4gr$, so the block won't reach the highest point of the track.

Q19: (C) $\frac{1}{3}r$

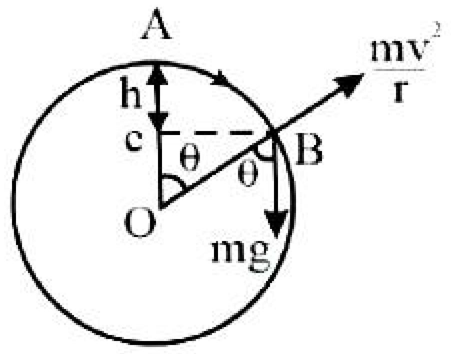
If v velocity acquired at B, then

$v^2 = 2\mathrm{g}h$

The particle will leave the sphere at B, when

$\frac{\mathrm{mv}^2}{\mathrm{r}} = \mathrm{mg}\cos\theta$

$\frac{2gh}{r} = \frac{g.(r-h)}{r}$



Which gives $h = \frac{r}{3}$

Q20: (B) $\mu = \frac{\sin\theta}{\cos\theta + \frac{v^2}{Rg}}$

Since the block is moving with a constant speed, at any instant its tangential acceleration is zero. Hence,

$f = mg\sin\theta$

Also,

$N - mg\cos\theta = \frac{mv^2}{R}$

$N = \frac{mv^2}{R} + mg\cos\theta$

$f = \mu N = \mu\frac{mv^2}{R} + \mu mg\cos\theta$

$mg\sin\theta = \mu\frac{mv^2}{R} + \mu mg\cos\theta$

$\Rightarrow \mu = \frac{\sin\theta}{\cos\theta + \frac{v^2}{Rg}}$

Q21: (C) $\frac{v_0}{2\pi R}\sqrt{\frac{2h}{g}}$

A hollow vertical

$h = \frac{1}{2}gt^2$

$n\left(2\pi R\right) = V_0 t$

$n = \frac{V_0}{2\pi R}\sqrt{\frac{2h}{g}}$

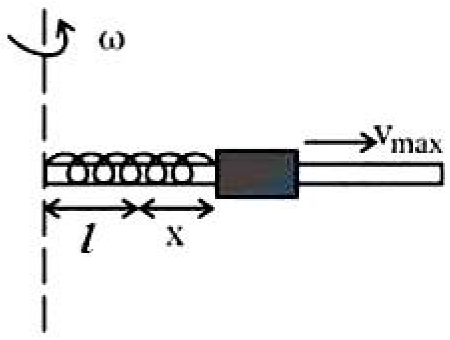
Q22: (B) $V_{\max} = \sqrt{\frac{m\omega^4 l^2}{k - m\omega^2}}$

Velocity will be maximum at the equilibrium position

$\Rightarrow kx = m\omega^2\left(l + x\right)$

$\Rightarrow \quad x = \frac{m\omega^2 l}{k - m\omega^2}$

Now, using work-energy theorem



$\Delta KE =$ Work done by all the forces

$$\frac{1}{2}mV_{\max}^2 = \int_0^x m\omega^2\,(l+x)dx - \frac{1}{2}kx^2$$

$$\Rightarrow \quad V_{\max}^2 = \frac{2m\omega^2 lx + m\omega^2 x^2 - kx^2}{m}$$

$$V_{\max}^2 = \frac{\left(m\omega^2 l + m\omega^2 (l+x) - kx\right)x}{m}$$

$$= \frac{m\omega^2 lx}{m}$$

$$\Rightarrow \quad V_{\max}^2 = \omega^2 lx = \frac{m\omega^4 l^2}{k - m\omega^2}$$

$$V_{\max} = \sqrt{\frac{m\omega^4 l^2}{k - m\omega^2}}$$